Multicolor Hubbard models with ultracold fermions: superfluidity and trionic liquids

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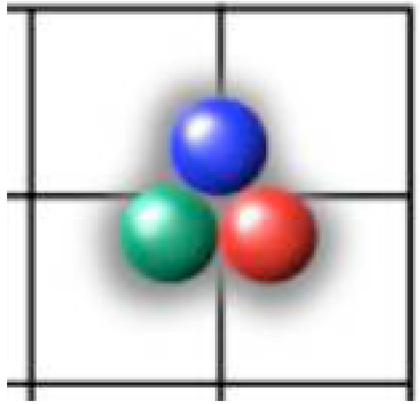


A new bridge between
ultracold atoms and QCD?

Disclaimer:

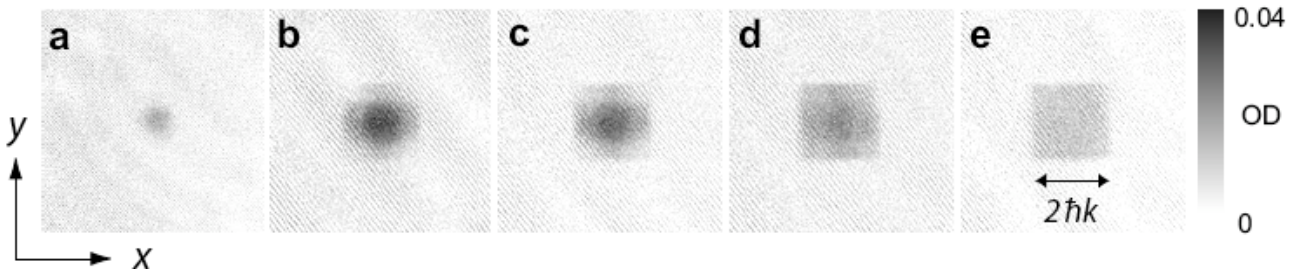
- proposal by solid state theorist

- oversimplified

But: Lots of fun for us!

CH & W. Hofstetter, PRL 2004, PRB 2005
A. Rapp, G. Zarand, CH, W. Hofstetter, PRL 2007
G. Klingschat, CH, ... in progress

Cold fermions on optical lattices



Köhl, Esslinger et al. (PRL 2005):
Band structure clearly observable, $T/T_F \sim 0.25$

Vol 443|26 October 2006|doi:10.1038/nature05224 nature

Evidence for superfluidity of ultracold fermions in an optical lattice

J. K. Chin[1], D. E. Miller[1], Y. Liu[1], C. Stan[1]†, W. Setiawan[1], C. Sanner[1], K. Xu[1] & W. Ketterle[1]

→ **Powerful lab for many-particle physics**

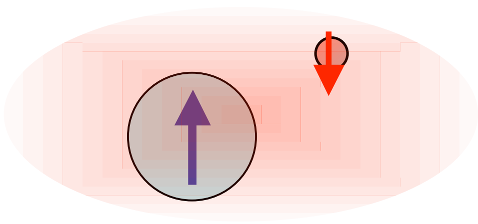
→ **Simulate solid state problems (high-T_c superconductivity!!!)**

→ **New aspects (imbalanced mixtures, time-dependent phenomena...)**

New possibilities?

Ultracold fermions: more than spin up and spin down

Cold atoms can have **more than 2 internal degrees of freedom**:
hyperfine spin $F \rightarrow 2F+1$ hyperfine states

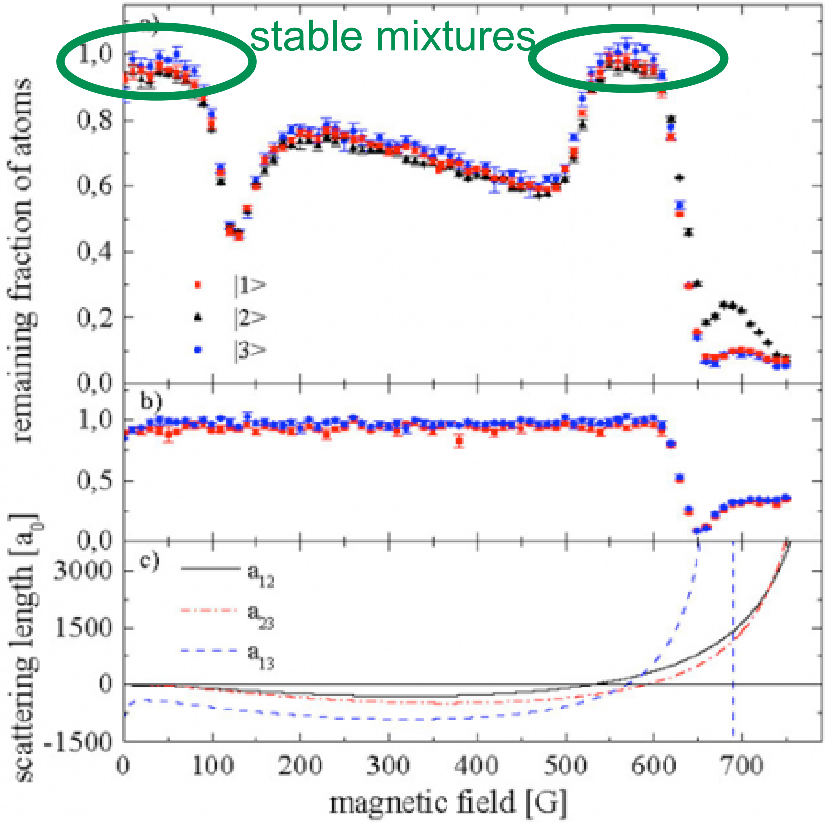


Fermions:

^{6}Li: max. F=3/2, attractive scattering length (e.g. S. Jochim et al.)

^{40}K: F=9/2 (e.g. ETH, Innsbruck)

Rare earth:
^{173}Yb: F=5/2 (Fukuhara et al.), electron configuration $4f^{14}6s^2$: no electron spin!



^{6}Li: MPI-HD group

SU(*N*) Hubbard model

Idealized system:

Take *N* hyperfine states (‘**colors**‘) of fermions on lattice and

- **equal nearest neighbor hopping** for all colors *m*
- **local density-density interaction** between different colors

→ SU(*N*) Hubbard model

$$H = -t \sum_{m,\langle ij \rangle} \left[c^\dagger_{i,m} c_{j,m} + c^\dagger_{j,m} c_{i,m} \right] + \frac{U}{2} \sum_i n_i^2$$

$$n_i = \sum_m n_{i,m}$$

invariant w.r.t. **global** SU(*N*) rotations among *m*=1…*N* fermion colors (fundamental representation)

$$c_{i,\alpha} = \sum_\beta U_{\alpha\beta} c_{i,\beta} \qquad U \in \mathrm{SU}(3)$$

Weak coupling picture

$$H = -t \sum_{m,\langle ij\rangle} \left[c^\dagger_{i,m} c_{j,m} + c^\dagger_{j,m} c_{i,m} \right] + \frac{U}{2} \sum_i n_i^2$$

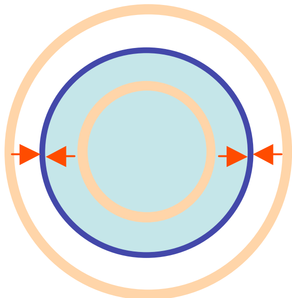
Effective interactions near Fermi surface?

Effective interactions from the functional renormalization group

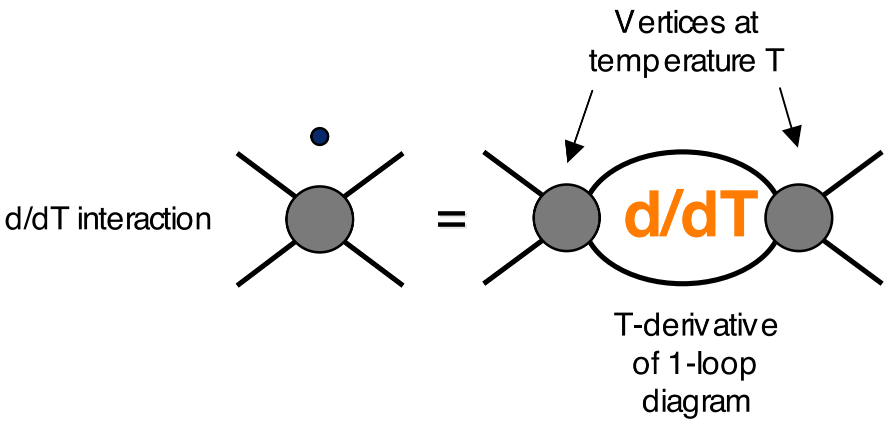
- Follow flow of interaction vertex with change of flow parameter

Choices for **flow parameter *k***:

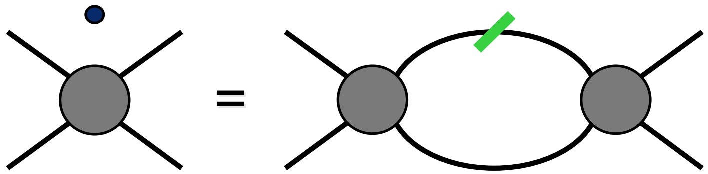
- Band energy cutoff $\Lambda \rightarrow$ momentum-shell schemes



- Temperature $\rightarrow$ *T*-flow scheme (CH& Salmhofer 2001)

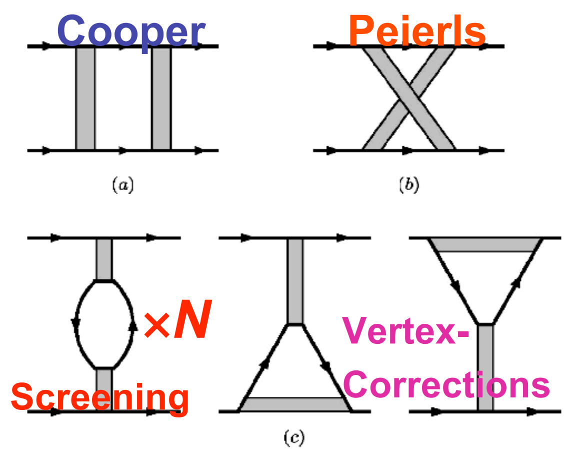


Wetterich 1993
Salmhofer 1998



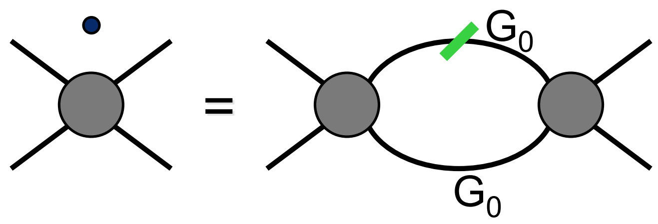
(truncated after γ_4)

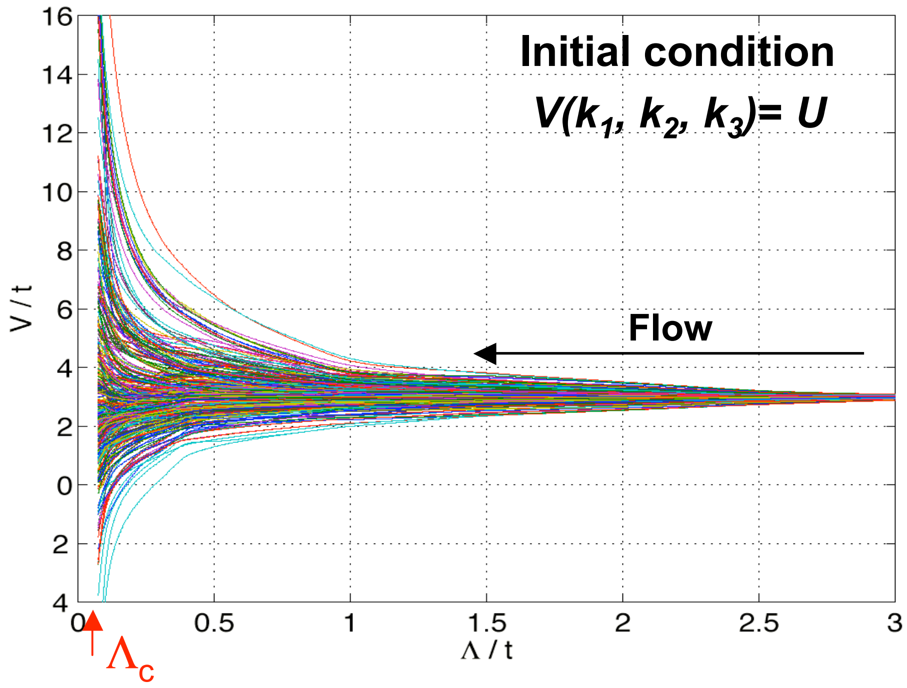
Includes all important fluctuation channels!



Flow to strong coupling

Flows without selfenergy feedback:
Analyze flow to strong coupling





Dominant interactions?

Leading correlations at low energy scales?

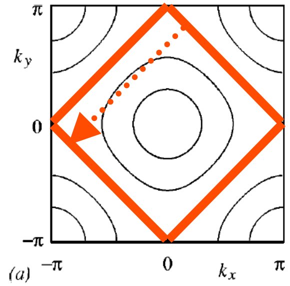
Critical energy scales?

Repulsive model: $U>0$

2D square lattice

Half band filling: N/2 fermions/site

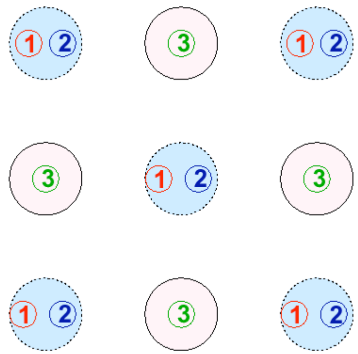
Repulsive SU(N) Hubbard model

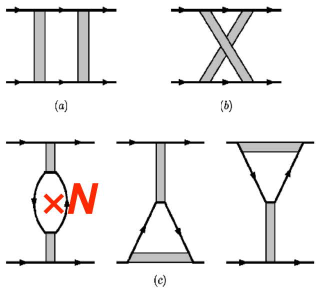


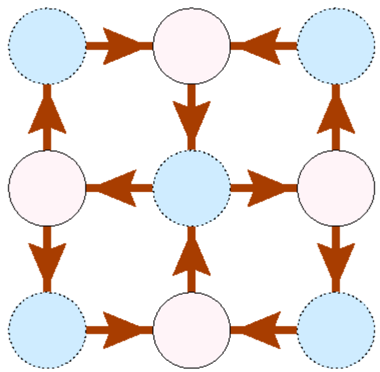
$U > 0$:

fRG for half band filling (N/2 fermions/site)

· N < 6: generalized antiferromagnetic order, color density wave







· N > 6: 'staggered current' (dDW) state, atoms hop around plaquettes

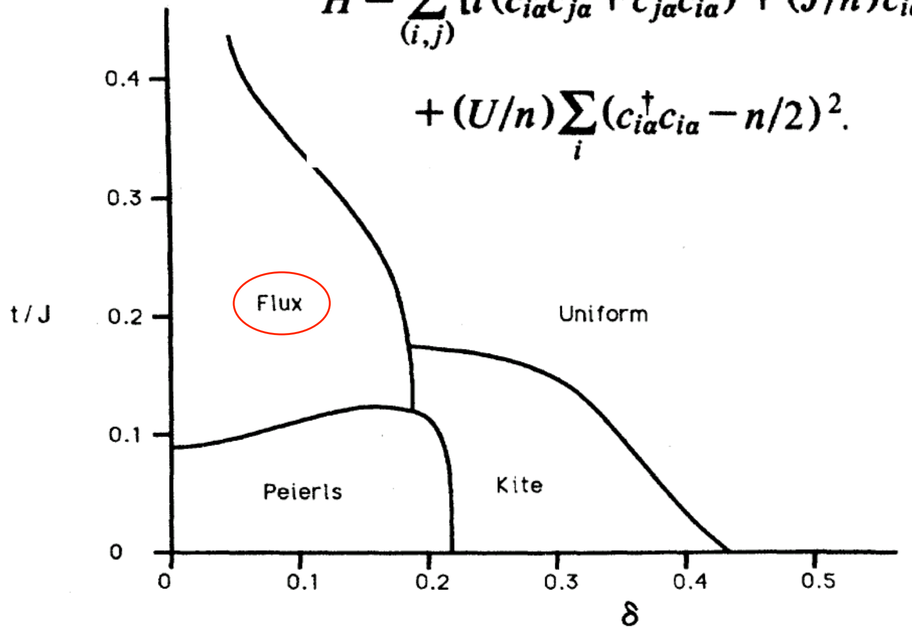
$$\Phi_{SF} = \sum_{\vec{k},m} (\cos k_x - \cos k_y) \langle c^{\dagger}_{\vec{k},m} c_{\vec{k}+\vec{Q},m} \rangle$$

Honerkamp, Hofstetter 2004

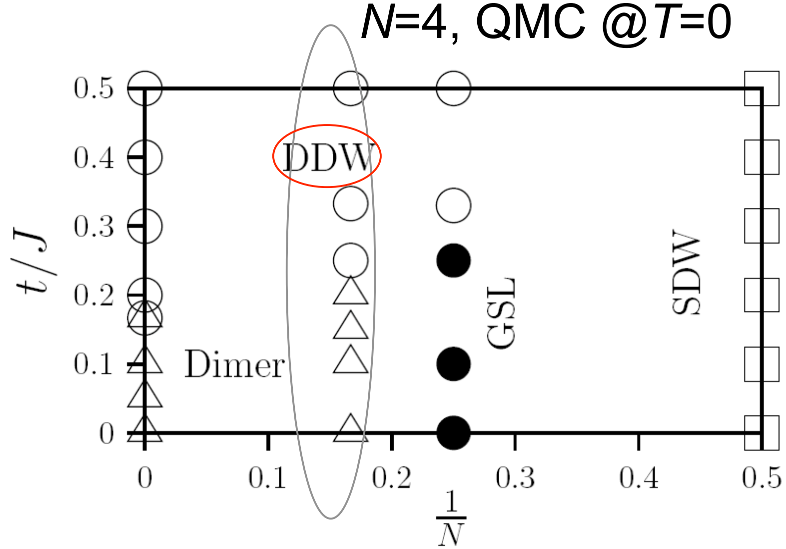
SU(N) Hubbard-Heisenberg model

Marston& Affleck 1988: $N \rightarrow \infty$

$$H = \sum_{(i,j)} [t(c^\dagger_{i\alpha} c_{j\alpha} + c^\dagger_{j\alpha} c_{i\alpha}) + (J/n) c^\dagger_{i\alpha} c_{i\beta} c^\dagger_{j\beta} c_{j\alpha}] + (U/n) \sum_i (c^\dagger_{i\alpha} c_{i\alpha} - n/2)^2.$$

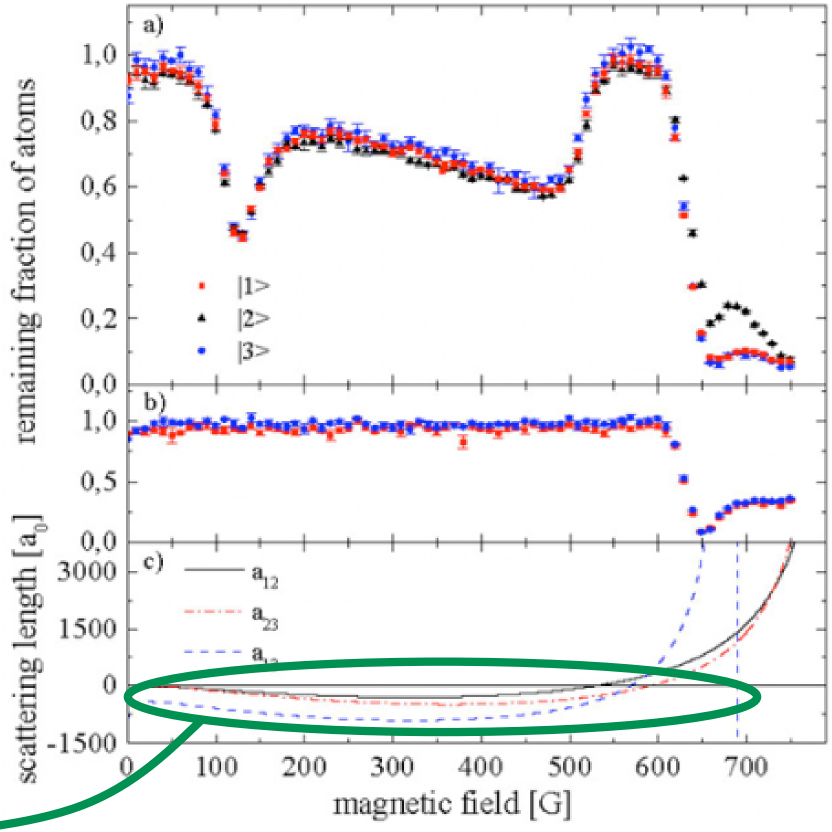


N=4, QMC @T=0



Assaad 2004, SU(4): DDW for $J<J_c$

Attractive SU(3) model: Pairing (and more) with 3 colors



Negative scattering lenghts!

^{6}Li: MPI-HD group

Pairing with 3 colors

Functional RG: *s*-wave Cooper pairing instability (off half filling)

Mean-field theory → decouple interaction in ***s*-wave even parity Cooper-channel** with onsite-pairing order parameter $\Delta_{\alpha\beta}$

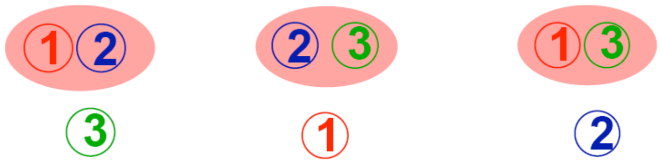
$$H_{U,\mathrm{mf.}} = -\frac{1}{2} \sum_{\vec{k},\alpha,\beta} c^\dagger_{\vec{k}\alpha} c^\dagger_{-\vec{k}\beta} \Delta_{\beta\alpha} + h.c.$$

$$\Delta_{\alpha\beta} = -U \sum_{\vec{k}} \langle c_{\vec{k}\alpha} c_{-\vec{k}\beta} \rangle = -\Delta_{\beta\alpha}$$

Even parity order parameter has 3 components:
Δ_{12}, Δ_{13}, Δ_{23}

What happens for 3 colors?

Do only 2 colors form condensate, or all 3?



$\bar{\Delta}_{12} \neq 0$	$\bar{\Delta}_{12} = 0$	$\bar{\Delta}_{12} = 0$
$\bar{\Delta}_{23} = 0$	$\bar{\Delta}_{23} \neq 0$	$\bar{\Delta}_{23} = 0$
$\bar{\Delta}_{13} = 0$	$\bar{\Delta}_{13} = 0$	$\bar{\Delta}_{13} \neq 0$

BCS for 3 colors, with $U_{12}= U_{13}= U_{23} < 0$

$$\Delta_{\alpha\beta} = -U \sum_{\vec{k}} \langle c_{\vec{k}\alpha} c_{-\vec{k}\beta} \rangle = -\Delta_{\beta\alpha}$$

Take SU(3)-transf. U (3D fundamental repres. of SU(3))

(1 2) 3	(2 3) 1	(1 3) 2
$\Delta_{12} \neq 0$	$\Delta_{12} = 0$	$\Delta_{12} = 0$
$\Delta_{23} = 0$	$\Delta_{23} \neq 0$	$\Delta_{23} = 0$
$\Delta_{13} = 0$	$\Delta_{13} = 0$	$\Delta_{13} \neq 0$

$$\tilde{c}_m = U_{m,m'} c_{m'} \qquad \longrightarrow \qquad \tilde{\Delta}_{ml} = U_{m,m'} U_{l,l'} \Delta_{m'l'}$$

Decompose product of 3dim representations

$$3 \otimes 3 = \bar{3} \oplus 6 \qquad \longrightarrow \qquad \tilde{D}_m = U^*_{m,m'} D_{m'}\ , \quad D = \begin{pmatrix} \Delta_{23} \\ -\Delta_{13} \\ \Delta_{12} \end{pmatrix}$$

→ **even parity order parameter transforms acc. to 3D representation**
no singlet! (cf. color-QCD: pions are quark-antiquark pairs)

→ **large ground state degeneracy, fulfilling** $\Delta_0^2 = \sum_{\alpha\beta} |\Delta_{\alpha\beta}|^2$

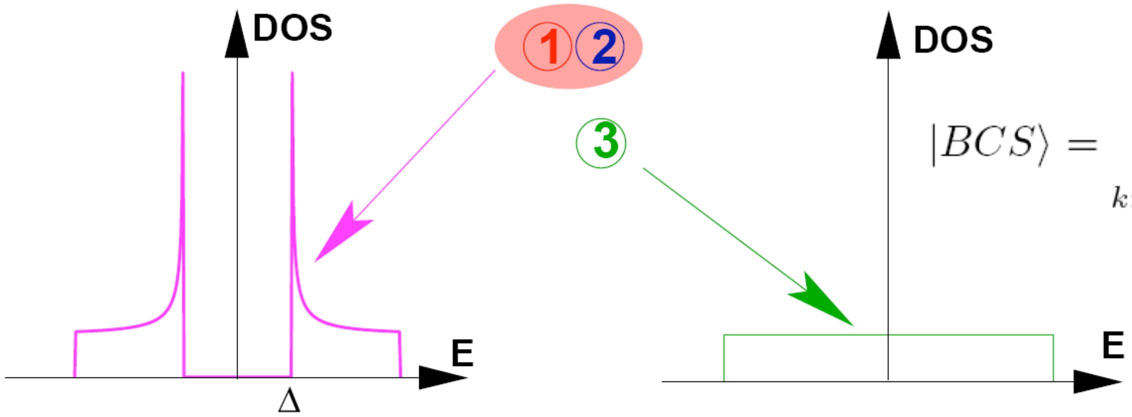
→ can always rotate onto (1,0,0), i.e. $\mathbf{\Delta_{12}=\Delta_0}$, $\mathbf{\Delta_{13}=\Delta_{23}=0}$!

Gapless superfluid

$$\Delta_{12} = \Delta_0 \quad \text{and} \quad \Delta_{13} = \Delta_{23} = 0$$

Single-particle excitations:

- Flavors 1 and 2 have gap, flavor 3 is gapless (→ 2-fluid model)
- Coexistence of pairing with **large Fermi surface**



$$|BCS\rangle = \prod_{k:\epsilon_k<\epsilon_F} c^+_{k3} \prod_{k'} (u_{k'} + v_{k'} c^+_{k'1} c^+_{-k'2})|0\rangle$$

Collective excitations:

Mean field solutions for the ground state with N=3: degeneracy of gap functions with fixed

$$\Delta_0^2 = \sum_{\alpha\beta} |\Delta_{\alpha\beta}|^2$$

5 Goldstone modes!
SU(2) ⊗ U(1) unbroken

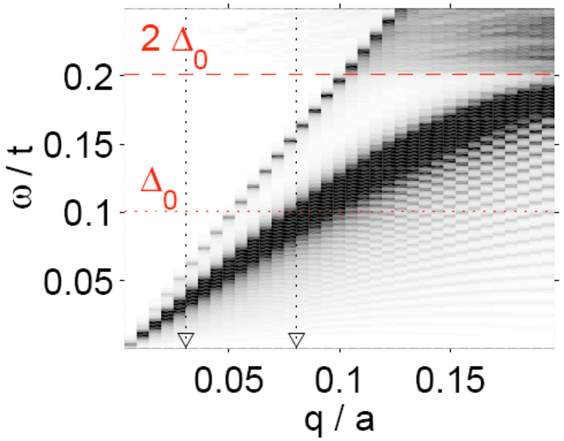
Experimental signatures

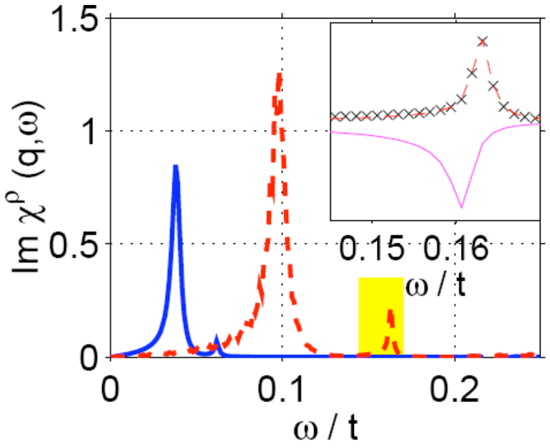
Signatures for color superfluid in density response $S(q,\omega)$:

- damping of phase mode for all frequencies due to gapless branch
- additional color mode above particle-hole continuum

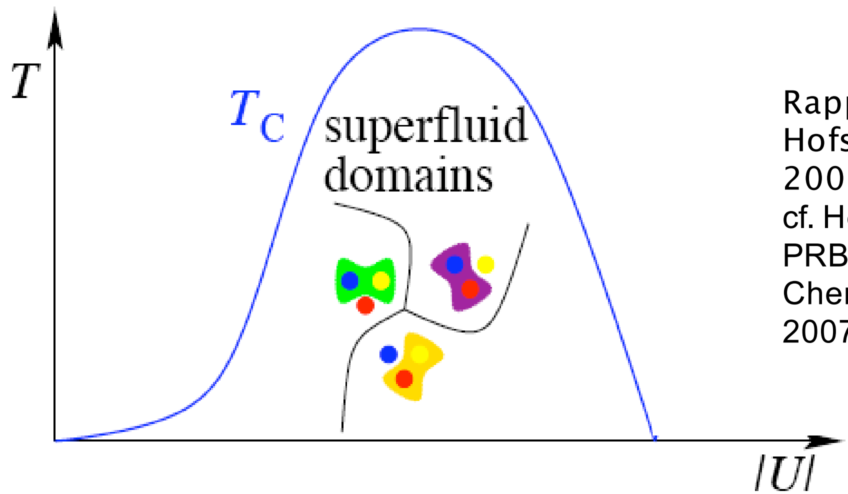
CH, Hofstetter PRB 2004

RPA for Im $S(q,\omega)$ in color superfluid:



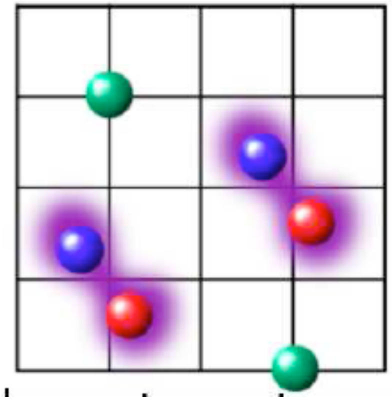


Phase separation and domain formation (light absorption?)

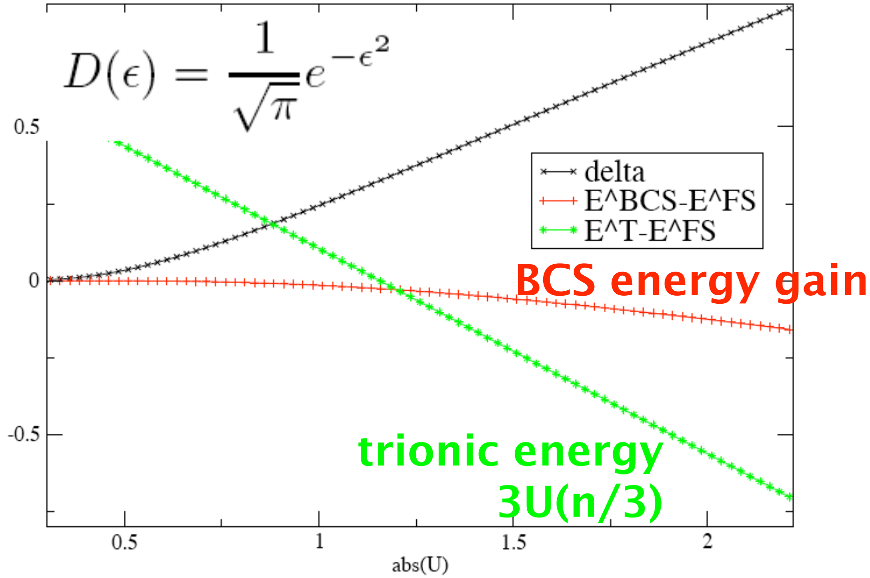


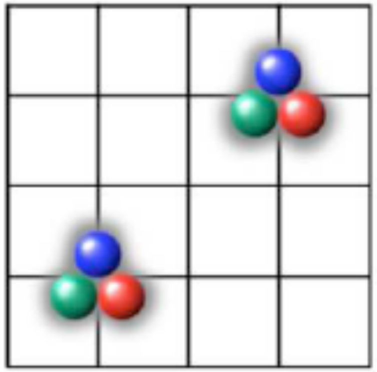
Rapp, Zarand, CH, Hofstetter PRL 2007
cf. He, Jin & Zhuang PRB 2006
Cherng et al., PRL 2007

From weak to strong attraction



Weak coupling:
Cooper pairing,
color superfluid





Strong coupling: 3 colors form **'trions'**

How to describe the transition?

Variational treatment for strong coupling

- Start with BCS paired state

$$|BCS\rangle = \prod_{k:\epsilon_k<\epsilon_F} c_{k3}^{+} \prod_{k'} (u_{k'} + v_{k'} c_{k'1}^{+} c_{-k'2}^{+})|0\rangle$$

- triple occupancy operator for site l

$$t_l = n_{l1} n_{l2} n_{l3}$$

- parameter *g* measures trionic component in wave function

$$\boxed{|G\rangle = \prod_l (1-(1-g)t_l)|BCS\rangle}$$

- Minimize energy with respect to Δ, *g* (and n_3)!

Rapp, Zarand, CH, Hofstetter PRL 2007

Evaluation of expectation values

- Rewrite expectation values as (equal time) functional integrals, e.g.

$$\langle \Psi_G | \Psi_G \rangle = \left\langle \Psi_0 \left| \prod_l (1-(1-g)t_l) \prod_{l'} (1-(1-g)t_{l'}) \right| \Psi_0 \right\rangle$$

$$\langle \Psi_G | \Psi_G \rangle = \int \mathcal{D}c\mathcal{D}\bar{c} \sum_m \frac{(g^2-1)^m}{m!} \sum_{i1 \neq i2 \neq \ldots \neq im} t_{i1} t_{i2} \ldots t_{im} \; e^{-\mathcal{S}_0}$$

given by
BCS (g=1)

- Full 'action' (no dynamics)

$$-\mathcal{S} = \frac{1}{2} \sum_{rr'} \bar{\Psi}_r M_{rr'} \Psi_{r'} + \frac{g^2-1}{48} \sum_r (\bar{\Psi}_r \tau_3 \Psi_r)^3$$

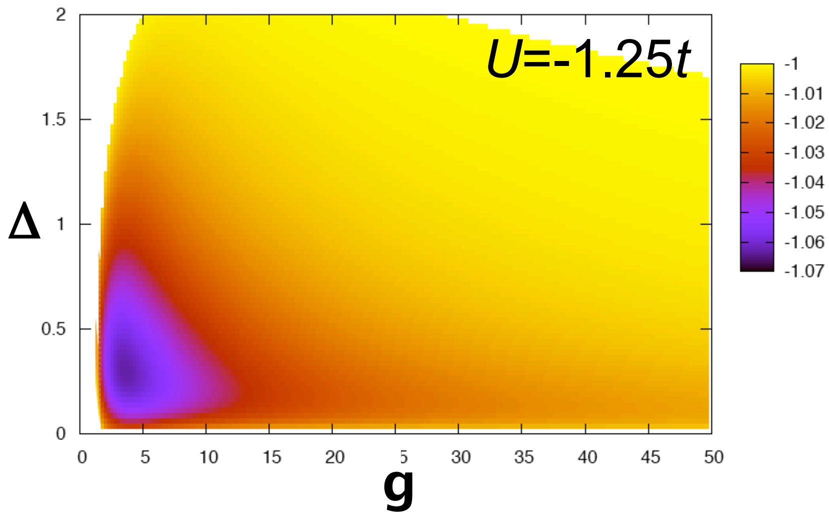
r = color & space index

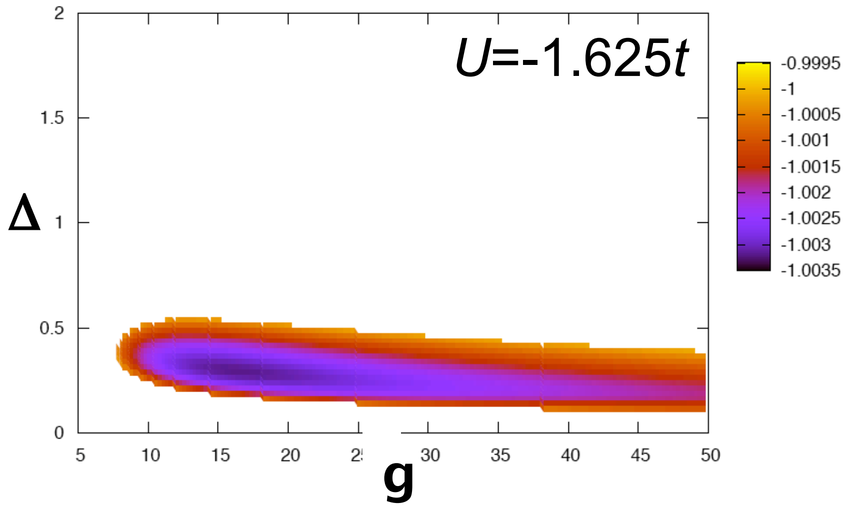
- Assume infinite dimensions: $\Sigma(k) = \Sigma$
- Local problem can be solved analytically, embedding à la DMFT

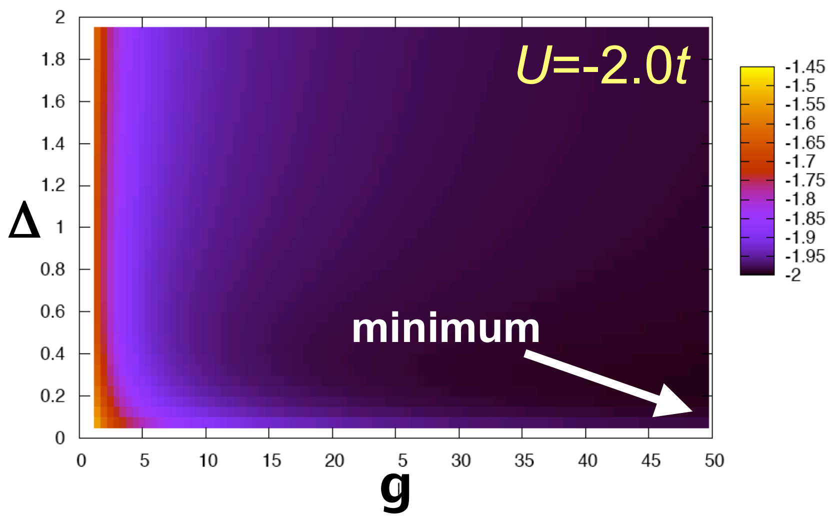
$$\mathcal{D} = \frac{\mathcal{D}_0}{1 + u\sqrt{-\det \mathcal{D}_0}} = \sum_k \left[D^{0-1}(k) - \Sigma(k) \right]^{-1}$$

Rapp, Zarand, CH, Hofstetter PRL 2007
Rapp, Zarand, Hofstetter PRB 2008

Ground state energy minimum







Optimal g diverges for $U > U_c$,
Δ vanishes at U_c:

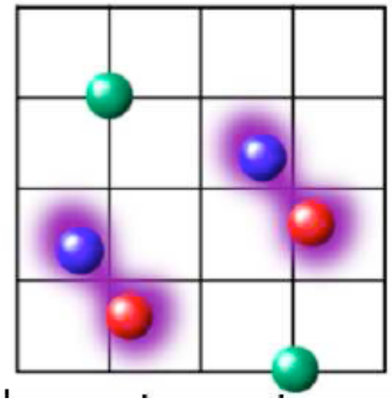
$$|G\rangle = \prod_l (1-(1-g)t_l)|BCS\rangle$$

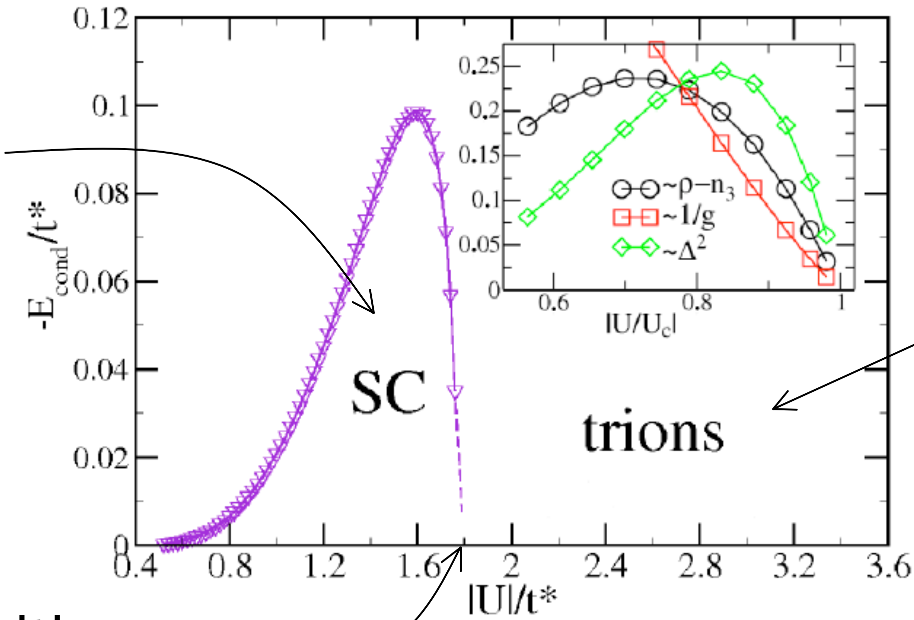
wave function becomes superposition of trions

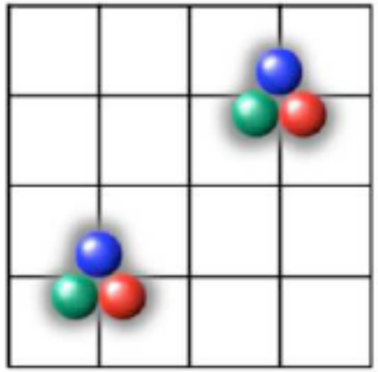
Rapp, Zarand, CH, Hofstetter PRL 2007
Rapp, Hofstetter, Zarand, PRB 2008

From Cooper pairs to trions

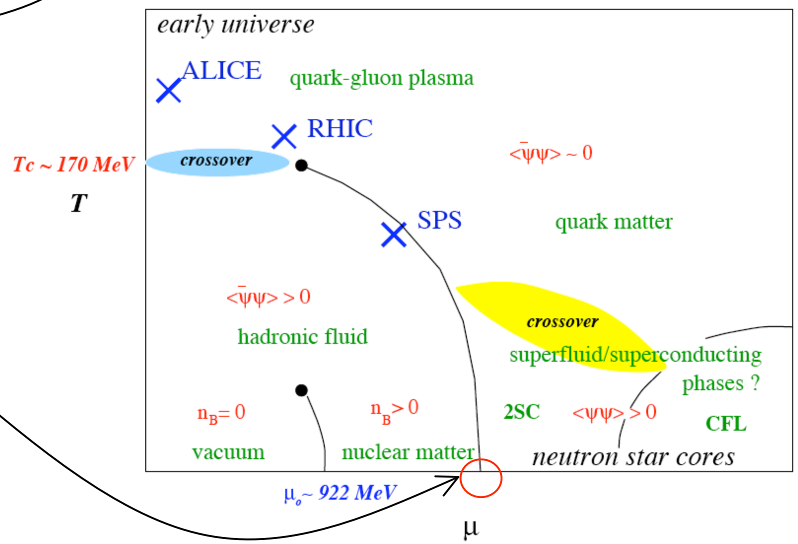
Rapp, Zarand, CH, Hofstetter PRL 2007







2nd order transition
(symmetry breaking -
replacing BCS/BEC crossover
of SU(2)-case)



Hands 2001

QCD phase transition
between baryonic matter
(color singlets) and color
superfluid

physicsweb

Physics news, jobs and resources

Optical lattices could give insight into QCD

27 April 2007

Physicists in Germany and Hungary claim that ultracold atoms in an optical lattice could be used to simulate certain aspects of quantum chromodynamics (QCD) -- the theory

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QUANTUM CHROMODYNAMICS

Lifestyles of the small and simple

Ultracold atoms in optical lattices are already used to simulate complex solid-state phenomena. But could the same platform also give us a better grasp of how quarks group together?

nature physics | VOL 3 | JUNE 2007 | www.nature.com/naturephysics

Frank Wilczek
is in the Department of Physics, Massachusetts Institute of Technology, 5 Cambridge Center, Cambridge, Massachusetts 02139, USA.
e-mail: wilczek@mit.edu

Quarks find other quarks — with different flavours and colours — attractive. They can choose either to form tight nuclear families, so-called baryons, or to pair up weakly with a large number of mates, in a phenomenon known as colour superconductivity. Theoretically, we expect the former behaviour to occur at low density, as in ordinary matter, and the latter at high density, as might be reached under enormous pressure inside neutron stars. Ákos Rapp and colleagues propose a terrestrial analogue, with cold atoms standing in for quarks, that should be much more accessible to experimental exploration (*Phys. Rev. Lett.* **98,** 160405; 2007).

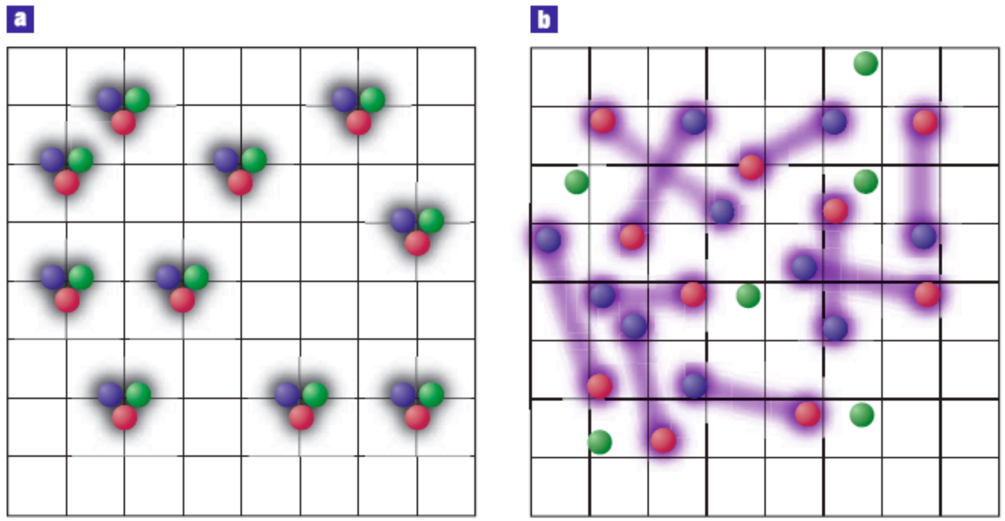


Figure 1 Cold fermionic atoms in an optical lattice might behave similarly to quarks. Three atoms with different internal quantum numbers (represented here by different colours) can, depending on the strength of the interactions between them, form either **a**, bound units of 'trions' or **b**, some form of Cooper pairs. The pairs can be extended and overlapping, and partner-swapping is common.

Incomplete account of other theoretical work …

Many interesting aspects ... rich new field of many-particle physics

PHYSICAL REVIEW A **73**, 053606 (2006)

Pairing in a three-component Fermi gas

T. Paananen,[1,2,*] J.-P. Martikainen,[1] and P. Törmä[2]

PHYSICAL REVIEW A **75**, 031603(R) (2007)

Superfluidity in three-species mixtures of Fermi gases across Feshbach resonances

Hui Zhai

PRL **99,** 130406 (2007) PHYSICAL REVIEW LETTERS week ending 28 SEPTEMBER 2007

Superfluidity and Magnetism in Multicomponent Ultracold Fermions

R. W. Cherng,[1] G. Refael,[2] and E. Demler[1]

PRL **100,** 200401 (2008) PHYSICAL REVIEW LETTERS week ending 23 MAY 2008

Magnetic Phase Transitions in One-Dimensional Strongly Attractive Three-Component Ultracold Fermions

X. W. Guan,[1] M. T. Batchelor,[1,2] C. Lee,[3] and H.-Q. Zhou[4]

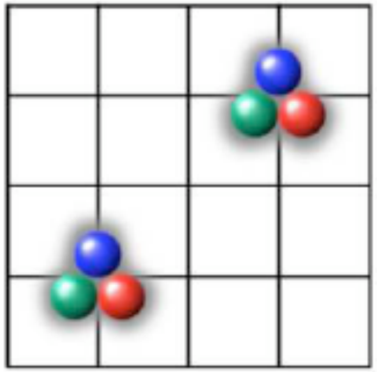
Functional renormalization for trion formation in ultracold fermion gases

S. Floerchinger, R. Schmidt, S. Moroz, and C. Wetterich
Institut für Theoretische Physik
Universität Heidelberg

Crystallization of trions in SU(3) cold atom gases trapped in optical lattices

Rafael A. Molina[1,2], Jorge Dukelsky[1], and Peter Schmitteckert[3]

Trion liquid: a strongly correlated state



Learn more about the trion liquid
from **exact diagonalization**!

(→ small systems, no transitions ...)

When is the ground state composed out of trions?

$$t_j^\dagger = c_{j1}^\dagger \, c_{j2}^\dagger \, c_{j3}^\dagger$$

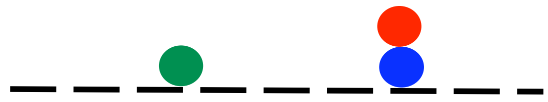
$$|\text{Ground state}\rangle \;=\; \sum_i \varphi_i \, c_{i,1}^\dagger c_{i,3}^\dagger c_{i,3}^\dagger |0\rangle \;+\; \text{non-trionic contribs.}$$

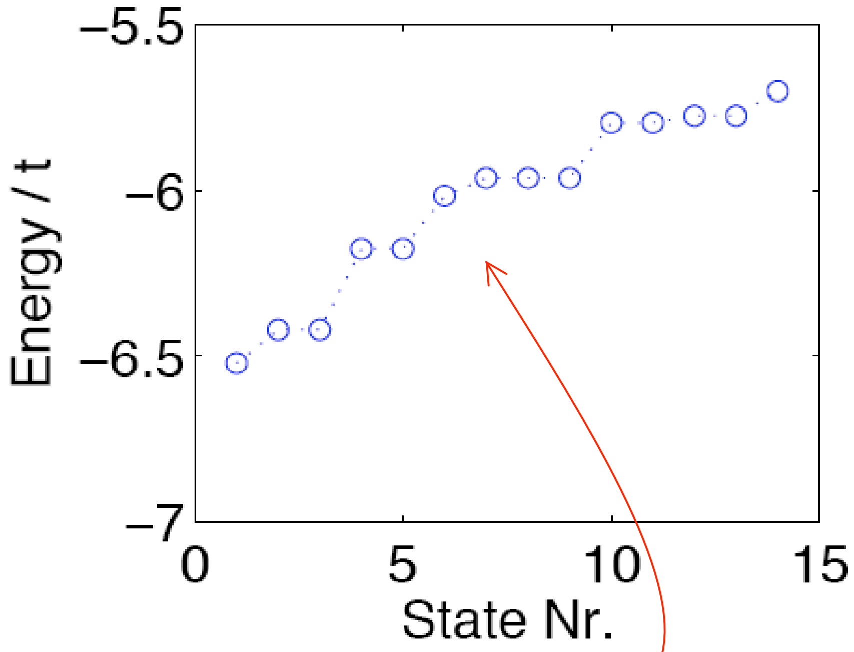
$$\text{trionic weight} \quad w_t = \sum_i |\varphi_i|^2$$

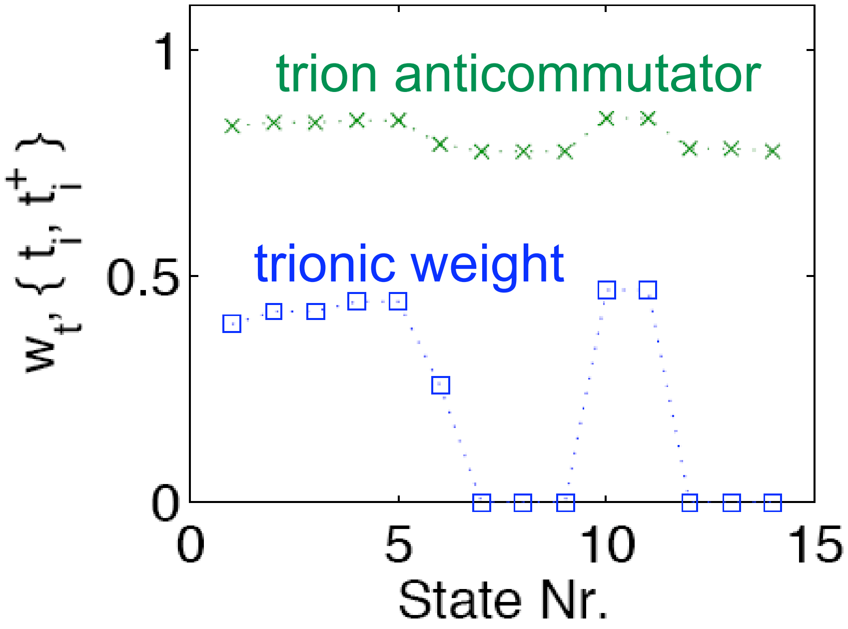
Properties of the trion liquid? Effective trion Hamiltonian?

Weak coupling: no trions

- Short chain (12 sites, PBC) at weak $U = -t$
- One fermion/color on 12 sites







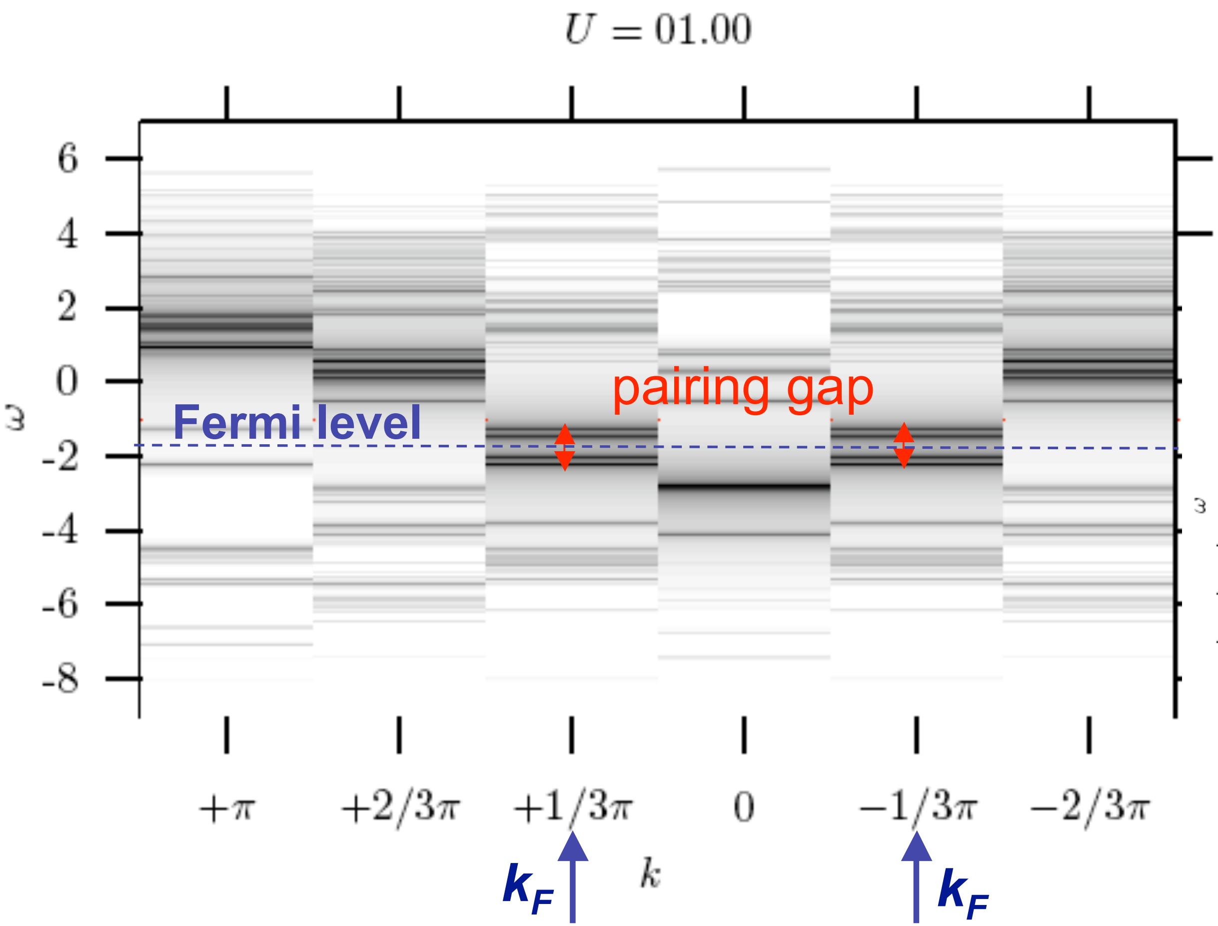
Lowest many-particle states

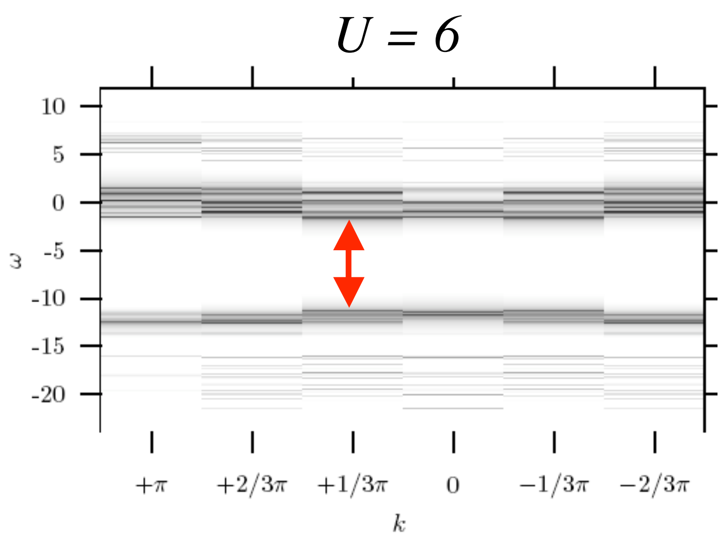
Trionic weight w_t: contribution of trionic basis states in many-particle state

Weak coupling: BCS spectrum

Single fermion spectral function A(*k*,ω), 1/3 filling:

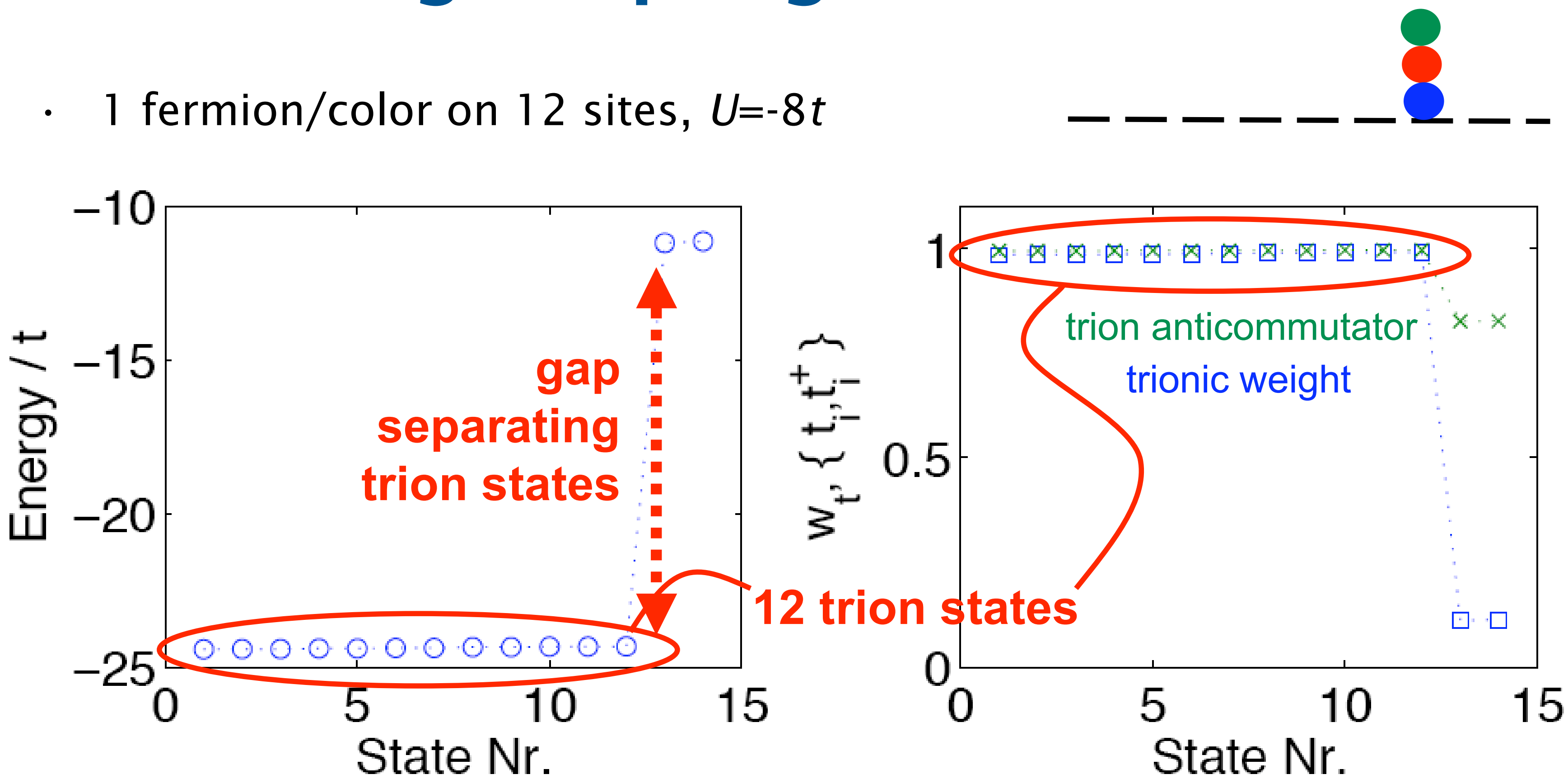
- Small pairing gap at Fermi level
- Gap grows continuously with *U:* no signature of trion formation





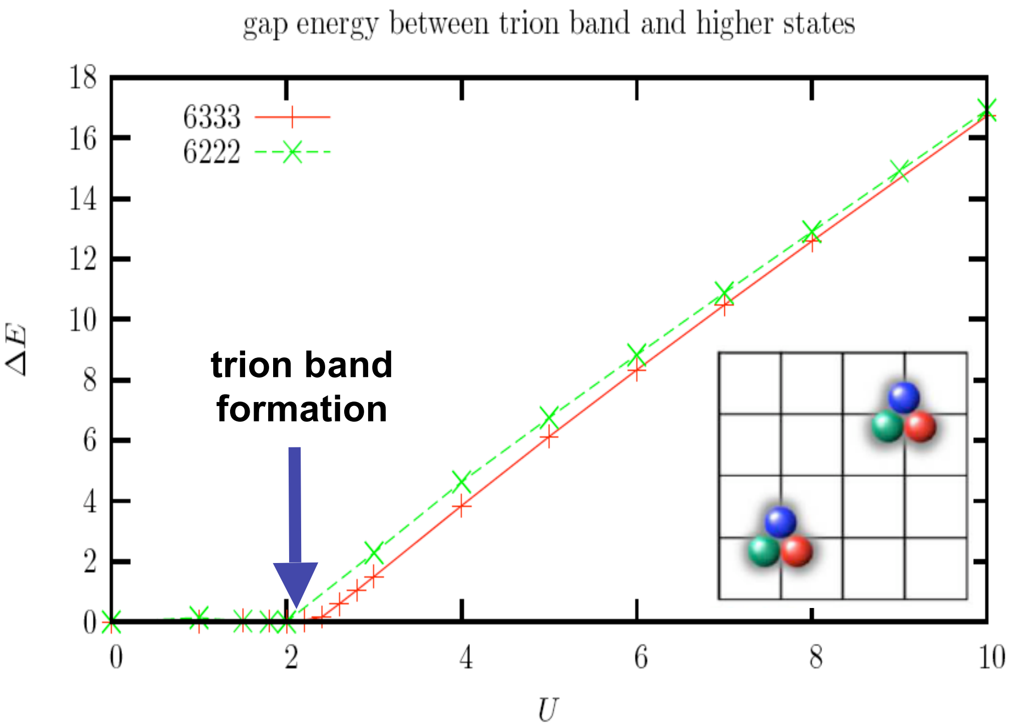
Strong coupling: trion band

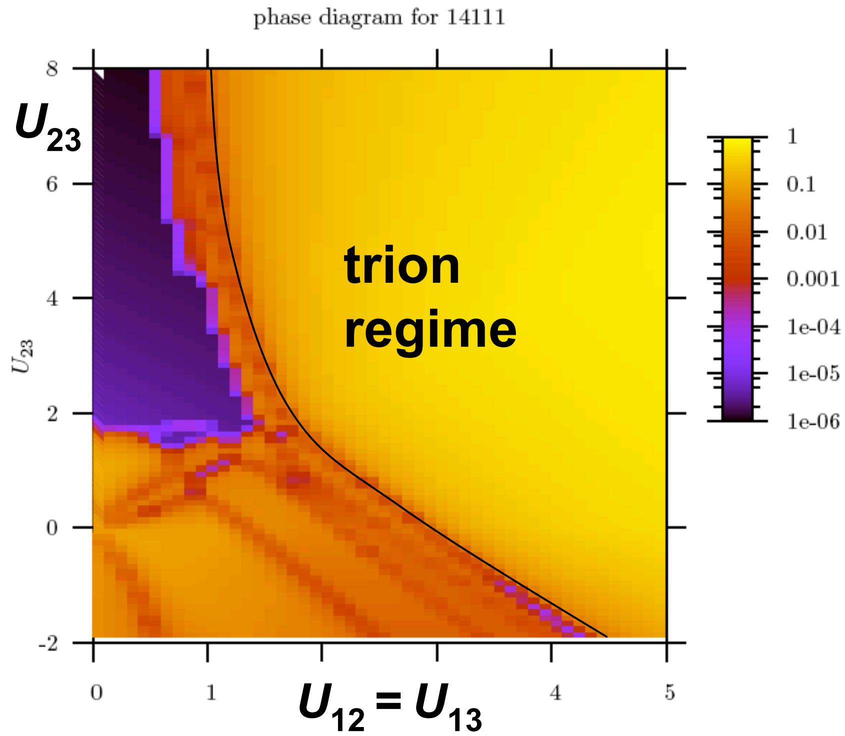
- 1 fermion/color on 12 sites, U=-8t



12 lowest states form trionic band
- high trionic weight
- trion anticommutator almost 1: effective fermionic particles!

Where´s the trionic crossover?

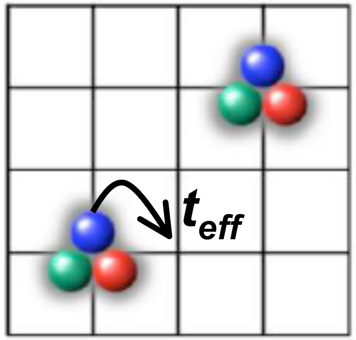




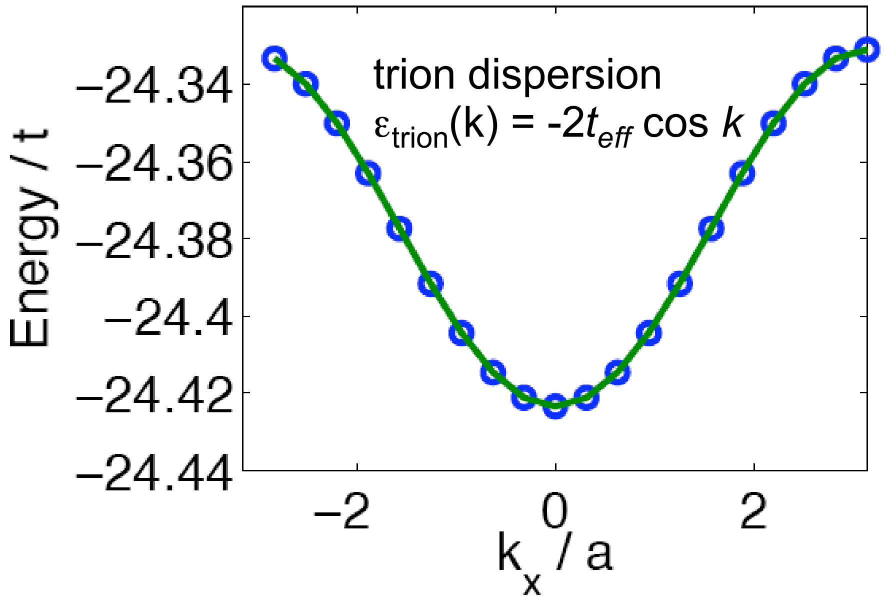
- Gap opens at $U \approx 2t$, i.e. at 2 × attraction ≈ bandwidth, increases linearly with slope ≈ $2U$
- Consistent with variational treatment of Rapp et al.
- Trionic regime also for non-symmetric interactions

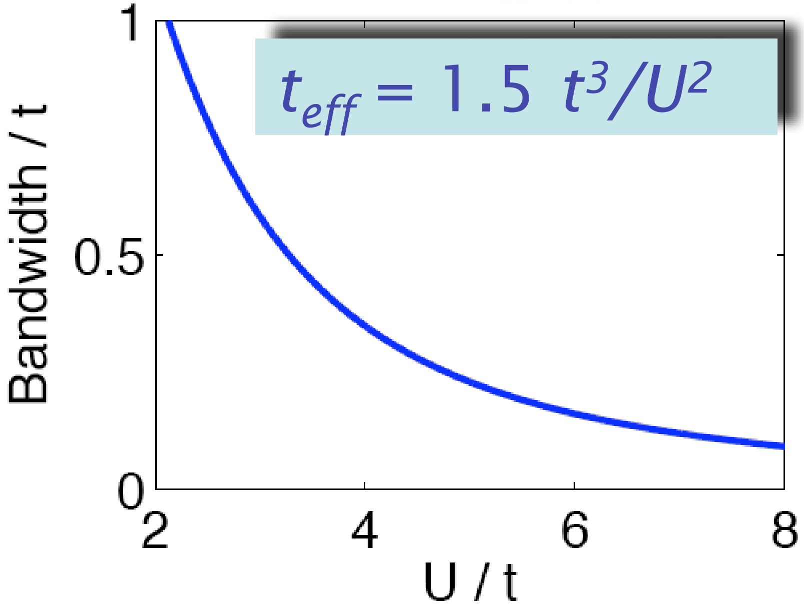
Effective trion Hamiltonian?

Strong coupling: trion band



- 1 fermion/color on 12 sites





Lowest states form trionic band with trion hopping

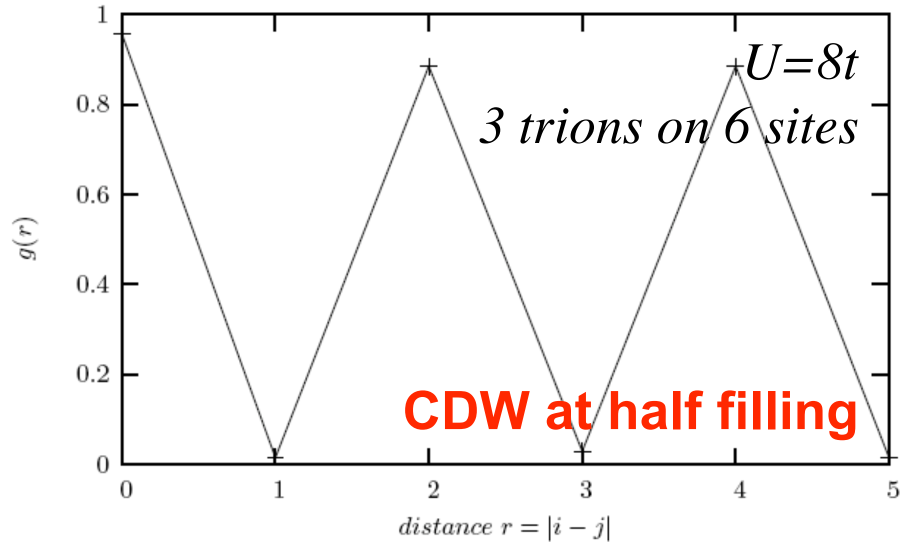
$t_{eff} \propto t^3/U^2$ (cf. Toke & Hofstetter: t/U-expansion)

→ Hopping via excited states with broken-up trions

→Trions form heavy Fermi liquid

Spatial trion correlations

g(r) = Probability to find trion at *r* if one trion is at *r*=0

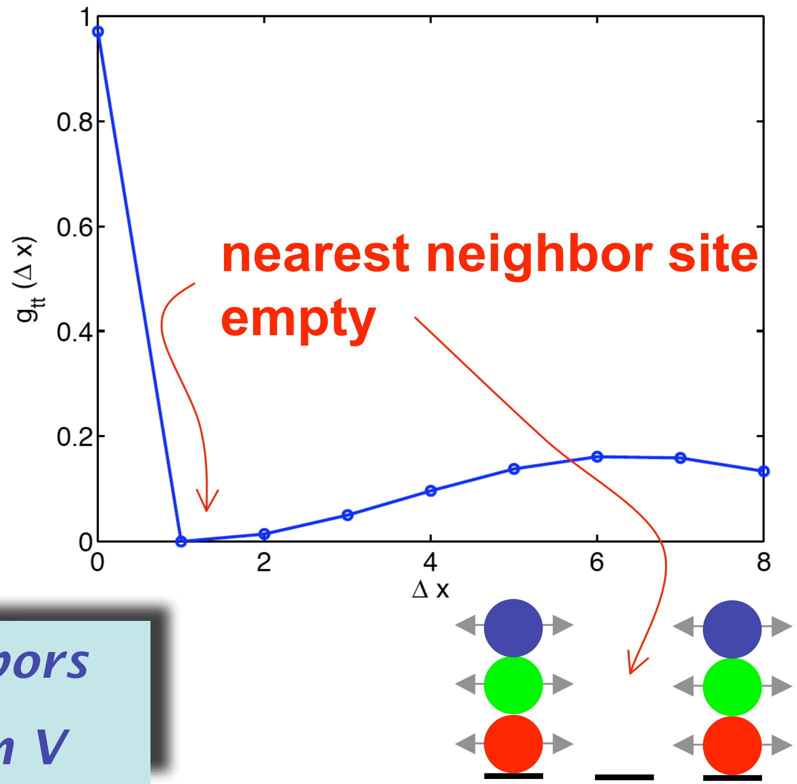


1/2-filling:

- CDW ground state (cf. Molina et al. 2008, DMRG)
- CDW gap also visible in trionic spectral function

1/8-filling:

- no CDW
- $g(1) \approx 0$, smaller than for noninteracting spinless fermions

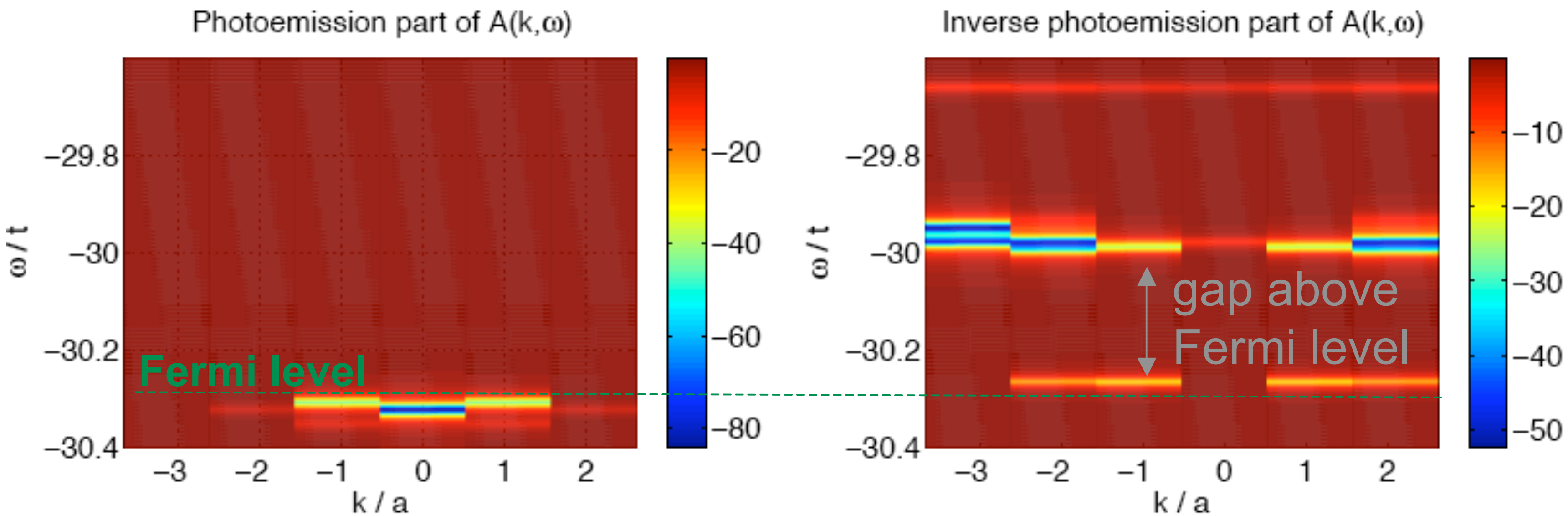


→ ***Trions avoid being nearest neighbors***

→ ***Strong nearest neighbor repulsion V***

Strongly correlated trion liquid

Close look at trionic spectral at 1/3 filling (2 trions on 6 sites), U=10t



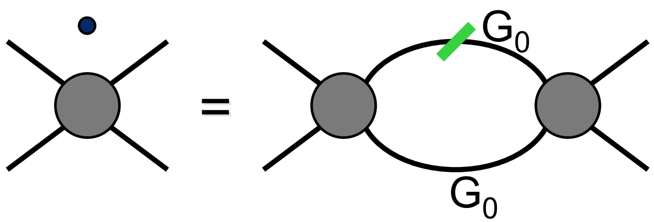
- Trionic band divided by CDW gap above Fermi level at k = $\pm\pi/2$
- Dispersive parts have width t^3/U^2, CDW gap scales with t^2/U
- **Trions interact strongly by nearest neighbor interactions**
- Comparison with spinless fermions gives $V_{eff} = 2\, t^2/|U|$

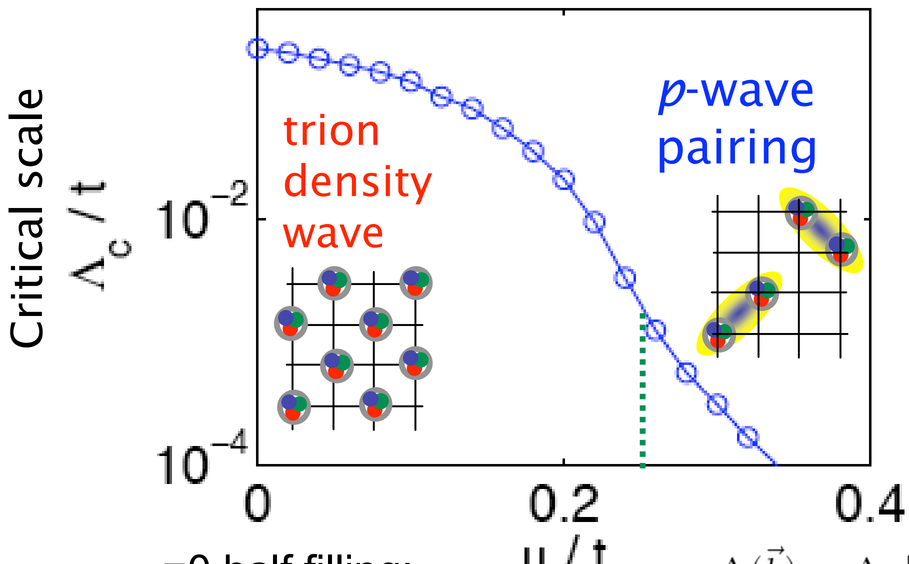
$$\mathcal{H}_{trion} = -t_{eff} \sum_{<i,j>} t_i^\dagger t_j + V_{eff} \sum_{<i,j>} n_i n_j$$

Instabilities of trionic liqiuid

Functional renormalization group for trions (spinless fermions) and moderate V/t on 2D square lattice

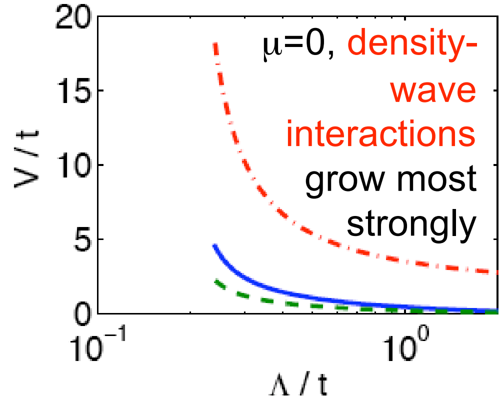
- Critical scale Λ_c highest for half filling
- Commensurate trion density wave near half filling
- p-wave pairing away from half filling

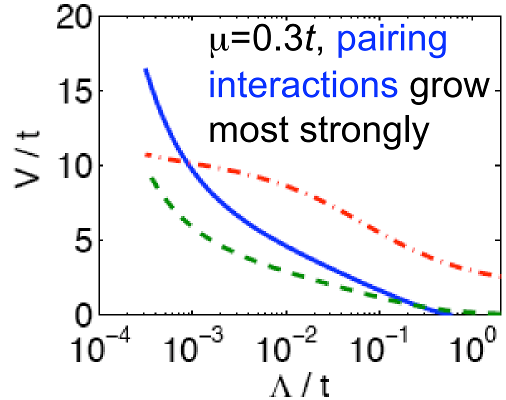




μ=0 half filling:
1/2 trion per site

$$\Delta(\vec{k}) = \Delta_0 \left[\sin(k_x + k_y) + i \sin(k_x - k_y)\right]$$

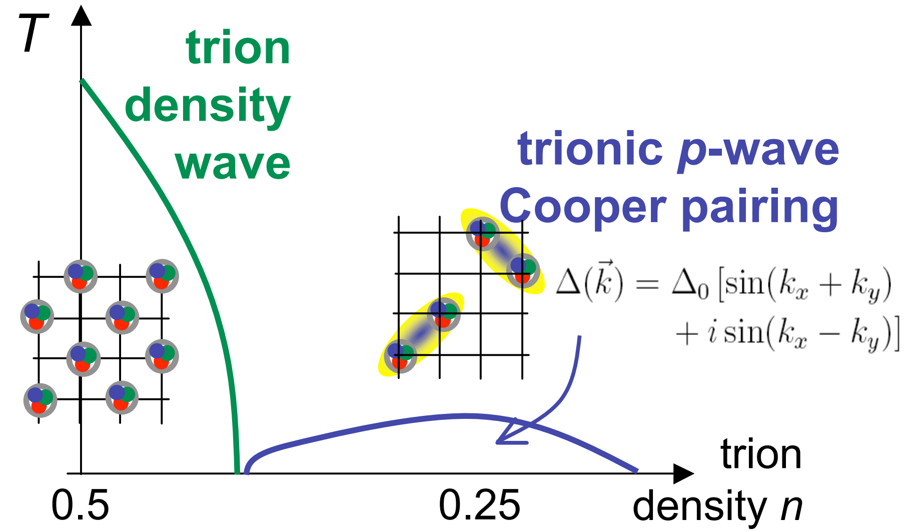


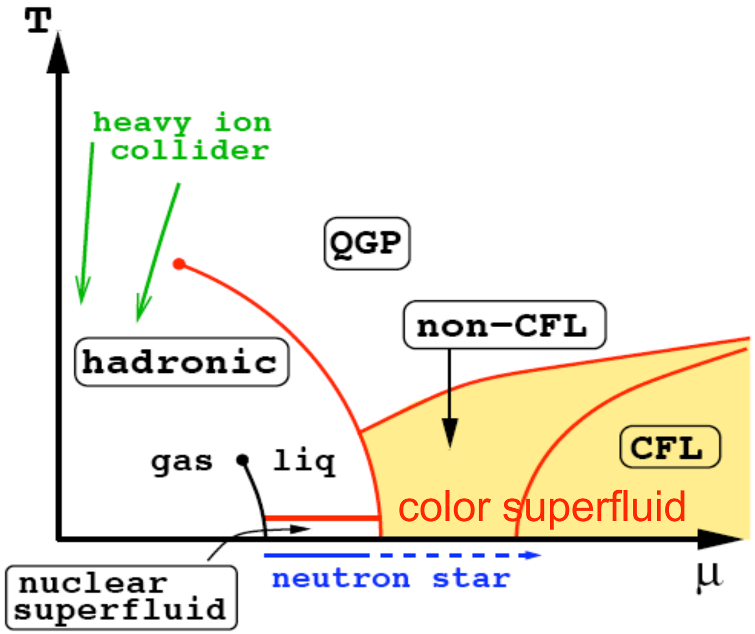


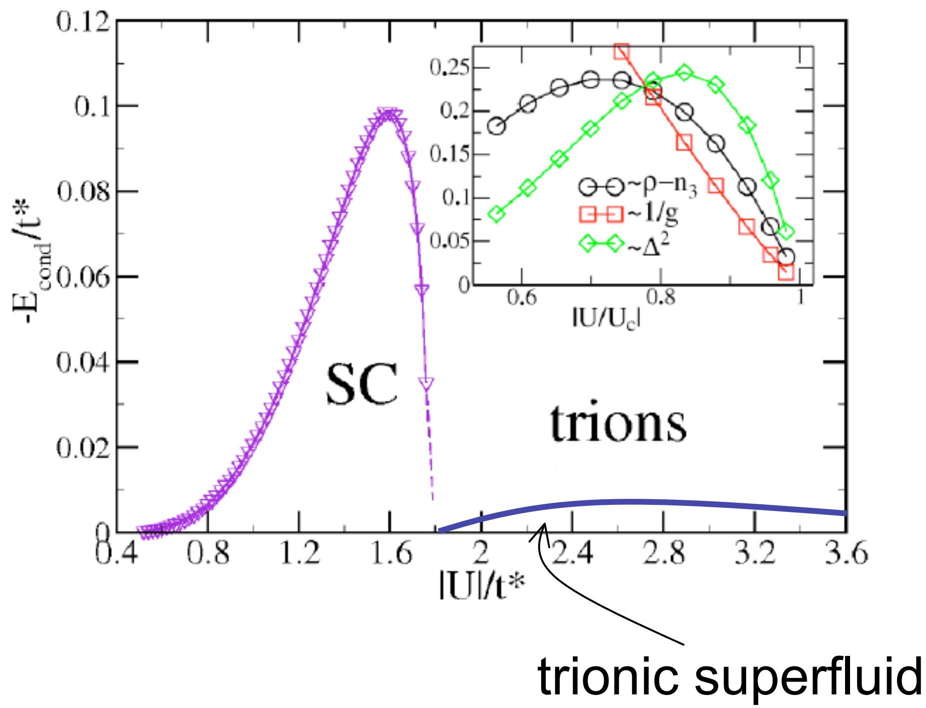
Trion phase diagram on square lattice

Effects of nearest neighbor repulsion V_{eff} in 2D:

- **trion density wave** near half filling
- ***p*-wave Cooper pairing** between diagonal neighbors away from density wave

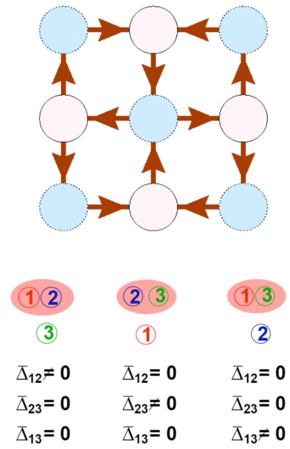




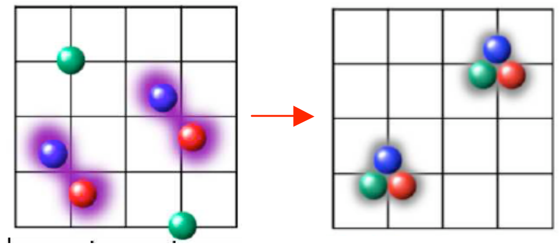


Conclusions

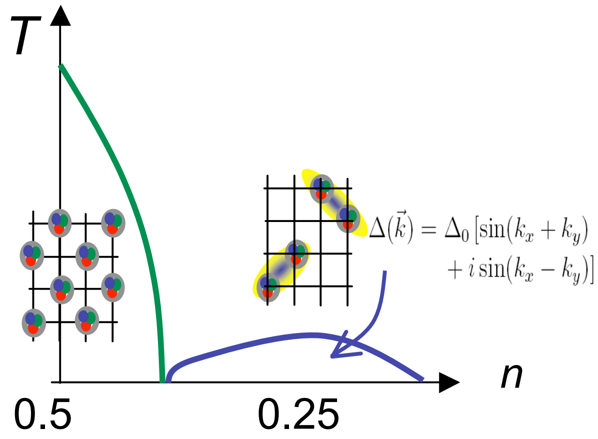
- Internal **hyperfine degree** of freedom of ultracold fermions may allow for new phases
  - color density waves
  - staggered flux phases
  - **color superfluids** with spontaneous polarization



- Attractive SU(3) models exhibits **superfluid-to-trion** transition resembling QCP phase diagram

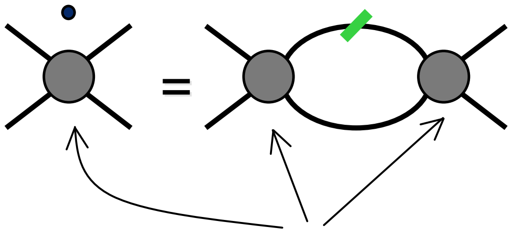


- Trions form strongly correlated heavy fermion liquid exhibiting
  - **trion density waves**
  - ***p*-wave superfluidity**

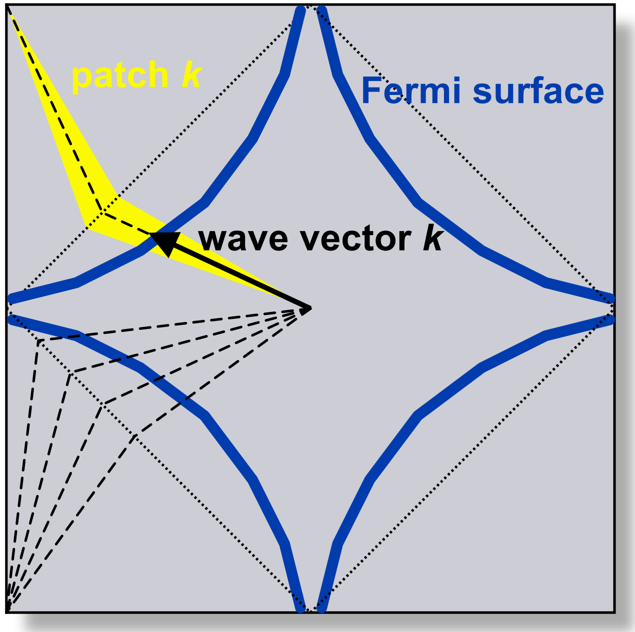


Thanks to Walter Hofstetter, Gergely Zarand, Akos Rapp, Guido Klingschat

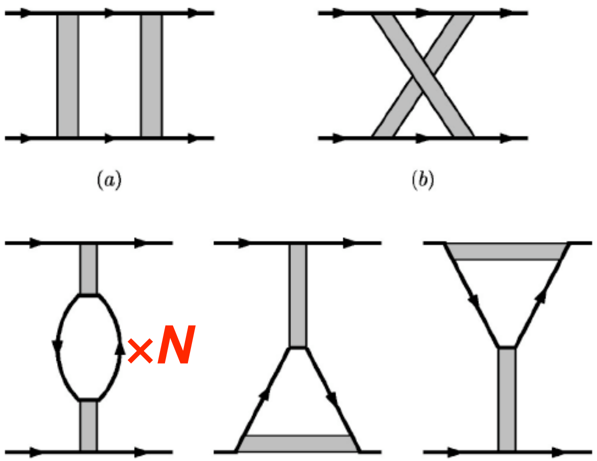
Implementation in 2D



- Coupling function $V(k_1,k_2,k_3)$ with incoming wavevectors k_1, k_2 and outgoing k_3 **on Fermi surface**
- Discretize: approximate $V(k_1,k_2,k_3)$ as **constant** for k_1, k_2 and k_3 **in same patch.**



Zanchi and Schulz 1997



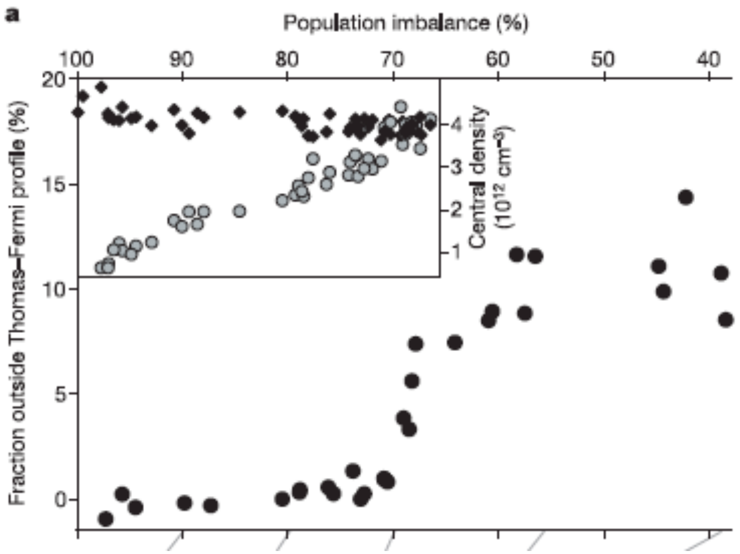
Ultracold fermions get cold enough & paired

VOLUME 92, NUMBER 4 PHYSICAL REVIEW LETTERS week ending 30 JANUARY 2004

Observation of Resonance Condensation of Fermionic Atom Pairs

C. A. Regal, M. Greiner, and D. S. Jin*
JILA, National Institute of Standards and Technology and University of Colorado, and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440, USA

(Received 13 January 2004; published 28 January 2004)



Vol 435|23 June 2005|doi:10.1038/nature03858 nature

Vortices and superfluidity in a strongly interacting Fermi gas

M. W. Zwierlein[1], J. R. Abo-Shaeer[1]†, A. Schirotzek[1], C. H. Schunck[1] & W. Ketterle[1]

nature Vol 442|6 July 2006

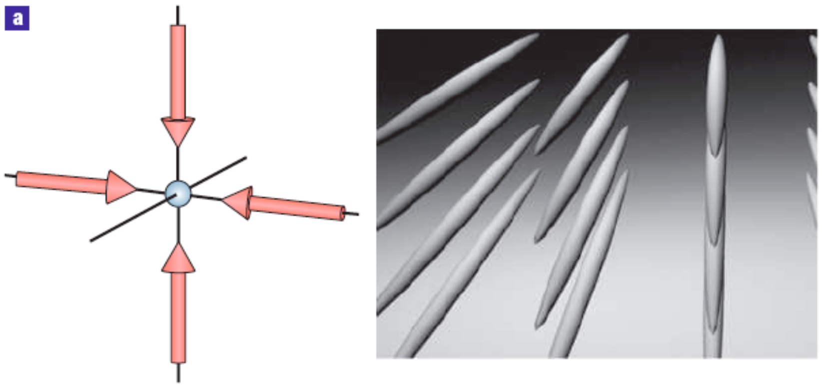
Direct observation of the superfluid phase transition in ultracold Fermi gases

Martin W. Zwierlein[1], Christian H. Schunck[1], André Schirotzek[1] & Wolfgang Ketterle[1]

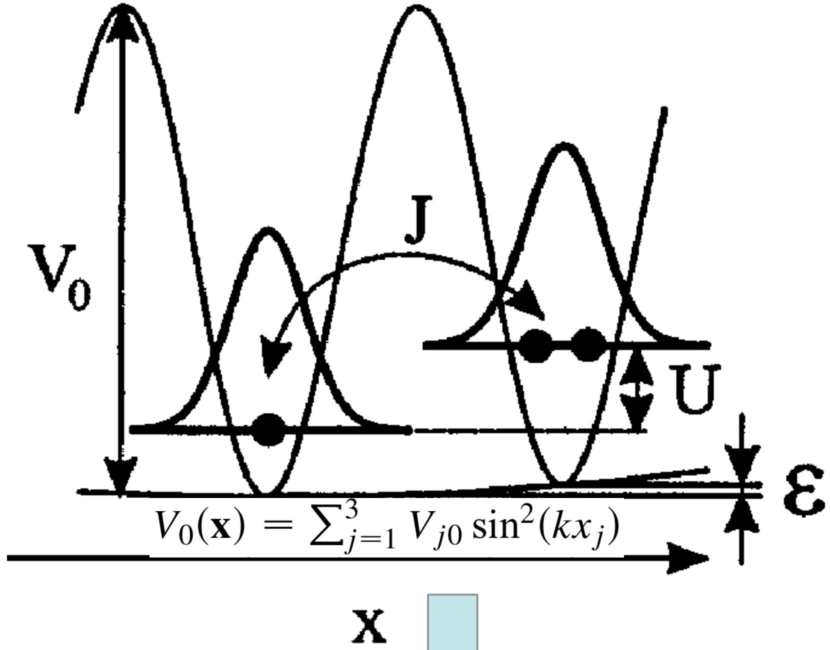
Cooper pairing and longer-range coherence in trapped ultracold fermionic gases experimentally established.
Quantum many-fermion physics open for exploration!

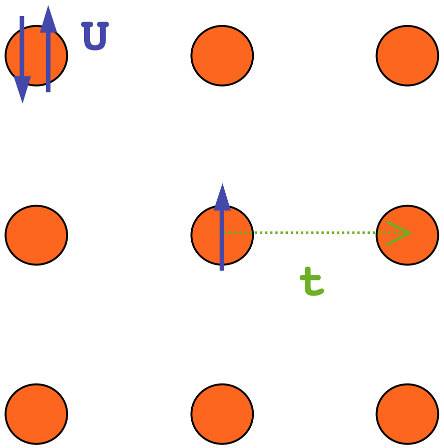
Optical lattices: Hubbard model

Load atomic gas into **optical lattice** made with **standing light waves**
$\rightarrow$ bosonic/fermionic **Hubbard model**



from Bloch, NPhys 2005





$$H = \int d^3x\, \psi^\dagger(\mathbf{x}) \left(-\frac{\hbar^2}{2m} \nabla^2 + V_0(\mathbf{x}) + V_T(\mathbf{x}) \right) \psi(\mathbf{x}) + \frac{1}{2} \frac{4\pi a_s \hbar^2}{m} \int d^3x\, \psi^\dagger(\mathbf{x}) \psi^\dagger(\mathbf{x}) \psi(\mathbf{x}) \psi(\mathbf{x}), \quad (1)$$

Jaksch et al. , PRL 1998