Critical Dynamics of Non-Equilibrium Phase Transitions

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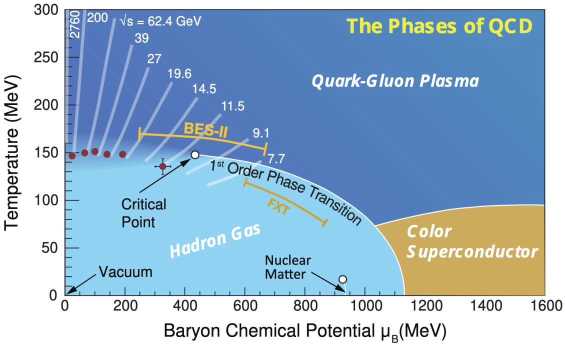
CRC–TR 211
Strong-interaction matter under extreme conditions

Motivation

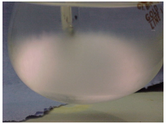
- Heavy-ion collisions probe QCD phase diagram



- Existence of critical point? $\rightsquigarrow$ Need to understand non-equilibrium phase transitions

Critical phenomena

- Strong **fluctuations** of order parameter
  - Sensitive Observables: Variance and higher non-Gaussian cumulants
  - Also experimentally accessible



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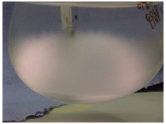

- Diverging correlation length $\xi(\tau) \sim |\tau|^{-\nu}$
  - Independence of microscopic interaction details $\rightsquigarrow$ Models fall in **static universality classes**
  - Observables become power laws with universal exponents

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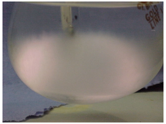

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  - System guaranteed to fall out of equilibrium in dynamic processes
  - **Dynamic universality classes**
    $\rightsquigarrow$ Universal dynamic critical exponent z, non-equilibrium scaling functions, etc.

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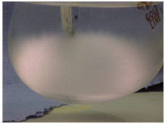

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Central idea: Measure universal quantities not in QCD but simpler models

Model

- Model in $\mathbb{Z}_2$ Ising universality class

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \, \partial^\mu \phi - m^2 \phi^2) - \frac{\lambda}{4!}\phi^4 - J\phi$$

with $m^2 = -1$, $\lambda = 1$

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- Observables:
  - Order Parameter $M = \frac{1}{V}\int_V d^d\vec{x}\;\phi(\vec{x})$
  - Susceptibility $\chi = \frac{V}{T}(\langle M^2\rangle - \langle M\rangle^2)$
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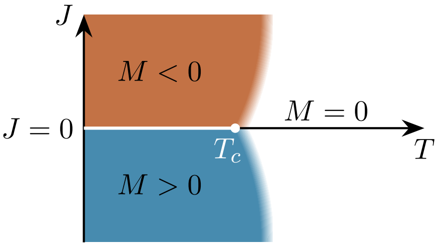
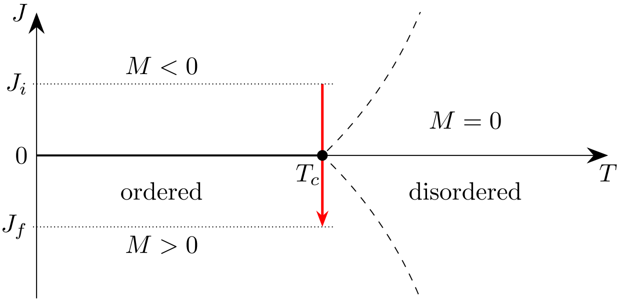


Non-equilibrium phase transitions

- Magnetic quenches $T(t) = T_c$, $J(t) = -r_J\, t$



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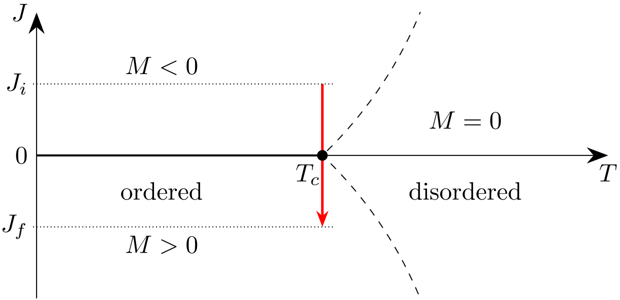
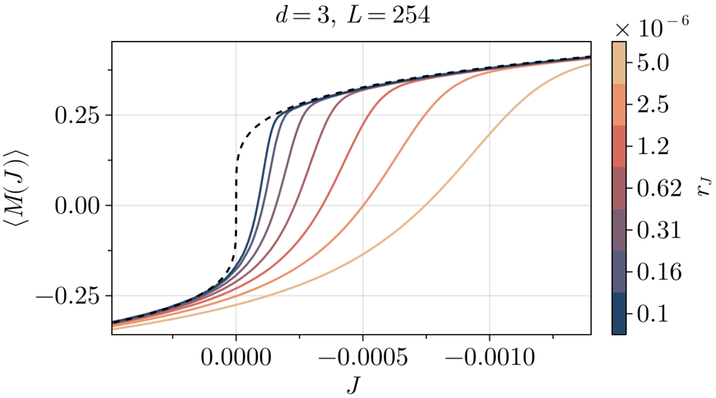


- Critical slowing down $\rightsquigarrow$ Non-equilibrium evolution near CP

Order parameter as function of time



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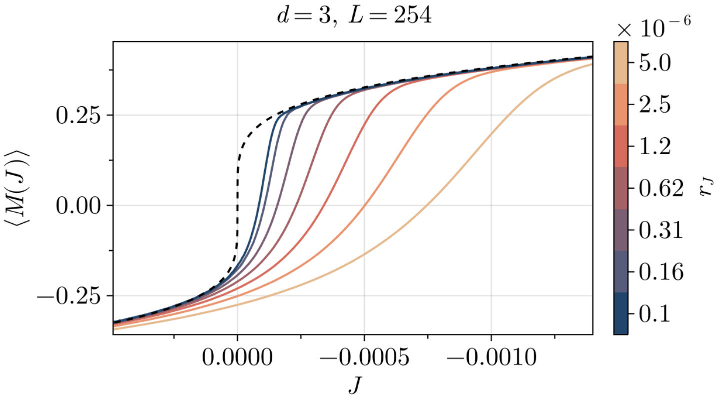

When does system fall out of equilibrium? How to describe these processes?

Kibble-Zurek mechanism

- System falls out of equilibrium when rate of change is greater than relaxation rate

$$\dot{\xi}_t/\xi_t \gtrsim 1/\xi_t \qquad \rightsquigarrow \qquad \text{Kibble-Zurek time } \dot{\xi}_t(t = t_{KZ}) = 1$$

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$$t_{KZ} \sim r_J^{-\nu_c z/(1+\nu_c z)}, \qquad J_{KZ} \sim r_J^{1/(1+\nu_c z)}$$

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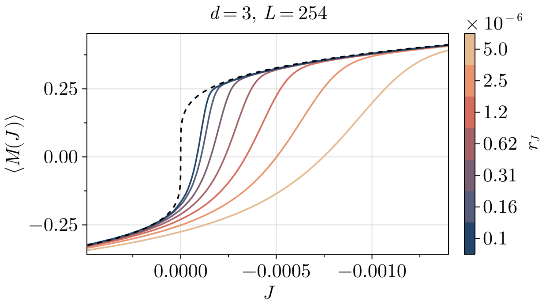
- Scaling Ansätze for all observables, e.g. $\langle M(J, r_J)\rangle \propto -r_J^{\beta/(\beta\delta+\nu z)} f_M(J/J_{KZ})$
  - Universal scaling functions
  - Normalization & characteristic time scale scale with power of quench rate r_J

Real-time observables collapse onto universal scaling functions

Order Parameter $\langle M(J, r_J)\rangle \propto -r_J^{\beta/(\beta\delta+\nu z)} f_M(J/J_{KZ})$



Extraction of universal scaling functions

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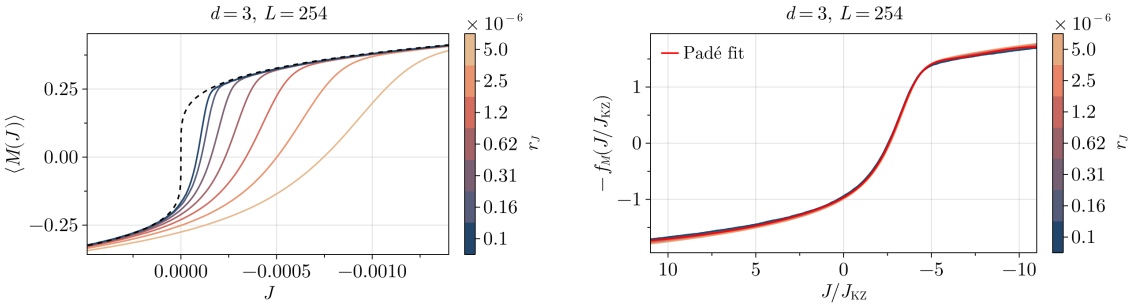
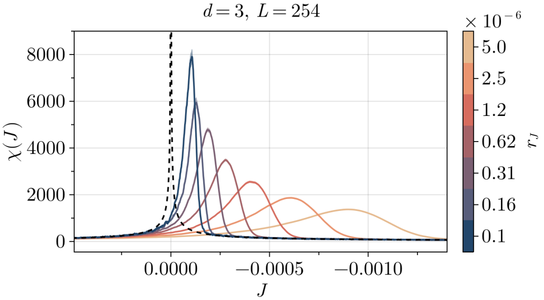
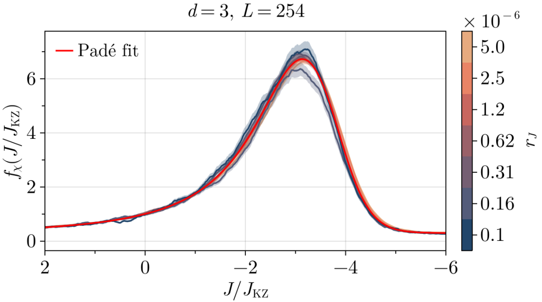


Real-time observables collapse onto universal scaling functions

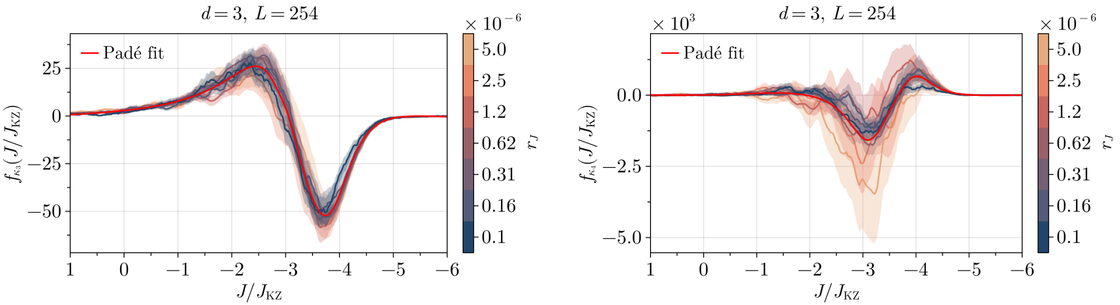
Susceptibility $$\langle \chi(J, r_J) \rangle \propto r_J^{-\gamma/(\beta\delta+\nu z)} f_\chi(J/J_{KZ})$$





Extraction of universal scaling functions

Skewness κ_3 | Kurtosis κ_4

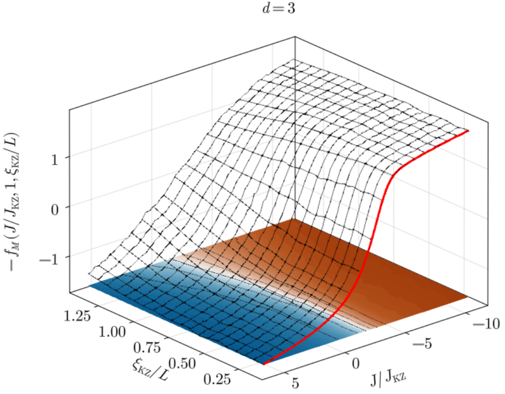


Dynamic finite-size scaling

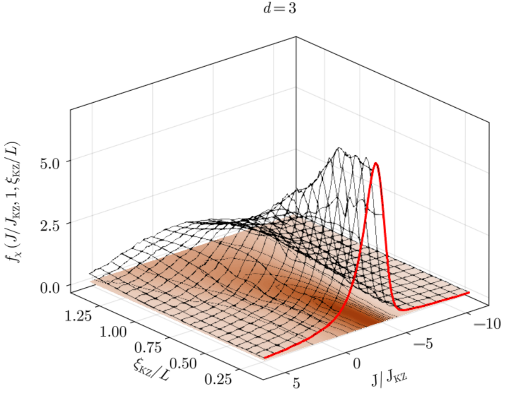
- Dependence on system size?
- Relevant cutoffs for diverging correlation length when approaching critical point:
  - Correlation length $\xi(t_{KZ})$ at time when system falls out of equilibrium
  - Finite system size L
- Make scaling functions also function of L^{-1}, e.g. $f_M(J/J_{KZ}) \to f_M(J/J_{KZ}, \xi_{KZ}/L)$

Dynamic finite-size scaling

Order parameter

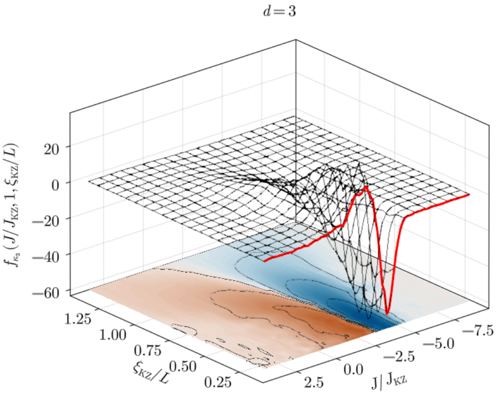


Susceptibility

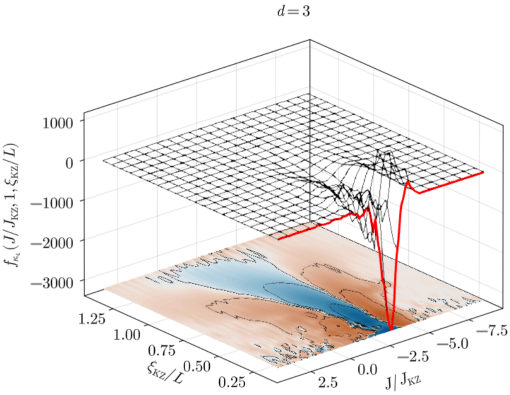


Dynamic finite-size scaling

Skewness



Kurtosis



Extraction of dynamic critical exponent z

- Dynamic critical exponent z sets scale:
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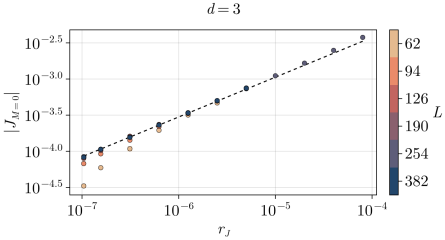
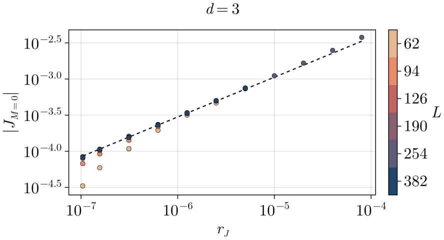


Extraction of dynamic critical exponent z

- Dynamic critical exponent z sets scale:
  Time & normalization of observables
- Extract from scaling of characteristic features
  (Zero crossings, minima/maxima, etc.)
- Difficulties:
  - Very slow quenches:
    Correlation length limited by system size
  - Very fast quenches:
    System cannot follow adiabatically anywhere
  - Sub-leading and regular corrections to power laws



Conclusion & Outlook

- Investigated non-equilibrium critical behavior of scalar relativistic field theory
  - Static $\mathbb{Z}_2$ Ising & dynamic Model A universality class in 2D and 3D
- Evolution explained by Kibble-Zurek scaling & universal functions
  - In principle, extraction of dynamic critical exponent z possible
  - Extracted universal scaling functions as function of J, L^{-1}
- Future directions
  - Realistic trajectories
  - Model H (hydrodynamic description)

Thank you for your attention!

Appendix

Lattice Setup

Lattice Hamiltonian

$$H = \sum_x a^d \left\{ \frac{1}{2}\pi_x^2 + \frac{1}{2a^2}\sum_{y\sim x}\phi_x\phi_y + \left(\frac{m^2}{2} + \frac{d}{a^2}\right)\phi_x^2 + \frac{\lambda}{4!}\phi_x^4 + J(t)\phi_x \right\}$$

Equations of motion (Langevin dynamics)

$$\partial_t \phi_x = \frac{\partial H}{\partial \pi_x}, \quad \partial_t \pi_x = -\frac{\partial H}{\partial \phi_x} - \gamma\pi_x + \sqrt{2\gamma T}\eta_x(t)$$

Observables

Order Parameter, Susceptibility, Skewness, Kurtosis

$$\langle M \rangle = \left\langle \frac{1}{V} \sum_x \phi_x \right\rangle$$

$$\chi = \frac{V}{T} \left(\langle M^2 \rangle - \langle M \rangle^2 \right)$$

$$\kappa_3 = \left(\frac{V}{T} \right)^2 \left(\langle M^3 \rangle - 3\langle M^2 \rangle \langle M \rangle + 2\langle M \rangle^3 \right)$$

$$\kappa_4 = \left(\frac{V}{T} \right)^3 \left(\langle M^4 \rangle - 4\langle M^3 \rangle \langle M \rangle - 3\langle M^2 \rangle^2 + 12\langle M^2 \rangle \langle M \rangle^2 - 6\langle M \rangle^4 \right)$$

General scaling Ansatz

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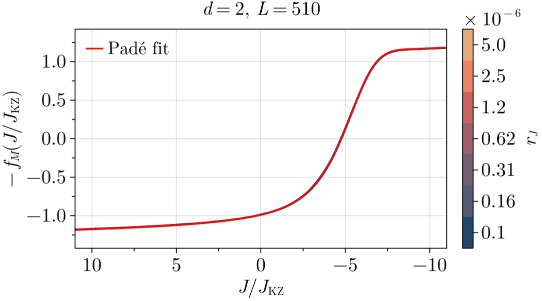
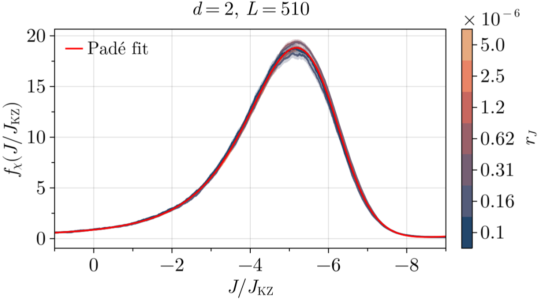
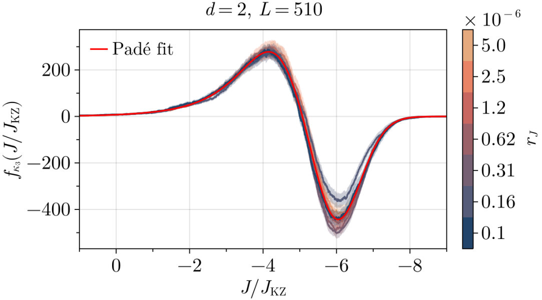
$$A(J, r_J) = A_0 s^{\Delta} f_A(s^{1/\nu_c} \bar{J}, s^{z+1/\nu_c} \bar{r_J})$$

$s > 0$: Length rescaling parameter (can be chosen arbitrarily)
Δ: Scaling dimension
Normalization condition $f_A(1,0) = f_A(0,1) = 1$

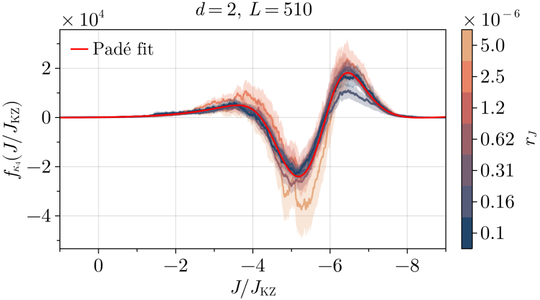
Scaling functions in 2D







$d = 2,\ L = 510$
Padé fit
$f_{\kappa_3}(J/J_{\rm KZ})$
$J/J_{\rm KZ}$
$\times 10^{-6}$
5.0
2.5
1.2
0.62
0.31
0.16
0.1
r_J



$d = 2,\ L = 510$
$\times 10^4$
Padé fit
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5.0
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1.2
0.62
0.31
0.16
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r_J

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