Strongly correlated systems: From QCD to ultracold atoms

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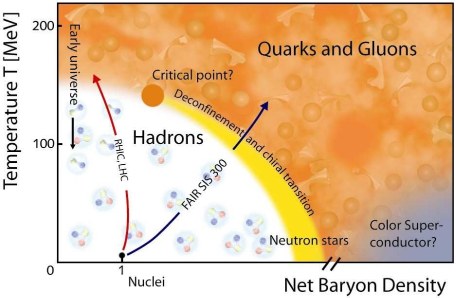
Quark-Gluon Plasma meets Cold Atoms
GSI, July 26th 2008

Overview

1. Motivation
2. Strongly correlated QCD
   - QCD phase diagram
   - Confinement-deconfinement phase transition
3. Strongly correlated ultracold atoms
   - BEC-BCS crossover
   - Phase diagram of ultracold atoms

Phase diagrams

QCD



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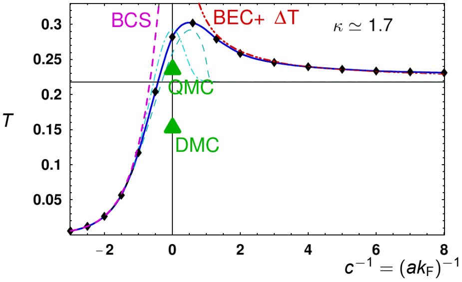
Ultracold atoms



Diehl, Gies, JMP, Wetterich'07

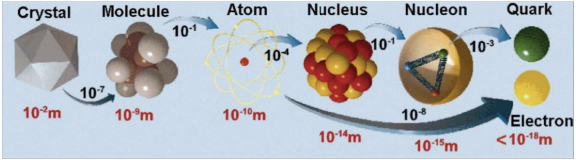
Phase structure

- Quark-Gluon phase
- Hadronic phase

$\Longleftrightarrow$

- Atomic phase
- Condensate phase

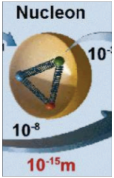
Bound States/Condensates



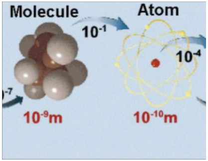
QCD

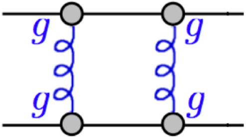
Ultracold Atoms

$\Rightarrow$

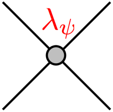


$\Longleftrightarrow$

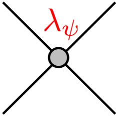




$\Rightarrow$

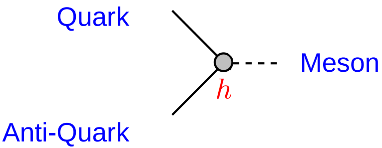


$\Longleftrightarrow$



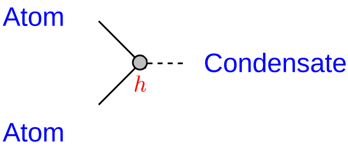
Dynamical

Hadronisation



+Baryons and Glueballs

Condensation

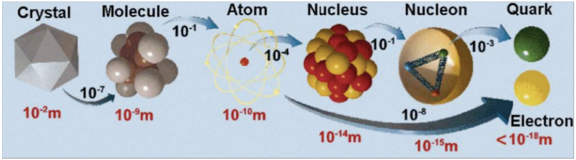


+Trions

Dynamical degrees of freedom

- Quarks, Gluons $\psi,\ A$
- Mesons, Baryons $\phi \sim \bar{\psi}\psi,\ b \sim \psi^3$
- Atoms ψ
- Condensates $\phi \sim \psi^2$

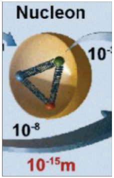
Bound States/Condensates



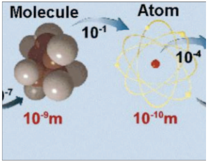
QCD

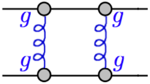
Ultracold Atoms

$\Rightarrow$

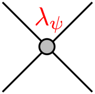


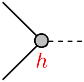
$\Longleftrightarrow$



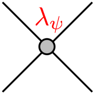


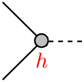
$\Rightarrow$





$\Longleftrightarrow$





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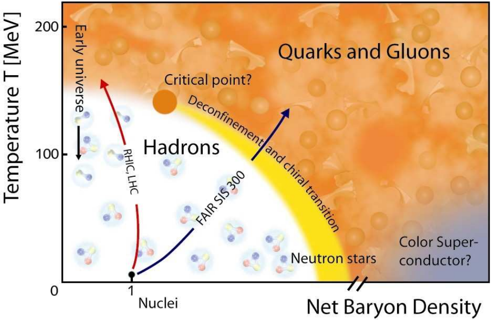
QCD phase diagram

- chiral phase transition:

  $SU_L(N_f) \times SU_R(N_f)$

  order parameter:

  $$\langle \bar{q}\, q \rangle = \begin{cases} 0, & T > T_{c,\chi} \\ > 0, & T < T_{c,\chi} \end{cases}$$



- confinement-deconfinement: Z_3

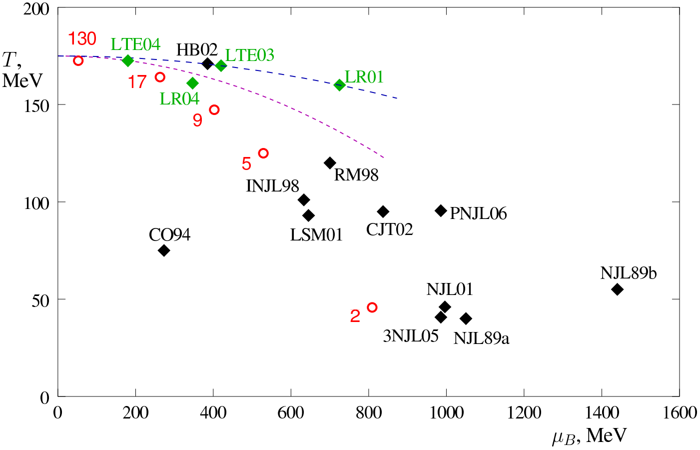
  order parameter: $\beta = 1/T$

$$\Phi[A_0] = \langle \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} e^{i \int_0^\beta dt\, A_0} \rangle = \begin{cases} > 0, & T > T_{c,\mathrm{conf}} \\ 0, & T < T_{c,\mathrm{conf}} \end{cases}$$

Polyakov loop $\Phi = e^{-\beta F_q}$ relates to a static quark state.

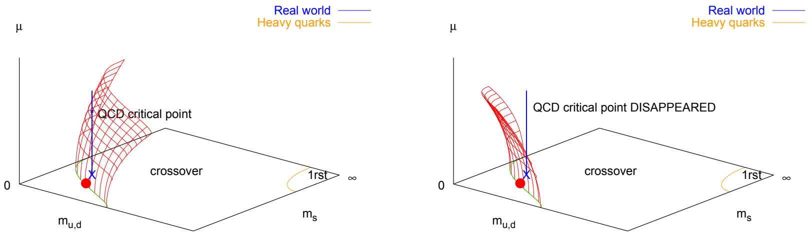
Critical point

Black: models   green, lines: lattice   Red: Freeze-out points for HIC



Stephanov '07

Critical point



Strategy: tune m_q for 2nd-order P.T. at $\mu = 0$, then turn on infinitesimal μ

Does the transition become 1rst-order (left) or crossover (right)?

Answer: little change (→ surface almost vertical)

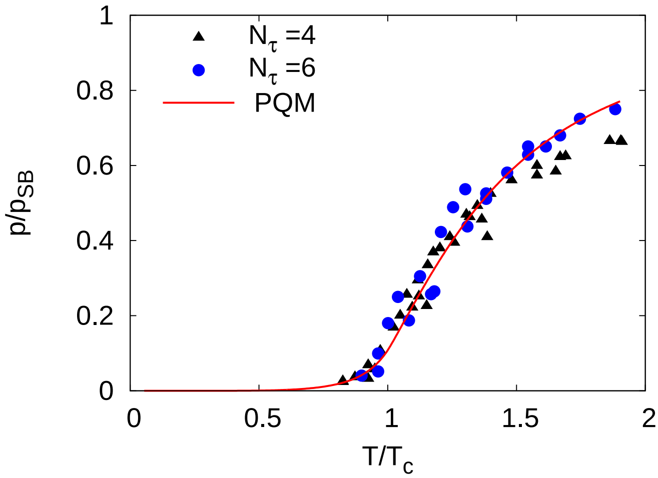
2007: measure δB_4 under $\delta\mu^2$ → crossover: $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(5)\left(\frac{\mu}{\pi T}\right)^2$

Polyakov–Quark-Meson Model

Schaefer, JMP, Wambach '07

EoM of: $U[\Phi, \bar{\Phi}]$ $+V[\sigma, \vec{\pi}]$ $+\Omega_{\bar{q}q}(\Phi, \bar{\Phi}, \sigma)$

Polyakov loop | meson | fermionic determinant

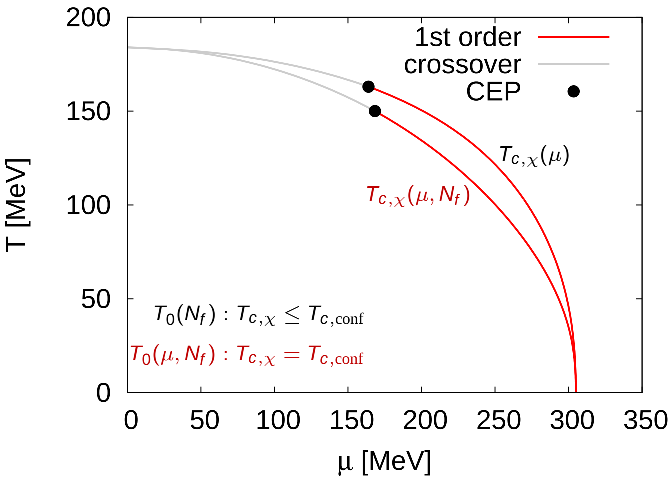


lattice data taken from Ali Khan et al. (CP-PACS), Phys. Rev. D **64** (2001)

Polyakov–Quark-Meson Model

Schaefer, JMP, Wambach ’07

T_0 is the critical temperature from the Polyakov loop potential



$N_f = 2$

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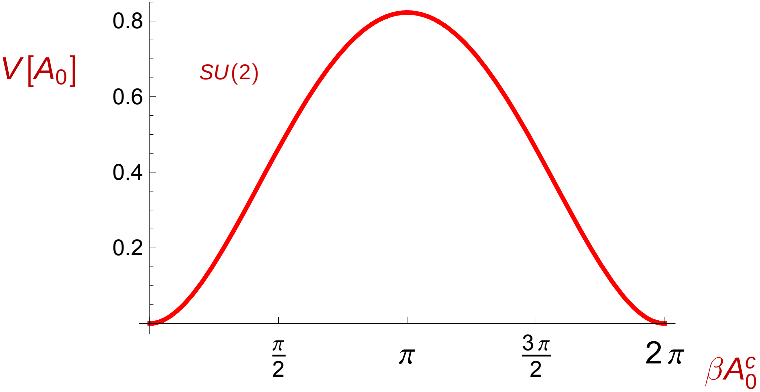
Weiss Potential

$V[A_0]$: one-loop effective potential



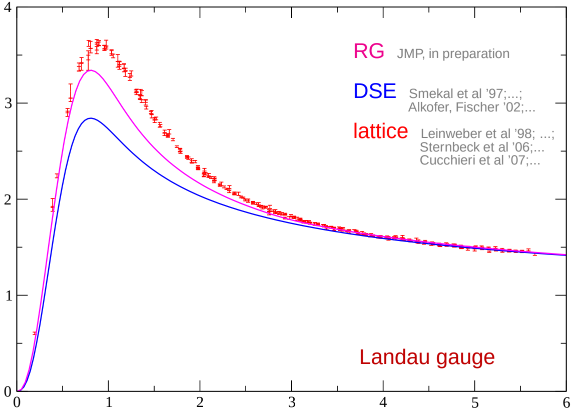
$SU(2)$: $\Phi[A_0] = \cos \frac{1}{2}\beta A_0^c$ with $A_0 = A_0^c \sigma_3$

Polyakov loop Potential

$$\text{RG-flow}: \quad V[\bar{A}_0] = -\frac{1}{2}\,\mathrm{Tr}\ln \langle AA\rangle[\bar{A}_0] + O(\partial_t \langle AA\rangle[\bar{A}_0]) + O(V''[A_0])$$

90% 10% +ghosts

$p^2\,\langle AA\rangle(p^2)$

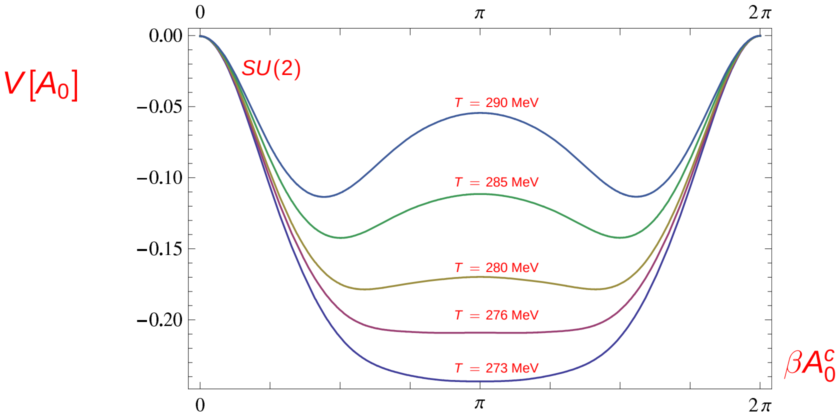


p [GEV]

Polyakov Loop Potential: $SU(2)$

Braun, Gies, JMP '07

$T_c \simeq 276 \pm 10\mathrm{MeV}$ $\quad T_c/\sqrt{\sigma} = 0.627 \pm 0.023$ $\quad$ lattice: $T_c/\sqrt{\sigma} = .709$



$$\Phi[A_0] = \cos \tfrac{1}{2}\beta A_0^c$$

Polyakov Loop Potential: $SU(2)$

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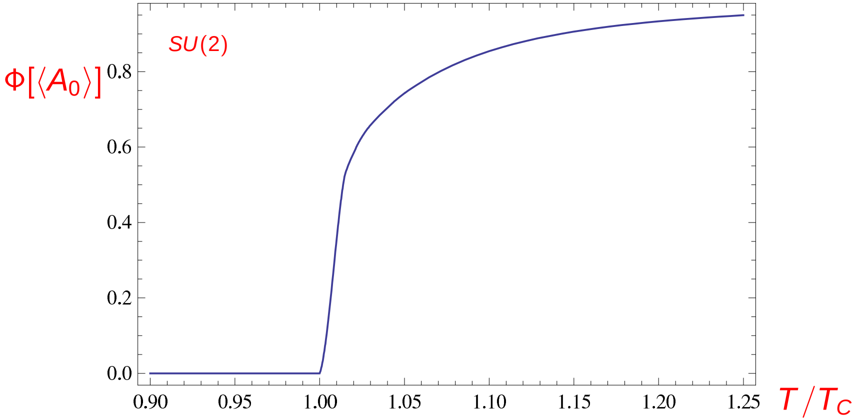
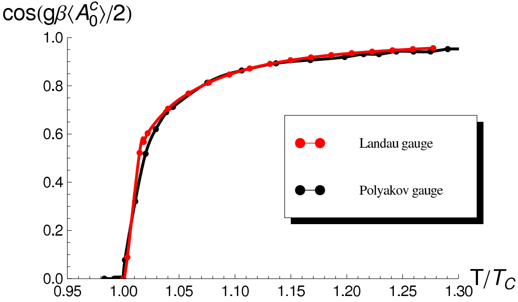


Universal properties

Marhauser, JMP, in preparation

RG-flow in Polyakov gauge: $A_0 = A_0^c(\vec{x})\sigma_3$

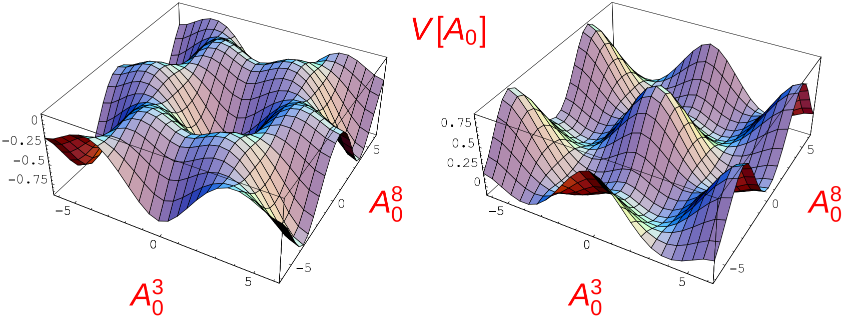


- ——: Polyakov gauge: crit. exp. $\nu = 0.65$ $\nu_{\text{Ising}} = 0.63$
- ——: Landau gauge propagators

Polyakov Loop Potential: $SU(3)$

Braun, Gies, JMP '07

$T_c \simeq 284 \pm 10\mathrm{MeV}$ $\quad$ $T_c/\sqrt{\sigma} = 0.646 \pm 0.023$ $\quad$ lattice: $T_c/\sqrt{\sigma} = .646$



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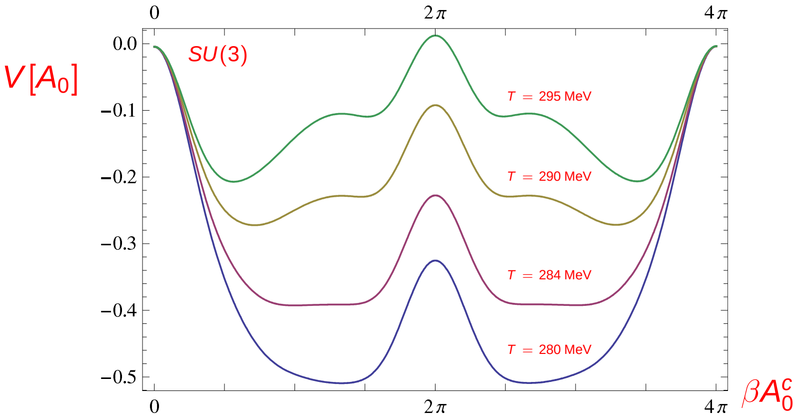

$$\Phi[\beta A_0^c = \tfrac{4}{3}\pi] = 0$$

Polyakov Loop Potential: $SU(3)$

Braun, Gies, JMP '07

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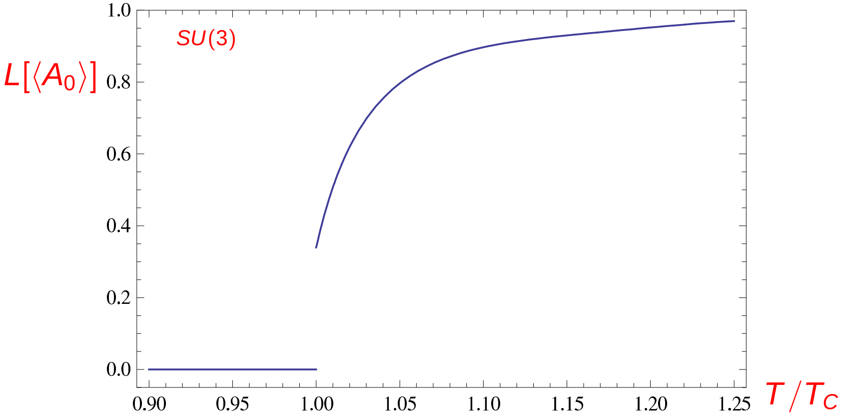

Summary & Outlook

- results
  - confinement-deconfinement phase transition
  - dynamical chiral symmetry breaking $\Leftrightarrow$ ultracold atoms
  - ’QCD phase diagram’ from models
- challenges
  - full QCD
  - QCD at finite temperature & density $\Leftrightarrow$ ultracold atoms
  - Non-equilibrium effects in QCD $\Leftrightarrow$ ultracold atoms

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BEC-BCS crossover

(Eagles'69; Leggett'80)

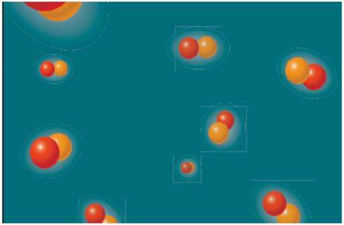
- Bound molecules of two atoms on microscopic scale

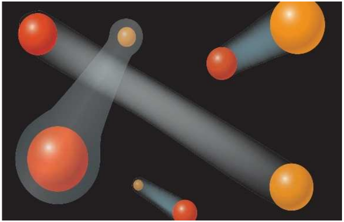
BEC at low *T*

- Fermions with attractive interactions

BCS superfluidity at low *T*

(Cho@Science'03)





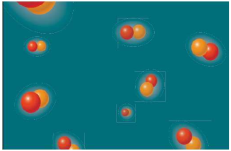
Crossover by means of a Feshbach resonance

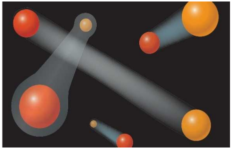
(Regal&'04; Bartenstein&'04; Zwierlein&'04; Kinast&'04; Bourdel&'04)

Relevant degrees of freedom

ψ: stable fermionic atom field

ϕ: bosonic molecule field / Cooper pair





- include all relevant dof as propagating fields

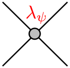
$$\Gamma[\psi] \rightarrow \Gamma[\psi, \phi]$$

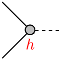
- dynamical condensation (Re-bosonisation)

Gies, Wetterich'01; JMP'05

Effective action

- interaction terms





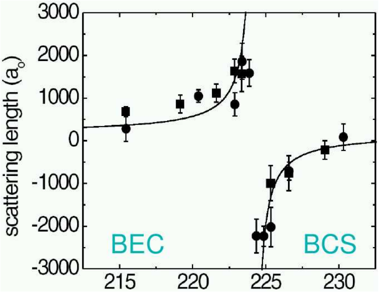
$$\int d\tau\, d^3x \left(m_\phi^2 \phi^* \phi + \frac{\lambda_\phi}{2} (\phi^* \phi)^2 - \frac{h_\phi}{2} (\phi^* \psi^T \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) + \ldots \right)$$

- relation to microphysics (via Hubbard-Stratonovich)

$$\lambda_\psi = \frac{4\pi a_{\mathrm{bg}}}{M}$$

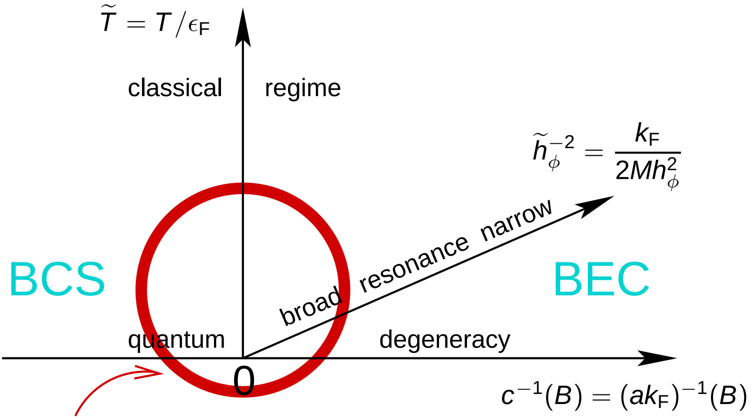
$$m_\phi^2 = \bar{\mu}(B - B_0) - 2\sigma$$

$$h_\phi^2 \sim \Delta B$$



Crossover Diagramm

in units of Fermi momentum $k_\mathrm{F} = (3\pi^2 n)^{1/3}$ and energy $\epsilon_\mathrm{F} = k_\mathrm{F}^2/(2M)$



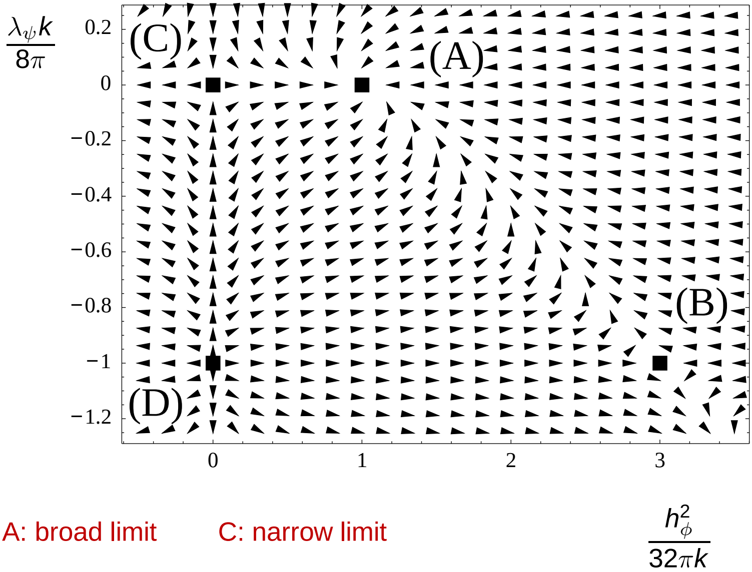
"concentration" parameter strong coupling, "broad-resonance universality" $h_\phi \sim \Delta B \to \infty$

Universal long-distance physics for ^{6}Li and ^{40}K ?

(Diehl, Wetterich'05; Nicolić, Sachdev'06)

Universality

Diehl, Gies, JMP, Wetterich'07



A: broad limit

C: narrow limit

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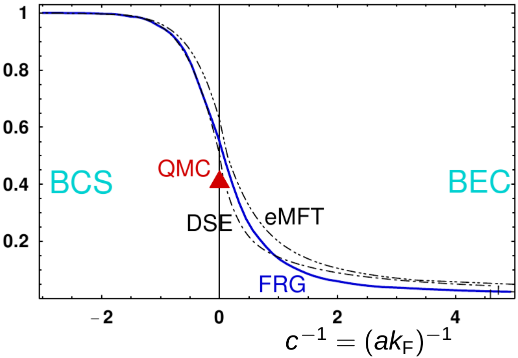
Results: Many body effects

Diehl, Gies, JMP, Wetterich'07

chemical potential $\tilde{\sigma}$ minus molecular binding energy $\tilde{\epsilon}_\mathrm{M}$ at $T = 0$

$\tilde{\sigma} - \tilde{\epsilon}_\mathrm{M}$

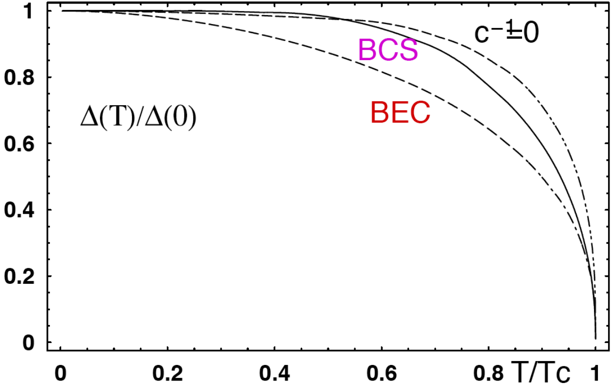


QMC (Giorgini&'04)	FRG (Diehl et al)	ϵNLO (Son&06)	DSE (Diehl&'05)	MFT	2PI (Zwerger&'06)	Padé (Baker'99)	NSR (Hu&'06)	Exp.
0.42(2)	0.55	0.475	0.50	0.63	0.36	0.33	0.40	0.32 -0.51

Results: 2nd order phase transition

Diehl, Gies, JMP, Wetterich'07

Fermionischer Gap $\Delta = h_\phi \sqrt{\rho_{\min}}$



BEC: $c^{-1} = 4$

BCS: $c^{-1} = -2$

Resonanz: $c^{-1} = 0$

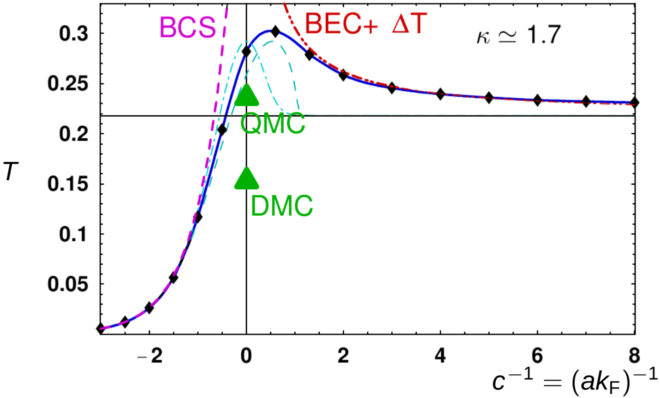
- Universal critical behaviour: $O(2)$-universality class

$$\eta(c^{-1}) = 0.05$$

Results: Phase diagram

Diehl, Gies, JMP, Wetterich'07

Broad resonance limit



shift ΔT of T_c in BEC regime:

(Baym, Blaizot, Holzmann, Laloë, Vautherin'99)
(Baym, Blaizot, Zinn-Justin'00)
(Blaizot, Mendez-Galain, Wschebor'05,06)

$$\frac{T_c - T_c^{\mathrm{BEC}}}{T_c^{\mathrm{BEC}}} = \kappa\, a_{\mathrm{B}} n^{1/3}, \quad \kappa \simeq 1.3$$

Summary & Outlook

- phase diagram of ultracold fermionic gases
- fermion-dimer scattering: include momentum-dep. $\psi^\dagger\psi\phi^*\phi$

  Diehl, Krahl, Scherer'07
- particle-hole fluctuations (T_c): a lesson in re-bosonisation

  Diehl, Flörchinger, Scherer, Wetterich'08
- dynamical condensation

  Gies, Wetterich'01; JMP'05
- non-equilibrium time evolution

  Gasenzer, JMP'07

Bonus material

Particle-hole fluctuations

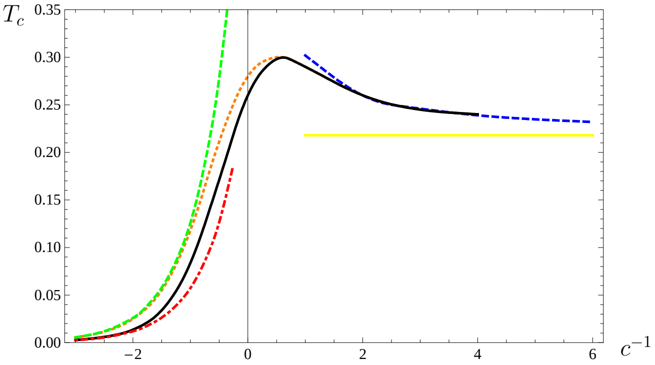


FIG. 1: Black line: FRG including particle-hole fluctuations; Orange line: FRG without particle-hole fluctuations; Green line: BCS result; Red line: Gorkov's correction; Yellow line: Free BEC; Blue line: Interacting BEC with FRG

Diehl, Flörchinger, Scherer, Wetterich'08