A three-fluid dynamical model simulations at FAIR energies

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Fluid dynamical description at energies of few GeV per NN pair

Why running hydro at low energies:

- to get theoretical description of excitation functions for various observables
- to have description of bulk dynamics close to the critical point (needs also fluctuations)

Issues with hydro at lower energies:

- no separation of scales with the initial conditions—the passage time of two nuclei is not order of magnitude shorter than the evolution of the fireball
- non-zero baryon (and other charges) density

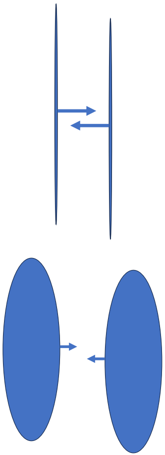
A formulation of the initial conditions

High-energy paradigm:

- separate initial conditions from the evolution
- works for 'thin pancakes' and passage times of the order 10^{-1} fm/c and less
- may have some hydrodynamisation phase included (how soon is the hydro applicable)

(Direct) applications to lower collision energies:

- Pick some model (parametrisation) for the initial conditions and start the hydro later (after the passage time)
  - does not catch the dynamics of the early phase - the most dense phase
- Run transport model at the beginning and convert it to fluid later
  - a lot of dynamical evolution happens during the initial transport phase
  - there may be very dense spots in the initial phase, that would ask for hydro description



Initial conditions integrated with the fluid dynamical evolution

Start hydrodynamics early!

Dynamical initialisation

Energy and momentum from particles gradually transferred into the fluid, when energy density is high.
To solve: how to treat initial state with non-fluid and fluid parts?

Multi-fluid hydrodynamics

- Fluid-dynamical description from the very beginning
- Three fluids represent non-zero regions of the kinetic distribution function

To solve: how to describe the initial state with the fluid

MUlti Fluid simulation for Fast IoN collisions (MUFFIN): the scheme

1. Initialisation

Create two incoming fluids from sampled nucleons within two nuclei.

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Evolve the fluids.

Transfer energy where fluids pass each other.

Use vHLLE.

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3. Particlisation

Determine the particlisation hypersurface with CORNELIUS.

Convert energy and momentum to hadrons.

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4. Hadron cascade

Evolve hadrons with SMASH.

Decay resonances

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Novelties in MUFFIN:

- Equation of State can be modified by exchanging the module
- use of hyperbolic coordinates allows simulating higher collision energies
- viscosity (can be) included

Initial setup: sampling of the incoming fluids

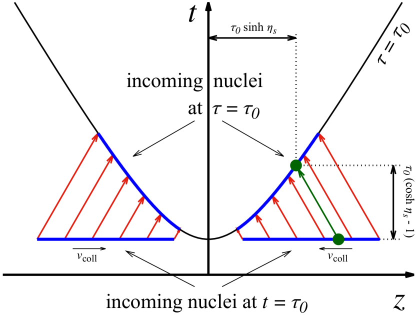
- incoming nucleons sampled along $t = \text{const.}$ hypersurface
- nucleons propagated to $\tau = \tau_0$ hyperbola
- nucleons smeared into the two fluids with the kernel

Makes E-by-E fluctuations possible!

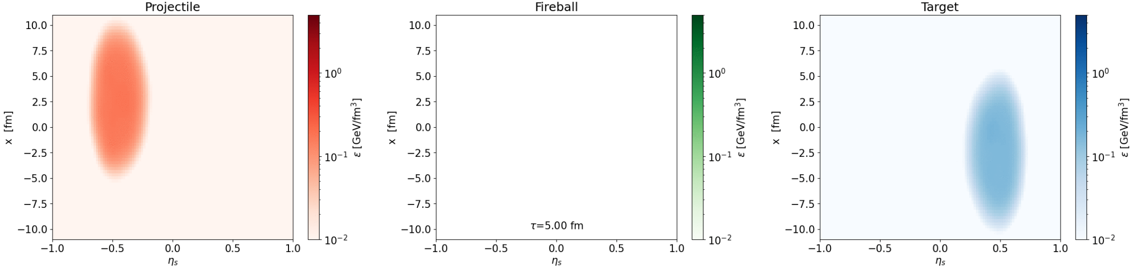
$$K(\Delta x, \Delta y, \Delta\eta_s) =$$
$$A \exp\left(-\frac{\Delta x^2 + \Delta y^2 + \Delta\eta_s^2\tau^2\cosh^2\eta_s\cosh^2 \mathrm{y}}{2\sigma^2}\right)$$

$$T^{0\mu}(x_{\text{cell}}, y_{\text{cell}}, \eta_{\text{cell}}) = \sum_{i\in\text{nucleons}} p_i^\mu K(\Delta x, \Delta y, \Delta\eta_s)$$

$$N_b^0(x_{\text{cell}}, y_{\text{cell}}, \eta_{\text{cell}}) = \sum_{i\in\text{nucleons}} B_i K(\Delta x, \Delta y, \Delta\eta_s)$$



An illustration of the evolution



Friction between the fluids

The original idea behind the treatment: nucleons collide and by loosing energy produce pions.
Transfer of energy and momentum, but no transfer of baryon number and charge.
Adopt the idea from Ivanov et al. (2006)
Energy and momentum transfer between the fluids

$$\partial_\mu T^{\mu\nu}_{\mathrm{p}}(x) = -F^\nu_{\mathrm{p}}(x) + F^\nu_{\mathrm{fp}}(x),$$
$$\partial_\mu T^{\mu\nu}_{\mathrm{t}}(x) = -F^\nu_{\mathrm{t}}(x) + F^\nu_{\mathrm{ft}}(x),$$
$$\partial_\mu T^{\mu\nu}_{\mathrm{f}}(x) = F^\nu_{\mathrm{p}}(x) + F^\nu_{\mathrm{t}}(x) - F^\nu_{\mathrm{fp}}(x) - F^\nu_{\mathrm{ft}}(x),$$

Total energy conservation

$$\partial_\mu \left[T^{\mu\nu}_{p}(x) + T^{\mu\nu}_{t}(x) + T^{\mu\nu}_{f}(x) \right] = 0$$

Y. B. Ivanov, V. N. Russkikh, and V. D. Toneev, Phys. Rev. C 73, 044904 (2006), [nucl-th/0503088]

The friction terms [Ivanov et al. (2006)]

Projectile-target friction: $F_{\alpha}^{\nu} = \vartheta^2 \rho_p^{\xi} \rho_t^{\xi} m_N V_{\text{rel}}^{pt} [(u_{\alpha}^{\nu} - u_{\bar{\alpha}}^{\nu}) \sigma_P(s_{pt}) + (u_p^{\nu} + u_t^{\nu}) \sigma_E(s_{pt})]$

ϑ limits friction for close cell velocities; m_N is the nucleon mass; V_{rel}^{pt} is relative velocity;
u_{α}^{ν} is projectile or target cell velocity;
$\sigma_P(s_{pt})$, $\sigma_E(s_{pt})$ are cross-sections for momentum and energy transfer;
tunable s_{pt} dependence of effective densities:

$$\rho_{\alpha}^{\xi}(s_{pt}) = \begin{cases} \rho_{\alpha}^{b} \xi_h(s_{pt}) & \varepsilon_{\alpha} < 0.7\ \text{GeV}/\text{fm}^3, \\ \frac{1}{3}\left(\rho_{\alpha}^{q} + \rho_{\alpha}^{g}\right) \xi_q(s_{pt}) & \varepsilon_{\alpha} > 0.7\ \text{GeV}/\text{fm}^3. \end{cases}$$

Projectile(target)-fireball friction: $F_{f\alpha}^{\nu} = \rho_{\alpha}^{b} \xi_{f\alpha}(s_{f\alpha}) V_{\text{rel}}^{f\alpha} \frac{T_{f(eq)}^{0\nu}}{u_f^0} \sigma_{tot}^{N\pi \to R}(s_{f\alpha})$

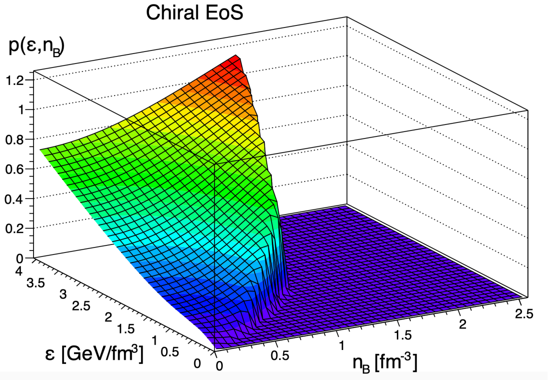
Energy dependence of the cell-cell friction (inferred from comparisons to data):

$$\xi_h = 1.8\sqrt{\frac{2m_N}{\sqrt{s_{pt}}}}, \qquad \xi_q = 30\sqrt{\frac{2m_N}{\sqrt{s_{pt}}}}, \qquad \xi_{f\alpha} = 0.15\frac{m_N^2}{s_{f\alpha}},$$

Equation of State

Chiral model Equation of State used in this paper
[J. Steinheimer *et al.*, J. Phys. G 38 (2011) 035001]

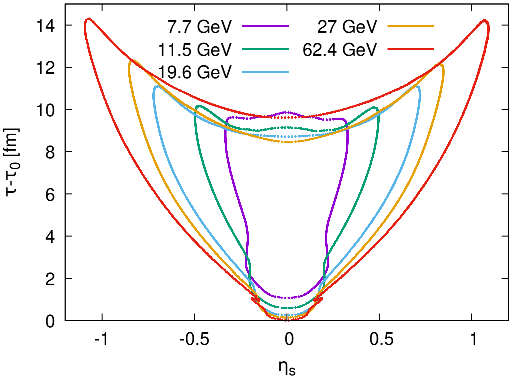
- good agreement with lattice QCD at $\mu_B = 0$
- crossover type phase transition do deconfined phase at all μ_B



Equation of State can be easily exchanged.
E.g. Hadron resonance gas plus bag model

The particlisation hypersurface

- determine "effective energy" ε_{sw} from diagonalised $T^{p}_{\mu\nu} + T^{f}_{\mu\nu} + T^{t}_{\mu\nu}$
- set one particlisation hypersurface for all fluids where $\varepsilon_{sw} = 0.5$ GeV/fm^3 [CORNELIUS]
- vanishing total energy flux through the (closed) FO hypersurface
- exclude from particlisation cells where energy flows in
- hadron sampling by SMASH hadron sampler



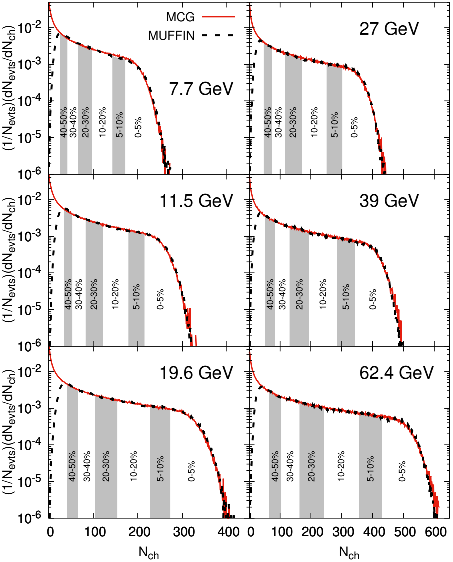
Centrality determination

- (Semi-)minimum bias in MUFFIN ($0 < b < 12$ fm) (MUFFIN not suitable for very peripheral collisions)
- two component Monte Carlo Glauber model

$$\frac{\mathrm{d}N_{\mathrm{ch}}}{\mathrm{d}\eta} = n_{pp}\left[(1-x)\frac{\langle N_{\mathrm{part}}\rangle}{2} + x\langle N_{\mathrm{coll}}\rangle\right]$$

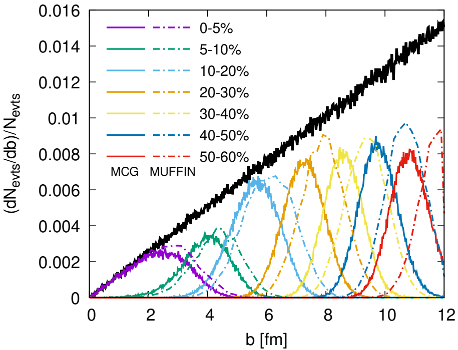
$$P_{\mathrm{NBD}}(n_{pp}, k; n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\frac{(n_{pp}/k)^n}{(n_{pp}/k+1)^{n+k}}$$

- match the normalisation of curves from MUFFIN and MCG, to account for missing peripheral collisions in MUFFIN



Determination of the impact parameter: MUFFIN vs. MC Glauber model

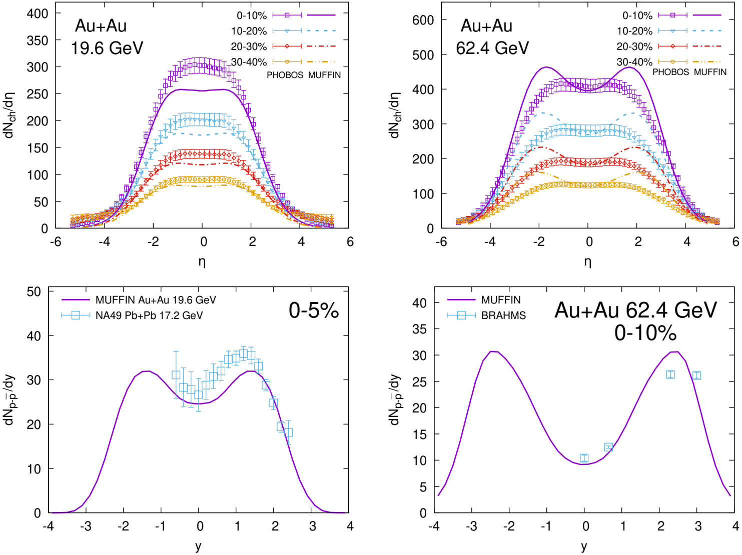
- at the same centrality class, MUFFIN gives larger impact paramater
- different models at the same centrality class
  - give different impact parameters
  - give different eccentricities
  - may lead to different predictions for flow anisotropies



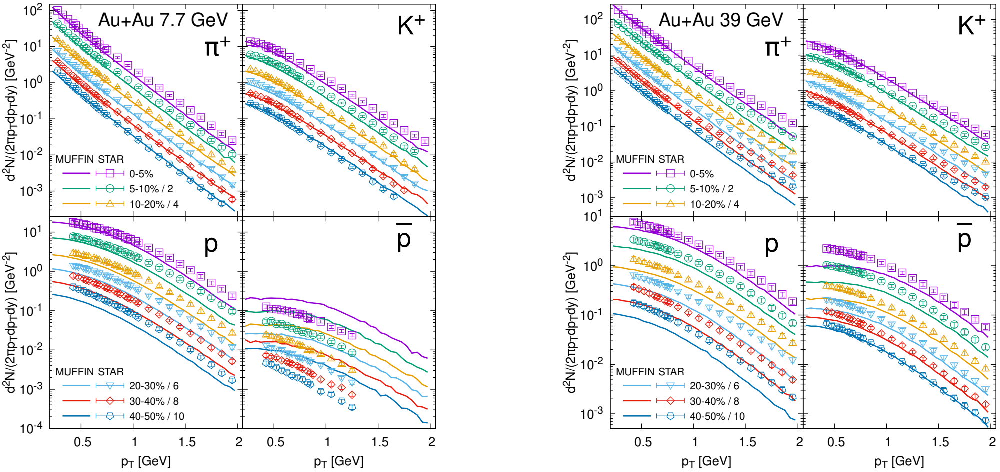
Results: η distributions of all charged hadrons and net protons

Tuned with the help of scaling factors in the friction terms $\xi_h(\sqrt{s_{pt}})$, $\xi_q(\sqrt{s_{pt}})$, $\xi_{f\alpha}(\sqrt{s_{pt}})$.

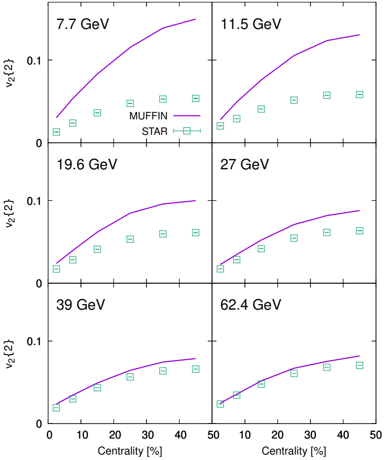
For each energy:
3000 hydro events
$\times$ 500 hadron transport oversampling

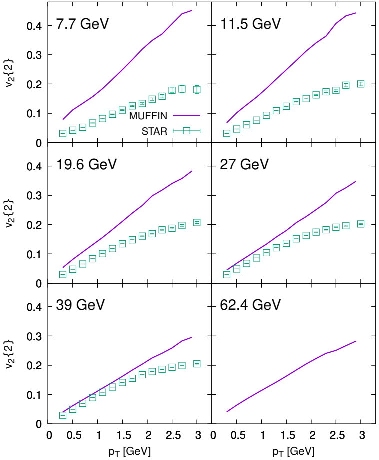


Results: transverse momentum spectra (selected)



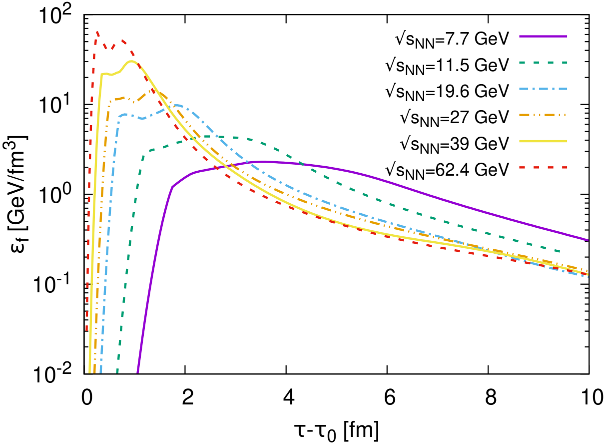
Results: elliptic flow





Viscosity has not yet been included! It tends to lower the elliptic flow.

Energy density in the central cell



Prolonged maximum at lower energy due to longer interpenetration of the nuclei.

Summary

- MUFFIN: a new generation three-fluid dynamical model
  - modular EoS—can be used to study the influence of EoS
  - hyperbolic coordinates—can be used up to higher energies
  - fluctuating initial state
  - (can include) viscosity
- tuned to reproduce bulk data from 7.7 – 62.4 GeV
- next steps—work in progress:
  - stopping of baryon number
  - viscosity
  - . . .

[J. Cimerman, I. Karpenko, B. Tomášik, P. Huovinen, Phys. Rev C **107** (2023) 044902 [2301.11894]]

BACKUP

Formulas for the friction terms

Unification factor:

$$\vartheta = 1 - \exp\left[-(V_{\mathrm{rel}}^{pt}/\Delta V)^4\right]$$

Relative velocities of fluid cells:

$$V_{\mathrm{rel}}^{pt} = \frac{\sqrt{s_{pt}(s_{pt} - 4m_N^2)}}{2m_N^2}, \qquad V_{\mathrm{rel}}^{f\alpha} = \frac{\sqrt{(s_{f\alpha} - m_N^2 - m_\pi^2)^2 - 4m_N^2 m_\pi^2}}{2m_N m_\pi}$$

centre-of-mass energies for fluid cells:

$$s_{pt} = m_N^2(u_p^\nu + u_t^\nu)^2, \qquad s_{f\alpha} = (m_\pi u_f + m_N u_\alpha)^2$$

densities of massless quarks and gluons:

$$\rho_\alpha^q = \frac{18\zeta(3)}{\pi^2} T^3 + 2\mu_q^3, \qquad \rho_\alpha^g = \frac{16\zeta(3)}{\pi^2} T^3$$