Hybrid Approaches for the Star BES energy range

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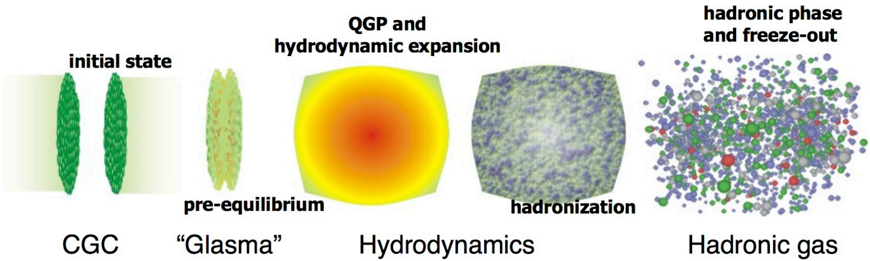
Status quo at high energies (LHC or top RHIC)

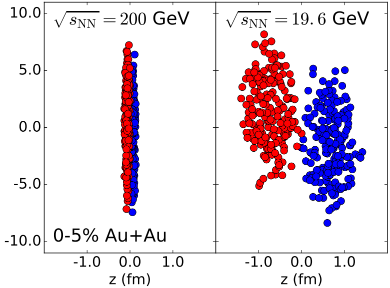
There is a relatively clear time separation between the initial state and the fluid stage.



When foraging into lower energies using the same tools:

The paradigm of “thin pancakes” gradually loses its applicability.

- There is no boost invariance
- Baryon and electric charge densities are significant
- **Nuclei pass through each other slowly**
  (the passage can last as long as subsequent fluid stage)
- There is no clear separation of the initial state and the fluid stage.

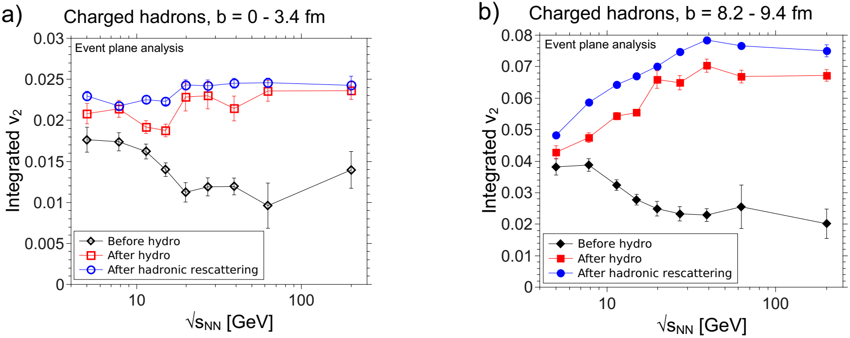


picture credit: C. Shen, B. Schenke, Phys. Rev. C 97, 024907 (2018)

From the last two bullet points:

A lot of evolution is happening before the nuclei have completely passed through each other.

Simulation: UrQMD IS + ideal hydro + UrQMD afterburner
J. Auvinen, H. Petersen, Phys.Rev.C 88 (2013), 064908



Not only the initial state model is important, but the way of switching from initial state to fluid-dynamical picture.

Taxonomy of existing hybrids for RHIC BES and FAIR energies

There is abundance of models for $\sqrt{s_{\mathrm{NN}}}$ =7...62 GeV!

Existing hydro/hybrid models can be classified as follows:

Initial state

1. Parametrized initial state + hydro
2. Transport + hydro, switch at fixed t or τ
3. EPOS
4. Transport + hydro, switch dynamically
5. Multi-fluid models

Equation of state

- Chiral model EoS
- BM + HRG EoS
- NEOS-B, NEOS-BQS
- BEST EoS

Final conditions

- Particlization at fixed energy density + hadronic cascade (except for multi-fluid models)

All the models feature 3D initial state, 3D hydro and EoS at finite n_B.

Existing hybrids do not contain criticality, and models with criticality aren't hybrids.

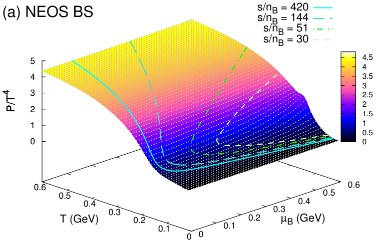
EoS in use (1)

All-crossover cases:

NEOS-B, NEOS-BQ, NEOS-BQS:

- lattice QCD at $\mu_B = 0$
- Tylor expansion in μ_i/T using susceptibilities
- free hadron-resonance gas at low T
- crossover transition at all μ_B

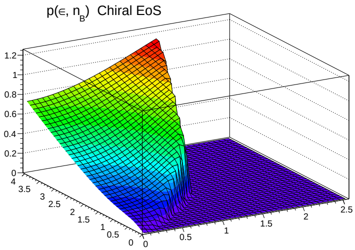
Monnai, Schenke, Shen,
Phys. Rev. C 100, 024907 (2019)



Chiral model EoS

- a single model for hadronic and quark phases
- hadronic SU(3) model
- extension to quark DOF in analogy to PNJL
- crossover transition at all densities

Steinheimer, Schramm, Stocker,
J. Phys. G 38 035001

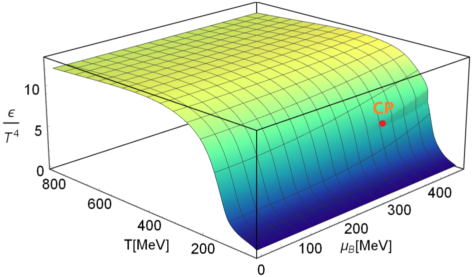


EoS in use (2)

BEST EoS

- lattice QCD at $\mu_B = 0$
- Tylor expansion in μ_i/T using susceptibilities
- critical contribution inspired by a 3D ising model

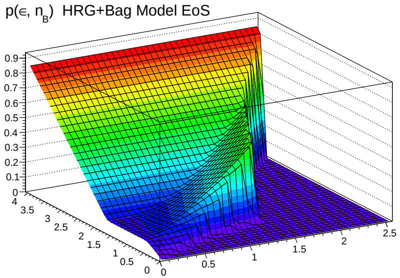
Parotto et al, Phys. Rev. C 101, 034901 (2020)



Bag model + HRG EoS a.k.a. EoS Q

- Bag model for QGP phase
- HRG model with repulsion
- Maxwell construction ⇒ 1st order PT.

P.F. Kolb, et al, Phys.Rev. C 62, 054909 (2000)



Hybrids and their initial states

Type 1: Parametrized initial state

Chun Shen, Sahr Alzhrani, Phys.Rev.C 102 (2020) 1, 014909

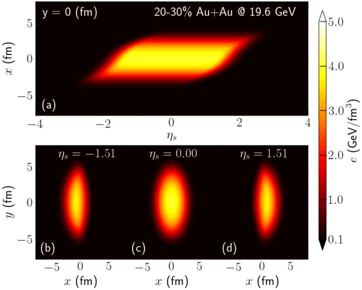
`superMC` (initial state) + `MUSIC` (hydro) + `iSS` (hadron sampling) + `UrQMD`

$$e(x,y,\eta_s;y_{\rm CM}) = \mathcal{N}_e(x,y)\times\exp\left[-\frac{(|\eta_s-y_{\rm CM}|-\eta_0)^2}{2\sigma_\eta^2}\theta(|\eta_s-y_{\rm CM}|-\eta_0)\right].$$

$$\mathcal{N}_e(x,y)=\frac{M(x,y)}{2\sinh(\eta_0)+\sqrt{\frac{\pi}{2}}\sigma_\eta e^{\sigma_\eta^2/2}C_\eta} C_\eta = e^{\eta_0}\mathrm{erfc}\left(-\sqrt{\frac{1}{2}}\sigma_\eta\right)+e^{-\eta_0}\mathrm{erfc}\left(\sqrt{\frac{1}{2}}\sigma_\eta\right).$$

+scaling initial flow, $v_z = z/t$

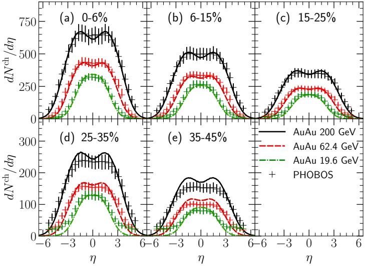
$\sqrt{s_{\rm NN}}$ (GeV)	τ_0 (fm/c)	η_0	σ_η	$\eta_{B,0}$	$\sigma_{B,\rm in}$	$\sigma_{B,\rm ou}$
AuAu & dAu @ 200	1.0	2.5	0.6	3.5	2.0	0.1
AuAu & dAu @ 62.4	1.0	2.25	0.3	2.7	1.9	0.2
AuAu & dAu @ 39	1.3	1.9	0.3	2.2	1.6	0.2
AuAu@27	1.4	1.6	0.3	1.8	1.5	0.2
AuAu & dAu @ 19.6	1.8	1.3	0.3	1.5	1.2	0.2
AuAu@14.5	2.2	1.15	0.3	1.4	1.15	0.2
AuAu@7.7	3.6	0.9	0.2	1.05	1.0	0.1
PbPb@17.3	1.8	1.25	0.3	1.6	1.2	0.2
PbPb@8.77	3.5	0.95	0.2	1.2	1.0	0.1

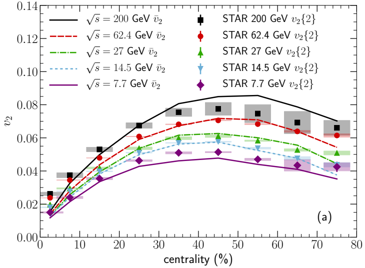


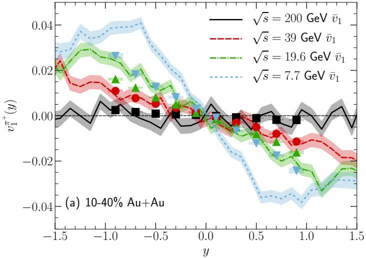
EoS: NEOS-BQS, 1902.05095.

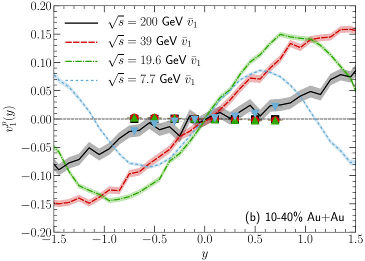
Chun Shen, Sahr Alzhrani, Phys.Rev.C 102 (2020) 1, 014909

Carve the initial state to fit the data.









Type 2: Transport model for initial state

`UrQMD` (initial state) + fluidisation at fixed τ_0 + `vHLLE` (hydro) + `UrQMD`
IK, Huovinen, Petersen, Bleicher, Phys.Rev. C91 (2015) no.6, 064901

- Initial state from $t = 0$ till $\tau = \tau_0$: `UrQMD`
- At $\tau = \tau_0$:
  Gaussian smearing of energy, momentum and charges at fluidization:

$$\Delta P^{\alpha}_{ijk} = P^{\alpha} \cdot C \cdot \exp\left(-(\Delta x_i^2 + \Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2 \gamma_\eta^2 \tau_0^2 / R_\eta^2\right)$$
$$\Delta N^{0}_{ijk} = N^{0} \cdot C \cdot \exp\left(-(\Delta x_i^2 + \Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2 \gamma_\eta^2 \tau_0^2 / R_\eta^2\right)$$

  from each hadron that crosses the $\tau = \tau_0$ surface

- Assume that the resulting energy and momentum corresponds to a flowing fluid:

$$E/\Delta V = (\varepsilon + p)(u^0)^2 - p, \quad P^i/\Delta V = (\varepsilon + p) u^0 u^i$$

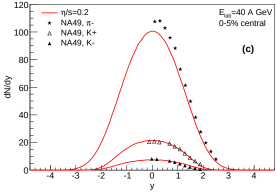
- Run fluid dynamics.
- ...
- compute the observables

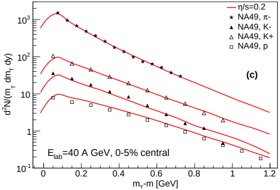
Type 2: Transport model for initial state

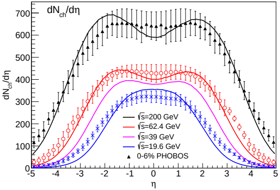
`UrQMD` (initial state) + fluidisation at fixed τ_0 + `vHLLE` (hydro) + `UrQMD`
IK, Huovinen, Petersen, Bleicher, Phys.Rev. C91 (2015) no.6, 064901

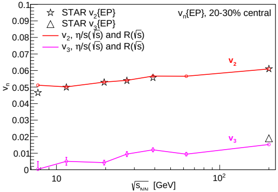
Fix parameters of fluidisation procedure ($R_\perp$, R_η) and shear viscosity η/s to fit the data.

- ⇒ decent agreement with a mix of RHIC BES + NA49 + PHOBOS data









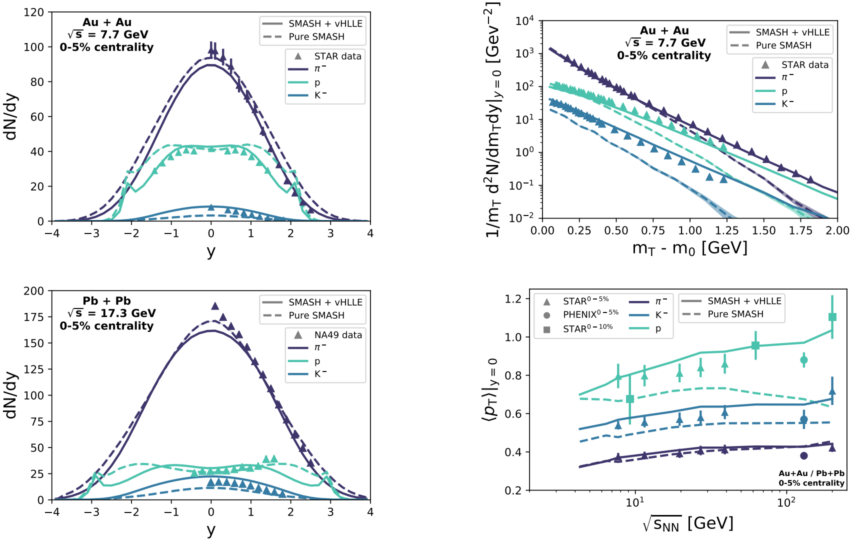
EoS: chiral model EoS.

Type 2: Transport model for initial state

`SMASH` (initial state) + fluidisation at fixed τ_0 + `vHLLE` (hydro) + `SMASH`

A. Schäfer, IK, Xiang-Yu Wu, J. Hammelmann, H. Elfner, Eur.Phys.J.A 58 (2022) 11, 230

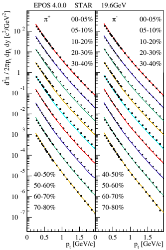
Same set of free parameters.

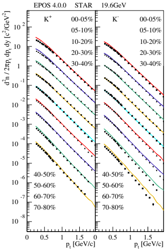


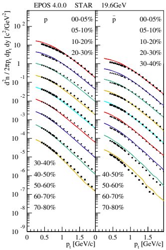
EPOS 4.0.0

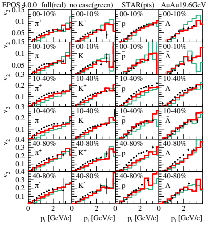
Werner, Jahan, IK, Pierog, Stefaniak, Vintache, 2401.11275

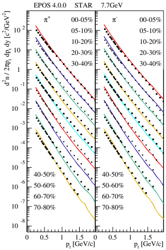
`EPOS` IS + fluidisation at τ_0 + `vHLLE` + `UrQMD`. core-corona included. BEST EoS

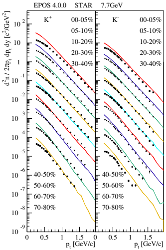


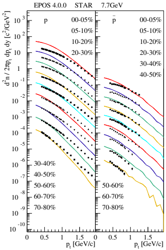


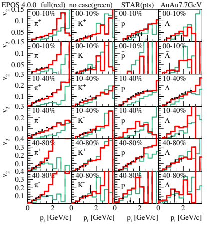








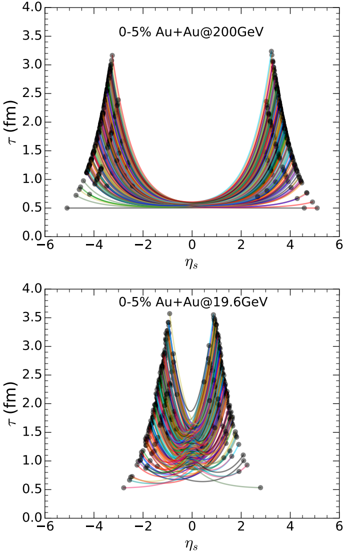




Type 3: Dynamical fluidisation (strings)

Chun Shen, Bjoern Schenke, Phys. Rev. C 97, 024907 (2018)

Decelerated strings melt into fluid, at different proper times. EoS: unclear.



- 1 string per colliding nucleon pair
- The string ends decelerate according to:

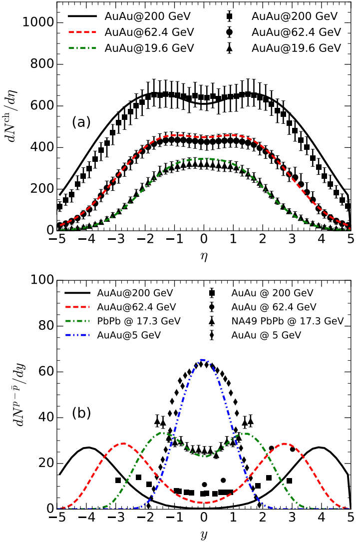
$$\frac{dE}{dz} = -\sigma, \quad \frac{dP_z}{dz} = -\sigma$$

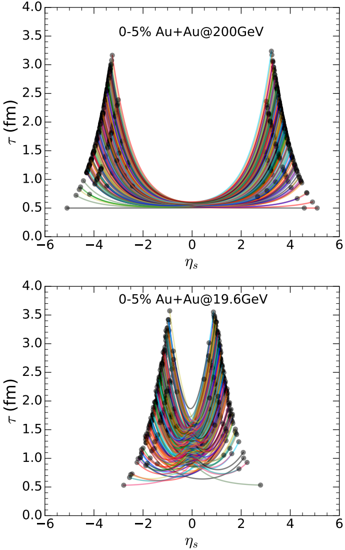
- Once string ends come to a halt, its evolution stops.
- The products fluidize.

Type 3: Dynamical fluidisation (strings)

Chun Shen, Bjoern Schenke, Phys. Rev. C 97, 024907 (2018)

Decelerated strings melt into fluid, at different proper times. EoS: unclear.





Type 3: Dynamical fluidisation

`JAM` + dynamical fluidisation + hydro + `JAM`

Y. Akamatsu, M. Asakawa, T. Hirano, M. Kitazawa, K. Morita, K. Murase, Y. Nara, C. Nonaka, A. Ohnishi, Phys. Rev. C 98, 024909 (2018)

JAM IS: HIJING string excitation + PYTHIA6 fragmentation + rescatterings of produced hadrons.

Hadrons are converted to fluid if the local energy density $e > e_f = 0.5$ GeV/fm^3.

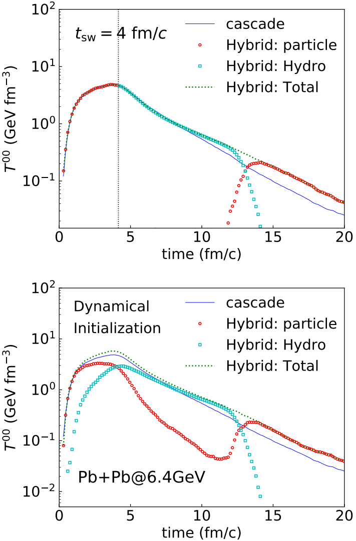
$$\partial_\mu T_f^{\mu\nu} = J^\nu, \quad \partial_\mu N_f^\mu = \rho$$

$$J^\mu(r) = \frac{1}{\Delta t} \sum_i p_i^\mu G(r - r_i(t))$$

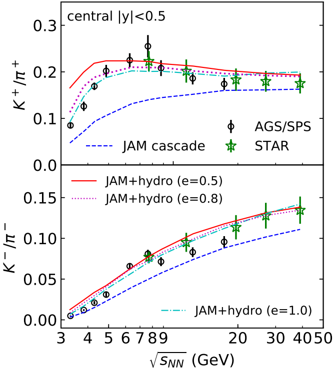
$$\rho(r) = \frac{1}{\Delta t} \sum_i B_i G(r - r_i(t))$$

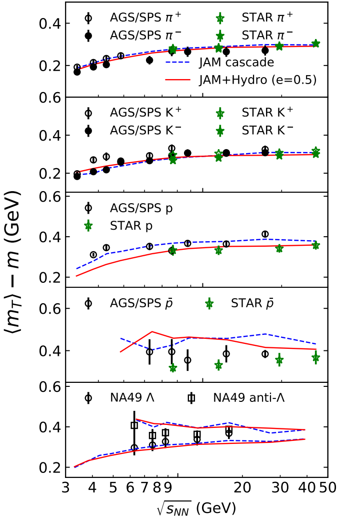
$G(r)$ is a Gaussian smearing profile.

EoS: EoS Q, a.k.a. BM+HRG EoS



`JAM` + dynamical fluidisation + hydro + `JAM`



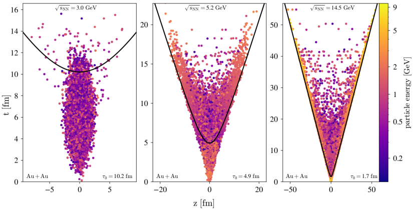


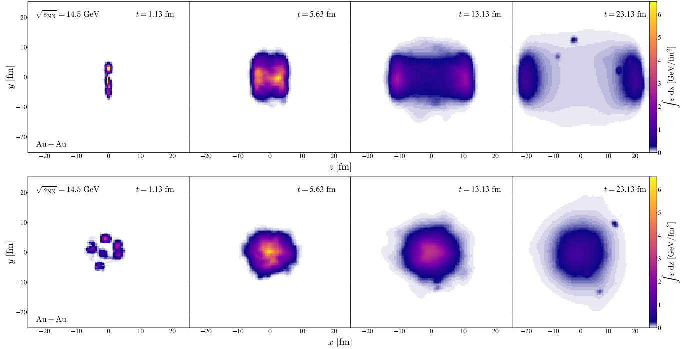
Type 3: Dynamical fluidisation (SMASH cascade)

`SMASH` + dynamical fluidisation + hydro + `SMASH`

WIP: Renan Hirayama, Zuzana Paulínyová, IK, Hannah Elfner

Same idea but with `SMASH` cascade.





Does transport prove the fluid dynamics is applicable?

Not really.

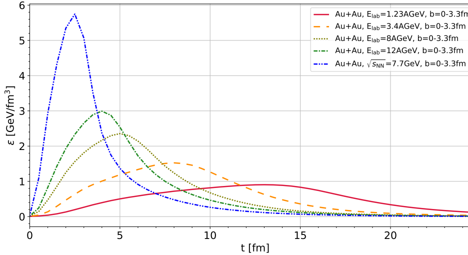
Gabriele Inghirami, Hannah Elfner, Eur. Phys. J. C 82, 796 (2022)

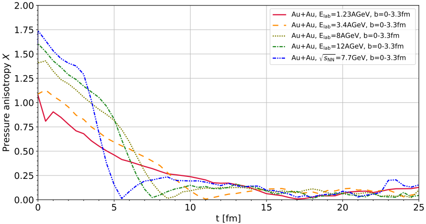
From $E_{\mathrm{lab}} = 1.23$ GeV up to $\sqrt{s} = 7.7$ GeV

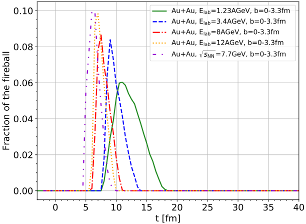
$$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\mathbf{r}-\mathbf{r}_i, \mathbf{p_i})$$

$$X \equiv \frac{|\langle T_L^{11}\rangle - \langle T_L^{22}\rangle| + |\langle T_L^{22}\rangle - \langle T_L^{33}\rangle| + |\langle T_L^{33}\rangle - \langle T_L^{11}\rangle|}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle},$$

$$Y \equiv \frac{3(|\langle T_L^{12}\rangle| + |\langle T_L^{23}\rangle| + |\langle T_L^{13}\rangle|)}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle}$$

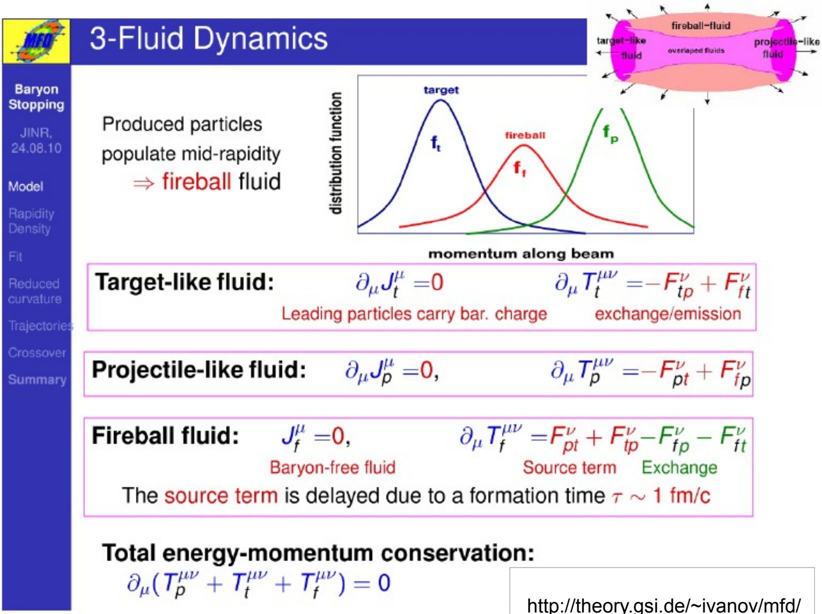






For most of the system, when simulated with transport, $T^{\mu\nu}$ does not look like a fluid.

Type 4: Everything is Fluid



3-Fluid Dynamics

Baryon Stopping

JINR, 24.08.10

Model

Rapidity Density

Fit

Reduced curvature

Trajectories

Crossover

Summary

Produced particles populate mid-rapidity

⇒ fireball fluid

Target-like fluid: $\partial_\mu J_t^\mu = 0$ $\quad \partial_\mu T_t^{\mu\nu} = -F_{tp}^\nu + F_{ft}^\nu$

Leading particles carry bar. charge exchange/emission

Projectile-like fluid: $\partial_\mu J_p^\mu = 0,$ $\quad \partial_\mu T_p^{\mu\nu} = -F_{pt}^\nu + F_{fp}^\nu$

Fireball fluid: $J_f^\mu = 0,$ $\quad \partial_\mu T_f^{\mu\nu} = F_{pt}^\nu + F_{tp}^\nu - F_{fp}^\nu - F_{ft}^\nu$

Baryon-free fluid Source term Exchange

The source term is delayed due to a formation time $\tau \sim 1$ fm/c

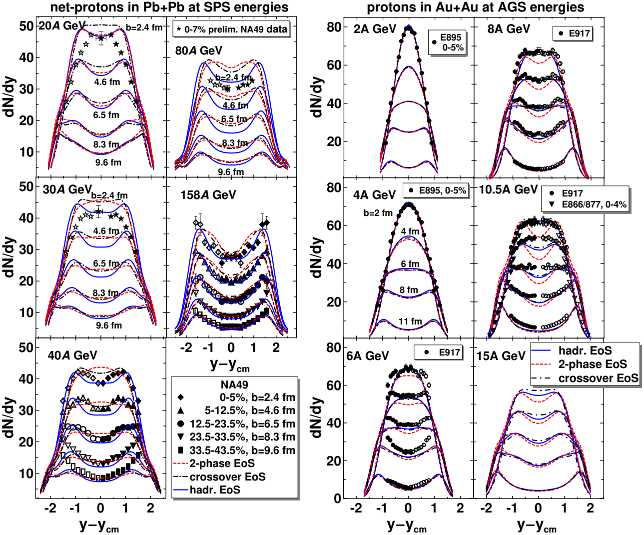
Total energy-momentum conservation:

$$\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0$$

http://theory.gsi.de/~ivanov/mfd/

Yu.B. Ivanov, V.N. Russkikh and V.D. Toneev, Phys. Rev. C73, 044904 (2006)

Yu.B. Ivanov, Phys. Rev. C 87, 064904



What also works:

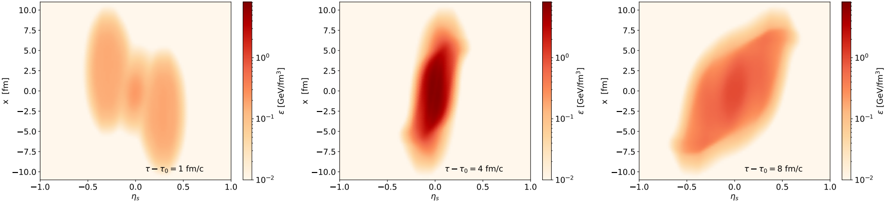
- pion dN/dy
- kaon dN/dp_T
  Yu.B. Ivanov, Phys. Rev. C 87, 064905
- proton, pion dN/dp_T
  Yu.B. Ivanov, Phys. Rev. C 89, 024903 (2014)
- elliptic flow
  Ivanov, Soldatov, Phys. Rev. C 91, 024914 (2015)

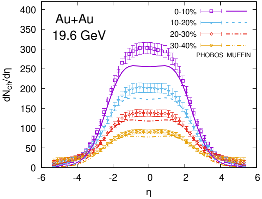
Relatively low sensitivity to the PT type in the EoS; rather worse agreement for purely hadronic EoS.

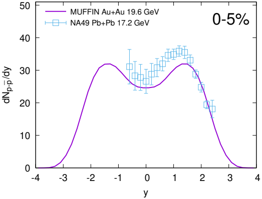
Type 4: Improved 3-fluid dynamics: `MUFFIN`

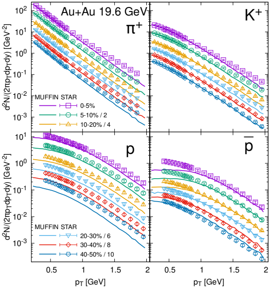
3-fluid version of `vHLLE` + hadron sampling + `SMASH`

WIP but published version corresponds to Ivanov's friction: Cimerman, IK, Tomasik, Huovinen, Phys.Rev.C 107 (2023) 4, 044902







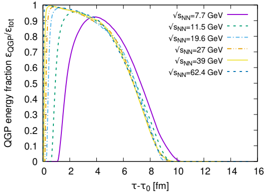


Acceptable data reproduction with chiral model EoS (crossover).

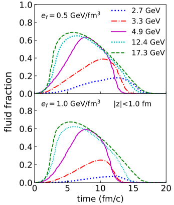
Generic conclusions from the modelling

- Hybrid models generally fit dN/dy, dN/dp_T and v_2 **together**
- high-density medium is formed down to 7.7 GeV. According to EoS it has to be in the QGP phase.

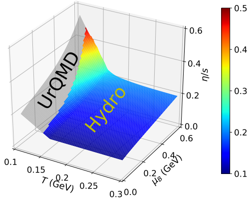
`MUFFIN` (MFH)

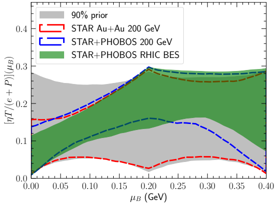


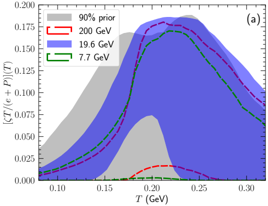
`JAM`+hydro



- η/s decreases with T (Geom.IS 2003.05852; String deceleration 2310.10787).



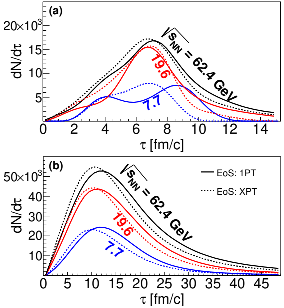


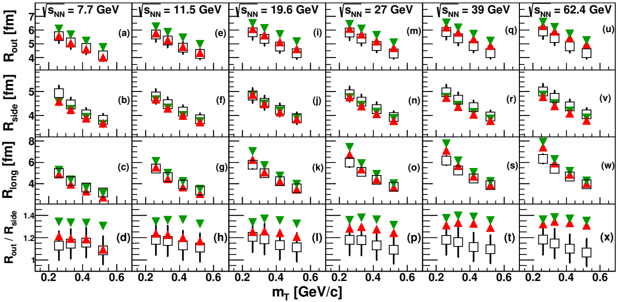


Some models (MFH-based) do without viscosity but overestimate v_2.

EoS sensitivity: HBT?

`UrQMD` (initial state) + fluidisation at fixed τ_0 + `vHLLE` (hydro) + `UrQMD`
Batyuk, IK, Lednicky, Malinina, Mikhaylov, Rogachevsky, Wielanek, Phys. Rev. C 96, 024911 (2017)





1PT = 1st order PT, XPT = crossover; crossover EoS is red, 1PT EoS is green

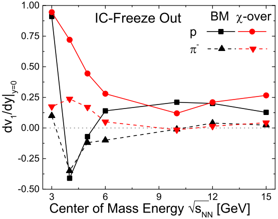
There is weak EoS sensitivity, crossover EoS is preferred.

EoS sensitivity: directed flow?

J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stöcker, Phys. Rev. C 89 (2014) 054913, arXiv:1402.7236

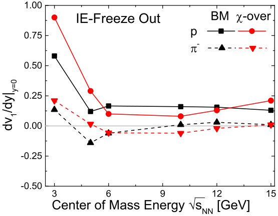
- 1-fluid model with iso-time freezeoout:

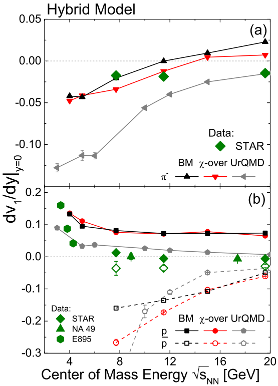
  sign change of dv_1/dy with 1^{st}-order PT EoS



- 1-fluid model with iso-T freezeoout:

  NO sign change of dv_1/dy with 1^{st}-order PT EoS





- Full hybrid model: no sign change of dv_1/dy, weak EoS dependence and no agreement with the data

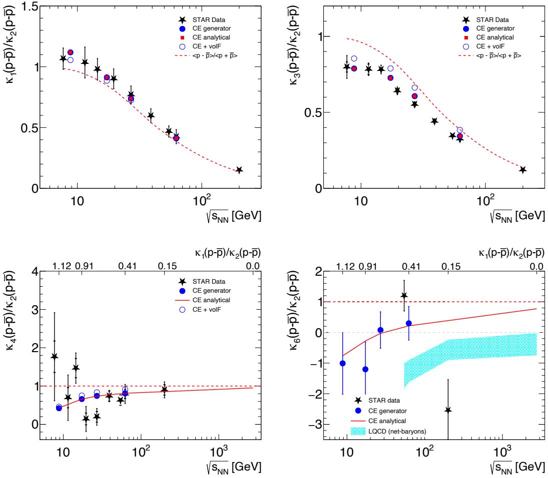
Proton Number Cumulants

Non-critical baseline for proton cumulants is non-trivial

...as a function of collision energy

Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, Nucl. Phys. A 1008, 122141 (2021)

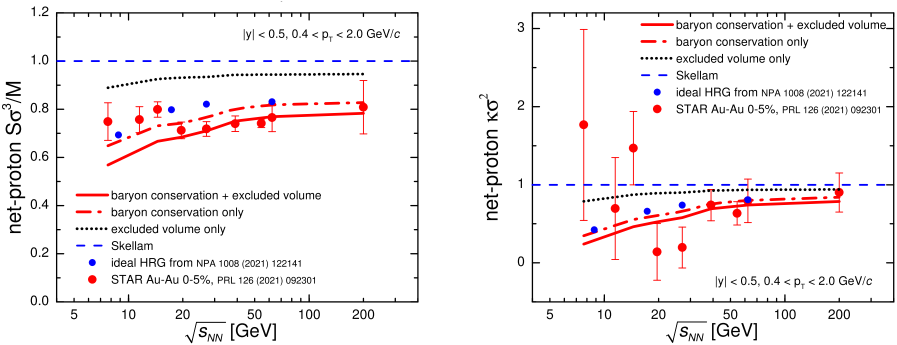
Effect of global baryon number conservation + volume fluctuations



Non-critical baseline from 3D hydro

Vovchenko, Koch, Shen, Phys. Rev. C 105, 014904 (2022)

Parametrized initial state (Type 1) + `MUSIC`
Global baryon conservation using SAM-2.0 method

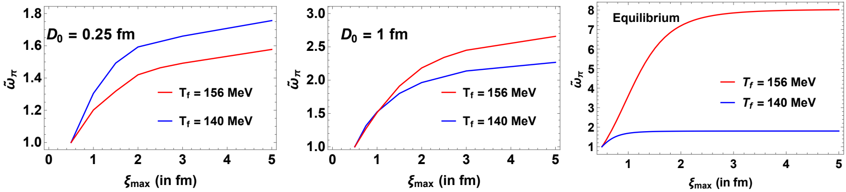


Progress on critical fluctuations

Maneesha Pradeep, Jamie Karthein, Krishna Rajagopal, Misha Stephanov, Yi Yin,
Phys.Rev.D 106 (2022) 3, 036017

Hydro+ : deterministic hydrodynamics + non-hydrodynamic modes ϕ
No concrete result just yet.

$\tilde{\omega}_\pi = \frac{\omega_\pi}{\omega_\pi^{\mathrm{nc}}}$, where $\omega_A = \frac{\langle \delta N_A^2 \rangle}{\langle N_A \rangle}$



where D is a diffusion coefficient which inversely scales with the correlation length ξ.

Conclusions

- There is abundance of hybrid models for the RHIC BES energy range, with different assumptions about initial state dynamics (or absence thereof).
- Most of the models desribe most of the basic observables.
- EoS sensitivity is generally not strong; it is not clear yet what observables would be very sensitive to the type of PT in the EoS.
- Probably a future Bayesian analysis will tell us whether some EoS are excluded by the coherent data reproduction
- Baryon/proton cumulants: we know a non-critical baseline.
- There are no hybrids with critical behaviour included yet.
- We need more modelling and **more consistent experimental data**.