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THE QCD PHASE DIAGRAM: A (PHENO)THEORETICAL OVERVIEW

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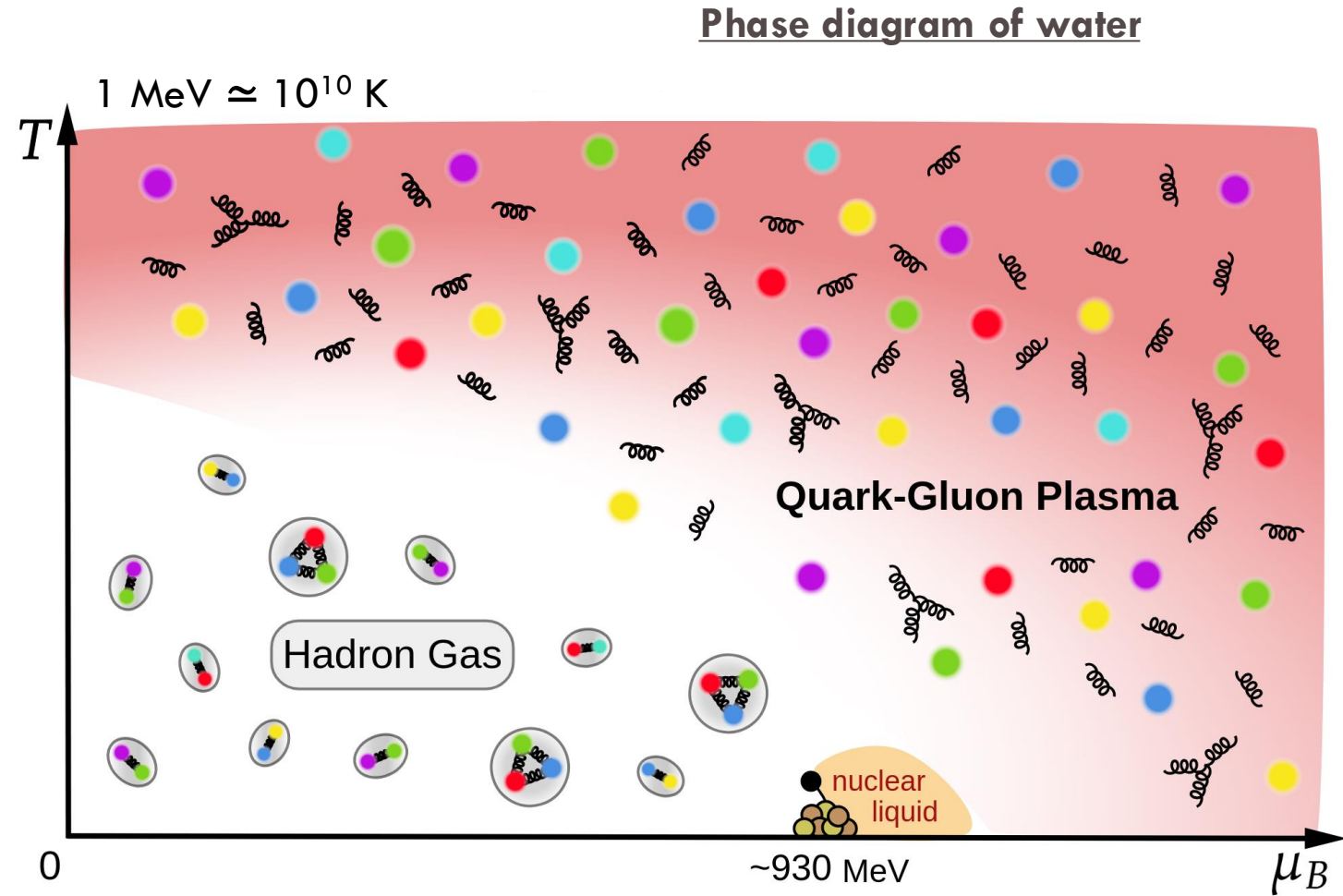
OUTLINE

1. Phase diagram & thermodynamics
2. First-principle calculations
3. 1st-order phase transition & critical point
4. Constraints from the experiments
5. MUSES: one framework to unify them all

PHASE DIAGRAM OF NUCLEAR MATTER

- Representing different phases and transitions as a function of thermodynamic variables
(*temperature, pressure, entropy density...*)
- What do we know about the nuclear phase diagram?
(for sure, from observation)
 - Atomic nuclei
 - Hadron gas / nuclear liquid
 - Quark-gluon plasma (QGP)

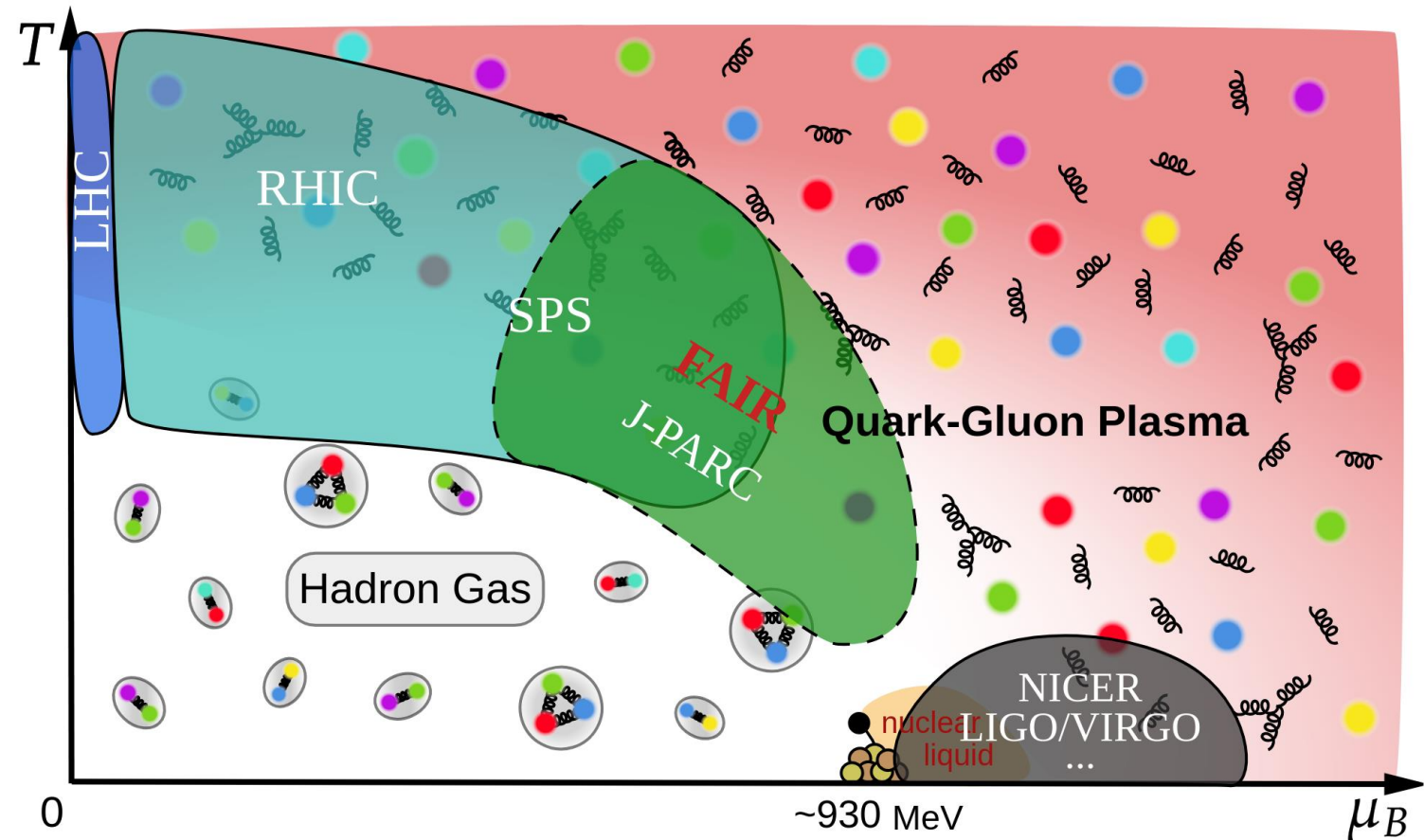
...and that's all.



HOW TO EXPLORE THE PHASE DIAGRAM?

Essentially 2 ways to learn about the structure of the phase diagram:

- Looking up at the sky
 - Reproducing the evolution of the matter in the early Universe
 - Study neutron star structure + mergers from gravitational waves
- Down on Earth, in colliders
 - LHC @ CERN
 - RHIC @ BNL / SPS @ CERN
 - J-PARC @ Tokai / **FAIR @ GSI**



THERMODYNAMICS RELATIONS

To compute an **equation of state (EoS)**, one usually start by calculating one quantity as a function $\mathcal{F}(T, \mu//n)$ and derive all other quantities from there.

Basic thermodynamic relations, in the **grand-canonical limit** (from the partition function \mathcal{Z}):

Pressure:
$$P = -T \frac{\partial \ln(\mathcal{Z})}{\partial V}$$

Entropy density:
$$s = \left(\frac{\partial P}{\partial T} \right)_{\mu_i}$$

Charge densities:
$$n_i = \left(\frac{\partial P}{\partial \mu_i} \right)_{T, \mu_{j \neq i}}$$

Energy density:
$$\varepsilon = sT - P + \sum_i \mu_i \cdot n_i$$

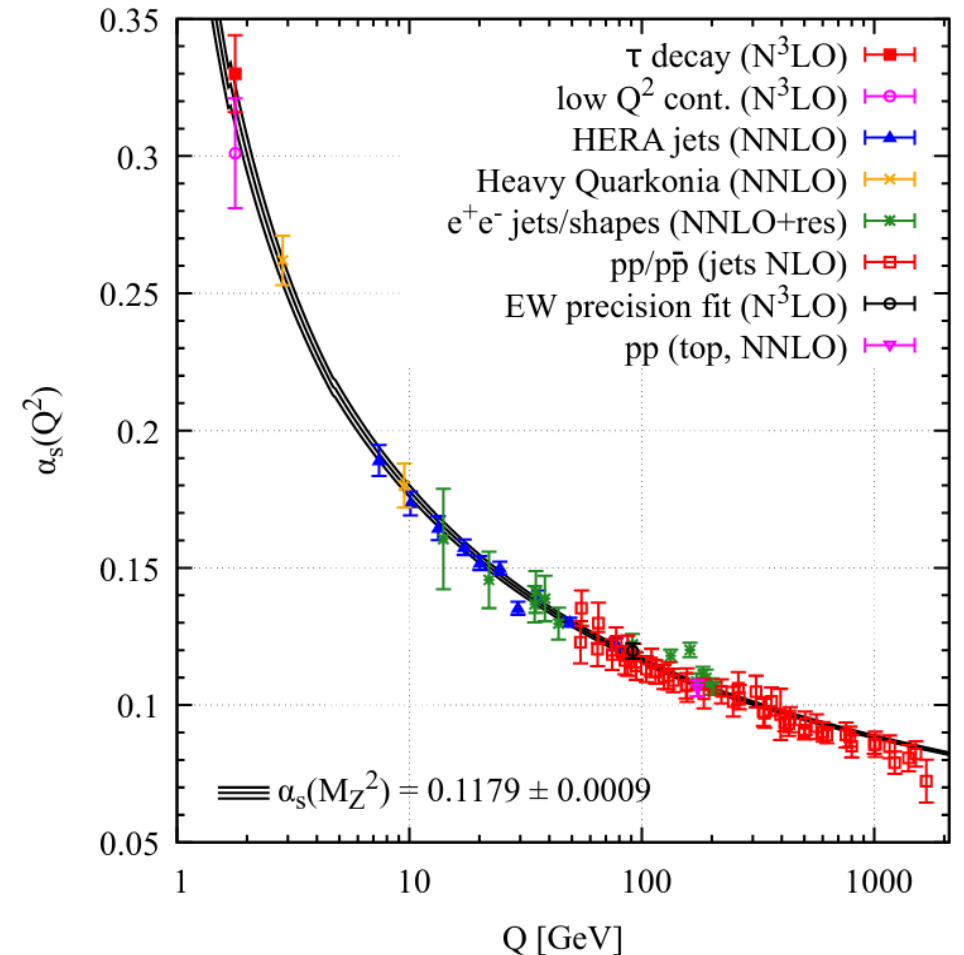
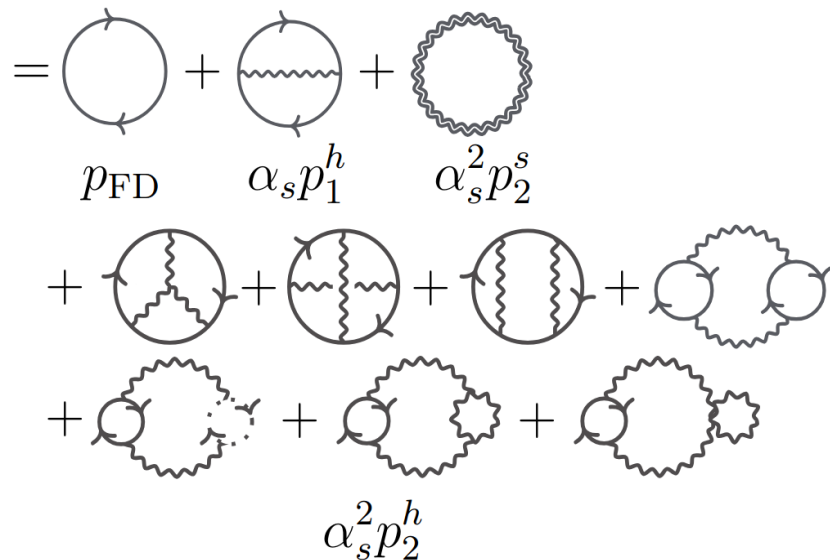
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1. Phase diagram & thermodynamics
2. First-principle calculations & baselines
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THE RUNNING COUPLING CONSTANT OF QCD

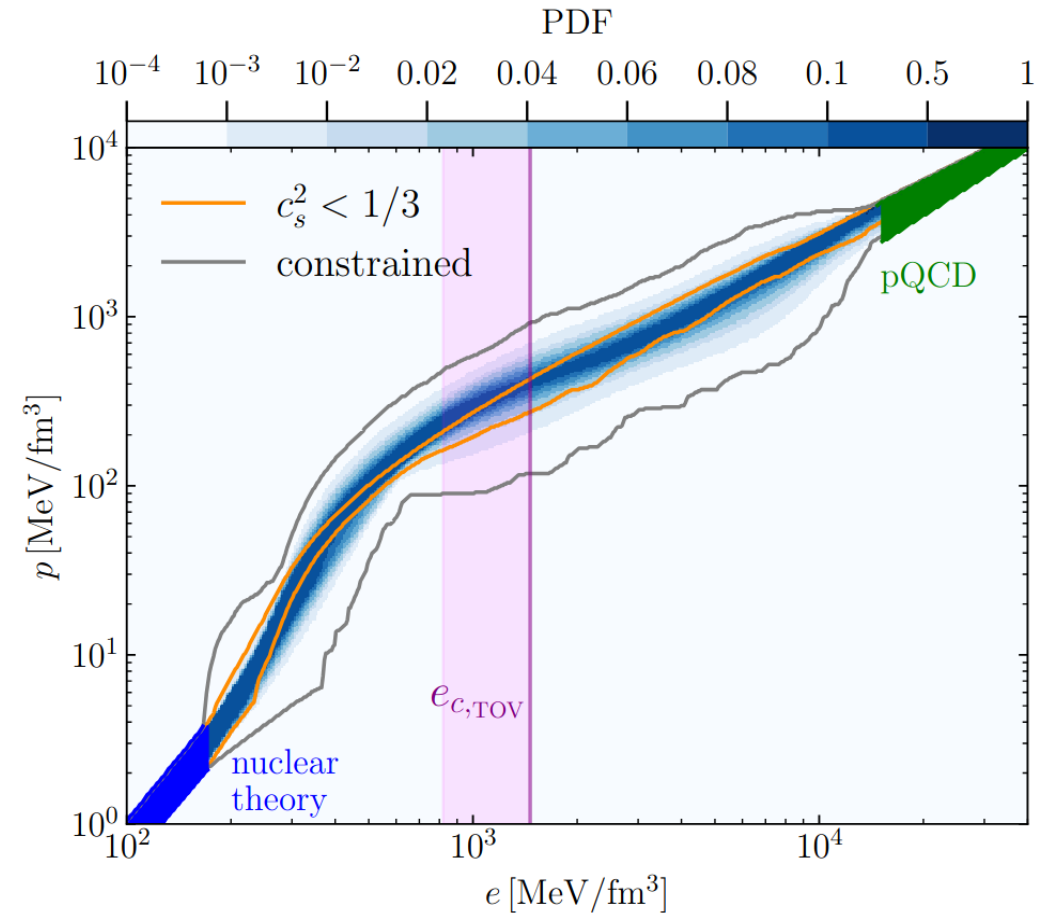
- One **fundamental property of QCD** is the fact that its **coupling α_s** is **changing magnitude** depending on the energy involved in the process considered.
- Direct calculation of the pressure from QCD is only possible through perturbation methods for small α_s :

$$p = p_{\text{FD}} + \alpha_s p_1^h + \alpha_s^2 p_2^h + \alpha_s^2 p_2^s + \dots$$



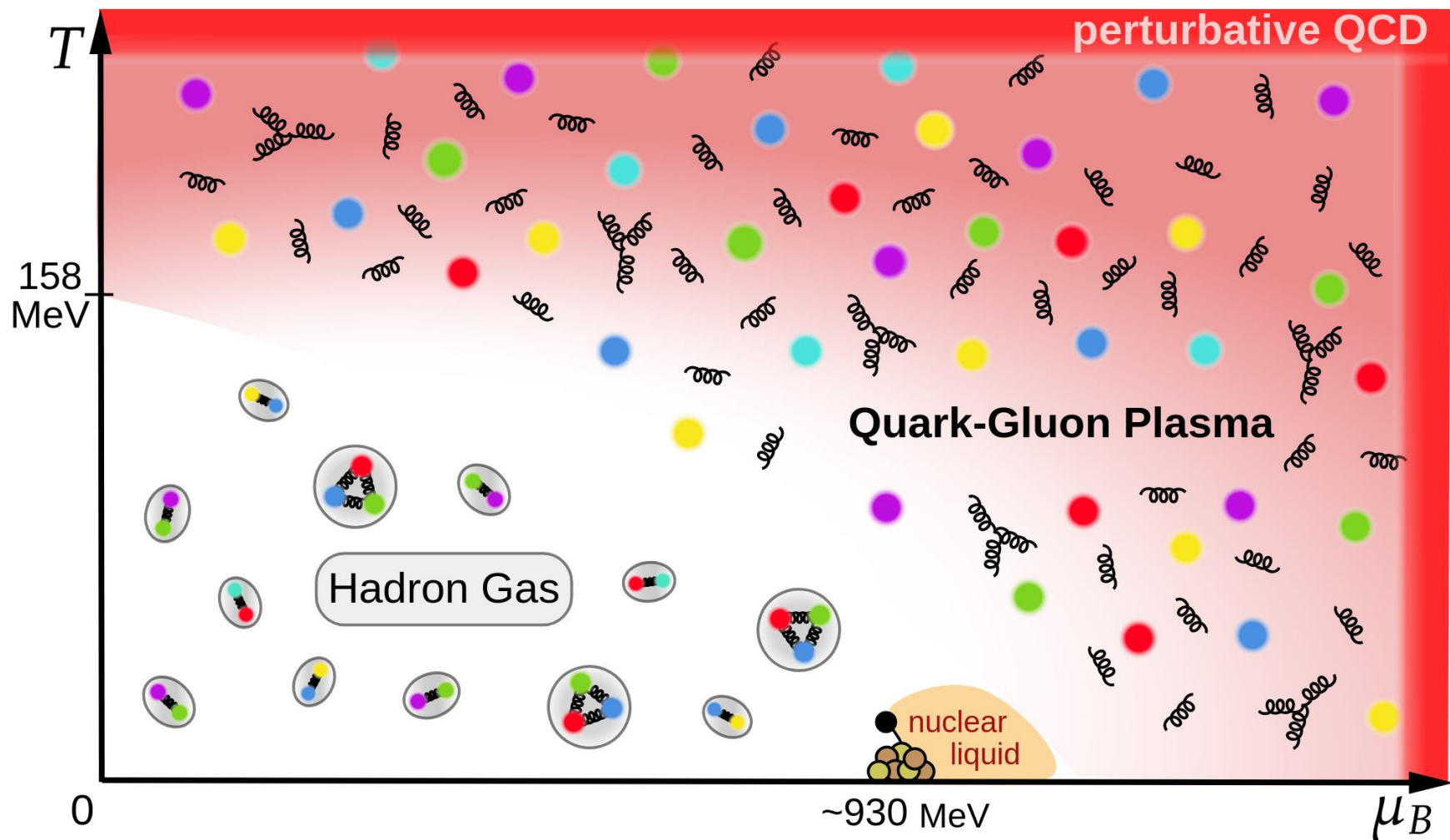
PERTURBATIVE QCD

- Perturbative calculation from Feynman diagrams are only possible at very high T/n_B
- In recent years, the approach have been generalized at NNLO to cover all T and n_B
[Gorda et al., Phys.Rev.D 104 \(2021\) 7, 074015](#)
- Only available for $n_B \geq 40 n_{sat}$ at low- T , pQCD helps to constrain the nuclear EoS through Bayesian analysis when considering:
 - Constraint from nuclear theory at low density
 - Imposing causal and stable EoS
 - Using known constraints from astronomical observations



[Altiparmak, Eckert & Rezzolla, Astrophys.J.Lett. 939 \(2022\) 2, L34](#)

NUCLEAR PHASE DIAGRAM



LATTICE QCD – QCD FROM FIRST PRINCIPLES

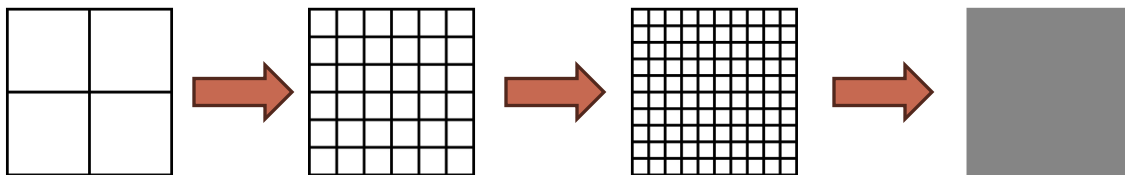
In the non-perturbative regime, no analytical solutions of QCD can be calculated.

Solution: discretizing space-time on a lattice and compute path integral using Monte-Carlo sampling for many configurations U

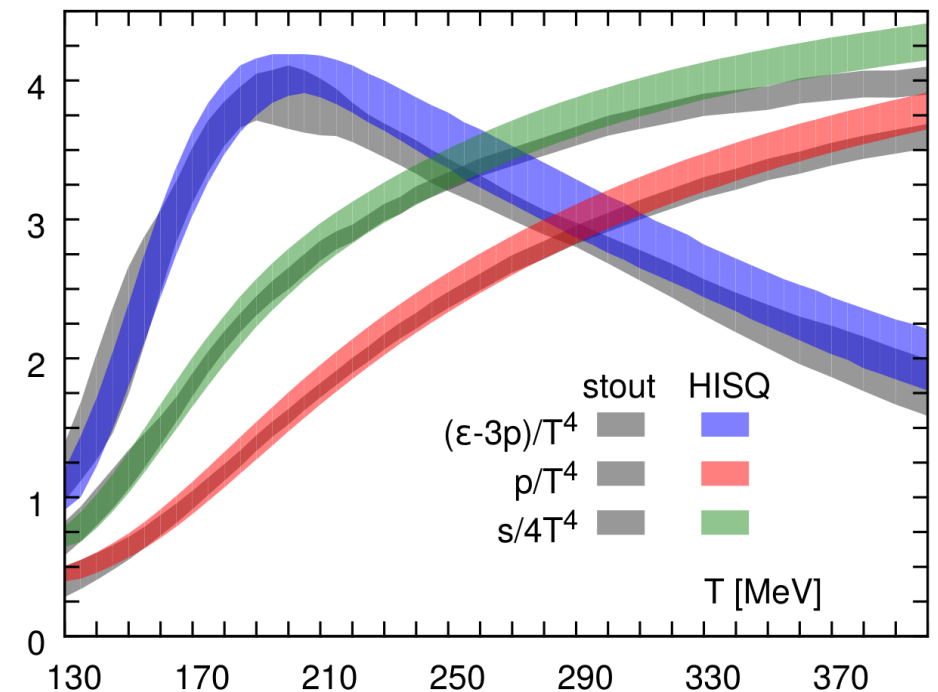
$$\langle \hat{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \hat{O} \det M[U] e^{-S_G[U]}$$

for any observable \hat{O} .

Simulations achieved at **different volumes V** and **different lattice spacings a** to obtain in the end **continuum limit results in an infinite volume**.



Results at $\mu_B = 0$ are consistent across different collaborations.



Bazavov et al., PRD 90 (2014),094503

LATTICE QCD - THE SIGN PROBLEM

When using Monte-Carlo sampling: $\langle \hat{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \hat{O} \det M[U] e^{-S_G[U]}$

$\det M[U] e^{-S[U]}$ is used as a **statistical weight**
(least probable configurations U are ignored)

- When $\mu_B^2 = 0$: $\det M[U] e^{-S[U]}$ is **real**
- When $\mu_B^2 > 0$: $\det M[U] e^{-S[U]}$ becomes **complex** and has **highly oscillating phase**

→ **Can't be interpreted as a statistical weight!**



THIS IS THE WORST!

BUT! For **purely imaginary μ_B** (when $\mu_B^2 < 0$), $\det M[U] e^{-S[U]}$ is real again: **simulations possible...**

LATTICE QCD — THE NATURE OF THE TRANSITION

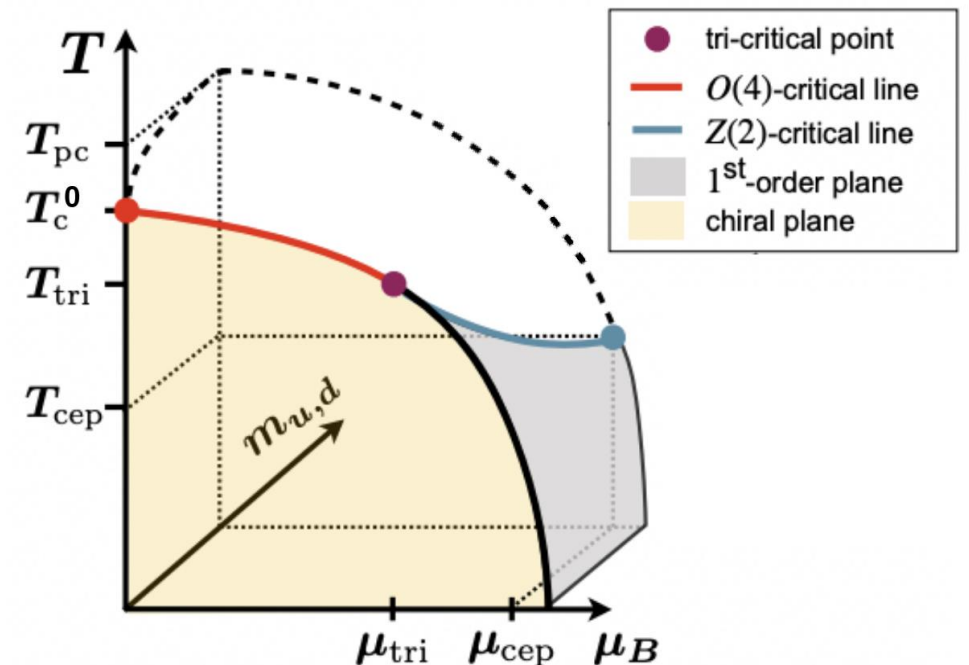
If one considers the case where $m_u = m_d = 0$ (chiral limit), the **phase transition** is expected to be of **2nd order**.

However, in real-life QCD, $m_u \simeq m_d \neq 0$, and the chiral symmetry is broken.

→ smooth crossover from hadron gas to QGP
(no discontinuity in 1st and 2nd order derivatives)



Hadrons basically melt like butter at room temperature



LATTICE QCD – THE NATURE OF THE TRANSITION

By extrapolating from complex μ_B lattice simulations to real μ_B , one can compute the shape of the transition line:

$$\frac{T_c(\mu_B)}{T_c(\mu_B = 0)} = 1 + \kappa_2 \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \kappa_4 \left(\frac{\mu_B}{T_c(\mu_B)} \right)^4 + \mathcal{O}(\mu_B^6)$$

(location, curvature, “hyper-curvature”, ...)

[Bazavov et al., PLB 795 \(2019\) 15-21](#)

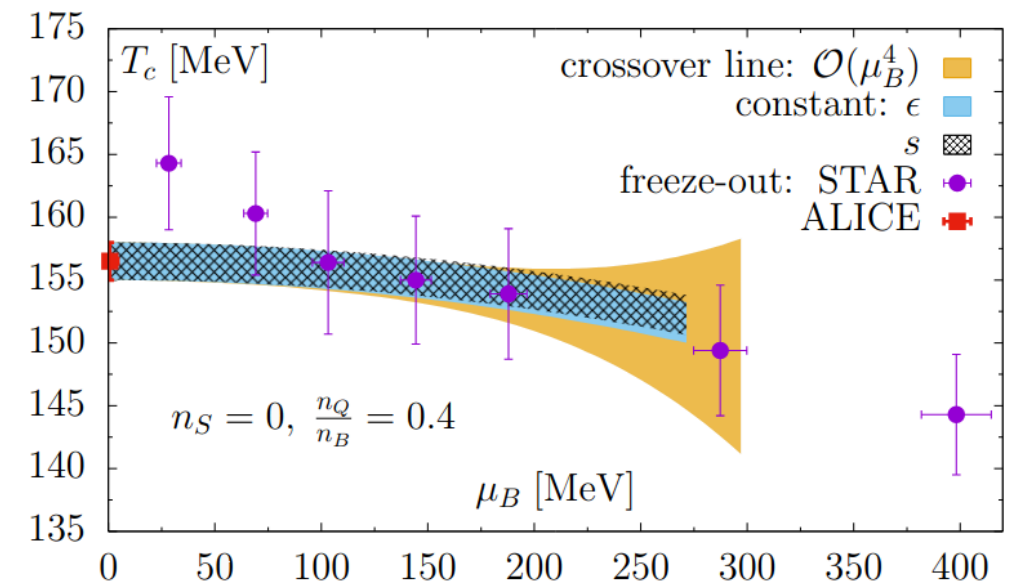
Most recent results:

$$T_c(\mu_B = 0) = 158.0 \pm 0.6 \text{ MeV}$$

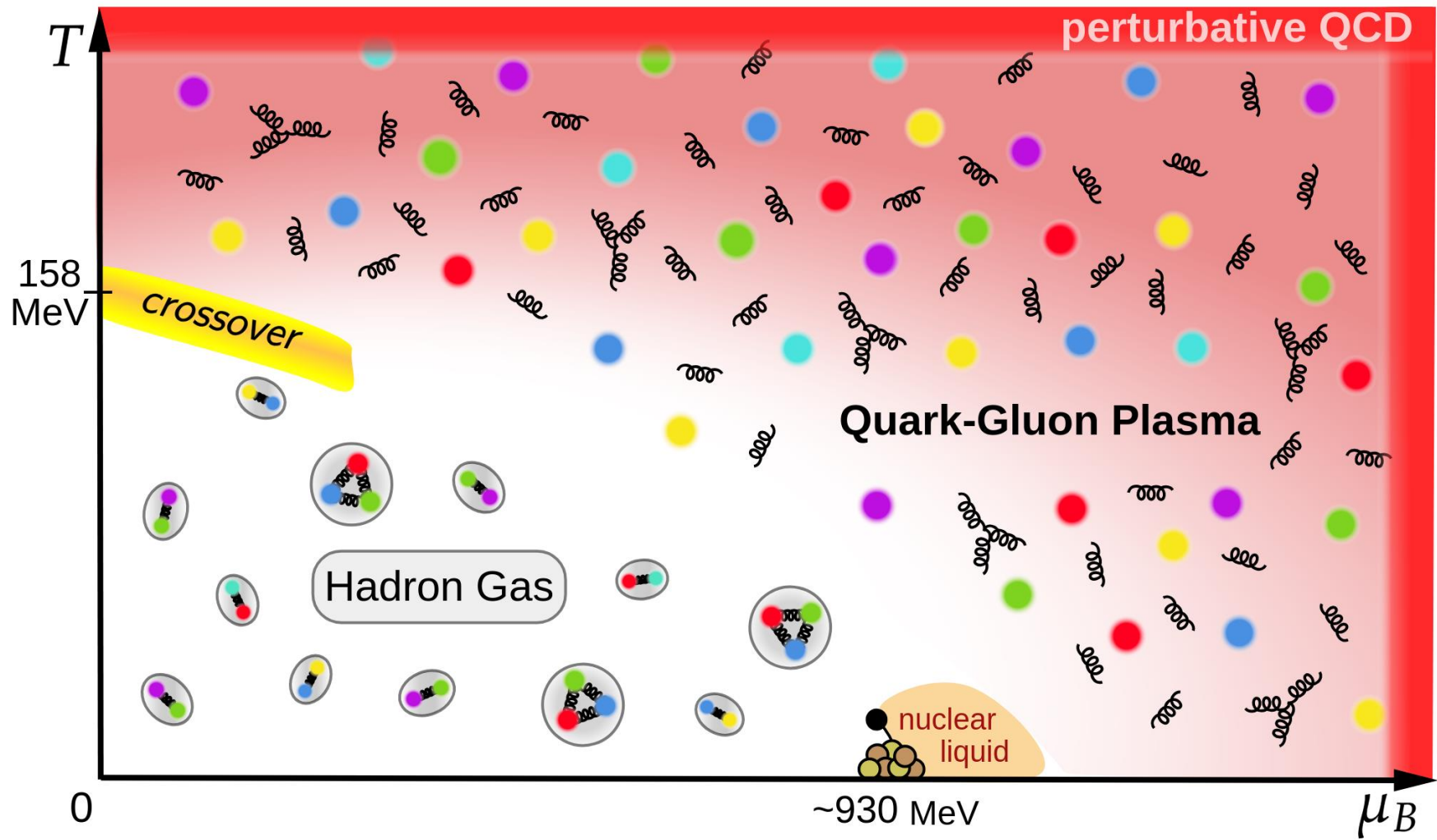
$$\kappa_2 = 0.0153 \pm 0.0018 \quad \kappa_4 = 0.00032 \pm 0.00067$$

[Borsányi et al., PRL 125 \(2020\), 052001](#)

Existence of a **critical point excluded** for $\mu_B < 300 \text{ MeV}$
by lattice QCD results.



NUCLEAR PHASE DIAGRAM



LATTICE QCD – GOING TO FINITE DENSITY

The **sign problem** in lattice QCD prevent from direct computation of thermodynamics at real finite $\mu_B \rightarrow$ need to employ **expansion methods**.

Taylor series expansion

$$\frac{P(T, \mu_B)}{T^4} = \sum_i \frac{1}{i!} \chi_i^B(T, \mu_B=0) \left(\frac{\mu_B}{T}\right)^i$$

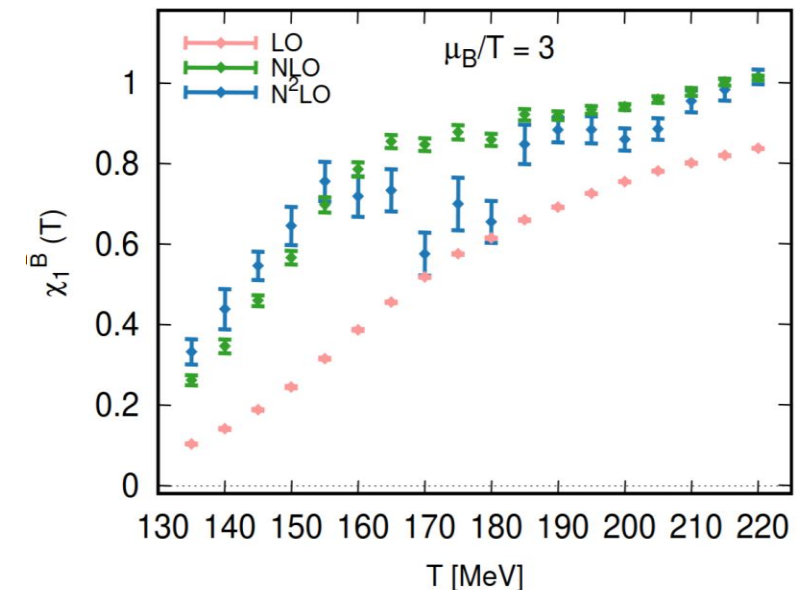
Allows to compute an EoS at **finite density** from expansion coefficients, called **susceptibilities**, computed at $\mu_{B/Q/S} = 0$ from lattice QCD.

$$\chi_i^B = \left. \frac{\partial^i (P/T^4)}{\partial \hat{\mu}_B^i} \right|_{\hat{\mu}_B=0}$$

Limitations:

- Expansion achieved at constant T , missing out curvature of transition line
- Large errors due to high-order terms leading at large μ_i/T

\rightarrow Expansion limited to $\mu_i/T \leq 2.5$



[Borsányi et al., JHEP 10 \(2018\) 205](#)

LATTICE QCD – GOING TO FINITE DENSITY

T-Expansion Scheme (TExS)

New method based on a resummation of Taylor expansion,

defined from the following ansatz: $\frac{\chi_1^B(T, \hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T', 0)$

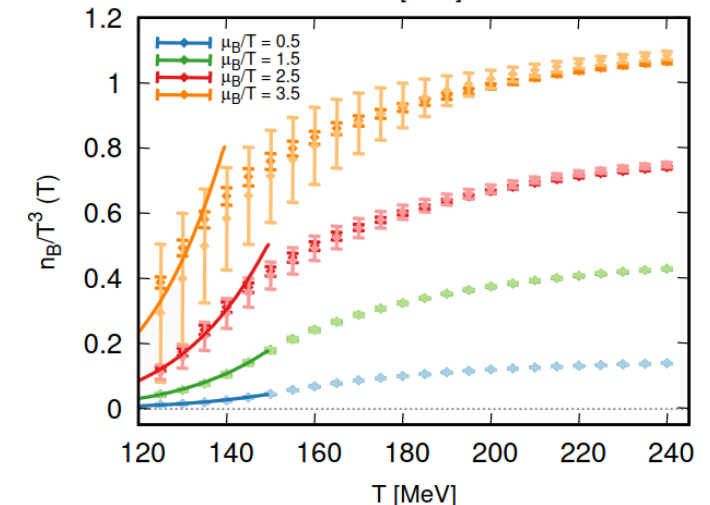
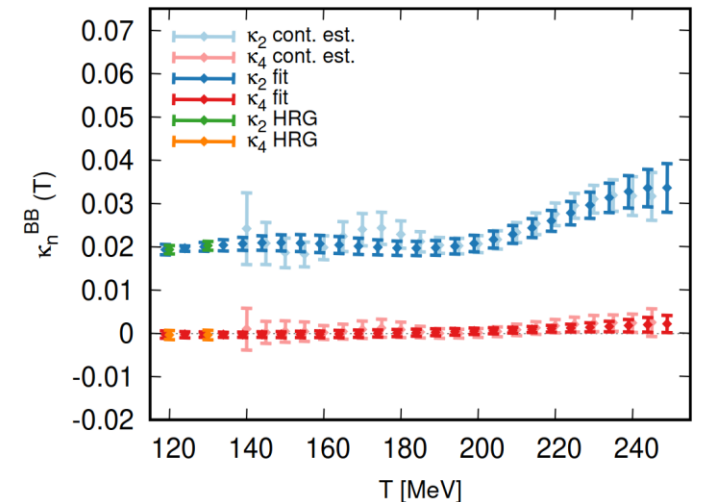
using a shifted temperature T' expanded in T and μ_B

$$T'(T, \mu_B) = T \left(1 + \kappa_2^{BB}(T) \hat{\mu}_B^2 + \kappa_4^{BB}(T) \hat{\mu}_B^4 \dots \right)$$

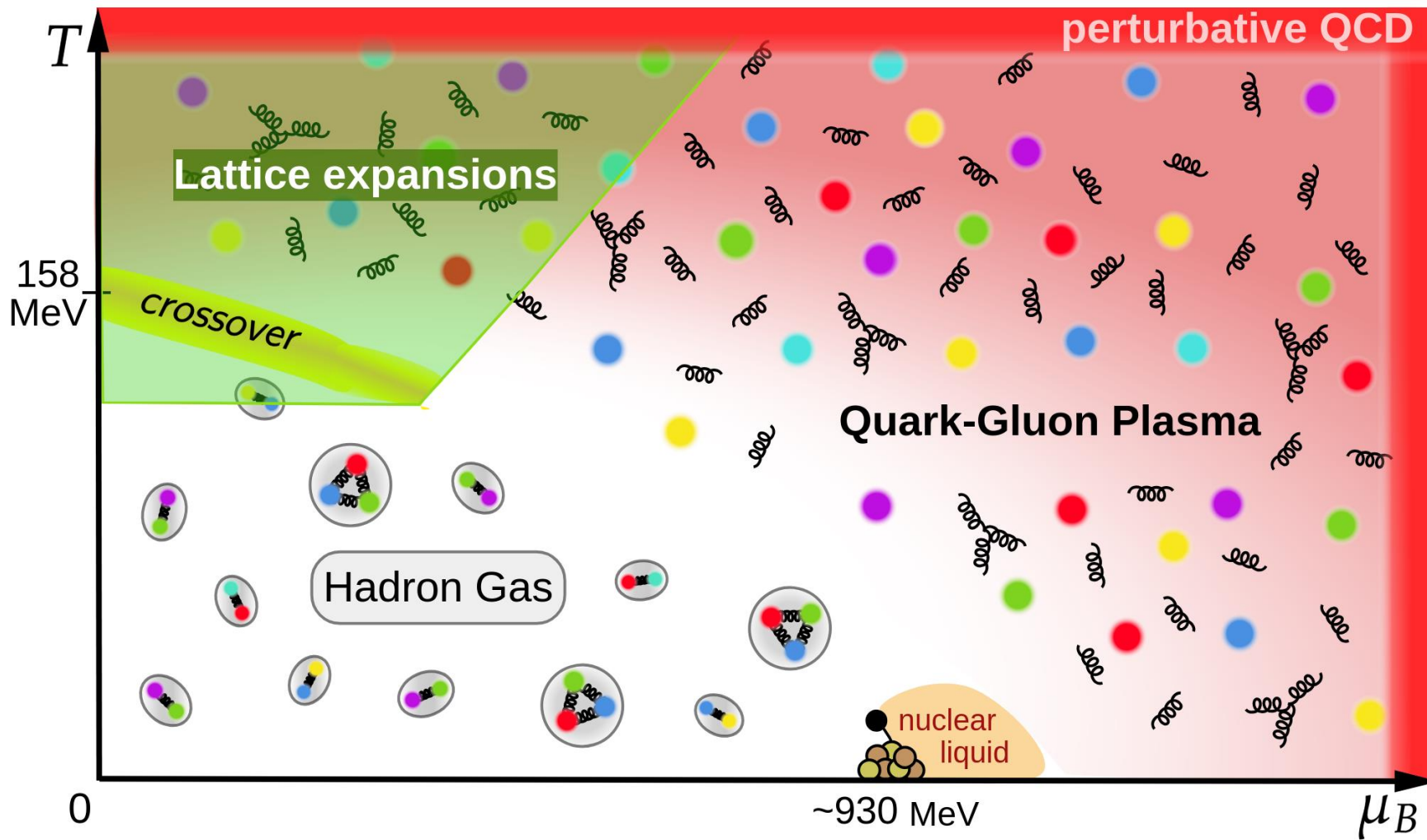
- To compute the complete EoS, one has to integrate χ_1^B to get pressure
- Separation in magnitude between expansion coefficients (related to χ_s) hints at better convergence than Taylor expansion
- Generalization to 4D (T, μ_B, μ_Q, μ_S) currently ongoing

→ **Trusted up to $\mu_B / T = 3.5$**

Borsányi et al., PRL 126 (2021) 23, 232001

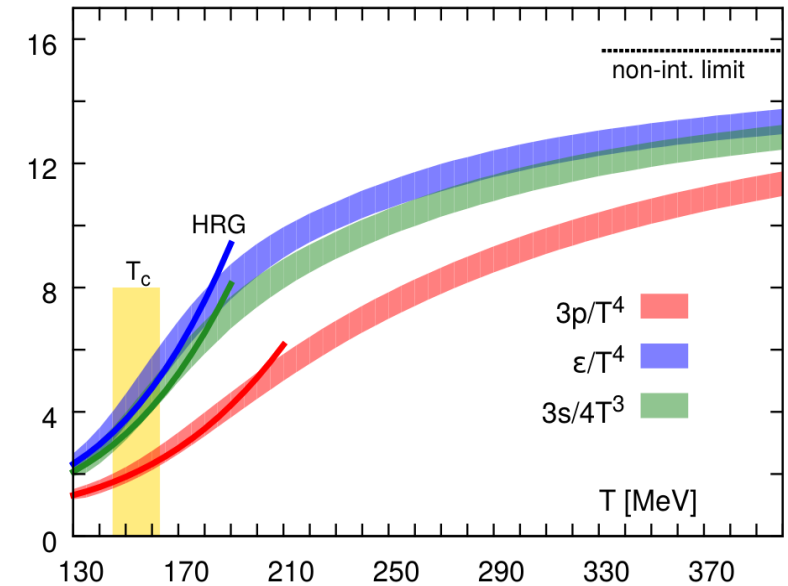


NUCLEAR PHASE DIAGRAM

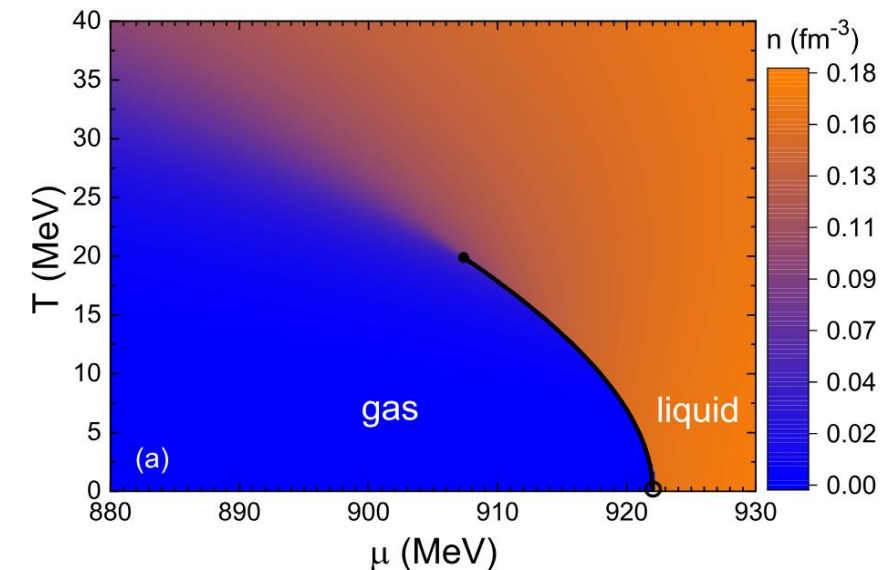


HADRON RESONANCE GAS MODEL

- Thermal model based on Fermi-Dirac & Bose-Einstein statistics, assuming a gas of interacting hadrons in their ground states can be modeled by a gas of non-interacting hadrons and resonances.
- Describes the hadronic phase only (*blows up at the transition*)
 - used as a reference for low-T QCD, as it matches with lattice QCD (*which is too costly to compute below $T \sim 120$ MeV*)
- Can be improved by adding excluded volume correction and van der Waals attractive interaction
 - describes the nuclear liquid-gas phase transition

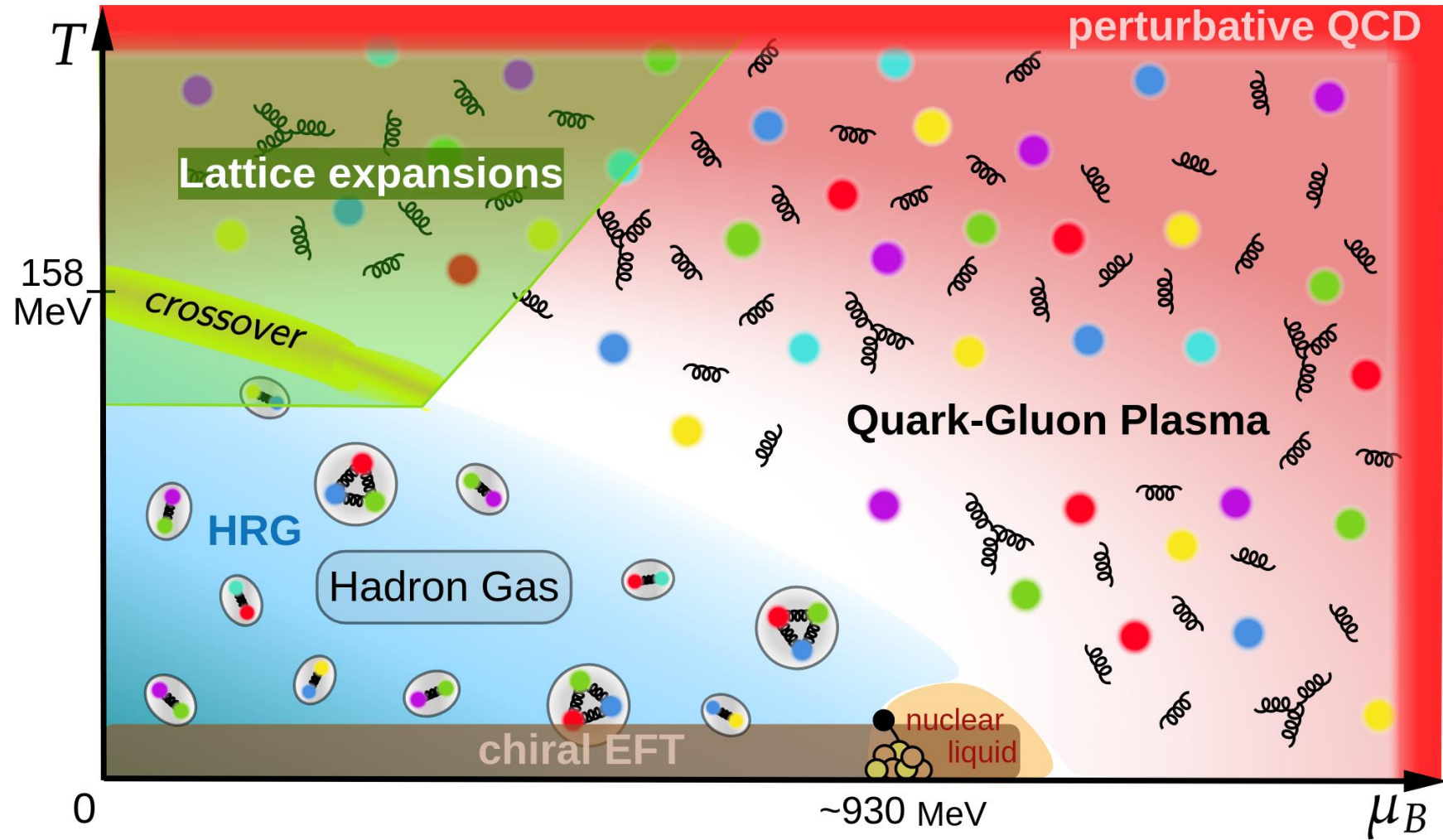


[Bazavov et al., PRD 90 \(2014\),094503](#)



[Vovchenko et al., Phys.Rev.C 92 \(2015\) 5, 054901](#)

NUCLEAR PHASE DIAGRAM



See [@Javier Menéndez' talk](#) for discussion about description of nuclear matter at very low T (chiral EFT, many-body theories, etc.)

OUTLINE

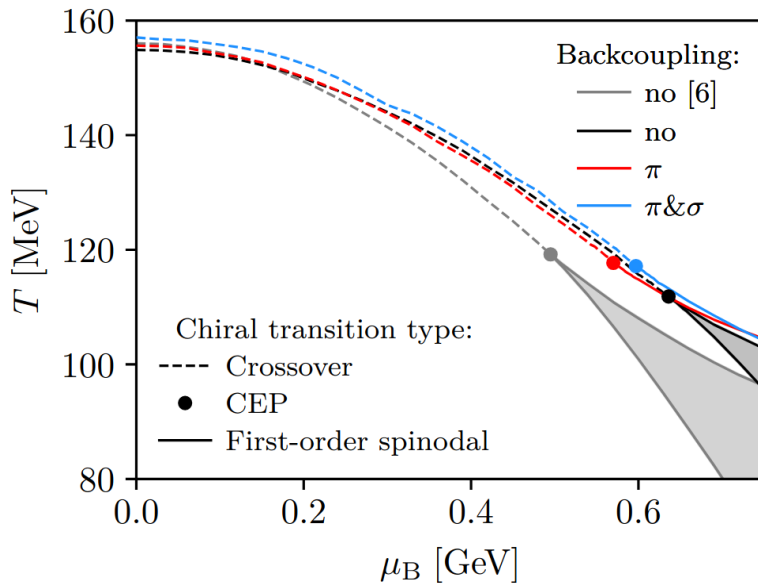
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QCD EFFECTIVE THEORIES

Several (radically) different **effective non-perturbative approaches to QCD** predict the **existence** of a **critical point** at finite baryon chemical potential, among which the most recent ones:

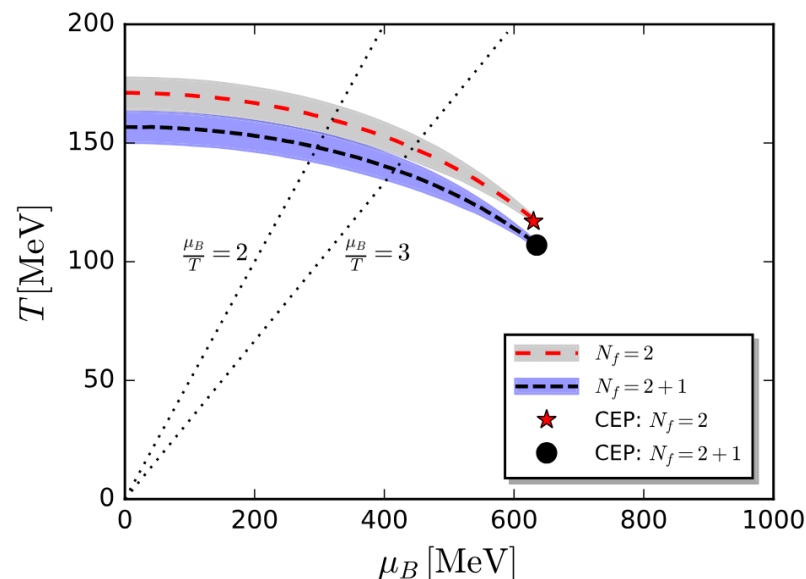
- Dyson-Schwinger equations

[Phys.Rev.D 104 \(2021\) 5, 054022](#)



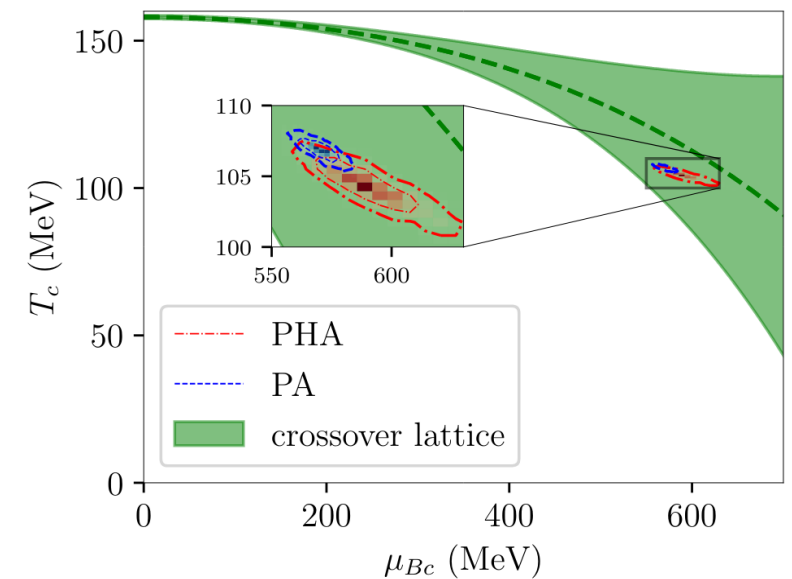
- Functional Renormalization Group

[Phys.Rev.D 101 \(2020\) 5, 054032](#)



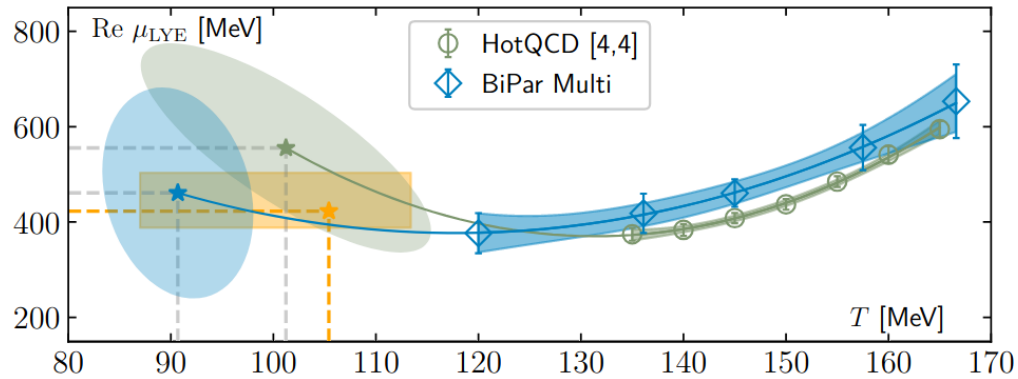
- Holographic model

[Hippert et al., arXiv:2309.00579](#)



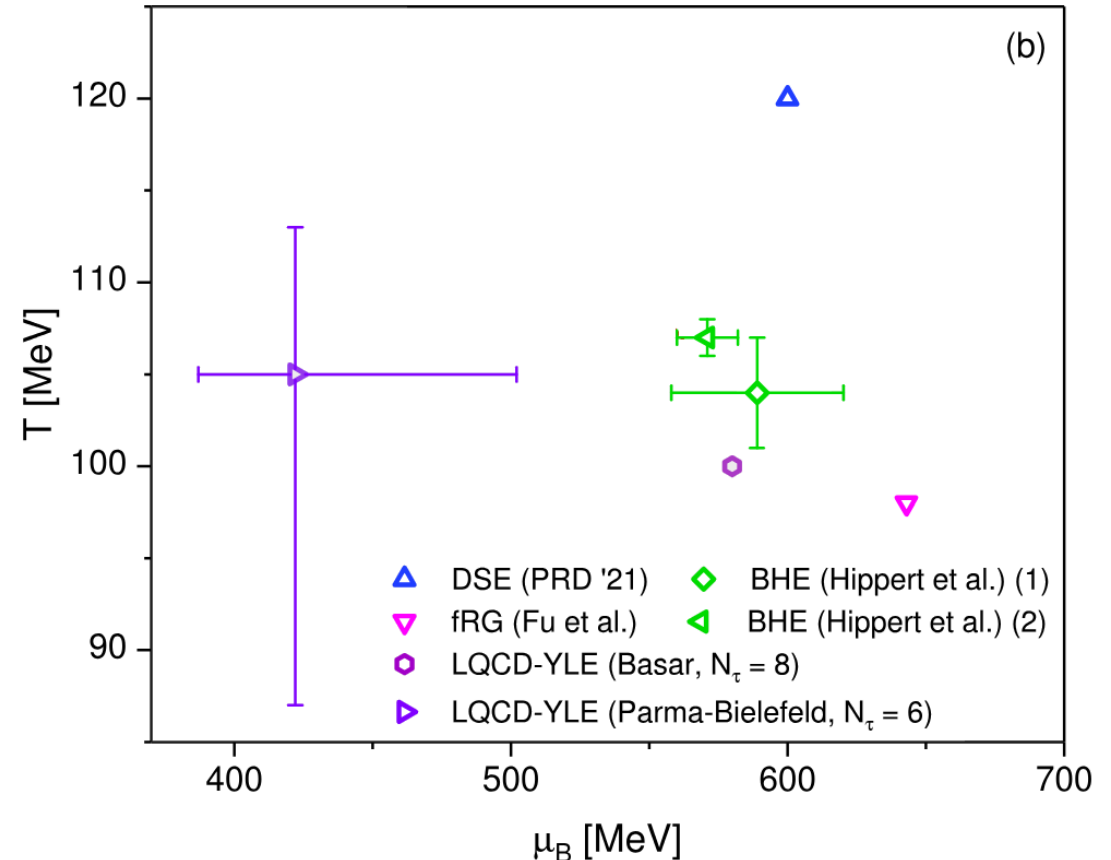
LOCATION OF A CRITICAL POINT FROM LATTICE QCD

Prediction of a **critical point** from **lattice QCD** from simulations at imaginary baryon chemical potential, extrapolated to the real plane.



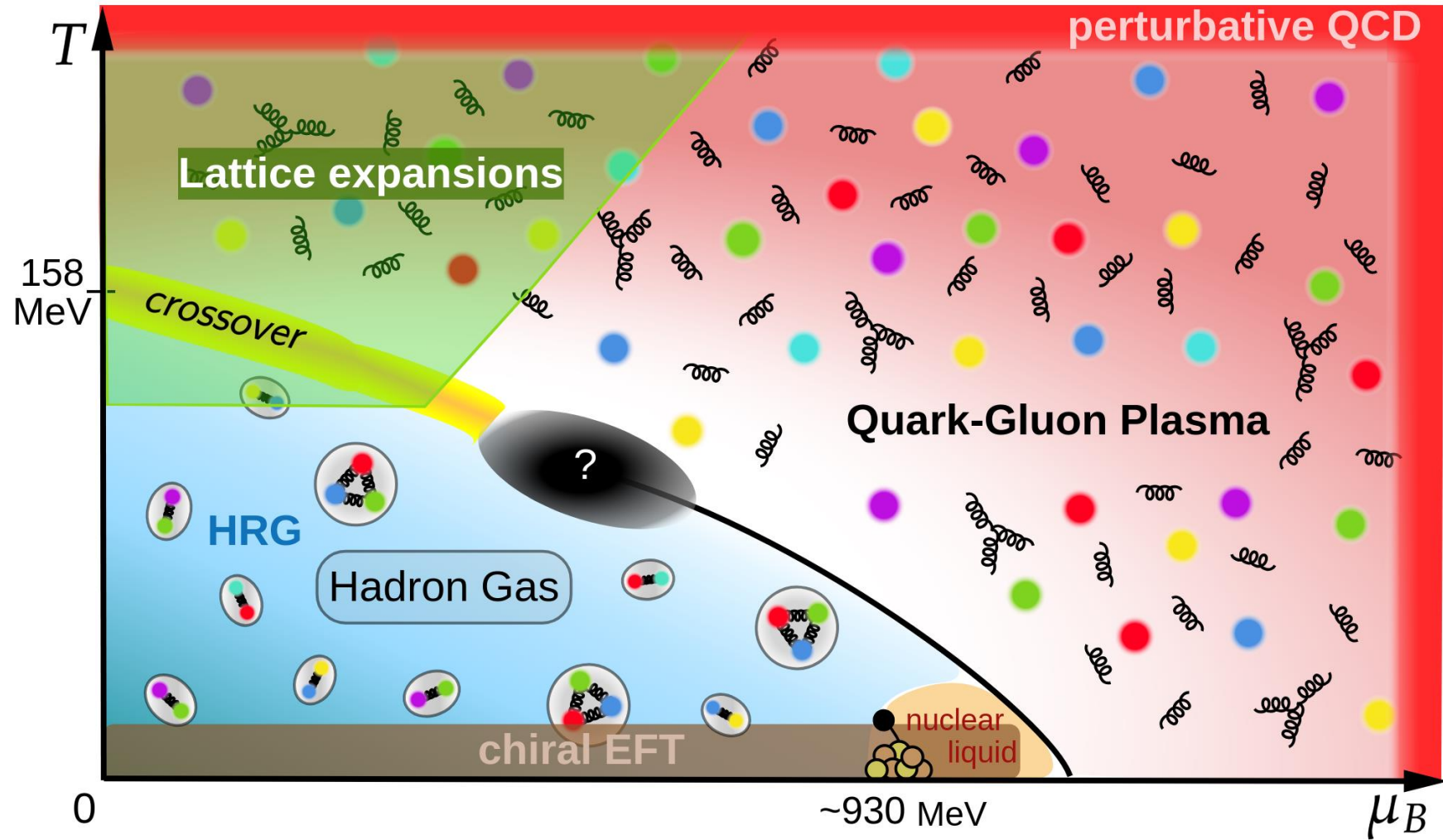
[D.A. Clarke et al., arXiv:2405.10196](#)

Compilation of predictions



Adapted from [Lysenko et al., arXiv:2408.06473](#)

NUCLEAR PHASE DIAGRAM



See talk from [@Gyöző K.](#) on mean-field model EoS
and [@Mathis H.](#) on behavior of the critical dynamics out-of-equil.

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HOW TO LINK THEORY TO THE EXPERIMENTS?

The best way to confront theoretically calculated EoS with experimental results is to run **simulations**.

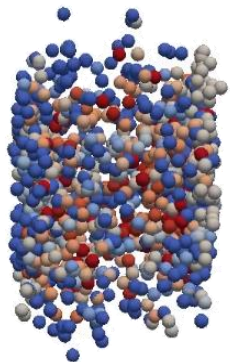
- Making data prediction/interpretation using pre-computed EoS
- Helps to constrain properties of the EoS by comparing to measurements

This is where FAIR enters the game!

Heavy-ion collisions

Microscopic transport

- Hadrons/quarks as d.o.fs



SMASH

Event generators:

- EPOS4
- HIJING
- Angantyr
- AMPT
- UrQMD
- SMASH

...

Relativistic hydrodynamics

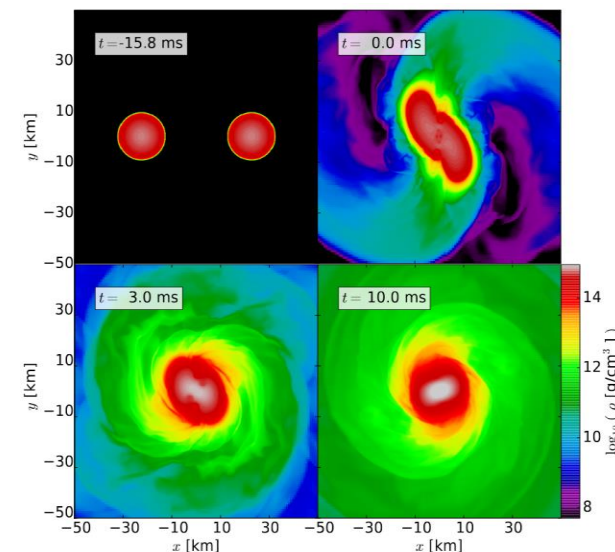
- Mesoscopic scale (using densities)



MADAI

Neutron star mergers & Kilonovae

Relativistic Magnetohydrodynamics



[Takami et al., Phys.Rev.D 91 \(2015\) 6, 064001](#)

See talks from [@Christine C.](#), [@Aristeidis N.](#), [@Vimal V.](#) & [@Luke S.](#)

FREEZE-OUT COORDINATES

Using the HRG model, one can extract information for **temperature** and **chemical potentials** at:

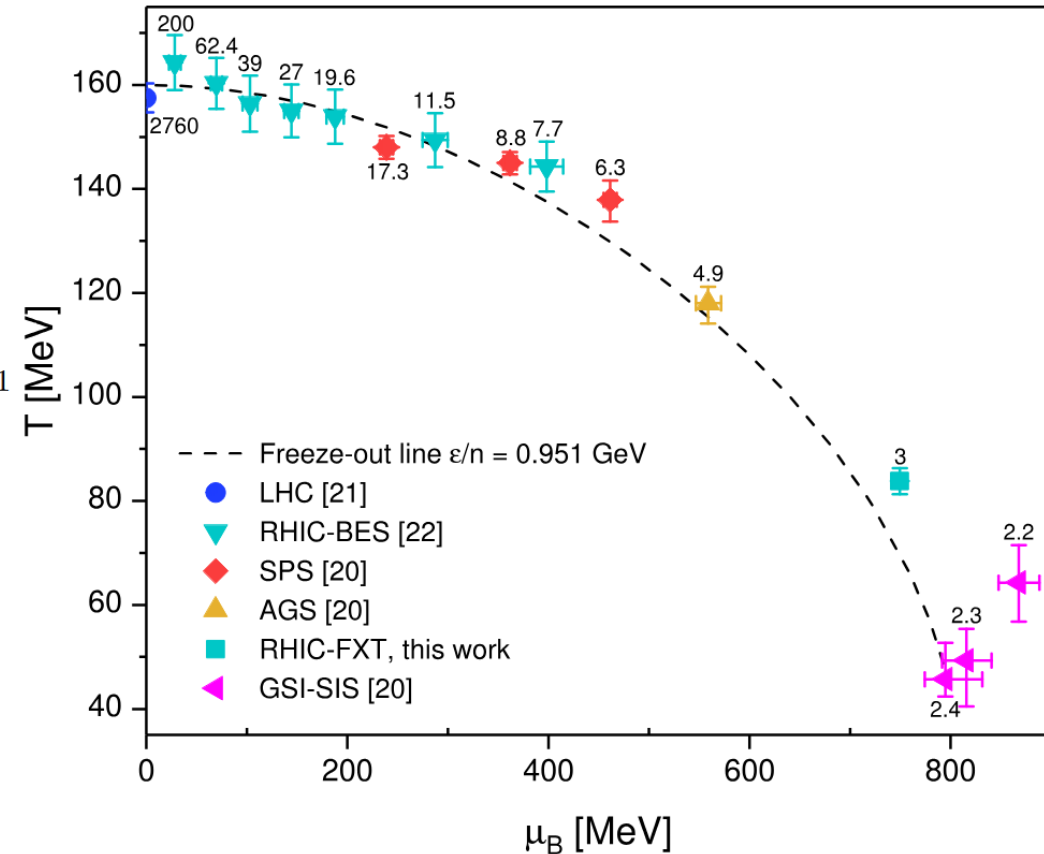
- **Chemical freeze-out:** when inelastic interactions cease, by fitting particle abundances through density

$$n_i^{\text{id}}(T, \mu_i) = \frac{d_i}{2\pi^2} \int dm f_i(m) \int_0^\infty k^2 dk \left[\exp\left(\frac{\sqrt{k^2 + m^2} - \mu_i}{T}\right) + \eta_i \right]^{-1}$$

- **Kinetic freeze-out:** when all interaction cease, by fitting the transvers momentum spectra

$$\frac{dN}{p_T dp_T} \propto \int_0^R r dr m_T I_0\left(\frac{p_T \sinh \rho(r)}{T_{\text{kin}}}\right) \times K_1\left(\frac{m_T \cosh \rho(r)}{T_{\text{kin}}}\right)$$

assuming a simple radial flow velocity profile $\beta = \beta_S(r/R)^n$



Lysenko et al., arXiv:2408.06473

EXPERIMENTAL SEARCH OF THE CRITICAL POINT

Fluctuations of conserved charges

Cumulants of conserved charges are sensitive to the critical point (diverge in its vicinity, in theory)

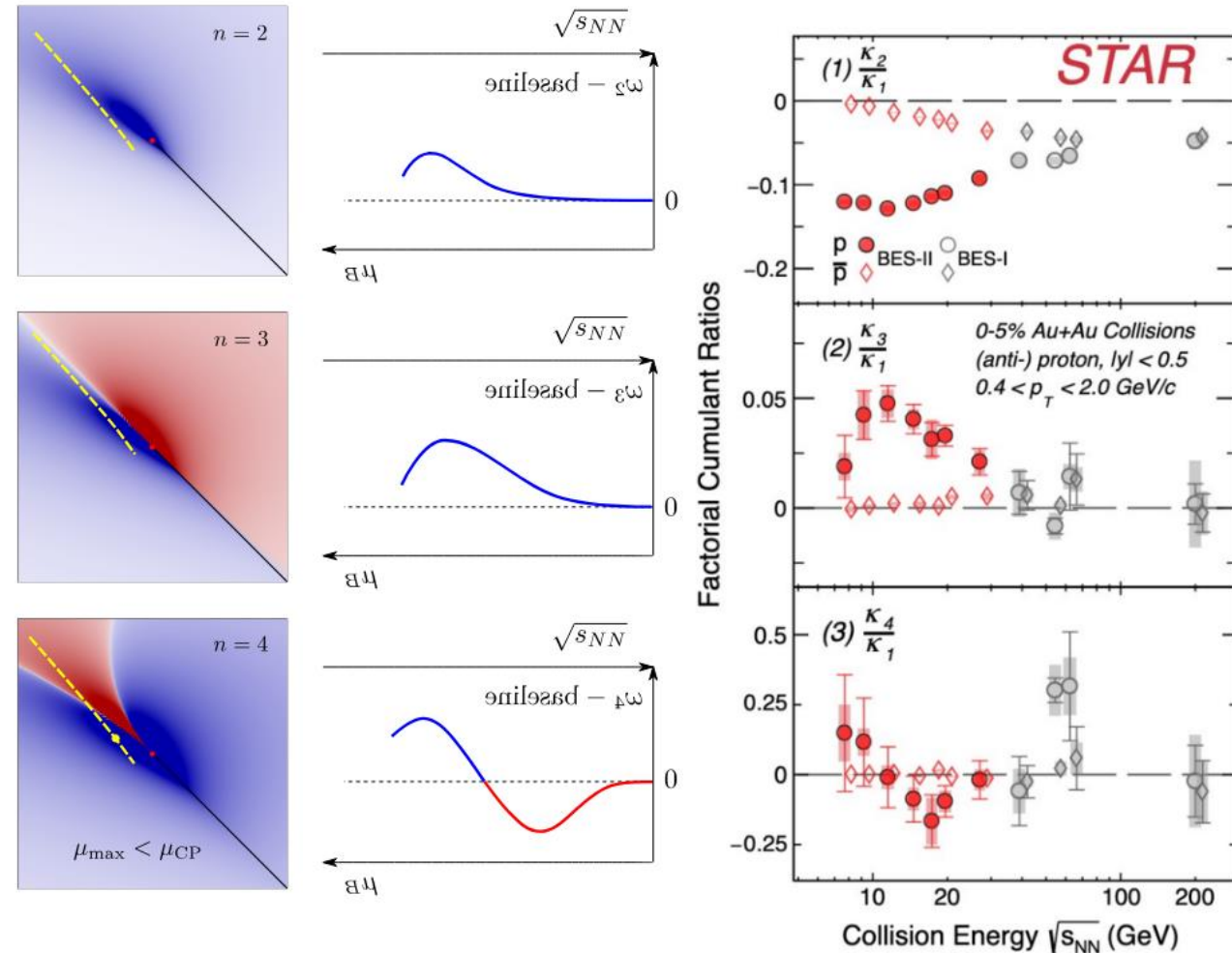
$$k_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial \mu^n}$$

→ using **hadron species** as proxies for **conserved charges**

- Differences between highly dynamic and short-lived HICs make comparison far from straight-forward:
 - choice of proxies
 - finite-volume effects
 - volume fluctuations
 - acceptance

...

See talks from @Rutik M., @Athira S., @Beatriz A.



INFERRING THE EOS FROM SPACE

Tolos & Fabbietti, *Prog.Part.Nucl.Phys.* 112 (2020) 103770

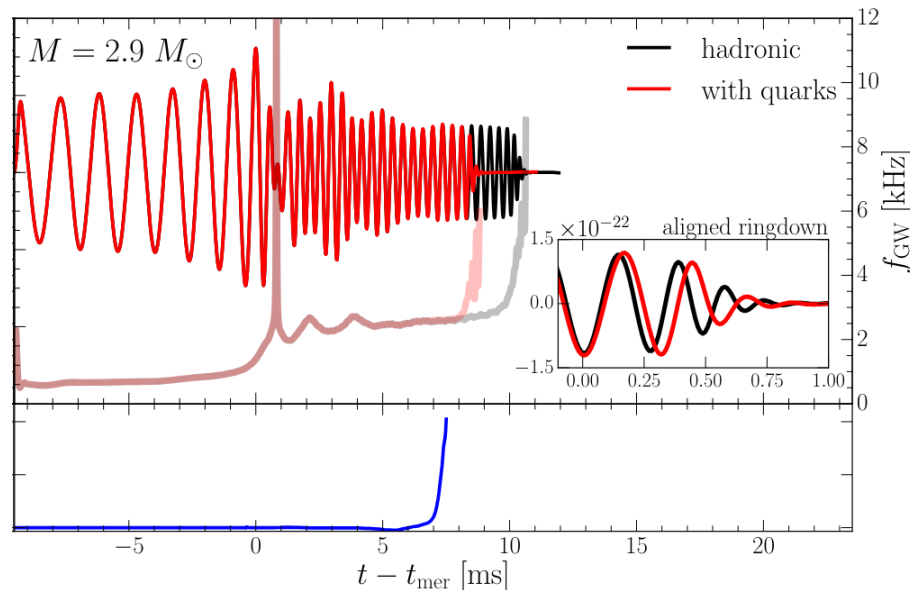
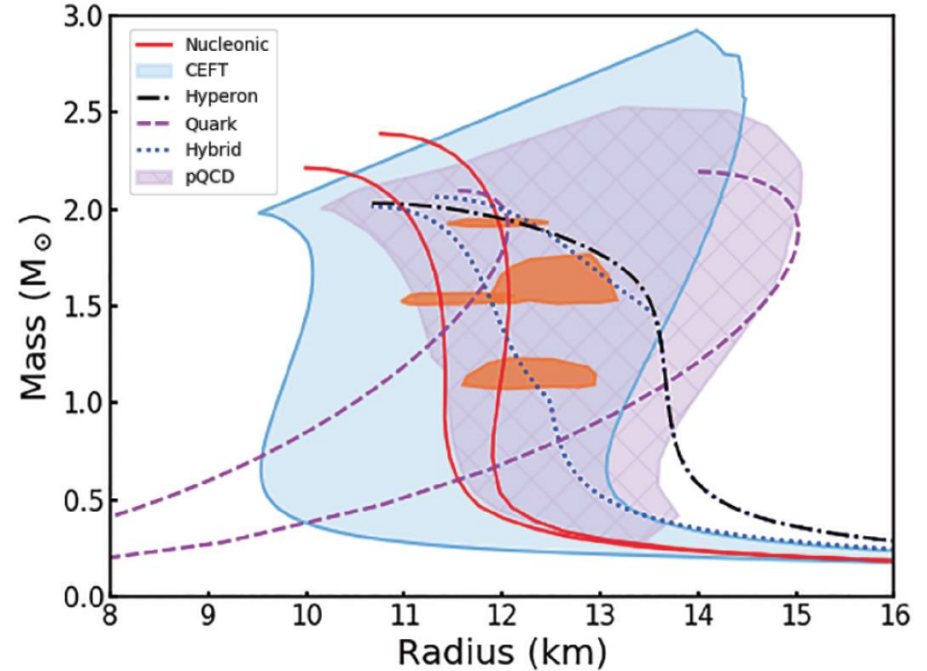
• Properties of neutron stars

Using the Tolman-Oppenheimer-Volkoff equation, one can compute (M,R) relations for a given nuclear EoS:

$$\frac{dP}{dr} = -\frac{G\epsilon(r)m(r)}{c^2 r^2} \left[1 + \frac{P(r)}{\epsilon(r)} \right] \left[1 + \frac{4\pi r^3 P(r)}{m(r)c^2} \right] \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1}$$

and confront the results to current measurements

+ other observables (*tidal deformability, quad. moment...*)



• Gravitational waves detection

GW signals detected from neutron star mergers can help to infer NS properties, as well as the nature of the composition, and hence of the transition type

TO LEARN EVEN MORE... ON THE MENU THIS WEEK

- Flow of dileptons

(probing chiral symmetry restoration, measuring T_{medium} , probing in medium interactions...)

See talks from [@Sukyung K.](#), [@Pawan K.S.](#), [@Karina S.](#) [@Cornelius F.-R.](#)

- Hyperon-nucleon/nucleon-nucleon interactions

(to compute mean-field potentials, understand matter composition at high densities...)

See talks from [@Snehankit P.](#) [@Konrad S.](#), [@Anna W.](#), [@Susanne G.](#)

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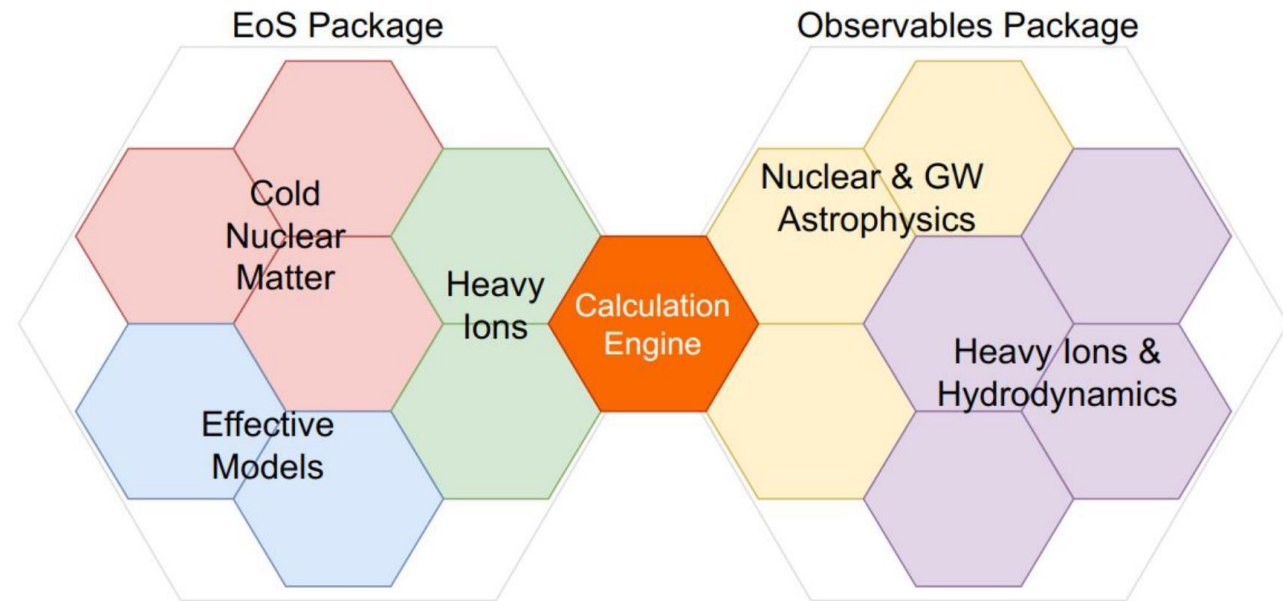
Click here for more



MODULAR UNIFIED SOLVER OF THE EQUATION OF STATE

Gathers physicists from [heavy-ion](#), [neutron star](#) and [low-energy nuclear physics](#) and [computer scientists](#)

- **Modular:** different modules computing EoS for different regions of the nuclear phase diagram + associated observables
- **Unified:** modules are integrated in a single framework, to ensure
 - i. Maximum coverage of phase space
 - ii. Respect of their constraints

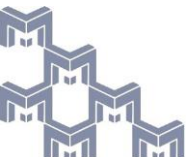


[A.T. Manning, MUSES Collaboration Meeting 2023](#)

Alpha release ongoing

(testing of the calculation engine by selected users)

PUBLIC RELEASE SOON!!! Stay tuned...



CONCLUSION

- Heavy-ion, low-energy nuclear physics and astro communities are working closer than ever to assess the structures of the phase diagram
- FAIR will play an important role to explore the high-density & low-temperature region
- Collaboration efforts are being led to improve theoretical modeling and help bridging the gap with experimental results (i.e. [BEST](#), [MUSES](#), [ELEMENTS](#)...)

BACKUP

BACKUP SLIDES

- Limitation of Taylor expansion
- Other phase diagrams (eB / isospin)
- Bayesian analysis on EoS + UrQMD (Manjunath @ SQM)



LATTICE QCD HIGH-ORDER SUSCEPTIBILITIES

LATTICE QCD – THE NATURE OF THE TRANSITION

One can simulate lattice QCD at purely imaginary chemical potential, and determine the transition line by looking at:

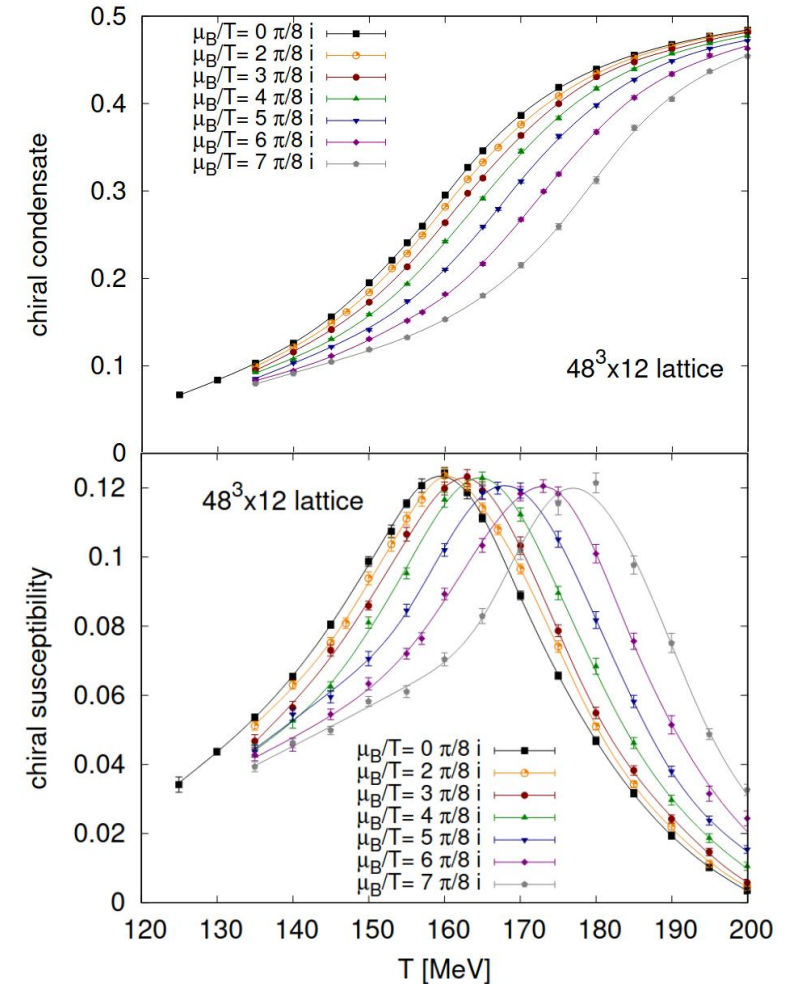
- inflection point of the chiral condensate $\langle \bar{\psi}\psi \rangle = \frac{T}{V} \frac{\partial \ln Z}{\partial m_{ud}}$

- peak of the chiral susceptibility $\chi = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial m_{ud}^2}$

→ smooth crossover from hadron gas to QGP
(no discontinuity in 1st and 2nd order derivatives)

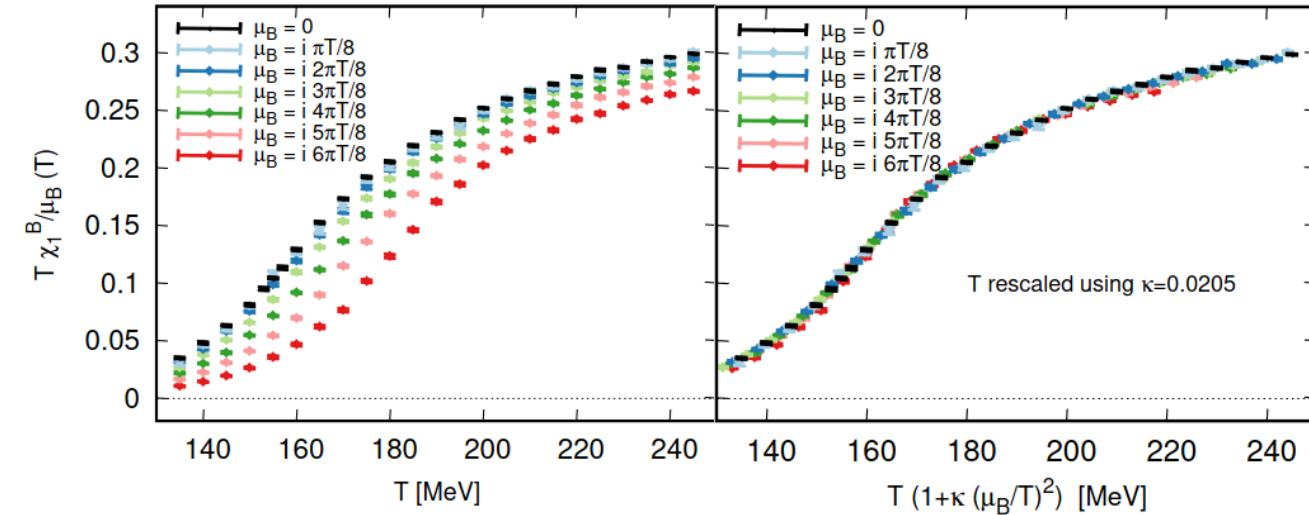


Hadrons basically melt like butter



2D ISING-T.EX.S EQUATION OF STATE FROM L-QCD

Range: $30 \text{ MeV} < T < 800 \text{ MeV}$; $\mu_{B/Q/S} < 700 \text{ MeV}$



one can thus redefine temperature and use an alternative expansion scheme:

$$T'(T, \hat{\mu}_B) = T \left(1 + \kappa_2^{BB}(T) \hat{\mu}_B^2 + \kappa_4^{BB}(T) \hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^6) \right)$$

with alternative expansion coefficients κ , related to susceptibilities:

$$\kappa_2^{BB}(T) = \frac{1}{6T} \frac{\chi_4^B(T)}{\chi_2^{B'}(T)} \quad \kappa_4^{BB}(T) = \frac{1}{360 \chi_2^{B'}(T)^3} \left(3 \chi_2^{B'}(T)^2 \chi_6^B(T) - 5 \chi_2^{B''}(T) \chi_4^B(T)^2 \right)$$

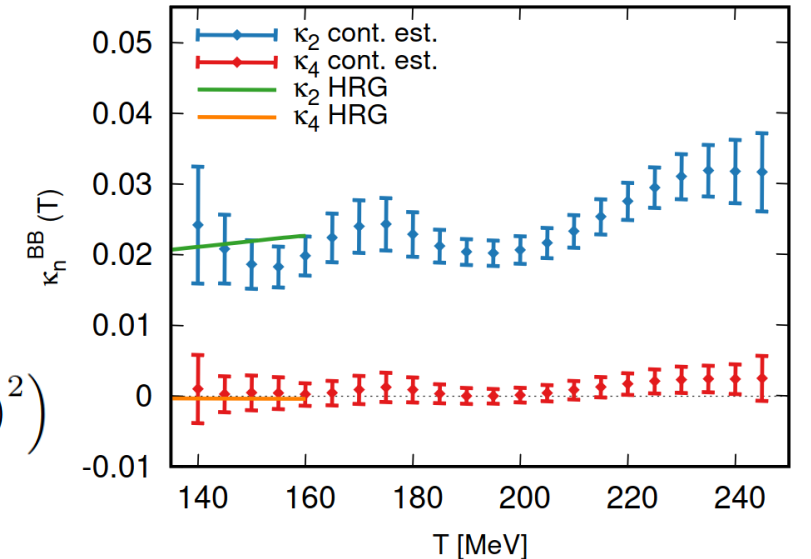
Empirical observation:

- all 1st order susceptibilities scale when defining a μ_B -dependent temperature $T'(T, \mu_B)$

- scales like:

$$\frac{\chi_1^B(T, \hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T', 0)$$

Main identity



2D ISING-T.EX.S EQUATION OF STATE FROM L-QCD

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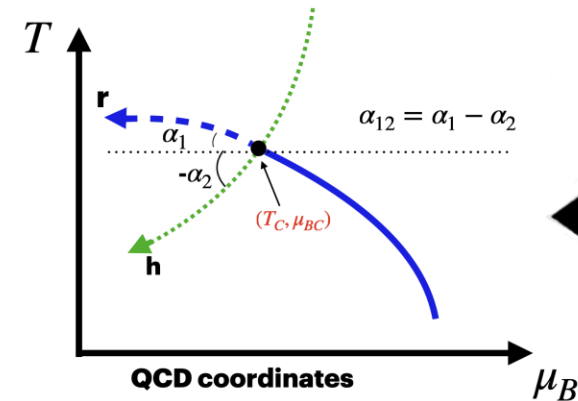
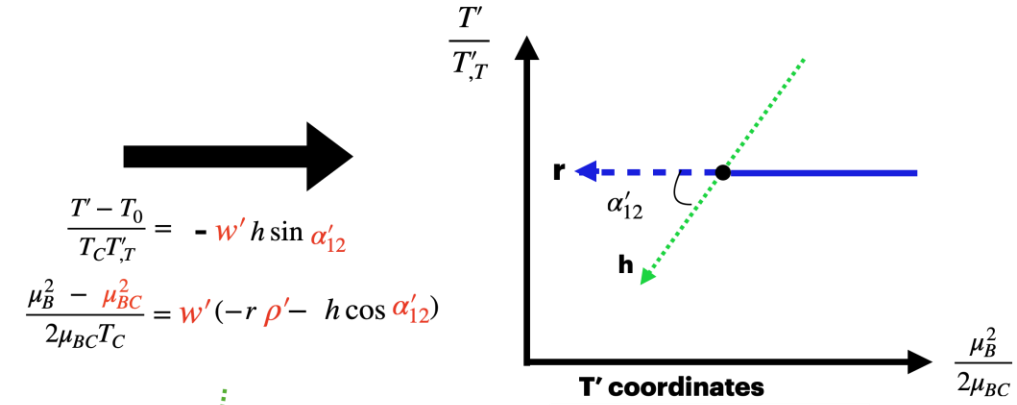
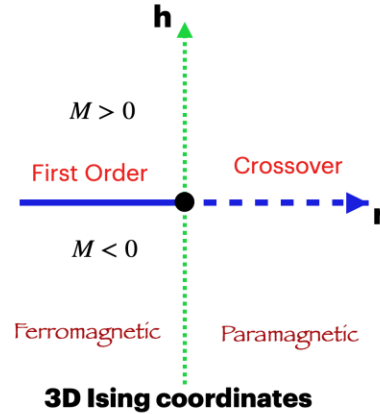
Implement scaling behavior of 3D-Ising model EoS:

- Define map from 3D-Ising model to QCD
- Estimate contribution to Taylor coefficients from 3D-Ising model critical point

- Reconstruct full baryon density $\frac{n_B^{full}(T, \mu_B)}{(\mu_B/T)T^3} = \chi_{2,lattice}^B(T'_{full}, 0)$

with

$$T'_{full}(T, \mu_B) = \underbrace{T'_{lattice}(T, \mu_B)}_{\text{lowest orders in } (\mu_B/T)} + \underbrace{T'_{crit}(T, \mu_B) - Taylor[T'_{crit}(T, \mu_B)]}_{\text{higher order in } (\mu_B/T)}$$



$$T = T \left[1 + \left(\frac{\mu_B}{T} \right)^2 \kappa_2^{BB}(T) + \mathcal{O} \left(\frac{\mu_B}{T} \right)^4 \right]$$

$$T'_{crit}(T, \mu_B) \approx \left(\frac{\partial \chi_{2,lat}^B(T, 0)}{\partial T} \Big|_{T_0} \right)^{-1} \frac{n_B^{crit}(T, \mu_B)}{(\mu_B/T)}$$

4D-T.Ex.S EQUATION OF STATE FROM L-QCD

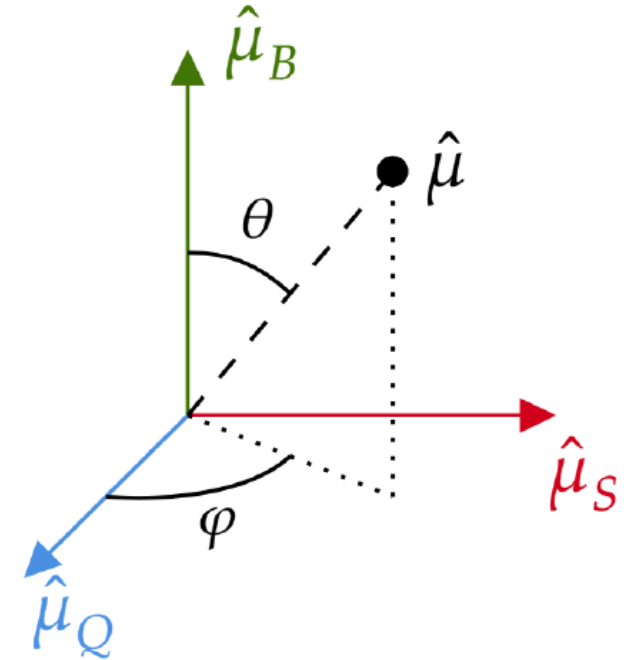
Range: $T < 800 \text{ MeV}$; $\mu_{B/Q/S} <? 700 \text{ MeV}$

- Generalization of the previous 2D T' -Expansion Scheme to 3 conserved charges by projecting the "cartesian" (μ_B, μ_Q, μ_S) coordinates to spherical ones

$$\hat{\mu} = \sqrt{\hat{\mu}_B^2 + \hat{\mu}_Q^2 + \hat{\mu}_S^2}$$

$$\begin{aligned}\hat{\mu}_B &= \hat{\mu} \cdot \cos(\theta) \\ \hat{\mu}_Q &= \hat{\mu} \cdot \sin(\theta) \cos(\varphi) \\ \hat{\mu}_S &= \hat{\mu} \cdot \sin(\theta) \sin(\varphi)\end{aligned}$$

→ still a 2D-TExS expansion, along a constant μ/T line



- Calculate expansion coefficient λ_2 based on so-called "generalized susceptibilities" $X_{2/4}$ (linear combinations of lattice QCD susceptibilities) + their Stefan-Boltzmann limits

$$\lambda_2^{\theta, \varphi}(T) = \frac{1}{6T} \frac{1}{X_2^{\prime \theta, \varphi}(T)} \times \left(X_4^{\theta, \varphi}(T) - \frac{\bar{X}_4^{\theta, \varphi}(0)}{\bar{X}_2^{\theta, \varphi}(0)} X_2^{\theta, \varphi}(T) \right)$$

$$\begin{aligned}X_2^{\theta, \varphi}(T) &= c_\theta^2 \cdot \chi_2^B(T) + s_\theta^2 c_\varphi^2 \cdot \chi_2^Q(T) + s_\theta^2 s_\varphi^2 \cdot \chi_2^S(T) + \dots \\ X_4^{\theta, \varphi}(T) &= c_\theta^4 \cdot \chi_4^B(T) + s_\theta^4 c_\varphi^4 \cdot \chi_4^Q(T) + s_\theta^4 s_\varphi^4 \cdot \chi_4^S(T) + \dots\end{aligned}$$

4D-T.Ex.S EQUATION OF STATE FROM L-QCD

Range: $T < 800 \text{ MeV}$; $\mu_{B/Q/S} <? 700 \text{ MeV}$

- Compute the "generalized charge density" X_1 along the projected line using the expanded temperature T' and the T.Ex.S main identity (modified to match with Stefan-Boltzmann limit at $T \rightarrow \infty$)

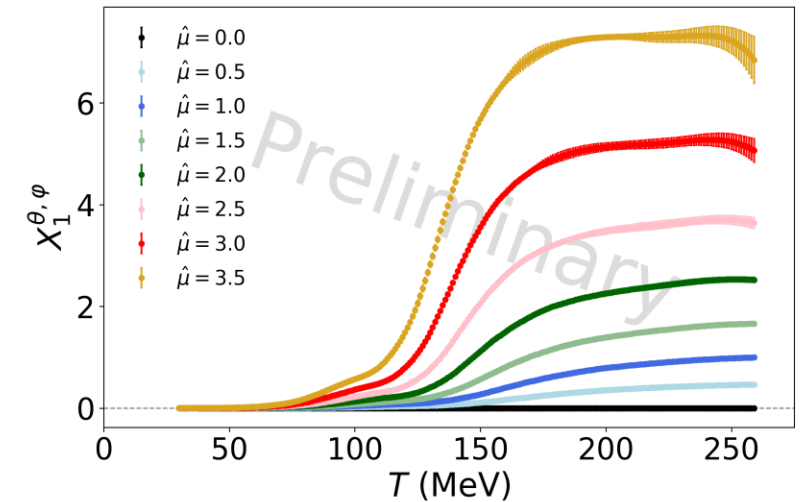
$$X_1^{\theta,\varphi}(T, \hat{\mu}) = \frac{\bar{X}_1^{\theta,\varphi}(\hat{\mu})}{\bar{X}_2^{\theta,\varphi}(0)} \times X_2^{\theta,\varphi}(T'^{\theta,\varphi}(T, \hat{\mu}), 0)$$

$$\text{with } T'^{\theta,\varphi}(T, \hat{\mu}) = T \left(1 + \lambda_2^{\theta,\varphi}(T) \hat{\mu}_B^2 \right)$$

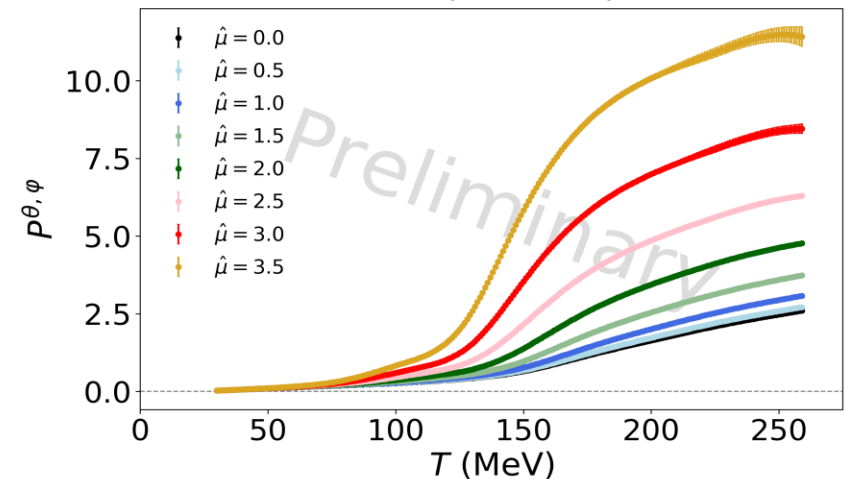
- Obtain pressure by integrating X_1 , allowing then to compute all thermodynamics

$$P^{\theta,\varphi}(T, \hat{\mu}) = P(T, 0) + \int_0^{\hat{\mu}} X_1^{\theta,\varphi}(T, \hat{\mu}') d\hat{\mu}'$$

$X_1(T)$ for $\theta = 90^\circ, \varphi = 90^\circ$ (μ_S direction)

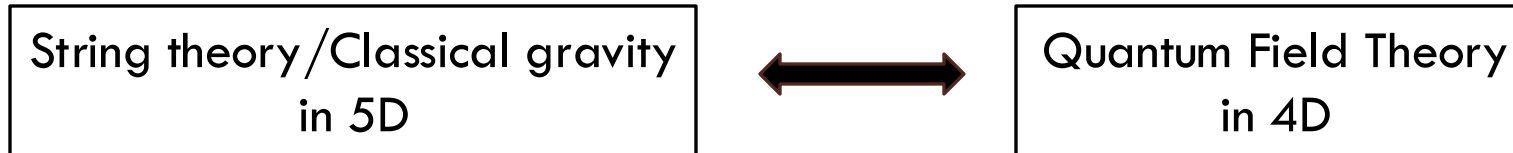


$P(T)$ for $\theta = 90^\circ, \varphi = 90^\circ$ (μ_S direction)



EQUATION OF STATE FROM HOLOGRAPHY

Range: $30 \text{ MeV} < T < 400 \text{ MeV}$; $\mu_B < 1100 \text{ MeV}$



By solving the equations of motion (EoM) for a 5D Einstein-Maxwell-Dilaton (EMD) model defined by the following action:

$$S = \frac{1}{2\kappa_5^2} \int_{\mathcal{M}_5} d^5x \sqrt{-g} \left[R - \frac{(\partial_\mu \phi)^2}{2} - V(\phi) - \frac{f(\phi) F_{\mu\nu}^2}{4} \right]$$

(simplest action reproducing a realistic 4D QCD EFT)

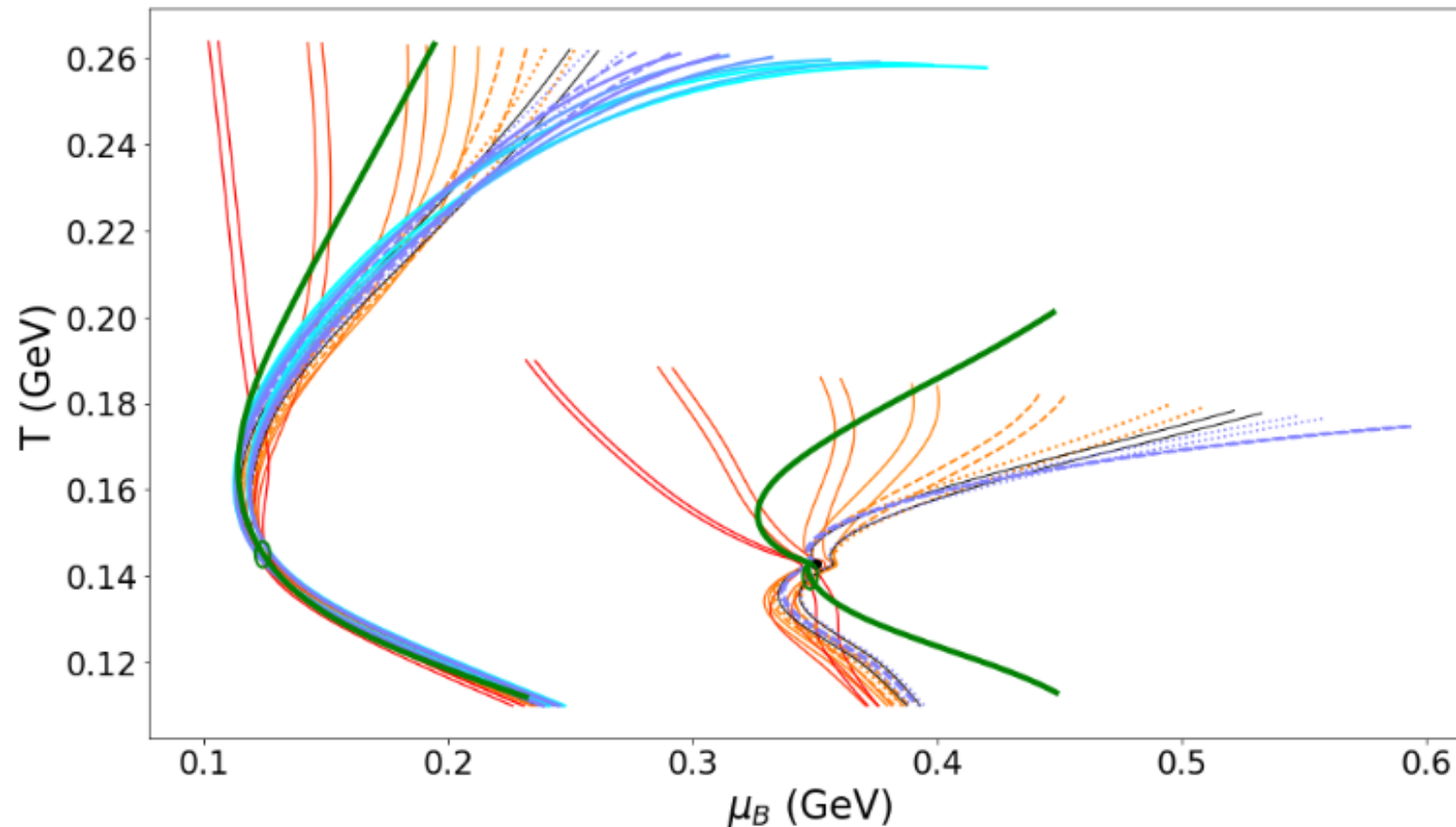
...one can obtain the following thermodynamic quantities by

- using the UV behavior of the EMD fields
- fixing free parameters Λ , κ_5 and the functional form of $V(\phi)$ and $f(\phi)$ by matching with IQCD results at $\mu_B = 0$

$$T = \frac{1}{4\pi \phi_A^{1/\nu} \sqrt{h_0^{\text{far}}}} \Lambda \quad s = \frac{2\pi}{\kappa_5^2 \phi_A^{3/\nu}} \Lambda^3$$

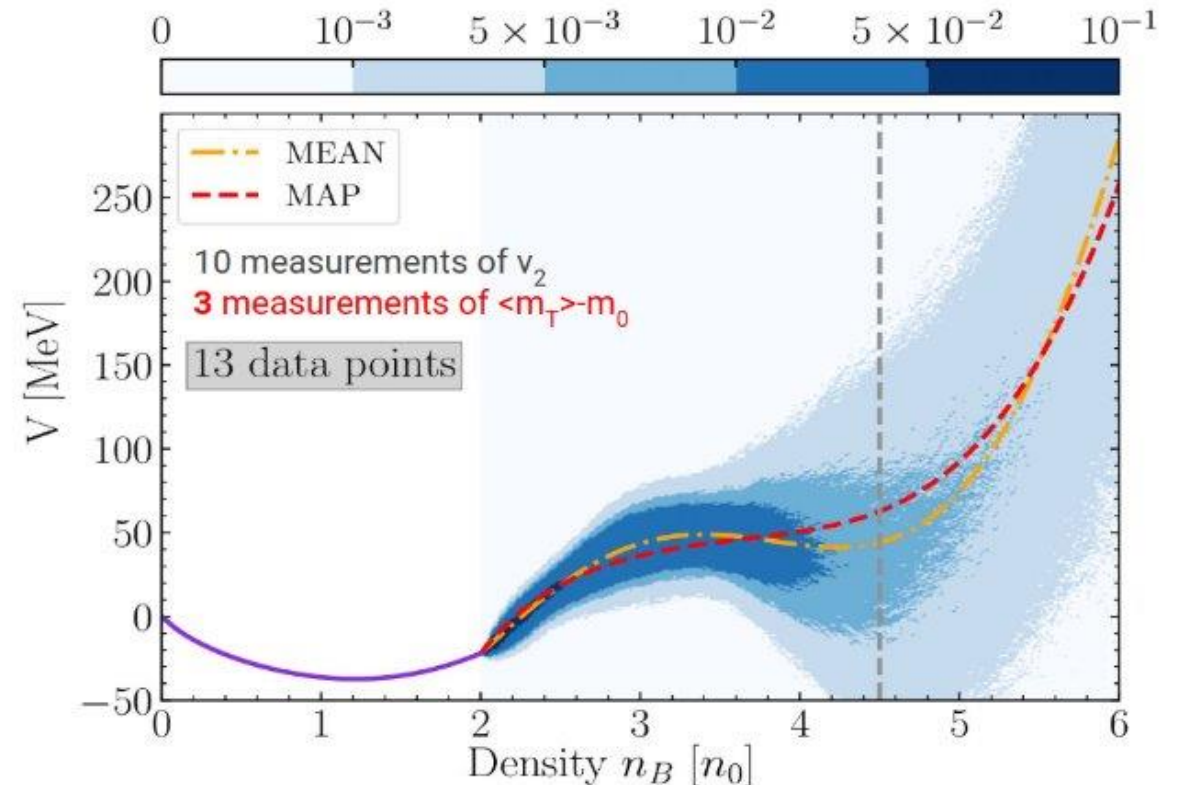
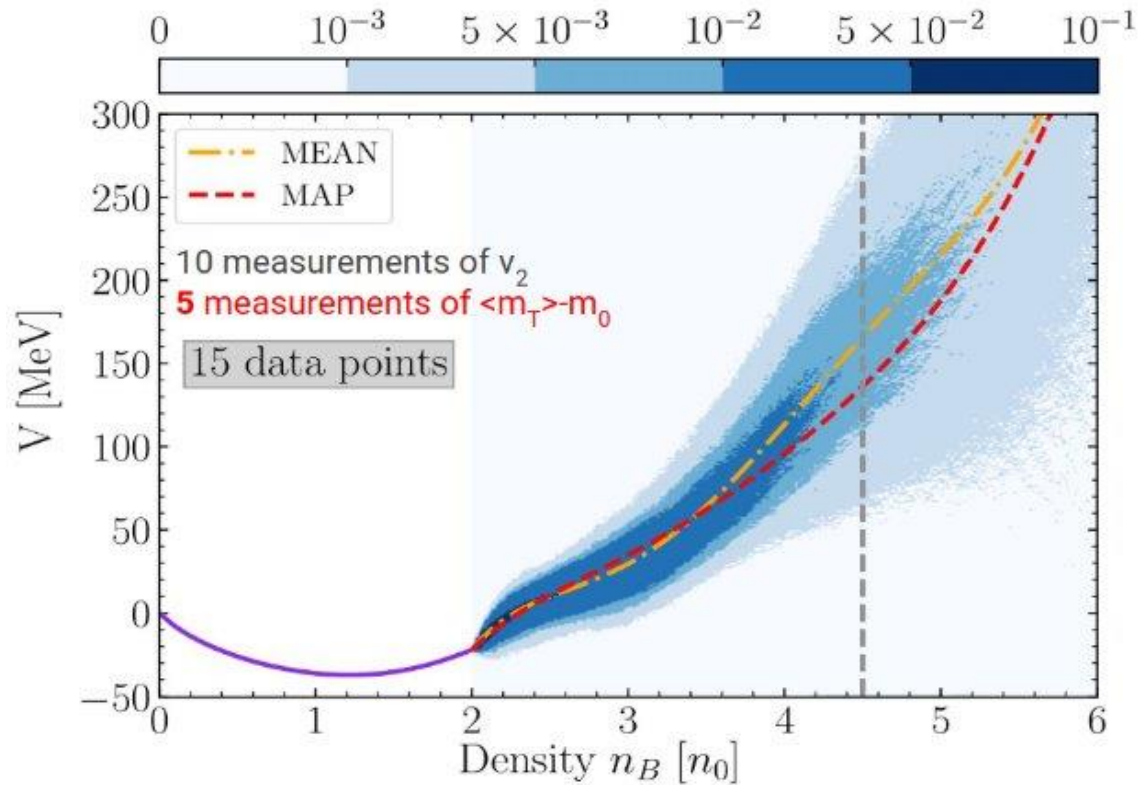
$$\mu_B = \frac{\phi_0^{\text{far}}}{\phi_A^{1/\nu} \sqrt{h_0^{\text{far}}}} \Lambda \quad \rho_B = -\frac{\phi_2^{\text{far}}}{\kappa_5^2 \phi_A^{3/\nu} \sqrt{h_0^{\text{far}}}} \Lambda^3$$

TRAJECTORIES ACROSS THE PHASE DIAGRAM



BAYESIAN INFERENCE OF THE EOS THROUGH HIC

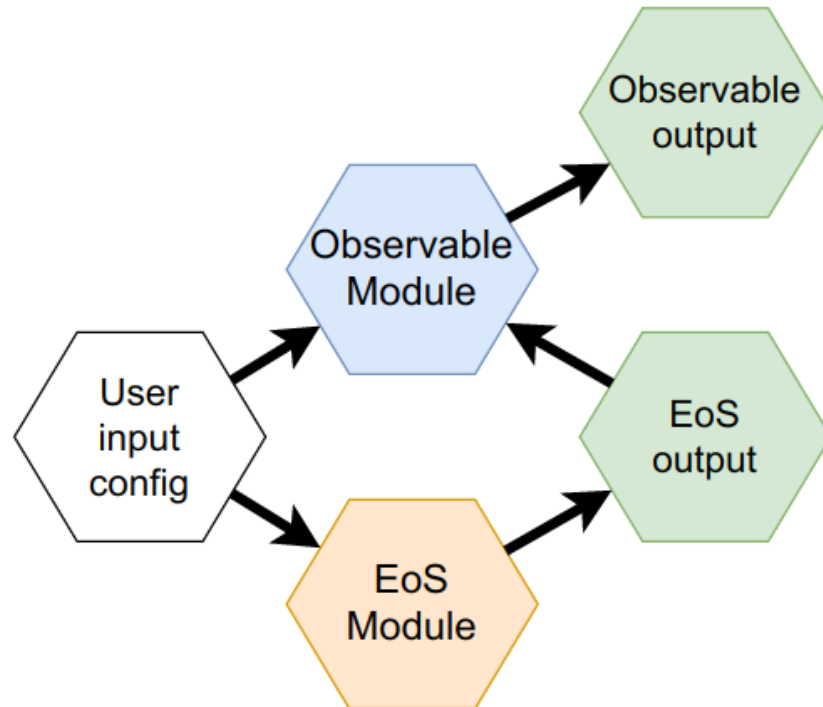
M.O. Kuttan, *Strangeness in Quark Matter 2024*



Simulation achieved with UrQMD at different collision energies, using CMF equations of state.

WORKFLOWS IN MUSES

- Example of a typical workflow within MUSES, implying EOS generation + observable calculation



- More complex workflows can also be defined

