Perfect Fluid Olympics

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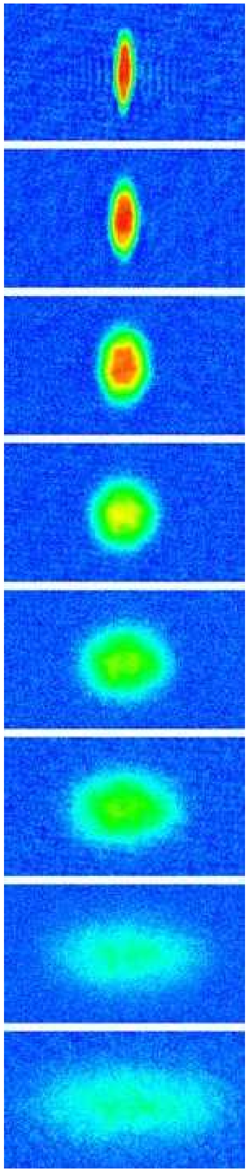
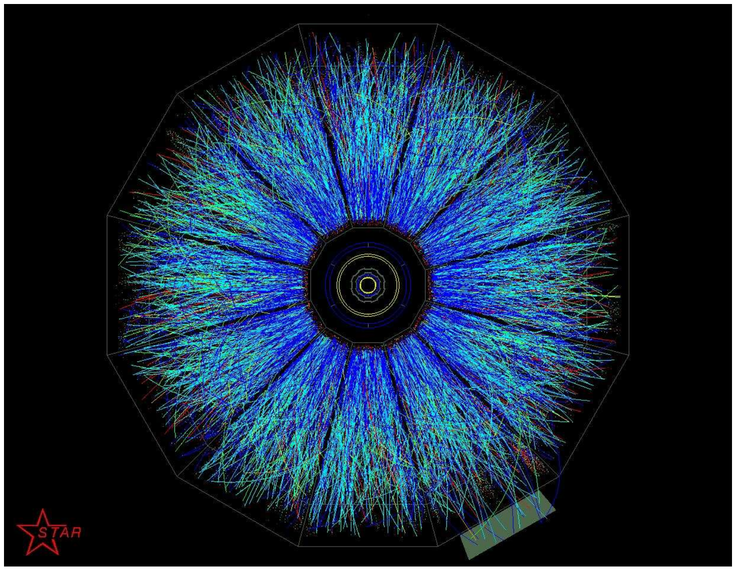
Perfect Fluids: The contenders



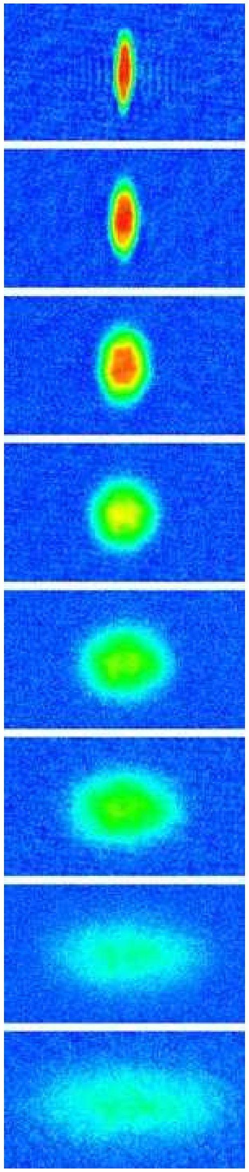
Trapped Atoms
(T=0.1 neV)

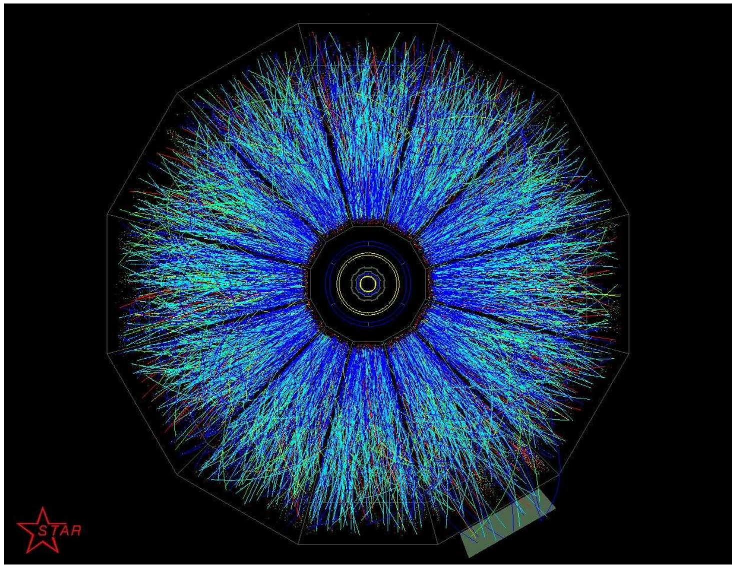


QGP (T=180 MeV)

Liquid Helium
(T=0.1 meV)

Perfect Fluids: The contenders

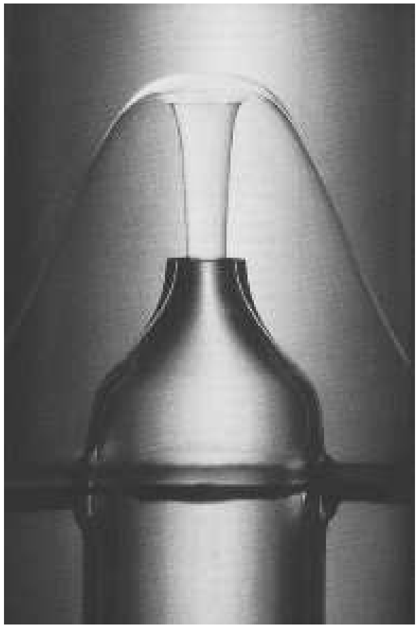




QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

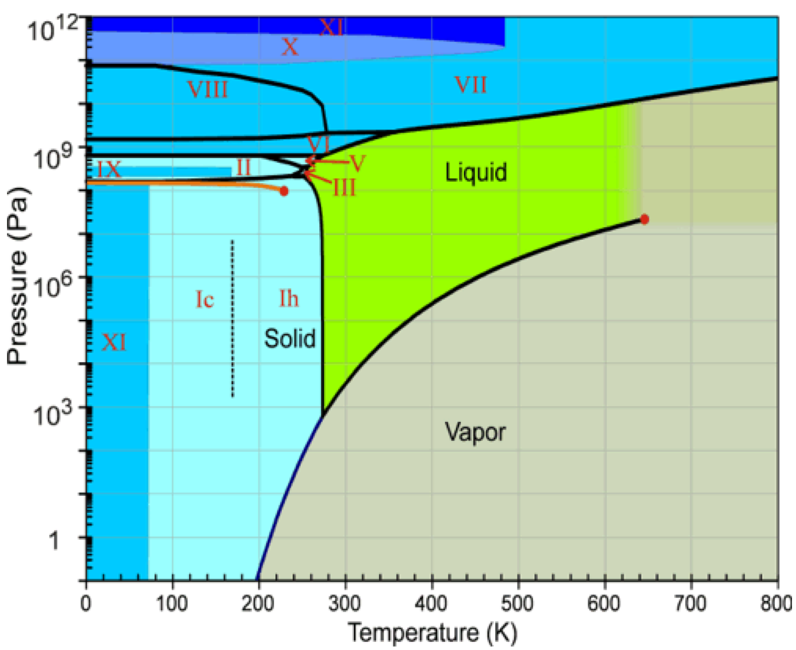
$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η / s

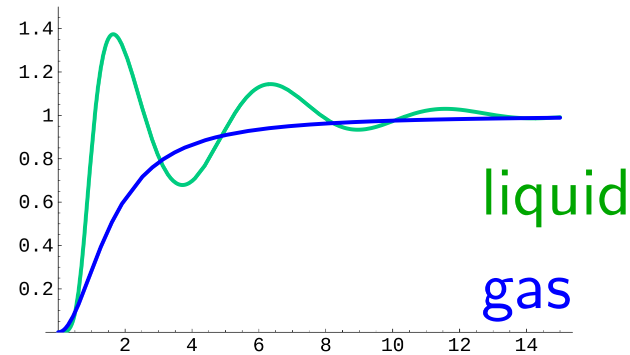
Transitions without change of symmetry: Liquid-Gas

Phase diagram of water

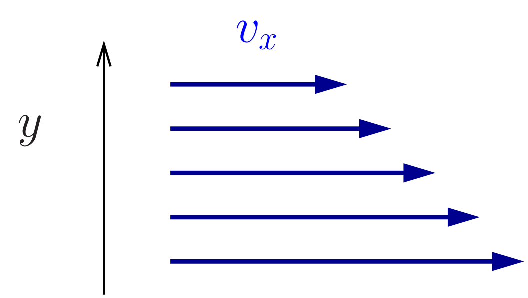


Characteristics of a liquid

Pair correlation function



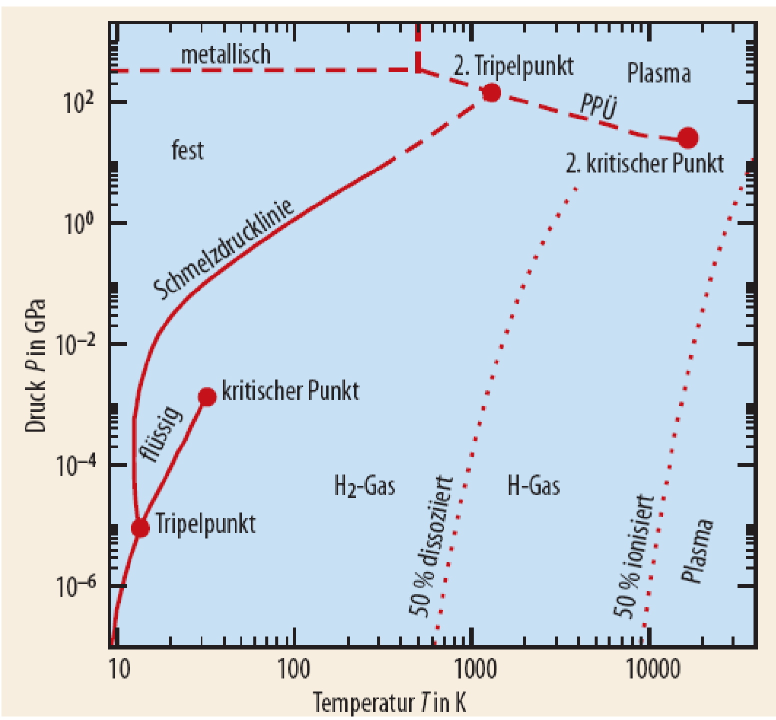
Good fluid: low viscosity



$$F_x = \eta A \frac{\partial v_x}{\partial y}$$

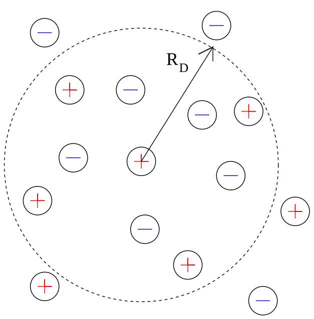
Transitions without change of symmetry: Gas-Plasma

Phase diagram of hydrogen



Plasma Effects

Debye screening



$$V(r) = -\frac{e}{r} e^{-m_D r}$$

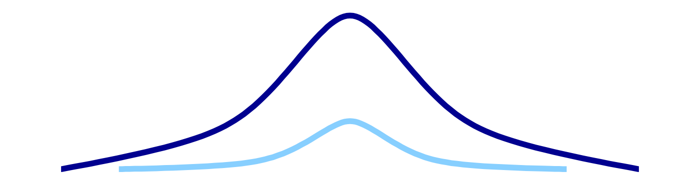
$$m_D^2 = \frac{4\pi e^2 n}{kT}$$

Plasma oscillations

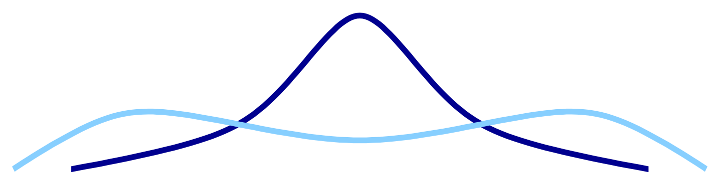
$$\omega_{pl} = \frac{4\pi e^2 n}{m}$$

Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



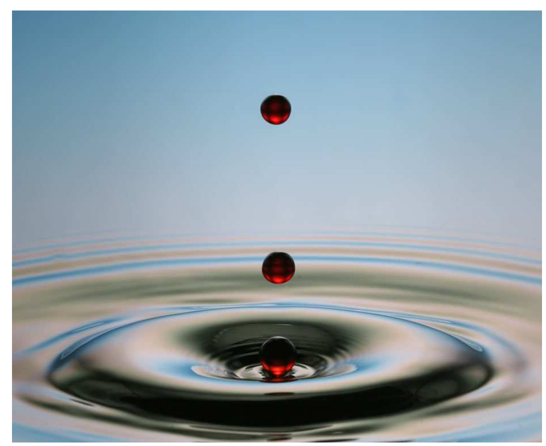
$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Example: Simple Fluid

Continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

Euler (Navier-Stokes) equation

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0$$

Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k \right) + \ldots$$

reactive dissipative

Kinetic Theory

Quasi-Particles ($\gamma \ll \omega$): introduce distribution function $f_p(x,t)$

$$N = \int d^3p\, f_p \qquad T_{ij} = \int d^3p\, \frac{p_i p_j}{E_p} f_p,$$

Boltzmann equation

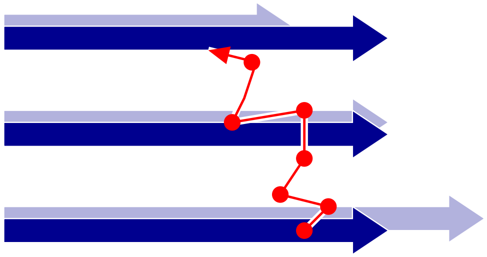
$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

Collision term $C[f_p] = C_{gain} - C_{loss}$

Linearized theory (Chapman-Enskog): $f_p = f_p^0 (1 + \chi_p / T)$

suitable for transport coefficients

shear viscosity $\chi_p = g_p p_i p_j v_{ij}$



Viscosity Bound: Rough Argument

Kinetic theory estimate of shear viscosity

$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp} \qquad\qquad (\mathrm{Note}: l_{mfp} \sim 1/(n\sigma))$$

Normalize to density. Uncertainty relation implies

$$\frac{\eta}{n} \sim \frac{n\,\bar{p}\,l_{mft}}{n} \geq \hbar$$

Also: $s \sim k_B n$ and $\eta/s \geq \hbar/k_B$

Validity of kinetic theory as $\bar{p}\,l_{mfp} \sim \hbar$?

What is the “right” ratio, η/s or η/n?

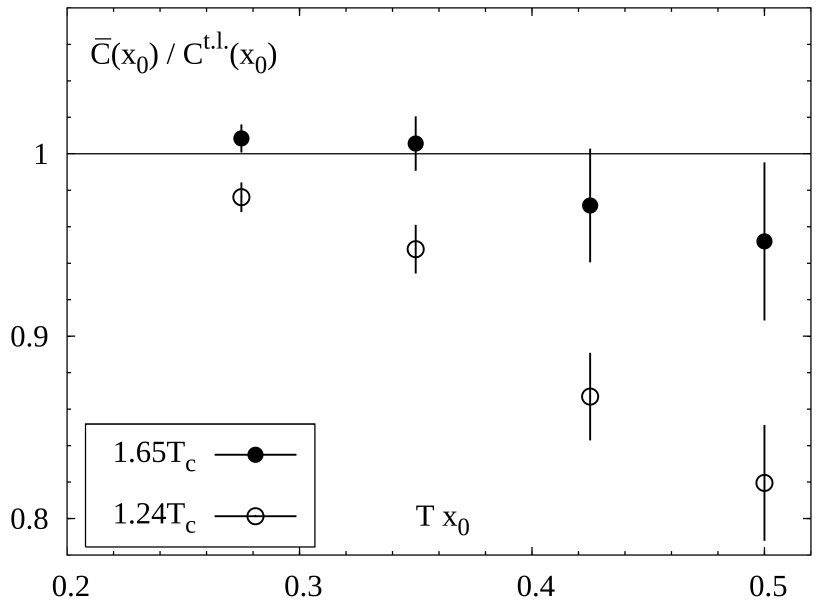
Kubo Formula

Linear response theory provides relation to Green functions

$$\eta = -\lim_{\omega\to 0} \frac{1}{\omega} G_R(\omega, 0)$$

$$G_R(\omega, 0) = \int dt\, d^3x\, e^{i\omega t}\Theta(t)\langle[T_{xy}(t,x), T_{xy}(0,0)]\rangle$$

Can be used in field theory (hard), lattice calculations (also hard)



H. Meyer (2007)

Dynamic Universality

Continuous phase transition: Dynamics of low energy modes universal

Universality for transport coefficients

Universal theory: Hydro (diffusive modes), order parameters (time dependent LG), stochastic forces (Langevin)

$$\frac{\partial}{\partial t}(\rho v_i) = P_{ij}^{\perp}\left[\eta_0 \nabla^2 \frac{\delta \mathcal{H}}{\delta(\rho v_j)} + w_0 (\nabla_j \phi)\frac{\delta \mathcal{H}}{\delta \phi} + \zeta_j\right]$$

Model H of Hohenberg and Halperin

$$\eta \sim \xi^{x_\eta} \; (x_\eta \simeq 0.06) \qquad\qquad \zeta \sim \xi^{x_\zeta} \; (x_\zeta \simeq 2.8)$$

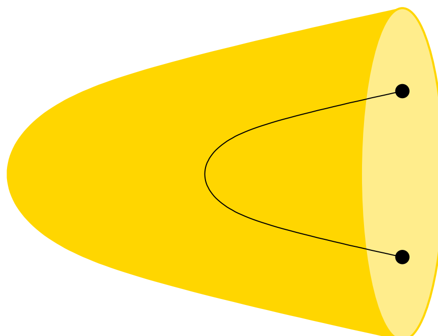
Gauge Theory at Strong Coupling: Holographic Duals

The AdS/CFT duality relates

large N_c (Conformal) gauge theory in 4 dimensions	$\Leftrightarrow$	string theory on 5 dimensional Anti-de Sitter space $\times S_5$
correlation fcts of gauge invariant operators	$\Leftrightarrow$	boundary correlation fcts of AdS fields

$$\langle \exp \int dx\ \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



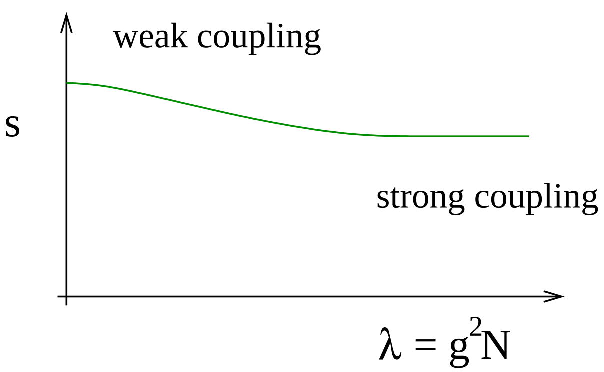
The correspondence is simplest at strong coupling $g^2 N_c$

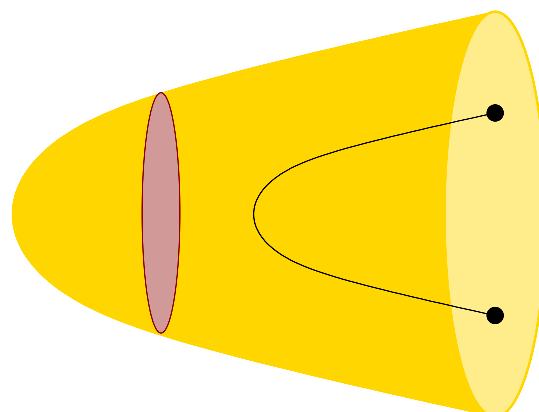
strongly coupled gauge theory $\Leftrightarrow$ classical string theory

Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon





$$s(\lambda \to \infty) = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s(\lambda = 0)$$

Gubser and Klebanov

Holographic Duals: Transport Properties

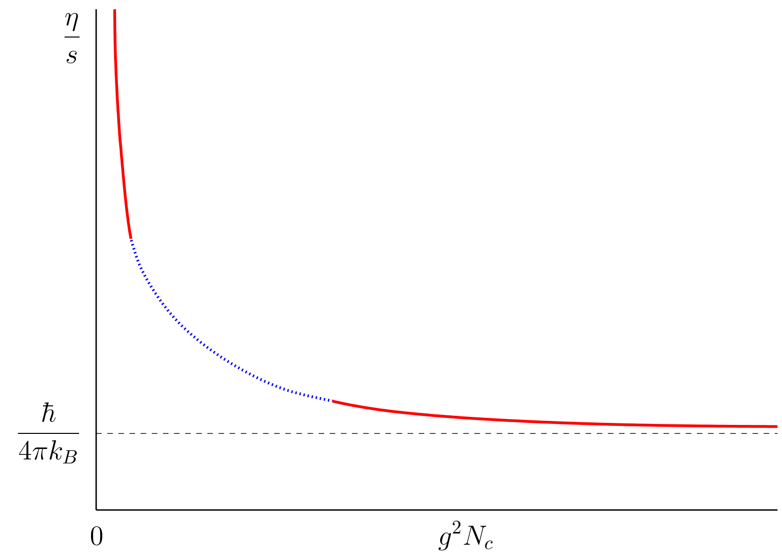
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets



Strong coupling limit universal? Provides lower bound for all theories?

Holographic Duals: Recent Work

Can we get $AdS/NRCFT$ using an embedding of non-relativistic conformal group?

$$Schr(d) \subset SO(d+1,2) \;\; (= Iso(AdS_{d+2}))$$

Not quite, but we can use $Schr(d) \subset SO(d+2,2)$

$$\text{New } 2+1 \text{ NRCFTs with } \frac{\eta}{s} = \frac{1}{4\pi}$$

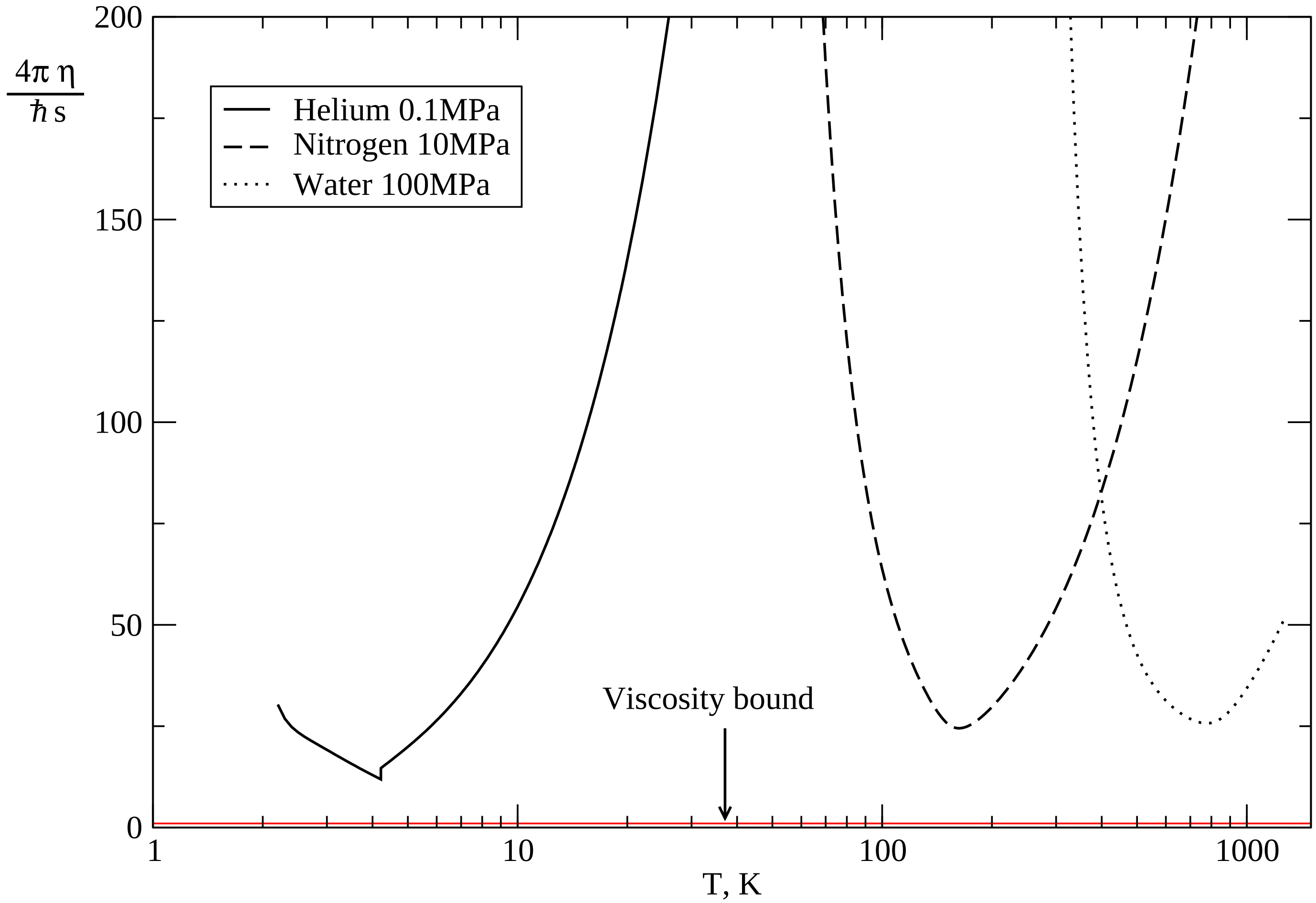
Son, McGreevy, Balasubramanian, . . .

Can we prove viscosity bound? Proven in certain classes of backgrounds, but some possible counterexamples exist

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\lambda_{GB}) \qquad \lambda_{GB} < \frac{9}{100}$$

Myers, Shenker, Kats, Petrov, . . .

Viscosity Bound: Common Fluids



Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

(Almost) scale invariant systems

I. Fermi gas at unitarity

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger\Big(i\partial_0 + \frac{\nabla^2}{2M}\Big)\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2$$

Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ ($C_0 \to \infty$)

This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$

$$\mathcal{L} = \Psi^\dagger\left[i\partial_0 + \sigma_3\frac{\vec{\nabla}^2}{2m}\right]\Psi + \Big(\Psi^\dagger\sigma_+\Psi\phi + h.c.\Big) - \frac{1}{C_0}\phi^*\phi\ ,$$

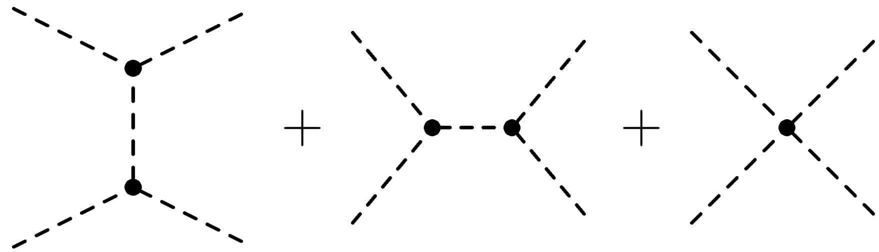
Low T ($T < T_c \sim \mu$): Pairing and superfluidity

Low T: Phonons Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

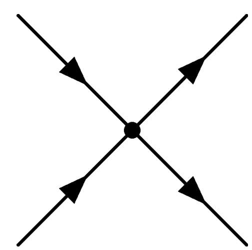
Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$



High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{2}}(mT)^{3/2}$$



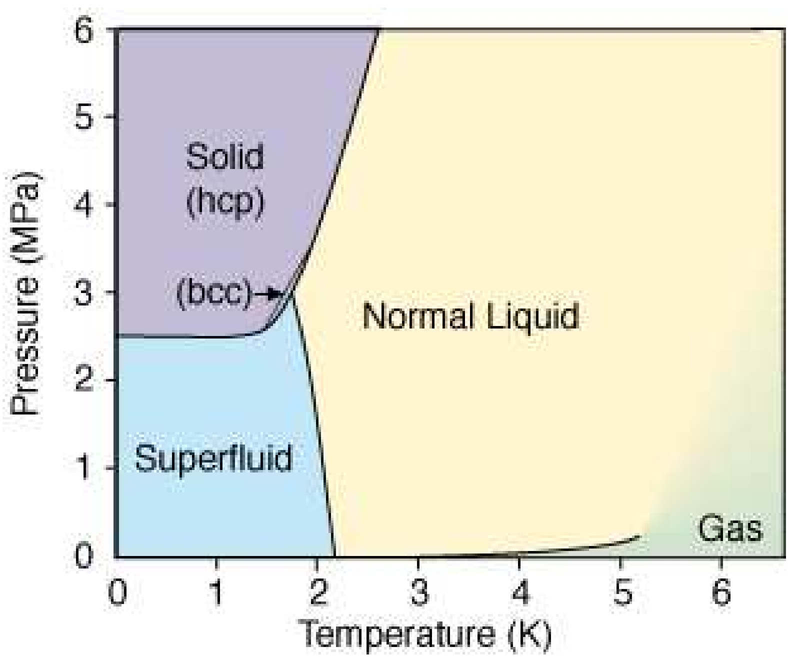
II. Liquid Helium

Bosons, van der Waals + short range repulsion

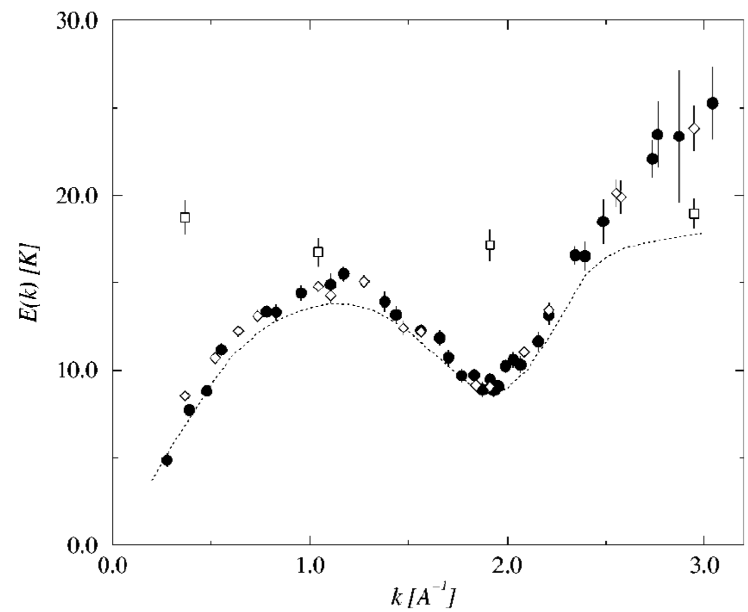
$$S = \int \Phi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \Phi + \int \int \left(\Phi^\dagger \Phi \right) V(x-y) \left(\Phi^\dagger \Phi \right)$$

with $V(x) = V_{sr}(x) - c_6/x^6$. Note: $a = 189 a_0 \gg a_0$

Phase Diagram



Excitations



Low T: Phonons and Rotons Effective lagrangian

$$\mathcal{L} = \varphi^*(\partial_0^2 - v^2)\varphi + i\lambda\dot{\varphi}(\vec{\nabla}\varphi)^2 + \ldots$$

$$+ \varphi^*_{R,v}(i\partial_0 - \Delta)\varphi_{R,v} + c_0(\varphi^*_{R,v}\varphi_{R,v})^2 + \ldots$$

Shear viscosity

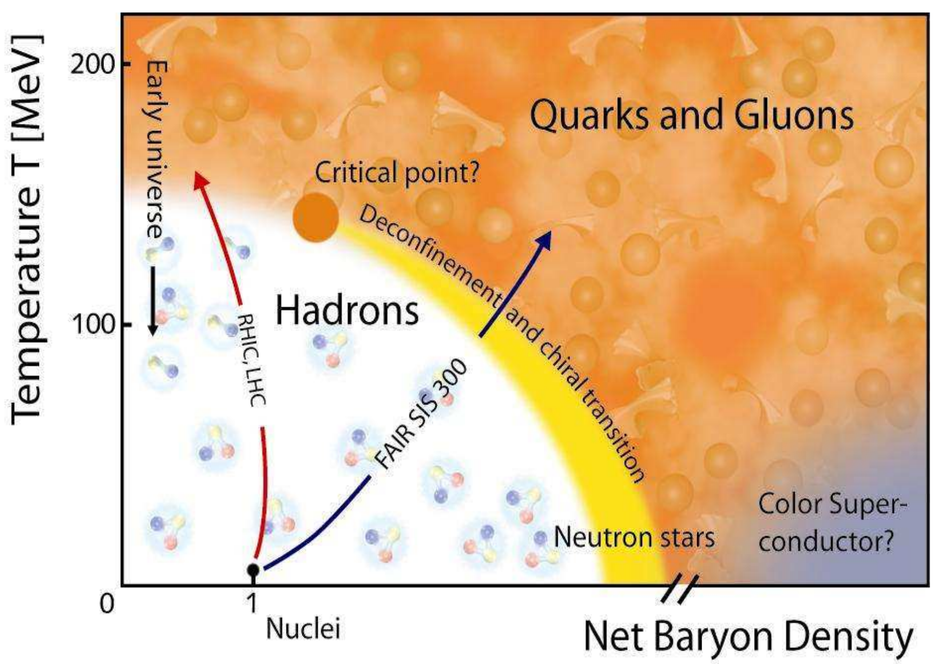
$$\eta = \eta_R + \frac{c_{Rph}}{\sqrt{T}}\, e^{\Delta/T} + \frac{c_{ph}}{T^5} + \ldots$$

High T: Atoms Viscosity governed by hard core ($V \sim 1/r^{12}$)

$$\eta = \eta_0 (T/T_0)^{2/3}$$

III. Quark Gluon Plasma

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

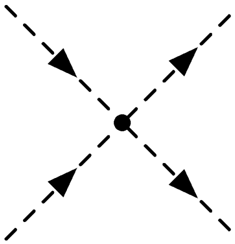


Low T: Pions Chiral perturbation theory

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + (B\mathrm{Tr}[MU] + h.c.) + \ldots$$

Viscosity dominated by $\pi\pi$ scattering

$$\eta = A\frac{f_\pi^4}{T}$$



High T: Quasi-Particles HTL theory (screening, damping, ...)

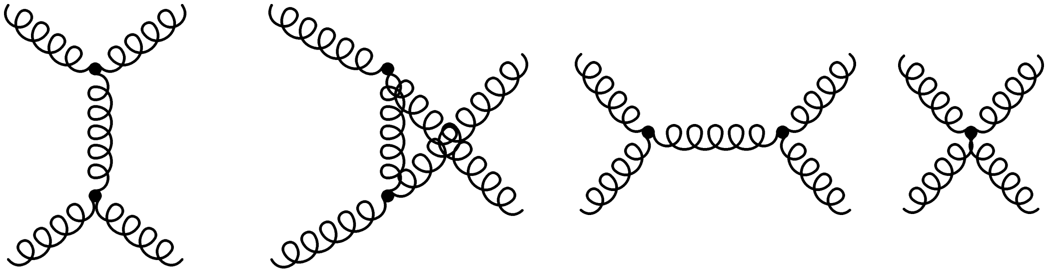
$$\mathcal{L}_{HTL} = \int d\Omega\, G^a_{\mu\alpha}\frac{v^\alpha v_\beta}{(v\cdot D)^2}G^{a,\mu\beta}$$

quasi-particle width

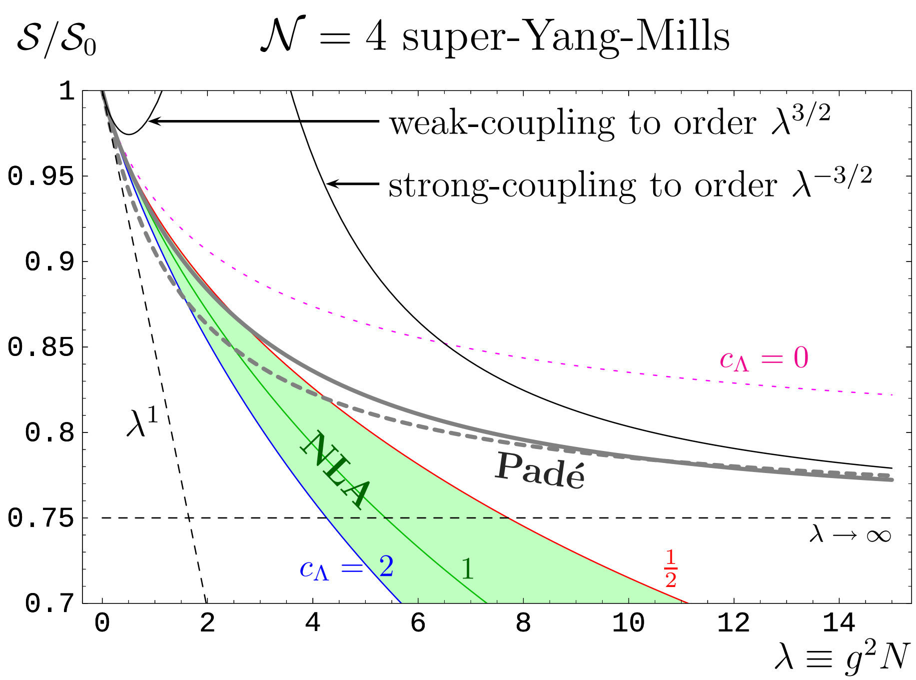
$$\gamma \sim g^2 T$$

Viscosity dominated by t-channel gluon exchange

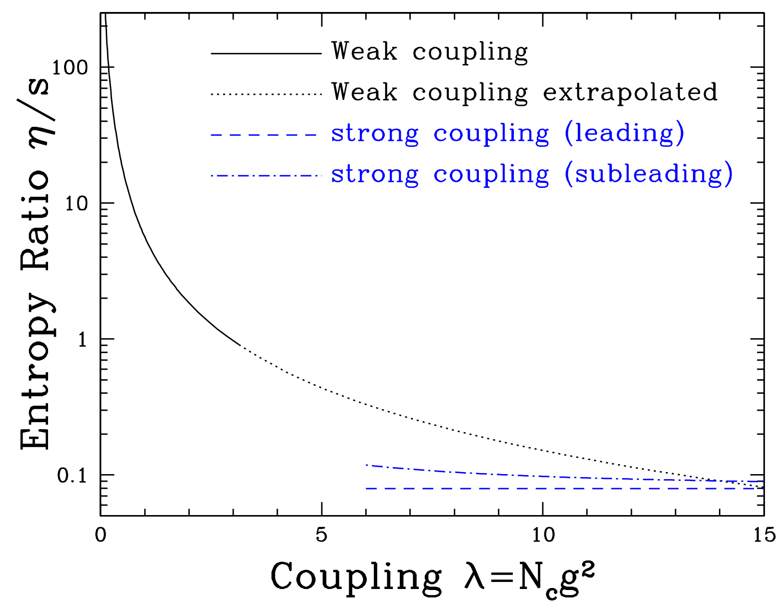
$$\eta = \frac{27.13T^3}{g^4\log(2.7/g)}$$



What is the value of the coupling?



Blaizot, Iancu, Rebhan, Kraeemmer, Rebhan (2006)



Huot, Jeon, Moore (2006)

I. Experiment (Liquid Helium)

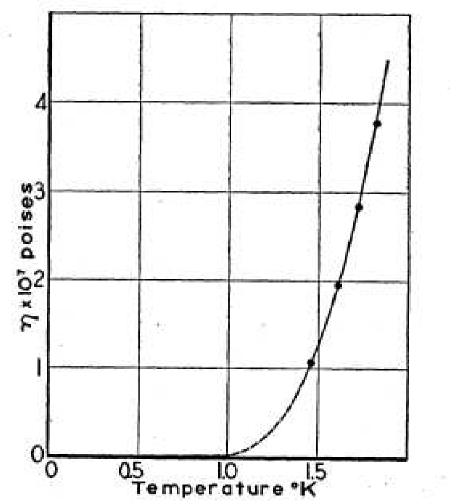


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.

2, § 3 LIQUID HELIUM II 23

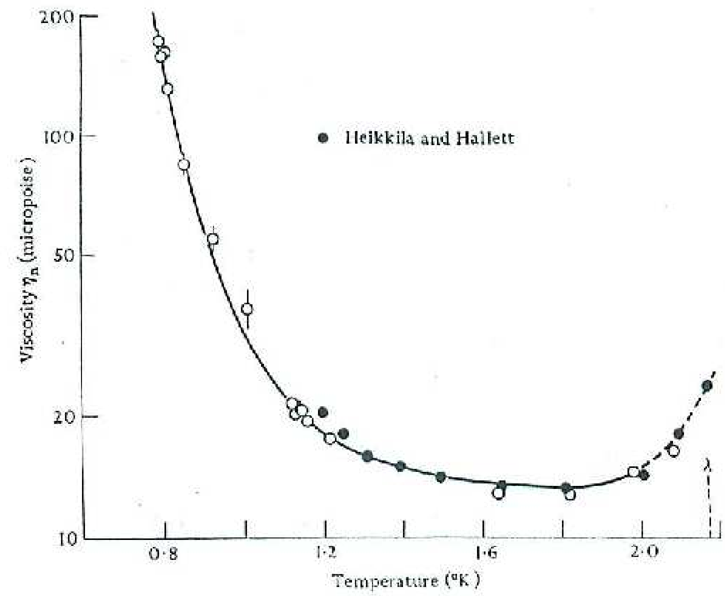


FIG. 11. The viscosity (η_n) of helium II as measured in a rotation viscometer (Woods and Hollis Hallett [50]). The full points show the earlier results of Heikkila and Hollis Hallett [51]).

Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

Hollis-Hallett (1955)

roton minimum, phonon rise

rotation viscometer

$$\eta/s \simeq 0.8\,\hbar/k_B$$

Viscosity near Liquid-Gas Transition

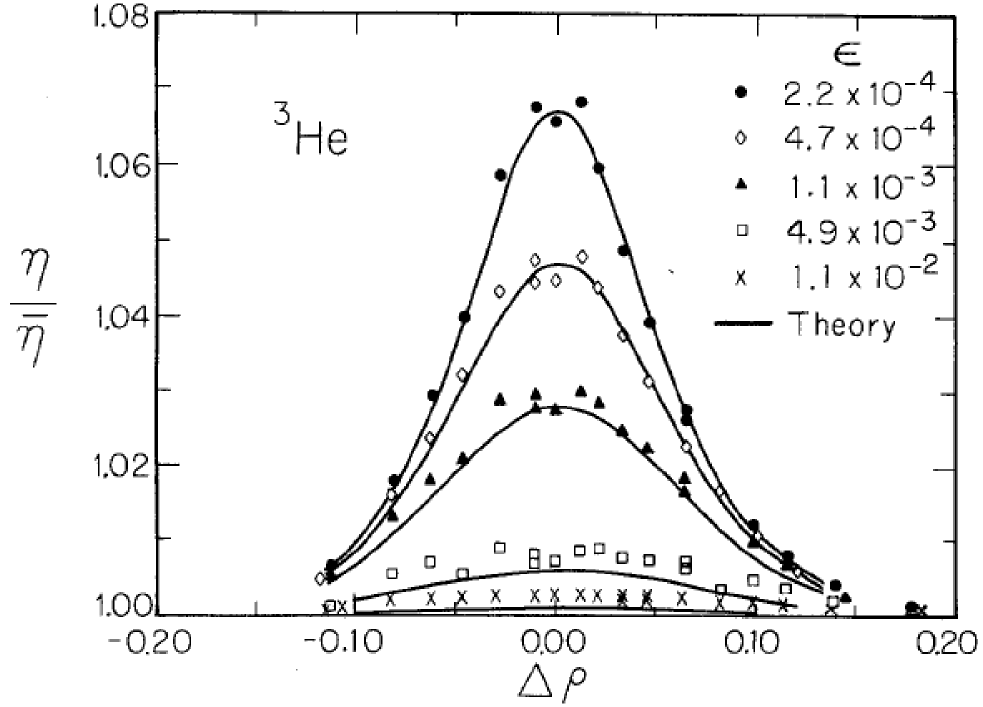
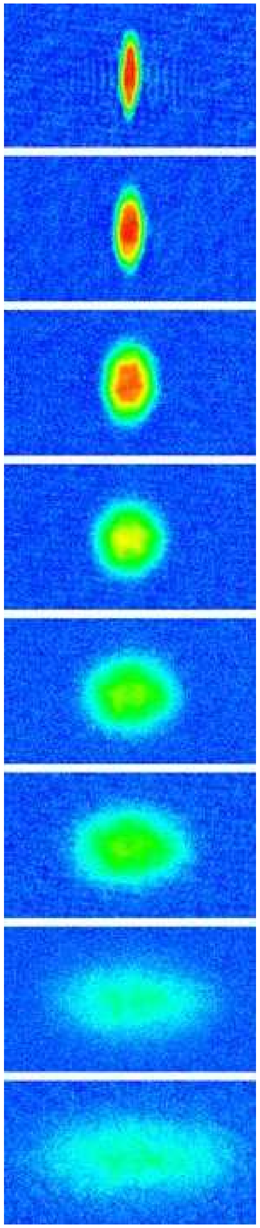
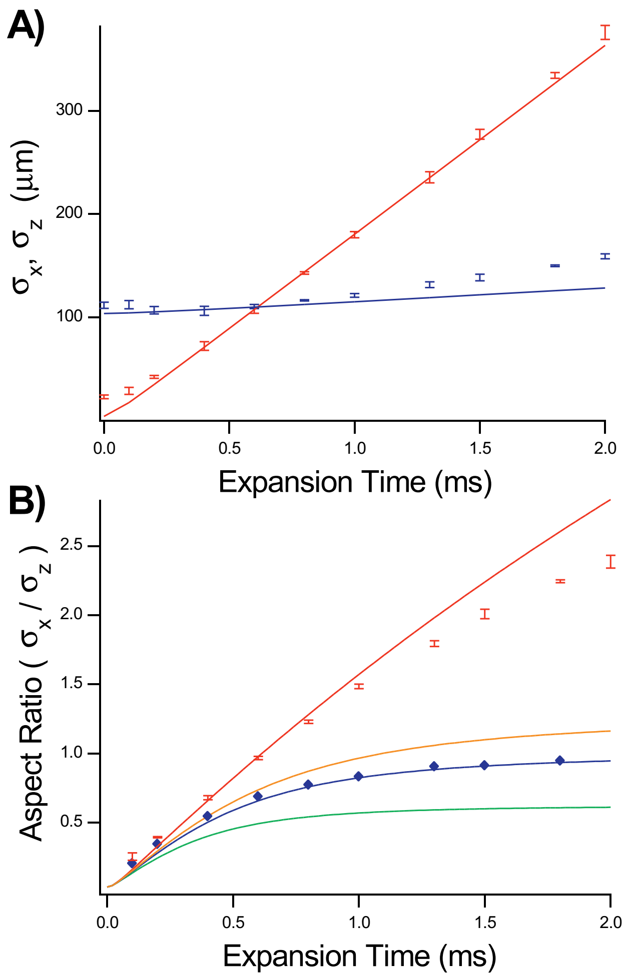


Fig. 14. The normalized viscosity $\tilde{\eta}/\bar{\eta}$ for ^{3}He versus $\Delta\rho_c$ along several isotherms. (—) Fits of the MC theory with $q_D = 3\times10^6\ \mathrm{cm}^{-1}$.

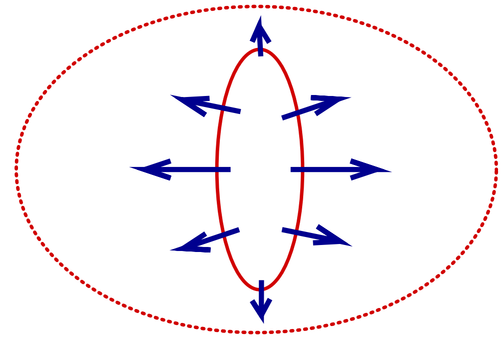
Agosta, Wang, Cohen, Meyer (1987)

II. Elliptic Flow (Fermions)



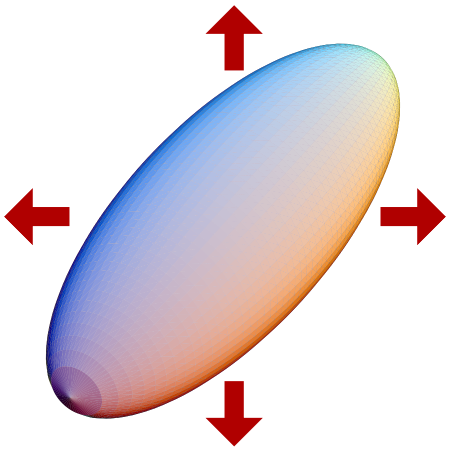


Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

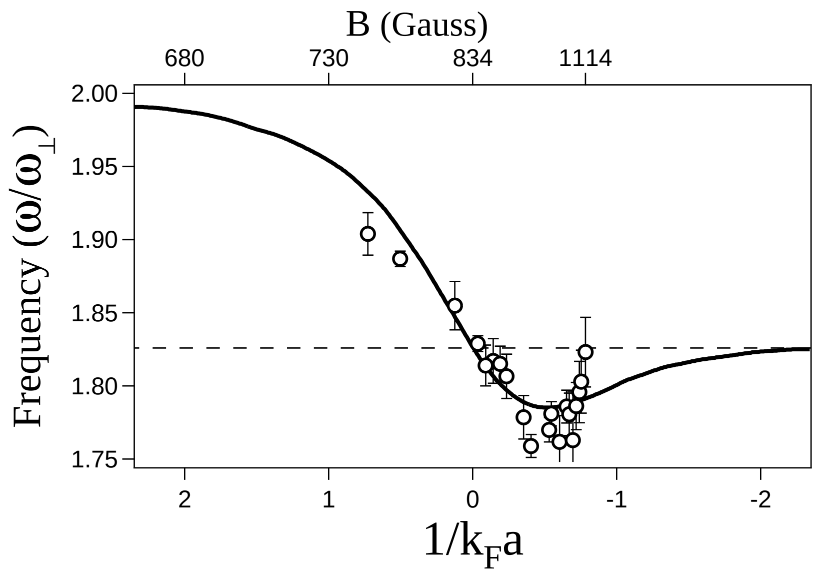


Collective Modes

Radial breathing mode



Kinast et al. (2005)



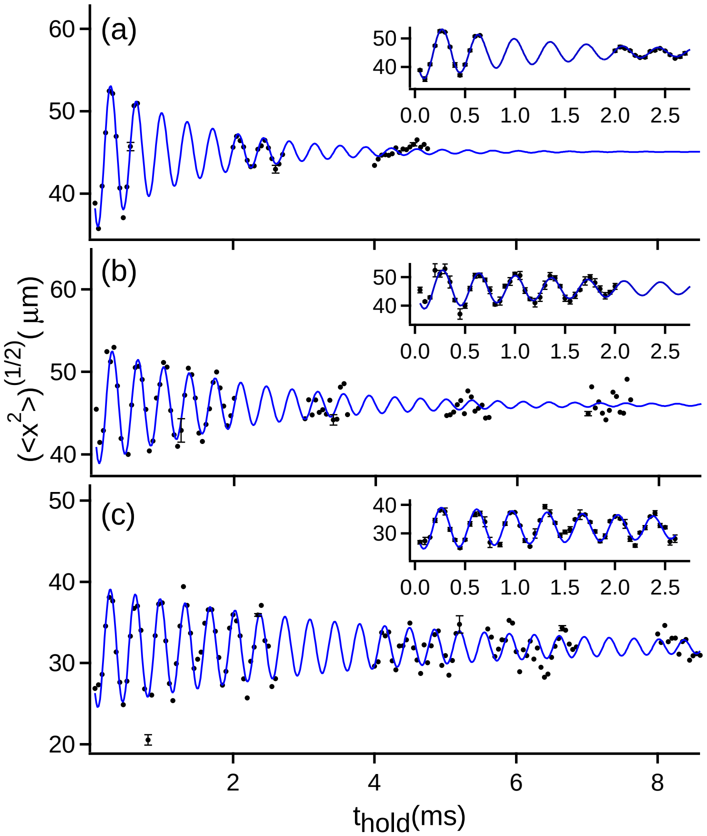
Ideal fluid hydrodynamics, equation of state $P \sim n^{5/3}$

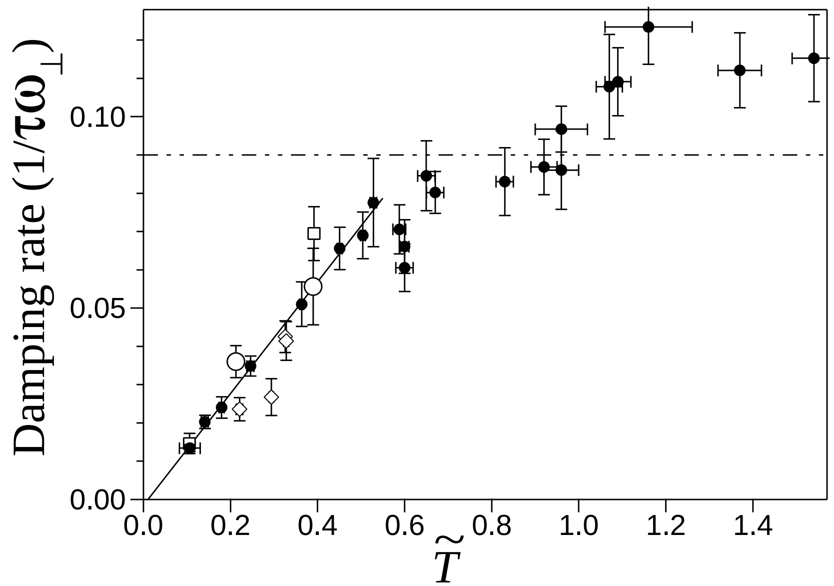
$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} = -\frac{1}{mn}\vec{\nabla} P - \frac{1}{m}\vec{\nabla} V$$

$$\omega = \sqrt{\frac{10}{3}}\omega_\perp$$

Damping of Collective Excitations





$T/T_F = (0.5, 0.33, 0.17)$

$\tau\omega$: decay time $\times$ trap frequency

Kinast et al. (2005)

Viscous Hydrodynamics

Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

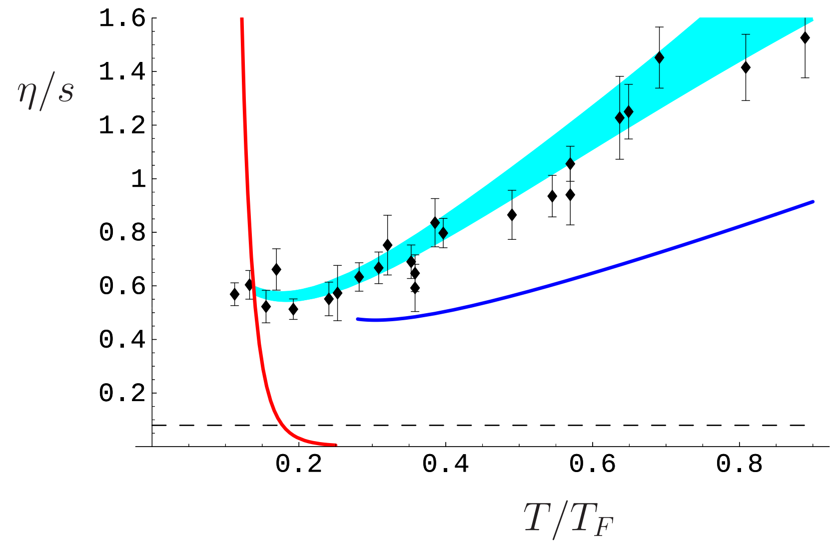
$$\dot{E} = -\frac{\eta}{2}\int d^3x \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \zeta \int d^3x \, (\partial_i v_i)^2 - \frac{\kappa}{T}\int d^3x \, (\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

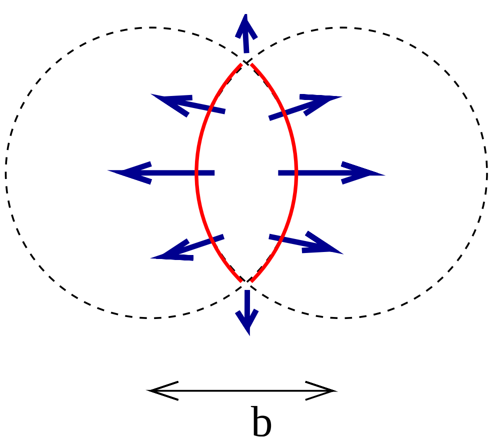
$$\frac{\eta}{s} = \frac{3}{4}\xi^{\frac{1}{2}}(3N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\,\frac{\bar{\omega}}{\omega_\perp}\,\frac{N}{S}$$

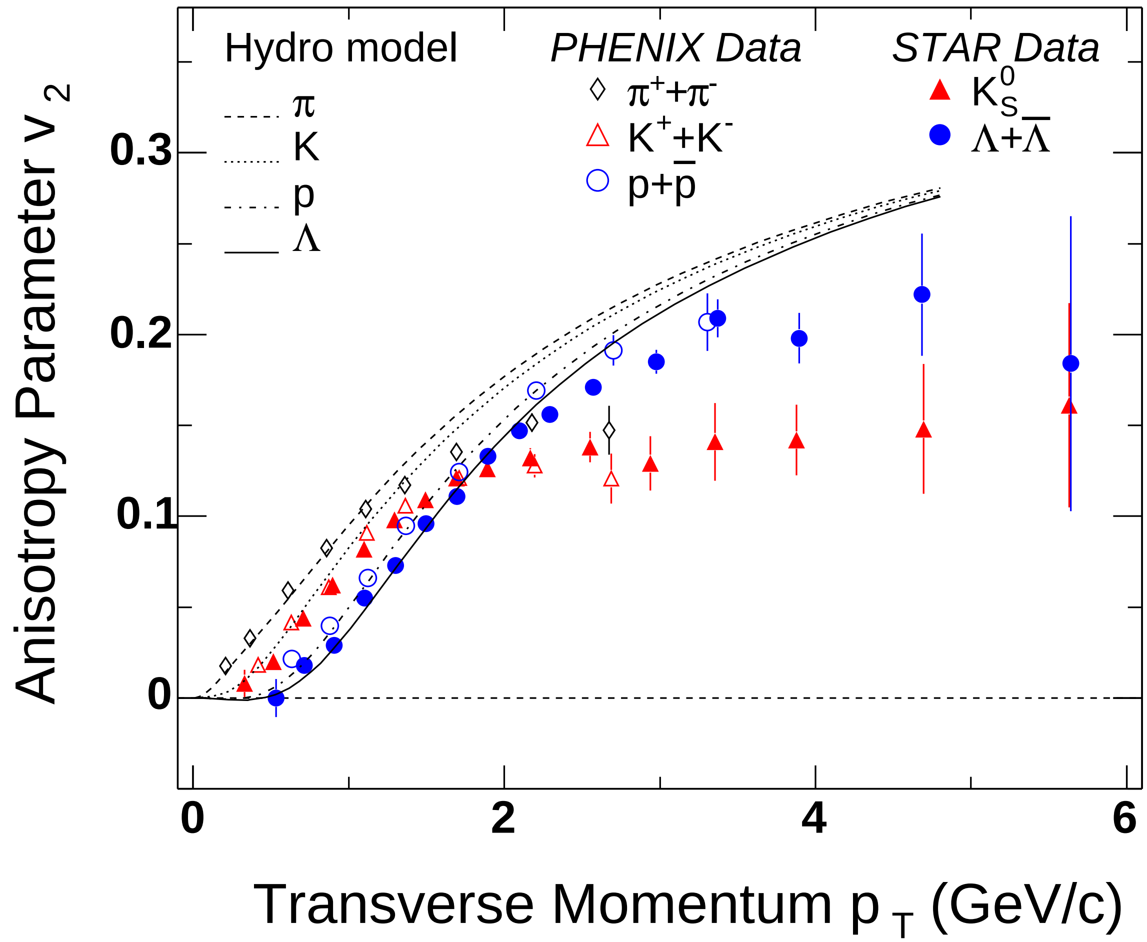
see also Bruun, Smith, Gelman et al.



III. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





source: U. Heinz (2005)

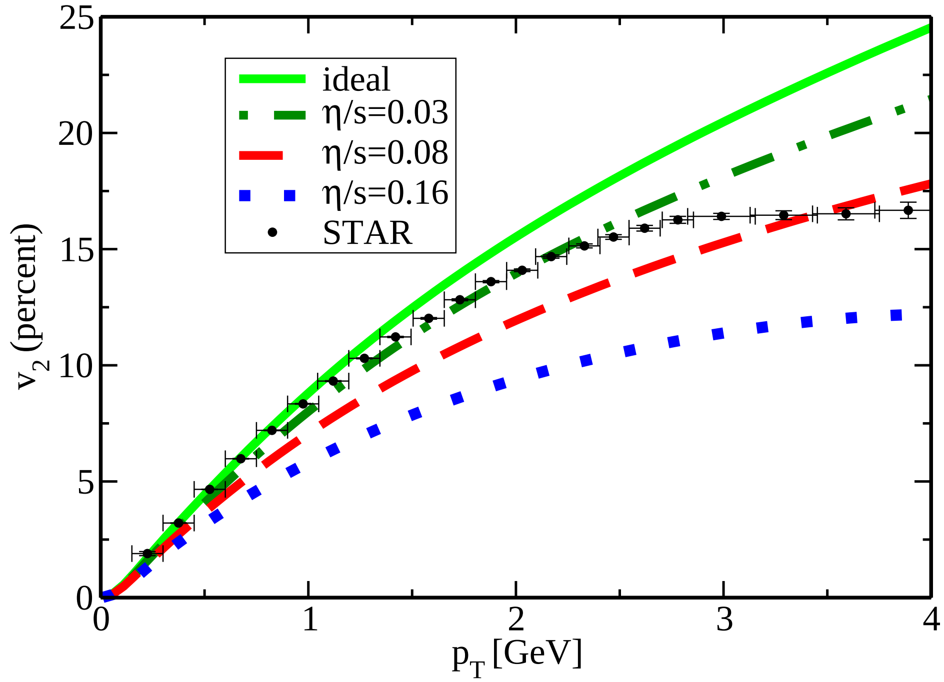
Viscosity and Elliptic Flow

Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$ (applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm



Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

Outlook

Too early to declare a winner.

$$\eta/s \simeq 0.8 \text{ (He)}, \qquad \eta/s \leq 0.5 \text{ (CA)}, \qquad \eta/s \leq 0.5 \text{ (QGP)}$$

Other experimental constraints, more analysis needed.

Existing theory tools: Kinetic theory in low/high T limit, Kubo+field theory tools (lattice, etc), dynamic universality, ...

New theory tools: AdS/Cold Atom correspondence? Field theory approaches in cross over regime (large N, epsilon expansions, ...)