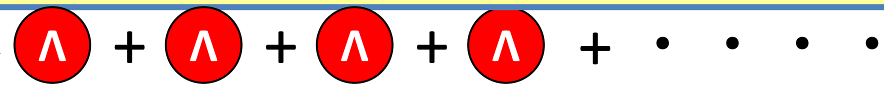
Four- and five-body calculation of double Λ hyperuclei

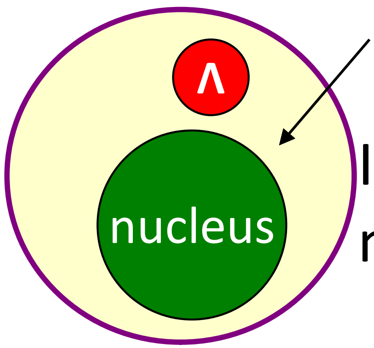
E. Hiyama (RIKEN)

One of major goals of hypernuclear physics

To study the structure of multi-strangeness systems

What is the structure when many Λs are added to a nucleus?





It is conjectured that extreme limit, which includes many Λs in nuclear matter, is the core of a neutron star.

In this meaning, the sector of S=-2 nuclei , double Λ hypernuclei and Ξ hypernuclei is just the entrance to the multi-strangeness world.

However, we have hardly any knowledge of the YY interaction because there exist no YY scattering data.

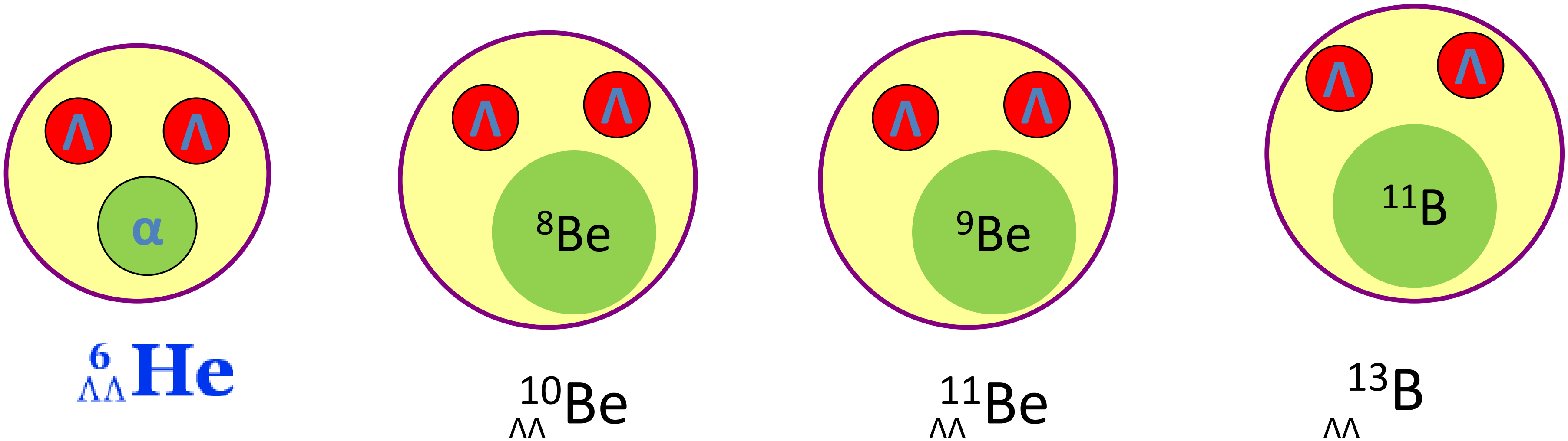
Then, in order to understand the YY interaction, it is crucial to study the structure of double Λ hypernuclei and Ξ hypernuclei.

Question: How many double Λ hypernuclei did we observe?

Up to now,

Only four double Λ hypernuclei

Observed by Emulsion



Since the number of observation of double Λ hypernuclei is too small, then, the highest priority in this sector is to observe many double Λ hypernuclei.

If we have many double Λ hypernuclei in the future, we can study interesting structure of double Λ hypernuclei.

Powerful tools to observe double Λ hypernuclei are

(1) to use anti-proton beam at GSI (Panda project).

(2) to have many double Λ hypernuclei by emulsion (J-PARC).

Advantage at PANDA project
We can select a target and produce double Λ hypernuclei which we want to have.

Advantage at Emulsion
We can have many double Λ hypernuclei (about 100 at J-PARC).
But, we do not know which kinds of double Λ hypernuclei will be produced.

・E07 Approved proposal at J-PARC

"Systematic Study of double strangness systems at J-PARC"

by Nakazawa and his collaborators

It is difficult to determine
- (1) spin-parity
- (2) whether the observed state is the ground state or an excited state

My theoretical contribution using few-body calculation

Emulsion experiment ⟷ **comparison** ⟷ **Theoretical calculation**
input: ΛΛ interaction to reproduce the observed binding energy of $_{\Lambda\Lambda}{}^{6}He$

↓

the identification of the state

A example

Calculation Method

Our few-body caluclation method

Gaussian Expansion Method (GEM) , since 1987,

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group,
Kamimura and his collaborators.

Review article :
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

High-precision calculations of various 3- and 4-body systems:

Exotic atoms / molecules ,
3- and 4-nucleon systems,
multi-cluster structure of light nuclei,
Light hypernuclei,
3-quark systems,

This method also successfully applied to four-body problems.

The merit of this method:

(1)To calculate the energy of bound state very accurately

(2) To calculate the wavefunction very precisely

one successful examples

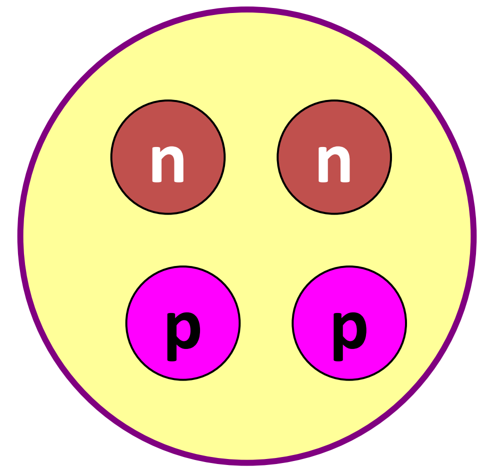
PRC **64**, 044001(2001)

Benchmark-test calculation to solve the 4-nucleon bound state

7 different groups (18 co-authors)

1. Faddeev-Yakubovski (Kamada et al.)
2. Gaussian Expansion Method
   (Kamimura and Hiyama)
3. Stochastic varitional (Varga et al.)
4. Hyperspherical variational (Viviani et al.)
5. Green Function Variational Monte Carlo
   (Carlson at al.)
6. Non-Core shell model (Navratil et al.)
7. Effective Interaction Hypershperical
   HarmonicsEIHH (Barnea et al.)

^{4}He



4-nucleon
bound state
NN**:** AV8’

Benchmark-test calculation of the 4-nucleon bound state

Good agreement among 7different methods

In the binding energy, r.m.s. radius and wavefunction density

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
GEM	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

ours

very different techniques and the complexity of the nuclear force chosen. Except for NCSM and EIHH, the expectation values of T and V also agree within three digits. The NCSM results are, however, still within 1% and EIHH within 1.5% of the others, but note that the EIHH results for T and V are

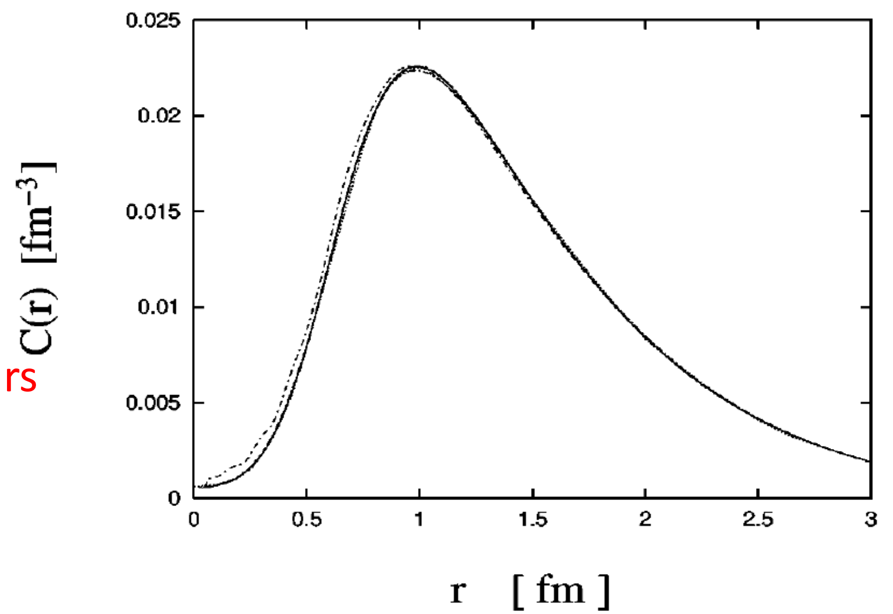


FIG. 1. Correlation functions in the different calculational schemes: EIHH (dashed-dotted curves), FY, CRCGV, SVM, HH, and NCSM (overlapping curves).

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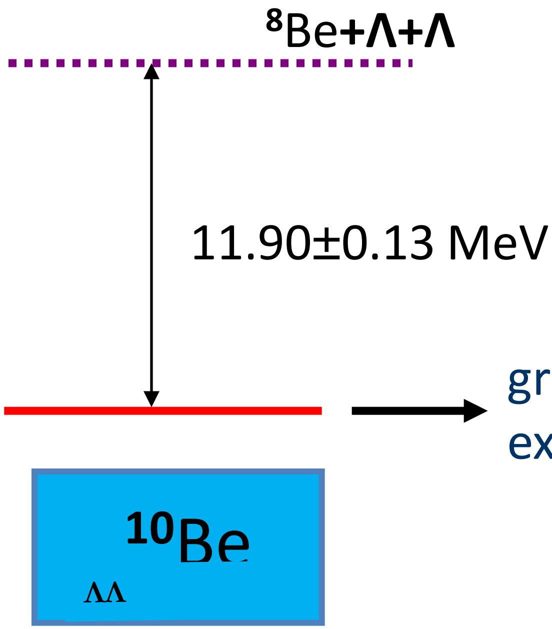
↓

the identification of the state

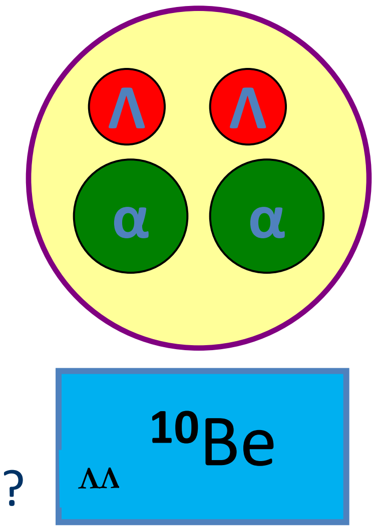
Two examples

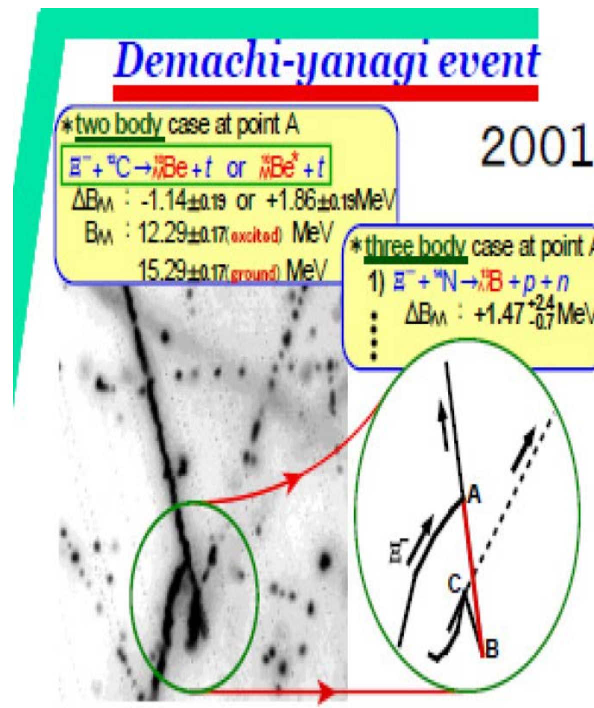
Successful example to determine spin-parity of double Λ hypernucleus --- Demachi-Yanagi event for ${}^{10}_{\Lambda\Lambda}Be$

Observation of ${}^{10}_{\Lambda\Lambda}Be$ --- KEK-E373 experiment



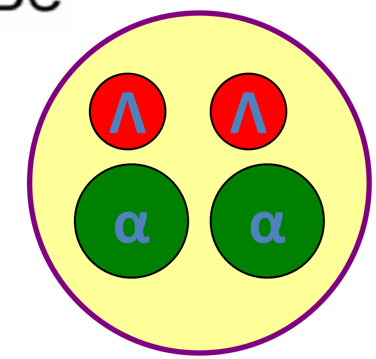
ground state ?
excited state ?

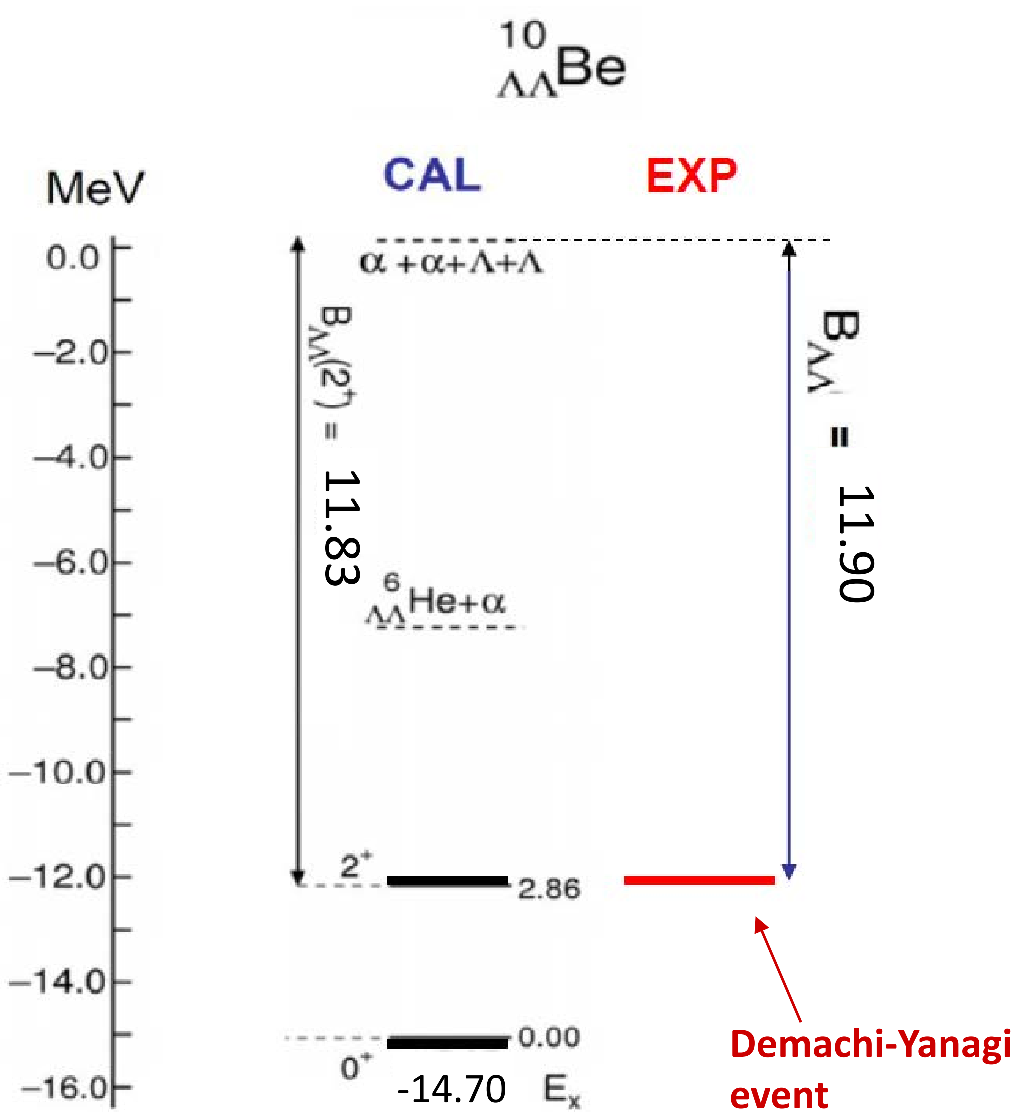




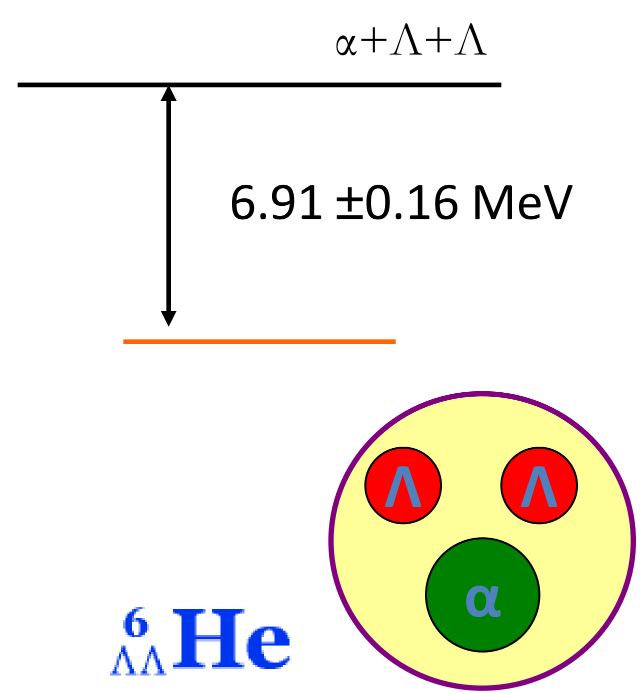
Demachi-Yanagi event

Successful interpretation of spin-parity of $^{10}_{\Lambda\Lambda}Be$





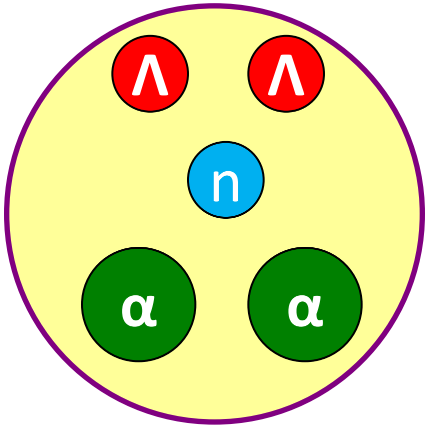
E. Hiyama, M. Kamimura,T.Motoba, T. Yamada and Y. Yamamoto
Phys. Rev. 66 (2002) , 024007



Observation of Hida event

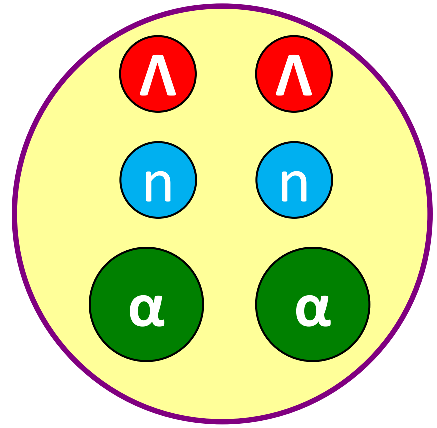
KEK-E373 experiment

Two years ago, observed



$^{11}_{\Lambda\Lambda}Be$

$B_{\Lambda\Lambda}$ = 20.49±1.15 MeV



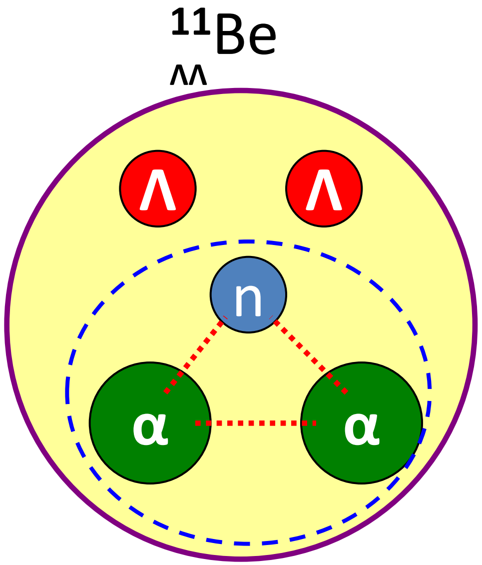
$^{12}_{\Lambda\Lambda}Be$

$B_{\Lambda\Lambda}$ = 22.06±1.15 MeV

Important issue:

Is the Hida event the observation of a ground state or an excited state?

It is neccesary to perform 5-body calculation of this system.
Why 5-body?



Core nucleus, ^{9}Be is well described as α+α+ n three-cluster model.

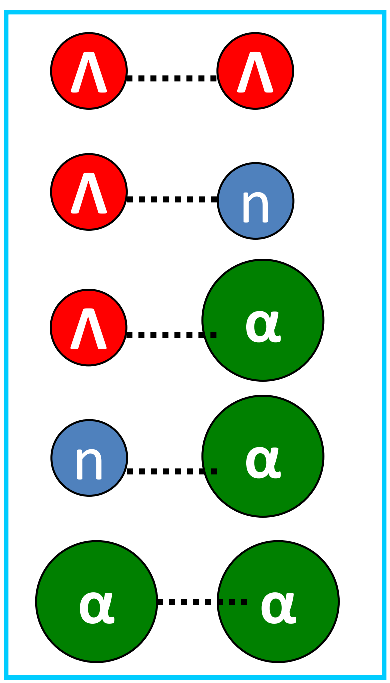
Then, $^{11}_{\Lambda\Lambda}$Be is considered to be suited for studying with α+α+ n +Λ+Λ 5-body model.

Difficult 5-body calculation:

1) 3 kinds of particles (**α**, Λ, **n**)

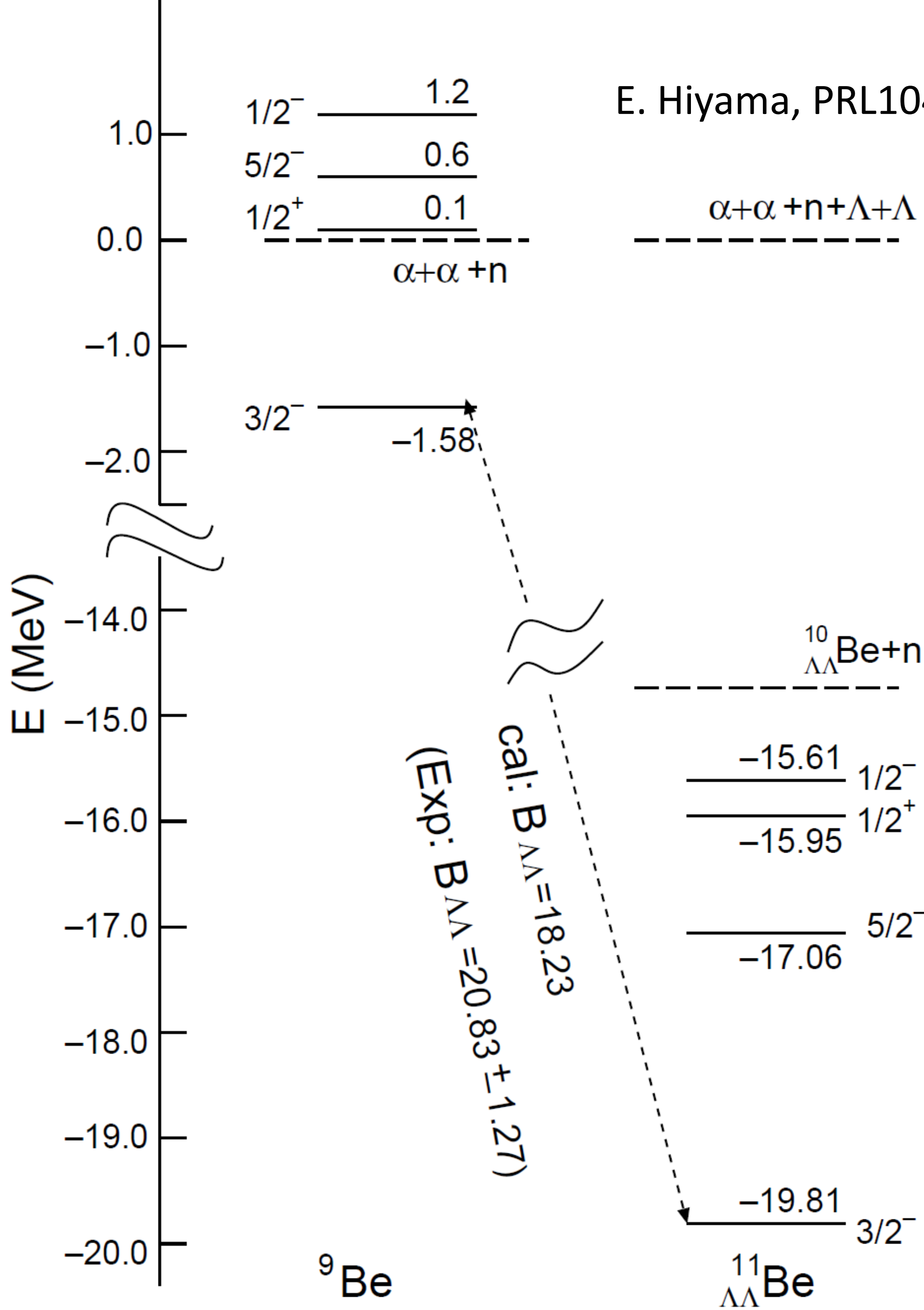
2) 5 different kinds of interactions

3) Pauli principle between **α** and **α**, and between **α** and **n**



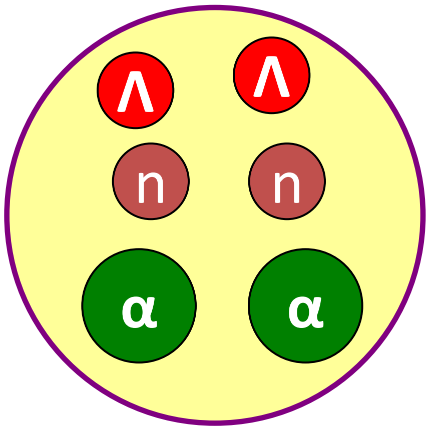
But, I have succeeded in performing this calculation.

E. Hiyama, PRL104, 212502(2010)



As mentioned before, Hida event has another possibility, namely, observation of ${}^{12}_{\Lambda\Lambda}Be$.

${}^{12}_{\Lambda\Lambda}Be$



$B_{\Lambda\Lambda}$= 22.06±1.15 MeV

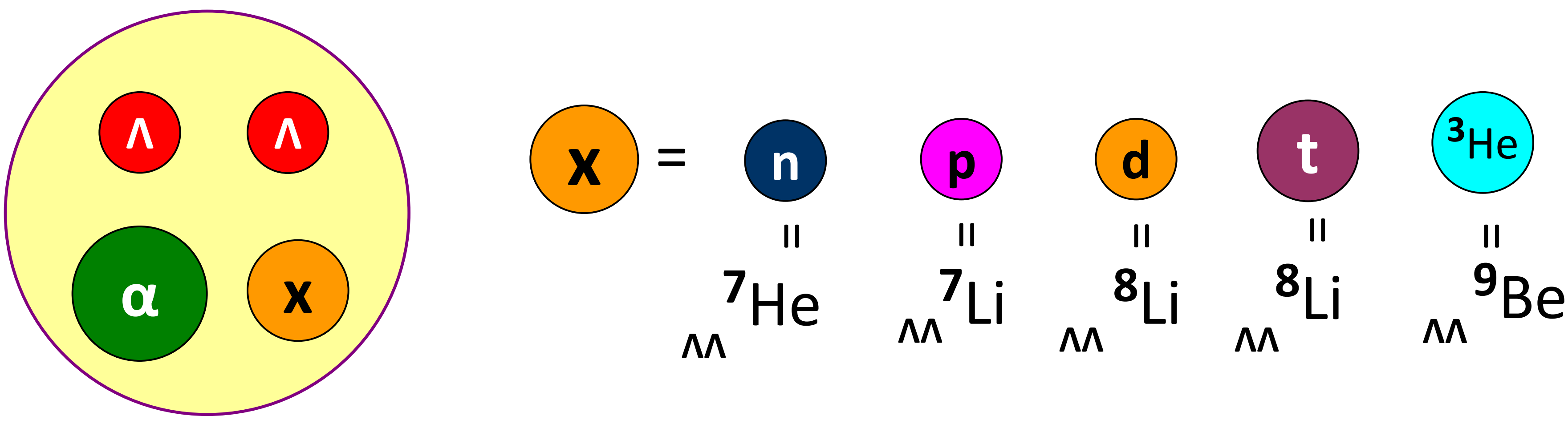
For this study, it is necessary to calculate 6-body problem.
It is good chance to develop my method
for 6-body problem. This is my future plan.

For the confirmation of Hida event, we expect to have more precise data at J-PARC.

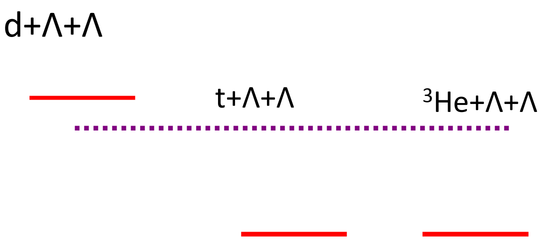
Hoping to observe new double **Λ** hypernuclei in future experiments, I predicted level structures of these double **Λ** hypernuclei within the framework of the α+x+**Λ**+**Λ** 4-body model.

E. Hiyama, M. Kamimura, T. Motoba, T.Yamada and Y. Yamamoto

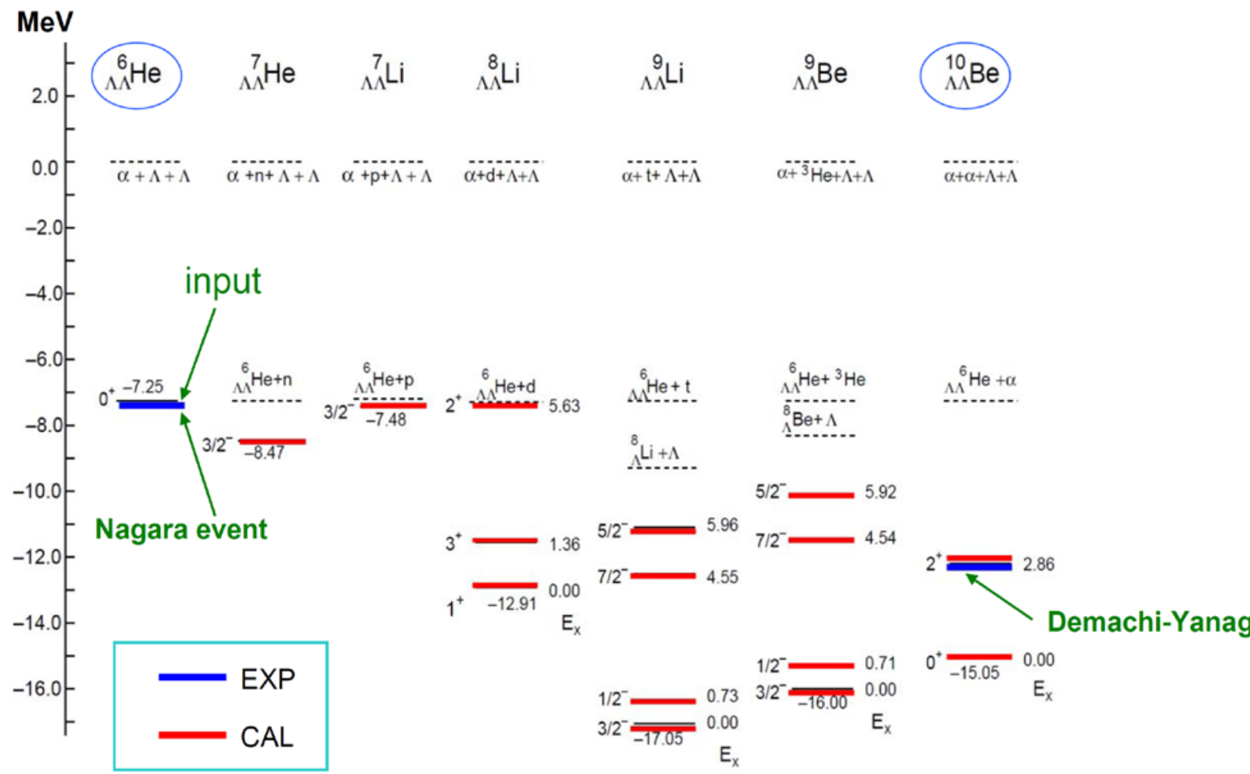
Phys. Rev. C66, 024007 (2002)



Spectroscopy of ΛΛ-hypernuclei



$_{\Lambda\Lambda}{}^{4}H$, $_{\Lambda\Lambda}{}^{5}H$, $_{\Lambda\Lambda}{}^{5}He$



In s-shell hypernuclei, probably, our prediction might be inconsistent with the future data.

In p-shell hypernuclei, probably, our prediction might be consistent with the future experimental data.

Why our prediction in s-shell double Λ hypernuclei
is inconsistent with the future data?

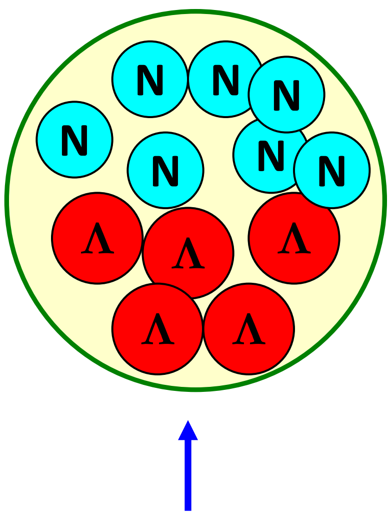
In the s-shell hypernuclei,
new component of interaction
should be taken into account.

A missing component is ΛΛ—ΞN coupling.

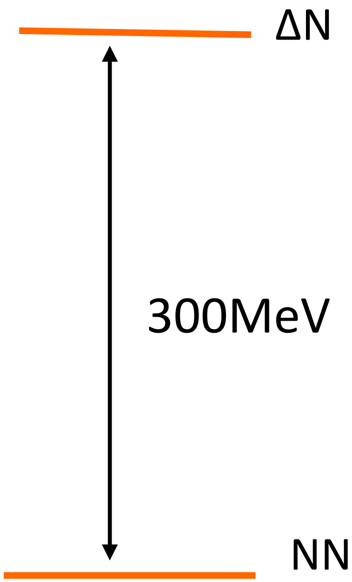
ΛΛ-ΞN coupling

One of the major goals in hypernuclear physics :
To study the structure of multi-strangeness systems (extreme limit : neutron star)

Multi-strangeness systems



threshold energy
difference
is very small !

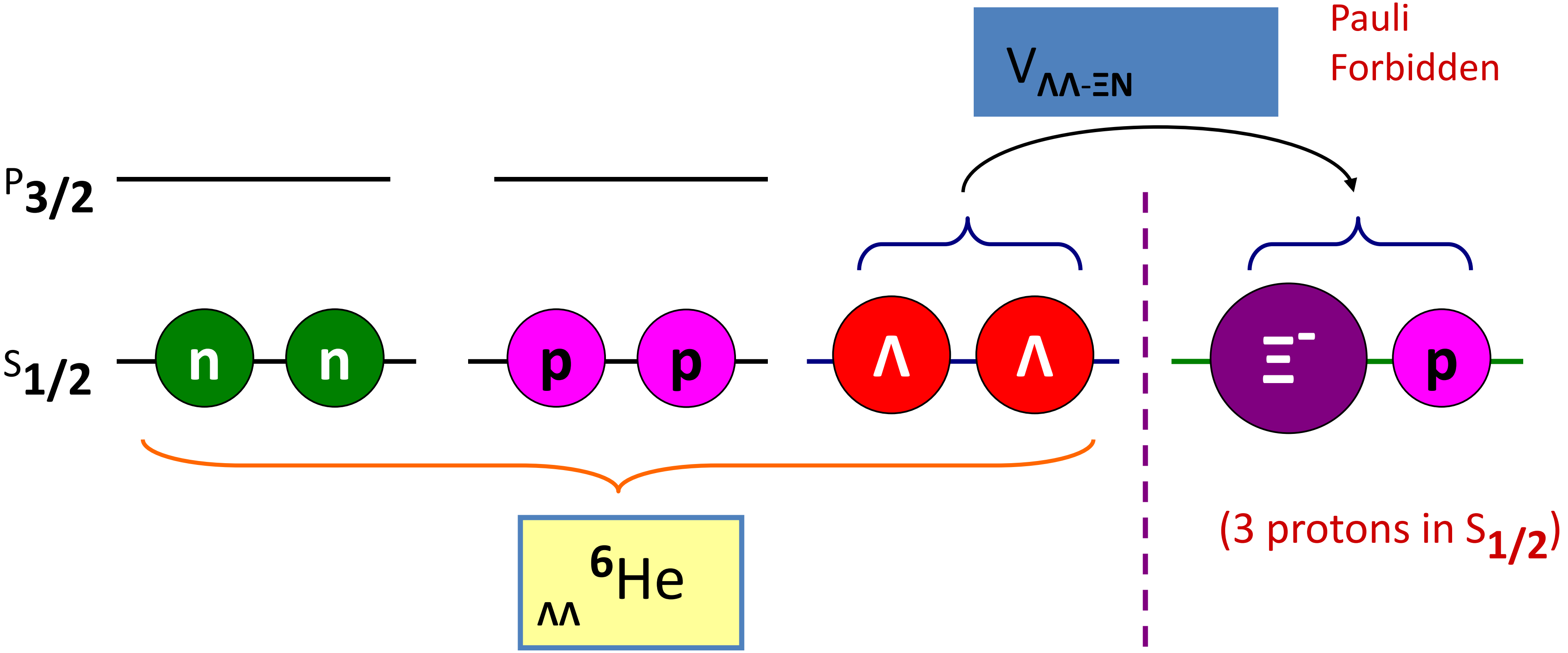


ΛΛ→ΞN particle conversion is
is strong in multi-strangeness system.

The ΛΛ-ΞN coupling is small in $^{6}_{\Lambda\Lambda}He$ and heavier double Λ Hypernuclei.

On the other hand, the ΛΛ-ΞN coupling is significant in S-shell double Λ hypernuclei such as $^{4}_{\Lambda\Lambda}H$ and $^{5}_{\Lambda\Lambda}H$ ($^{5}_{\Lambda\Lambda}He$).

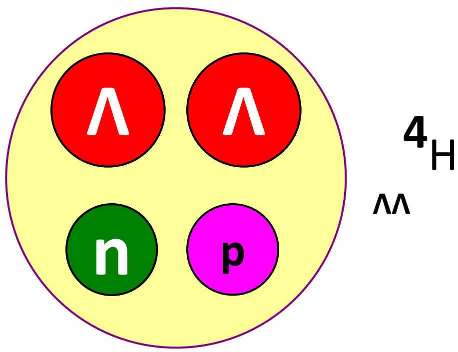
Effect of ΛΛ - ΞN coupling is small in ${}^{6}_{\Lambda\Lambda}He$

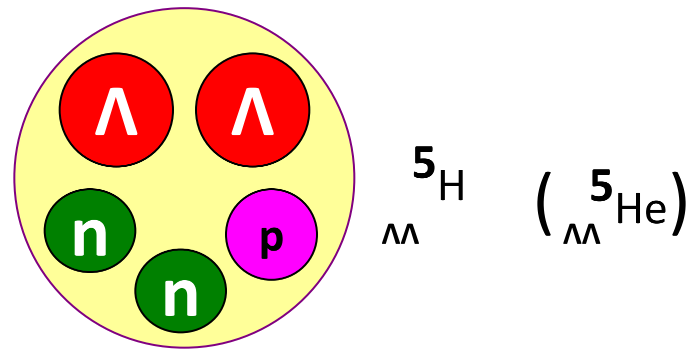


- I.R. Afnan and B.F. Gibson, Phys. Rev. C67, 017001 (2003).
- Khin Swe Myint, S. Shinmura and Y. Akaishi, nucl-th/029090.
- T. Yamada and C. Nakamoto, Phys. Rev.C62, 034319 (2000).

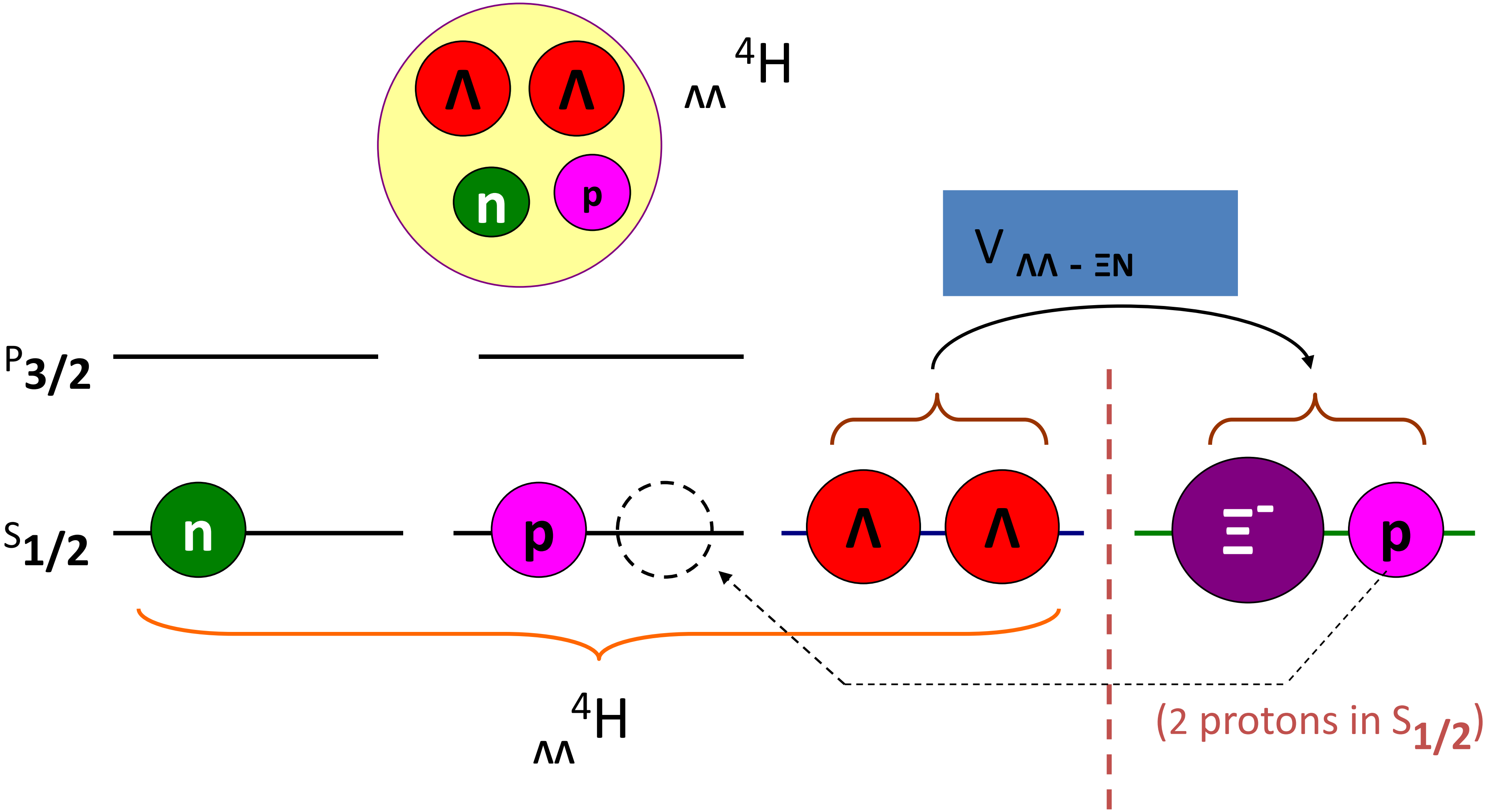
For the study of **ΛΛ - Ξ**N coupling interaction,

${}^{4}_{\Lambda\Lambda}$H and ${}^{5}_{\Lambda\Lambda}$H (${}^{5}_{\Lambda\Lambda}$He) are very suitable.



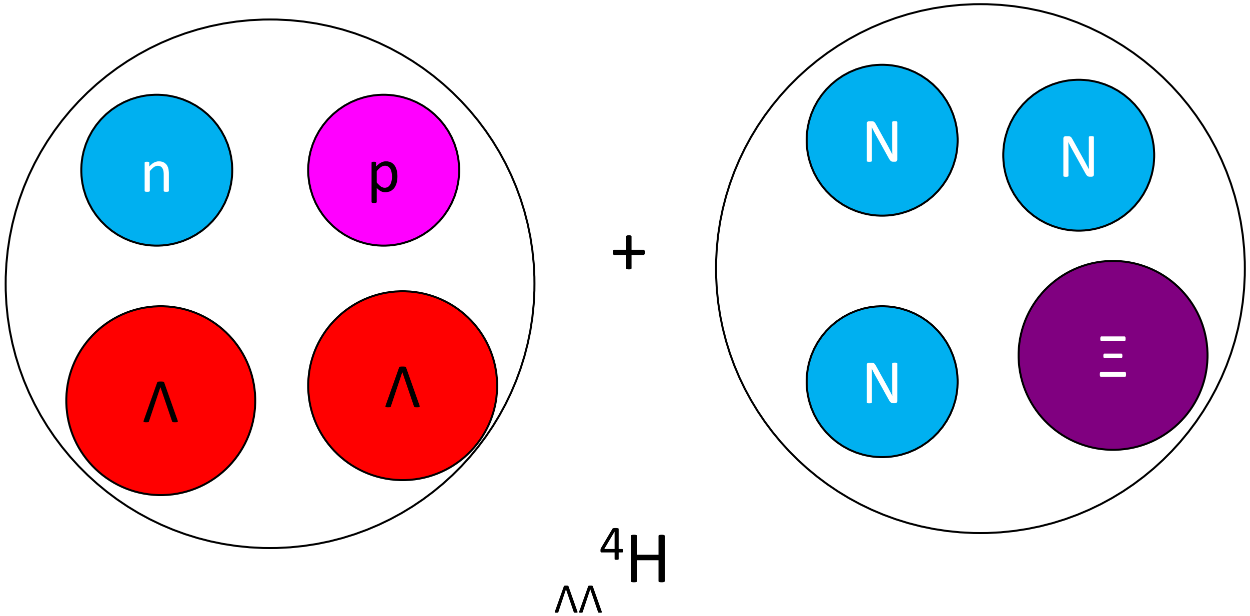


- I.N. Filikhin and A. Gal, Phys. Rev. Lett. 89, 172502 (2002)
- Khin Swe Myint, S. Shinmura and Y. Akaishi, Eur. Phys. J. A16, 21 (2003).
- D. E. Lanscoy and Y. Yamamoto, Phys. Rev. C69, 014303 (2004).
- H. Nemura, S. Shinmura, Y. Akaishi and Khin Swe Myint, Phys. Rev. Lett. 94, 202502 (2005).

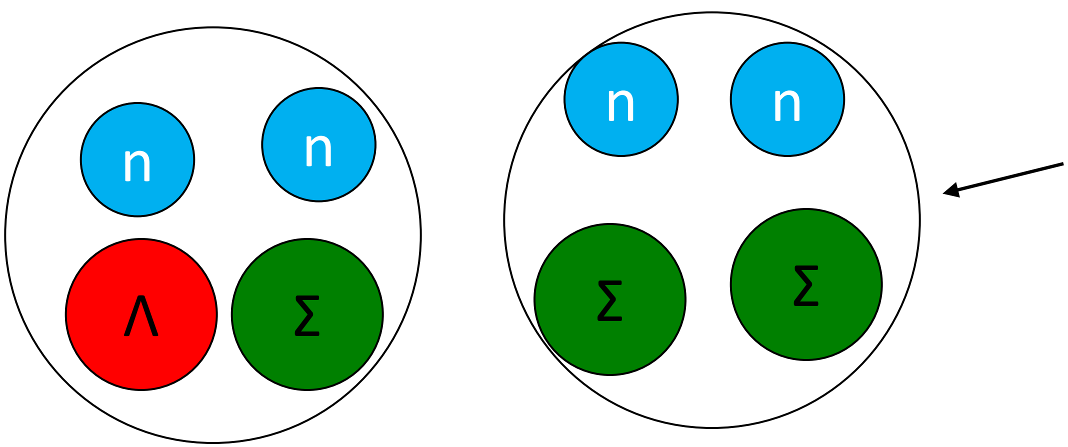


Due to NO Pauli plocking, the **ΛΛ - Ξ**N coupling can be large in $^{4}_{\Lambda\Lambda}H$

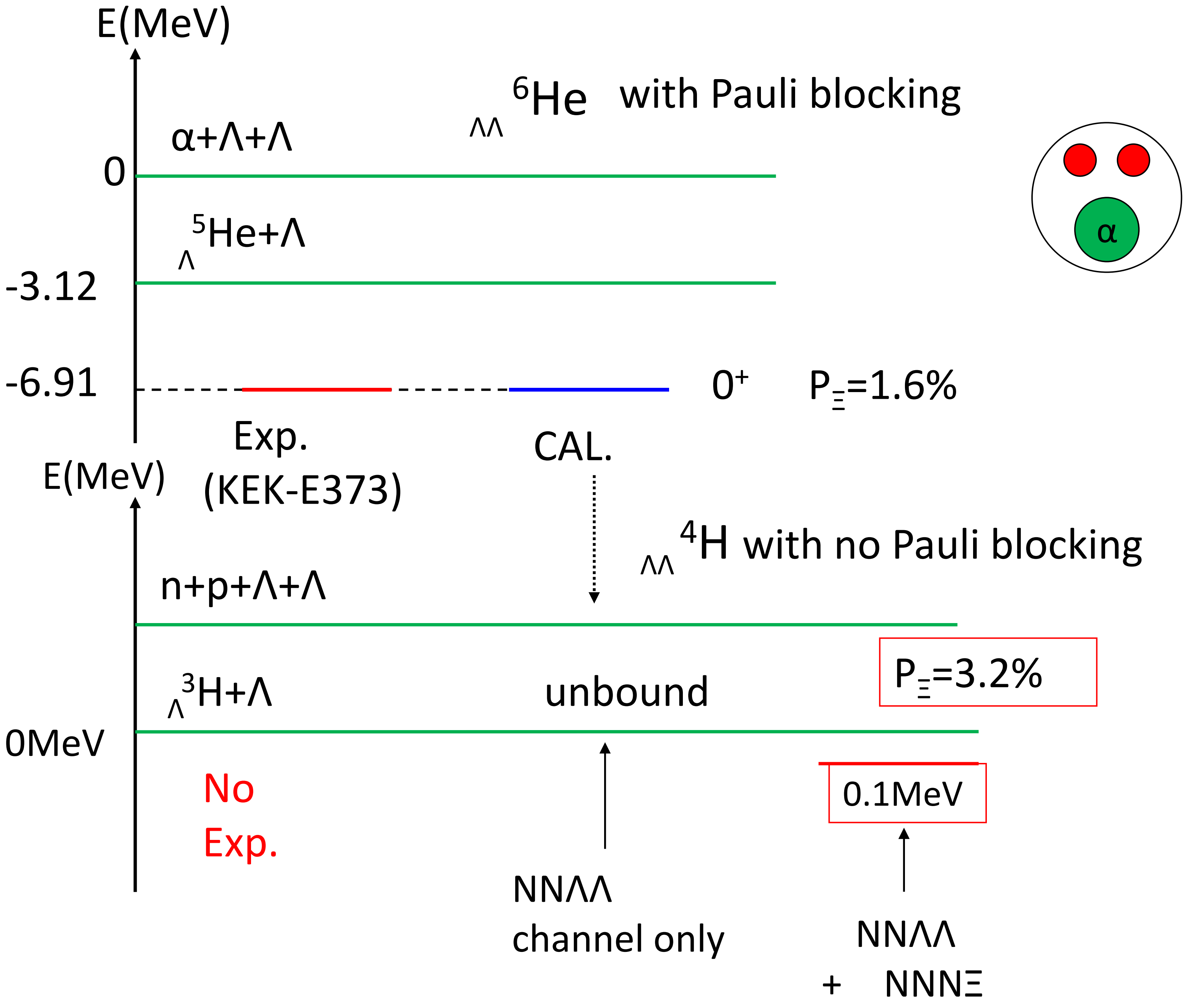
B.F. Gibson, I.R. Afnan, J.A.Carlson and D.R.Lehman, Prog. Theor. Phys. Suppl. 117, 339 (1994).



One of the most numerically difficult 4-body problem



E. Hiyama
Dr. Nemura



E(MeV)
$^{6}_{\Lambda\Lambda}He$ with Pauli blocking
α+Λ+Λ
0
$^{5}_{\Lambda}He$+Λ
-3.12
-6.91
0^+
P_Ξ=1.6%
α
Exp.
(KEK-E373)
CAL.
E(MeV)
$^{4}_{\Lambda\Lambda}H$ with no Pauli blocking
n+p+Λ+Λ
P_Ξ=3.2%
$^{3}_{\Lambda}H$+Λ
unbound
0MeV
No
Exp.
0.1MeV
NNΛΛ
channel only
NNΛΛ
+ NNNΞ

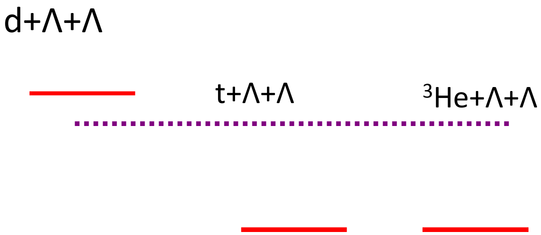
It is possible that ΛΛ—ΞN coupling potential makes the lightest double Λ hypernucleus, ${}_{\Lambda\Lambda}^{4}H$ bound.

If the bound state of ${}_{\Lambda\Lambda}^{4}H$ is observed ,

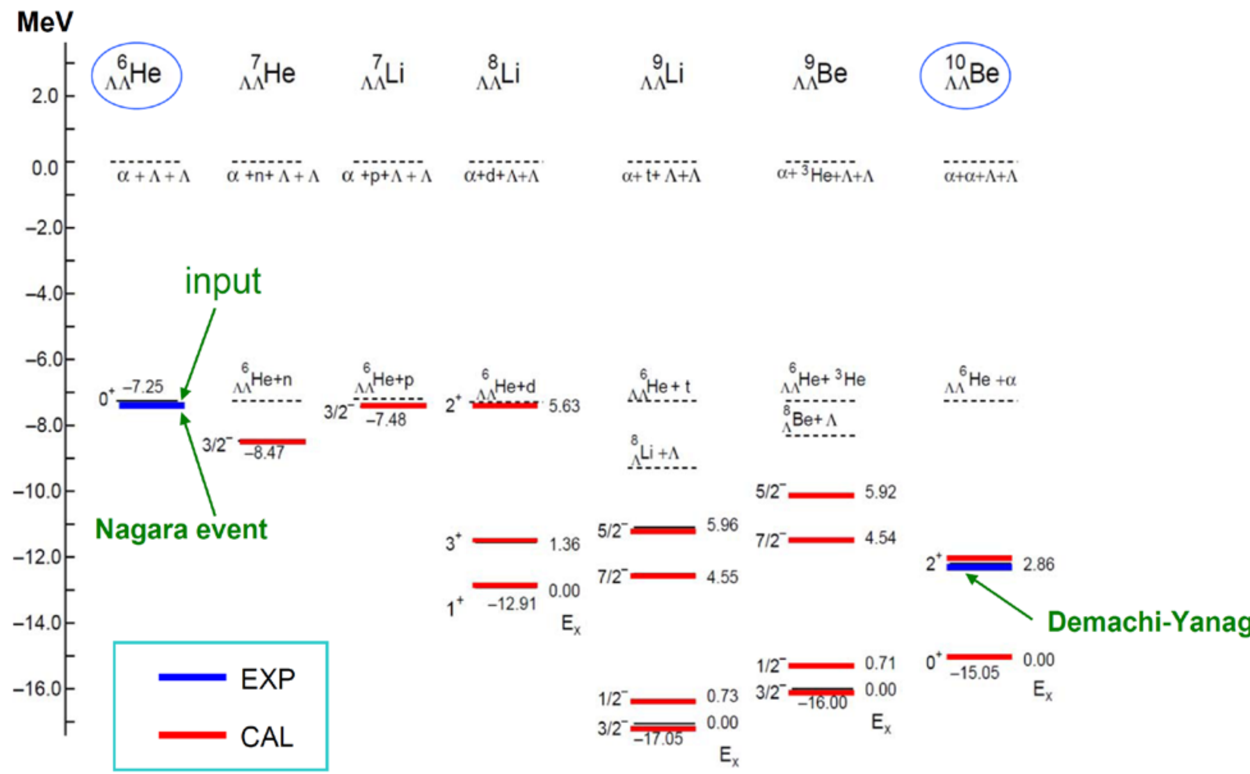
we can obtain useful information on ΛΛ—ΞN coupling mechanism. At PANDA project, if we use ${}^{9}Be$ target, we can produce ${}_{\Lambda\Lambda}^{4}H$.

I hope that PANDA project can produce this hypernucleus in the future.

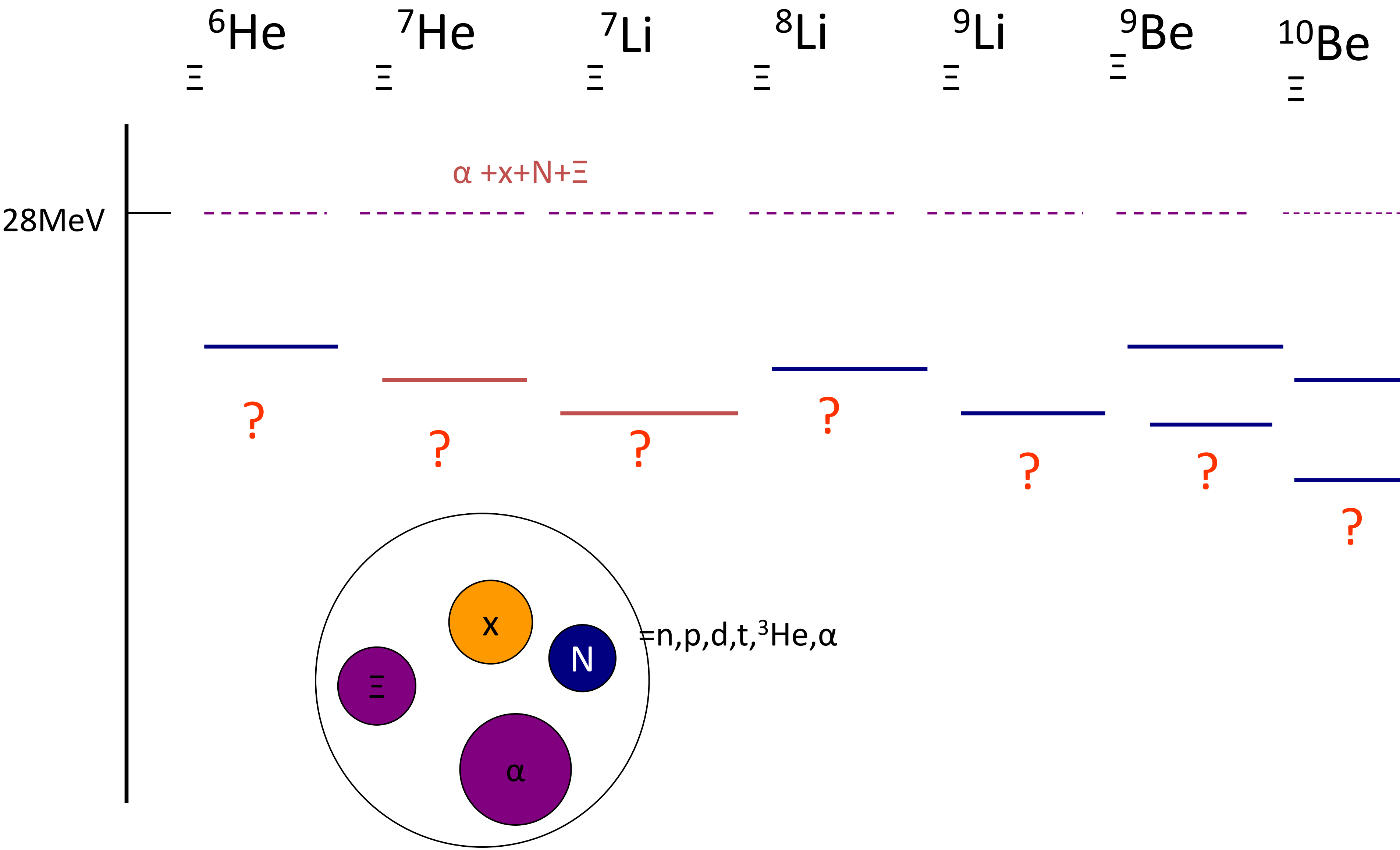
Spectroscopy of ΛΛ-hypernuclei



${}_{\Lambda\Lambda}^{4}H$, ${}_{\Lambda\Lambda}^{5}H$, ${}_{\Lambda\Lambda}^{5}He$



Spectroscopy of Ξ hypernuclei



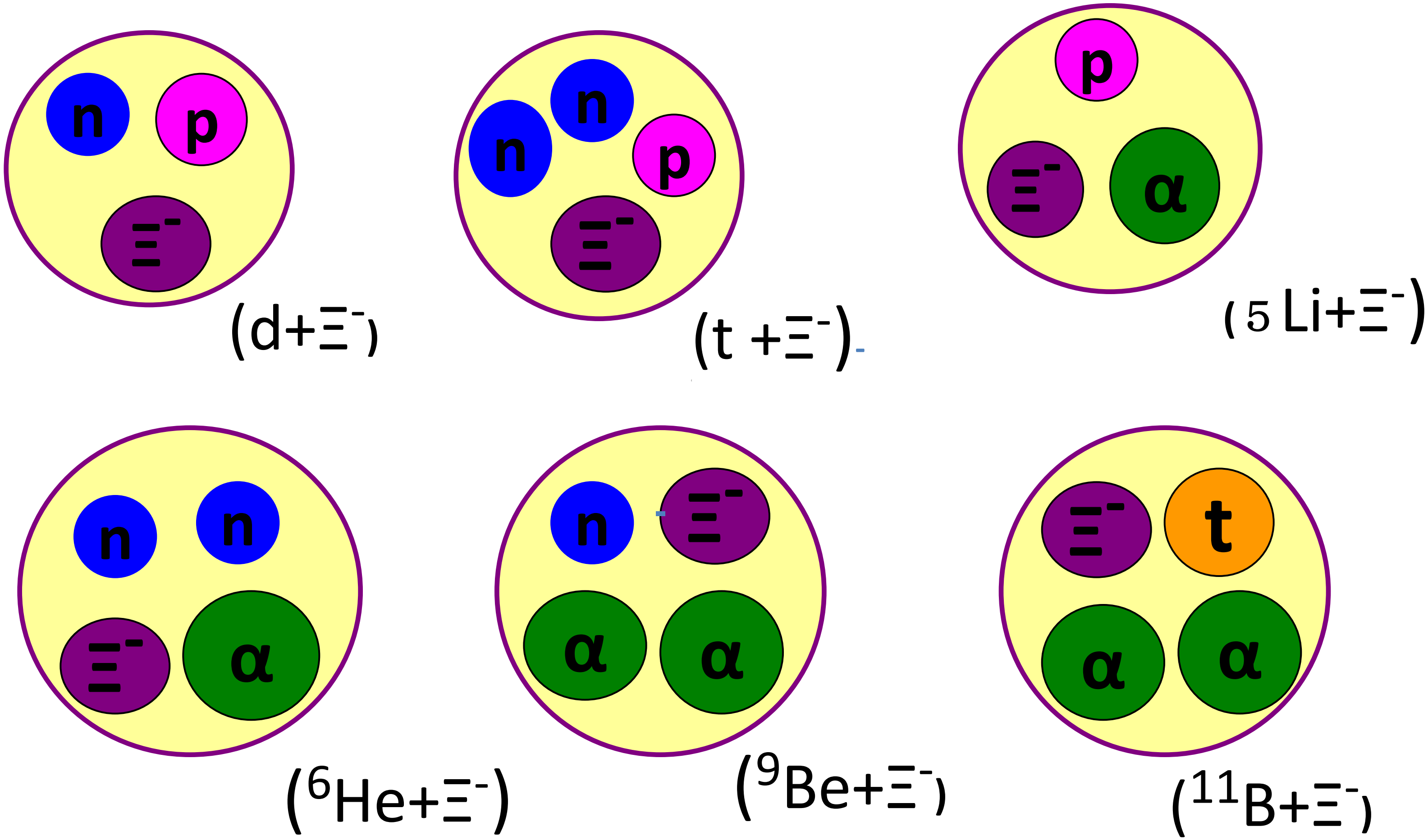
Ξhypernuclei

So far, how many Ξ hypernuclei did they observe with NO ambiguity?

Answer: 0 There was NO experimental data about Ξ hypernuclei

Therefore, we do not know whether ΞN potential is attractive or repulsive.

For this purpose, recently, I studied these Ξ hypernuclei.

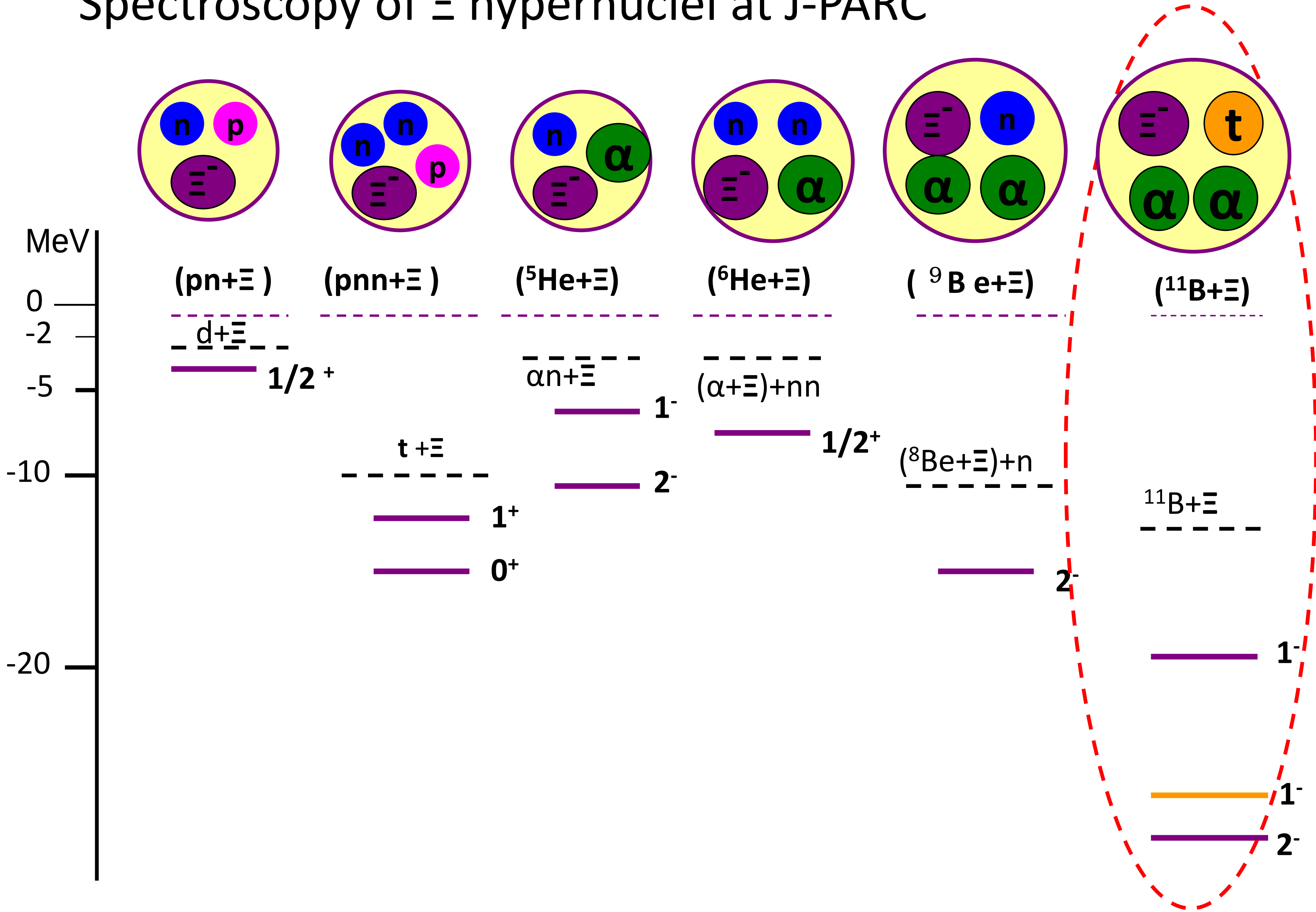


(ESC04)

ΞN interaction to reproduce the experimental data of
$^{12}C(K^-,K^+)$ reaction

- T. Fukuda *et al*. Phys. Rev. C58, 1306, (1998);
- P.Khaustov *et al*. Phys. Rev. C61, 054603 (2000).

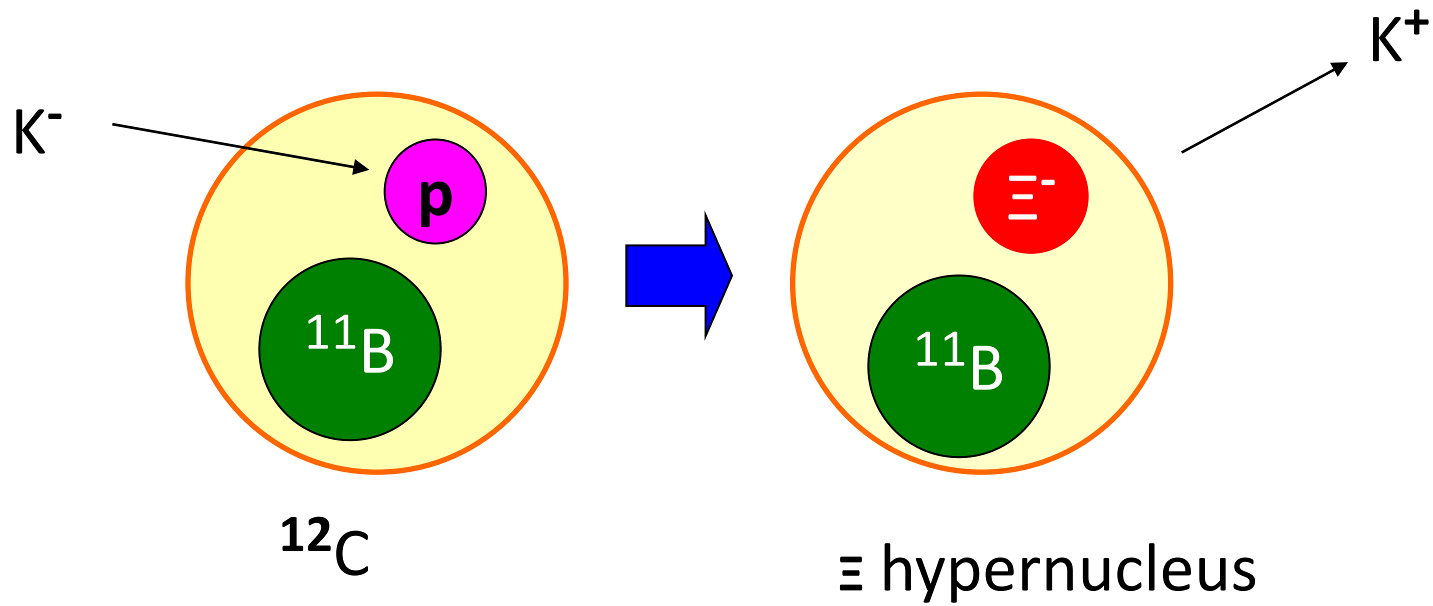
Spectroscopy of Ξ hypernuclei at J-PARC



For the study of ΞN interaction, it is important to study the structure of Ξ hypernuclei.

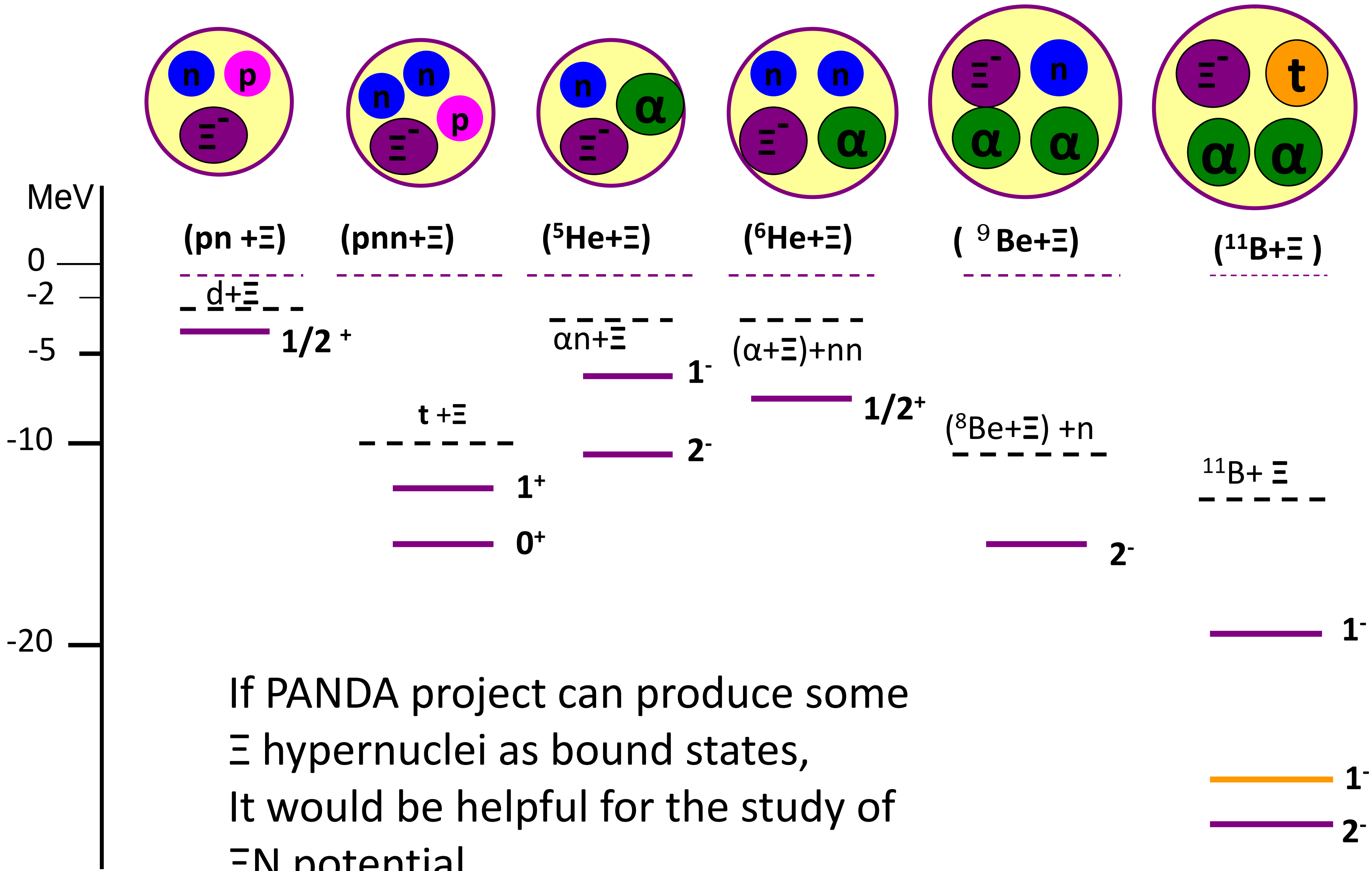
Approved proposal at J-PARC : Day-1 experiment

- E05 "Spectroscopic study of Ξ-Hypernucleus, $^{12}_{\Xi^-}Be$, via the $^{12}C(K^-,K^+)$ Reaction" by Nagae and his collaborators



This will be the first observation of Ξ hypernucleus

Spectroscopy of Ξ hypernuclei



If PANDA project can produce some
Ξ hypernuclei as bound states,
It would be helpful for the study of
ΞN potential.

Summary

We introduce some experimental data of double Λ hypernuclei , $^{10}_{\Lambda\Lambda}Be$ $^{11}_{\Lambda\Lambda}Be$ and by emulsion.

We need more double Λ hypernuclei. In this meaning, PANDA project is one of powerful method. Then, I hope that they can observe many double Λ hypernuclei.
Especially, observation of s-shell double Λ hypernuclei is important for the study of ΛΛ−ΞN coupling.
I hope to have these hypernuclei in the future.

In S=-2, sector, observation of Ξ hypernuclei is also essential.
Using p-bar beam, we have Ξ-atomic state as an internal state.
At PANDA project, if they can observe Ξ hypernuclei,
it would be very great!

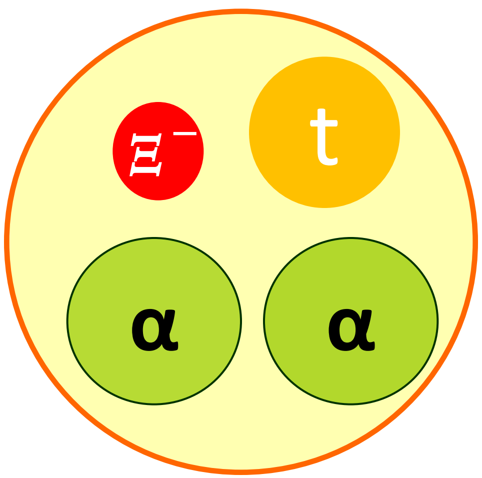
Thank you !

${}^{12}C(K^-, K^+)\,{}^{12}_{\Xi^-}Be$ Day-1 experiment at J-PARC

What part’s information of the ΞN interaction do we extract?

$$V_{\Xi N} = V_0 + \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}\, V_{\sigma\cdot\sigma} + \boldsymbol{\tau}\cdot\boldsymbol{\tau}\, V_{\tau\cdot\tau} + (\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})(\boldsymbol{\tau}\cdot\boldsymbol{\tau})\, V_{\sigma\cdot\sigma\ \tau\cdot\tau}$$

All of the terms contribute to binding energy of ${}^{12}_{\Xi^-}Be$ (${}^{11}B$ is not spin-, isospin- saturated).



Then, even if we observe this system as a bound state, we shall get only information that $V_{\Xi N}$ itself is attractive.

${}^{12}_{\Xi^-}Be$

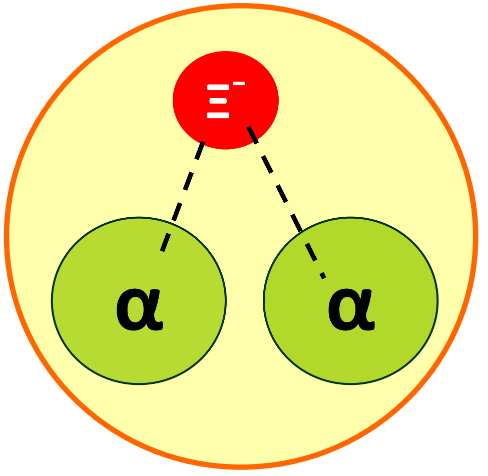
(T=1, J=1⁻)

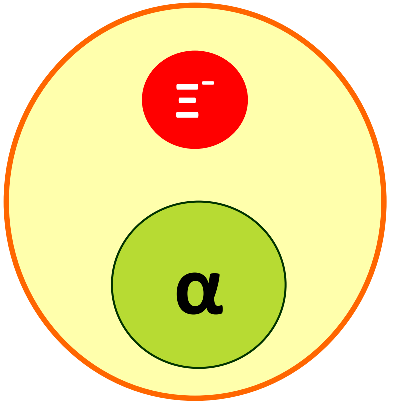
Therefore, after the Day-1 experiment, next, we want to know desirable strength of V_0, the spin-,isospin-independent term.

$$V_{\Xi N} = V_0 + \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}\, V_{\sigma\cdot\sigma} + \boldsymbol{\tau}\cdot\boldsymbol{\tau}\, V_{\tau\cdot\tau} + (\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})(\boldsymbol{\tau}\cdot\boldsymbol{\tau})\, V_{\sigma\cdot\sigma\ \tau\cdot\tau}$$

In order to obtain useful information about V_0,
the following systems are suited, because

the **(σ・σ)**, **(τ・τ)** and **(σ・σ) (τ・τ)** terms of $V_{\Xi N}$ vanish by folding them into the **α**-cluster wave function that are spin-, isospin-satulated.





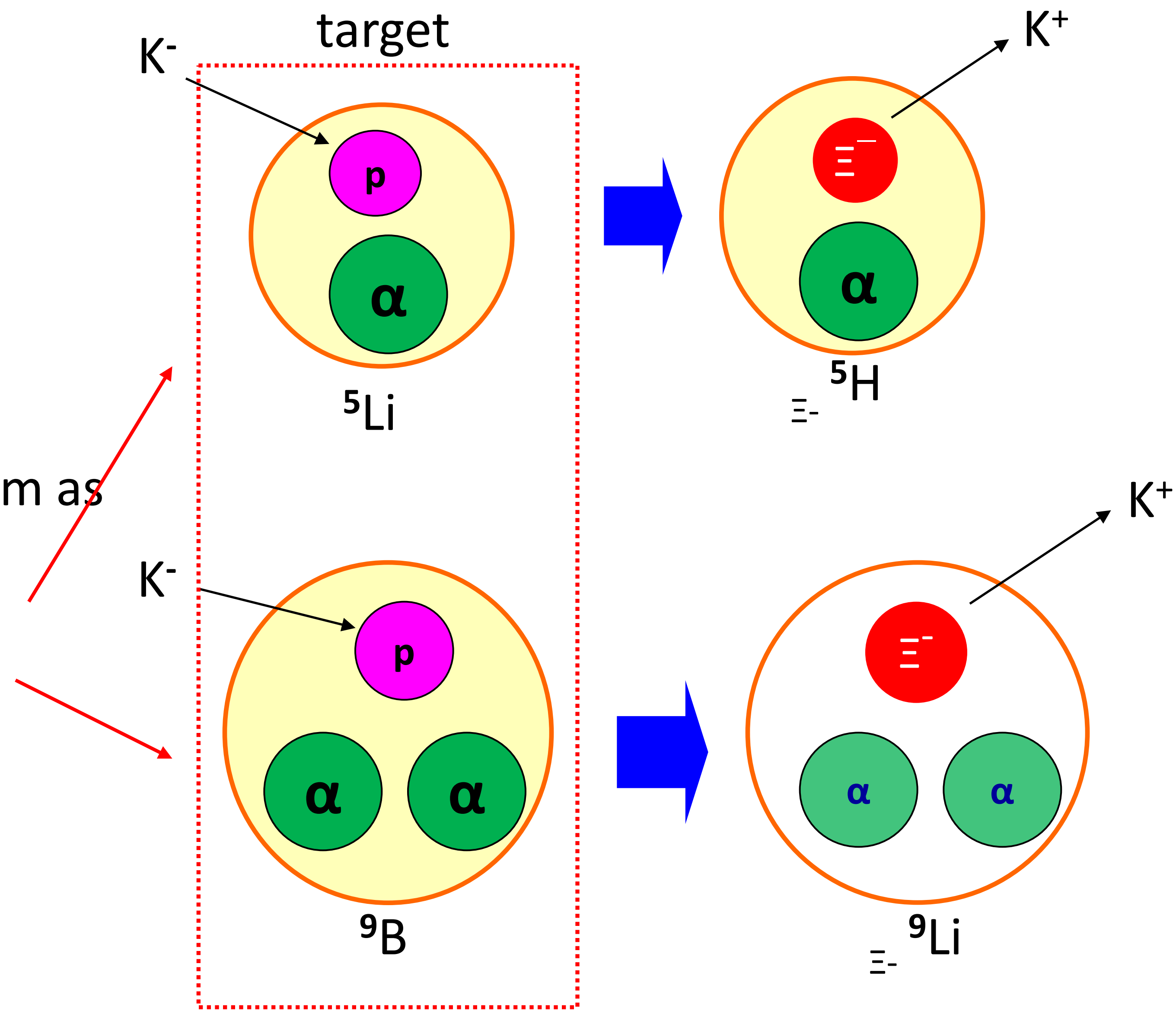
problem : there is NO target to produce them by the (K^-, K^+) experiment .

Because, ・ ・ ・

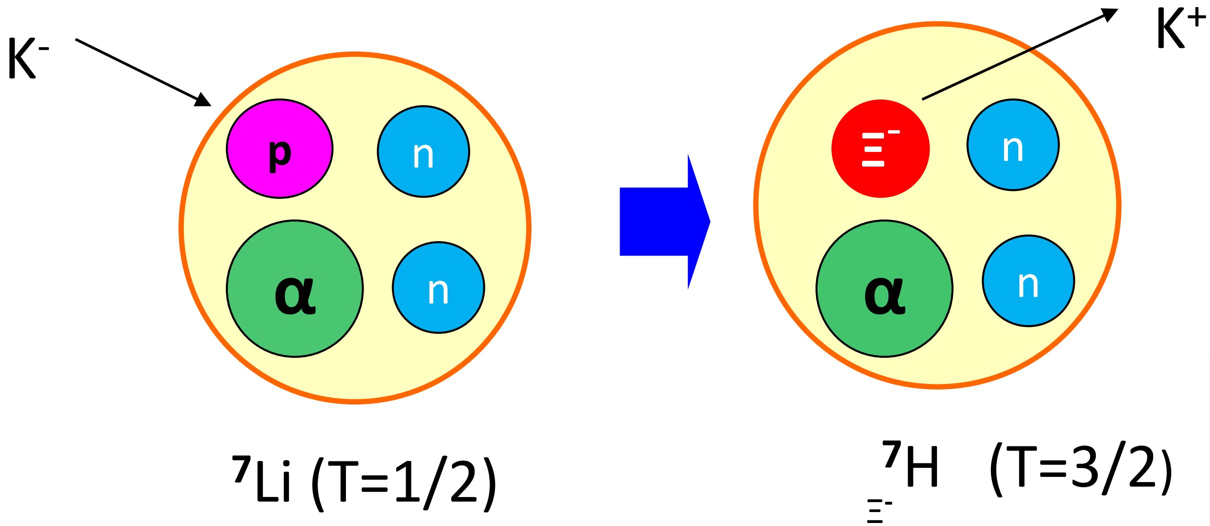
To produce **αΞ⁻** and **ααΞ⁻** systems by (K^-, K^+) reaction,

These systems are unbound.

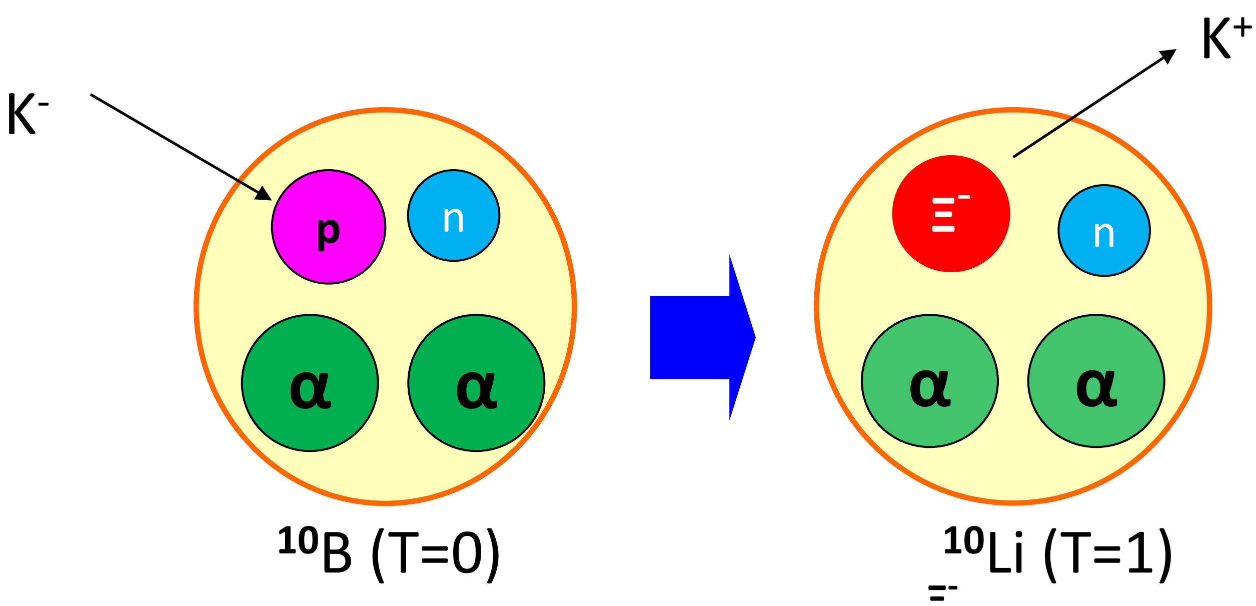
Then, we cannot use them as targets.



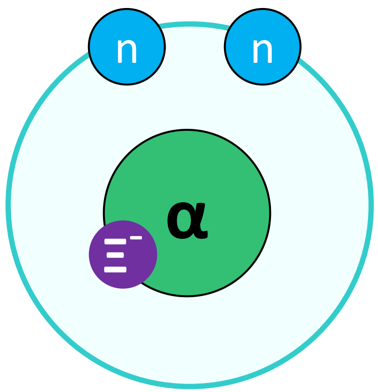
As the second best candidates to extract information about the spin-, isospin-independent term V_0, we propose to perform…



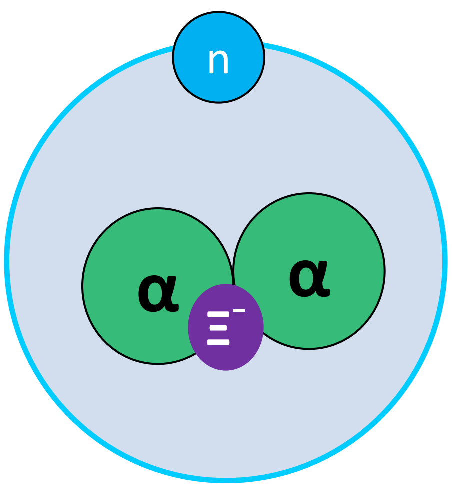
Why they are suited for investigating V_0?



(more realistic illustration)



${}^{7}_{\Xi^-}H$ (T=3/2)



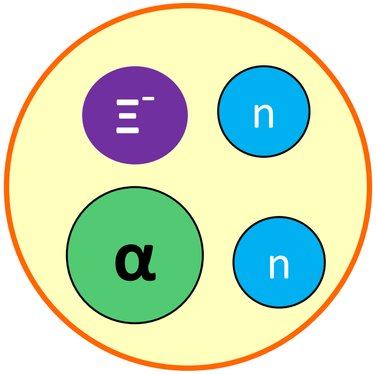
${}^{10}_{\Xi^-}Li$ (T=1)

Core nucleus ^{6}He is known to be halo nucleus. Then, valence neutrons are located far away from α particle.

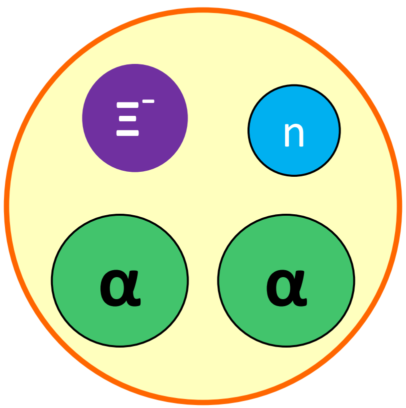
Valence neutrons (n) are located in p-orbit, whereas Ξ particle (Ξ^-) is located in 0s-orbit.

Then, distance between Ξ and **n** is much larger than the interaction range of Ξ and **n**.

Then, αΞ potential, in which only V_0 term works, plays a dominant role in the binding energies of these system.



$^{7}_{\Xi^-}H$ (T=3/2)



$^{10}_{\Xi^-}Li$ (T=1)

Before the experiments will be done, we should predict whether these hypernuclei will be observed as bound states or not.

Namely, we calculate the binding energies of these hypernuclei.

ΞN interaction

Only one experimental information about ΞN interaction

Y. Yamamoto, Gensikaku kenkyu 39, 23 (1996),

T. Fukuda *et al*. Phys. Rev. C58, 1306, (1998);

P.Khaustov *et al*., Phys. Rev. C61, 054603 (2000).

Well-depth of the potential between Ξ and ^{11}B: -14 MeV

Among all of the Nijmegen model,

ESC04 (Nijmegen soft core) and ND (Nijmegen Model D)

reproduce the experimental value.

OtherΞN interaction are repulsive or weak attractive.

We employ ESC04 and ND.

The properties of ESC04 and ND are quite different from each other.

Property of the spin- and isospin-components of ESC04 and ND

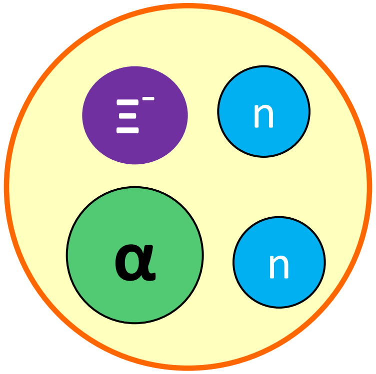
V(T,S)	ESC04	ND
T=0, S=1	strongly attractive (a bound state)	weakly attractive (all four components)
T=0, S=0	weakly repulsive	
T=1, S=1	weakly attractive	
T=1, S=0	weakly repulsive	

Although the spin- and isospin-components of these two models are very different between them (due to the different meson contributions

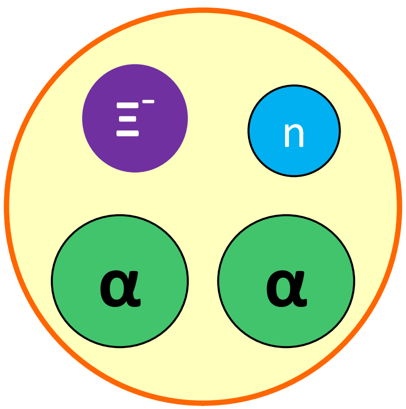
we find that the spin- and isospin-averaged property,

$$V_0 = [\, V(0,0) + 3V(0,1) + 3V(1,0) + 9V(1,1) \,] / 16,$$

namely, strength of the V_0- term is similar to each other.



$^{7}_{\Xi^-}H$ (T=3/2)



$^{10}_{\Xi^-}Li$ (T=1)

As mentioned before,

αΞ potential, in which only $\mathbf{V_0}$ term works,

plays a dominant role in the binding

energies of these system.

Therefore, interestingly,

we may expect to have similar binding energies

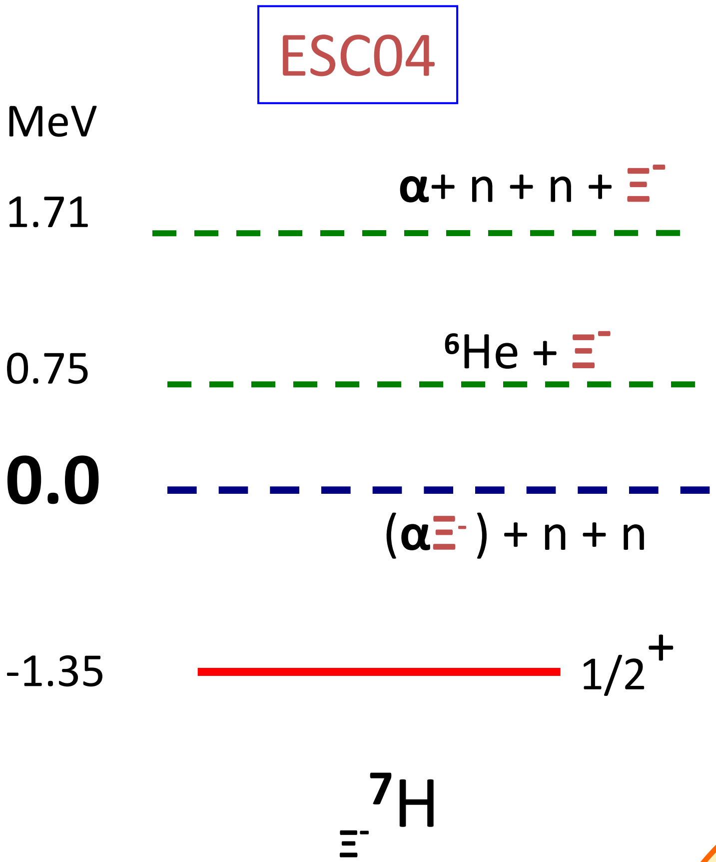
between ESC04 and ND,

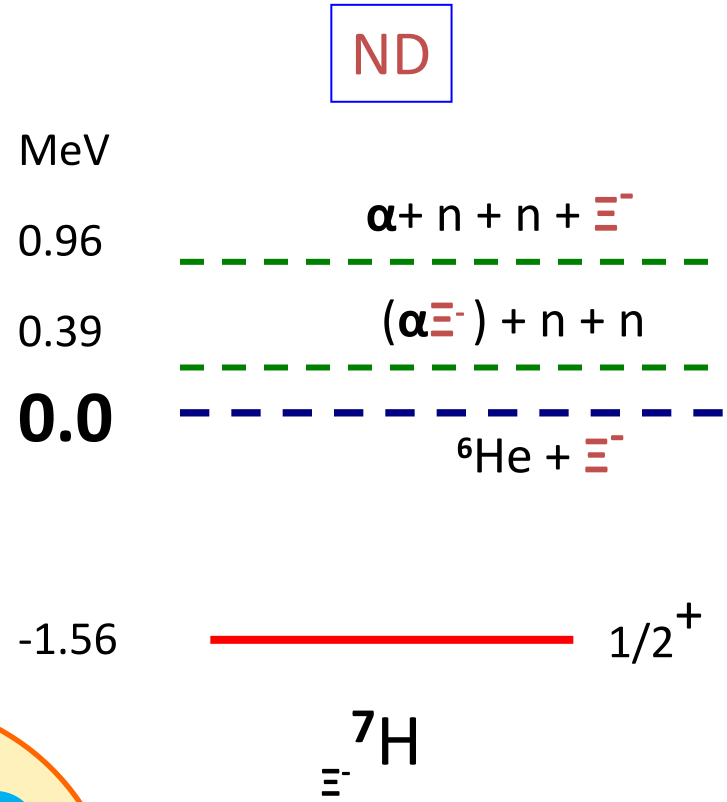
although the spin- and isospin-components are

very different between the two.

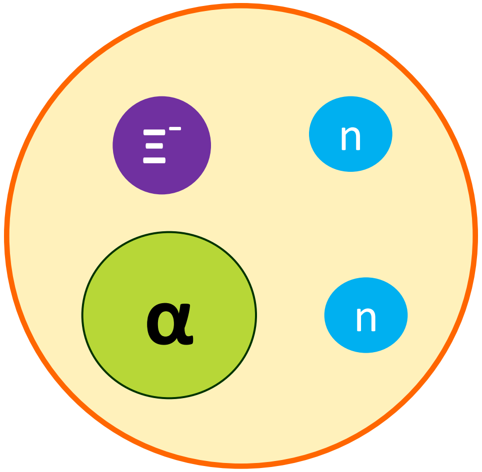
4-body calculation of $^{7}_{\Xi^-}H$

E. Hiyama et al., PRC**78** (2008) 054316





In experiments, we can expect a bound state.

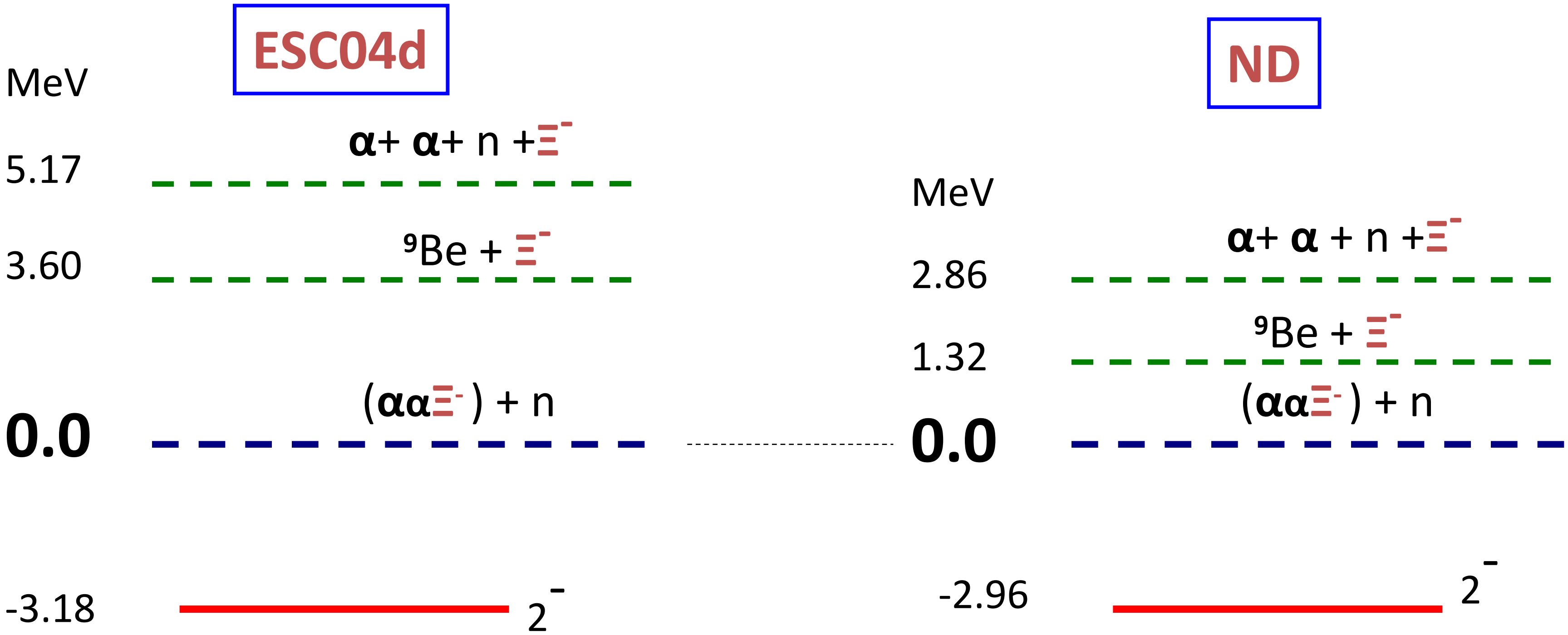


Similar binding energies using ND and ESC04.

Independent on employed ΞN potential

4-body calculation of $^{10}_{\Xi^-}Li$

E. Hiyama et al., PRC**78** (2008) 054316



In this way, the binding energies of Ξ hypernuclei with A=7 and 10 are dominated by αΞ potential, namely, spin-, and iso-spin independent ΞN interaction (V_0).

Then, to get information about this part, we propose to perform the (K^-,K^+) experiment by using ^7Li and ^{10}B targets at J-PARC after the Day-1 experiment with ^{12}C target.