

LEAP 2013, Uppsala

Induced Resonance Transitions between Gravitational States of Antihydrogen

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Plan of the talk

- Ultracold Antihydrogen quantum reflection
- Gravitational states above material surface
- Magnetic field induced transitions
- Gravitational mass of antihydrogen

$\bar{H} + \text{Wall}$

$\neq$

ANNIHILATION ?

A. Yu. Voronin, P. Froelich, and B. Zygelman, [Phys. Rev. A **72**, 062903 \(2005\)](#).

A.Yu. Voronin, P. Froelich and V.V. Nesvizhevsky, “Gravitational quantum states of antihydrogen”, **Phys. Rev. A** **83**, 032903 (2011)

Quantum reflection=

Reflection from the **abruptly changing attractive** potential

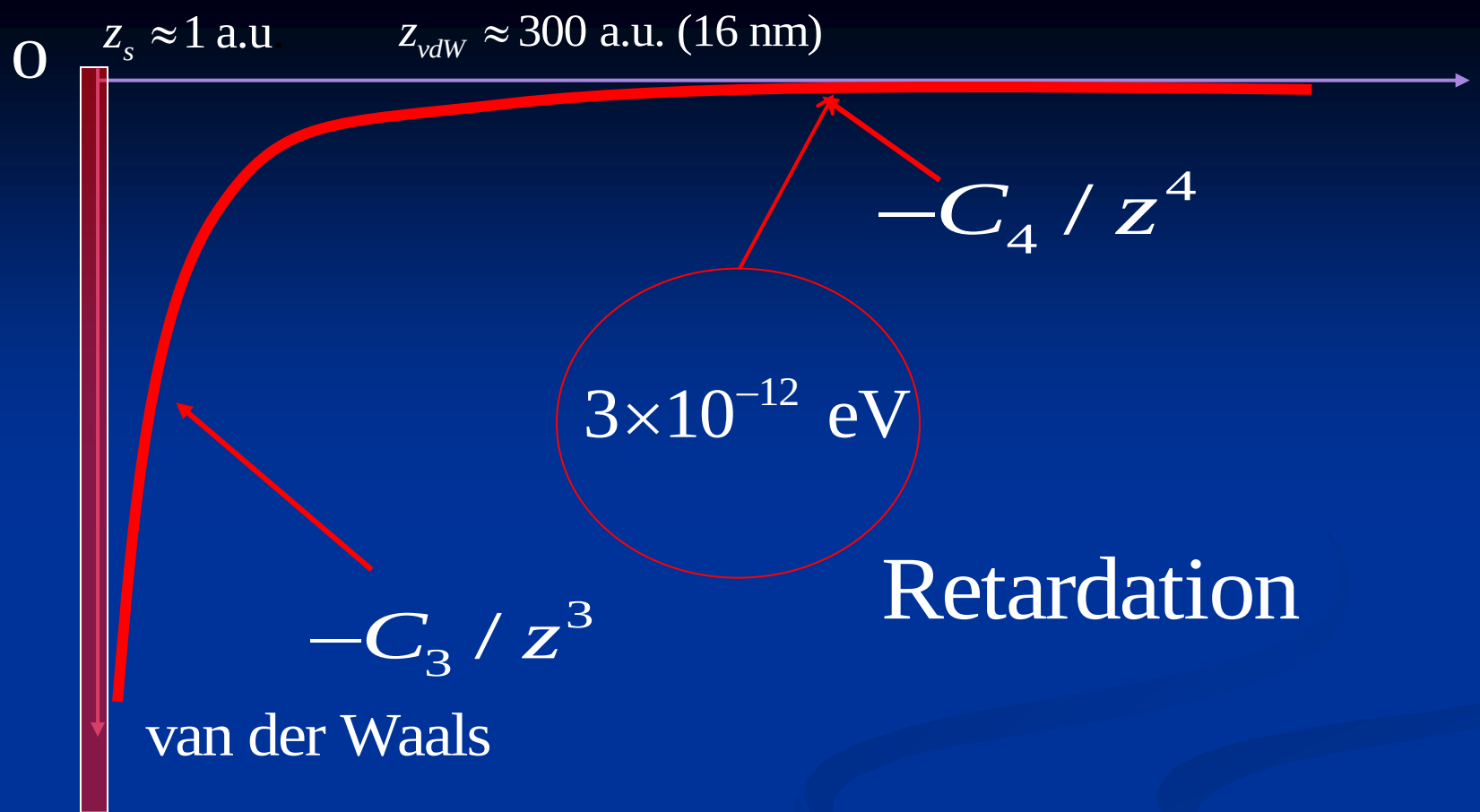
$$\frac{d\lambda_B(z)}{dz} \geq 1; \quad \lambda_B(z) = \frac{2\pi\hbar}{\sqrt{2m(E - V(z))}}$$

$$\frac{d\lambda_B(z)}{dz} = \frac{m\lambda_B^3}{\hbar^2} \frac{dV(z)}{dz}$$

$$\Psi(z \rightarrow \infty) = e^{-ikz} - Se^{ikz}$$

$$\lim_{k \rightarrow 0} S = 1 - 2ika \quad a = \text{Re } a - i|\text{Im } a|$$

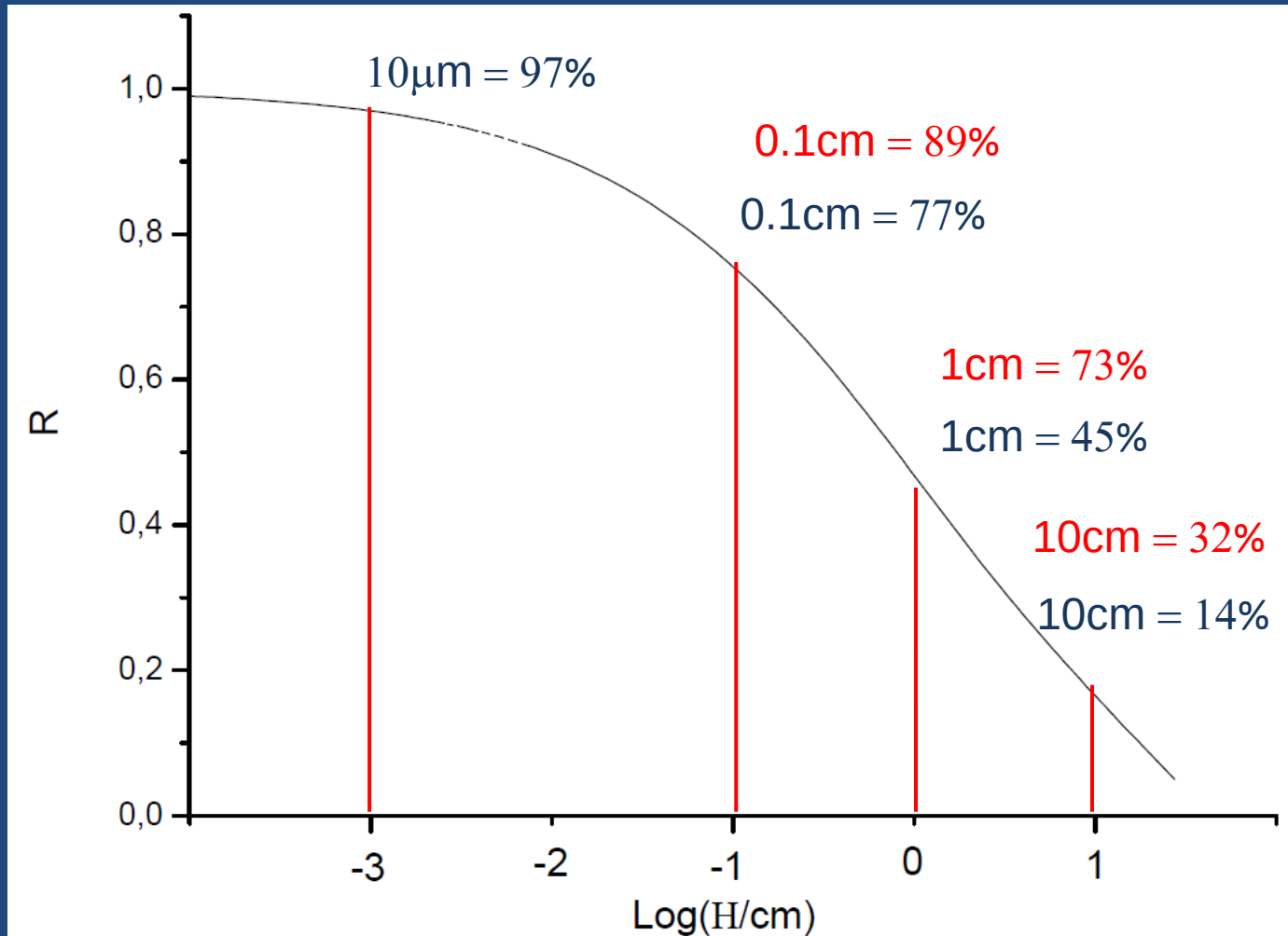
$$\lim_{k \rightarrow 0} R_{\text{qr}} \equiv |S|^2 = 1 - 4k|\text{Im } a| \rightarrow 1; \quad P_{\text{ann}} = 4k|\text{Im } a| \rightarrow 0$$



CASIMIR-POLDER POTENTIAL

$$a = -0,0028 - i0,027 \mu m$$

From which height can we drop antihydrogen?



Reflection coefficient as a function of incident energy

Gravitational states

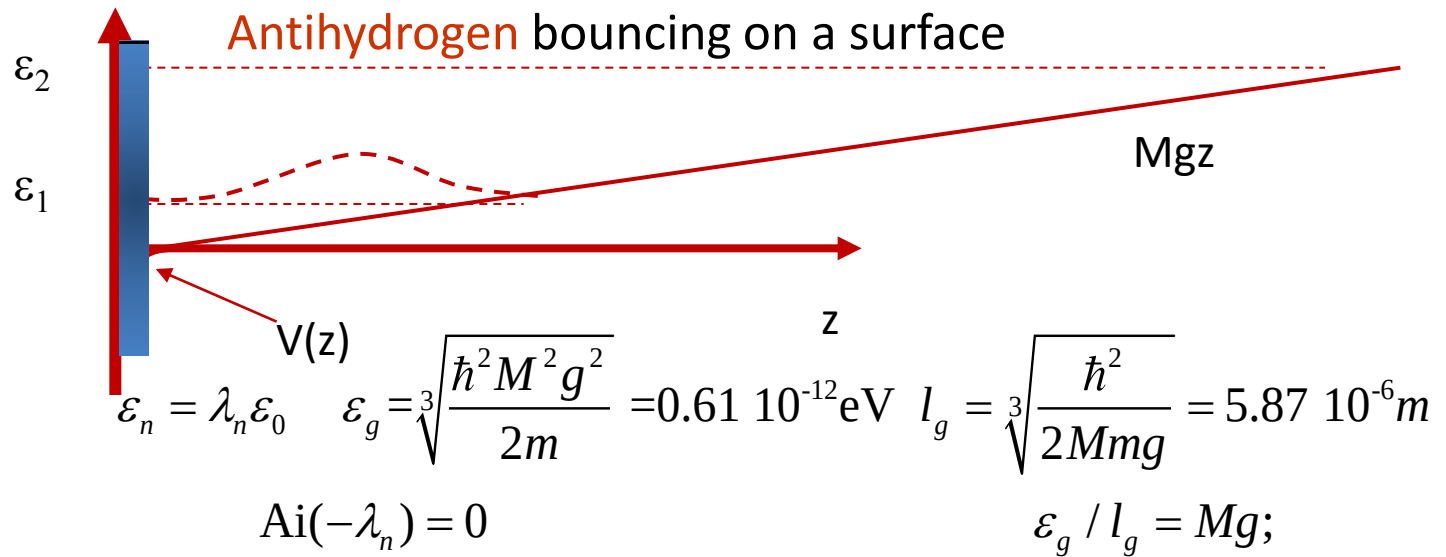
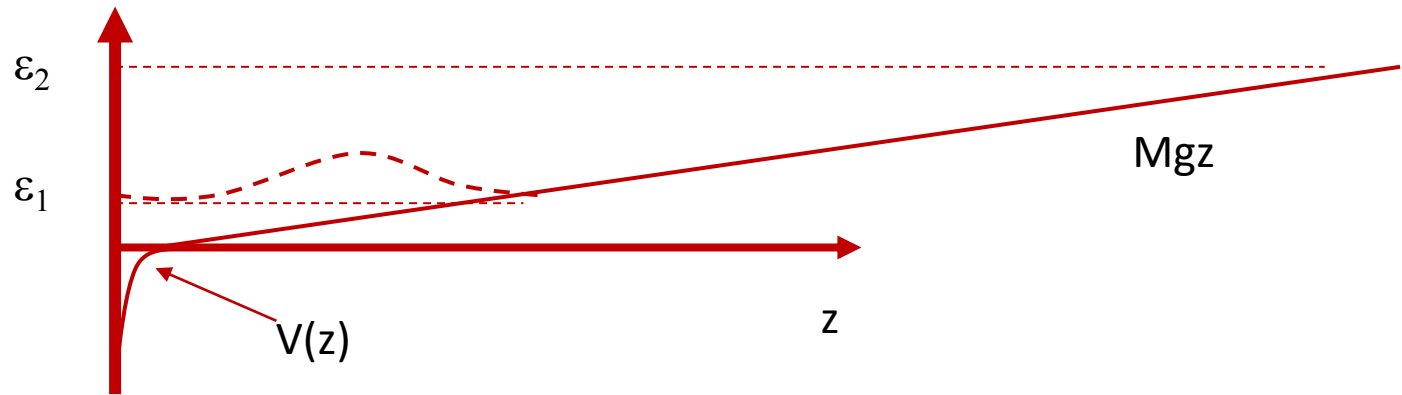


TABLE I. The eigenvalues, gravitational energies, and classical turning points of a quantum bouncer with the mass of (anti)hydrogen in the Earth's gravitational field.

n	λ_n^0	E_n^0 (peV)	z_n^0 (μm)
1	2.338	1.407	13.726
2	4.088	2.461	24.001
3	5.521	3.324	32.414
4	6.787	4.086	39.846
5	7.944	4.782	46.639
6	9.023	5.431	52.974
7	10.040	6.044	58.945

Correction by Casimir-Polder potential + annihilation



$$|a| \approx \sqrt{2mC_4 / \hbar} ; l_0 = \sqrt[3]{\hbar^2 / (2Mmg)};$$

$$|a| / l_0 \approx 0.005$$

Correction by Casimir-Polder $V(z)$: $\varepsilon_n = \varepsilon \tilde{\lambda}_n$

$$\tilde{\lambda}_n = \lambda_n + a / l_0$$

$$\varepsilon_n = \varepsilon_0 (\lambda_n + \text{Re } a / l_0) \quad \Gamma = 2\varepsilon_0 |\text{Im } a| / l_0$$

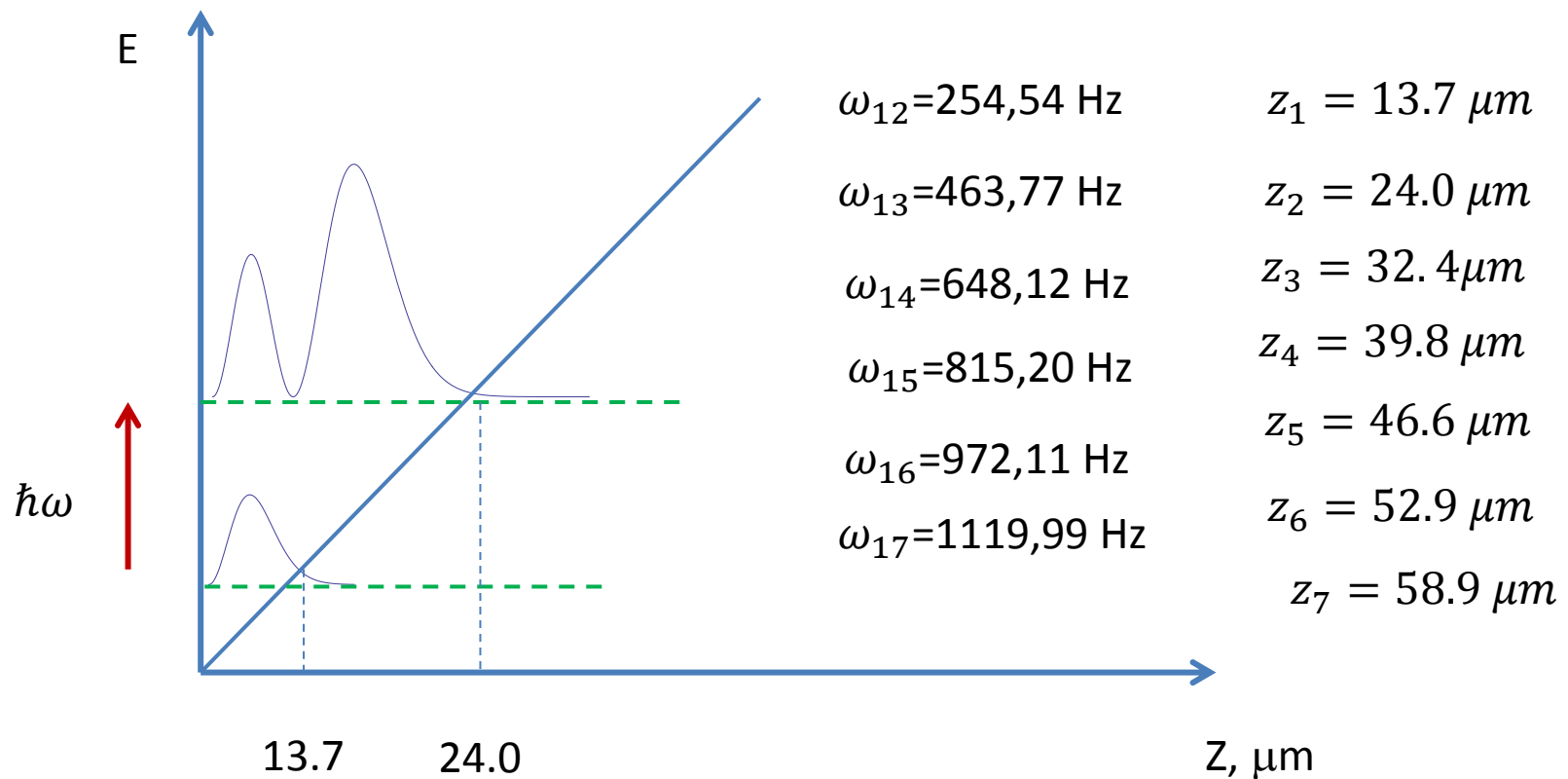
$$\tau = \frac{l_0}{\varepsilon_0} \frac{\hbar}{2|\text{Im } a|} = \frac{\hbar}{2Mg|\text{Im } a|} \approx 0.1s$$

Lifetime is determined by gravitational force Mg and $|\text{Im } a|$.

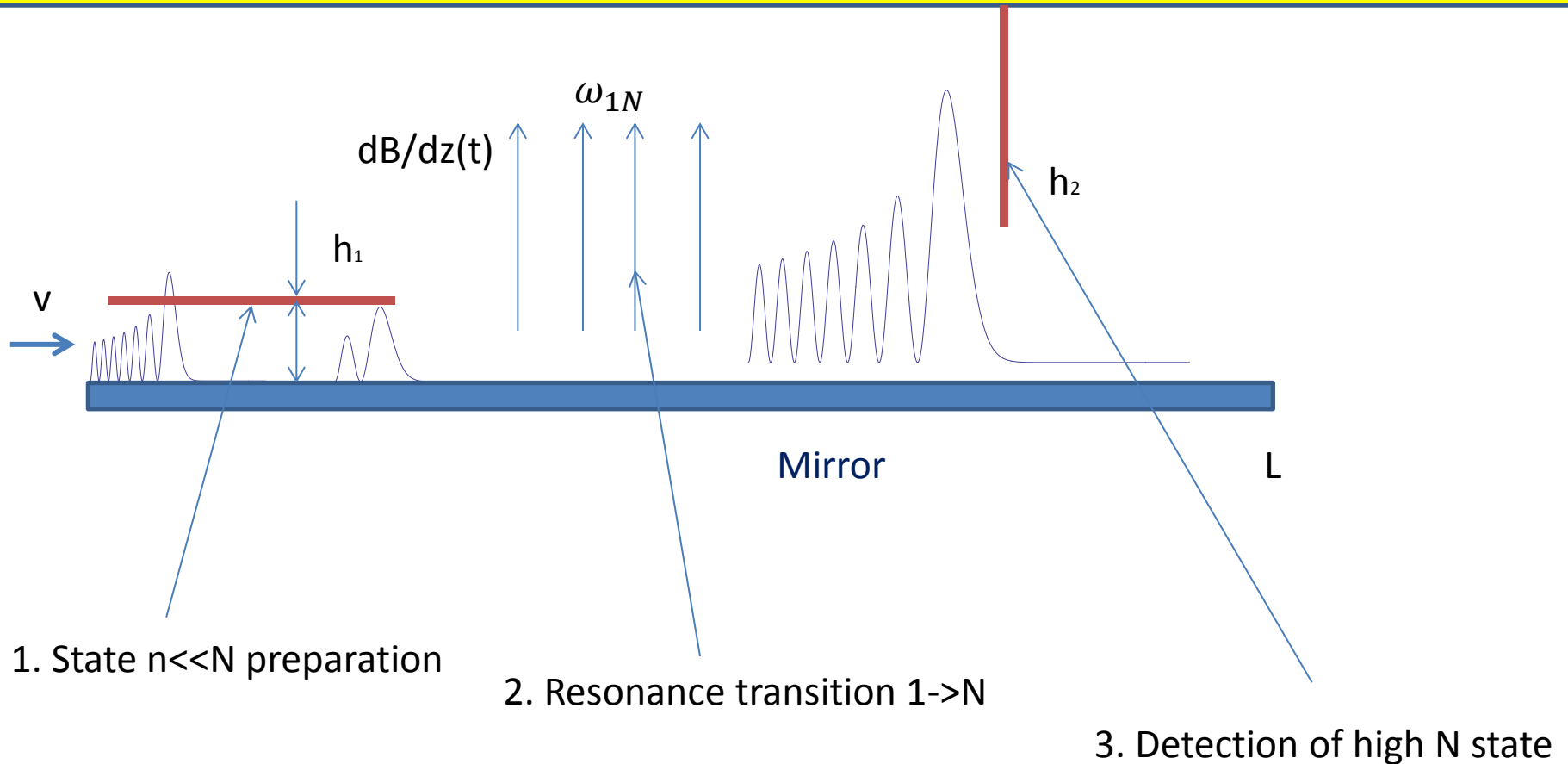
Numbers: $\varepsilon_0 = 0.6 \text{ peV}$; $l_0 = 5.9 \mu\text{m}$; $|\text{Im } a| \approx 20 \text{ nm}$

Idea- to induce transitions between gravitational states with alternating magnetic field

Developed for neutrons by V. Nesvizhevsky, S. Baessler, G. Pignol, K. Protasov, A.Voronin



Principle of observation



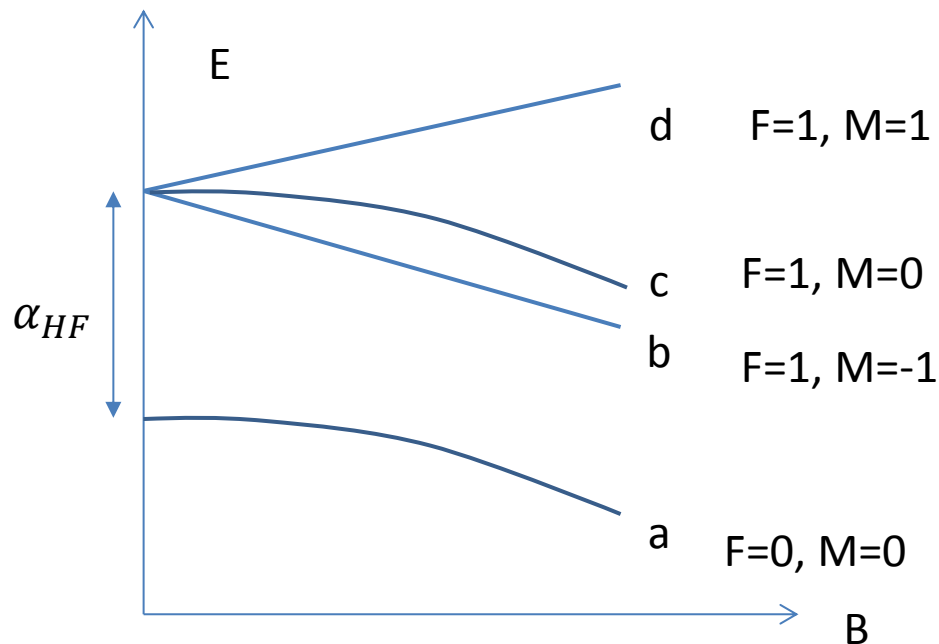
To measure flux of annihilation events at height of N th state as a function of frequency

$$v \sim 1 \frac{m}{s}, h_1 \sim 30 \mu m, h_2 \sim 150 \mu m, L \sim 30 cm$$

Antihydrogen in Magnetic Field

$$\hat{T}_z + Mgz + \hat{H}_c + \frac{\alpha_{HF}}{2} (\mathbf{F}^2 - \frac{3}{2}) + \mathbf{B}(z, t) (\mu_B \hat{\mathbf{s}}_{\bar{e}} + \mu_{\bar{p}} \hat{\mathbf{s}}_{\bar{p}}) + \hat{H}_{st}$$

$\hat{T}_z + Mgz$ → Center-of-mass fall
 $\hat{H}_c$ → Coulomb
 $\frac{\alpha_{HF}}{2} (\mathbf{F}^2 - \frac{3}{2})$ → HF splitting
 $\mathbf{B}(z, t) (\mu_B \hat{\mathbf{s}}_{\bar{e}} + \mu_{\bar{p}} \hat{\mathbf{s}}_{\bar{p}})$ → Field-magnetic moment interaction
 $\hat{H}_{st}$ → Effective Stark-effect



$$\frac{\alpha_{HF}}{\hbar} \cong 1420 \text{ MHz}$$

$$\epsilon_{grav} \ll \mu B \ll \alpha_{HF}$$

Magnetic Field

- $B(t, z, x) = (B_{0z} + \beta z \cos(\omega t))\vec{e}_z - \beta x \cos(\omega t)\vec{e}_x$

We need
 $B_{0z} \gg \beta L$

$$\omega \sim 10^3 \text{ Hz}$$

$$\text{div} \vec{B} = 0$$

$$E_b \cong \alpha_{HF} \frac{1}{4} - \frac{|B(t, z, x)(\mu_B + \mu_p)|}{2}$$

$$E_a \cong -\alpha_{HF} \frac{3}{4} - \frac{|B(t, z, x)(\mu_B - \mu_p)|^2}{4\alpha_{HF}}$$

$$E_d \cong \alpha_{HF} \frac{1}{4} + \frac{|B(t, z, x)(\mu_B + \mu_p)|}{2}$$

$$E_c \cong \alpha_{HF} \frac{1}{4} + \frac{|B(t, z, x)(\mu_B - \mu_p)|^2}{4\alpha_{HF}}$$

Transitions between states

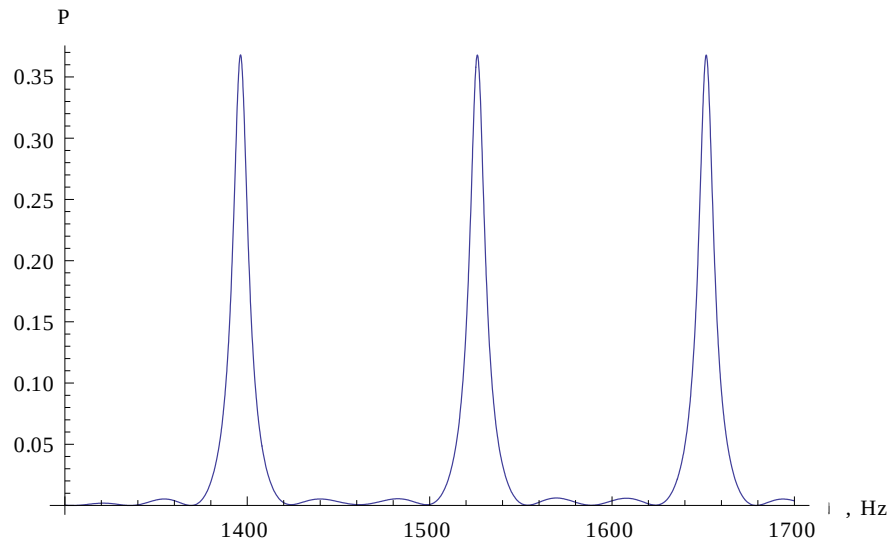
- $\frac{e^2 m_e}{\hbar^2} \gg \alpha_{HF} \gg \mu_B B \gg \varepsilon_g$ *adiabaticity*
- $i\hbar \frac{\partial \Psi(z, \mathbf{r}, t)}{\partial t} = H \Psi(z, \mathbf{r}, t)$
- $\Psi(z, \mathbf{r}, t) = \sum_{\alpha, i} C_{\alpha, i}(t) g_i(z) \varphi_{\alpha}(\mathbf{r}, z, t) \exp(-i\varepsilon_i t - iE_{\alpha, 0} t)$
- $\left(\hat{H}_c + \frac{\alpha_{HF}}{2} \left(\mathbf{F}^2 - \frac{3}{2} \right) + \mathbf{B}(z, t) (\mu_B \hat{\mathbf{s}}_{\bar{e}} + \mu_{\bar{p}} \hat{\mathbf{s}}_{\bar{p}}) - E_{\alpha}(z, t) \right) \varphi_{\alpha}(\mathbf{r}, z, t)$
- $= 0$
- $i\hbar \frac{\partial C_{\alpha, i}}{\partial t} = \sum_{k=1}^N C_{\alpha k} \langle g_i | E_{\alpha}(t, z) | g_k \rangle \exp(-i\omega_{ik} t)$

$$E_{(b,d)}(t, z) = \pm (\mu_B + \mu_{\bar{p}}) \sqrt{(B_0 + \beta z \cos(\omega t))^2 + \beta^2 v^2 t^2}$$

Transition amplitudes

$$P_{ik} = \frac{\Omega_{ik}^2}{\Omega_{ik}^2 + \hbar^2 \Delta^2} \sin^2\left(\frac{t}{\hbar} \sqrt{\Omega_{ik}^2 + \hbar^2 \Delta^2}\right) \exp(-\Gamma t)$$

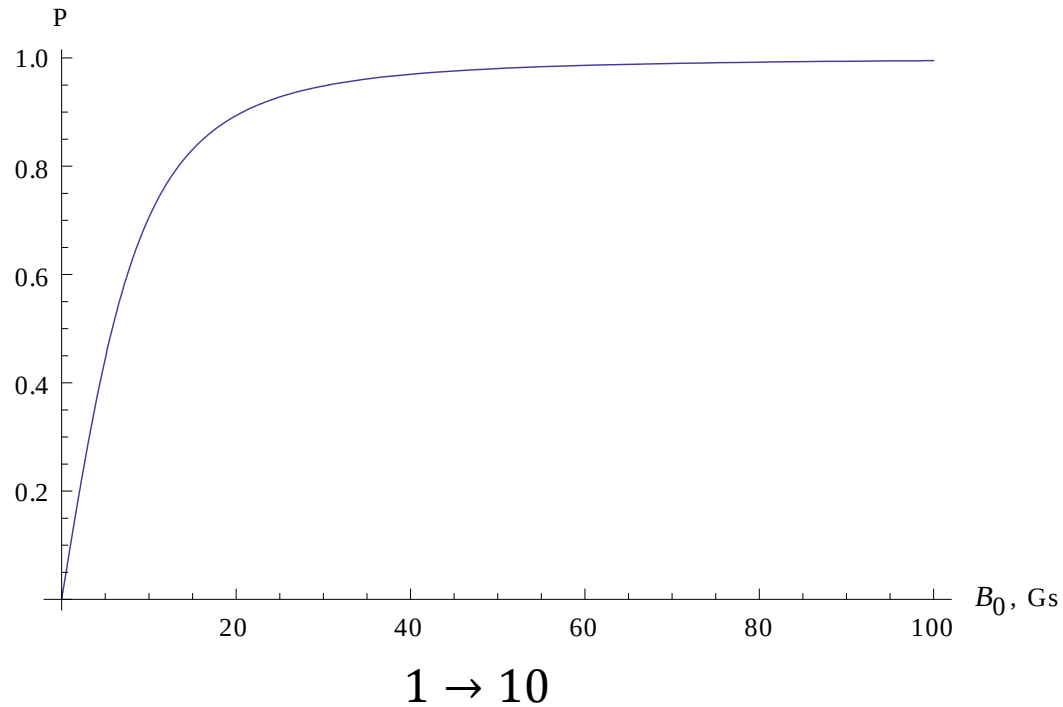
$$\Omega_{ik} = \frac{(\mu_B + \mu_{\bar{p}}) \beta l_g^3}{\hbar (z_k - z_i)^2}$$



Transition probability as a function of frequency. Transitions 1->9,1->10,1->11

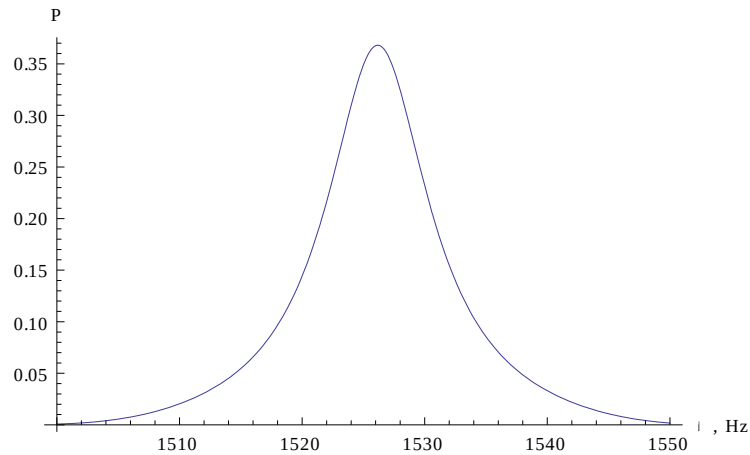
Time of observation $t=0.1$ s

Guiding field



Transition probability as a function of guiding field B_{0z}

Spectral line



$$\beta = 33 \text{ Gs/m} \quad B_0 = 100 \text{ Gs}$$

$$\Delta \cong 20 \text{ Hz}$$

$$1 \rightarrow 10 \quad z_{10} = 75.29 \mu m$$

$$\omega_{1-10} = 1526.17 \text{ Hz} \quad \Gamma \approx 20 \text{ Hz}$$

$$\text{Line shift } \delta\omega_{nk} = (\mu_B + \mu_{\bar{p}}) \frac{8\beta^2(z_n^2 - z_k^2)}{15\hbar B_0}$$

$$\delta\omega_{1-10} = 0.05 \text{ Hz}$$

Time, space, gravitational mass

$$\frac{\hbar\omega_{ik}}{z_k - z_i} = Mg$$

$$M = m \rightarrow \frac{\omega_{ik}}{(\lambda_k - \lambda_i)^{3/2}} = \sqrt{\frac{g}{2(z_k - z_i)}}$$

$$\omega_{ik} = \sqrt[3]{\frac{\hbar^2 M^2 g^2}{2m}} (\lambda_k - \lambda_i) \rightarrow M = \sqrt{\frac{2m\omega_{ik}^3}{g^2 \hbar^2 (\lambda_k - \lambda_i)^3}}$$

$$\varepsilon \approx \frac{3\Gamma}{2\sqrt{N_{\bar{H}}}\omega}; N_{\bar{H}} = 10^3$$

$$\varepsilon_{1-10} \approx 10^{-3}$$

Conclusions

- Resonance transitions induced by alternating field enable us to measure energy spacing between gravitational quantum states and extract gravitational mass M
- Typical parameters: field gradient $dB/dz \sim 30$ Gs/m, guiding field $B_z \sim 100$ Gs
- Time of life limitation for the line-width $\Delta \cong 20$ Hz
- Preferred transitions $1 \rightarrow N$ with N maximum accessible (N easy ~ 10 ; N resolved ~ 300)