Glueball phenomenology, the eLSM, and the $\rho\pi$ puzzle

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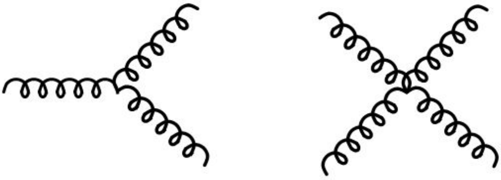
Overview

Introduction

The QCD lagrangian contains gluon self-interaction due to its non-abelian SU(3) symmetry



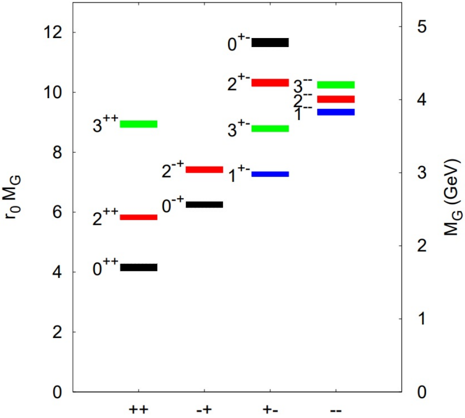
This begs the question: is there a bound state consisting of only gluons? These states, "glueballs" have not been found yet experimentally, but there is much theoretical research on it.

Lattice QCD

Lattice calculations have found a large spectrum of pure gluon states.

The two lightest ones are the scalar ($J^{PC} = 0^{++}$) and the tensor ($J^{PC} = 2^{++}$) and such they are some of the best candidates for experimental verification.

Lattice calculations have some difficulties computing decay rates, so there is room for us to find new information using our chiral model.



Chen et al, 2005

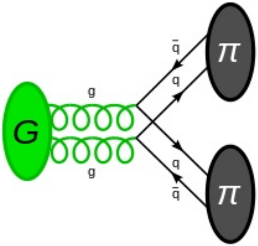
Glueball width

Glueballs are expected to have relatively small decay widths, from large N_c scaling:

$$A_{gg\to\bar{q}q+\bar{q}q} \propto N_c^{-1}$$

$$A_{\bar{q}q\to\bar{q}q+\bar{q}q} \propto N_c^{-\frac{1}{2}}$$

All processes glueball $\to$ hadrons are also suppressed because of the OZI rule



Experimental Search

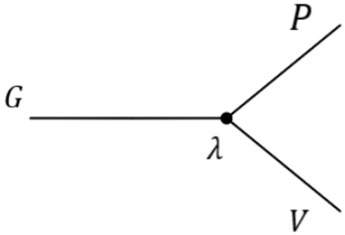
Numerous experiments work on data related to glueballs

- BESIII
- LHCb
- GlueX
- Compass
- Clas 12
- PANDA

Experimentally J/ψ decays are one of the best places to search for glueballs.

The extended Linear Sigma Model

The extended Linear Sigma Model is an effective model, so diagrams are of the form:



The structure of the vertex λ is found using the symmetries of QCD and how they are broken, especially chiral symmetry.

The scalar glueball

The scalar glueball is an integral part of the Linear Sigma Model, as it is responsible for the breaking of dilatation invariance

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left[G^4 \log(\frac{G}{\Lambda_G}) - \frac{G^4}{4}\right]$$

The best candidates for the scalar glueball are the $f_0(1500)$ and the $f_0(1710)$. The eLSM points to the $f_0(1710)$:

$$\begin{aligned} f_0(1370): &\quad 83\%\,\sigma_N, \quad 6\%\,\sigma_S, \quad 11\%\,G\,, \\ f_0(1500): &\quad 9\%\,\sigma_N, \quad 88\%\,\sigma_S, \quad 3\%\,G\,, \\ f_0(1710): &\quad 8\%\,\sigma_N, \quad 6\%\,\sigma_S, \quad 86\%\,G\,. \end{aligned}$$

Janowski et al, 2014

The pseudoscalar glueball

- The pseudoscalar glueball has quantum numbers $J^{(PC)} = 0^{-+}$, and lattice QCD predicts a mass of about 2.3 - 2.6 GeV.
- Recently, BESIII found 2 new states, the $X(2370)$ in the $\gamma K_S^0 K_S^0 \eta'$ channel and the $X(2600)$ in the $\gamma\pi\pi\eta'$ channel of J/ψ decays,
- eLSM calculations (Eshraim et al, 2013) also suggest that the $KK\pi$ and $\eta'\pi\pi$ are dominant
- Calculations on instanton interactions (Giacosa et al, 2024) show that the axial anomaly enhances these channels

Spin 2 in the eLSM

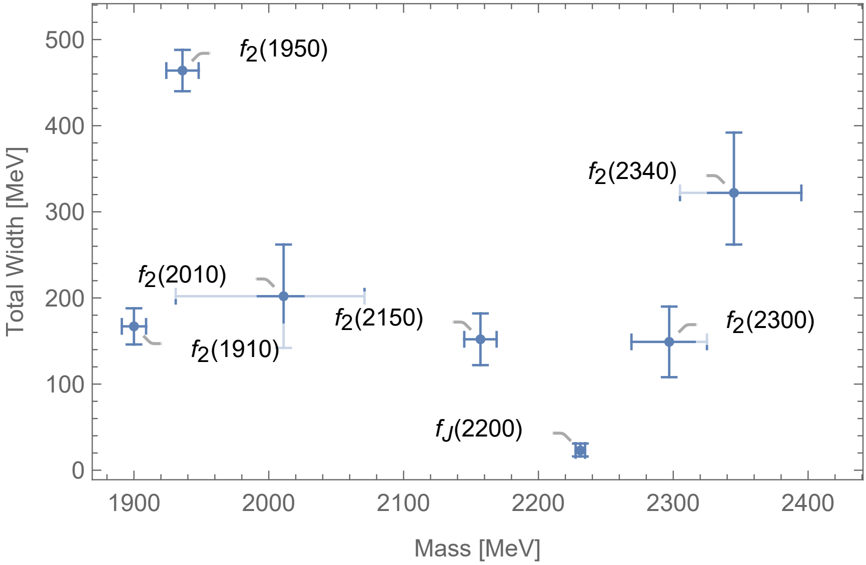
We have extended the eLSM to spin 2 mesons, where we could use data on tensor mesons ($J^{PC} = 2^{++}$) to predict masses and decays of axial tensor mesons ($J^{PC} = 2^{--}$)

Resonances	Masses (in MeV)	Resonances	Masses (in MeV)
$a_2(1320)$	**1317**	$\rho_2(?)$	1663
$K_2^*(1430)$	**1427**	$K_2(1820)$	**1819**
$f_2(1270)$	1297	$\omega_{2,N}(?)$	1663
$f_2'(1525)$	1538	$\omega_{2,S}(?)$	1971

PDG inputs, consistency checks and predictions
in summary for the axial tensor states:

- Masses are found, $\sim$ 300-400 MeV larger than their chiral partners
- Quite sizeable decays in some channels
- Potentially difficult to recognize since the sum of various decay channels for axial tensor is quite large

Tensor glueball candidates



For the tensor glueball, there are many states that are potential candidates, but no strong candidates so far

Tensor glueball decay ratios

Decay Ratio	theory	Decay Ratio	theory	Decay Ratio	theory
$\frac{G_2(2369)\longrightarrow\bar{K}\,K}{G_2(2369)\longrightarrow\pi\,\pi}$	0.4	$\frac{G_2(2369)\longrightarrow\rho(770)\,\rho(770)}{G_2(2369)\longrightarrow\pi\,\pi}$	51	$\frac{G_2(2369)\longrightarrow a_1(1260)\,\pi}{G_2(2369)\longrightarrow\pi\,\pi}$	0.26
$\frac{G_2(2369)\longrightarrow\eta\,\eta}{G_2(2369)\longrightarrow\pi\,\pi}$	0.1	$\frac{G_2(2369)\longrightarrow\bar{K}^*(892)\,\bar{K}^*(892)}{G_2(2369)\longrightarrow\pi\,\pi}$	44	$\frac{G_2(2369)\longrightarrow K_{1\,,A}\,K}{G_2(2369)\longrightarrow\pi\,\pi}$	0.12
$\frac{G_2(2369)\longrightarrow\eta\,\eta'}{G_2(2369)\longrightarrow\pi\,\pi}$	0.005	$\frac{G_2(2369)\longrightarrow\omega(782)\,\omega(782)}{G_2(2369)\longrightarrow\pi\,\pi}$	17	$\frac{G_2(2369)\longrightarrow f_1(1285)\,\eta}{G_2(2369)\longrightarrow\pi\,\pi}$	0.03
$\frac{G_2(2369)\longrightarrow\eta'\,\eta'}{G_2(2369)\longrightarrow\pi\,\pi}$	0.01	$\frac{G_2(2369)\longrightarrow\phi(1020)\,\phi(1020)}{G_2(2369)\longrightarrow\pi\,\pi}$	7	$\frac{G_2(2369)\longrightarrow f_1(1420)\,\eta}{G_2(2369)\longrightarrow\pi\,\pi}$	0.008

Decay ratios of G_2 w.r.t. $\pi\pi$ for a mass of 2369 MeV. The columns are sorted as PP on the left, VV in the middle, and A_1P on the right. Vector channels are dominant, in particular $\rho\rho$ and K^*K^*

Data Comparison

Resonance	Decay Ratio	PDG	Model Prediction
$f_2(1910)$	$\rho\rho/\omega\omega$	2.6 ± 0.4	3.1
$f_2(1910)$	$f_2(1270)\eta/a_2(1320)\pi$	0.09 ± 0.05	0.07
$f_2(1910)$	$\eta\eta/\eta\eta'$	< 0.05	~ 8
$f_2(1910)$	$\omega\omega/\eta\eta'$	2.6 ± 0.6	~ 200
$\mathbf{f_2(1950)}$	$\boldsymbol{\eta\eta/\pi\pi}$	$\mathbf{0.14 \pm 0.05}$	**0.081**
$\mathbf{f_2(1950)}$	$\boldsymbol{K\bar{K}/\pi\pi}$	$\mathbf{\sim 0.8}$	**0.32**
$\mathbf{f_2(1950)}$	$\boldsymbol{4\pi/\eta\eta}$	$\mathbf{> 200}$	$\mathbf{> 700}$
$f_2(2150)$	$f_2(1270)\eta/a_2(1320)\pi$	0.79 ± 0.11	0.1
$f_2(2150)$	$K\bar{K}/\eta\eta$	1.28 ± 0.23	~ 4
$f_2(2150)$	$\pi\pi/\eta\eta$	< 0.33	~ 10

Decay ratios for the decay channels with available data.

Tensor glueball candidates

Resonances	Interpretation status
$f_2(1910)$	Agreement with some data, but large discrepancies in $\eta\eta\prime$ mode
$f_2(1950)$	**$\eta\eta/\pi\pi$ agrees with data, no contradictions to data** **but broad tensor glueball** **Best fit as predominantly glueball**
$f_2(2010)$	Likely primarily strange-antistrange content
$f_2(2150)$	All available data contradicts theoretical prediction
$f_J(2220)$	Data on $\pi\pi/K\bar{K}$ disagrees with theory Only smallest predicted decay channels are seen
$f_2(2300)$	Likely primarily strange-antistrange content
$f_2(2340)$	Likely primarily strange-antistrange content would also imply a broad glueball

Spin 2 resonances and their status as the tensor glueball.

The vector glueball

- For the vector glueball, with mass 3.8 - 4 GeV, there are currently no candidates
- But the vector glueball may play a role in the decay of known states due to its mixing
- The J/ψ and $\psi(2S)$ are bound $c\bar{c}$ states that are quite stable, and primarily decay through a 3-gluon channel, making them sensitive to glueballs
- They are supposed to follow the "12% rule", but this rule is broken for yet unknown reasons

The 12% rule

In perturbative, non-relativistic QCD the partial decay width to some final state X is proportional to the wave function at the origin

$$\Gamma(J/\psi \to X) = |\psi(0)|^2 |M_X|^2$$

Similar expression for $\psi(2S)$, neglecting phase space, M only depends on the final state so it divides out in

$$Q_h = \frac{\mathcal{B}(\psi(2S) \to h)}{\mathcal{B}(J/\psi \to h)} \approx \frac{\mathcal{B}(\psi(2S) \to e^+e^-)}{\mathcal{B}(J/\psi \to e^+e^-)} \approx 12\%$$

This is the "12% rule", the ratio of branching fractions is independent of the final state and equals approximately 12%.

The $\rho\pi$ puzzle

The 12% rule is severely violated in some decay channels, in particular for the $\rho\pi$ channel

$\eta\omega$	$< 6.3\%$
$\eta'\omega$	$16.93 \pm 12.28\%$
$\phi\eta$	$4.19 \pm 0.54\%$
$\phi\eta'$	$3.35 \pm 0.57\%$
$\rho\pi$	$0.19 \pm 0.073\%$
$\eta\rho$	$11.4 \pm 3.39\%$

VP channel, PDG values

In other channels, particularly involving kaons, the branching ratio is larger

$\pi^+\pi^-$	$5.31 \pm 1.84\%$
K^+K^-	$26.2 \pm 2.6\%$
$K_S^0K_L^0$	$27.4 \pm 2.3\%$

PP channel, PDG values

Vector glueball mixing

The $\psi(2S)$ has a mass of 3.686 GeV, close to the vector glueball mass of $3.8 - 4$ GeV, so it is possible for them to mix

$$\begin{pmatrix} \psi(2S) \\ \mathcal{O}' \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} c\bar{c} = \psi \\ ggg = \mathcal{O} \end{pmatrix}$$

Leads to interference term in amplitude that can give both suppression and enhancement for different channels

Preliminary results

Currently, mixing of the $\psi(2S)$ with the vector glueball does not seem to solve the puzzle

- for small mixing angle which is expected, χ^2 is of order $\sim 10^5$
- The optimal mixing angle for the fit is around $\theta \sim 80\,\mathrm{deg}$, which does not fit current picture of the $\psi(2S)$

The puzzle remains, but we can still improve the model by adding electromagnetic channels via Vector Meson Dominance.

Summary

- Glueballs are a yet undiscovered state of QCD, with a lot of theoretical work and some promising experimental prospects.
- The eLSM is used to study various glueballs. The scalar and pseudoscalar have promising candidates in the $f_0(1710), X(2370)$ and the $X(2600)$.
- We extended the eLSM to spin 2 particles. Predictions on axial tensors could be made and the $f_2(1950)$ is clearly favored as a tensor glueball candidate by the eLSM, but data on other resonances is limited.
- At this moment, the vector glueball by itself does not explain the $\rho\pi$ puzzle, but we can add electromagnetic contributions and further improve the model.

Thank you for your attention!

Backup slides

$f_J(2220)$

$f_J(2220)$ is historically seen as a good candidate for the tensor glueball

$f_J(2220)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	not seen
Γ_2	$\pi^+\pi^-$	not seen
Γ_3	$K\overline{K}$	not seen
Γ_4	$p\overline{p}$	not seen
Γ_5	$\gamma\gamma$	not seen
Γ_6	$\eta\eta'(958)$	seen
Γ_7	$\phi\phi$	not seen
Γ_8	$\eta\eta$	not seen

- Only $\eta\eta\prime$ is seen, but we find it is $\sim 10^{-3}$ times $\pi\pi$ mode.
- PDG lists decay ratio $\pi\pi/\bar{K}K = 1.0 \pm 0.5$, we find $\pi\pi/\bar{K}K \sim 2.5$

Estimating glueball width

- A rough guess on the width of the tensor glueball can be made.
- Consider $f_2 \equiv f_2(1270) \simeq \sqrt{1/2}(\bar{u}u + \bar{d}d)$ and $f_2' \equiv f_2'(1525) \simeq \bar{s}s$, with $\Gamma_{f_2 \to \pi\pi} = 157.2$ MeV and $\Gamma_{f_2' \to \pi\pi} = 0.71$ MeV.
- The amplitude for $f_2 \to \pi\pi$ requires the creation of a single $\bar{q}q$ pair from the vacuum and scales as $1/\sqrt{N_c}$, where N_c is the number of colors. On the other hand, the amplitude for $f_2' \to \pi\pi$ scales as $1/N_c^{3/2}$ and goes schematically like

$$\bar{s}s \to gg \to \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

Estimating glueball width

- Consider a transition Hamiltonian

$$H_{int} = \lambda \left(|\bar{u}u\rangle \langle gg| + \left|\bar{d}d\right\rangle \langle gg| + |\bar{s}s\rangle \langle gg| + h.c. \right), \;\; \lambda \propto 1/\sqrt{N_c}.$$

  Then: $A_{f_2' \to \pi\pi} \simeq \sqrt{2}\lambda^2 A_{f_2 \to \pi\pi}$, hence $\Gamma_{f_2' \to \pi\pi} \simeq 2\lambda^4 \Gamma_{f_2 \to \pi\pi}$,
- Tensor glueball decay into $\pi\pi$ intuitively speaking, is at an 'intermediate stage', since it starts with a *gg* pair. One has:

$$A_{G_2 \to \pi\pi} \simeq \sqrt{2}\lambda A_{f_2 \to \pi\pi},$$
$$\Gamma_{G_2 \to \pi\pi} \simeq 2\lambda^2 \Gamma_{f_2 \to \pi\pi} \simeq \sqrt{2}\sqrt{\Gamma_{f_2 \to \pi\pi}\Gamma_{f_2' \to \pi\pi}} \simeq 15\text{MeV}.$$

- **rough estimate**, based on large N_c scaling.
- Similar results to some holographic models: very large decay widths in vector modes.

Tensor glueball decays

The Lagrangian leads to three kinematically allowed decay channels

- Decaying of the tensor glueball to the two pseudoscalar mesons have the following decay rate formula

$$\Gamma_{G_2 \longrightarrow P^{(1)} P^{(2)}} = \frac{\kappa_{gpp,i}\, \lambda^2\, |\vec{k}_{p^{(1)},p^{(2)}}|^5}{60\, \pi\, m_{G_2}^2} \; ;$$

- while for two vector mesons

$$\Gamma_{G_2 \to V^{(1)} V^{(2)}} = \frac{\kappa_{gvv,i} \lambda^2 |\vec{k}_{v^{(1)},v^{(2)}}|}{120\, \pi\, m_{G_2}^2} \Big(15 + \frac{5|\vec{k}_{v^{(1)},v^{(2)}}|^2}{m_{v^{(1)}}^2} + \frac{5|\vec{k}_{v^{(1)},v^{(2)}}|^2}{m_{v^{(2)}}^2} + \frac{2|\vec{k}_{v^{(1)},v^{(2)}}|^4}{m_{v^{(1)}}^2\, m_{v^{(2)}}^2} \Big) \; ;$$

- and for the axial-vector and pseudoscalar mesons

$$\Gamma_{G_2 \longrightarrow A_1 P} = \frac{\kappa_{gap,i}\, \lambda^2\, |\vec{k}_{a_1,p}|^3}{120\, \pi\, m_{G_2}^2} \big(5 + \frac{2\, |\vec{k}_{a_1,p}|^2}{m_{a_1}^2} \big) \; .$$