From BCS-BEC to quark-baryonic matter crossover in a boson-fermion Model

Qun Wang

University of Science and Technology of China

- Introduction
- CSC: from weak to strong couplings
- Boson-fermion model for BCS-BEC crossover
- Baryon formation in quark-diquark model
- Discussions and outlooks

• J. Deng, A. Schmitt, QW, Phys.Rev.D76:034013,2007
• J. Deng, J.-C. Wang, QW, Phys.Rev.D78:034014,2008
• J.-C. Wang, et al, in preparation

Where does QGP meet cold atoms

Nucleon pairings in nuclear shell structure ⟷ ***BCS pairings in Superconductivity***

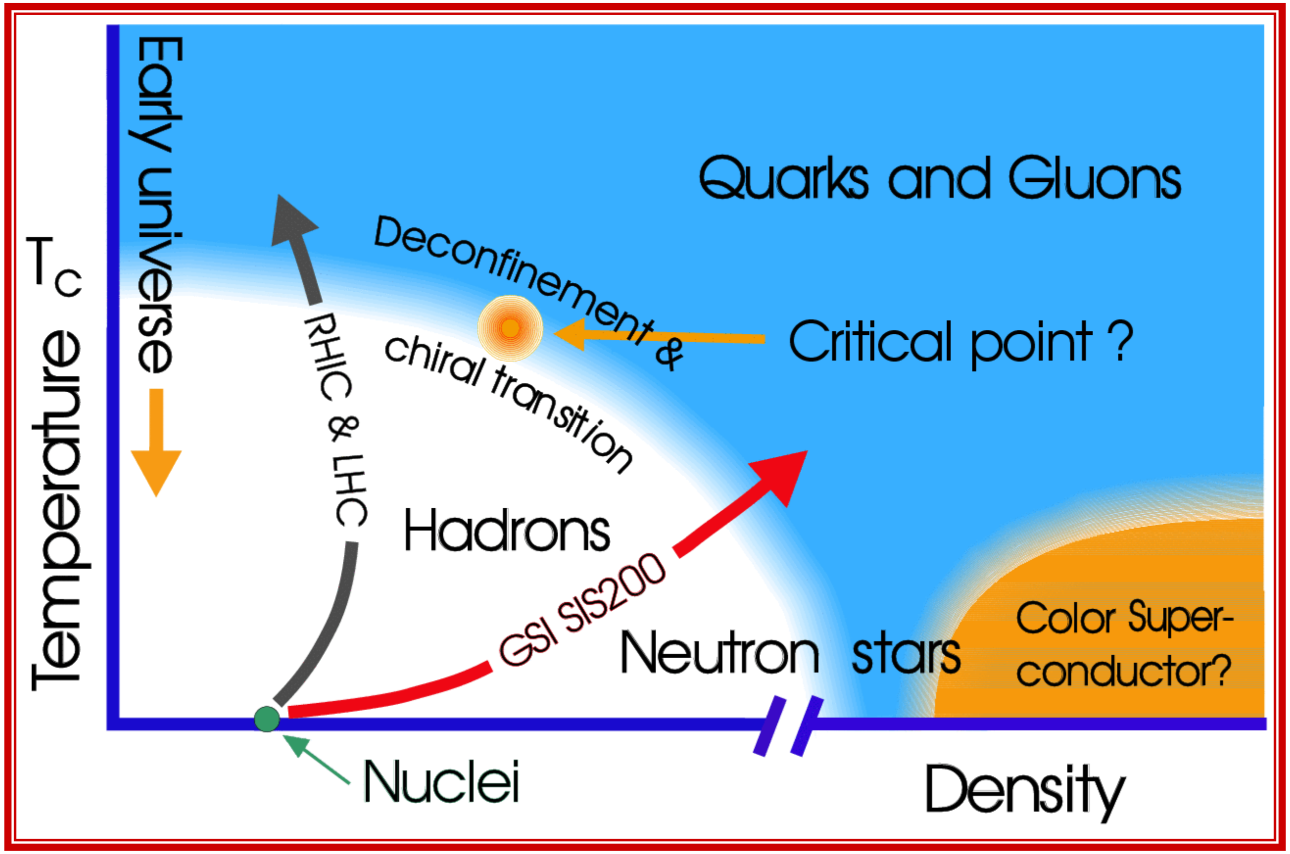
Feschbach resonance In baryon resonances ⟷ ***Feschbach resonance In cold atom system***

	QGP	cold atoms
Strongly coupled many body system	Yes Relativistic	Yes Non-relativistic
Collective flow [Thomas' talk, Schaefer's talk]	Yes	Yes
AdS/CFT [Schaefer's talk]	Maybe	Maybe
BCS pairing/BEC [this talk, Grimm's talk]	Yes	Yes
Three particle bound state [this talk, also posters: Johim, Floerchinger]	Yes, baryon	Yes, trimer

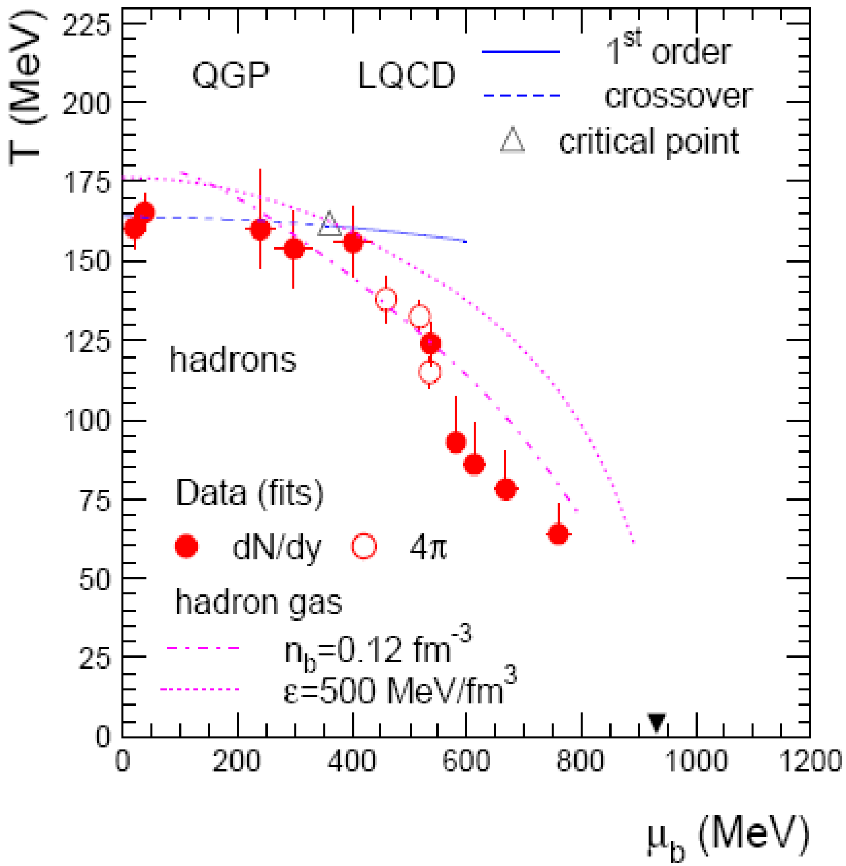
Phase diagram of
Strongly interacting Quark Gluon Plasma

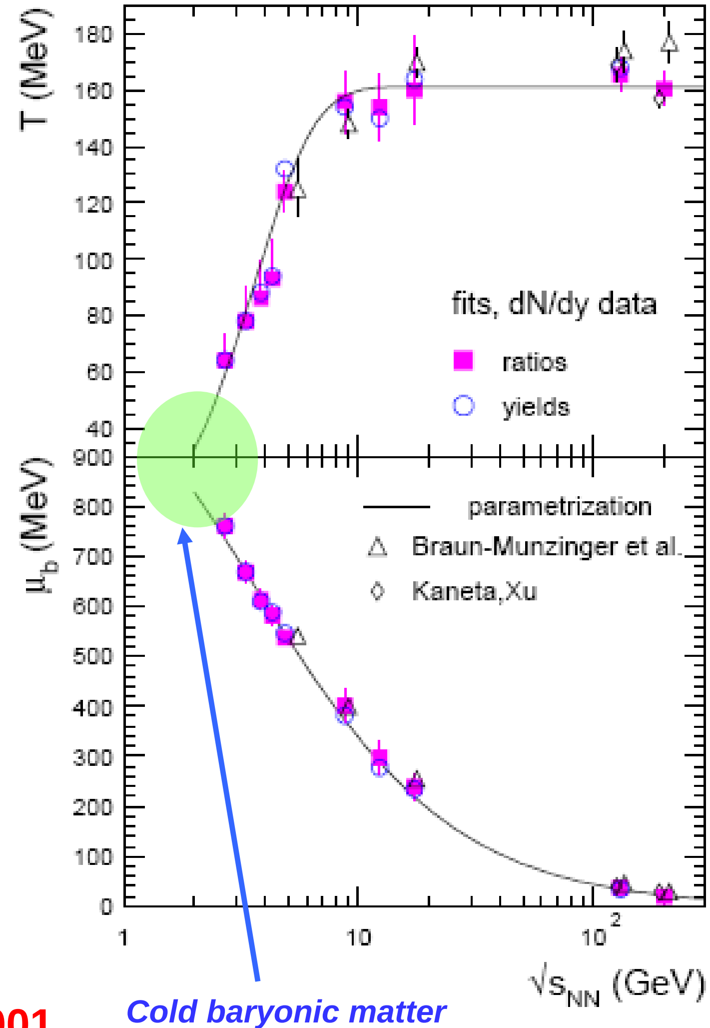
See e.g.

- ***Braun-Munzinger, Wambach, 2008* (review)**
- ***Ruester,Werth,Buballa, Shovkovy,Rischke,2005***
- ***Fukushima, Kouvaris, Rajagopal, 2005***
- ***Blaschke, Fredriksson, Grigorian, Oztas, Sandin, 2005***



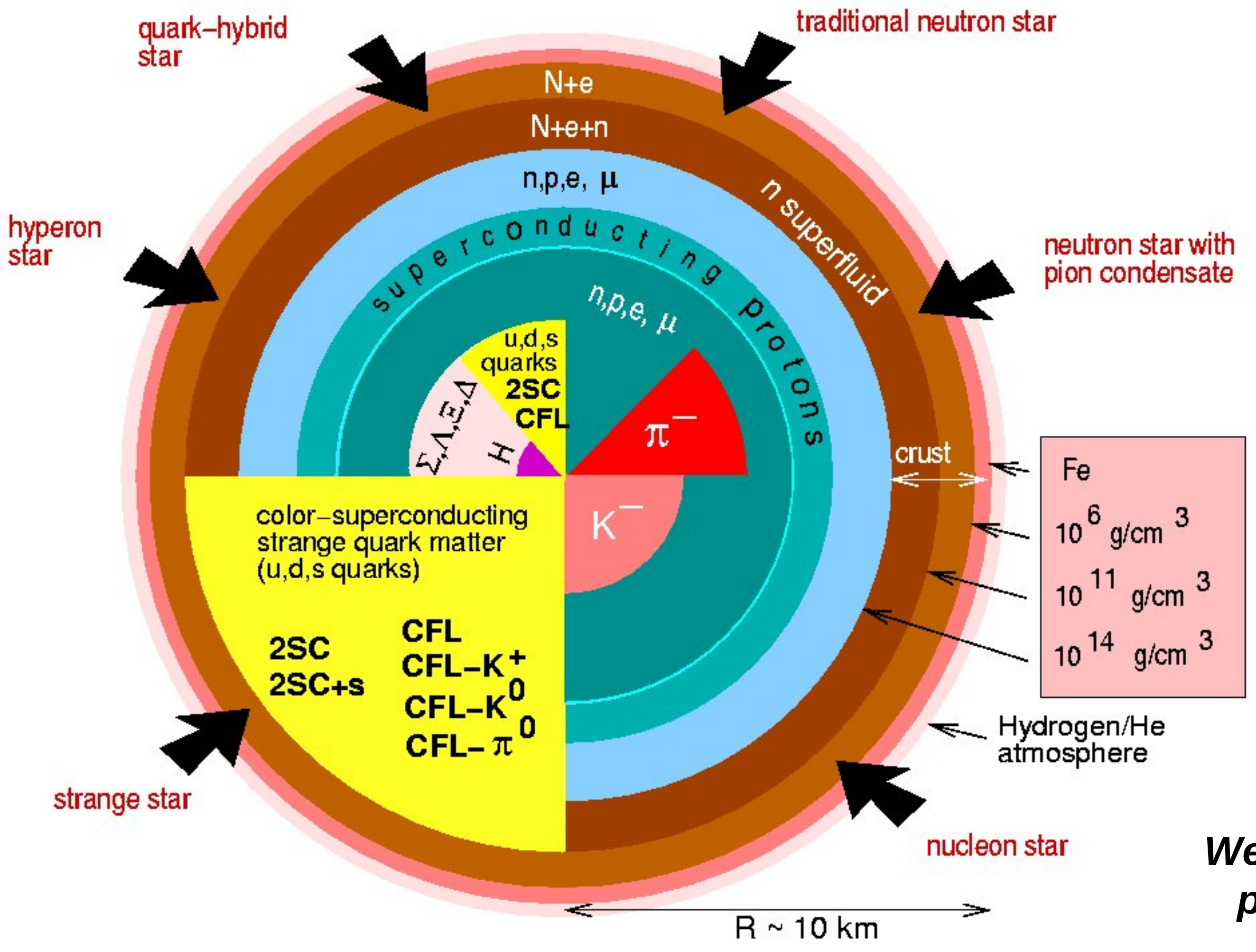
Freezeout temperature and chemical potential in Heavy Ion Collisions





Andronic, Braun-Munzinger, Stachel, 2006
Braun-Munzinger,Magestro,Redlich,Stachel, 2001

Color superconductivity in neutron stars



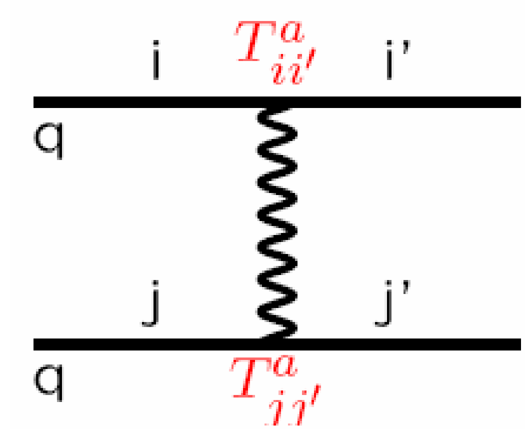
Webber, astro-ph/0407155

Why color superconductivity

Also see talks: Huang, Shovkovy

$$T^a_{ii'}T^a_{jj'} = \frac{1}{2}\delta_{ij'}\delta_{i'j} - \frac{1}{2N_c}\delta_{ii'}\delta_{jj'}$$

$$= \frac{1}{6}(\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{i'j}) - \frac{1}{3}(\delta_{ii'}\delta_{jj'} - \delta_{ij'}\delta_{i'j})$$

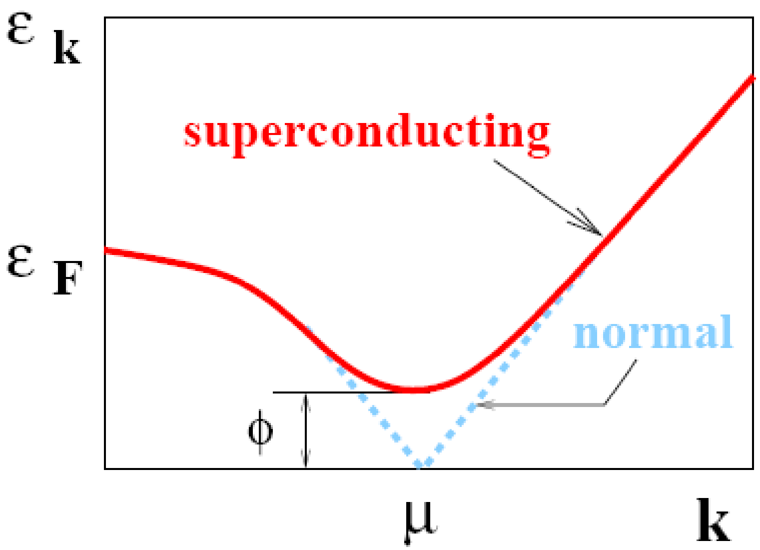


Anti-symmetric channel:
attractive interaction

Energy gap in quasi-particle excitation

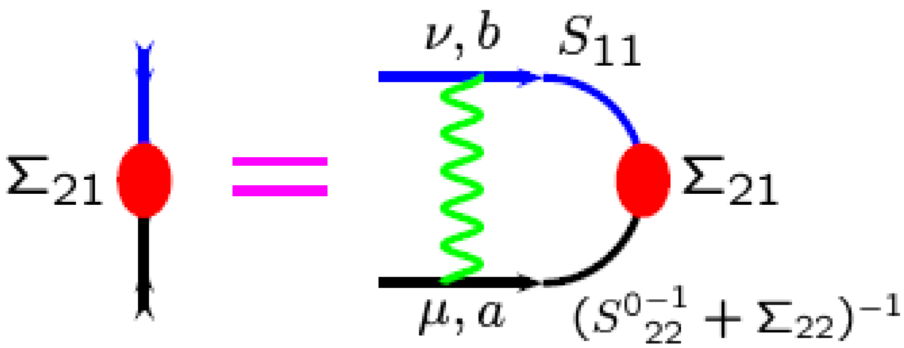
No gap: $\epsilon_{\mathrm{k}} = |k - \mu|$;

With gap: $\epsilon_{\mathrm{k}} = \sqrt{(k-\mu)^2 + \phi^2}$



Color superconductivity - weak coupling

Weak coupling gap equation (DS equation) in asymptotically high density



Nambu-Gorkov matrix
$\hat{\Gamma}^{\mu}_{a} = \mathrm{diag}(\gamma^{\mu} T_a, -\gamma^{\mu} T_a^{T})$

$$
\Sigma_{21}(K) = -g^2 \frac{T}{V} \sum_{Q} \Delta^{ab}_{\mu\nu}(K-Q) \left[\hat{\Gamma}^{\mu}_{a22} S_{21}(Q) \hat{\Gamma}^{\nu}_{b11} \right]
$$

Weak coupling solution to gap equation

[Son 1999; Schafer,Wilczek 2000; Hong et al. 2000; Pisarski,Rischke 2000; Brown et al 2000; Wang,Rischke 2002; Schmitt,Wang,Rischke 2003]

$$\boxed{\phi_0 = 2\,\tilde{b}\,b_0\,b_1\,\mu\,\exp\left(-\frac{\pi}{2\bar{g}}\right)}$$

$\tilde{b} \equiv 256\pi^4[2/(N_f g^2)]^{5/2}$

$\exp\left(-\frac{4+\pi^2}{8}\right)$

Depends on phases

Schematical gap equation

$$\boxed{\phi = g^2\phi_0\left[\xi \ln^2\left(\frac{\mu}{\phi_0}\right) + \beta \ln\left(\frac{\mu}{\phi_0}\right) + \alpha\right]}$$

$O(1/g^2)$
leading
static magnetic

$O(1/g)$–subleading
non-static magnetic
static electric
quark-selfenergy

$O(g^2)$–subsubleading

Gauge parameter dependence

- The gap is NOT invariant to subleading order in covariant gauge,

[D.K. Hong, V.A. Miransky, I.A. Shovkovy, L.C.R. Wijewardhana, Phys. Rev. D61, 056001(2000)]

$$\boxed{\phi = e^{3/2\xi}\phi_0}$$

From 1-loop contribution to the gap eqn. at subleading order

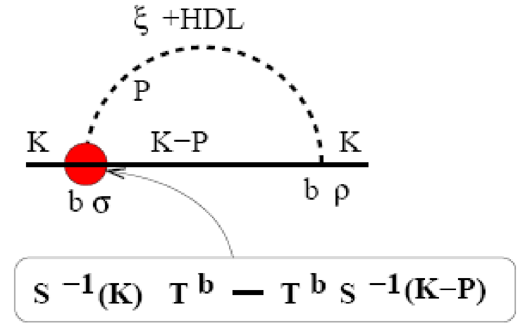
- Hint to solution of the problem: Impose on-shell condition!

$$\begin{aligned} K_{on} = (\epsilon_{\mathbf{k}}, \mathbf{k}) &\rightarrow (\phi, \mu\hat{\mathbf{k}}) \\ \mathrm{S}^{-1}(K_{on})\Psi_{on}(K_{on}) &= 0 \\ \overline{\Psi}_{on}(K_{on})\mathrm{S}^{-1}(K_{on}) &= 0 \end{aligned}$$

Gerhold, Rebhan, 2003
Hou, QW, Rischke, 2004

Generalised Ward identity with condensate

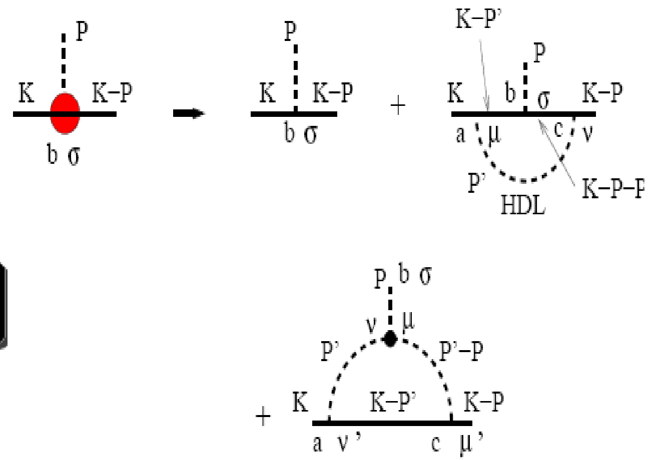
$$P_\sigma i\Gamma^b_\sigma = ig[\mathbf{S}^{-1}(K)\mathbf{T}^b - \mathbf{T}^b\mathbf{S}^{-1}(K-P)] + I^b_X$$



$$\begin{aligned} I^\xi[a,c,d] \quad \sim \quad & -\xi g^2 \int \frac{d^4P}{(2\pi)^4}\frac{1}{P^4} \\ & \times \left[\mathbf{S}^{-1}(K)\mathbf{T}^b - \mathbf{T}^b\mathbf{S}^{-1}(K-P)\right] \\ & \times \mathbf{S}(K-P)\mathbf{T}^b\gamma_\rho P_\rho \end{aligned}$$

$$\begin{aligned} & \overline{\Psi}_{on}(K_{on}) I^\xi[a,c,d]\Psi(K_{on}) \\ & \to \overline{\Psi}_{on}(K_{on})\mathbf{S}^{-1}(K_{on}) = 0 \end{aligned}$$

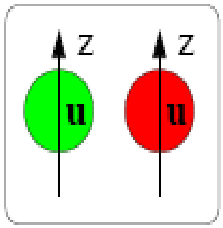
$$\int d^4P P_\rho/P^4 = 0$$



It can be proved that ***the contribution is of subsubleading order if all excitations are gapped***

H.J.Xu and QW, in preparation

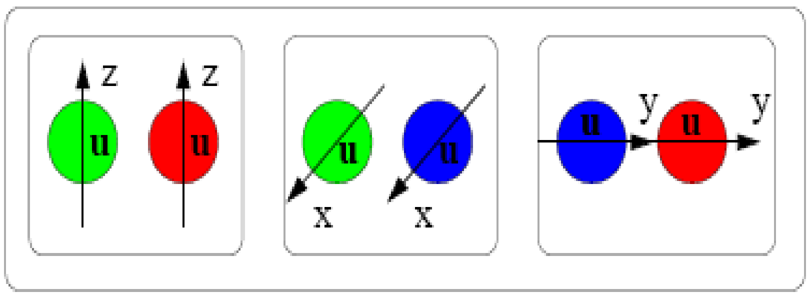
Pairings within the same flavor



- Polar phase, $N_f = 1$, spin-1 gap

$SU(3)_c \times SO(3)_J \to SU(2)_c \times U(1)_J$

[T. Schafer, Phys. Rev. D62, 094007(2000)]



- Color spin locking, CSL

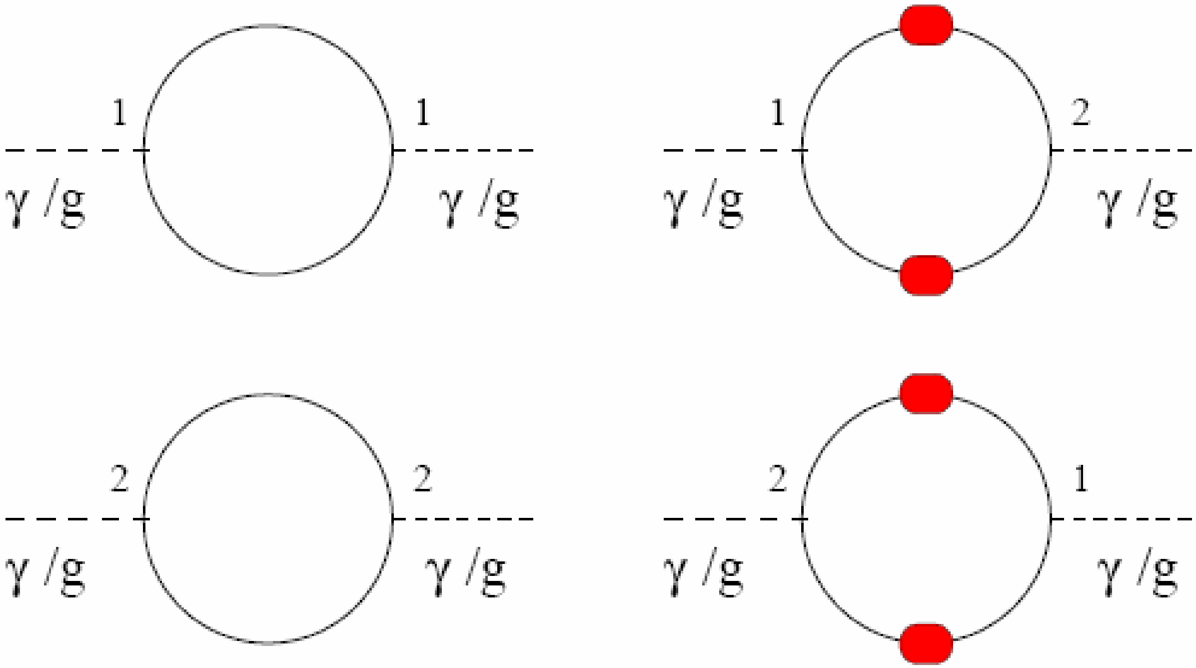
$N_f = 1$, spin-1 gap

$SU(3)_c \times SO(3)_J \to SO(3)_{c+J}$

[D. Bailin, A. Love, Phys. Rep. 107, 325(1984), T. Schafer, Phys. Rev. D62, 094007(2000)]

Schmitt, QW, Rischke, Phys.Rev.Lett.91, 242301(2003)
Schmitt, Phys.Rev.D71, 054016(2005)

Meissner effects in weak coupling



diagonalize (9,9) matrix Π^{ab}

Son, Stephanov, Phys.Rev.D61,074012(2000);
Schmitt, QW, Rischke, Phys. Rev. D69, 094017(2004);
Phys. Rev. Lett.91, 242301(2003)

Meissner effects in weak coupling: results

- Zero-temperature rotated Debye masses

	$\tilde{m}^2_{D,88}$	$\tilde{m}^2_{D,\gamma\gamma}$	$\cos^2\theta_D$
2SC	$3\,g^2$	$2\,e^2$	1
CFL	$(4\,e^2+3\,g^2)\,\zeta$	0	$3g^2/(3g^2+4e^2)$
polar	$3\,g^2$	$18\,q^2e^2$	1
CSL	$3\,\beta\,g^2$	$18\,q^2e^2$	1

$$\begin{aligned} \text{Unit} &= N_f\mu^2/6\pi \\ \zeta &\equiv (21-8\ln 2)/54 \\ \alpha &\equiv (3+4\ln 2)/27 \\ \beta &\equiv (6-4\ln 2)/9 \end{aligned}$$

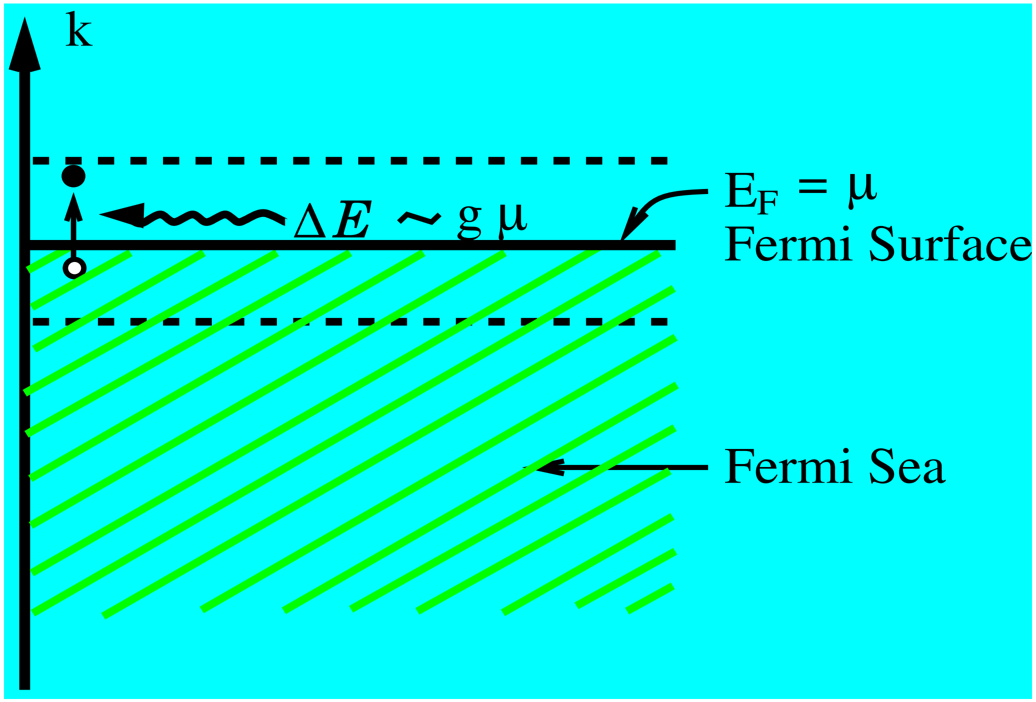
- Zero-temperature rotated Meissner masses

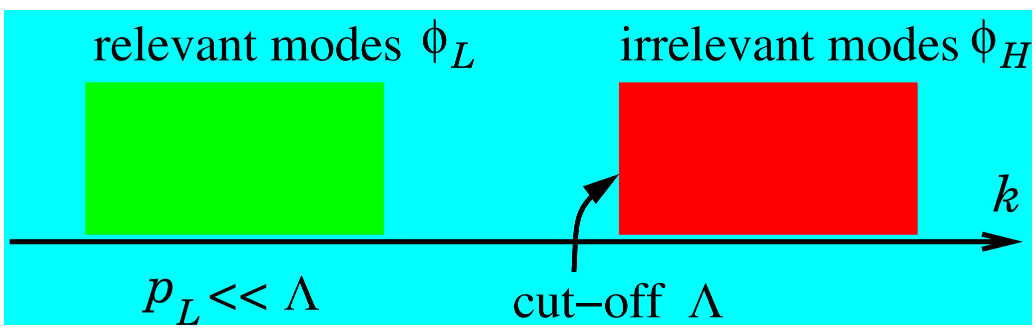
	$\tilde{m}^2_{M,88}$	$\tilde{m}^2_{M,\gamma\gamma}$	$\cos^2\theta_M$
2SC	$\frac{1}{3}g^2+\frac{1}{9}e^2$	0	$3g^2/(3g^2+e^2)$
CFL	$\left(\frac{4}{3}e^2+g^2\right)\,\zeta$	0	$3g^2/(3g^2+4e^2)$
polar	$\frac{1}{3}g^2+4\,q^2e^2$	0	$g^2/(g^2+12\,q^2e^2)$
CSL	$\beta\,g^2$	$6\,q^2e^2$	1

Schmitt, QW, Rischke, 2003, 2004

- **Rotated photon in CSL phase has a non-vanishing mass: Electromagnetic Meissner effect.**
- **Although rotated photon in polar phase has a zero mass but a system with 2 or 3 favors still exhibits Electromagnetic Meissner effect because of different chemical potential or no single mixing angle for all favors.**

Effective Theory of dense matter





$\mu \gg \Lambda_{QCD} \Rightarrow g(\mu) \ll 1$

$\Rightarrow$ Controlled calculation in QCD

$\mu \gg g\mu \gg \phi$

$\Rightarrow$ physics dominated by strip close to Fermi surface

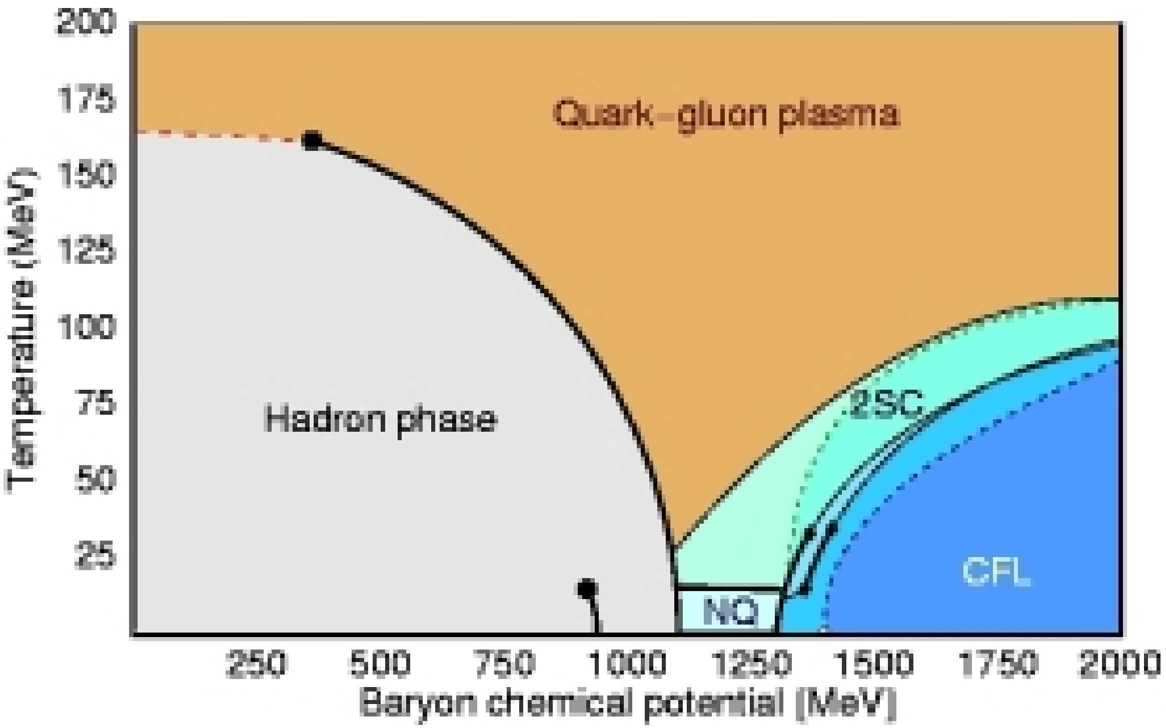
$\Rightarrow$ separation of scales and need for EFT

Hong, 2000
Schaefer, 2002, 2003
Reuter, QW, Rischke, 2004

Color superconductivity – intermediate coupling (NJL)

NJL model

Buballa, 2005; (Review)
Ruester, Werth, Buballa, Shovkovy, Rischke, 2005;
Fukushima, Kouvaris, Rajagopal, 2005;
Blaschke, Fredriksson, Grigorian, Oztas, Sandin, 2005

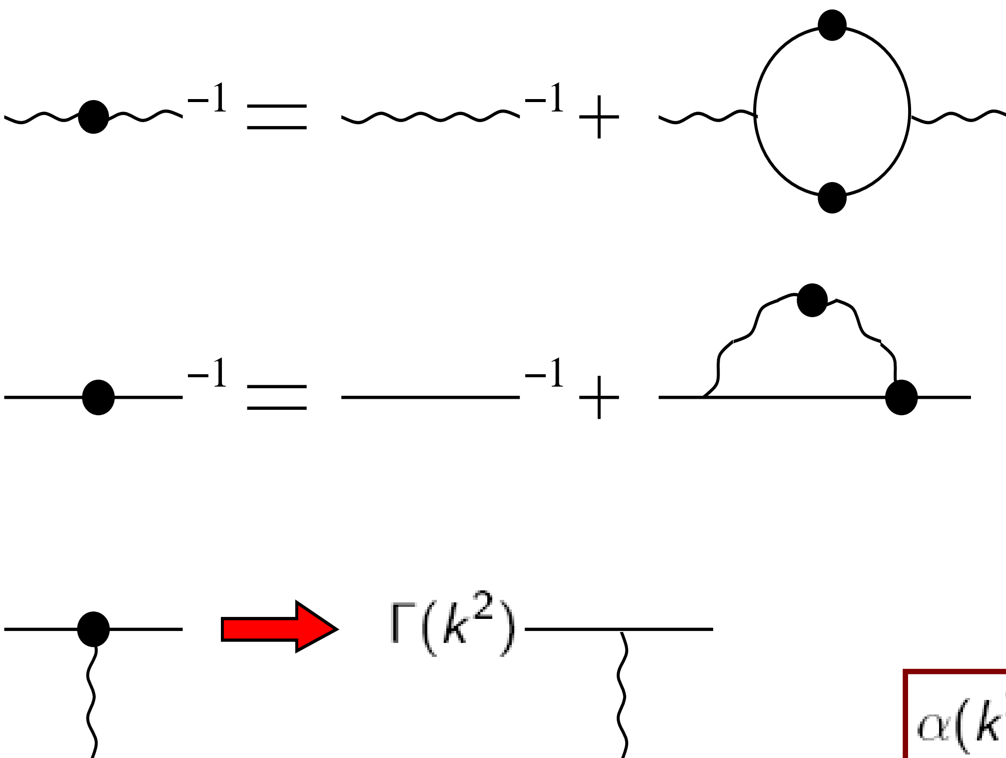


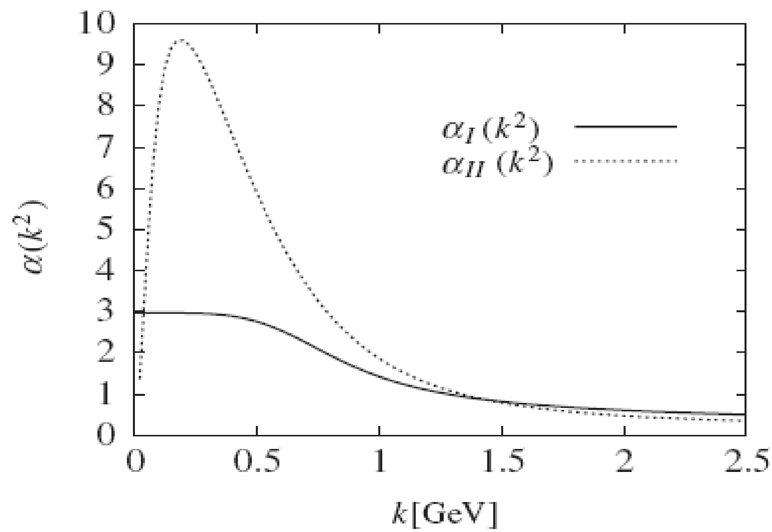
Color superconductivity – intermediate coupling (DS)

DS approach

See Fischer's talk

Nickel, Wambach, Alkofer, 2006 (2SC,CFL)
Marhauser, Nickel, Buballa, Wambach, 2007 (CSL)
Nickel, Wambach, Alkofer, 2008 (CFL+neutrality)

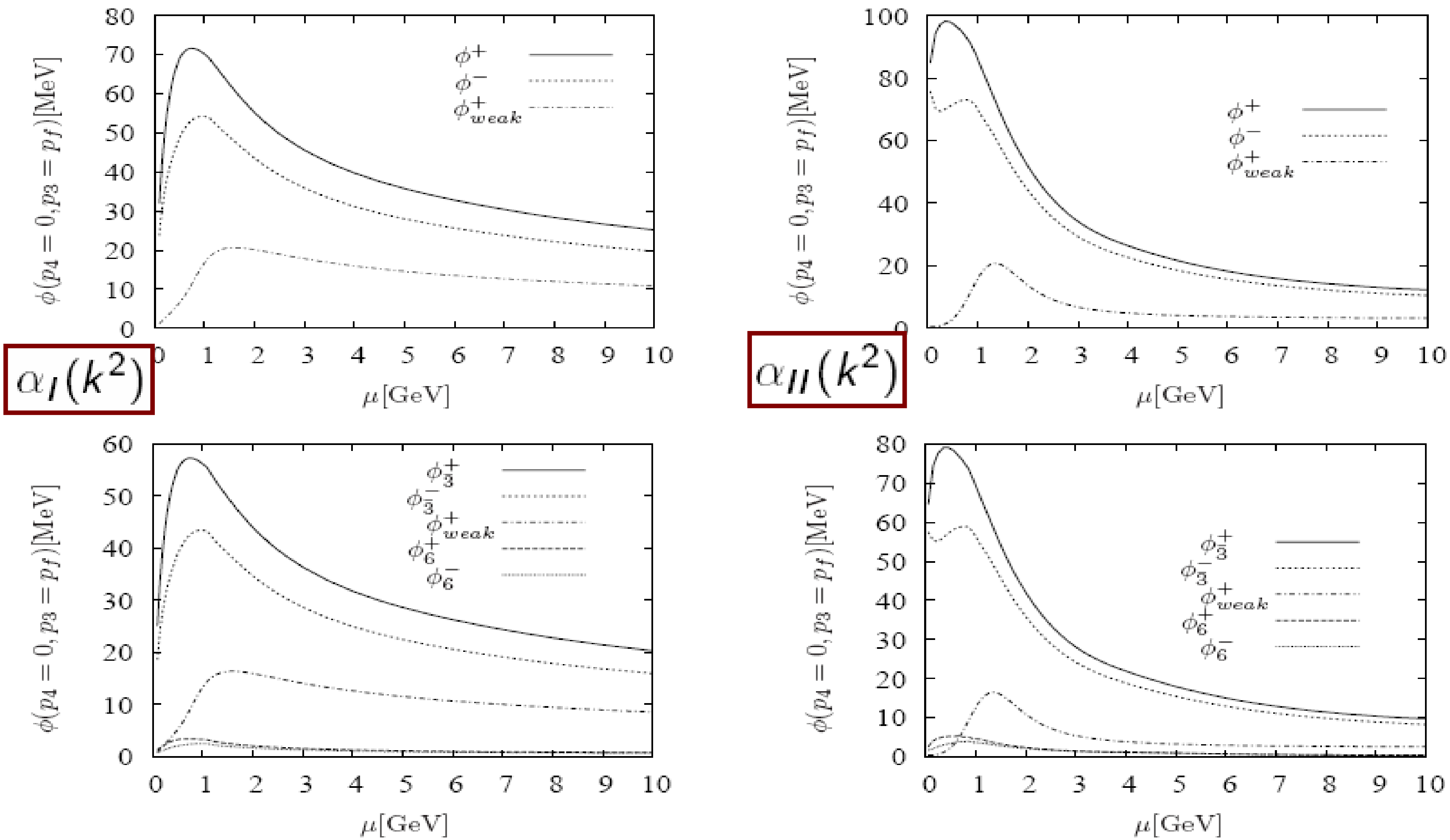




$$\boxed{\alpha(k^2) \sim \Gamma(k^2)}$$

Color superconductivity – intermediate coupling (DS)

Nickel, Wambach, Alkofer, 2006 ***(2SC,CFL)***



It (QCD) provides the answer to a child-like question: What happens to matter, as you squeeze it harder and harder?

-- Wilczek

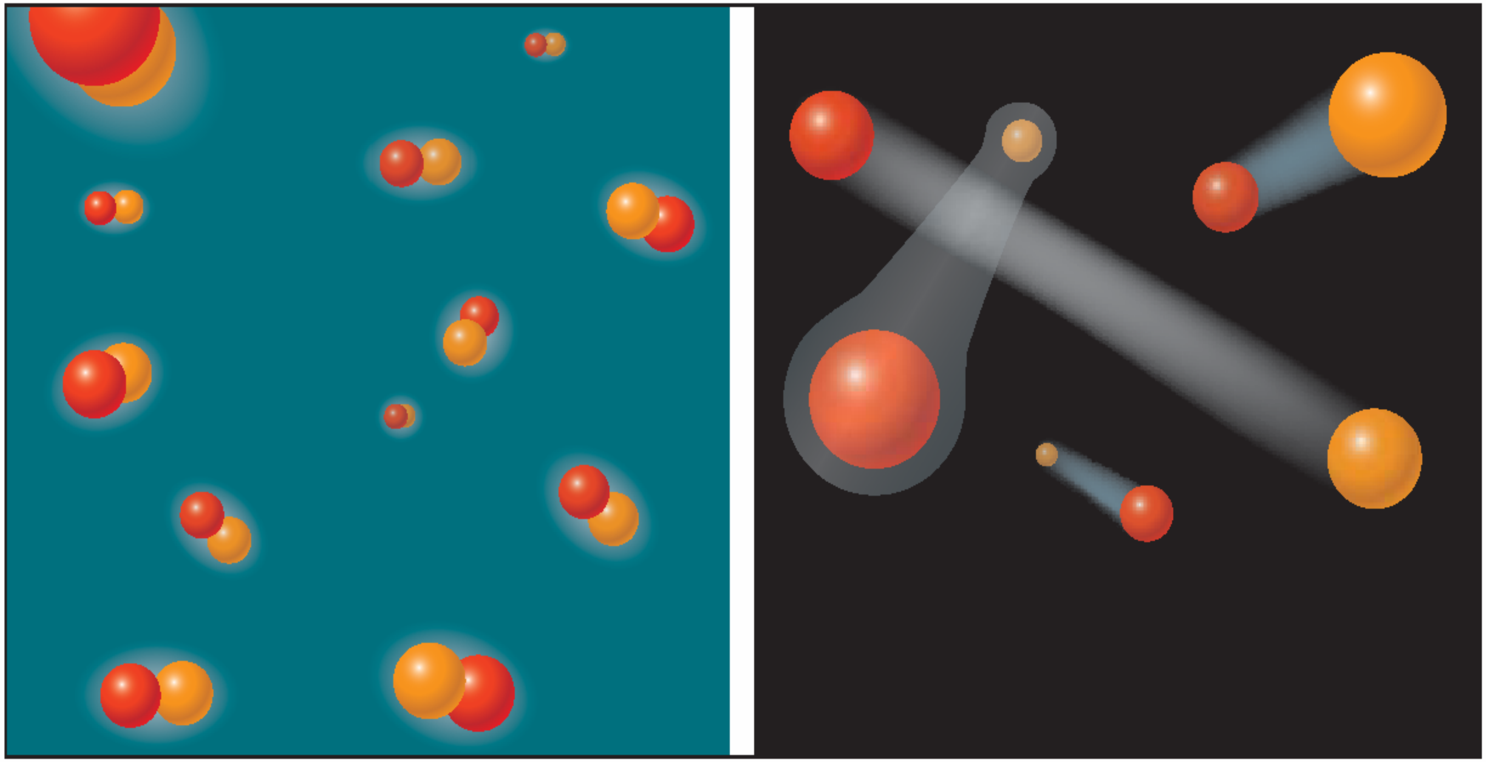
Answer: Perturbation in QCD in weak coupling

An opposite question: What happens to matter, as you increase interactions stronger and stronger?

What happens to quark-quark pairings: do they survive stronger and stronger interactions?

Answer: unclear

BCS-BEC Crossover



Tango or twist? In a magnetic field, atoms in different spin states can form molecules (*left*). Vary the field, and they might also form loose-knit Cooper pairs.

Science

Relativistic BCS-BEC crossover

Recent works by other group:

- **Nishida & Abuki, PRD 2007 -- NJL approach**
  - **– Abuki's Talk**
- **Abuki, NPA 2007 – Static and Dynamic properties**
- **Sun, He & Zhuang, PRD 2007 – NJL approach**
- **He & Zhuang, PRD 2007 – Beyond mean field**
- **Kitazawa, Rischke & Shovkovy, arXiv:0709.2235v1 – NJL+phase diagram**
  - **– Kitazawa's Talk**
- **Brauner, arXiv:0803.2422 – Collective excitations**
  - **– Brauner's Talk**
- **Chatterjee, Mishra, Mishra, arXiv:0804.1051 -- Variational approach**

Relativistic boson-fermion model (MFA)

With bosonic and fermionic degrees of freedom and their coupling, but neglect the coupling of thermal bosons and fermions as Mean Field Approximation ***Friedberg-Lee model, 1989***

$$\mathcal{L}(\Phi,\Psi) = -\frac{1}{2}\overline{\Psi}S_0^{-1}\Psi + |(\partial_\nu - i\mu_b\delta_{\nu 0})\Phi|^2 - m_b^2|\Phi|^2 + \frac{1}{2}\left(\Phi^\dagger\overline{\Psi}\widehat{\Gamma}\Psi - \Phi\overline{\Psi}\widehat{\Gamma}^\dagger\Psi\right)$$

$$\boxed{\Phi = \phi + \varphi}$$

$$\mathcal{L} = \frac{1}{2}\overline{\Psi}\mathcal{S}^{-1}\Psi + [\mu_b^2 - m_b^2]|\phi|^2 + |(\partial_t - i\mu_b)\varphi|^2 - |\nabla\varphi|^2 - m_b^2|\varphi|^2 .$$

$$\mathcal{S}^{-1}(P) = \begin{pmatrix} P_\mu\gamma^\mu + \mu\gamma_0 - m & 2ig\gamma_5\phi^* \\ 2ig\gamma_5\phi & P_\mu\gamma^\mu - \mu\gamma_0 - m \end{pmatrix} .$$

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix} \qquad \overline{\Psi} = (\overline{\psi}, \overline{\psi}_C)$$

zero mode of boson

J. Deng, A. Schmitt, QW, Phys.Rev.D76:034013,2007

Thermodynamic potetial

$$\mathcal{Z} = \int [d\Psi][d\overline{\Psi}][d\varphi][d\varphi^*] \exp\left[\int_0^{1/T} d\tau d^3x\, \mathcal{L}\right]$$

$$\boxed{\begin{aligned} \epsilon_k^e &= \sqrt{(\epsilon_{k0} - e\mu)^2 + \Delta^2} \\ \epsilon_{k0} &= \sqrt{k^2 + m^2} \\ \Delta &= g\phi, \\ \omega_k^e &= \sqrt{k^2 + m_b^2} - e\mu_b \end{aligned}}$$

$$\begin{aligned} \Omega = & \frac{(m_b^2 - \mu_b^2)\Delta^2}{4g^2} \\ & - \sum_{e=\pm} \int \frac{d^3k}{(2\pi)^3} \left\{ \epsilon_k^e + 2T \ln\left[1 + \exp\left(-\frac{\epsilon_k^e}{T}\right)\right]\right\} \\ & + \frac{1}{2} \sum_{e=\pm} \int \frac{d^3k}{(2\pi)^3} \left\{ \omega_k^e + 2T \ln\left[1 - \exp\left(-\frac{\omega_k^e}{T}\right)\right]\right\} \end{aligned}$$

Density and gap equations

$$\begin{aligned} n &= -\frac{\partial\Omega(\mu,\Delta)}{\partial\mu} \\ 0 &= \frac{\partial\Omega(\mu,\Delta)}{\partial\Delta} \end{aligned}$$

$$\begin{aligned} n &= n_F + n_B \\ -x &= \sum_{e=\pm}\int\frac{d^3k}{(2\pi)^3}\left(\frac{1}{2\epsilon_k^e}\tanh\frac{\epsilon_k^e}{2T} - \frac{1}{2\epsilon_{k0}}\right) \end{aligned}$$

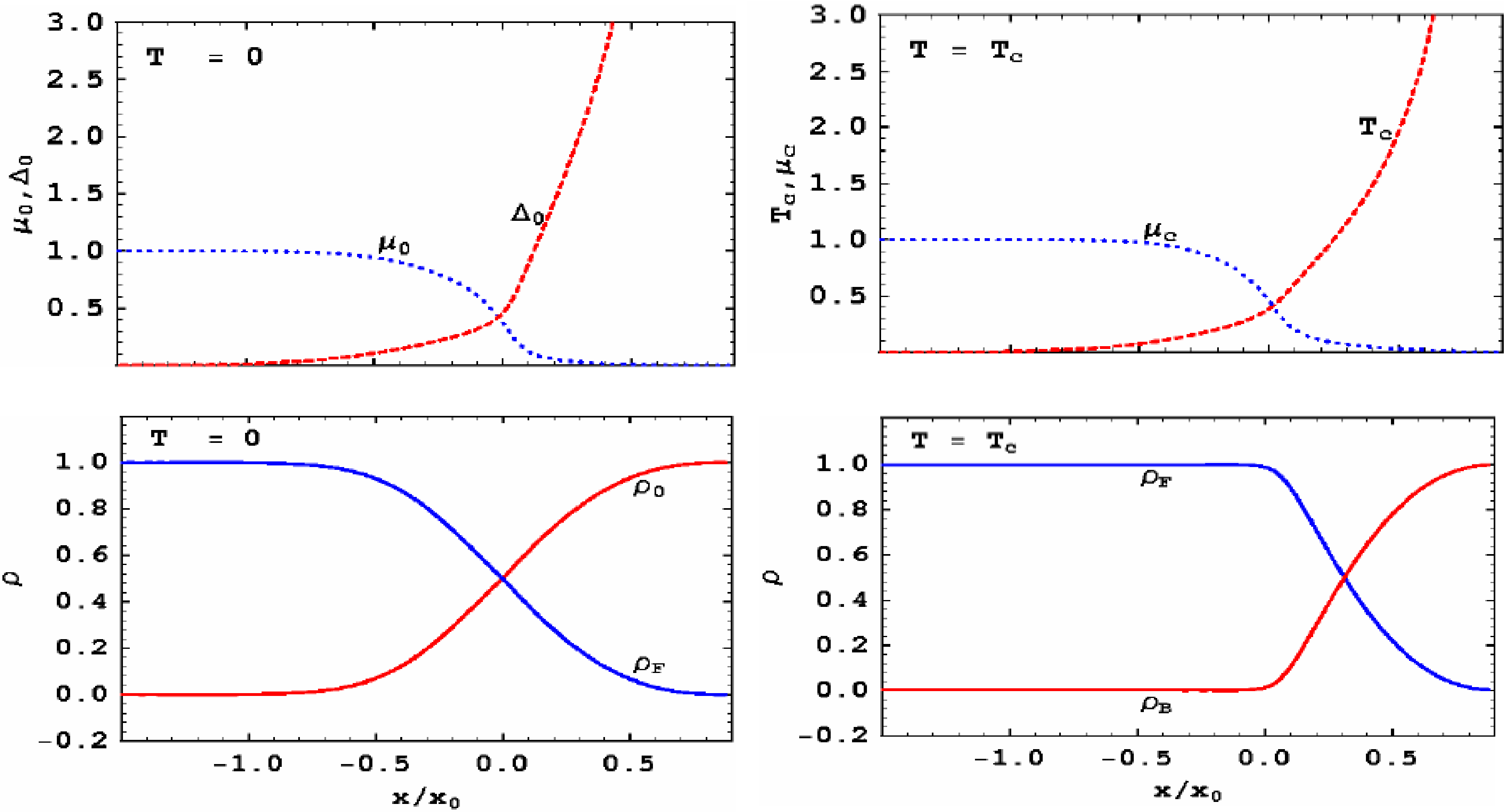
Crossover parameter

$$x \equiv -\frac{m_{b,r}^2 - \mu_b^2}{4g^2}$$

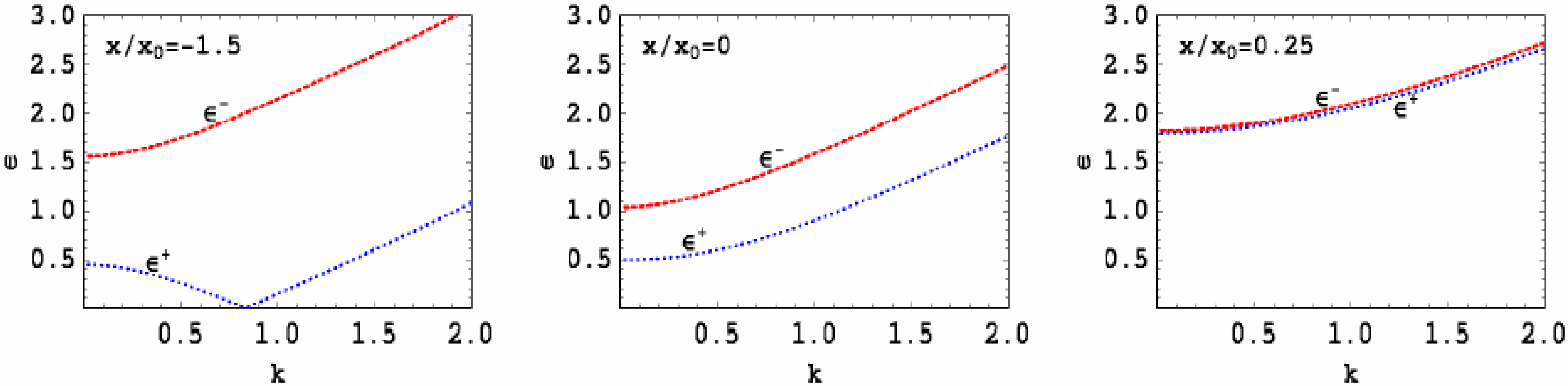
$$(\mu, \Delta)$$

x<0, BCS regime
x>0, BEC regime

At zero T or critical T



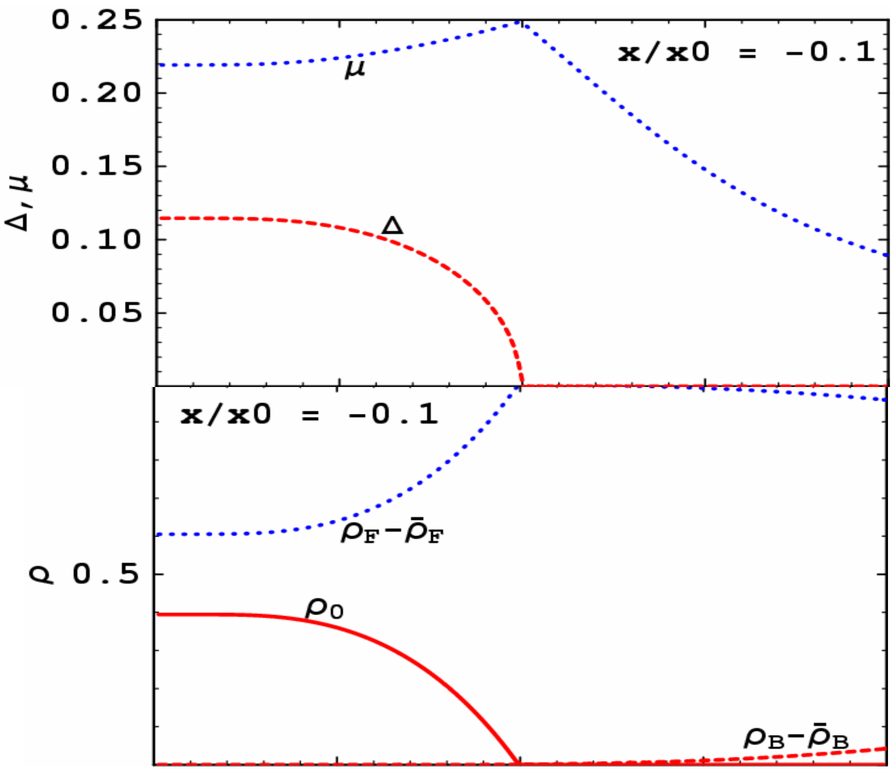
Dispersion relation

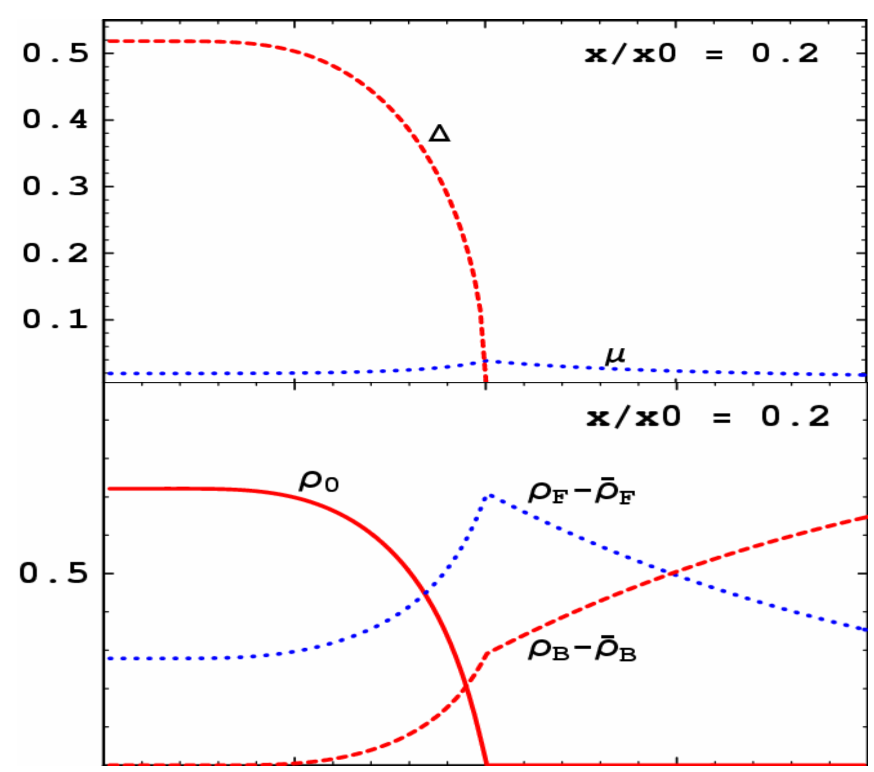


In BCS regime, fermions are slightly gapped, anti-fermions are strongly gapped.

In BEC regime, both are strongly gapped, indicating the formation of bound states with large binding energy

Finite T

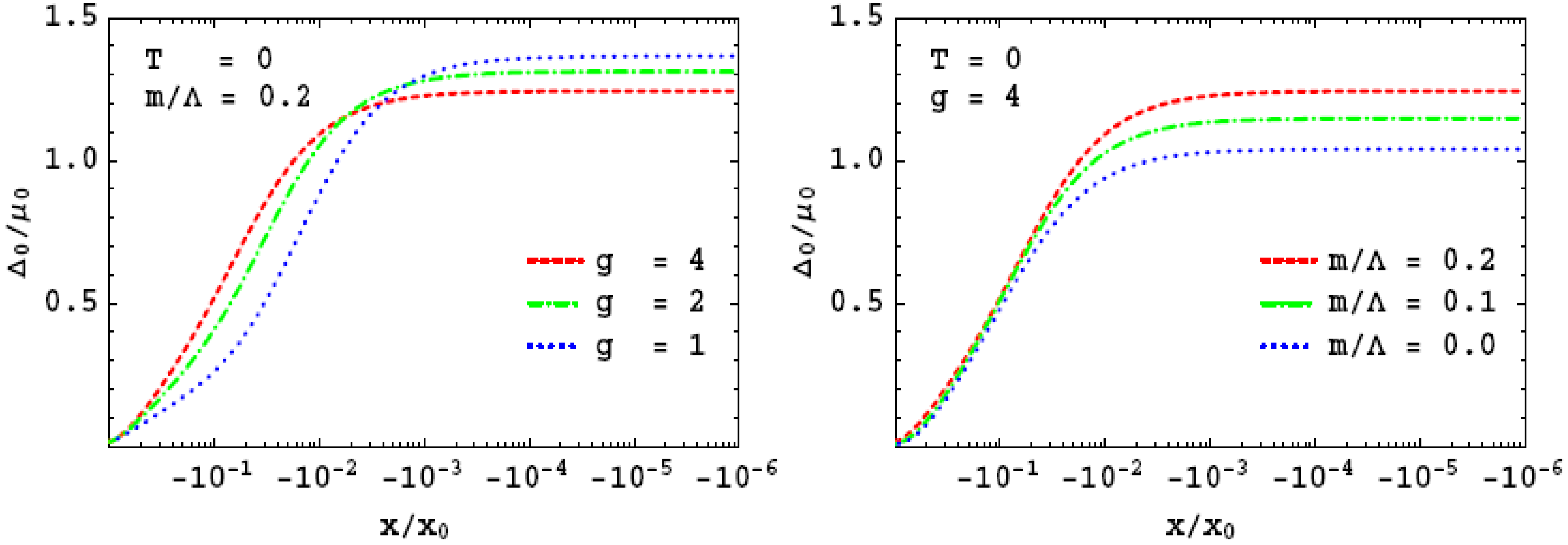




BCS regime:
Melting condensation of fermion pairs

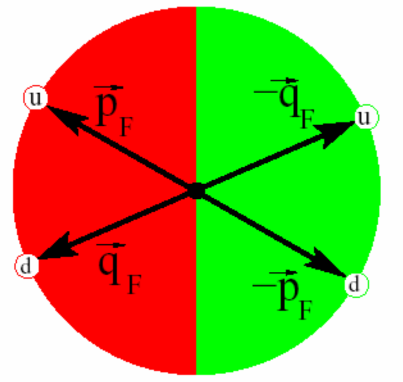
BEC regime:
Melting condensation of bosons

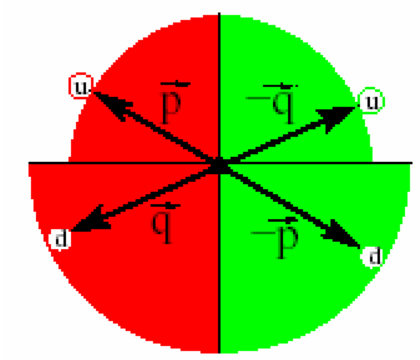
Unitary Regime



$$\frac{\Delta_0}{\mu_0} \sim 1.2 - 1.4, \qquad T = 0, \frac{m}{\Lambda} = 0.2$$

Pairing with imbalance population

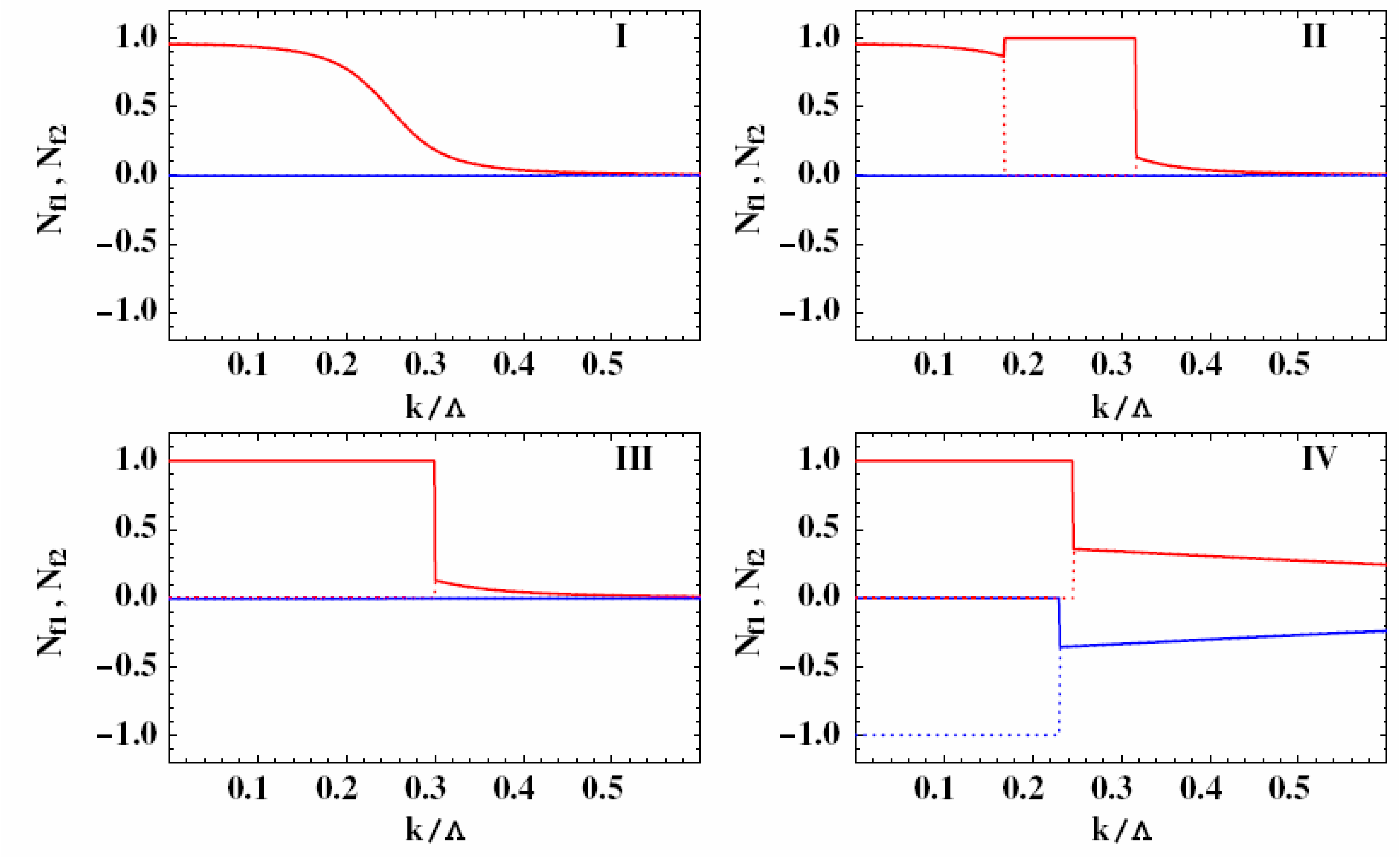




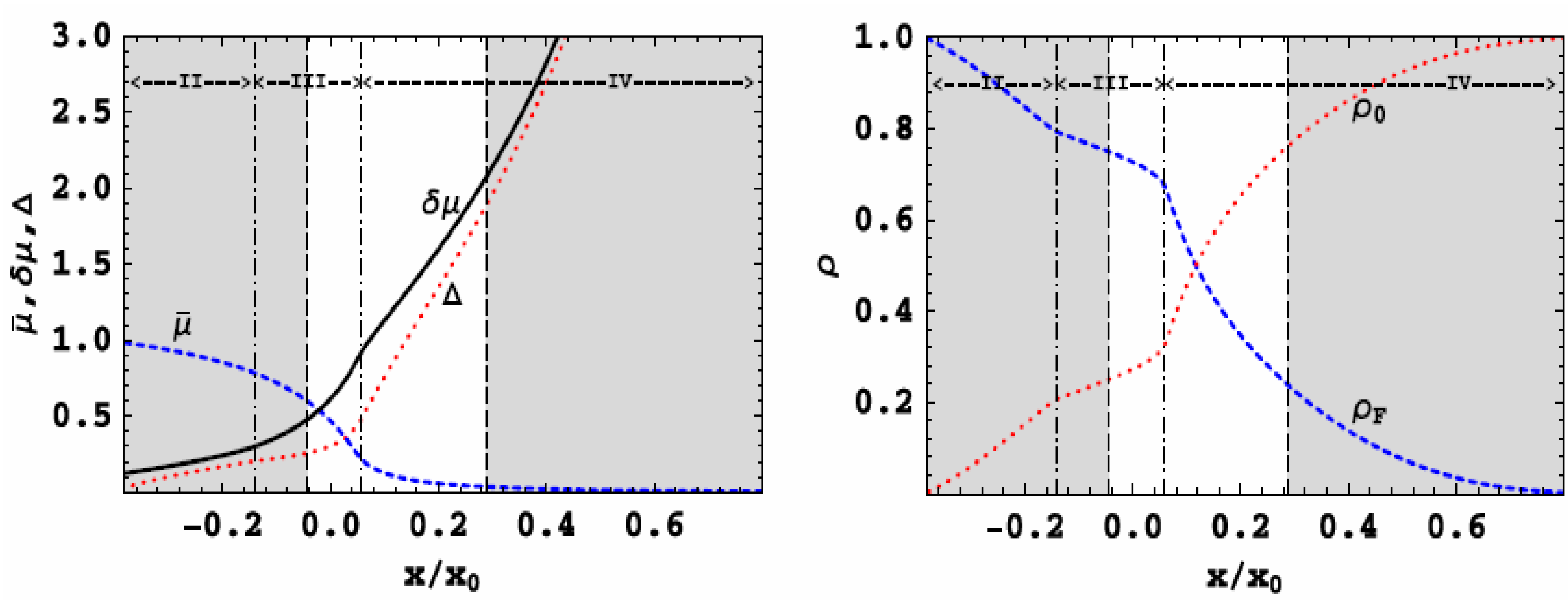
- **Alford, Berges & Rajagopal, PRL 2000; Alford, Kouvaris & Rajagopal, PRL 04, PRD 05 -- Gapless and crystalline color superconductivity (LOFF)**
- **Huang, Shovkovy, PLB 2003 and NPA 2003; PRD 04; PRD 04 -- Gapless color superconductivity in 2SC, instablility in Meissner masses**
- **Many others ……**

---- see Mei Huang's talk

Fermi surface topologies



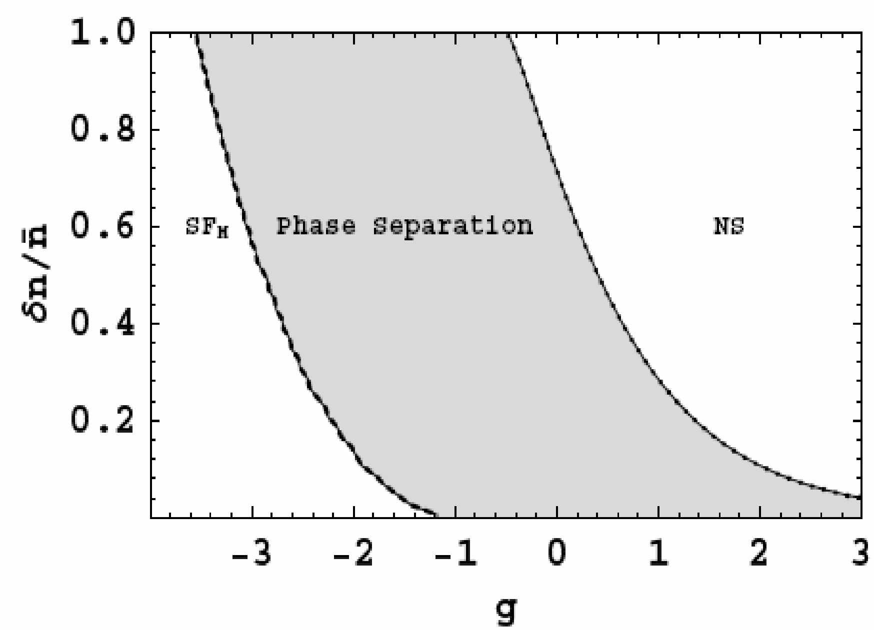
Homogeneous solution



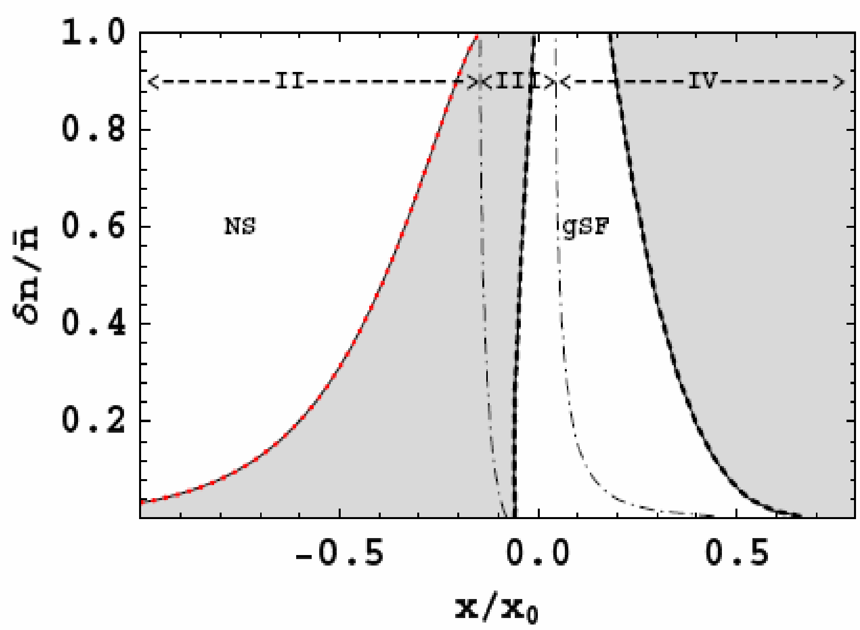
The fermion-boson mixture in BCS-BEC regime has been found in cold atomic system. Stable gapless phase in strong coupling (see also Kitazawa,Rischke, Shovkovy, 2006)

[Realization of a strongly interacting Bose-Fermi mixture from a two-component Fermi gas, MIT group, arXiv:0805.0623]

Phase diagram



Non-relativistic



relativistic

Shaded area: unstable, with negative susceptibility

Beyond mean-field aproximation

$$\mathcal{L}(\Phi,\Psi) = -\frac{1}{2}\overline{\Psi}S_0^{-1}\Psi + |(\partial_\nu - i\mu_b\delta_{\nu 0})\Phi|^2 - m_b^2|\Phi|^2 + \frac{1}{2}\left(\Phi^\dagger\overline{\Psi}\widehat{\Gamma}\Psi - \Phi\overline{\Psi}\widehat{\Gamma}^\dagger\Psi\right)$$

$$\boxed{\Phi \equiv \frac{1}{\sqrt{2}}(\varphi_R + i\varphi_I) + \varphi_0}$$

J. Deng, J.-C. Wang, QW, Phys.Rev.D78:034014,2008

$$\mathcal{L}(\Delta,\varphi,\Psi) = -\frac{1}{2}\overline{\Psi}S^{-1}\Psi + \frac{\mu_b^2 - m_b^2}{4g^2}|\Delta|^2 - \frac{1}{2}(\varphi_R,\varphi_I)D^{-1}\begin{pmatrix}\varphi_R\\ \varphi_I\end{pmatrix} + \frac{1}{2}(\varphi_R,\varphi_I)\overline{\Psi}\begin{pmatrix}\widehat{\Gamma}_R\\ \widehat{\Gamma}_I\end{pmatrix}\Psi$$

Lagrangian in Higgs and Goldstone fields

Propagator inverse

$$S^{-1} = -\begin{pmatrix} \gamma_\mu P^\mu + \mu\gamma^0 - m & i\gamma_5\Delta \\ i\gamma_5\Delta^* & \gamma_\mu P^\mu - \mu\gamma^0 - m \end{pmatrix}$$

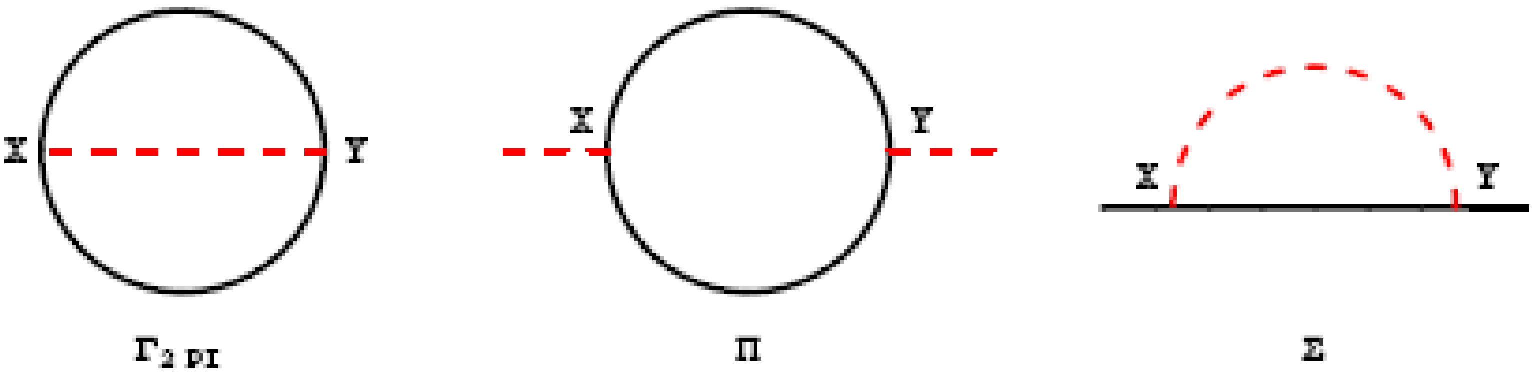
$$D^{-1} = -\begin{pmatrix} P_\mu P^\mu + \mu_b^2 - m_b^2 & 2\mu_b i p_0 \\ -2\mu_b i p_0 & P_\mu P^\mu + \mu_b^2 - m_b^2 \end{pmatrix}$$

Global U(1) symmetry, Higgs and Goldstone fields

$$\boxed{\Phi = \frac{1}{\sqrt{2}}(\eta + \eta_0)e^{2i\theta}}$$

$$\begin{aligned}
\mathcal{L}(\Delta, \varphi, \Psi) = & -\frac{1}{2}\overline{\Psi}S^{-1}\Psi + \frac{\mu_b^2 - m_b^2}{4g^2}\Delta^2 + \frac{1}{2}\left[(\partial_\nu\eta)^2 + (\mu_b^2 - m_b^2)\eta^2\right] \\
& +2(\eta + \eta_0)^2\left[(\partial_\nu\theta)^2 - \mu_b\partial_0\theta\right] \\
& -\frac{1}{2}(\partial^\mu\theta)\overline{\Psi}\sigma_3\gamma_\mu\Psi + \frac{1}{2}\eta\overline{\Psi}\widetilde{\Gamma}\Psi
\end{aligned}$$

The fluctuation of condensate
CJT formalism



$$\begin{aligned}\Gamma(\Delta,\overline{\mathcal{D}},\overline{\mathcal{S}}) &= -I(\Delta)+\frac{1}{2}\Big\{\mathrm{Tr}\ln\overline{\mathcal{D}}^{-1}+\mathrm{Tr}(D^{-1}\overline{\mathcal{D}}-1)\\ &\quad -\mathrm{Tr}\ln\overline{\mathcal{S}}^{-1}-\mathrm{Tr}(S^{-1}\overline{\mathcal{S}}-1)-2\Gamma_{2PI}(\overline{\mathcal{D}},\overline{\mathcal{S}})\Big\}\end{aligned}$$

Gasenzer 's talk

$$\Gamma_{2PI}(\overline{\mathcal{D}},\overline{\mathcal{S}}) \approx -\frac{1}{4}\mathrm{Tr}\left\{\overline{\mathcal{D}}_{XY}\mathrm{Tr}\left[\widehat{\Gamma}_X\overline{\mathcal{S}}_{XY}\widehat{\Gamma}_Y\overline{\mathcal{S}}_{YX}\right]\right\}$$

Dyson-Schwinger equations

Taking derivatives of the effective potential with respect to propagator Dyson-Schwinger eqs

$$\left.\frac{\delta\Gamma}{\delta\overline{\overline{\mathcal{D}}}}\right|_{\overline{\mathcal{D}}=\mathcal{D},\overline{\mathcal{S}}=\mathcal{S}} = 0$$

$$\left.\frac{\delta\Gamma}{\delta\overline{\overline{\mathcal{S}}}}\right|_{\overline{\mathcal{D}}=\mathcal{D},\overline{\mathcal{S}}=\mathcal{S}} = 0$$

$$\mathcal{D}^{-1} = D^{-1} - 2\frac{\delta\Gamma_{2PI}}{\delta\mathcal{D}} = D^{-1} + \Pi(\mathcal{D},\mathcal{S})$$

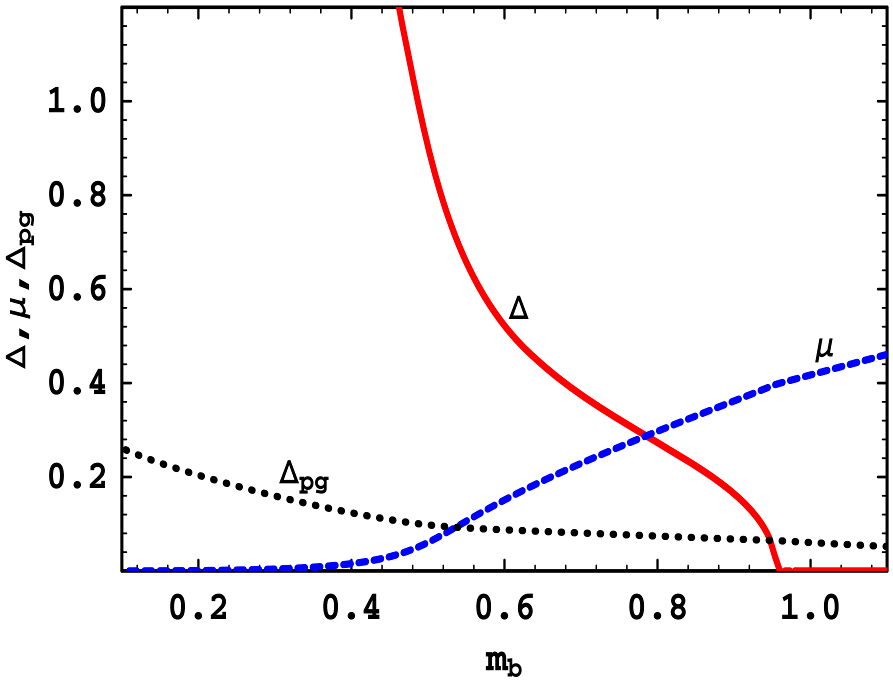
$$\mathcal{S}^{-1} = S^{-1} + 2\frac{\delta\Gamma_{2PI}}{\delta\mathcal{S}} = S^{-1} - \Sigma(\mathcal{D},\mathcal{S})$$

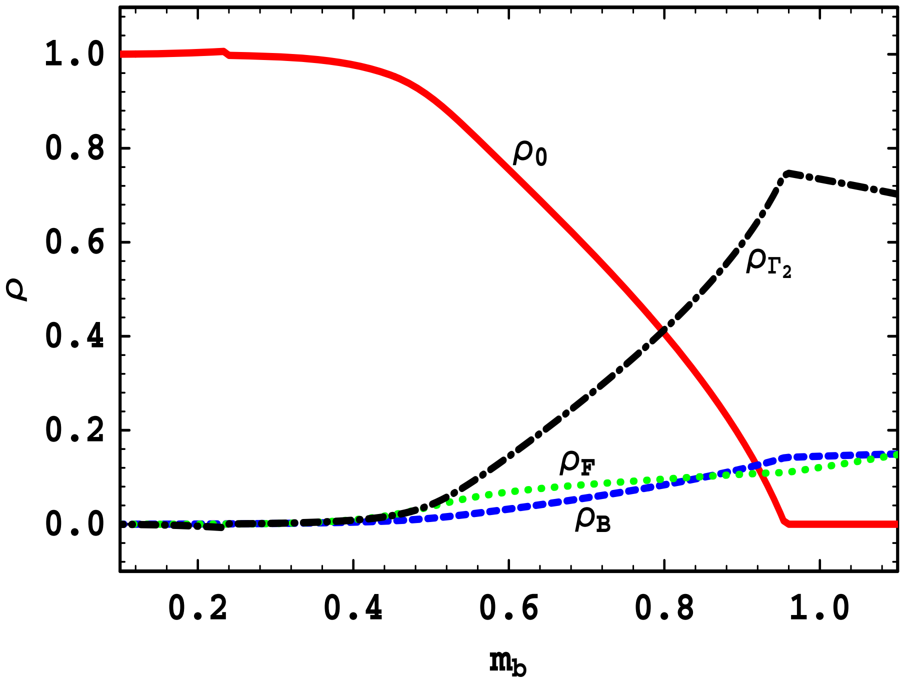
Gap & Density equations

$$\frac{\partial\Omega}{\partial\Delta} = \left\{ \frac{m_b^2 - \mu_b^2}{4g^2} - \sum_{e=\pm} \int \frac{d^3k}{(2\pi)^3} \frac{\tanh[\epsilon_k^e/(2T)]}{2\epsilon_k^e} - \frac{\partial\Gamma_{2PI}}{\partial(\Delta^2)} \right\} 2\Delta = 0$$

$$\begin{aligned} n &= -\frac{\partial\Omega}{\partial\mu} = \frac{2\mu\Delta^2}{g^2} + 2\sum_{e=\pm} \int \frac{d^3k}{(2\pi)^3} \frac{e\xi_k^e}{2\epsilon_k^e} [f_F(\epsilon_k^e) - f_F(-\epsilon_k^e)] \\ &\quad +2\sum_{e=\pm} \int \frac{d^3k}{(2\pi)^3} e f_B(E_k^b - e\mu_b) + \frac{\partial\Gamma_{2PI}}{\partial\mu} \end{aligned}$$

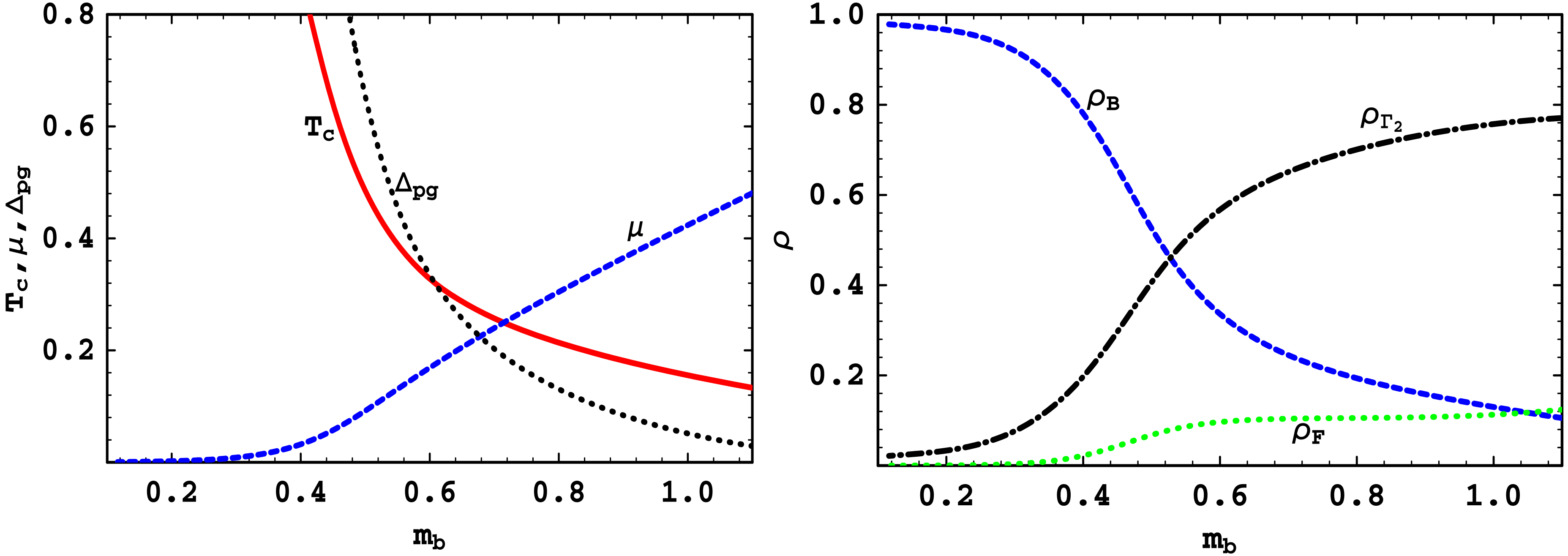
At small T





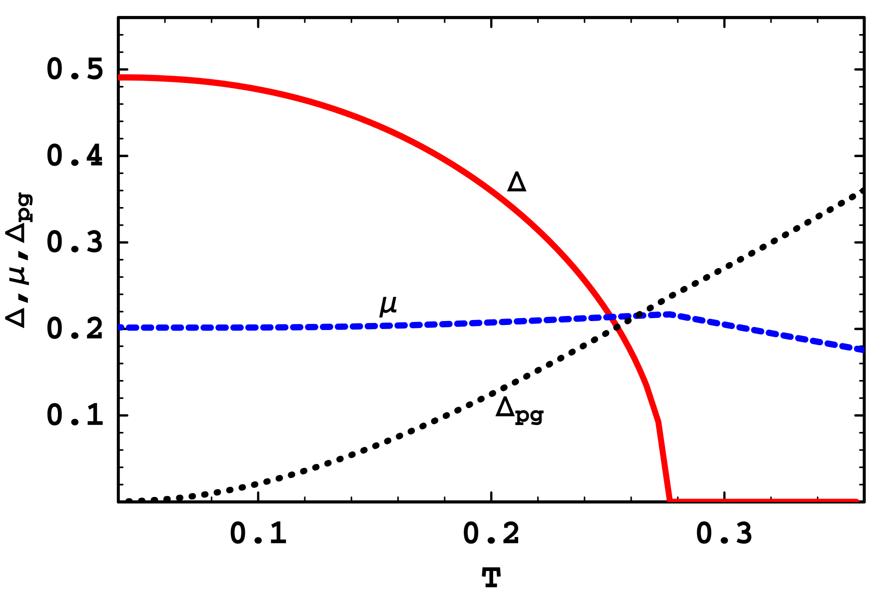
The results are similar to the MFA results

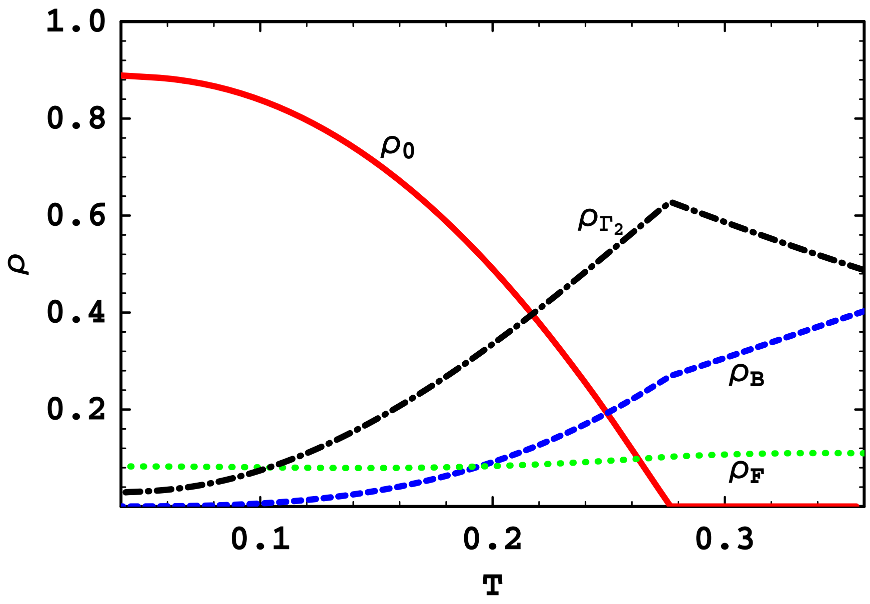
At T=Tc



Fluctuations become important in BEC regime.
In BEC regime T*>Tc.

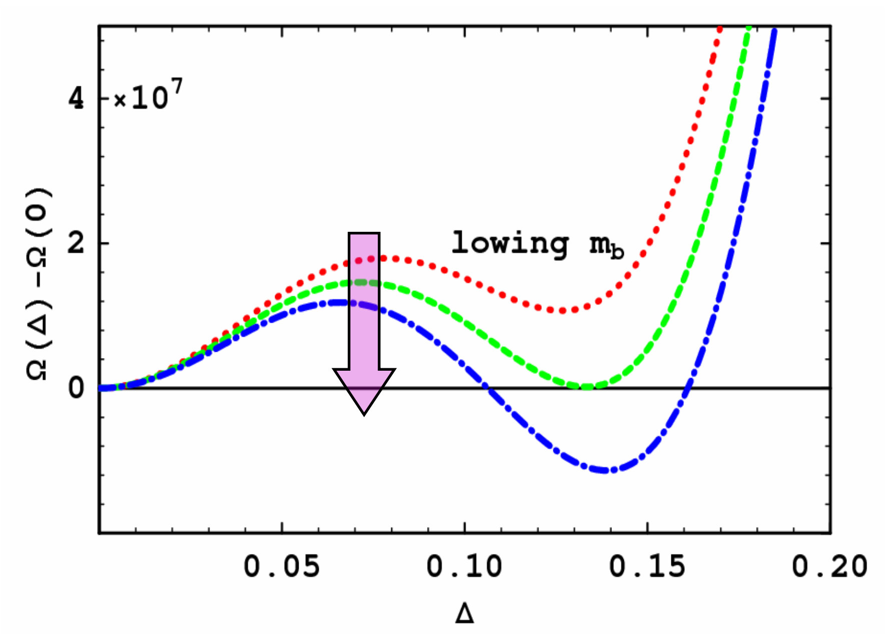
T-dependence

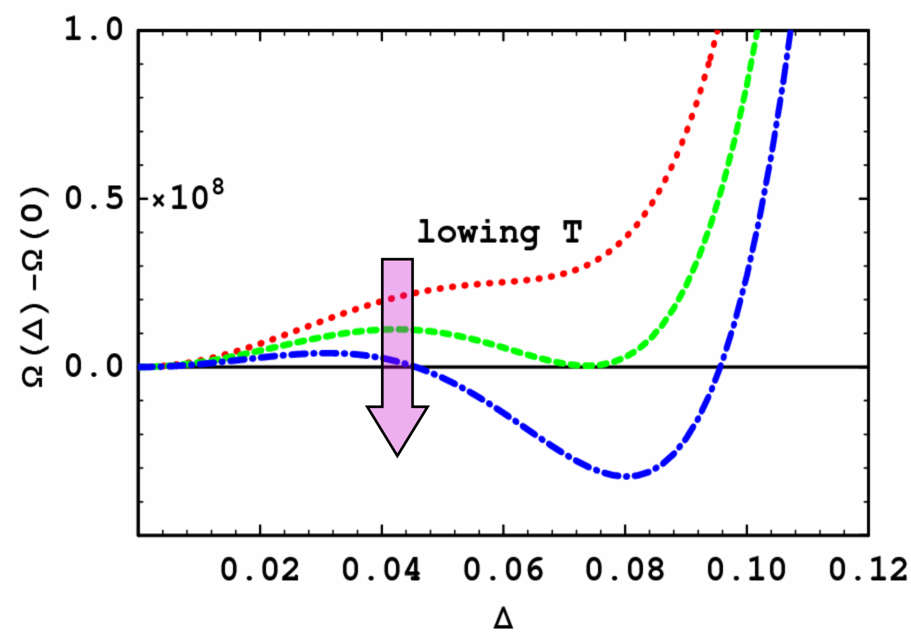




The fluctuation effects become larger.
BEC criterion is related to the minimization of the thermodynamic potential.

1st order phase transition from fluctuations: fixed μ

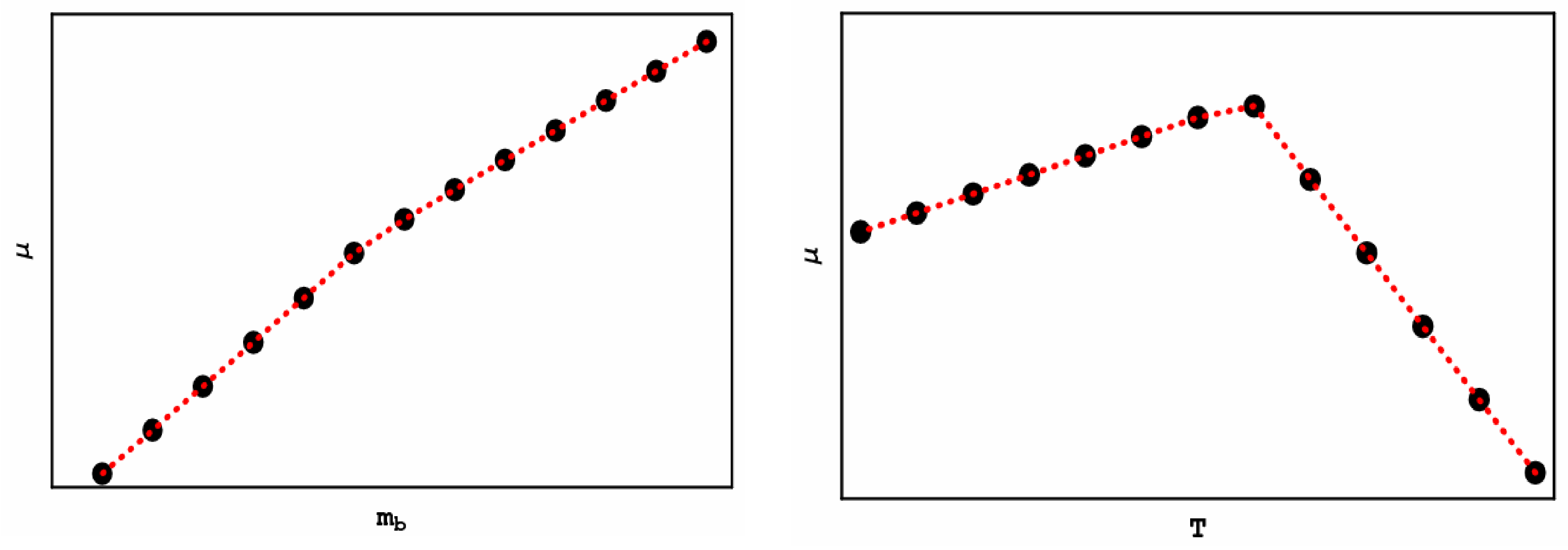




Introduction of Δ^3 term in $\Omega(\Delta)$:

B.I.Halperin, T.C.Lubensky and S. Ma 1974 (magnetic field fluctuations)
I. Giannakis, D. f. Hou, H. c. Ren and D. H. Rischke, 2004
(Gauge Field Fluctuations)
Sasaki, Friman, Redlich, 2007
(baryon number fluctuation in 1st chiral phase transition)

First order phase transition with fixed particle number

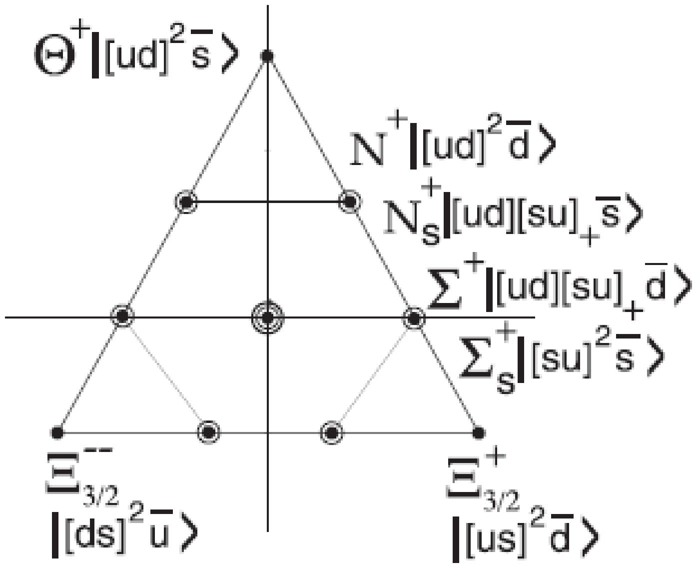


$$s = \frac{\partial \mu}{\partial T}, \qquad L = T_c(s_1 - s_2)$$

Diquarks in baryons

$$(qqq)_{[1]} = \frac{1}{\sqrt{6}}\left[\underbrace{\boxed{(RB - BR)}}_{\varphi_{\overline{G}}}G + \underbrace{\boxed{(BG - GB)}}_{\varphi_{\overline{R}}}R + \underbrace{\boxed{(GR - RG)}}_{\varphi_{\overline{B}}}B\right]$$

Diquark models:
Anselmino, et al., 1993
Abu-Raddad,Hosaka,Ebert,Toki, 2002
Many other papers……

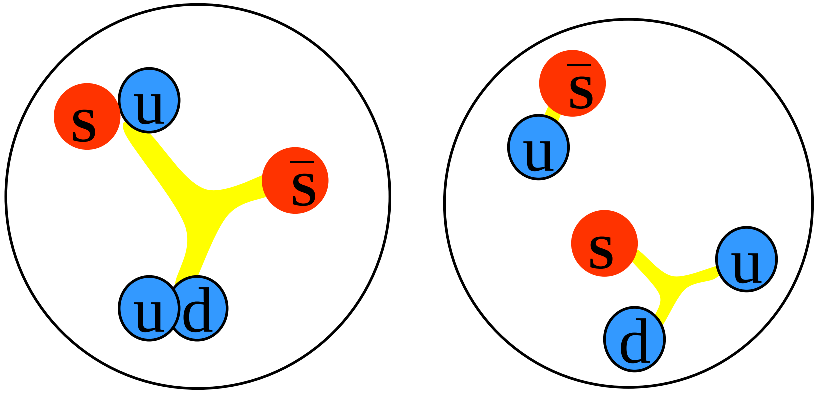


Quarks, diquarks and pentaquarks,
Jaffe, Wilczek, 2004
[Diquarks as building blocks of exotic hadrons]

Diquarks in baryons

Diquark configuration in proton: positive magnetic moments from strange quarks
Zou, Riska, 2005

$$
\begin{aligned}
|\text{proton}\rangle &= |uud\rangle \\
&+\varepsilon_1 \left|[ud][ud]\bar{d}\right\rangle \\
&+\varepsilon_2 \,|[ud][us]\bar{s}\rangle \\
&+\cdots
\end{aligned}
$$



Diquark-cluster **Meson Cloud**

Diquark configuration: inverse mass order in resonances
Zou, 2007

$$N^*(1535),\ J^P = \frac{1}{2}^-,\quad |uud\rangle_{L=1} + \varepsilon\,|[ud][us]\bar{s}\rangle_{ground} + \cdots$$

$$N^*(1440),\ J^P = \frac{1}{2}^+,\quad |uud\rangle_{n=1} + \varepsilon\left|[ud][ud]\bar{d}\right\rangle_{L=1} + \cdots$$

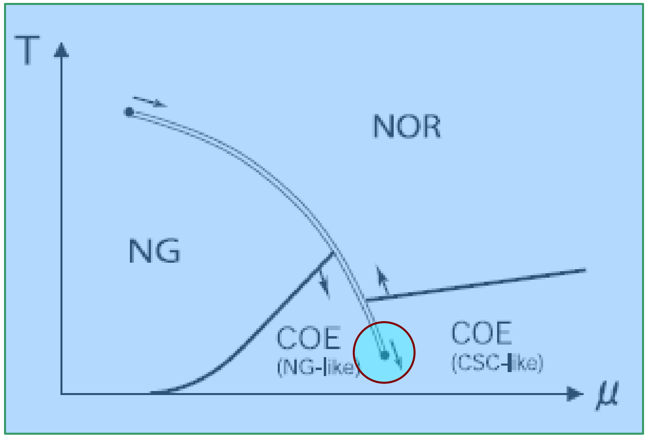
$$\Lambda(1405),\ J^P = \frac{1}{2}^-,\quad |uds\rangle_{L=1} + \varepsilon\,|[ud][us]\bar{u}\rangle_{ground} + \cdots$$

Quark-baryonic matter crossover in N_f=3 dense matter

■ ***Continuity of quark and hadron matter, Schafer, Wilczek, 2000***
[CFL-hadronic matter: a weak coupling realization of confinement and chiral symmetry breaking in idealization of QCD]

BCS-BEC crossover with boson-fermion model	⇒	crossover of quark-baryonic matter

■ ***New critical point induced by the axial anomaly in dense QCD, Hatsuda, Tachibana, Yamamoto, Baym, 2006***



■ *N_f=3, there is a new critical point near chemical potential axis due to coupling of chiral and diquark condensate: quark-nuclear matter crossover*

■ *N_f=2, no critical point: quark-nuclear matter transition*

Baryonic pole structure in quark and nuclear matter: quark-diquark model

Two flavor case

diquark condensate

chiral condensate

$$\mathcal{L}_{NJL} \approx -\frac{1}{2}\overline{\Psi}S^{-1}\Psi - \frac{1}{4G_D}\sum_a |\Delta_a|^2 - G_S(\sigma_u^2 + \sigma_d^2)$$
$$-\frac{1}{2}(\varphi_{aR}, \varphi_{aI})D^{-1}\begin{pmatrix} \varphi_{aR} \\ \varphi_{aI} \end{pmatrix} + \frac{1}{2}\overline{\Psi}\varphi_{ai}\widehat{\Gamma}_{ai}\Psi$$
$$-\frac{1}{G_B}\overline{\mathbf{B}}\mathbf{B} + \overline{\mathbf{B}}\widehat{\Gamma}_{Bi}\Psi_a\varphi_{ai} + \varphi_{ai}\overline{\Psi}_a\widehat{\Gamma}^*_{Bi}\mathbf{B},$$

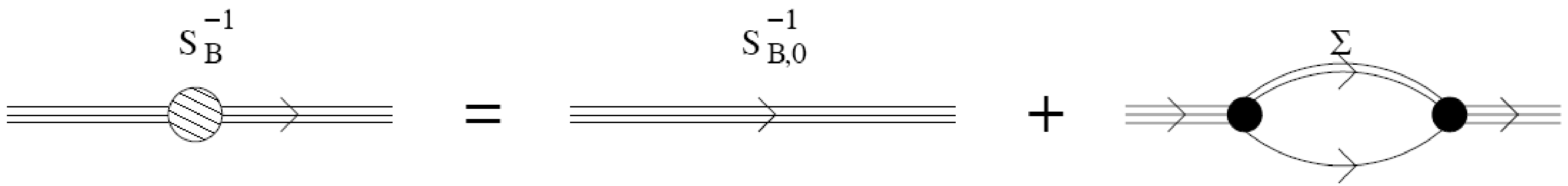
J.-C. Wang, et al, in preparation

Relativistic version of trimer

$$\mathbf{B} = (B, B_c)^T$$
$$\overline{\mathbf{B}} = (\overline{B}, \overline{B}_c)$$

$$\widehat{\Gamma}_{aR} = \frac{i}{\sqrt{2}}g\gamma_5\tau_2 J_a\tau_1^{NG}, \quad \widehat{\Gamma}_{aI} = \frac{i}{\sqrt{2}}g\gamma_5\tau_2 J_a\tau_2^{NG},$$
$$\widehat{\Gamma}_{BR} = \frac{1}{\sqrt{2}}1_{NG}, \quad \widehat{\Gamma}_{BI} = i\frac{1}{\sqrt{2}}\tau_3^{NG}.$$

Baryonic pole structure in quark and nuclear matter



$$S_B^{-1} = \frac{1}{2G_B} + \Sigma$$

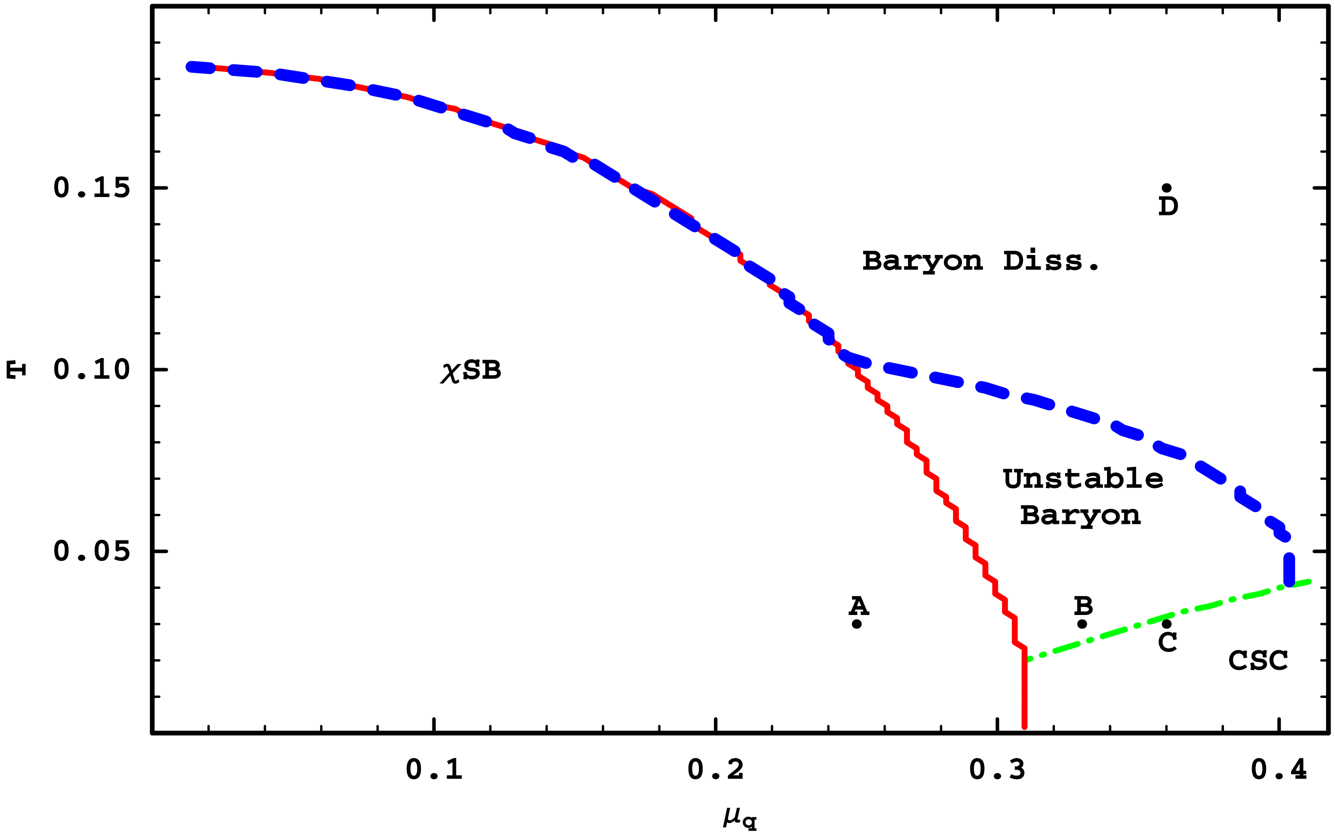
$$S_B^{-1}(P) = S_{B,s}^{-1}(P)\gamma_0\Lambda_{Bp}^s$$

$$S_{B,s}^{-1}(P) = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{1}{2G_B} + 2\Sigma_{11}^{(1)} + \Sigma_{11}^{(2)}\right)\Lambda_{Bp}^s\gamma_0\right]$$

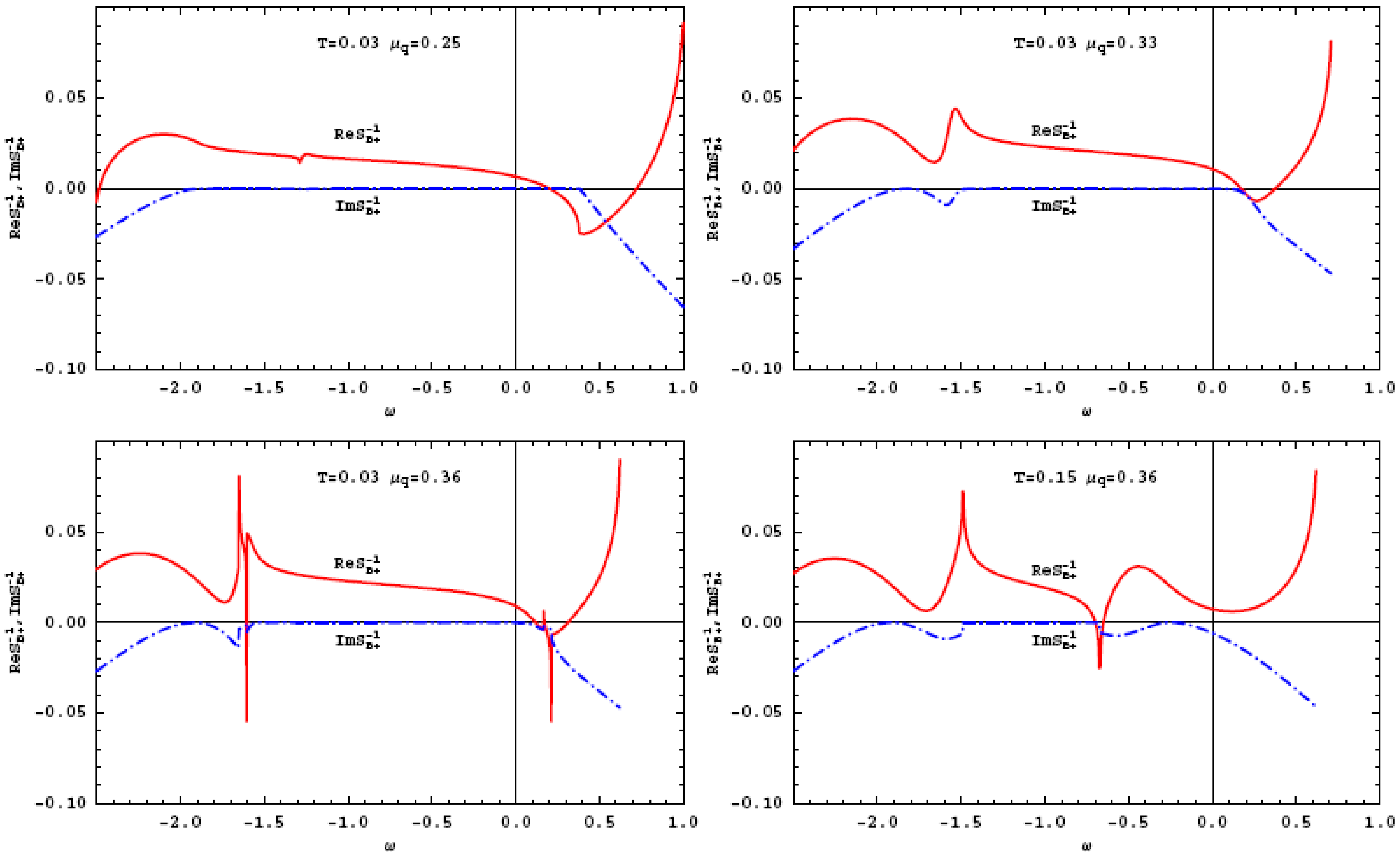
$\boxed{\mathrm{Re}[S_{B,s}^{-1}(\omega + i\frac{\Gamma}{2}, \mathbf{p} = 0)] = 0}$ → **baryonic pole**

$\boxed{\mathrm{Im}[S_{B,s}^{-1}(\omega + i\frac{\Gamma}{2}, \mathbf{p} = 0)]}$ → **pole width**

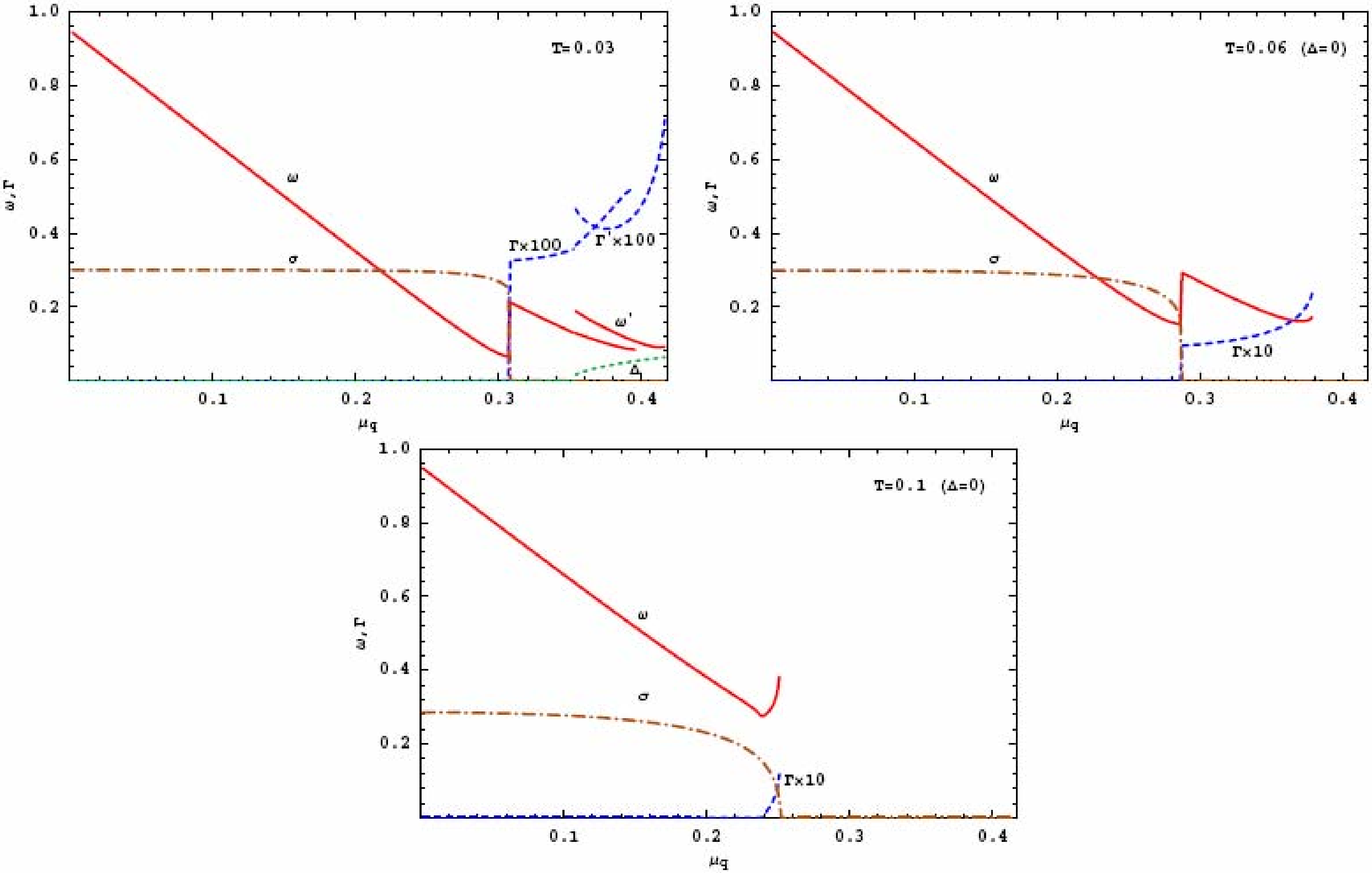
Baryonic pole structure in quark and nuclear matter: phase diagram



Analytic property of baryonic pole



Baryonic pole and width



Summary

CSC in weak and intermediate couplings has been extensively studied.

Relativistic BCS-BEC crossover can be well described in boson-fermion model within or beyond MFA.

With chemical potential mismatch, part of gapless solutions are stable in strong couplings.
[Recent experiments in cold atom system]

Fluctuation effects lead to first order transition.

Baryon formation is controlled by chiral symmetry and can be described by quark-diquark model in dense matter. [99% of nucleon mass from χ-symmetry, see Stachel's talk]

Outlook

Our model can be extended to discuss quarkoynic continuity with finite chemical potential where the confinement and chiral symmetry breaking do not coincide (L. Mclerran and R. D. Pisarski).

Quark-baryonic matter crossover for three flavor case in quark-diquark picture.