Modelling the behavior of the positron plasma temperature in antihydrogen experiments

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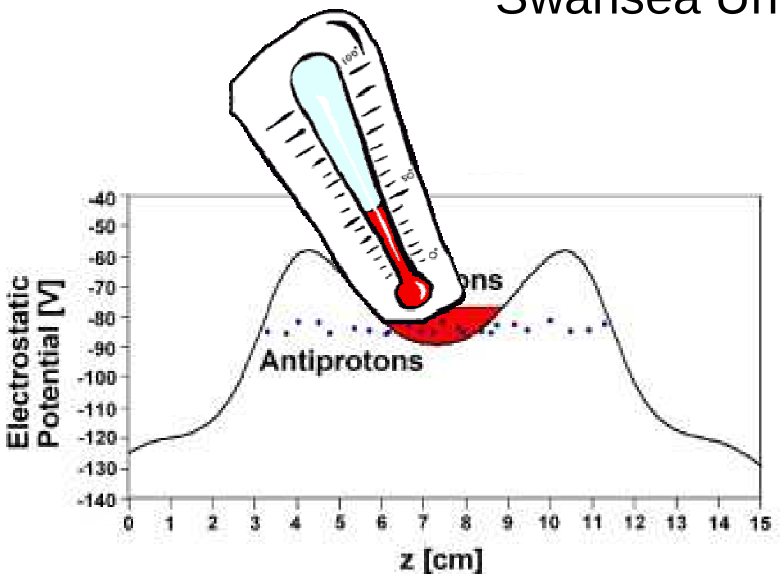
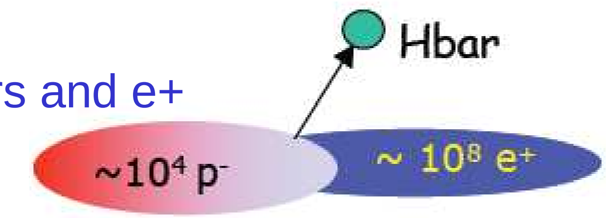


LEAP2013
Uppsala, June 11, 2013

Why e+ plasma temperature is important?

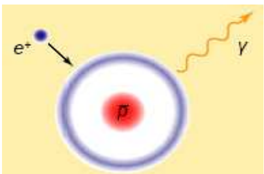
For experiments where Hbars are produced by overlapping pbars and e+



Hbar production rates depend on e+ plasma temperature

in both the expected Hbar production mechanisms:

Radiative recombination

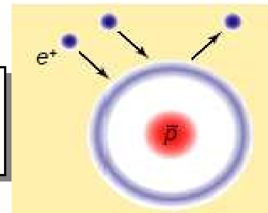


$$\overline{\mathrm{p}} + e^{+} \rightarrow \overline{\mathrm{H}} + \gamma$$

$$\lambda_{Hbar} \propto n_e T_e^{-0.67}$$

3-body recombination

$$\overline{\mathrm{p}} + e^{+} + e^{+} \rightarrow \overline{\mathrm{H}} + e^{+}$$



$$\lambda_{Hbar} \propto n_e^2 T_e^{-4.5}$$

λ_{Hbar} = Hbar production rate $\quad n_e$ = e+ density $\quad T_e$ = e+ temperature

e+ plasma temperature evaluations

- Evaluations of pbar temperature in Hbar experiments exist

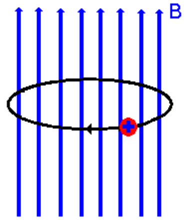
See, for example: S.L. Rolston, G. Gabrielse, Hyperf. Inter. 44 (1988) 233.
J. Bernard et al., NIM A 532 (2004) 224

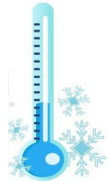
- No direct evaluation of e+ plasma temperature for Hbar experiments

we present a simple model to evaluate T_e

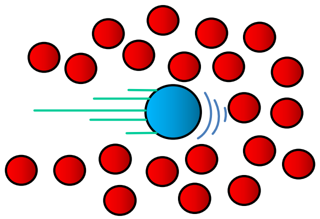
The processes affecting the e+ plasma temperature

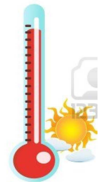
- synchrotron radiation (→ plasma cooling)



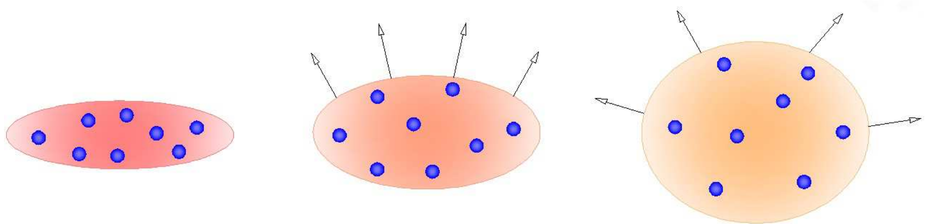


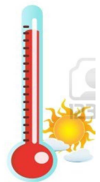
- antiprotons energy loss in the plasma (→ plasma heating)





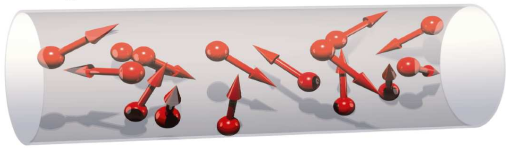
- e+ plasma expansion (→ plasma heating)





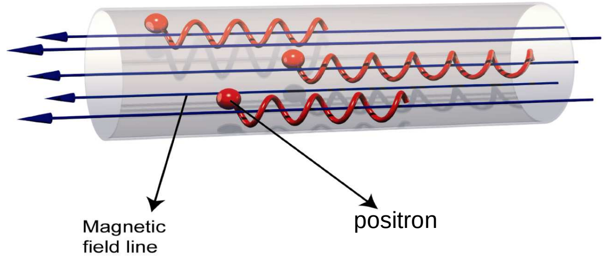
e+ plasma temperatures: 1 or 2?

B=0



T_e ← only 1 temperature

B>0



For strongly magnetized plasma

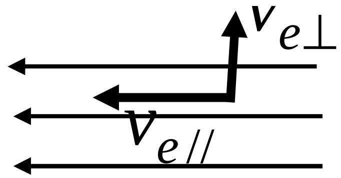
possibly $T_{e\perp} \neq T_{e//}$

But when $\lambda_{coll} >> \dfrac{1}{\tau_e}$

e+-e+ collision frequency

synchrotron cooling time

$$\Rightarrow \quad T_{e\perp} = T_{e//} = T_e$$



$$T_{e\perp} = \frac{m_e v_{e\perp}^2}{k_B}$$

$$T_{//} = \frac{m_e v_{e//}^2}{k_B}$$

$$\tau_e = \frac{4\pi\varepsilon_0 m_e^3 c^2}{e^4 B^2}$$

λ_{coll}

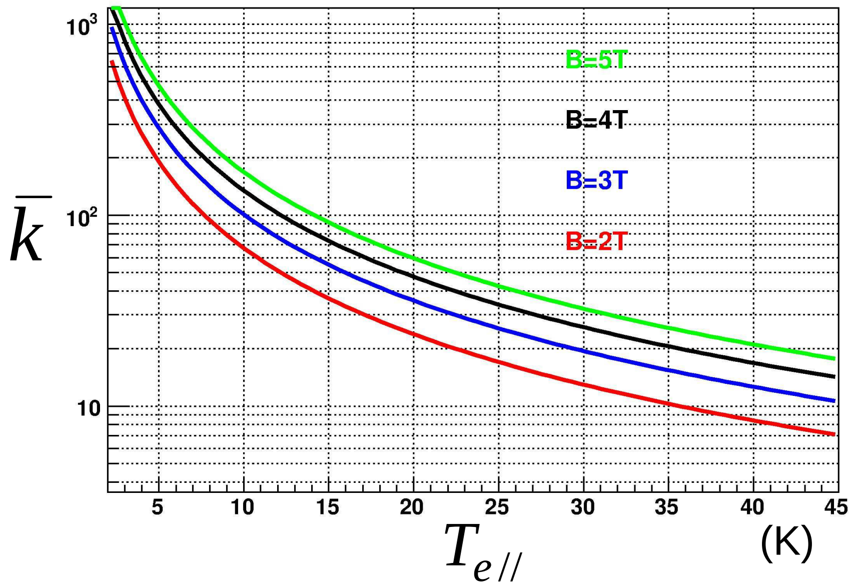
$$\lambda_{coll} = n_e \bar{v} \bar{b}^2 I(\bar{k})$$ ← See, for example Robicheaux JPB 2008

$$\bar{v} = \sqrt{2k_B \frac{T_{//}}{m_e}}$$

$$\bar{b} = \frac{2e^2}{4\pi\varepsilon_0 k_B T_{//}}$$

$$\bar{k} = \frac{eB\bar{b}}{m\bar{v}}$$

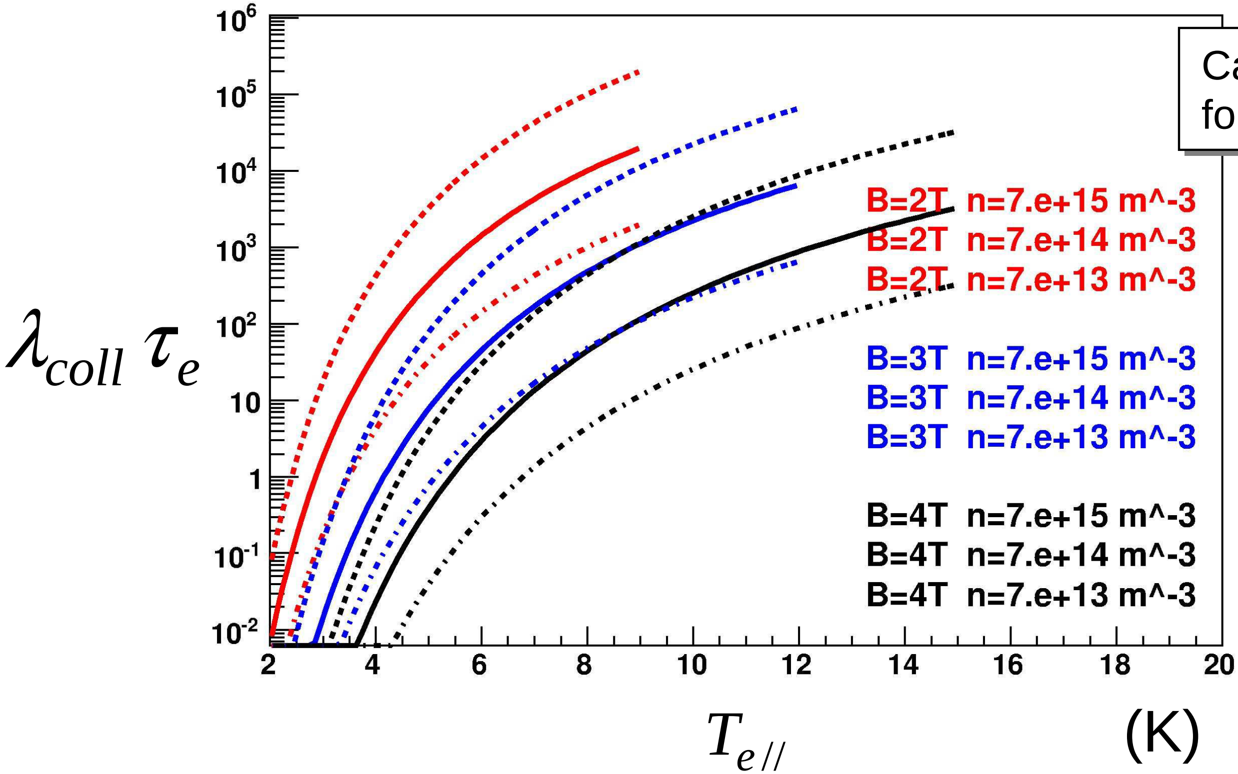
$I(\bar{k})$ = suppression due to *B*



If $\bar{k} >> 1$ (i.e. for strong magnetized plasma, see Glinsky et al. PhysFluids1992).

$$I(\bar{k}) \cong \exp\left[-\frac{5(3\pi\bar{k})^{2/5}}{6}\right] \times \left[\frac{1.83}{\bar{k}^{7/15}} + \frac{20.9}{\bar{k}^{11/15}} + \frac{0.347}{\bar{k}^{13/15}} + \frac{87.8}{\bar{k}^{15/15}} + \frac{6.68}{\bar{k}^{17/15}}\right]$$

$$\lambda_{coll} \gg \tau_e^{-1} \quad ? \qquad \Rightarrow \qquad T_{e\perp} = T_{e//} = T_e$$



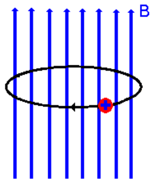
Calculations only for $\bar{k} \gg 1$

- For ATHENA, ALPHA, ASACUSA: $\lambda_{coll}\,\tau_e \gg 1$

- For ATRAP (B=5.4 T, n_e= 4 10^{13} m^{-3}, assuming environmental temperature =4.2 K):

$\lambda_{coll}\,\tau_e < 1$ for low temperatures $\Rightarrow T_{e\perp} \neq T_{e//}$ ← Not considered in the model

Synchrotron radiation cooling



due to e+ cyclotron rotation in **B**

From Larmour formula for Maxwellian distribution:

Assuming $T_{e\perp} >> T_{res}$ (↑ environmental temperature)

$$\frac{dT_{e\perp}}{dt} = -\frac{3}{2}\frac{1}{\tau_e}T_{e\perp} \qquad \text{(s1)}$$

$$\tau_e = \frac{4\pi\varepsilon_0 m_e^3 c^2}{e^4 B^2}$$

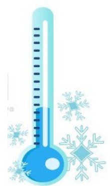
if $\lambda_{coll} >> \frac{1}{\tau_e}$ then $T_{e\perp} = T_{e//} = T_e$

(s1) becomes $\frac{dT_e}{dt} = -\frac{1}{\tau_e}T_e$

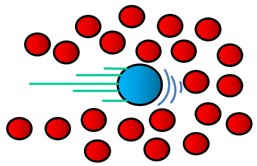
If $T_e \approx T_{res}$ $\Rightarrow$

$$\boxed{\frac{dT_e}{dt} = -\frac{T_e - T_{res}}{\tau_e}}$$

$\left(\lambda_{coll} >> \frac{1}{\tau_e}\right)$

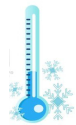


Antiproton energy loss



due to Rutherford collisions

$$\frac{dT_i}{dt} = -\frac{1}{\tau_i}\left(T_i - T_e\right)$$



T_i = pbar temperature

with:

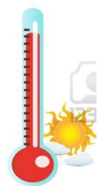
$$\tau_i = \frac{3 m_e m_i c^3}{8\sqrt{2\pi} n_e Z^2 \ln(\Lambda)} \left(\frac{4\pi\varepsilon_0}{e^2}\right)^2 \left(\frac{k_B T_i}{m_i c^2} + \frac{k_B T_e}{m_e c^2}\right)^{\frac{3}{2}}$$

L. Spitzer, Physics of Fully Ionised Gases, 1956.

$$\ln(\Lambda) = \ln\left(\frac{b_{\max}}{b_{\min}}\right) = \ln\left(4\pi\left(\frac{\varepsilon_0 k_B}{e^2}\right)^{\frac{3}{2}} \frac{1}{Z}\sqrt{\frac{T_e}{n_e}}\left(T_e + \frac{m_e}{m_i}T_i + 2\sqrt{\frac{m_e}{m_i}}\sqrt{T_e T_i}\right)\right)$$

when T_i decreases, T_e increases

$$\frac{dT_e}{dt} = \frac{1}{\tau_i}\frac{N_i}{N_e}\left(T_i - T_e\right)$$

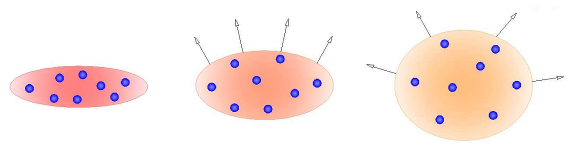


N_i = pbars number

N_e = e+ number

Sharing the same volume

e+ plasma expansion



due to not optimal vacuum and/or e.m. field asymmetry/misalignement

From energy conservation (electrostatic energy → internal energy):

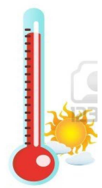
$$\frac{dT_e}{dt} = \frac{e^2}{6\varepsilon_0 k_B} n_e r_e^2 \frac{1}{r_e} \frac{dr_e}{dt}$$

r_e = e+plasma radius

n_e = e+plasma density

Since in Hbar experiment $n_e r_e^2$ is costant =>

$$\boxed{\frac{dT_e}{dt} = \frac{e^2}{6\varepsilon_0 k_B} \left(n_e r_e^2\right)_0 \frac{1}{r_e} \frac{dr_e}{dt}}$$



expansion velocity

The model: equations system

Considering: synchrotron radiation + pbar energy loss + plasma expansion

the e+ temperature must satisfy the system:

$$\frac{dT_e}{dt} = -\frac{T_e - T_{res}}{\tau_e} + \frac{1}{\tau_i}\frac{N_i}{N_e}\left(T_i - T_e\right) + \frac{e^2}{6\varepsilon_0 k_B}\left(n_e r_e^2\right)_0 \frac{1}{r_e}\frac{dr_e}{dt}$$

$$\frac{dT_i}{dt} = -\frac{1}{\tau_i}\left(T_i - T_e\right) \qquad \left(\lambda_{coll} >> \frac{1}{\tau_e}\right)$$

- The equation system is linear in T_{res} => the T_e behaviors are independent of T_{res}

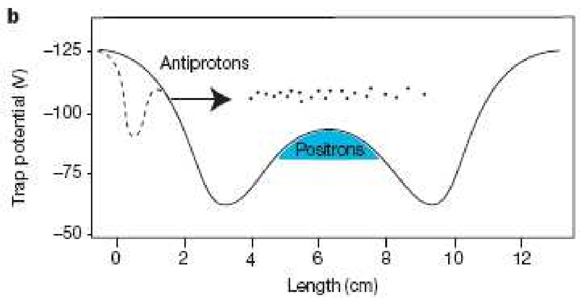
The model: modified equations system

2 parameters to consider the effect of the partial pbars-positrons overlap:

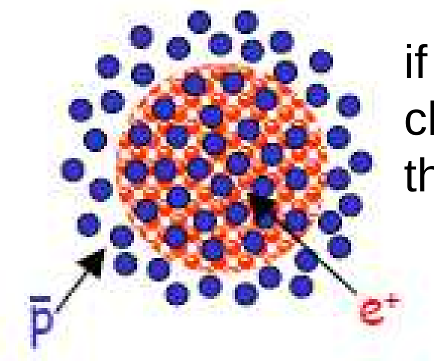
P parameter

pbars are not continuosly inside the e+ plasma

$$P \leq 1$$



M parameter



if the antiprotons cloud is larger than the e+ cloud

$$M \leq 1$$

$$\frac{dT_e}{dt} = -\frac{T_e - T_{res}}{\tau_e} + M\,P\,\frac{1}{\tau_i}\frac{N_i}{N_e}(T_i - T_e) + \frac{e^2}{6\varepsilon_0 k_B}\left(n_e r_e^2\right)_0 \frac{1}{r_e}\frac{dr_e}{dt}$$

$$\frac{dT_i}{dt} = -P\frac{1}{\tau_i}(T_i - T_e) \qquad \left(\lambda_{coll} >> \frac{1}{\tau_e}\right)$$

The solutions (T_e, T_i) depend on many parameters:

$$B \quad E_0 \quad T_0 \quad T_{res} \quad M \quad P \quad N_e \quad N_i \quad v_0 = \frac{dr_e}{dt} \quad \left(n_e r_e^2\right)_0$$

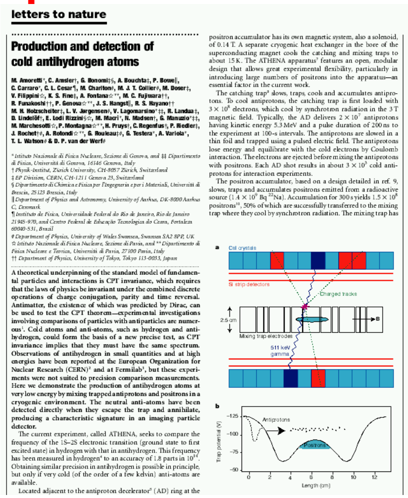
$\Longrightarrow$ no universal solution exists

Each particular case has to be considered

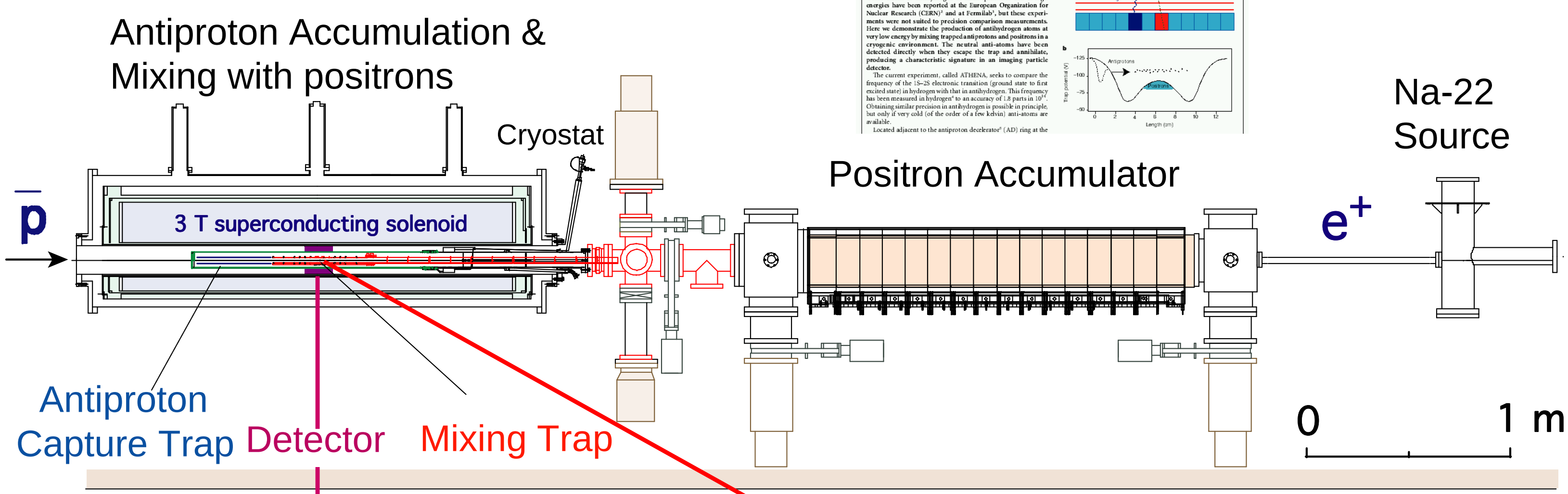
The ATHENA case …

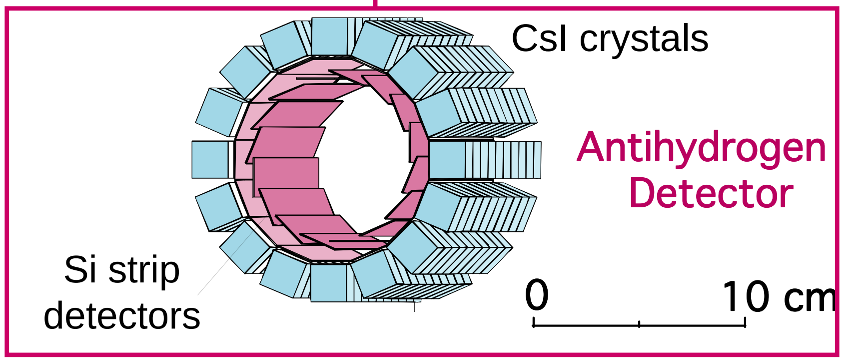
The ATHENA Experiment

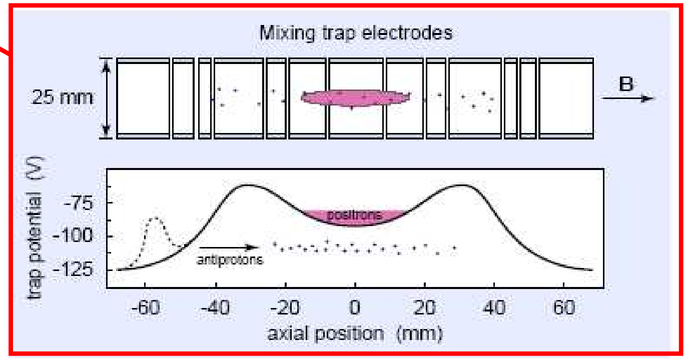
Main goal: production & studing of cold $\overline{H}$



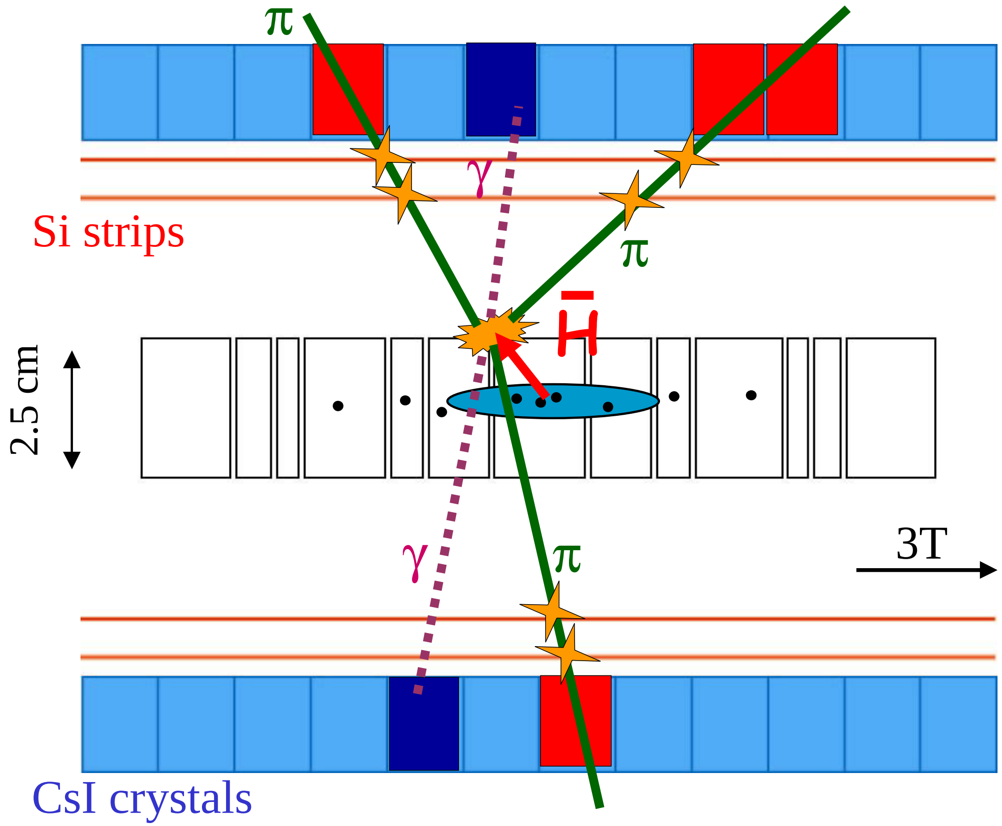
letter to Nature
Nature 419, 456-459 (2002)







How ATHENA worked



- 10^4 $\bar{p}$ & 10^8 e^+ are mixed in Penning trap
- $\bar{H}$ form and fly away
- $\bar{H}$ annihilate on trap wall

Offline selection of $\bar{H}$ annihilation:

Coincidence in space & time (<5ms) of:

- $\bar{p}$ annihilation (charged pion vertex)
- e^+ annihilation (2 back-to-back 511 keV γ)

The ATHENA case

COLD MIX 2003

- The condition $\lambda_{coll}\ \tau_e >> 1\ (\Rightarrow\ T_{e\perp} = T_{e//} = T_e)$ is always satisfied

- Values used in the simulations:

$$B = 3\,\mathrm{T} \qquad \tau_e = \frac{1}{0.39\,B^2}\ \mathrm{s} = 0.3\ \mathrm{s}$$

$$M = 1 \qquad P = 0.3$$

N.B. We have found that the P value affects only the time of the pbars slowing down

E_0=16 eV ← pbar initial energy

T_0=Tres=15 K ← e^+ initial temperature

N_i=1 10^4 ← pbars number

N_e=7.5 10^7 ← e+ number

T_{res}=15 K ← environment temperature

Not measured, lower limit

Quantities modified:

$v_0 = \frac{dr_e}{dt} = 0 - 20\ 10^{-6}$ m/s ← e^+ expansion velocity

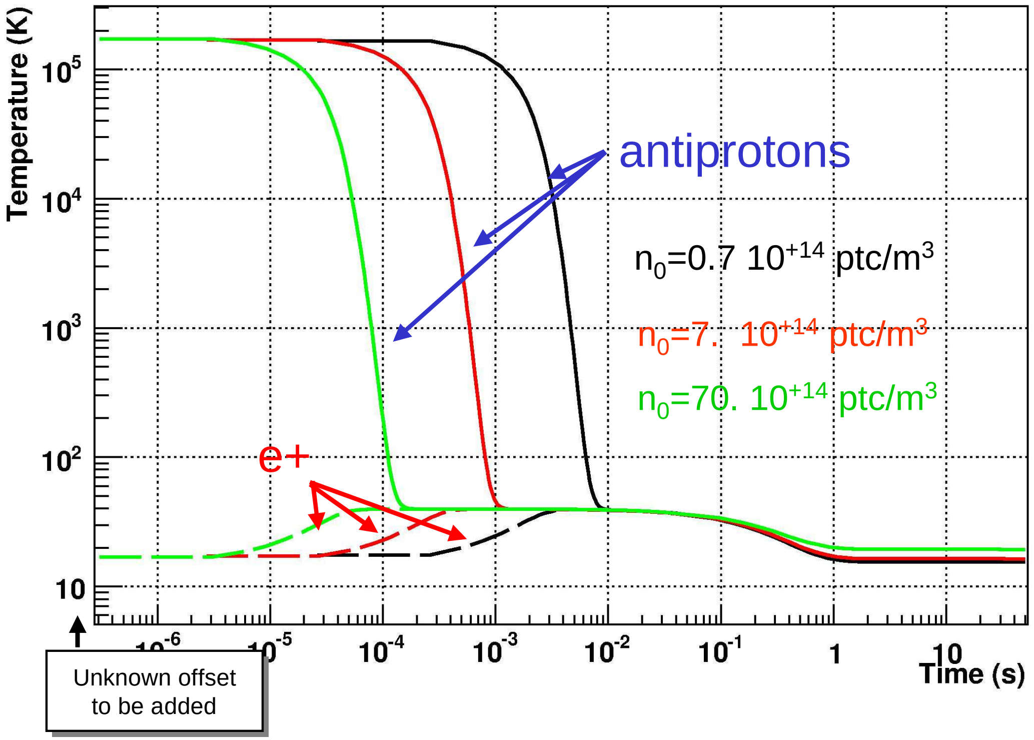
n_0=0.7-70 10^{+14} ptc/m^3 ← e^+ initial density

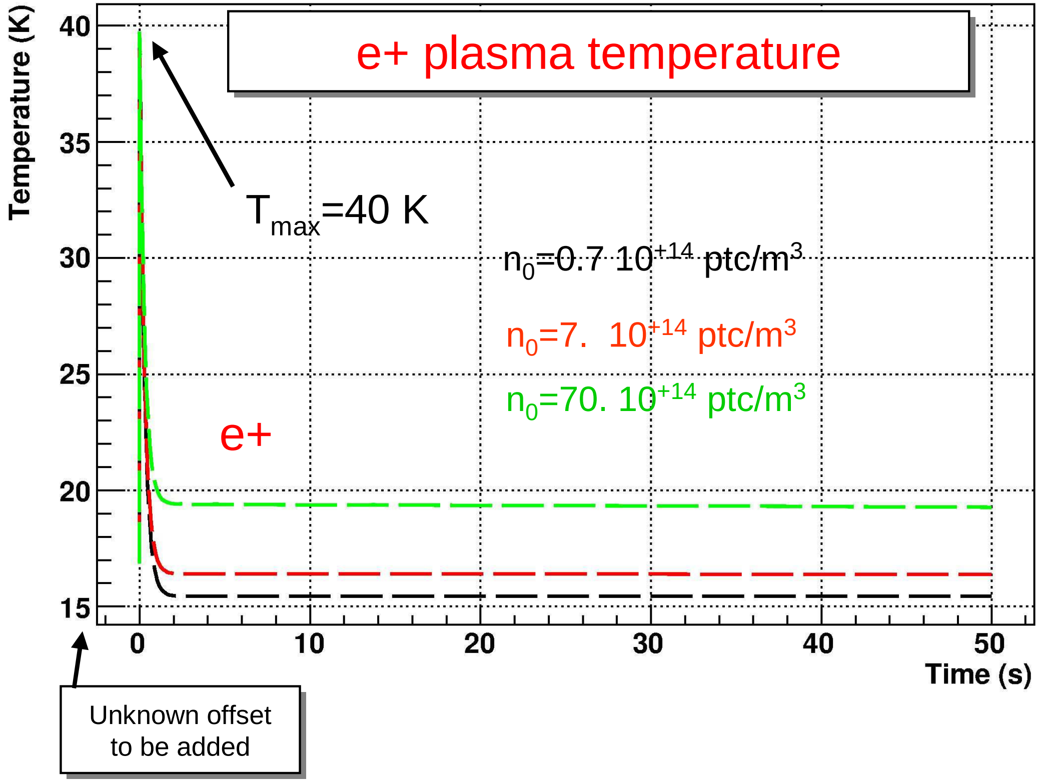
R_0=1. 10^{-3} *(7. 10^{+14} / n_0) m ← e^+ initial radius

The ATHENA case

COLD MIX 2003

$$v_0 = 0.2\,10^{-6}\ \mathrm{m/s}$$ ← experimental

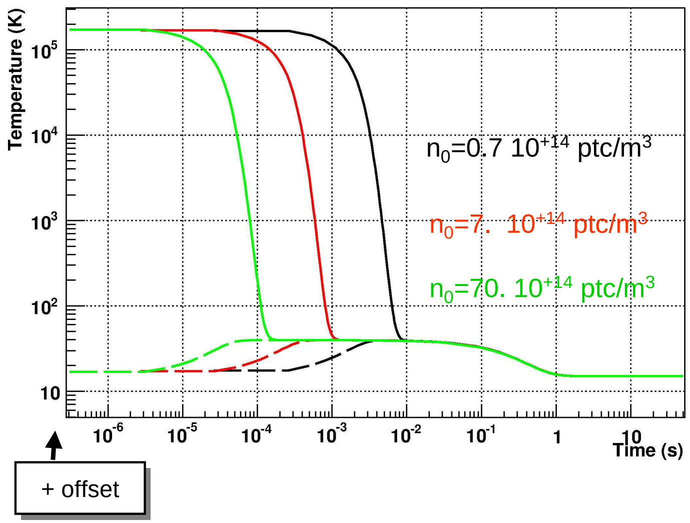


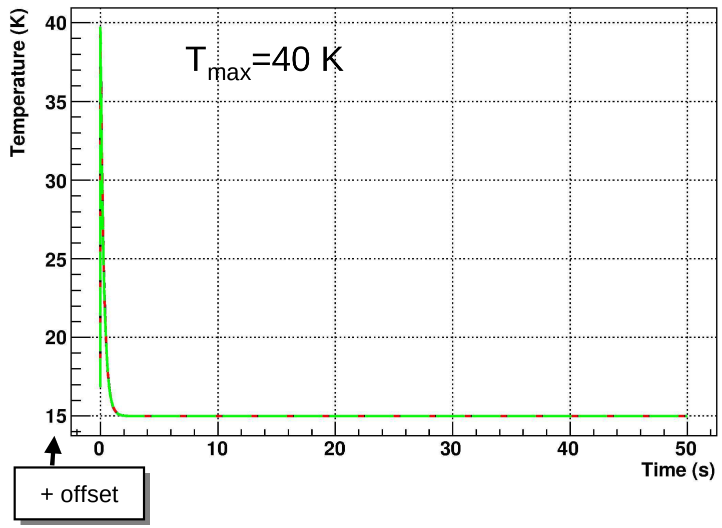


- $T_{pbar} \approx T_e$ after $\Delta t \approx \frac{10^{12}}{n_e}\,\mathrm{s}$
- T_e maximum increase is $\approx 25\ \mathrm{K}$
- Equilibrium of T_e after less than 1 s
- T_e values at equilibrium depend on the density (← expansion), but the differences are small

The ATHENA case – different expansion velocity

$v_0 = 0$ no expansion



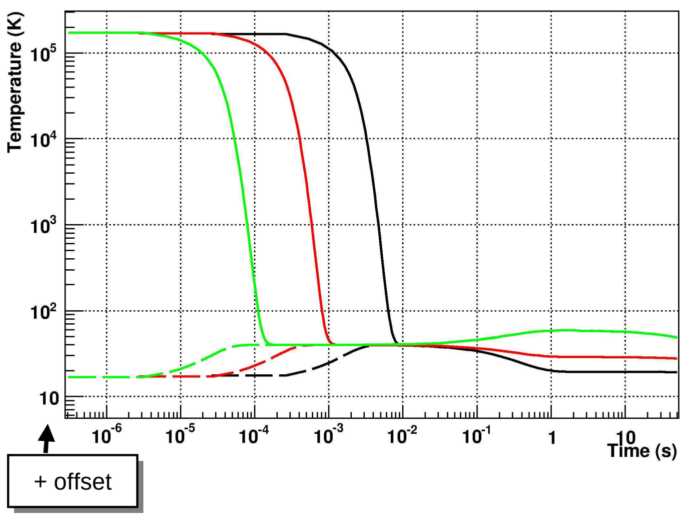


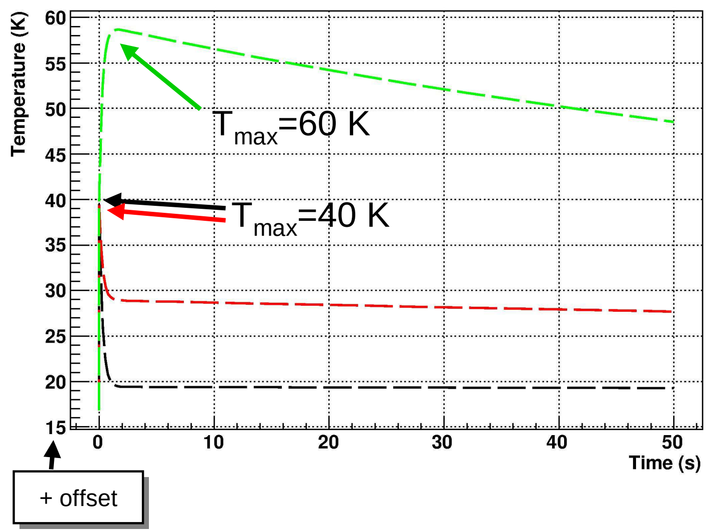
Similar to the case with

$v_0 = 0.2\,10^{-6}$ m/s

but same T_e at late times $(T_e \approx T_{res})$

$v_0 = 2\,10^{-6}$ m/s ← x10 experimental value





Different maximum T_e values

Different T_e at late time

For high densities T_e slowly decreases

⇒ expansion process can strongly influence T_e

The ATHENA case - How to check the model?

Direct check of the model with the experimental data is difficult

We can use the “reinjection data”

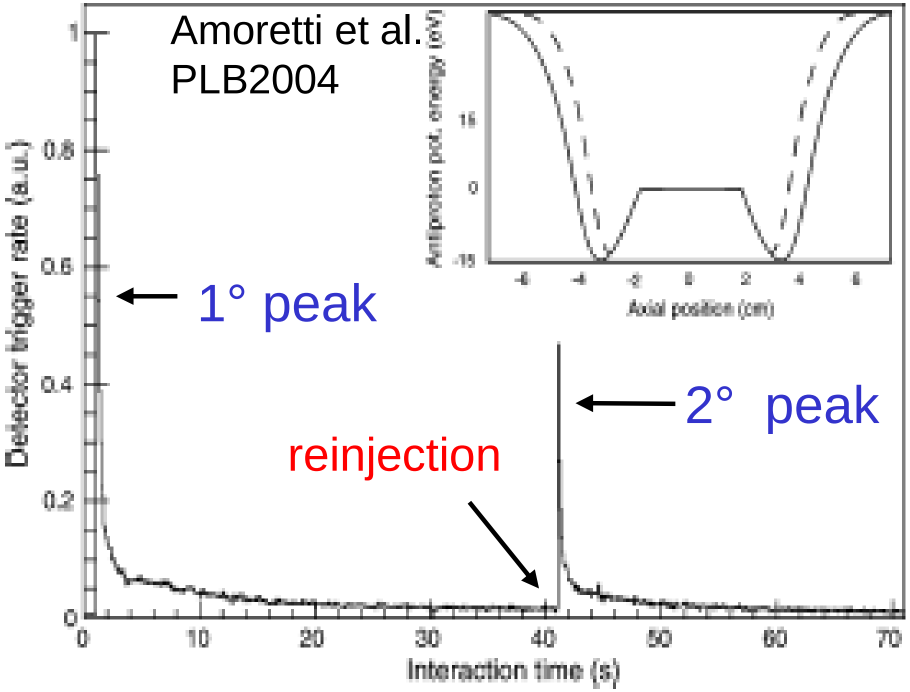


Fig. 5. Trigger rate during re-injection after 40 s. The corresponding peak is mainly due to antihydrogen production. In the inset, the on-axis potentials applied to re-inject the antiprotons into the positron plasma, are displayed (dashed lines).

The experiment

- At t=0 standard e+-pbar mixing. The 1° peak corresponds to the detected Hbars. Other Hbars are field-ionized and their pbars can fill the lateral wells
- After 40s pbars are re-injected into e+ plasma by adiabatically compressing the 2 side wells. Hbar formation restart and 2° peak appears.

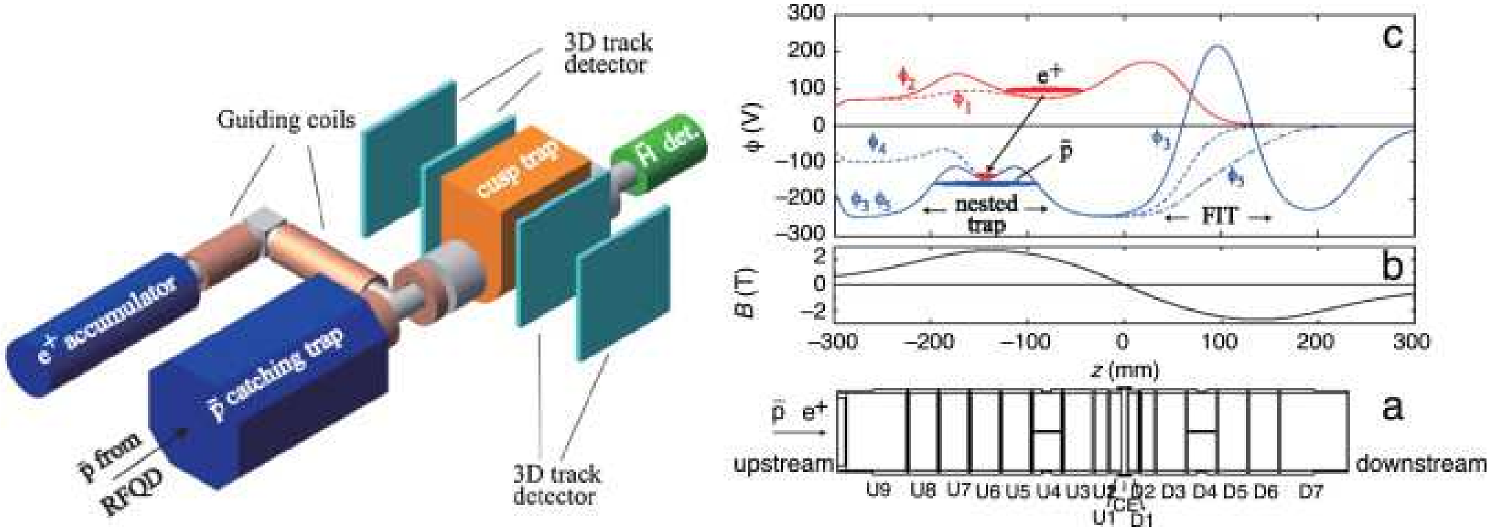
2° peak is similar to the 1° peak

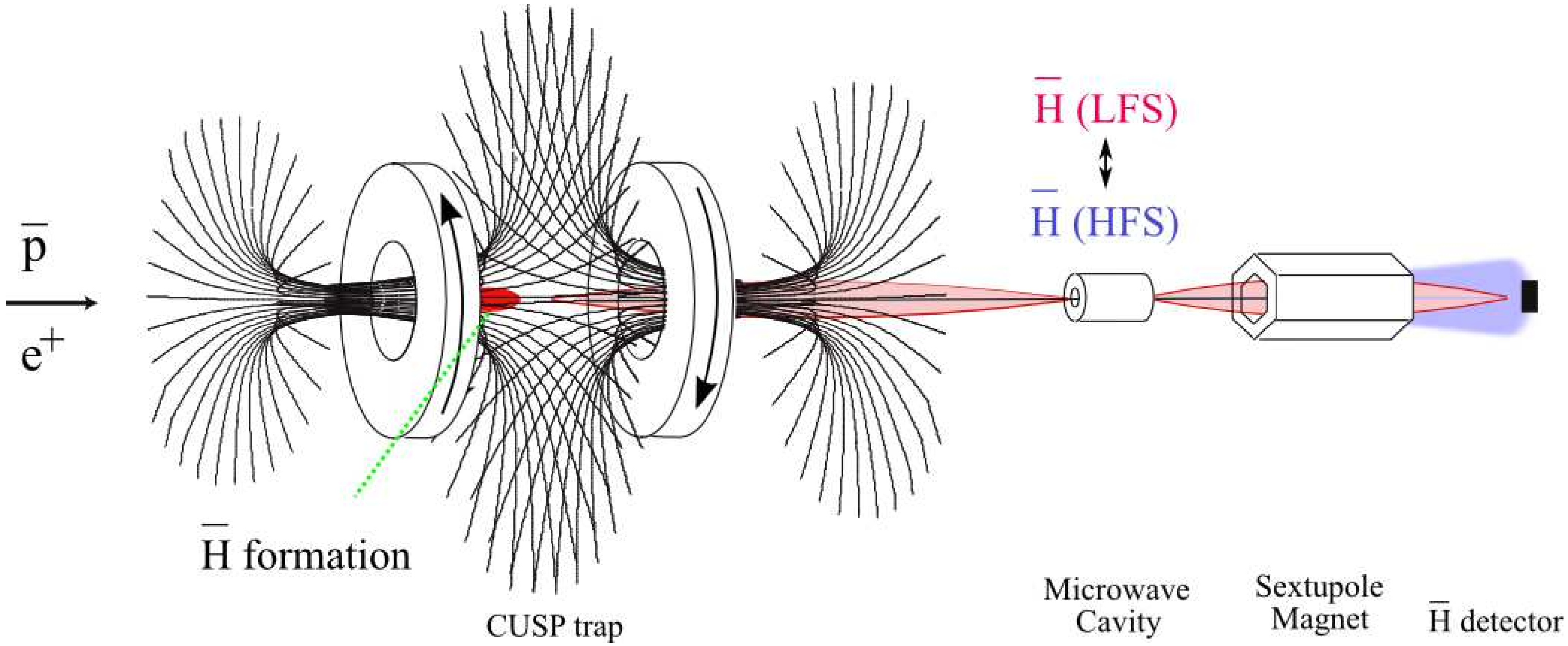
⇔ e+ plasma conditions (@40s) are similar to the initial ones (t=0)

In agreement with simulations (where $T_e \rightarrow T_{res}$ after 2s)

The ASACUSA case …

The ASACUSA Experiment



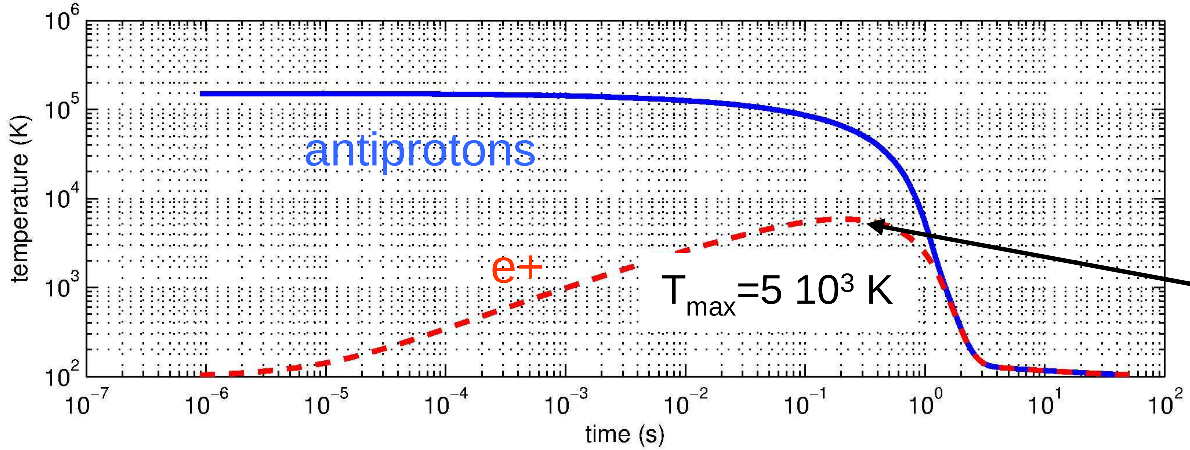


The ASACUSA case 2010

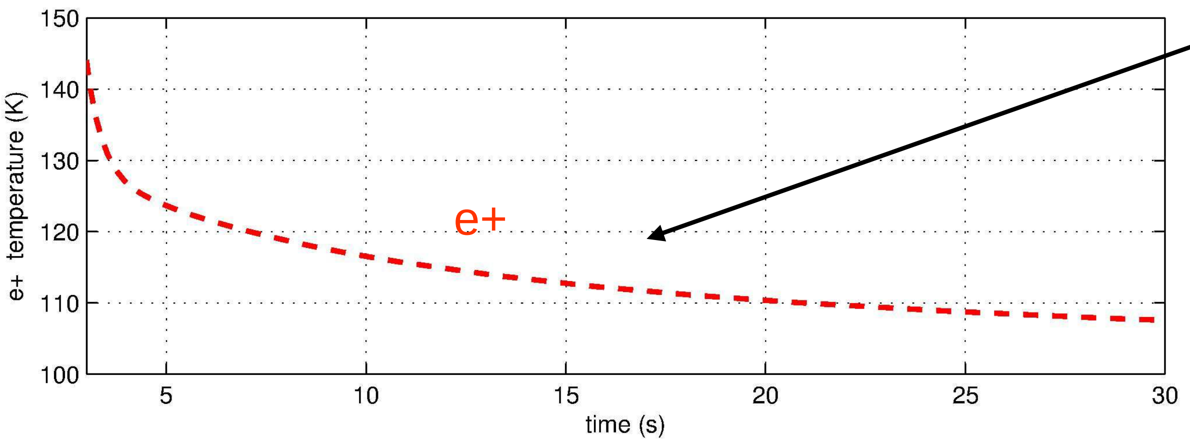
$B = 2.7\ \mathrm{T}$ $\tau_e = \frac{1}{0.39\ B^2}\ \mathrm{s}$ $P = 0.3$ $M = 1$ E_0=13 eV ← pbar initial energy

R_0=0.5 10^{-3} m ← e+ initial radius
N_e=3 10^6 ← e+ number
n_0=1. 10^{+13} ptc/m^3 ← e+ initial density

T_0=Tres=100 K ← e+ initial temperature
N_i=3 10^5 ← pbars number
v_0=70 10^{-6} m/s ← e+ expansion velocity
T_{res}=100 K ← environment temperature
↖ experimental



Large number of pbars strongly increases T_e

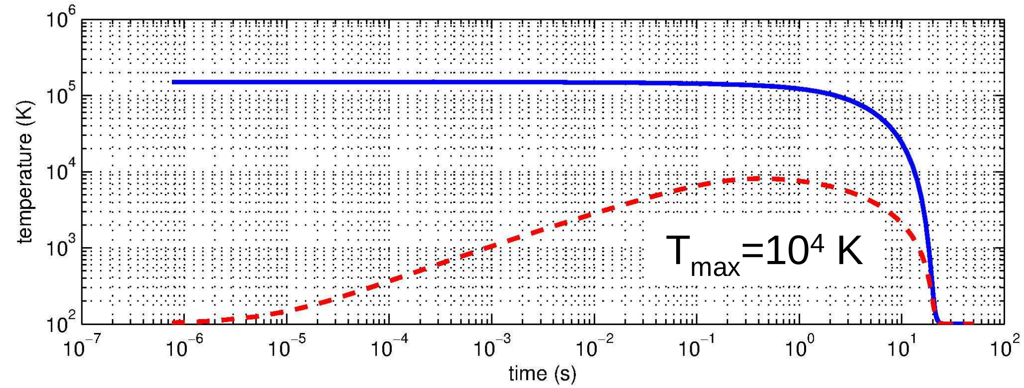


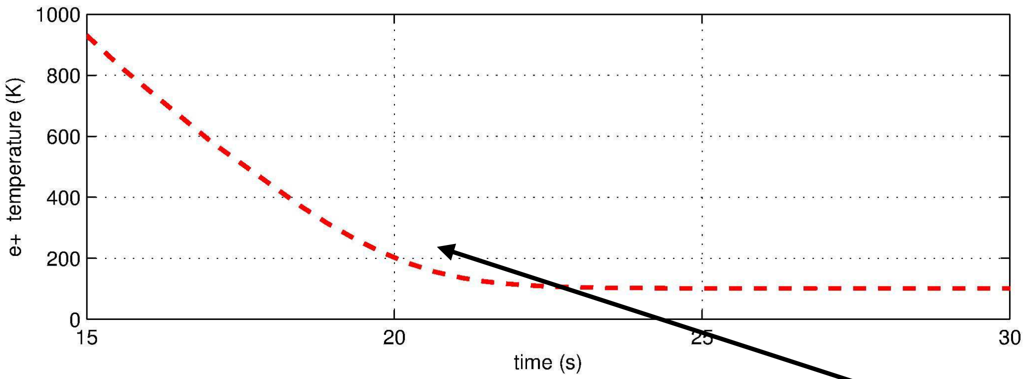
T_e slowly decreases due to the fast expansion and large pbars number

The ASACUSA case - different e+ values 2010

lower e+ values (number & dens.)

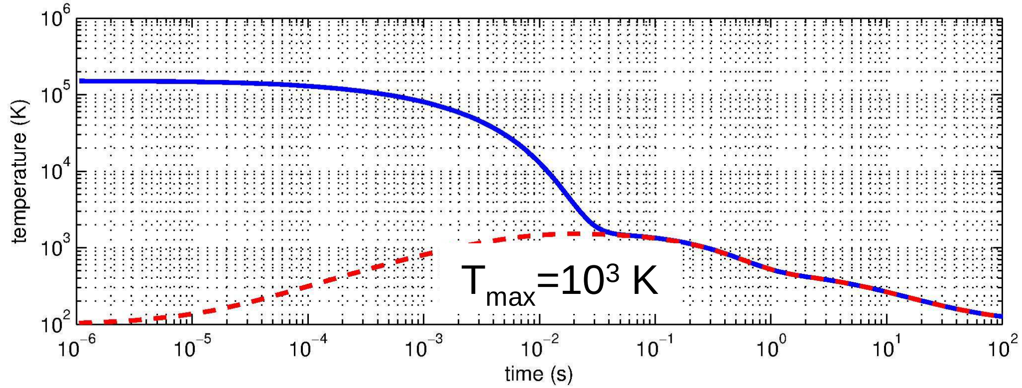
n_0=1. 10^{+12} ptc/m^3 e^+ initial density
N_e=3 10^5 e+ number

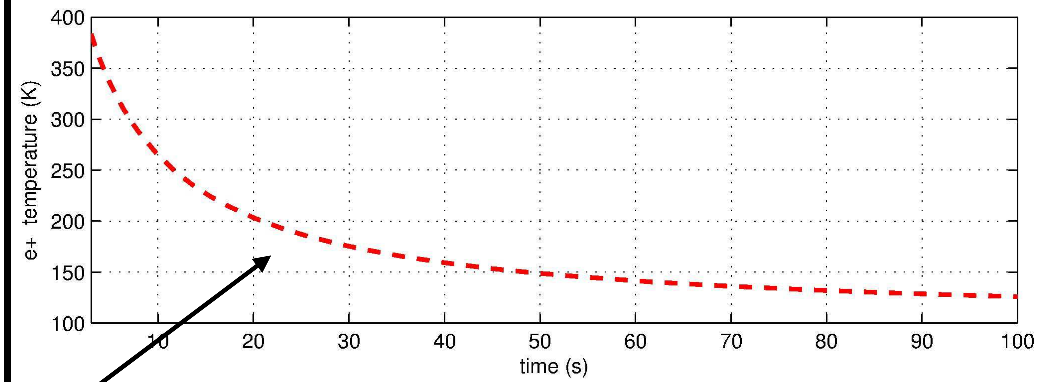




higher e+ values

n_0=1. 10^{+14} ptc/m^3 e^+ initial density
N_e=3 10^7 e+ number





T_e decreases slower

(in respect to the exp.case)

due to larger heating by pbars

due to larger heating by e+ expansion (more electrostatic energy to be converted)

=> The exp. case is a lucky compromise between heating from pbar energy loss and from e+ expansion

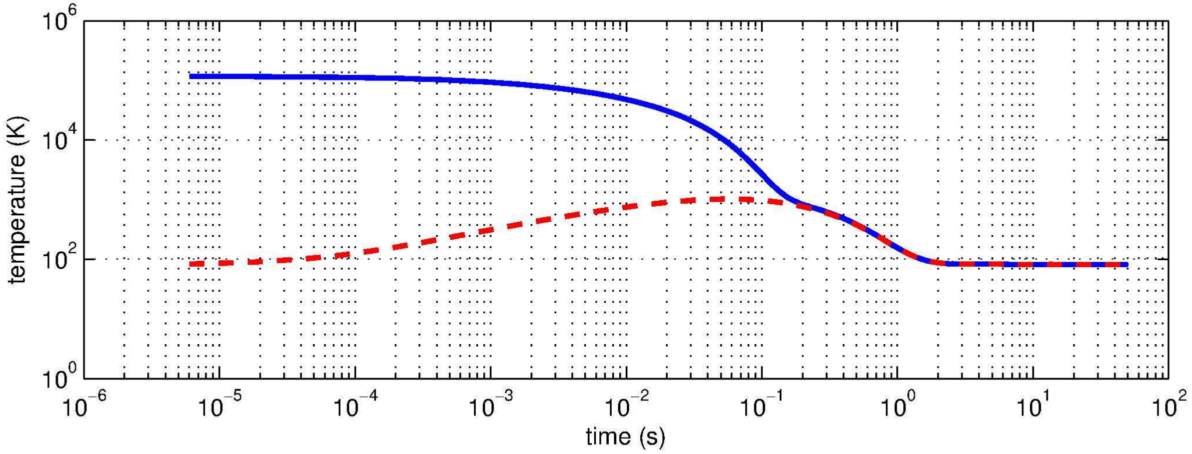
=> More e+ is not always the best

The ASACUSA case 2012

Same conditions as in 2010 but:

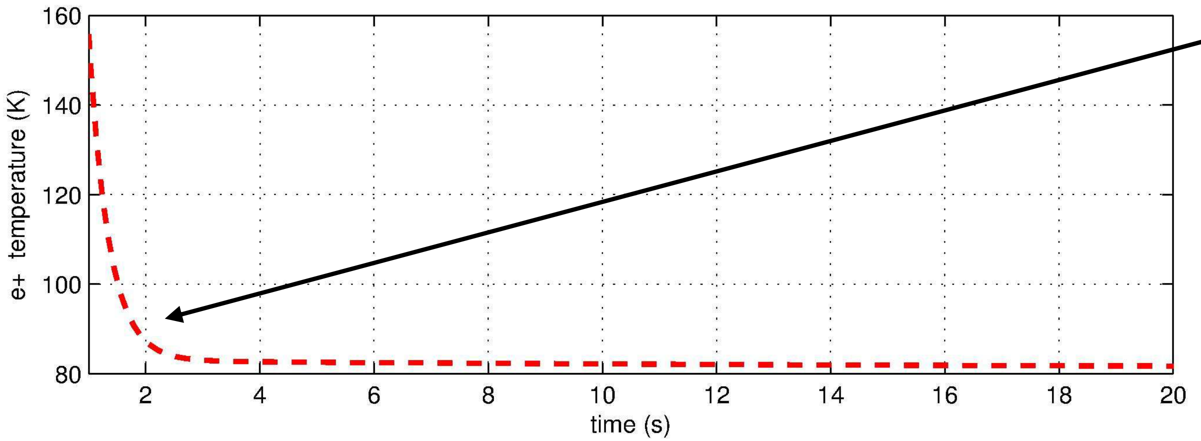
R_0=0.25 10^{-3} m ← e^+ initial radius
v_0=10 10^{-6} m/s ← e^+ expansion velocity
n_0=1. 10^{+14} ptc/m^3 ← e^+ initial density
N_e=3 10^7 ← e+ number

E_0=10 eV ← pbar initial energy
T_0=80 K ← e^+ initial temperature
T_{res}=80 K ← environment temperature



Better e+ conditions in respect to 2010 case

⇒ T_e quickly decreases



Conclusion

A simple model to evaluate T_e has been performed

The model includes collisional, radiative and plasma expansion effects (together for the 1° time)

Some results from the model qualitatively agree with the experimental data

The analysis of the model results suggest that some naive expectations can be wrong (for example: “more e+ ⇔ lower T_e”)

The model can also be used for the ALPHA case and it can be extended to include the ATRAP case