

Modelling the behavior of the positron plasma temperature in antihydrogen experiments

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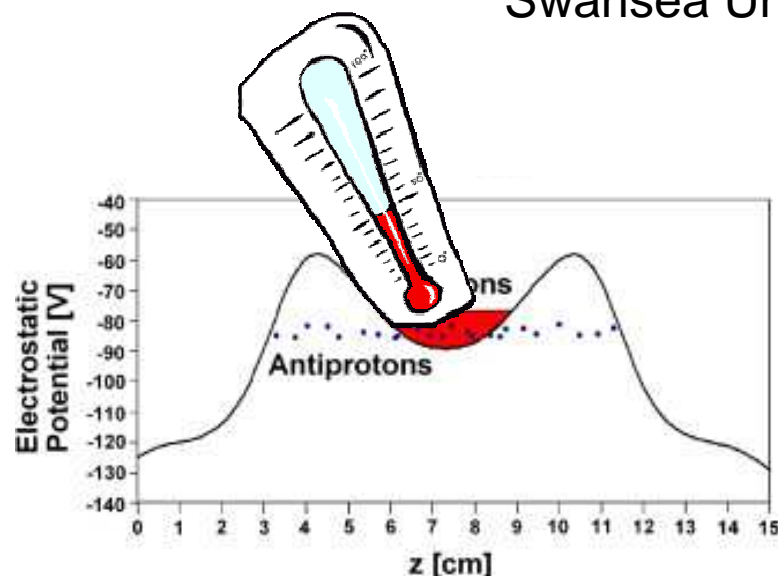


Michael Charlton

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Prifysgol Abertawe

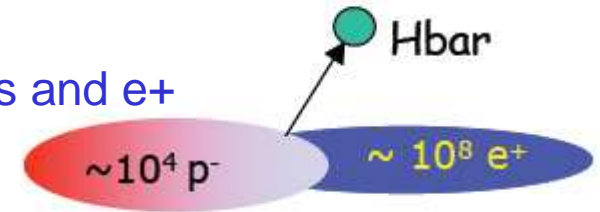


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Why e+ plasma temperature is important?

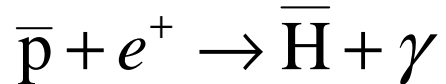
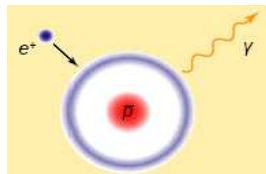
For experiments where Hbars are produced by overlapping pbars and e+



Hbar production **rates** depend on e+ plasma **temperature**

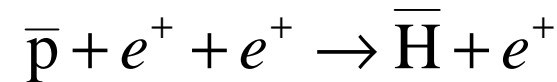
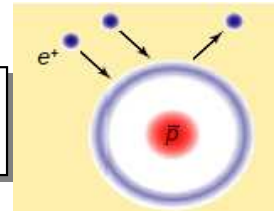
in both the **expected Hbar production mechanisms**:

Radiative recombination



$$\lambda_{Hbar} \propto n_e T_e^{-0.67}$$

3-body recombination



$$\lambda_{Hbar} \propto n_e^2 T_e^{-4.5}$$

λ_{Hbar} = Hbar production rate

n_e = e+ density

T_e = **e+ temperature**

e+ plasma temperature evaluations

- Evaluations of pbar temperature in Hbar experiments exist

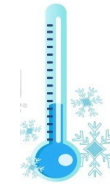
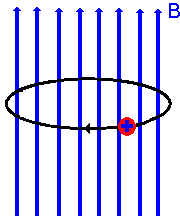
See, for example: S.L. Rolston, G. Gabrielse, Hyperf. Inter. 44 (1988) 233.
J. Bernard et al., NIM A 532 (2004) 224

- No direct evaluation of e+ plasma temperature for Hbar experiments

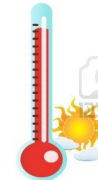
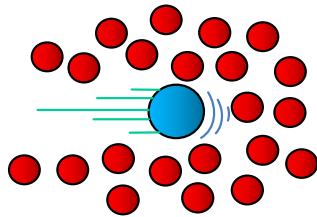
we present a simple model to evaluate T_e

The processes affecting the e^+ plasma temperature

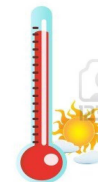
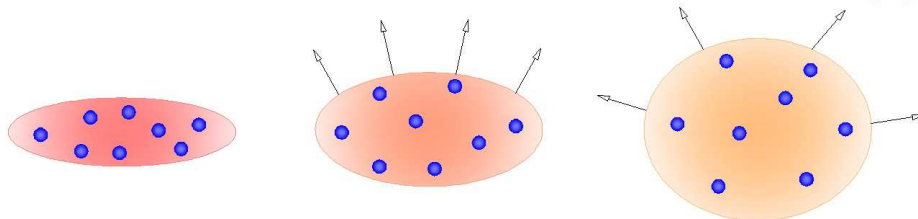
- **synchrotron radiation** (\rightarrow plasma cooling)



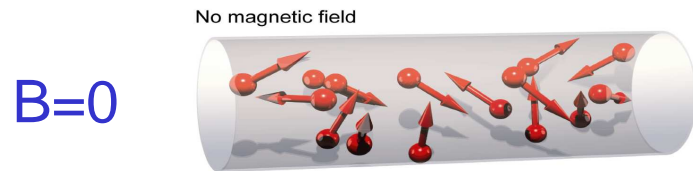
- **antiprotons energy loss** in the plasma (\rightarrow plasma heating)



- **e^+ plasma expansion** (\rightarrow plasma heating)



e+ plasma temperatures: 1 or 2?



T_e ← only 1 temperature

For strongly magnetized plasma

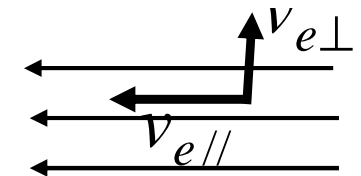
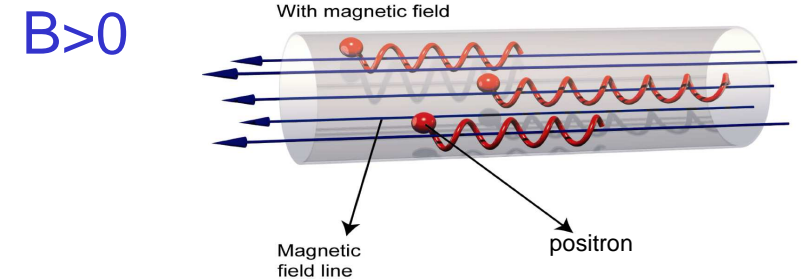
possibly $T_{e\perp} \neq T_{e\parallel}$

But when $\lambda_{coll} \gg \frac{1}{\tau_e}$

↑
e+-e+ collision
frequency

←
synchrotron
cooling time

$$\Rightarrow T_{e\perp} = T_{e\parallel} = T_e$$



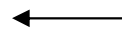
$$T_{e\perp} = \frac{m_e v_{e\perp}^2}{k_B}$$

$$T_{\parallel} = \frac{m_e v_{e\parallel}^2}{k_B}$$

$$\tau_e = \frac{4\pi\epsilon_0 m_e^3 c^2}{e^4 B^2}$$

$$\lambda_{coll}$$

$$\lambda_{coll} = n_e \bar{v} \bar{b}^2 I(\bar{k})$$



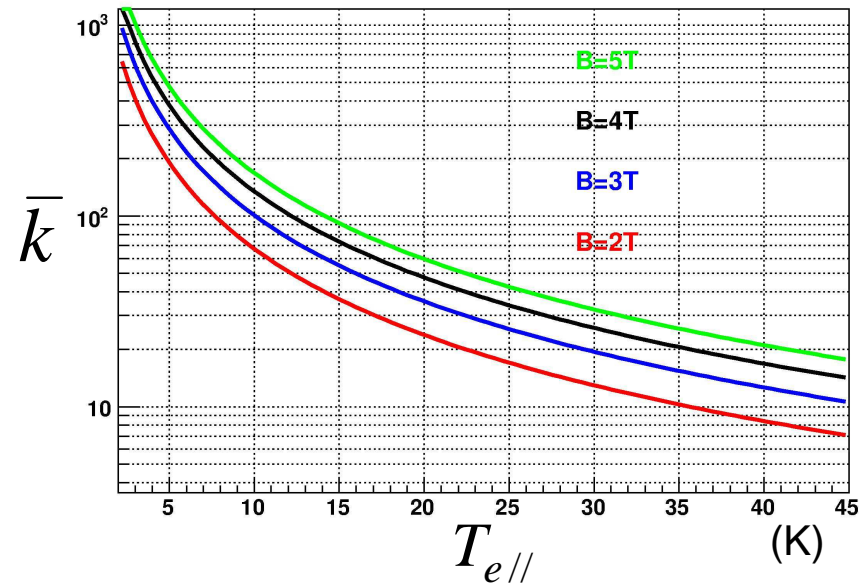
See, for example
Robicheaux JPB 2008

$$\bar{v} = \sqrt{2k_B \frac{T_{||}}{m_e}}$$

$$\bar{b} = \frac{2e^2}{4\pi\epsilon_0 k_B T_{||}}$$

$$\bar{k} = \frac{eB\bar{b}}{m\bar{v}}$$

$I(\bar{k})$ = suppression due to B

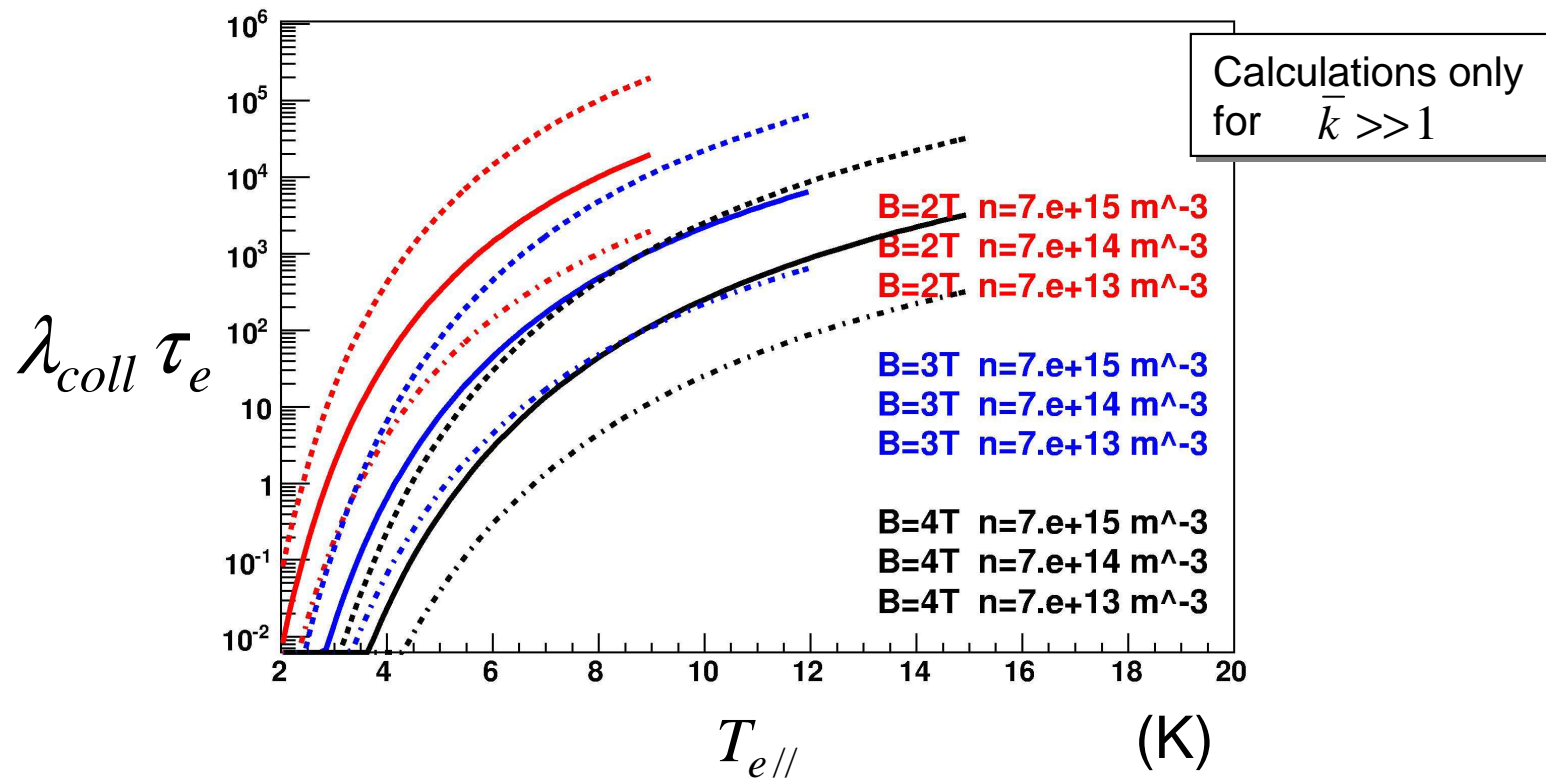


If $\bar{k} \gg 1$

(i.e. for strong magnetized plasma,
see Glinsky et al. PhysFluids1992).

$$I(\bar{k}) \cong \exp\left[-\frac{5(3\pi\bar{k})^{2/5}}{6}\right] \times \left[\frac{1.83}{\bar{k}^{7/15}} + \frac{20.9}{\bar{k}^{11/15}} + \frac{0.347}{\bar{k}^{13/15}} + \frac{87.8}{\bar{k}^{15/15}} + \frac{6.68}{\bar{k}^{17/15}}\right]$$

$$\lambda_{coll} \gg \tau_e^{-1} \quad ? \quad \Rightarrow \quad T_{e\perp} = T_{e\parallel} = T_e$$



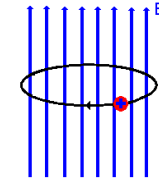
- For ATHENA, ALPHA, ASACUSA: $\lambda_{coll} \tau_e \gg 1$

- For ATRAP ($B=5.4 \text{ T}$, $n_e = 4 \cdot 10^{13} \text{ m}^{-3}$, assuming environmental temperature = 4.2 K):

$$\lambda_{coll} \tau_e < 1 \quad \text{for low temperatures} \quad \Rightarrow \quad T_{e\perp} \neq T_{e\parallel}$$

← Not considered in the model

Synchrotron radiation cooling



due to e⁺ cyclotron rotation in **B**

From Larmour formula for Maxwellian distribution:

Assuming $T_{e\perp} \gg T_{res}$
 \uparrow
 environmental temperature

$$\frac{dT_{e\perp}}{dt} = -\frac{3}{2} \frac{1}{\tau_e} T_{e\perp} \quad (s1)$$

$$\tau_e = \frac{4\pi\epsilon_0 m_e^3 c^2}{e^4 B^2}$$

if $\lambda_{coll} \gg \frac{1}{\tau_e}$ then $T_{e\perp} = T_{e\parallel} = T_e$

(s1) becomes $\frac{dT_e}{dt} = -\frac{1}{\tau_e} T_e$

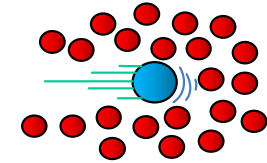
If $T_e \approx T_{res} \Rightarrow$

$$\frac{dT_e}{dt} = -\frac{T_e - T_{res}}{\tau_e}$$

$$\left(\lambda_{coll} \gg \frac{1}{\tau_e} \right)$$



Antiproton energy loss



due to Rutherford collisions

$$\frac{dT_i}{dt} = -\frac{1}{\tau_i} (T_i - T_e)$$



T_i = pbar temperature

with:

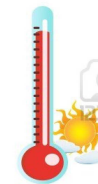
$$\tau_i = \frac{3m_e m_i c^3}{8\sqrt{2\pi} n_e Z^2 \ln(\Lambda)} \left(\frac{4\pi\epsilon_0}{e^2} \right)^2 \left(\frac{k_B T_i}{m_i c^2} + \frac{k_B T_e}{m_e c^2} \right)^{\frac{3}{2}}$$

L. Spitzer, Physics of Fully Ionised Gases, 1956.

$$\ln(\Lambda) = \ln\left(\frac{b_{\max}}{b_{\min}}\right) = \ln\left(4\pi\left(\frac{\epsilon_0 k_B}{e^2}\right)^{\frac{3}{2}} \frac{1}{Z} \sqrt{\frac{T_e}{n_e}} \left(T_e + \frac{m_e}{m_i} T_i + 2\sqrt{\frac{m_e}{m_i}} \sqrt{T_e T_i}\right)\right)$$

when T_i decreases, T_e increases

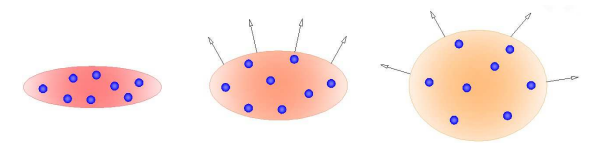
$$\frac{dT_e}{dt} = \frac{1}{\tau_i} \frac{N_i}{N_e} (T_i - T_e)$$



N_i = pbars number
 N_e = e+ number

Sharing the same volume

e+ plasma expansion



due to not optimal vacuum and/or e.m. field asymmetry/misalignment

From energy conservation (electrostatic energy \rightarrow internal energy):

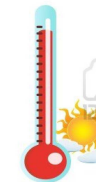
$$\frac{dT_e}{dt} = \frac{e^2}{6\epsilon_0 k_B} n_e r_e^2 \frac{1}{r_e} \frac{dr_e}{dt}$$

r_e = e+plasma radius

n_e = e+plasma density

Since in Hbar experiment $n_e r_e^2$ is constant \Rightarrow

$$\frac{dT_e}{dt} = \frac{e^2}{6\epsilon_0 k_B} (n_e r_e^2)_0 \frac{1}{r_e} \frac{dr_e}{dt}$$



expansion velocity

The model: equations system

Considering: **synchrotron radiation** + **pbar energy loss** + **plasma expansion**
the e+ temperature must satisfy the system:

$$\frac{dT_e}{dt} = -\frac{T_e - T_{res}}{\tau_e} + \frac{1}{\tau_i} \frac{N_i}{N_e} (T_i - T_e) + \frac{e^2}{6\epsilon_0 k_B} \left(n_e r_e^2 \right)_0 \frac{1}{r_e} \frac{dr_e}{dt}$$
$$\frac{dT_i}{dt} = -\frac{1}{\tau_i} (T_i - T_e) \quad \left(\lambda_{coll} \gg \frac{1}{\tau_e} \right)$$

- The equation system is linear in T_{res} \Rightarrow the T_e behaviors are independent of T_{res}

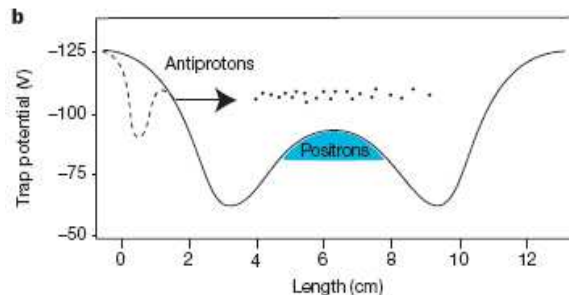
The model: modified equations system

2 parameters to consider the effect of the partial pbars-positrons overlap:

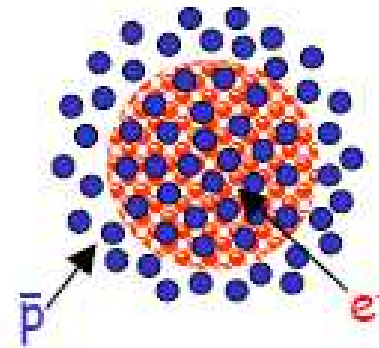
P parameter

pbars are not continuously inside the e+ plasma

$$P \leq 1$$



M parameter



if the antiprotons cloud is larger than the e+ cloud

$$M \leq 1$$

$$\frac{dT_e}{dt} = -\frac{T_e - T_{res}}{\tau_e} + \overset{\downarrow}{M} \overset{\downarrow}{P} \frac{1}{\tau_i} \frac{N_i}{N_e} (T_i - T_e) + \frac{e^2}{6\epsilon_0 k_B} \left(n_e r_e^2 \right)_0 \frac{1}{r_e} \frac{dr_e}{dt}$$

$$\frac{dT_i}{dt} = -\overset{\downarrow}{P} \frac{1}{\tau_i} (T_i - T_e) \quad \left(\lambda_{coll} \gg \frac{1}{\tau_e} \right)$$

The solutions (T_e, T_i) depend on many parameters:

$$B \quad E_0 \quad T_0 \quad T_{res} \quad M \quad P \quad N_e \quad N_i \quad v_0 = \frac{dr_e}{dt} \quad (n_e r_e^2)_0$$

\Rightarrow no universal solution exists

Each particular case has to be considered

The ATHENA case ...

The ATHENA Experiment

Main goal: production & studing of cold \bar{H}

letters to nature

Production and detection of cold antihydrogen atoms

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A theoretical understanding of the standard model of fundamental particles and interactions is CPT invariance, which requires that the laws of physics be invariant under the combined discrete operations of charge conjugation, parity and time reversal. Antimatter, the existence of which was predicted by Dirac, can be used to test the CPT theorem—experimental investigations involving comparisons of particles with antiparticles are numerous. Cold atoms and anti-atoms, such as hydrogen and antihydrogen, could form the basis of a new precise test, as CPT invariance implies that they must have the same spectrum. Observations of antihydrogen in small quantities and at high energies have been reported at the European Organization for Nuclear Research (CERN) and at Fermilab¹, but these experiments were not suited to precision comparison measurements. Here we demonstrate the production of antihydrogen atoms at very low energy by mixing trapped antiprotons and positrons in a cryogenic environment. The neutral anti-atoms have been detected directly when they escape the trap and annihilate, producing a characteristic signature in an imaging particle detector.

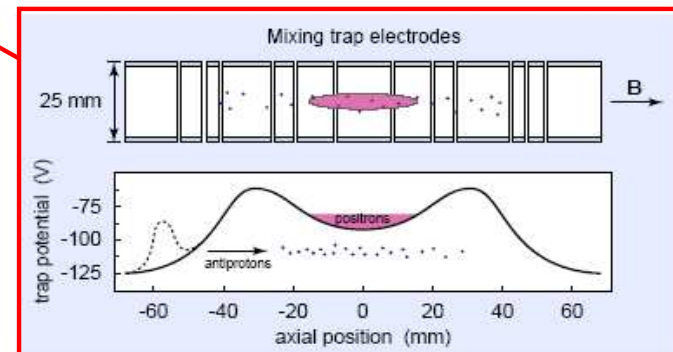
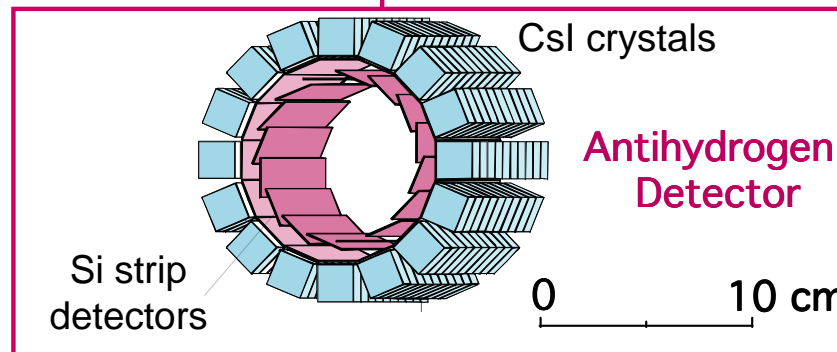
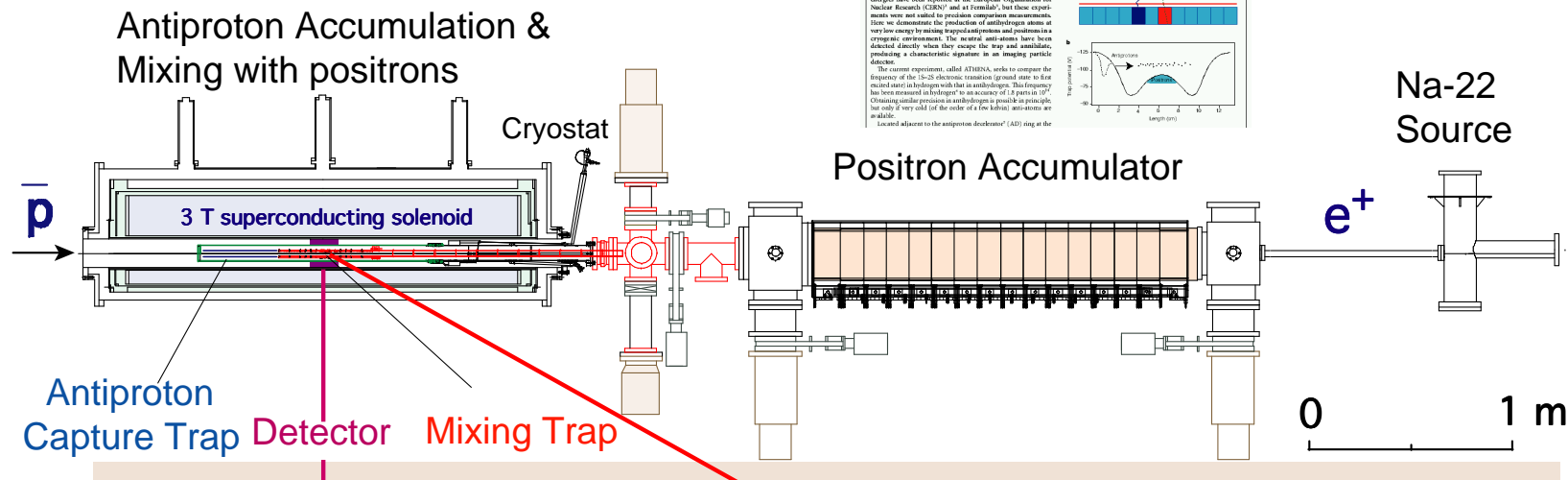
The current experiment, called ATHENA, seeks to compare the frequency of the 1–20 electronic transition (ground state to first excited state) in hydrogen with that in antihydrogen. This frequency has been measured in hydrogen to an accuracy of 1 part in 10¹⁴. Obtaining similar precision in antihydrogen is possible in principle, but only if very cold (of the order of a few kelvin) anti-atoms are available.

positron accumulator has its own magnetic system, also a solenoid, of 0.1 T. A separate cryogenic heat exchanger is the heart of the superconducting magnet which the catching and mixing trap is about 15 K. The ATHENA apparatus features an open, modular design that allows great experimental flexibility, particularly in introducing large numbers of positrons into the apparatus—essential feature in the current work.

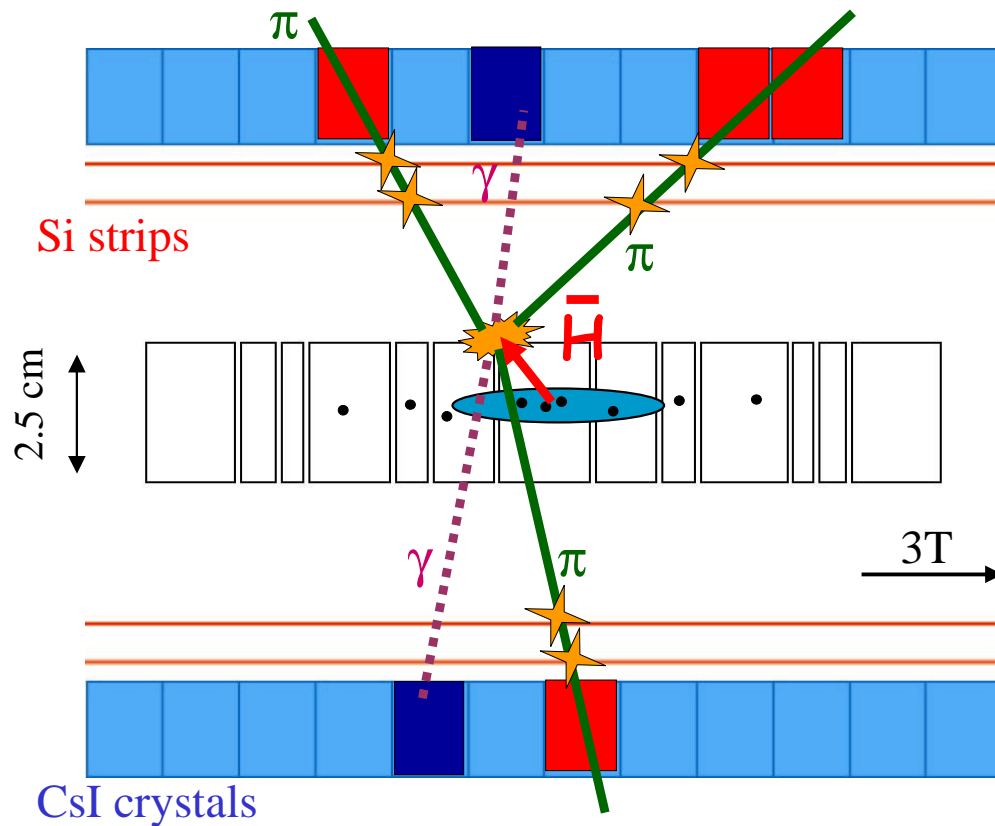
The catching trap² down traps cooled and accumulated antiprotons. To cool antiprotons, the catching trap is first loaded with 3 × 10⁸ deuterons which cool by muonion radiation in the 3 T magnetic field. Typically, the AD delivers 2 × 10⁷ antiprotons having kinetic energy 5.5 MeV and a pulse duration of 200 ns to the experiment at three intervals. The antiprotons are slowed in a thin foil and trapped using a pulsed electric field. The antiprotons lose energy and equilibrate with the cold deuterons by Coulombic interaction. The deuterons are ejected before mixing the antiprotons with positrons. Each AD shot results in about 1 × 10⁶ cold anti-protons for interaction experiments.

The positron accumulator, based on a design described in ref. 3, slows, traps and accumulates positrons created from a radioactive source (1.8 × 10⁶ Bq ²²Na). Accumulation for 300 yields 1.5 × 10⁷ positrons⁴, 50% of which are successfully transferred to the mixing trap where they cool by synchrotron radiation. The mixing trap has

letter to Nature
Nature 419, 456–459 (2002)



How ATHENA worked



- $10^4 \bar{p}$ & $10^8 e^+$ are mixed in Penning trap
- \bar{H} form and fly away
- \bar{H} annihilate on trap wall

Offline selection of \bar{H} annihilation:

Coincidence in space & time (<5ms) of:

- \bar{p} annihilation (charged pion vertex)
- e^+ annihilation (2 back-to-back 511 keV γ)

- The condition $\lambda_{coll} \tau_e \gg 1$ ($\Rightarrow T_{e\perp} = T_{e\parallel} = T_e$) is always satisfied
- Values used in the simulations:

$$B = 3 \text{ T} \quad \tau_e = \frac{1}{0.39 B^2} \text{ s} = 0.3 \text{ s}$$

$$M = 1 \quad P = 0.3 \quad \text{N.B. We have found that the P value affects only the time of the pbars slowing down}$$

$E_0 = 16 \text{ eV}$ \leftarrow pbar initial energy

$T_0 = T_{res} = 15 \text{ K}$ \leftarrow e⁺ initial temperature

$N_i = 1 \cdot 10^4$ \leftarrow pbars number

$N_e = 7.5 \cdot 10^7$ \leftarrow e⁺ number

$T_{res} = 15 \text{ K}$ \leftarrow environment temperature

\nwarrow Not measured, lower limit

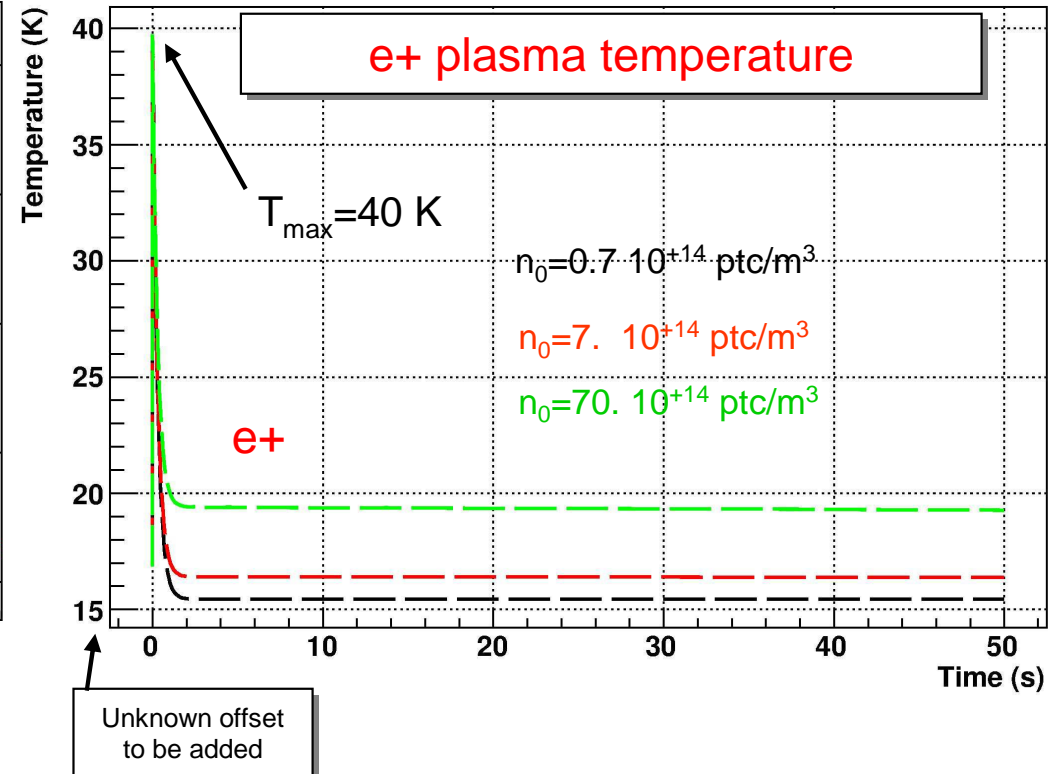
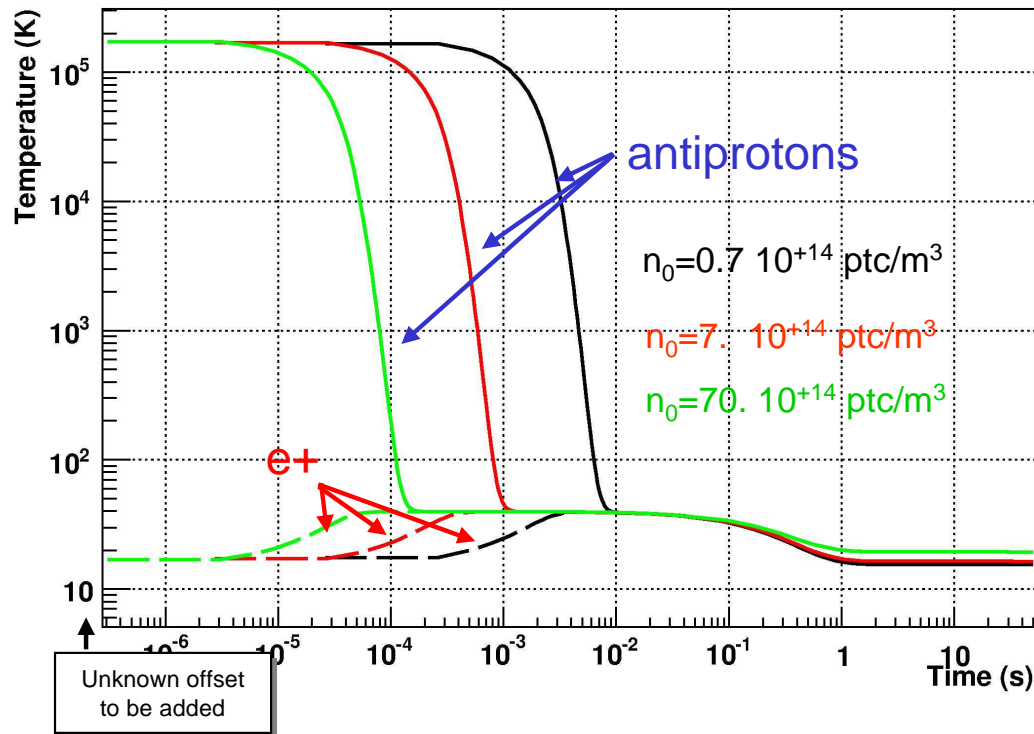
Quantities modified:

$v_0 = \frac{dr_e}{dt} = 0 - 20 \cdot 10^{-6} \text{ m/s}$ \leftarrow e⁺ expansion velocity

$n_0 = 0.7 - 70 \cdot 10^{14} \text{ ptc/m}^3$ \leftarrow e⁺ initial density

$R_0 = 1 \cdot 10^{-3} \cdot (7 \cdot 10^{14} / n_0) \text{ m}$ \leftarrow e⁺ initial radius

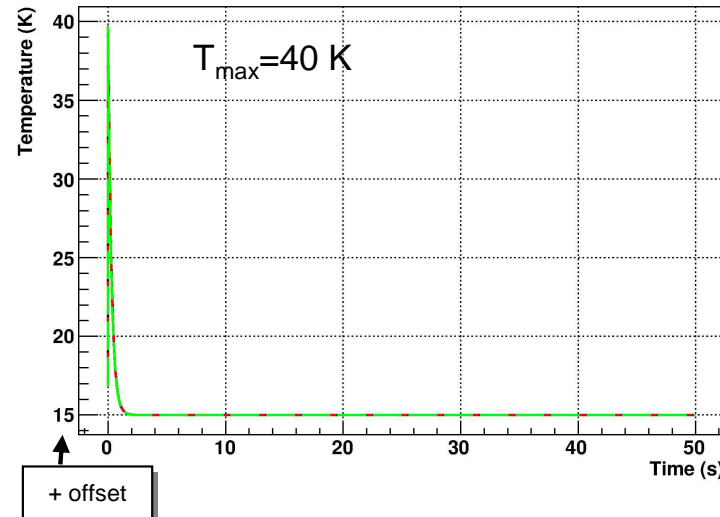
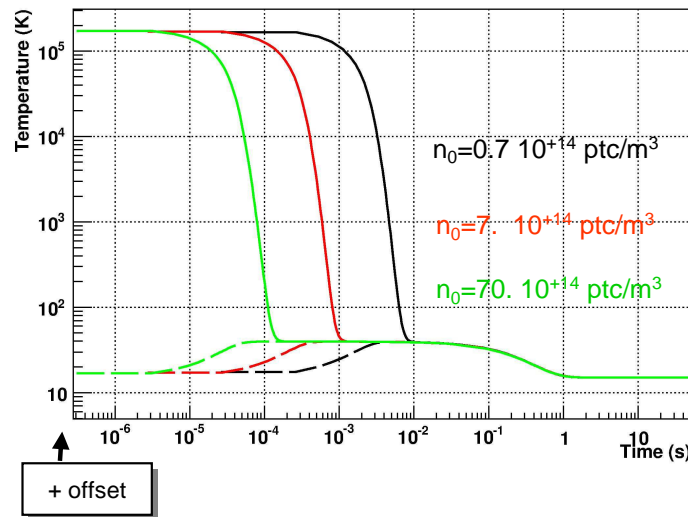
$$v_0 = 0.2 \cdot 10^{-6} \text{ m/s} \quad \leftarrow \text{experimental}$$



- $T_{pbar} \approx T_e$ after $\Delta t \approx \frac{10^{12}}{n_e} \text{ s}$
- T_e maximum increase is $\approx 25 \text{ K}$
- Equilibrium of T_e after less than 1 s
- T_e values at equilibrium depend on the density (\leftarrow expansion), but the differences are small

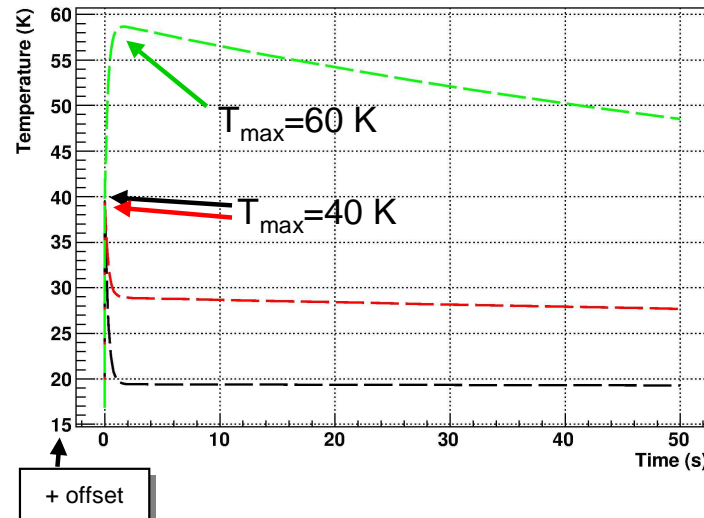
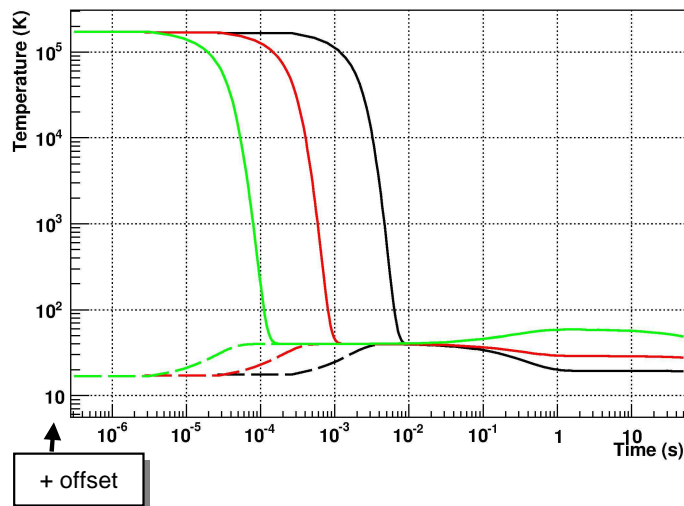
The ATHENA case – different expansion velocity

$v_0 = 0$ no expansion



Similar to the case with $v_0 = 0.2 \cdot 10^{-6}$ m/s but same T_e at late times ($T_e \approx T_{res}$)

$v_0 = 2 \cdot 10^{-6}$ m/s \leftarrow x10 experimental value



Different maximum T_e values

Different T_e at late time

For high densities T_e slowly decreases



expansion process can strongly influence T_e

The ATHENA case - How to check the model?

Direct check of the model with the experimental data is difficult

We can use the “reinjection data”

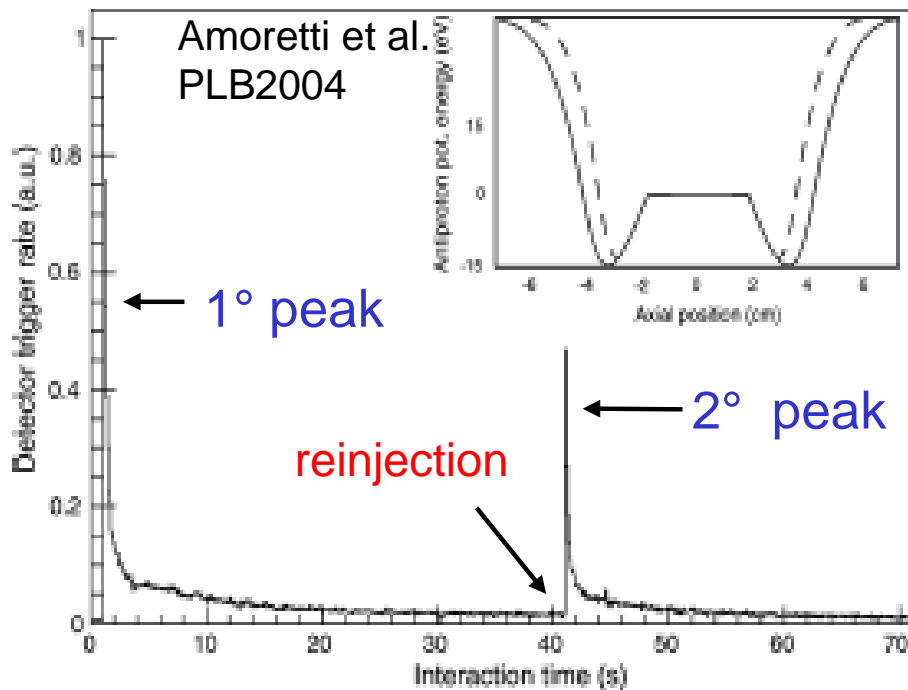


Fig. 5. Trigger rate during re-injection after 40 s. The corresponding peak is mainly due to antihydrogen production. In the inset, the on-axis potentials applied to re-inject the antiprotons into the positron plasma, are displayed (dashed lines).

The experiment

- At $t=0$ standard e^+ - $pbar$ mixing. The 1° peak corresponds to the detected Hbars. Other Hbars are field-ionized and their pbars can fill the lateral wells
- After 40s pbars are re-injected into e^+ plasma by adiabatically compressing the 2 side wells. Hbar formation restart and 2° peak appears.

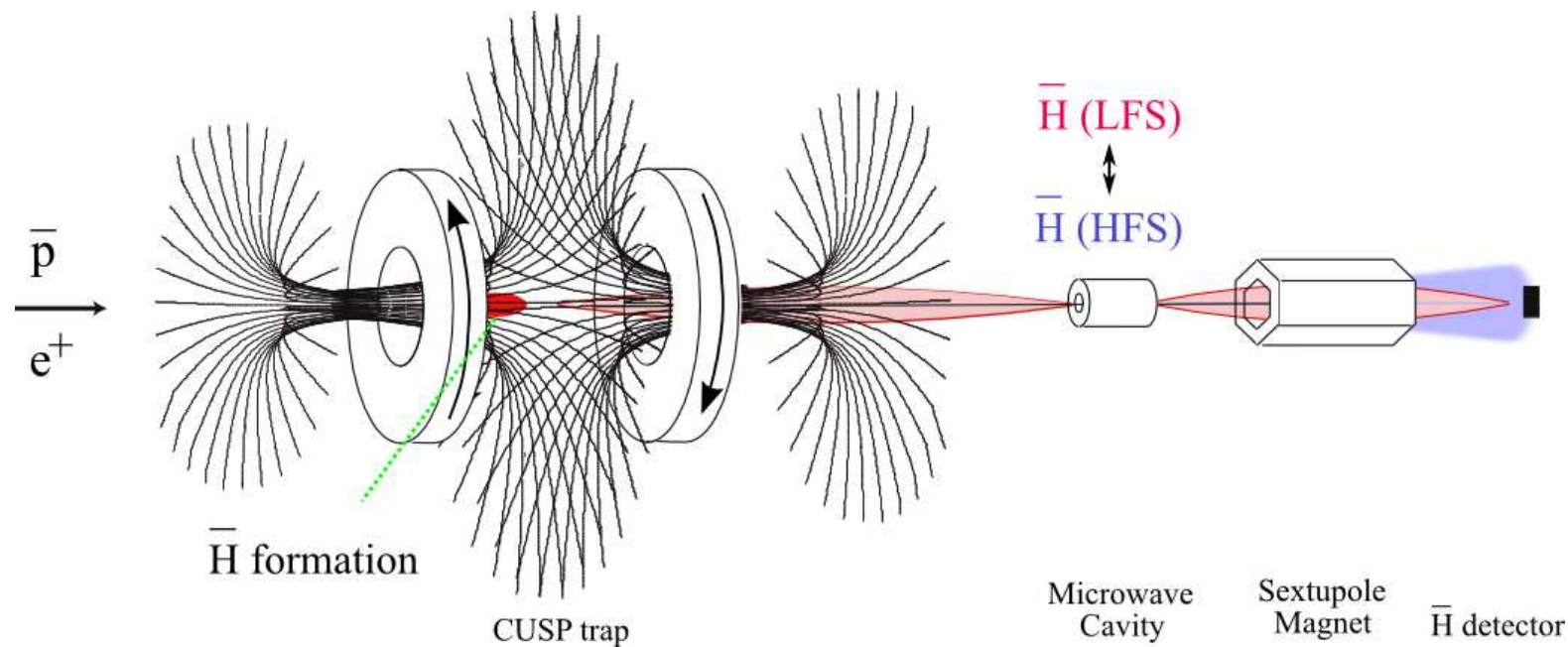
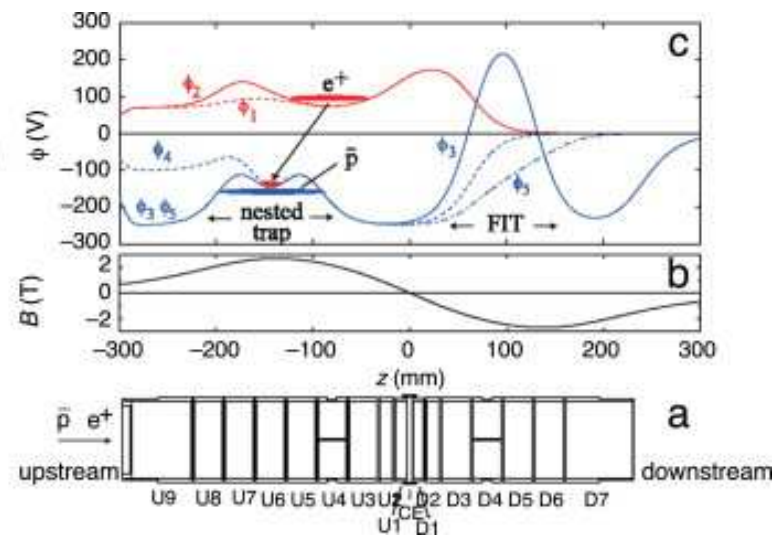
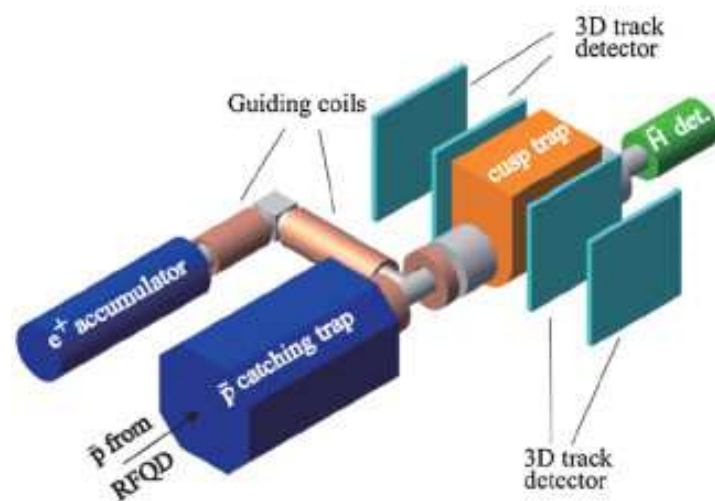
2° peak is similar to the 1° peak

⇔ e^+ plasma conditions (@40s) are similar to the initial ones ($t=0$)

In agreement with simulations
(where $T_e \rightarrow T_{res}$ after 2s)

The ASACUSA case ...

The ASACUSA Experiment



The ASACUSA case

2010

$$B = 2.7 \text{ T} \quad \tau_e = \frac{1}{0.39 B^2} \text{ s} \quad P = 0.3 \quad M = 1 \quad E_0 = 13 \text{ eV} \quad \leftarrow \text{pbar initial energy}$$

$$R_0 = 0.5 \cdot 10^{-3} \text{ m} \quad \leftarrow \text{e+ initial radius}$$

$$N_e = 3 \cdot 10^6 \quad \leftarrow \text{e+ number}$$

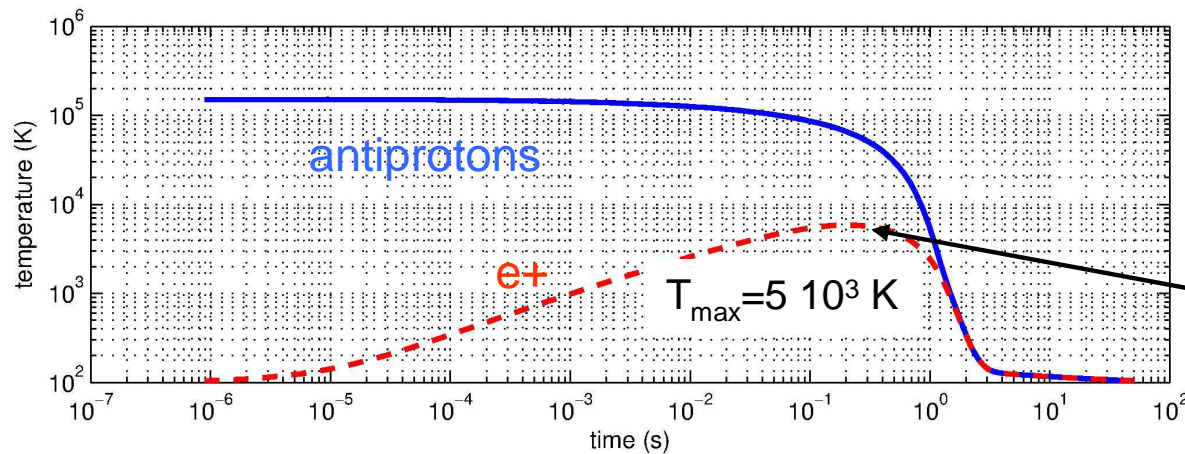
$$n_0 = 1 \cdot 10^{+13} \text{ ptc/m}^3 \quad \leftarrow \text{e+ initial density}$$

$$T_0 = T_{\text{res}} = 100 \text{ K} \quad \leftarrow \text{e+ initial temperature}$$

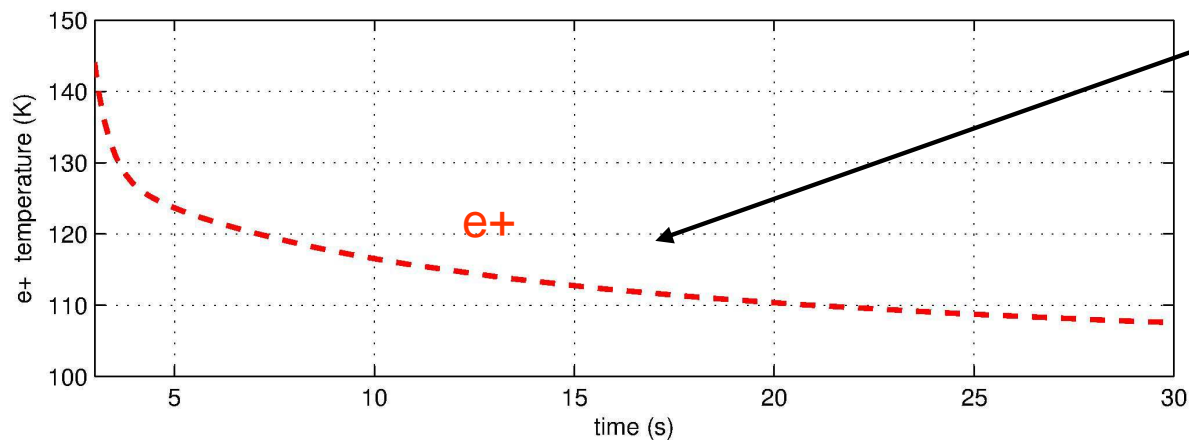
$$N_i = 3 \cdot 10^5 \quad \leftarrow \text{pbars number}$$

$$v_0 = 70 \cdot 10^{-6} \text{ m/s} \quad \leftarrow \text{e+ expansion velocity}$$

$$T_{\text{res}} = 100 \text{ K} \quad \leftarrow \text{environment temperature}$$



Large number of pbars strongly increases T_e

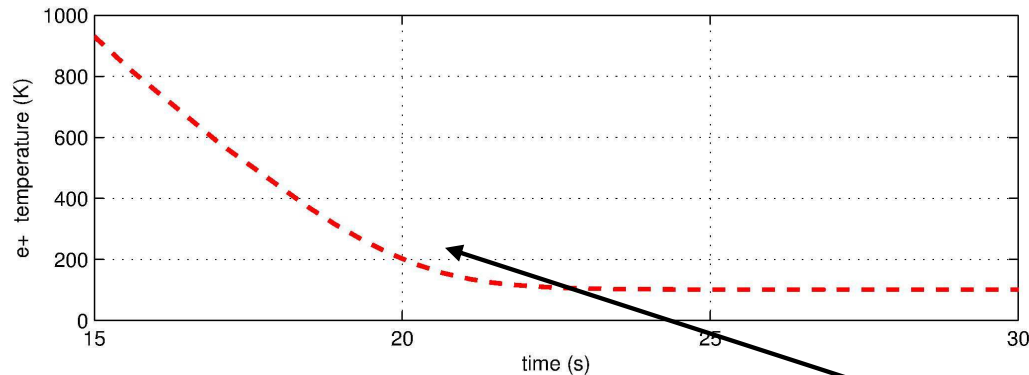
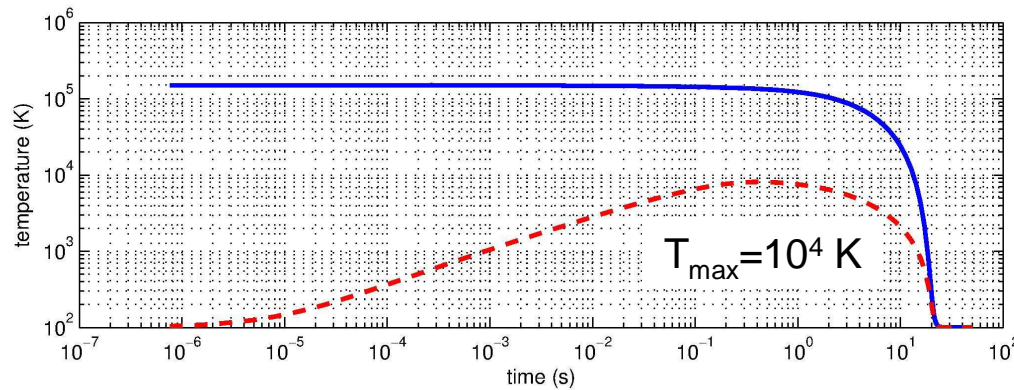


T_e slowly decreases due to the fast expansion and large pbars number

The ASACUSA case - different e+ values 2010

lower e+ values (number & dens.)

$n_0 = 1. \cdot 10^{+12}$ ptc/m³ e+ initial density
 $N_e = 3 \cdot 10^5$ e+ number



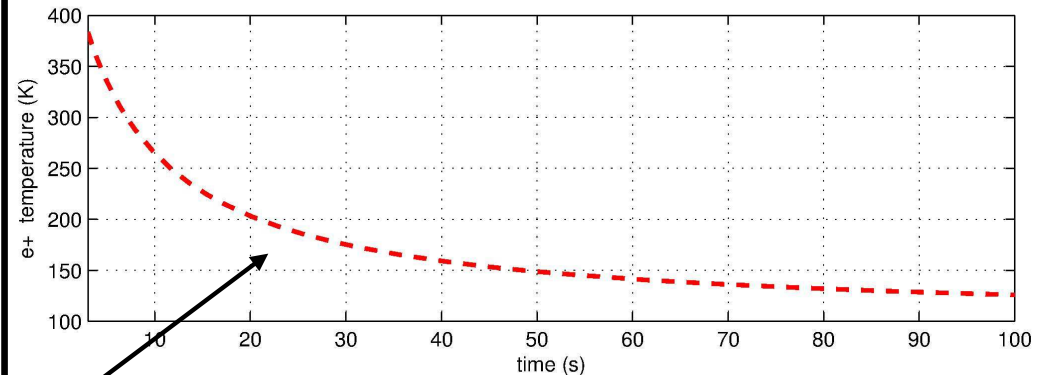
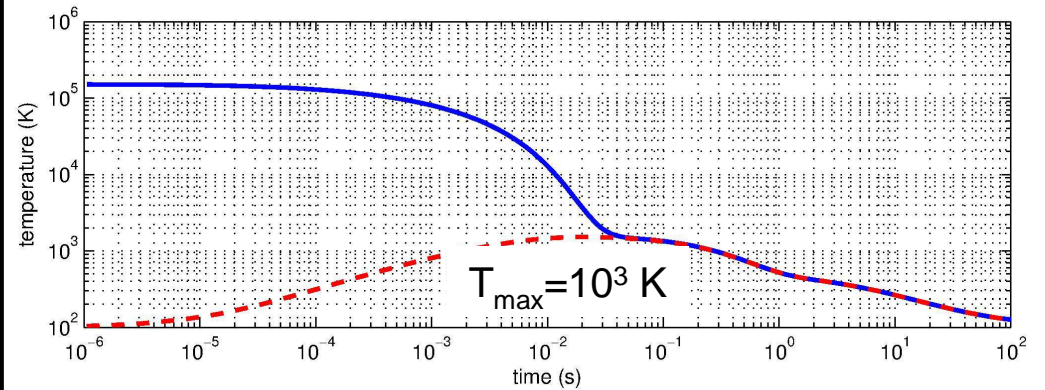
due to larger heating by pbars

T_e decreases slower

(in respect to the exp.case)

higher e+ values

$n_0 = 1. \cdot 10^{+14}$ ptc/m³ e+ initial density
 $N_e = 3 \cdot 10^7$ e+ number



due to larger heating by e+ expansion (more electrostatic energy to be converted)

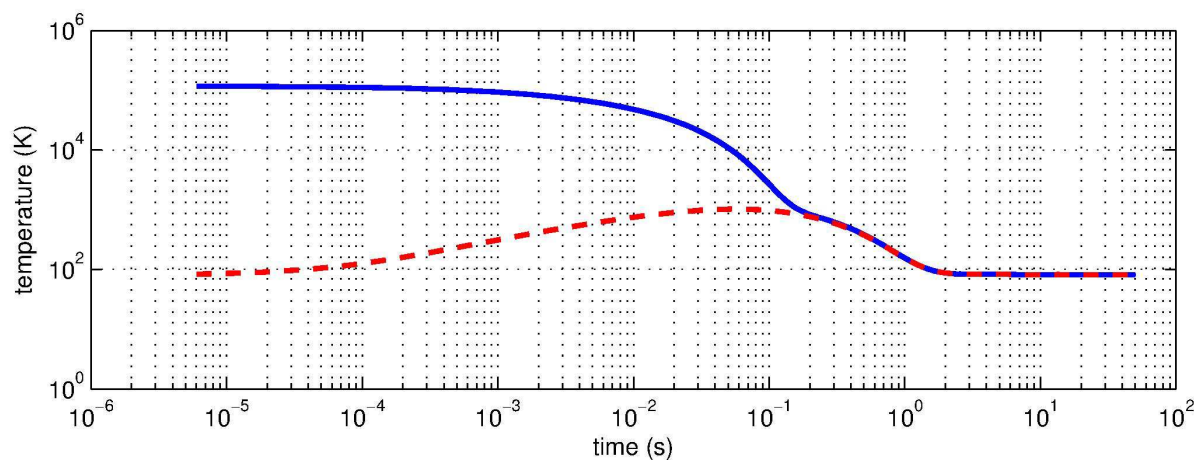
=> The exp. case is a lucky compromise between heating from pbar energy loss and from e+ expansion

=> More e+ is not always the best

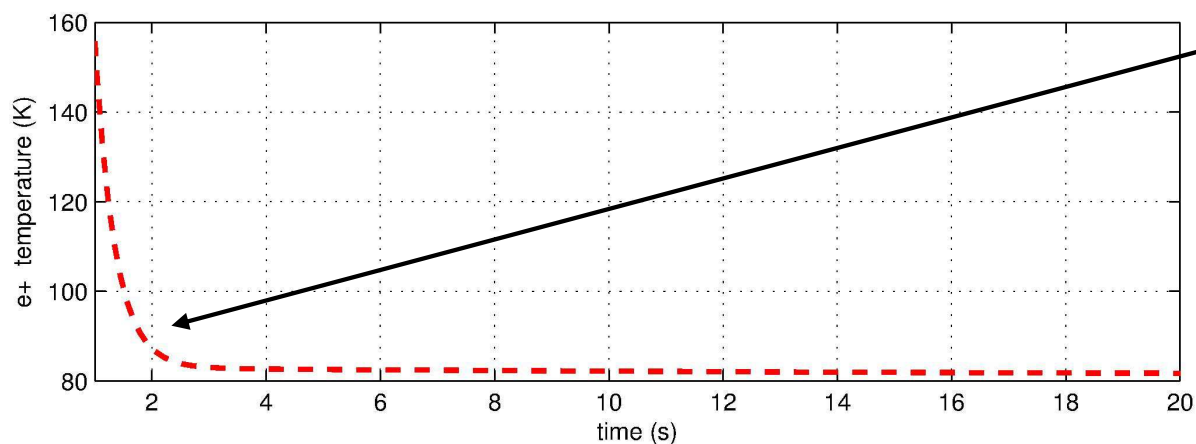
Same conditions as in 2010 but:

$R_0 = 0.25 \cdot 10^{-3} \text{ m}$ \leftarrow e⁺ initial radius
 $v_0 = 10 \cdot 10^{-6} \text{ m/s}$ \leftarrow e⁺ expansion velocity
 $n_0 = 1 \cdot 10^{14} \text{ ptc/m}^3$ \leftarrow e⁺ initial density
 $N_e = 3 \cdot 10^7$ \leftarrow e⁺ number

$E_0 = 10 \text{ eV}$ \leftarrow pbar initial energy
 $T_0 = 80 \text{ K}$ \leftarrow e⁺ initial temperature
 $T_{\text{res}} = 80 \text{ K}$ \leftarrow environment temperature



Better e⁺ conditions in respect to 2010 case



\Rightarrow

T_e quickly decreases

Conclusion

A simple model to evaluate T_e has been performed

The model includes collisional, radiative and plasma expansion effects (together for the 1° time)

Some results from the model qualitatively agree with the experimental data

The analysis of the model results suggest that some naive expectations can be wrong (for example: “more e^+ \Leftrightarrow lower T_e ”)

The model can also be used for the ALPHA case and it can be extended to include the ATRAP case