Instabilities in Imbalanced Quark Pairing Systems

Institute of High Energy Physics
Chinese Academy of Sciences
Beijing, China

黄 梅

Mei Huang

Quark-gluon plasma meets cold atoms, Sept.25-27, 2008, GSI

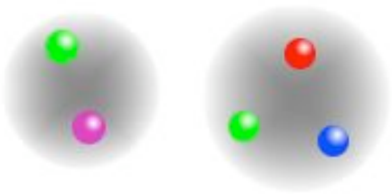
Content

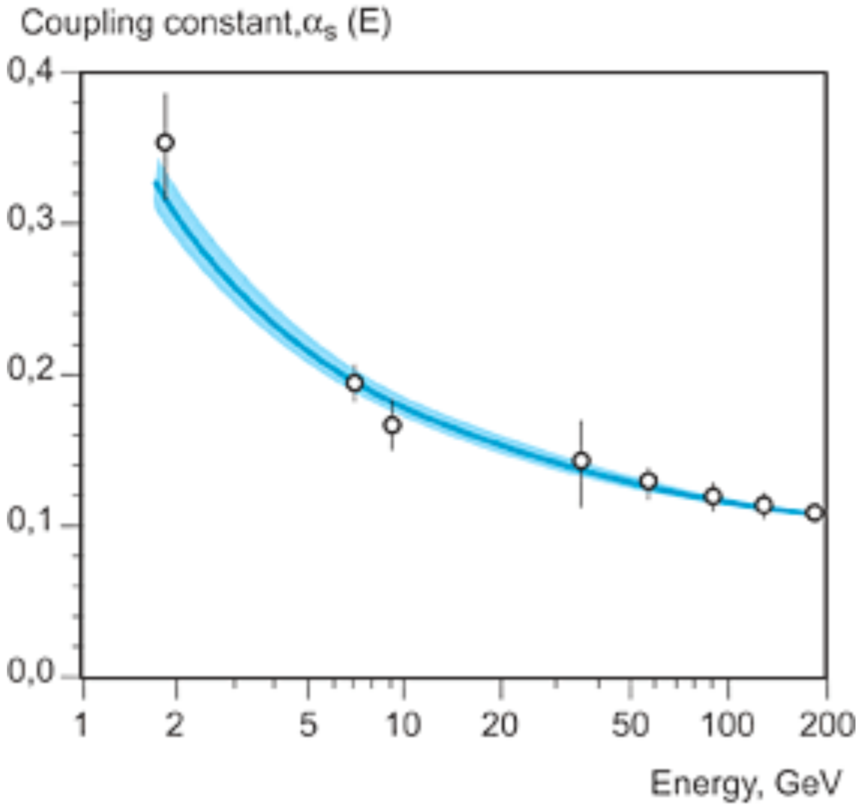
Apologize if I missed some important work.

QCD

Strong coupling:

Confinement



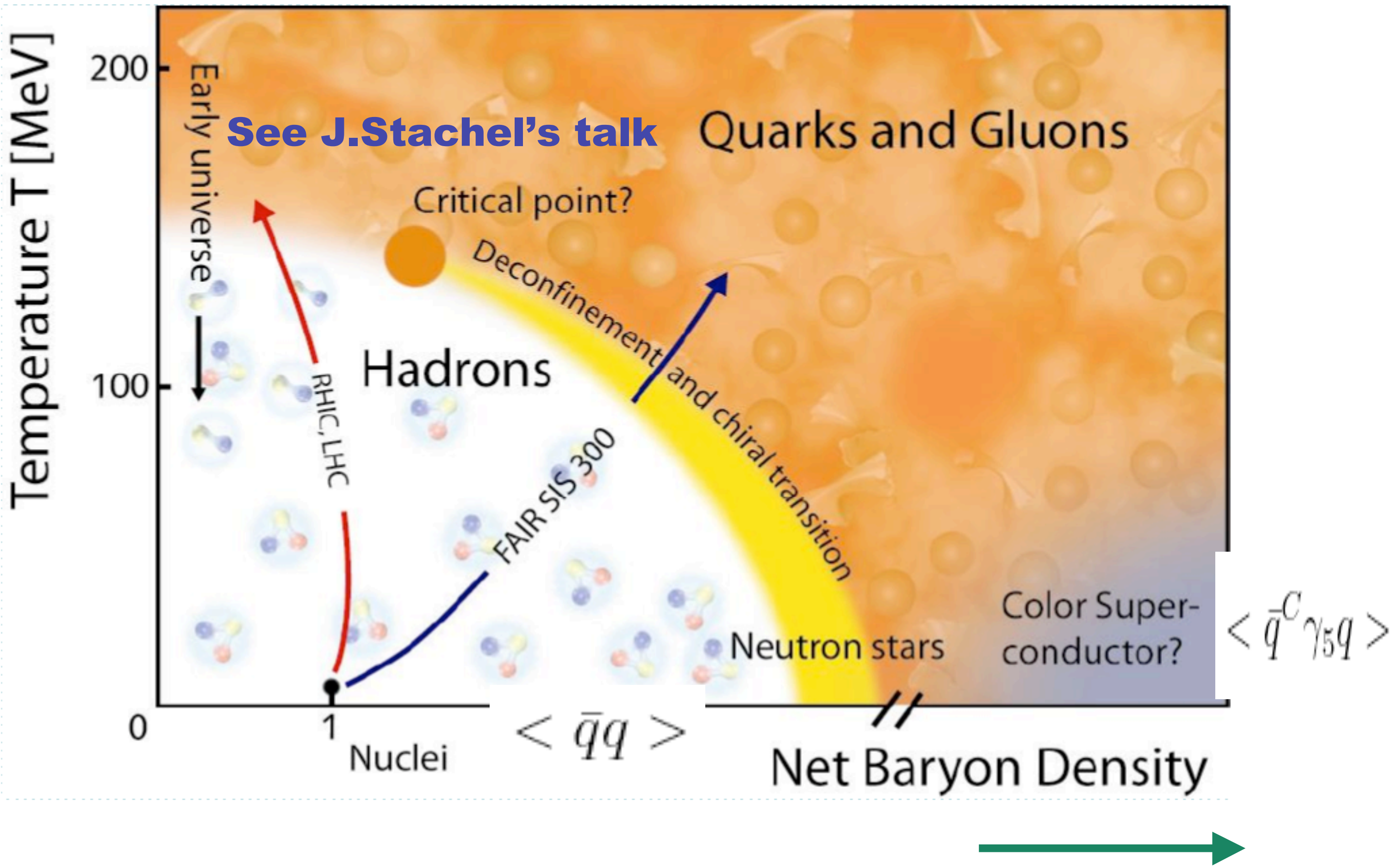


Weak coupling:

Asymptotic freedom

Nobel Prize 2004

QCD phase diagram

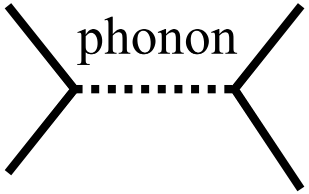


Connection between quark matter and condensed matter

1911

Superconductor

1957



$\langle e^{-} e^{-} \rangle$

BCS Theorem

1973

QCD

1977-1984

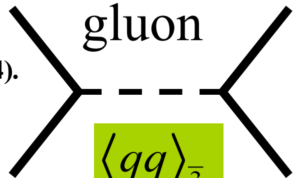
$\Delta \propto 1 MeV$

Barrois, NPB129 (1977),390;
Bailin, Love, Phys. Rep. 107, 325(1984).

1998

$\Delta \propto 100 MeV$

Rapp, Schaefer, Shuryak,Velkovsky, PRL81, 53 (1998);
Alford, Rajagopal, Wilczek, PLB422, 247 (1998).



$\langle qq \rangle_{\bar{3}_c}$

Color superconductivity

Around 2003--

Perfect fluid (J.Stachel & J.Thomas & T.Schafer)

BCS to BEC crossover (Q.Wang & H.Abuki & T.Brauner)

Imbalanced pairing (This talk & H.Stoof),

I. A brief introduction on CSC

Reviews:

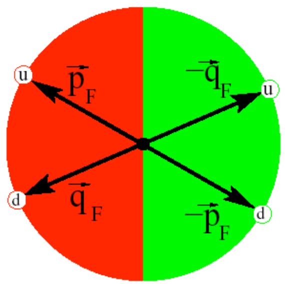
K. Rajagopal and F. Wilczek, hep-ph/0011333;
D. K. Hong, Acta Phys.Polon. B32, 1253 (2001);
M. Alford, Ann. Rev.Nucl. Part.Sci. 51, 131 (2001);
T. Schaefer, hep-ph/0304281;
D.H. Rischke, Prog.Part. Nucl. Phys. 52, 197 (2004);
M. Buballa, Phys. Rept. 407, 205 (2005);
H.-C. Ren, hep-ph/0404074;
M. Huang, Int. J. Mod.Phys. E14, 675 (2005);
I.A. Shovkovy, Found. Phys. 35, 1309 (2005);
M.Alford, A.Schmitt, K.Rajagopal, T.Schafer, arXiv:0709.4635;
……

Standard BCS pairing csc

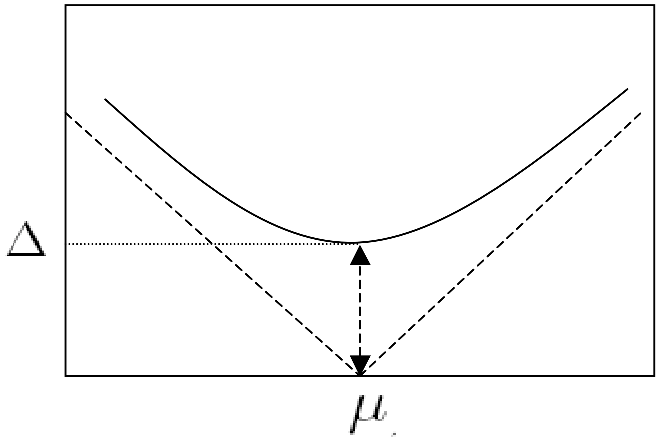
e.g. 2SC

$$\langle u_p d_{-p} \rangle = - \langle u_q d_{-q} \rangle \neq 0$$

Ideal BCS pairing



1) Quasiparticle excitation

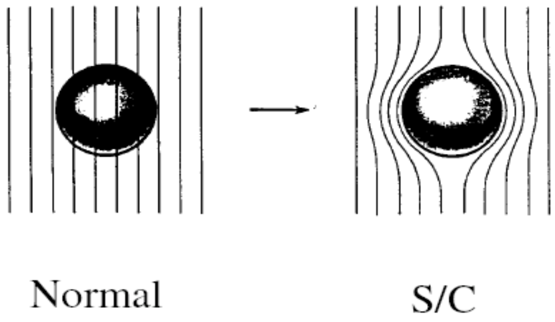


$$E_\Delta^{\pm} = \pm\sqrt{(p-\mu)^2 + \Delta^2}$$

$$E_b = \pm(p-\mu)$$

2) Meissner effect

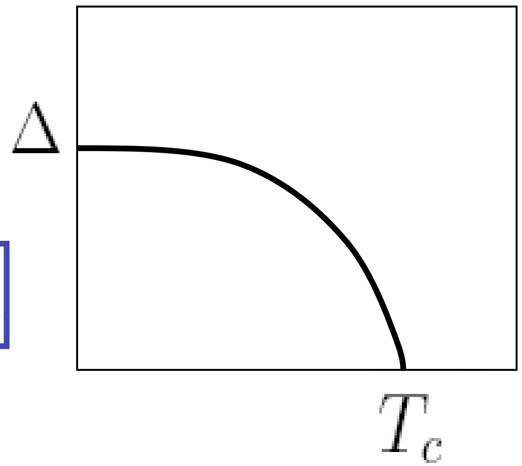
SU(3) → SU(2)



a=1,2,3	**massless**	
a=4,5,6,7	**massive**	$\frac{1}{2} m_g^2$
a=8	**massive**	$\frac{1}{3} m_g^2$

Rischke, PRD62:034007,2000

3) Finite temperature behavior



$$r_{\mathrm{BCS}} = T_c^{\mathrm{BCS}} / \Delta_0^{\mathrm{BCS}} \approx 0.567$$

$$\Delta_{BCS} \quad 100 MeV ,$$

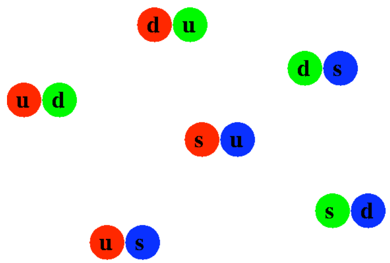
$$T_{BCS}^{c} = 0.567 \Delta_{BCS}$$

Rich Structure of BCS pairing $3_c \otimes 3_c = \boxed{\bar{3}_c} \oplus 6_c$

spin-0 CSC

2-flavor **2SC**

3-flavor **CFL**



Alford,Rajagopal,Wilczek

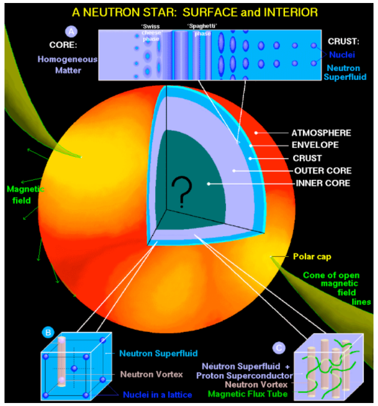
spin-1 CSC but with some non-BCS properties

Schafer

Schmitt, Wang, Rischke

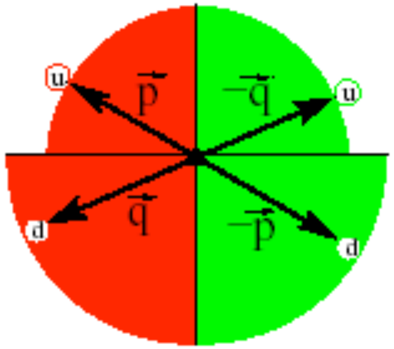
CSL, Polar, …

Imbalanced pairing CSC



beta-equilibrium, charge neutrality

$$\delta\mu, \delta m \to \delta p_F$$



Pair breaking?

Neutral dense quark matter
Imbalanced cold atom system,
Asymmetric nuclear matter,
Electric SC under external magnetic field
……

Rich Structure of imbalanced paring system

$$3_c \otimes 3_c = \bar{3}_c \oplus 6_c$$

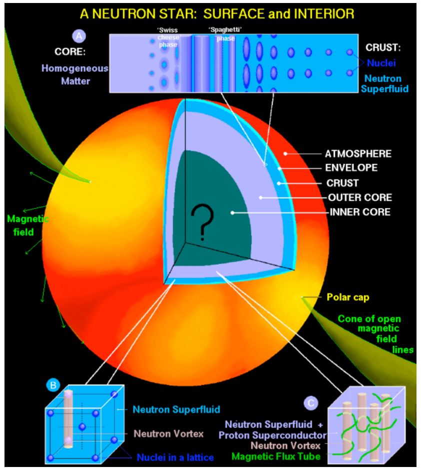
	2-flavor	3-flavor
Pairing without mismatch	2SC	CFL
Pairing with mismatch	g2SC	CFL+K, gCFL, uSC, dSC, sSC

LOFF (Larkin Ovchinnikov Fulde Ferrell)

and many more possibilities ……

II. Pairing with mismatched Fermi surfaces

One Example: charge neutrality condition on 2SC

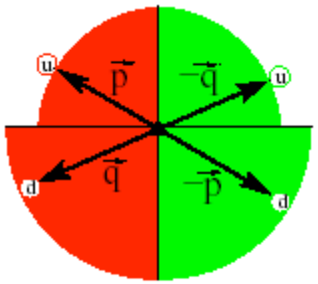


$$n_Q^{\rm el} = 0, \qquad n_Q^{\rm color} = 0$$

$$E_{\rm Coulomb} \sim n_Q^2 R^5$$

Color neutrality: easily satisfied

Electric neutrality: $n_d \approx 2 n_u$



$$\mu_e \approx \mu / 4$$

Pair breaking?

Gapless 2SC: result from BCS at Mean-field

$$\mathcal{L} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m_0)q + G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] + G_D\left[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)\right]$$

Beta-equilibrium:

$$\mu_{ij,\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij}) + \frac{2}{\sqrt{3}}\mu_8\delta_{ij}(T_8)_{\alpha\beta}$$

$$\bar{\mu} = \mu - \mu_e/6 + \mu_8/3$$
$$\delta\mu = \mu_e/2$$

Mean-field(MF):

$$\Delta = |\Delta|$$

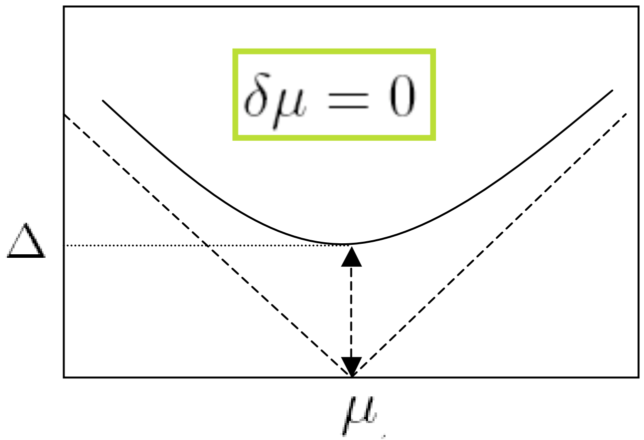
$$\Omega = \Omega_0 - \frac{1}{12\pi^2}\left(\mu_e^4 + 2\pi^2T^2\mu_e^2 + \frac{7\pi^4}{15}T^4\right) + \frac{(m-m_0)^2}{4G_S} + \frac{\Delta^2}{4G_D} - \sum_a\int\frac{d^3p}{(2\pi)^3}\left[E_a + 2T\ln\left(1+e^{-E_a/T}\right)\right]$$

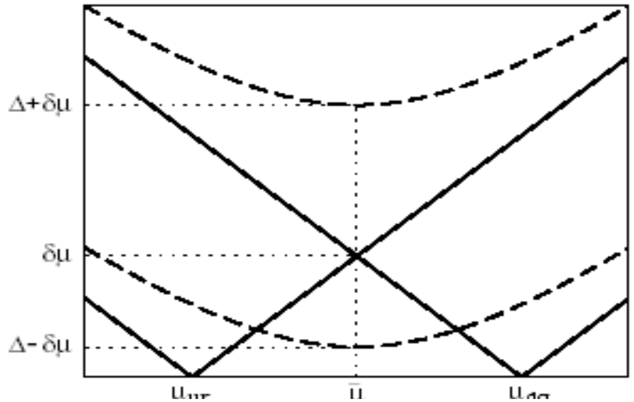
$$E_{ub}^{\pm} = E(p) \pm \mu_{ub},$$
$$E_{db}^{\pm} = E(p) \pm \mu_{db},$$

$$E_{\Delta^{\pm}}^{\pm} = E_{\Delta}^{\pm}(p) \pm \delta\mu.$$

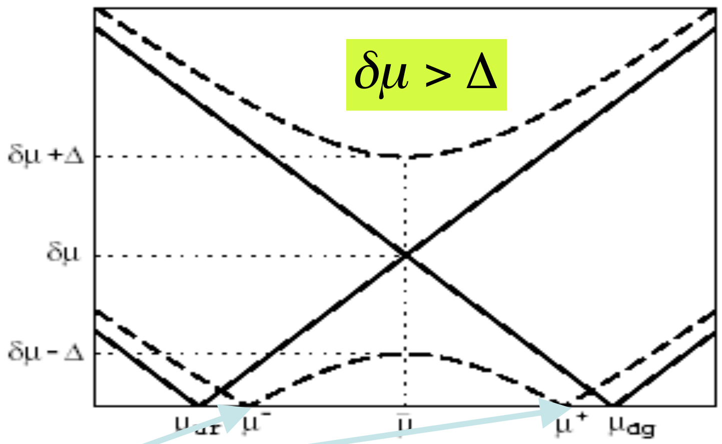
How does mismatch affect the excitation spectrum?

$$E_\Delta^\pm = \pm\sqrt{(p-\mu)^2 + \Delta^2} \quad \longrightarrow \quad E_{\Delta\pm}^\pm = E_\Delta^\pm(p) \pm \delta\mu.$$





δμ < Δ

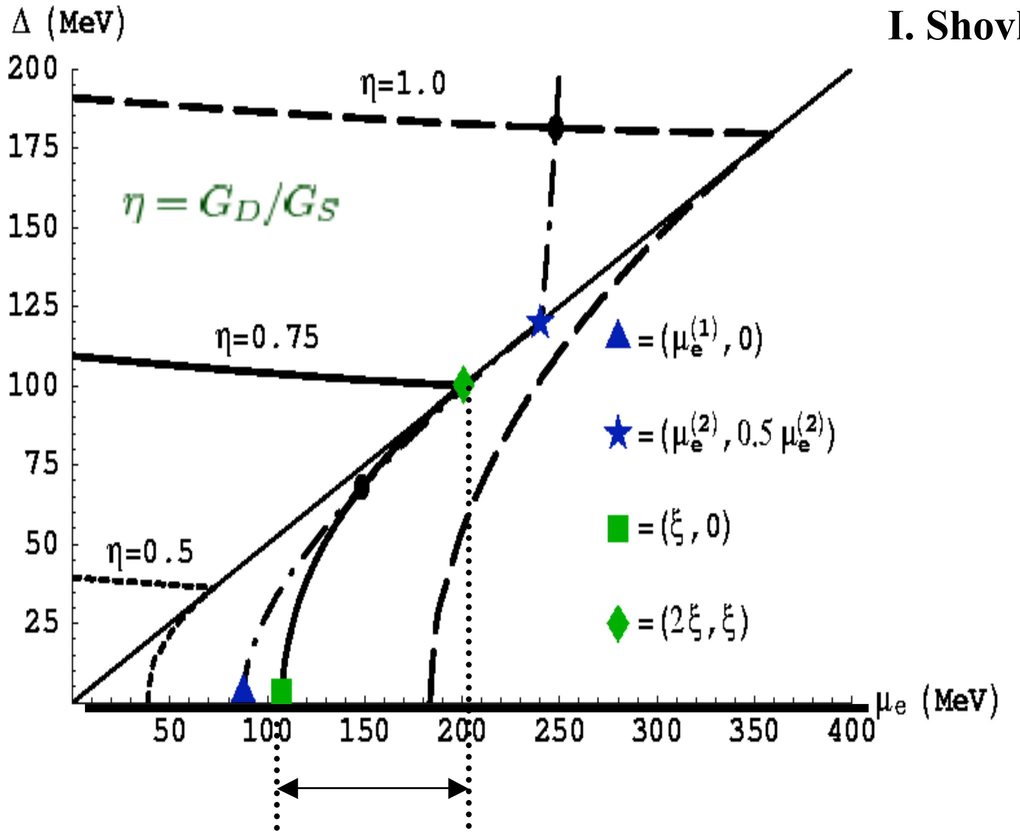


Gapless Mode !

I. Shovkovy, M.H, Phys.Lett.B564:205,2003

Ground state

I. Shovkovy, M.H, Phys.Lett.B564:205,2003



$$\frac{\partial \Omega}{\partial \Delta} = 0$$

$$\frac{\partial \Omega}{\partial \mu_8} = 0, \frac{\partial \Omega}{\partial \mu_e} = 0$$

$$\Delta_0/2 < \delta\mu < \Delta_0$$

$$\left(\frac{\partial^2 \Omega}{\partial \Delta^2}\right)_{\mu_e} < 0$$

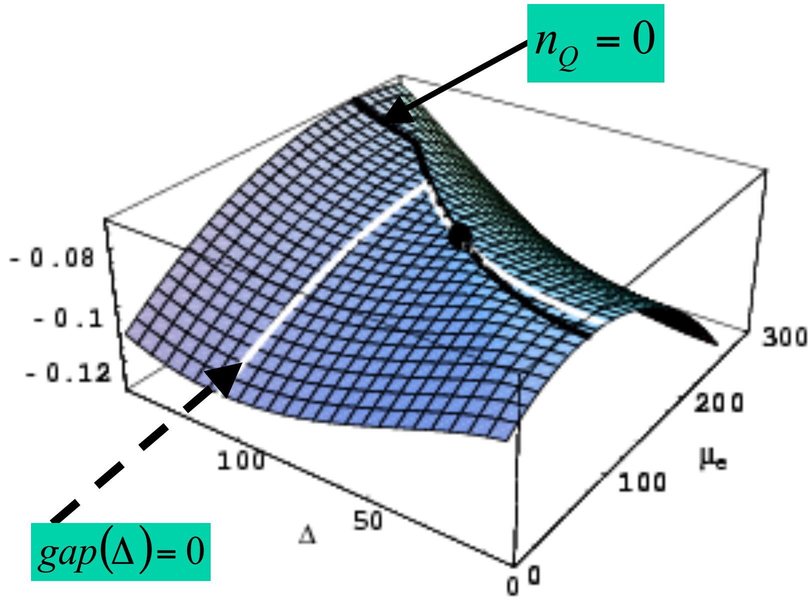
Sarma instability

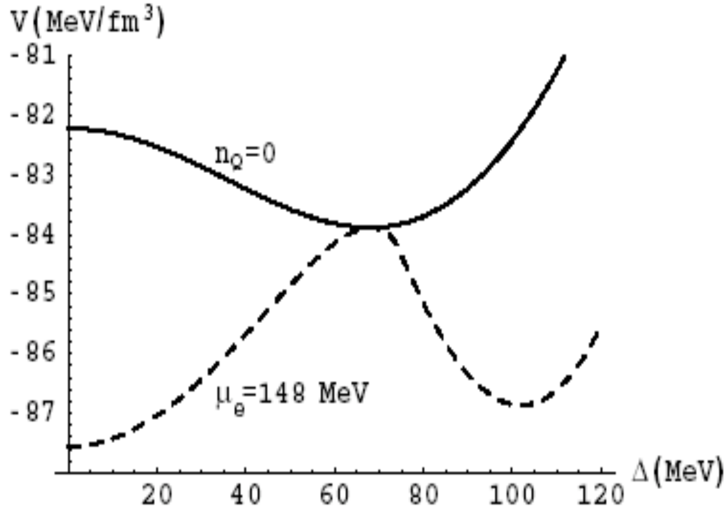
G.Sarma, J.Phys.Chem.Solids 24, 1029 (1963)

Thermal Stability along charge neutrality line

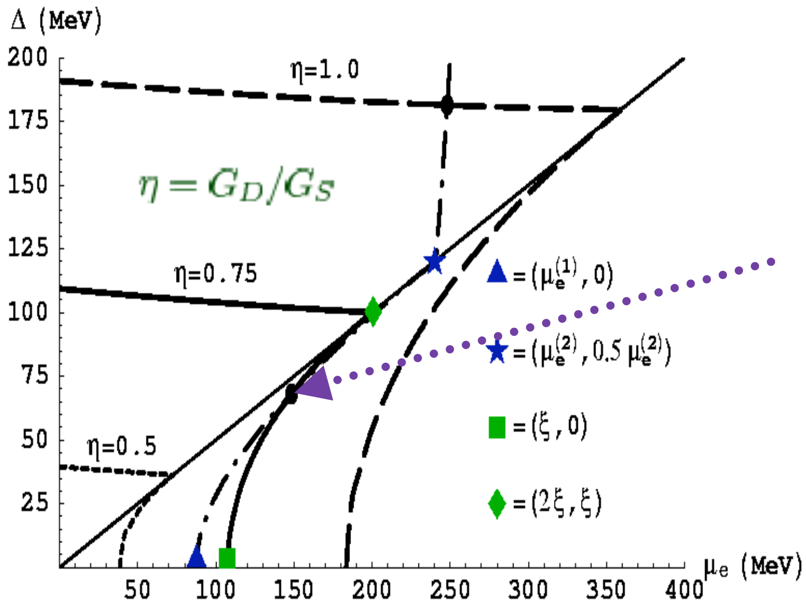
I. Shovkovy, M.H, Phys.Lett.B564:205,2003

$$E_{\rm Coulomb} \sim n_Q^2 R^5$$



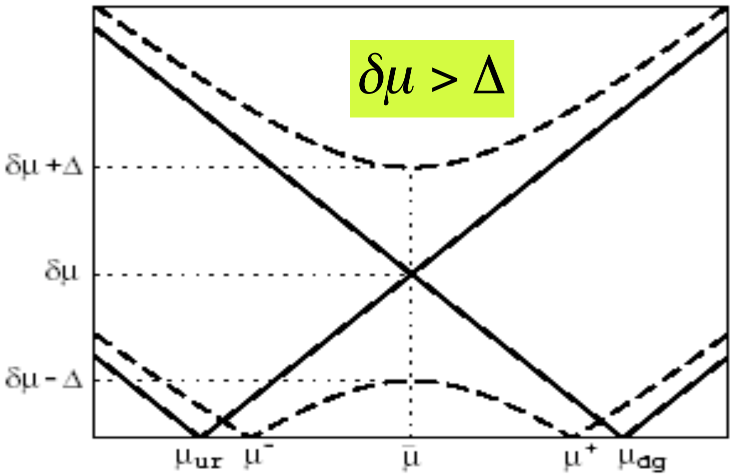


Gapless 2SC



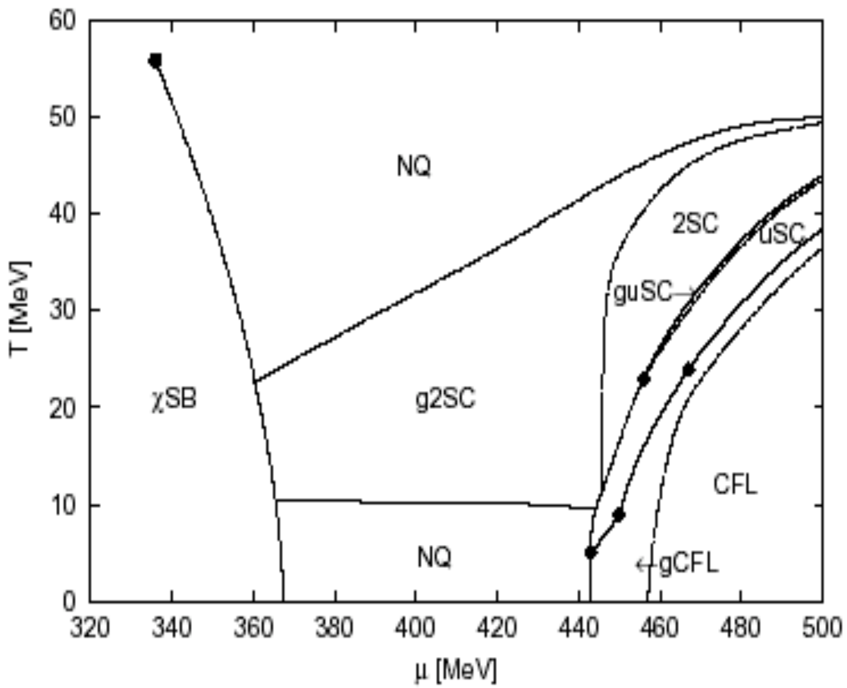
M.H, P.Zhuang, W.Chao, hep-ph/0207008

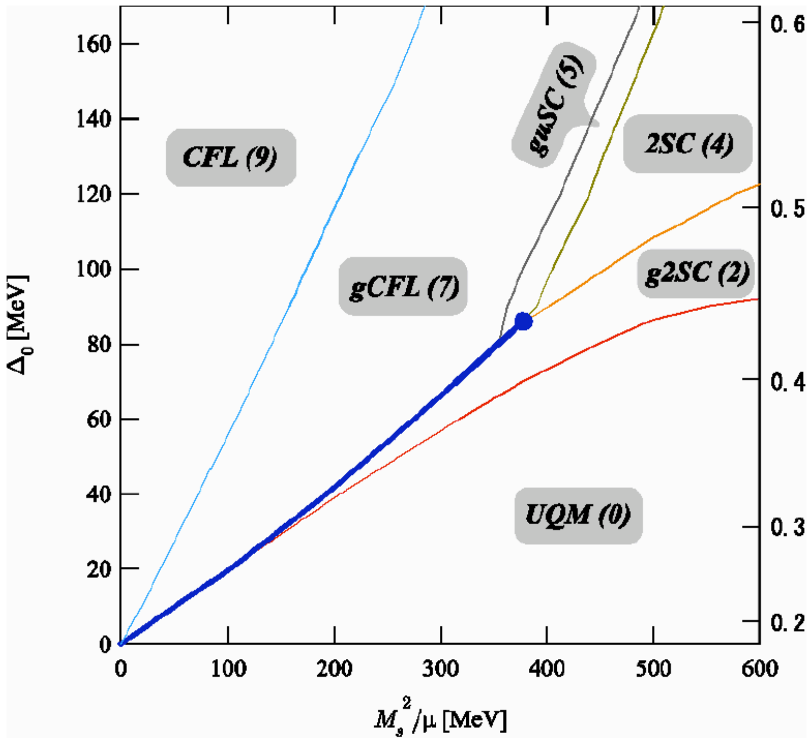
g2SC



I. Shovkovy, M.H,
Phys.Lett.B564:205,2003

Rich Structure of imbalanced pairing quark matter





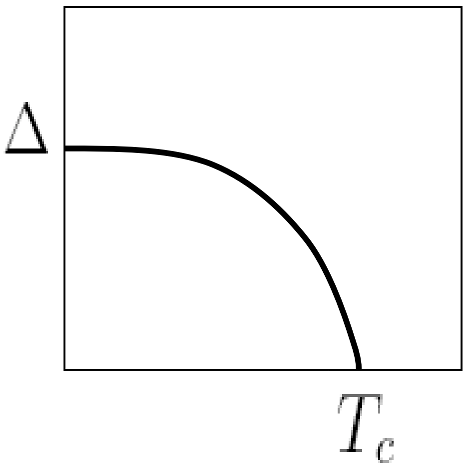
Darmstadt and Frankfurt CSC group
Phys.Rev.D72:034004,2005

Abuki, Kitazawa, & Kunihiro,
PLB 615, 102 (2005)

How does mismatch affect temperature properties?

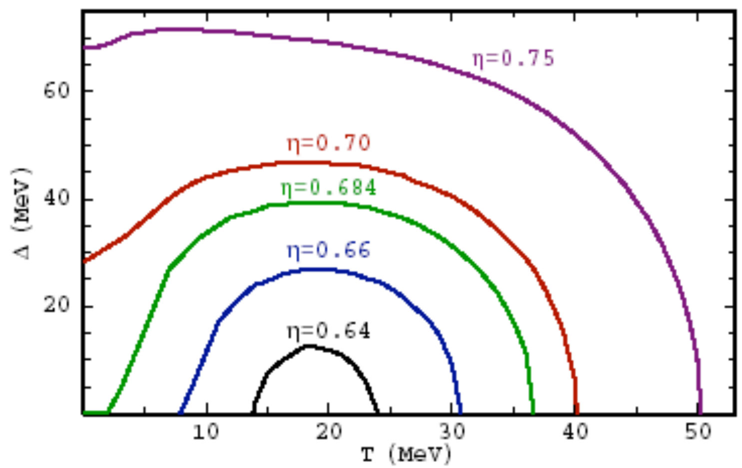
Finite temperature behavior

2SC



Gap magnitude monotonically drops to zero

g2SC



Extremely non-monotonic temperature dependence

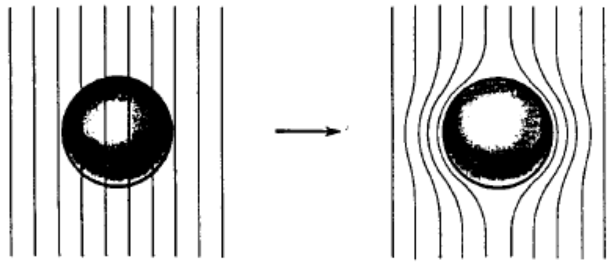
MH, I. Shovkovy, NPA729:835,2003

How does mismatch affect Meissner effect?

Meissner effect in g2SC ?

1933: Meissner & Ochsenfeld

Linear response theory



$$J_i^{ind}(\omega,\vec{q}) = \Pi_{ij}(\omega,\vec{q})A^j(\omega,\vec{q})$$

Normal

S/C

$$\Pi_{ij} = \Pi_{ij}^p + \Pi_{ij}^d$$

$$\Pi_{ij}^p(0,\vec{0}) + \Pi_{ij}^d(0,\vec{0}) = 0$$

$$\Pi_{ij}^p(0,\vec{0}) < \Pi_{ij}^d(0,\vec{0})$$

Perfect Diamagnet

Chromomagnetic instability driven by mismatch!

Ideal 2SC

a=1,2,3	massless	
a=4,5,6,7	massive	$\frac{1}{2} m_g^2$
a=8	massive	$\frac{1}{3} m_g^2$

Rischke, PRD62:034007,2000

g2SC

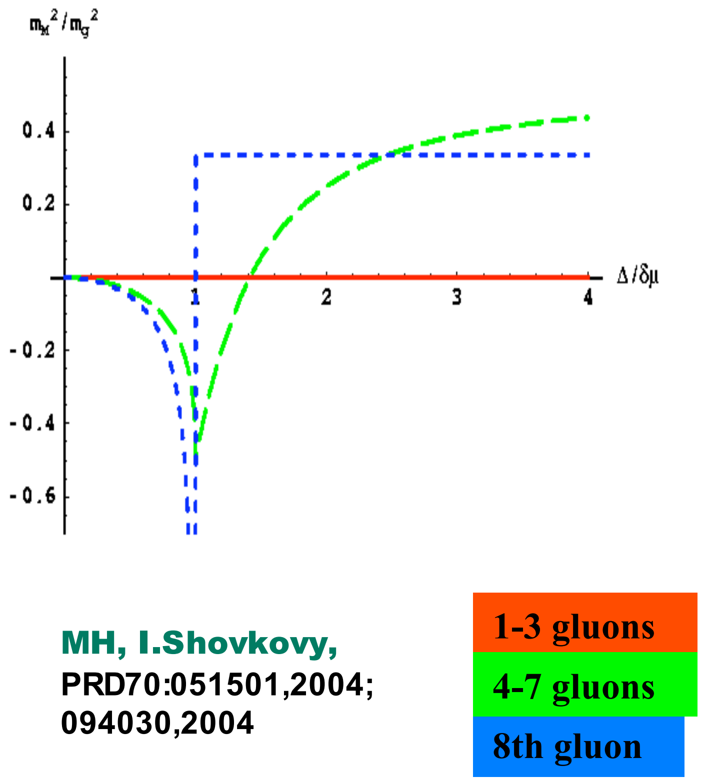
a=1,2,3	massless
a=4,5,6,7	negative
a=8	negative

2SC with mismatch

$$1 < \Delta/\delta\mu < \sqrt{2}$$

a=1,2,3	massless
a=4,5,6,7	negative
a=8	positive

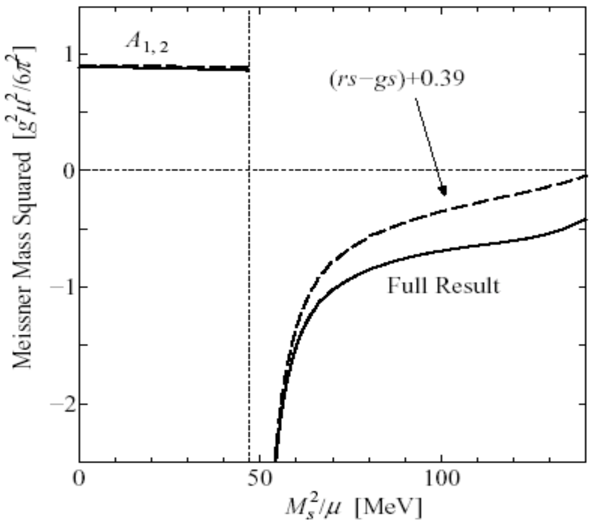
Anti-Meissner effect / chromomagnetic instability



MH, I.Shovkovy,
PRD70:051501,2004;
094030,2004

(Chromo)Magnetic instability in other gapless phases

gCFL



Casalbuoni, et.al., PLB605:362-368,2005

Alford, Wang, J.Phys.G31:719-738,2005

K. Fukushima, hep-ph/0506080

BP (gapless phase in imbalanced cold atom system)

Liu, Wilczek 2003

Superfluid density is negative **Wu, Yip,** PRA67: 053603, 2003

Possible ground states:

1. LOFF (Larkin Ovchinnikov Fulde Ferrell)

$\Delta(r) = |\Delta_q| e^{(iqr)}$ (FF)

P. Fulde and A. Ferrell Phys. Rev. 135, A550 (1964).

$\Delta(r) = |\Delta_q| \cos(qr)$ (LO)

A.I. Larkin and Yu.N. Ovchinnikov Zh. Eksp. Teor. Fiz. 47, 1136 (1964).

also multi-plane wave

Krishna Rajagopal, Rishi Sharma, **Phys.Rev.D74:094019,2006**

Nambu-Goldstone current state

1. **Single-plane wave FF**

   Giannakis, Ren, PLB611:137-146,2005; NPB723:255-280,2005

   Casalbuoni, Gatto, Ippolito, Nardulli, Ruggieri, hep-ph/0507247

2. **Goldstone current:**

   MH, hep-ph/0504235
   Hong, hep-ph/0506097;
   Kryjevski, hep-ph/0508180;
   Schaefer, hep-ph/0508190

3. **Gluon condensate:**

   Gorbar, Hashimoto, Miransky, hep-ph/0507303, ……

2. Phase separation

M.Alford, K.Rajagopal, S.Reddy, F.Wilczek,PRD64(2001), 074017;
F. Neumann, M. Buballa, M. Oertel, NPA 714, 2003;
I. Shovkovy, M. Hanauske, M.H, PRD67:103004,2003;
D.Aguilera,D.Blaschke,H.Grigorian, astro-ph/0212237;
S. Reddy and G. Rupak, nucl-th/0405054, ……

3. Rotation symmetry breaking state

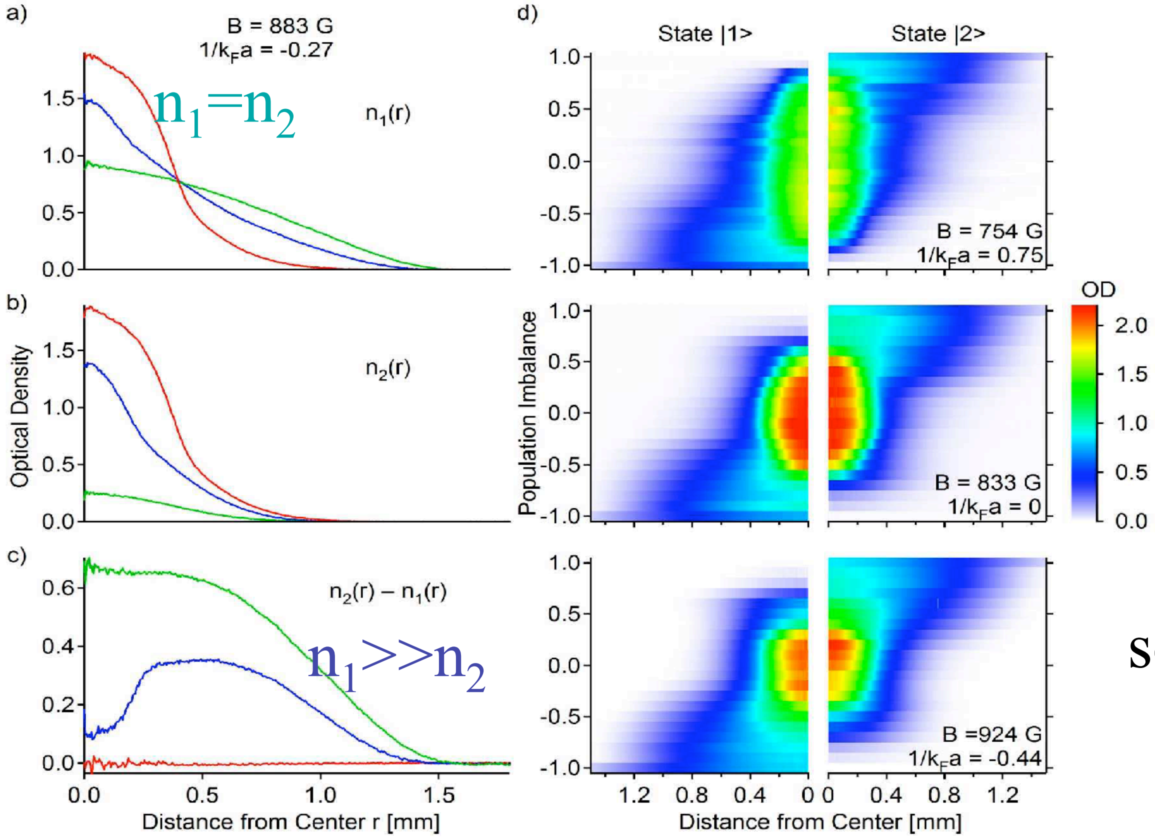
H. Muther, A. Sedrakian, Phys.Rev.D67:085024,2003A

$$\mu_f = \sum_{l=0}^{\infty} \mu_{f,l} P_l(\mathrm{Cos}\,\theta).$$

4. More ……

Experiments in imbalanced cold atoms told us:

$\delta = 0\%$ (red), $\delta = 46\%$ (blue) and $\delta = 86\%$ (green)



phase separation

Zwierlein, Schirotzek, Schunck, & Ketterle, Science 2005, cond-mat/0511197
Partridge, Li, Kamar, Liao, & Hulet, Science 2005, cond-mat/0511752.

III. Framework beyond MF for mismatched systems

III. Nonlinear realization framework (beyond MF)

III.1. Instability of NG bosons & FF-like state
III.2. Higgs instability & spatial inhomogeneity

Based on work:

M.H. PRD73:045007, 2006; Int.J.Mod.Phys.A21, 910, (2006)

I. Giannakis, D.F.Hou, M.H., H.C.Ren,

PRD75:011501,2007, PRD75:014015,2007

Group G → **Subspace: Group H**

Coset space: M=G/H

$$\Delta(\vec{r}) = (\Delta + H(\vec{r}))e^{i\varphi(\vec{r})}$$

$$\Omega = \Omega_M + \Omega_H + \Omega_{NG}$$

III.1. Instability of NG bosons & FF-like state

U(1): Hong, hep-ph/0506097 **SU(3)->SU(2): M. H.** PRD73:045007, 2006

2SC phase:

$$\begin{pmatrix} \Delta^1(\vec{r}) \\ \Delta^2(\vec{r}) \\ \Delta^3(\vec{r}) \end{pmatrix} = \exp\left[i \sum_{a=4}^{8} \varphi_a(\vec{r}) T_a \right] \begin{pmatrix} 0 \\ 0 \\ \Delta + H(\vec{r}) \end{pmatrix}$$

nonlinear realization framework

new quark field:

$$q = \mathcal{V}\,\chi \quad , \quad \bar{q} = \bar{\chi}\,\mathcal{V}^{\dagger}$$

$$X \equiv \begin{pmatrix} \chi \\ \chi_C \end{pmatrix} \quad , \quad \bar{X} \equiv (\bar{\chi}\,,\,\bar{\chi}_C)$$

$$\mathcal{L}_{nl} \equiv \bar{X}\,\mathcal{S}_{nl}^{-1}\,X - \frac{\Phi^{+}\Phi^{-}}{4G_D}$$

$$\mathcal{S}_{nl}^{-1} \equiv \begin{pmatrix} [G_{0,nl}^{+}]^{-1} & \Phi^{-} \\ \Phi^{+} & [G_{0,nl}^{-}]^{-1} \end{pmatrix}$$

$$[G_{0,nl}^{+}]^{-1} = i\,\not{D} + \hat{\mu}\,\gamma_0 + \gamma_\mu\,V^\mu ,$$

$$V^\mu \simeq -\sum_{a=4}^{8} (\partial^\mu \varphi_a)\,T_a$$

Shovkovy, Rischke, Phys.Rev.D66:054019,2002
M.H. PRD73:045007

NG sector

$$\Omega_{NG} = \frac{1}{2}\int d^3\vec{r}\sum_{a=1}^{8} m_a^2 (\vec{\mathbf{A}}^a - \frac{1}{g}\vec{\nabla}\varphi^a)(\vec{\mathbf{A}}^a - \frac{1}{g}\vec{\nabla}\varphi^a) + higher\, orders$$

NG currents & (LO)FF-like state

$$(m_a)^2 < 0, \quad a = 4,5,6,7 \qquad \sum_{a=4}^{7} < \vec{\mathbf{A}}^a - \frac{1}{g}\vec{\nabla}\varphi^a > \neq 0$$

Gluon phase, Gorbar, Hashimoto, Miransky, **hep-ph/0507303**

$$(m_8)^2 < 0 \qquad < \vec{\mathbf{A}}^8 - \frac{1}{g}\vec{\nabla}\varphi^8 > \neq 0.$$

U(1) (LO)FF-state, Giannakis, Ren, **hep-ph/0412015**

For the ground state, multi-plane wave might be more favorable

III.2. Higgs instability & spatial inhomogeneity

I. Giannakis, D.F.Hou, M.H., H.C.Ren, hep-ph/0606178; hep-ph/0609098

Higgs sector

$$\Omega_M = -\frac{T}{2}\sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \mathrm{Tr}\ln([\mathcal{S}_M(P)]^{-1}) + \frac{\Delta^2}{4G_D}$$

$$\Omega_H = \frac{T}{2}\sum_{k_0} \int \frac{d^3\vec{k}}{(2\pi)^3} H^*(\vec{k})\Pi_H(k)H(\vec{k}).$$

$\Pi_H =$

inhomogeneous field $\Delta_{\vec{k}}$

$$\delta\mathcal{F} = \frac{1}{2}\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^2}\right)_n \delta\Delta^2 + \frac{1}{2}\sum_{\vec{k}\neq 0}\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^*_{\vec{k}}\partial\Delta_{\vec{k}}}\right)_n \delta\Delta^*_{\vec{k}}\delta\Delta_{\vec{k}}$$

$$\Pi(k) = A_H + B_H k^2$$

for $k << \Delta$

In gapless region

$$A_H = \left(\frac{\partial^2\Omega}{\partial\Delta^2}\right)_{\mu,} = \frac{4\bar{\mu}^2}{\pi^2}\left[1 - \frac{\delta\mu}{\sqrt{(\delta\mu)^2 - \Delta^2}}\right]$$

Sarma Instability

$$B_H = \frac{2\bar{\mu}^2}{9\pi^2\Delta^2}\left[1 - \frac{(\delta\mu)^3}{((\delta\mu)^2 - \Delta^2)^{\frac{3}{2}}}\right]$$

Higgs Instability:
induce spatial inhomogeneity

Inhomogeneous Higgs field induces inhomogeneous charge distribution

Coulomb energy

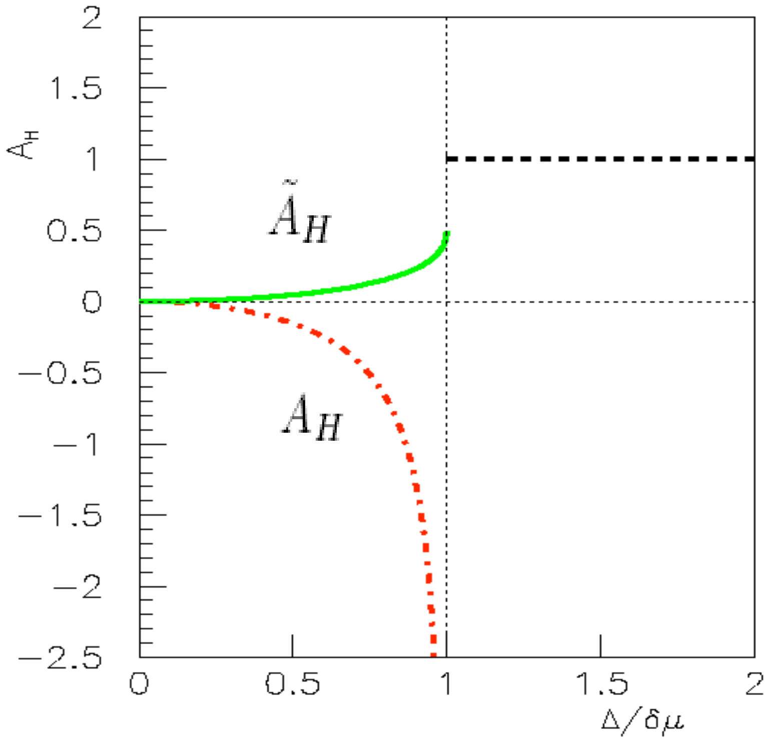
$$\delta\mathcal{F} = \frac{1}{2}\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^2}\right)_n \delta\Delta^2 + \frac{1}{2}\sum_{\vec{k}\neq 0}\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^*_{\vec{k}}\partial\Delta_{\vec{k}}}\right)_n \delta\Delta^*_{\vec{k}}\delta\Delta_{\vec{k}} + E_{\mathrm{coul.}}$$

$$\delta\rho(\vec{k}) = \kappa(k)H(\vec{k})$$

$$E_{\mathrm{coul.}} = \frac{1}{2V}\sum_{\vec{k}\neq 0}\frac{\delta\rho(\vec{k})^*\delta\rho(\vec{k})}{k^2 + m_D^2(k)}$$

$$\tilde{\Pi}(k) \equiv \left(\frac{\partial^2\mathcal{F}}{\partial H^*(\vec{k})\partial H(\vec{k})}\right)_{n_Q} = \Pi(k) + \frac{\kappa^*(k)\kappa(k)}{k^2 + m_D^2(k)}$$

Sarma Instability can be removed by Coulomb energy



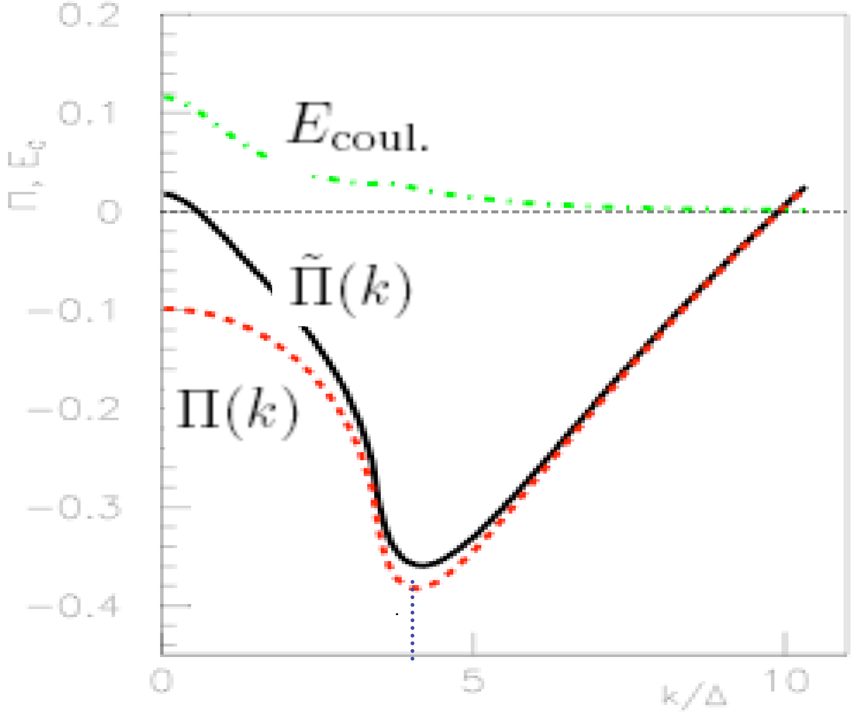
$$\left(\frac{\partial^2\Omega}{\partial\Delta^2}\right)_\nu < 0$$

$$\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^2}\right)_n = \left(\frac{\partial^2\Omega}{\partial\Delta^2}\right)_\nu + \frac{\left(\frac{\partial n}{\partial\Delta}\right)_\nu^2}{\left(\frac{\partial n}{\partial\nu}\right)_\Delta} > 0.$$

$$\tilde{A}_H = \frac{4(b^2-3a^2)\bar{\mu}^2(\delta\mu-\sqrt{\delta\mu^2-\Delta^2})}{\pi^2[3a^2\sqrt{\delta\mu^2-\Delta^2}+b^2(2\delta\mu+\sqrt{\delta\mu^2-\Delta^2})]} > 0$$

Electric Coulomb energy is not strong enough to compete the Higgs Instability in g2SC

numerical results in whole momentum space



results in whole momentum space

$$\xi \simeq \Delta^{-1}$$

$$l \simeq k_{min}^{-1}$$

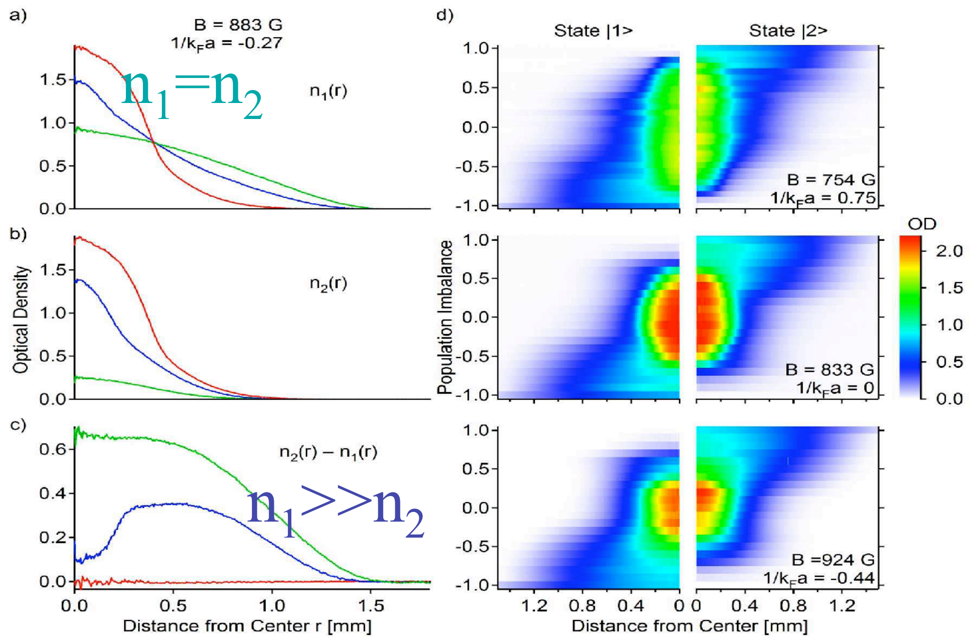
$l/\xi < 1$,

phase separation

$$\Delta/\delta\mu = 1/2 \text{ and } \alpha_e \bar{\mu}^2/\Delta^2 = 1$$

Giannakis, D.F.Hou, M.H., H.C.Ren, hep-ph/0609098

For gapless superfluid systems, Higgs instability remains, phase separation is favored



phase separation

Zwierlein, Schirotzek, Schunck, & Ketterle, Science 2005, cond-mat/0511197
Partridge, Li, Kamar, Liao, & Hulet, Science 2005, cond-mat/0511752.

Higgs instability in cold atom system

E.Gubankova, M.Mannarelli,R.Sharma, arXiv: 0804.0782[cond-mat]

Summary

I. Two instabilities in gapless phases: NG-current & (LO)FF state, Higgs instability & spatial inhomogeneity

II. Gapless superfluidity (BP) state, no other mechanism compete with Higgs instability, phase separation is more favored.

III. G2SC phase, the Sarma instability can be removed, but electric Coulomb energy is not strong enough to compete the Higgs instability in the whole momentum space.

IV. gCFL phase, whether color Coulomb energy is strong enough to remove Higgs instability?

V. Further efforts needed to find the real ground state.