Fluctuation Analysis at HADES

19/02/2024 EMMI – STAR/CBM-eTOF Workshop | R. Holzmann (GSI/HADES)

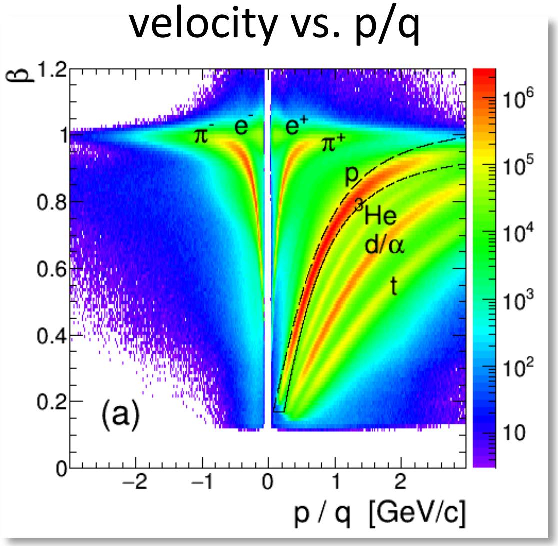
Experimental challenges in particle fluctuation measurements:

- achieve large event statistics ✓
- correct for acceptance/efficiency effects ✓
- control purity of event sample, e.g. correct for pile-up ✓
- **correct for volume fluctuations** caused by the centrality selection ☠

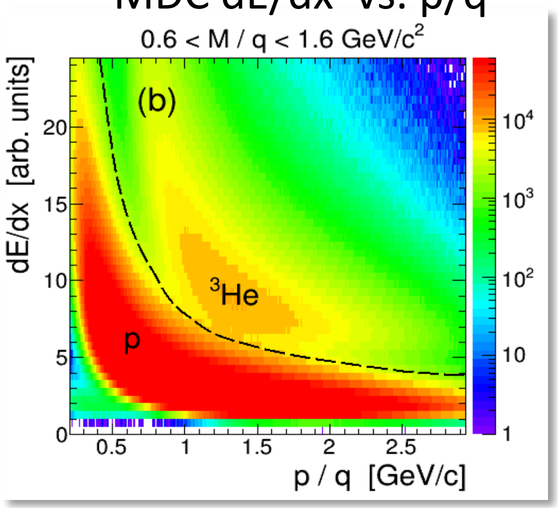
→ Main focus of this talk

Particle ID in HADES

velocity vs. p/q



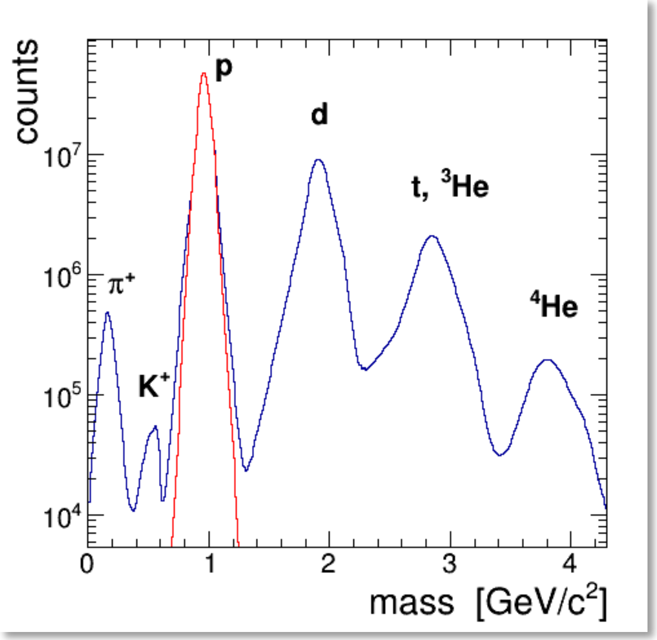
MDC dE/dx vs. p/q



Hadron ID based on

- ToF
- momentum
- dE/dx

Mass spectrum and accepted protons

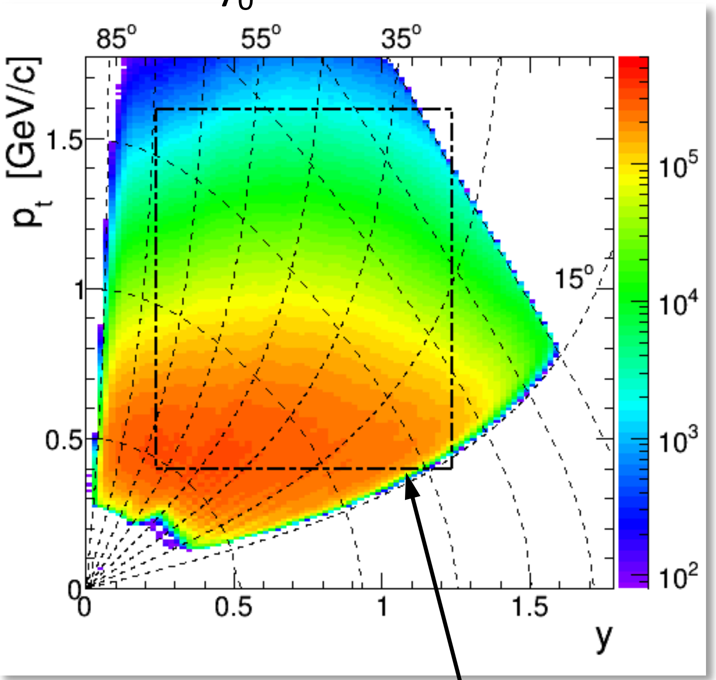


for details see PRC 102, 024914 (2020), EPJA 56 (2020)

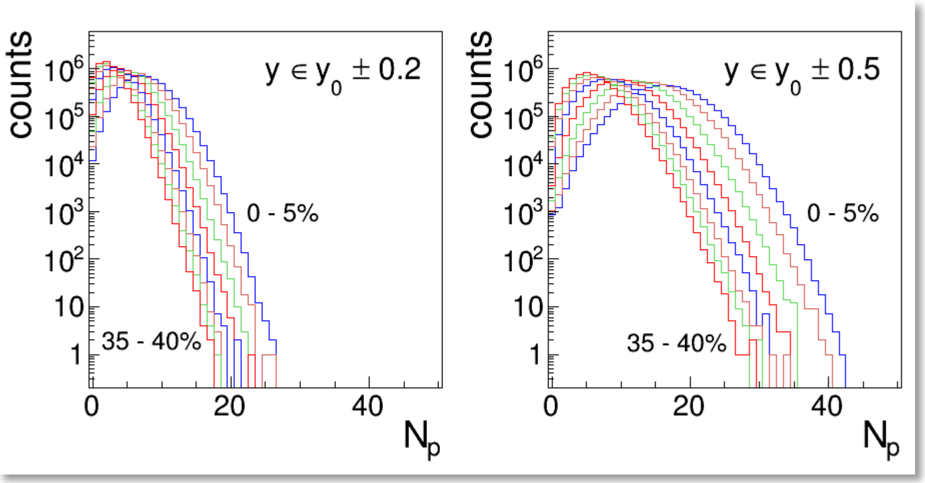
Proton distributions in Au+Au at $\sqrt{s} = 2.41\ GeV$

HADES $y - p_t$ coverage for protons

$y_0 = 0.74$



Proton multiplicity distributions



Analysis based on $1.6 \cdot 10^8$ Au+Au events divided into 5%-centrality bins in the range of the 0 - 40% most central events

Useful acceptance for fluctuation analysis

$\mathbf{y = y_0 \pm 0.5}$

$\boldsymbol{p_t = 0.4 - 1.6}$ **GeV/c**

for details see PRC 102, 024914 (2020), EPJA 56 (2020)

Centrality selection with the Forward Wall

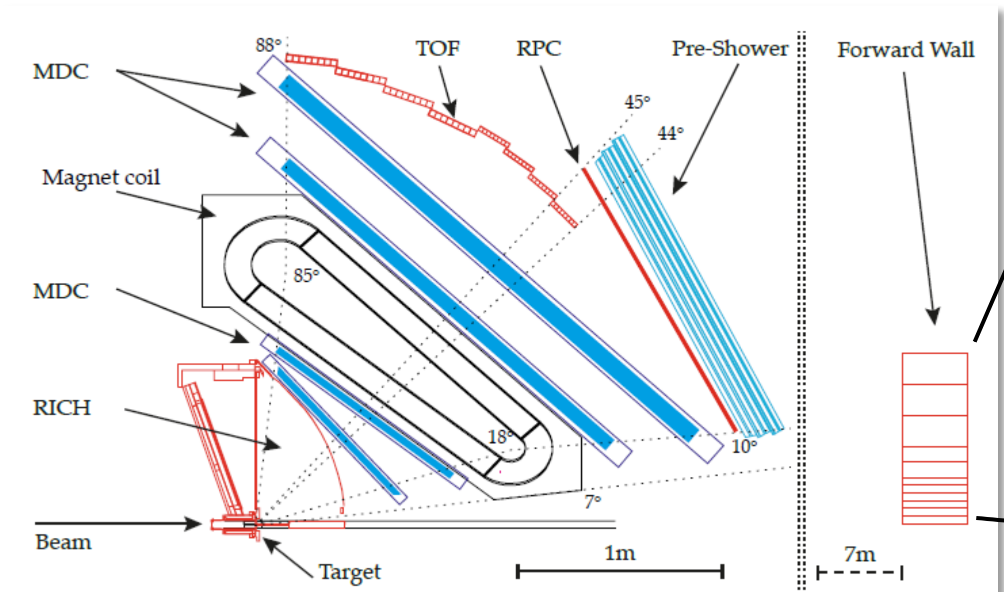
In 1.23 GeV/u Au+Au collisions:

- protons & clusters dominate
- centrality selection based on
  - hit mult in TOF & RPC
  - or track mult
  - or **FW sum of charges**

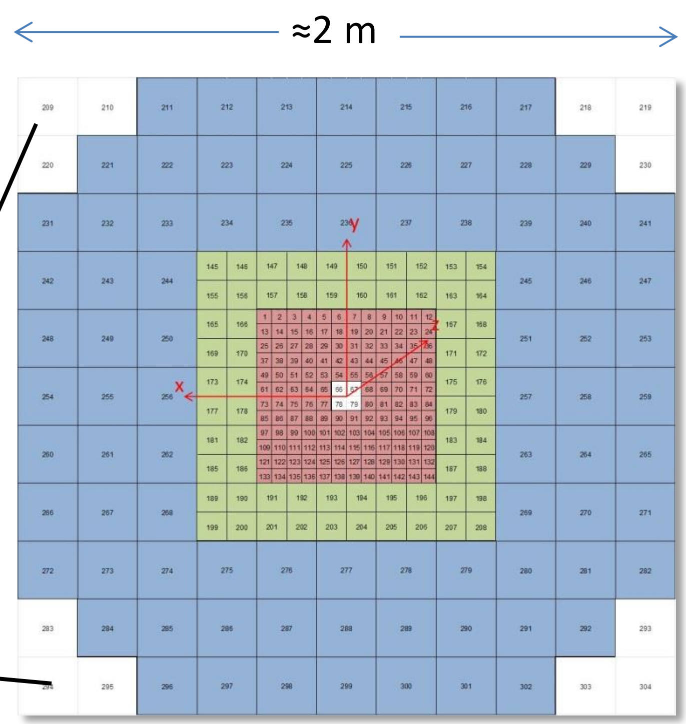
    ➔ reduce autocorrelations!

FW made of plastic scintillator tiles covering polar angles $\theta = 0.5^o - 7.5^o$ i.e. a pseudorapidity of $\eta = 2.7 - 5.4$ (HADES itself covers $y \approx 0 - 1.8$)

➔ Used for event-plane reconstruction



cross section of 1 of 6 HADES sectors

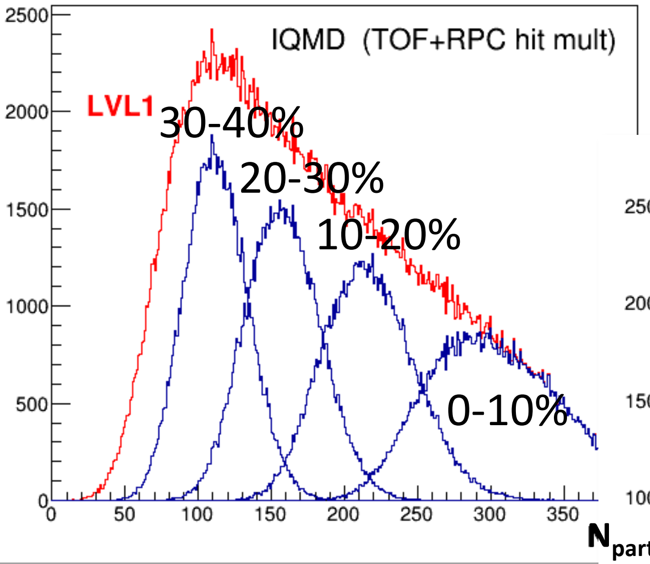


4x4, 8x8, 16x16 cm² tiles

Centrality estimators in the HADES experiment

see EPJA 54 (2018) 85

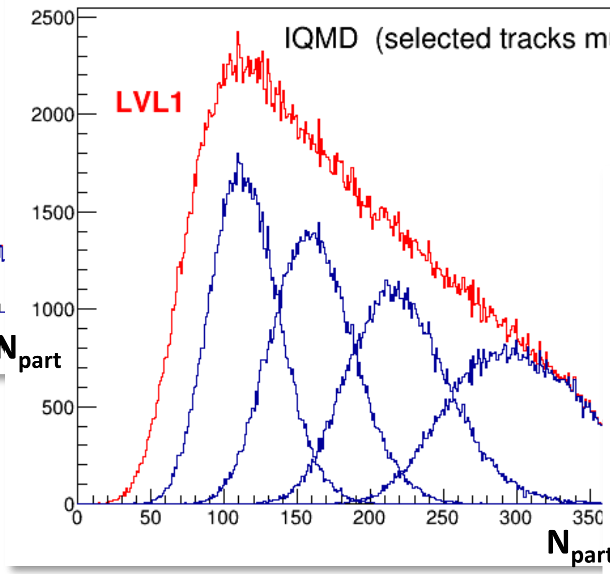
centrality based on N_{hit}



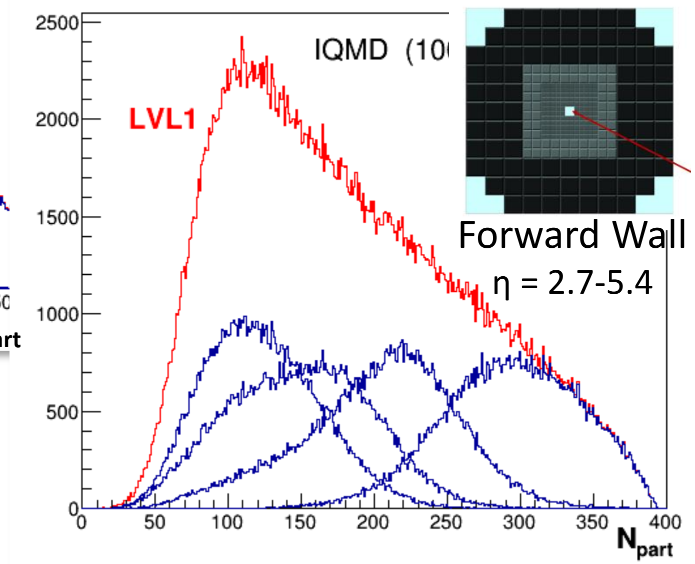
see also EPJA 54 (2018) 85

Based on a full simulation with events from **IQMD + MST** clusterizer

centrality based on N_{trk}



centrality based on Σ_{FW}



➔ Centrality selections lead to large volume fluctuations $\delta V \equiv \delta N_{part}$, characterized by volume cumulants V_j

➔ We use FW for fluct. analysis

to avoid autocorrelations!

Volume fluctuations in an independant-source model

Literature: V. Skokov, B. Friman & K. Redlich PRC 88 (2013), A. Rustamov et al. NPA 960 (2017), Sugiura, Nonaka & Esumi PRC 100 (2019), Esumi & Nonaka NIM A987 (2021)

Averaging the proton number moments over volume

$$< N_{prot}^{j} > = \int P(V) \underbrace{\sum N_{prot}^{j} P\left(N_{prot} \middle| V\right)}_{< N_{prot}^{j} >_{V}} dV$$

adds volume contributions to the observed proton cumulants $\kappa_j[N]$:

$$\kappa_1[N] = \langle N_w \rangle \, \kappa_1[n] = \langle N_w \rangle \, \langle n \rangle = \langle N \rangle \tag{1}$$

$$\kappa_2[N] = \langle N_w \rangle \, \kappa_2[n] + \langle n \rangle^2 \, \kappa_2[N_w] = \bar{\kappa}_2[N] + \langle N \rangle^2 \, \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} \tag{2}$$

$$\kappa_3[N] = \langle N_w \rangle \, \kappa_3[n] + 3 \, \langle n \rangle \, \kappa_2[n] \kappa_2[N_w] + \langle n \rangle^3 \, \kappa_3[N_w] = \bar{\kappa}_3[N] + 3 \, \langle N \rangle \, \bar{\kappa}_2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + \langle N \rangle^3 \, \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} \tag{3}$$

$$\begin{aligned} \kappa_4[N] &= \langle N_w \rangle \, \kappa_4[n] + 4 \, \langle n \rangle \, \kappa_3[n] \kappa_2[N_w] + 3\kappa_2^2[n] \kappa_2[N_w] + 6 \, \langle n \rangle^2 \, \kappa_2[n] \kappa_3[N_w] + \langle n \rangle^4 \, \kappa_4[N_w] \\ &= \bar{\kappa}_4[N] + 4 \, \langle N \rangle \, \bar{\kappa}_3[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 3\bar{\kappa}_2^2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 6 \, \langle N \rangle^2 \, \bar{\kappa}_2[N] \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} + \langle N \rangle^4 \, \frac{\kappa_4[N_w]}{\langle N_w \rangle^4} \end{aligned} \tag{4}$$

where the $\bar{\kappa}_j[N]$ are the proton cumulants at fixed volume, the $\kappa_j[N]$ are the observed cumulants, and the $V_j \equiv \kappa_j[N_w]$ characterize the volume fluctuations.

Illustration of the VF problem with STAR data

Higher-order cumulants and correlation functions of proton multiplicity distributions in $\sqrt{s_{NN}} = 3$ GeV Au+Au collisions at the RHIC STAR experiment

see also Abdallah et al. PRL 128 (2022)

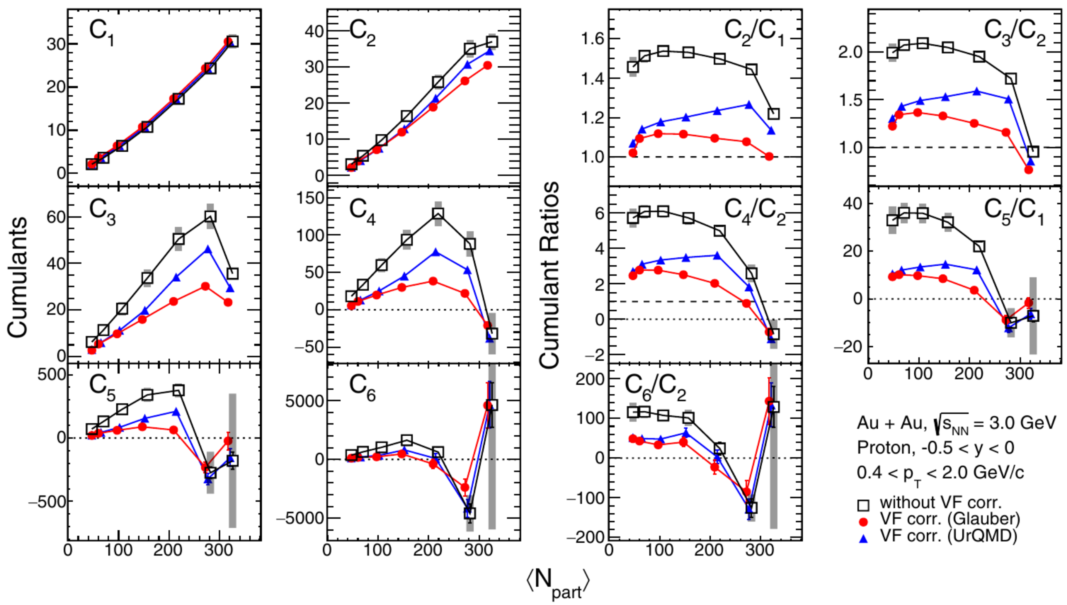


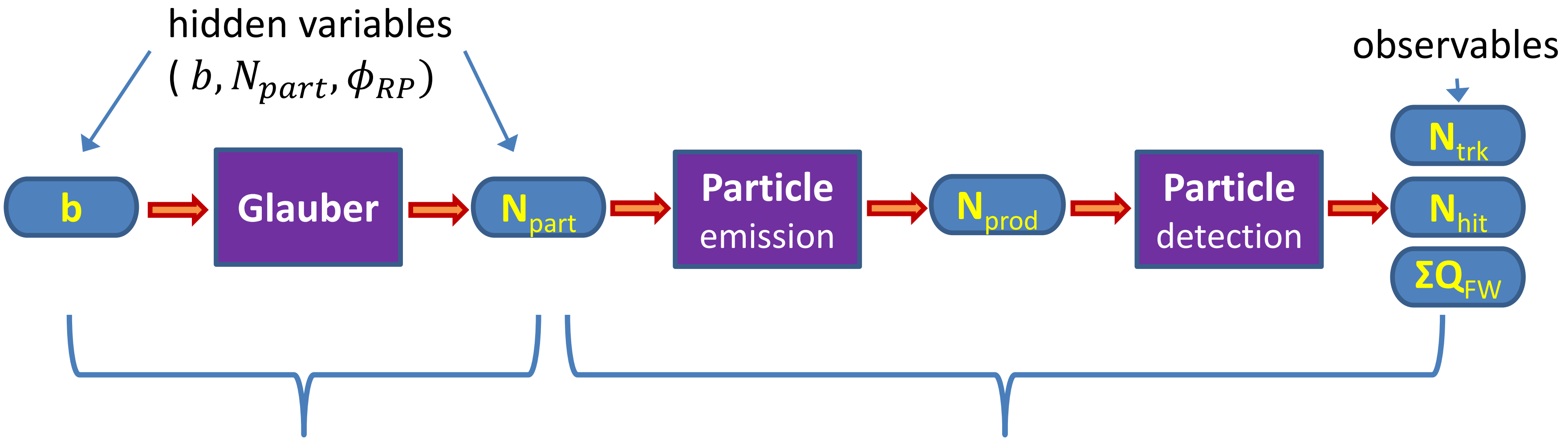
FIG. 14. Experimental results on centrality dependence of cumulants (left panels) and cumulant ratios (right panels) up to sixth order of the proton multiplicity distributions in Au+Au collisions at $\sqrt{s_{NN}} = 3$ GeV. The open squares are data without VF correction while red circles and blue triangles are results with VF correction with N_{part} distributions from Glauber and UrQMD models, respectively.

Volume corrections can be

- very large
- strongly model-dependent

➔ A data-driven approach would be preferrable !

$N_{hit} \rightarrow N_{part}$ deconvolution using Glauber MC



A minimal MC model:

e.g. Poisson(λ N_{part}) x (1 – α N^2_{part})

→ do a Bayesian reconstruction of the N_{part} distribution

Bayesian reconstruction of centrality

PHYSICAL REVIEW C **97**, 014905 (2018)

Relating centrality to impact parameter in nucleus-nucleus collisions

Sruthy Jyothi Das,[1,2] Giuliano Giacalone,[3] Pierre-Amaury Monard,[1] and Jean-Yves Ollitrault[1]

PHYSICAL REVIEW C **98**, 024902 (2018)

Editors' Suggestion

Reconstructing the impact parameter of proton-nucleus and nucleus-nucleus collisions

Rudolph Rogly,[1,2] Giuliano Giacalone,[1] and Jean-Yves Ollitrault[1]

PHYSICAL REVIEW C **104**, 034609 (2021)

Model independent reconstruction of impact parameter distributions for intermediate energy heavy ion collisions

J. D. Frankland,[1,*] D. Gruyer,[2] E. Bonnet,[3] B. Borderie,[4] R. Bougault,[2] A. Chbihi,[1] J. E. Ducret,[1] D. Durand,[2] Q. Fable,[2] M. Henri,[1] J. Lemarié,[1] N. Le Neindre,[2] I. Lombardo,[5] O. Lopez,[2] L. Manduci,[2,6] M. Pârlog,[2,7] J. Quicray,[2] G. Verde,[5,8] E. Vient,[2] and M. Vigilante[9]
(INDRA Collaboration)

Ollitrault et al.

- validated with simulations and applied to LHC data

Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment
Volume 987, 21 January 2021, 164802

Reconstructing particle number distributions with convoluting volume fluctuations

ShinIchi Esumi[a], Kana Nakagawa[a], Toshihiro Nonaka[a b]

use Bayes' theorem → $P(B|A) = P(A|B)\,P(B)/P(A)$

Setting $A = N_{hit}$, $B = N_{part}$ with

P(B|A)	↔	prob of N_{part} for given N_{hit}	←	to be reconstructed
P(A|B)	↔	prob of N_{hit} for given N_{part}	←	Glauber fit to N_{hit} data
P(A)	↔	min bias N_{hit} distribution	←	data
P(B)	↔	min bias N_{part} distribution	←	Glauber

} minimal model (Glauber MC)

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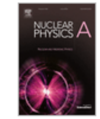
Nuclear Physics A

Volume 1034, June 2023, 122641



Anar‘s novel approach:

A model-free procedure to correct for volume fluctuations in E-by-E analyses of particle multiplicities

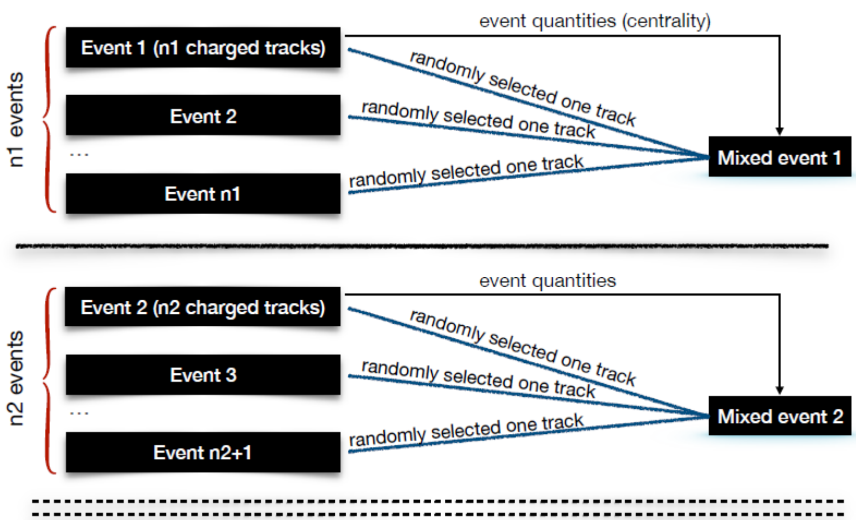
Anar Rustamov [a], Romain Holzmann [a], Joachim Stroth [b a c]

Main idea: 1) Use event mixing to remove (critical) correlations from the data, allowing to extract volume cumulants from the mixed-evt particle cumulants $\kappa_n^{mix}[N]$

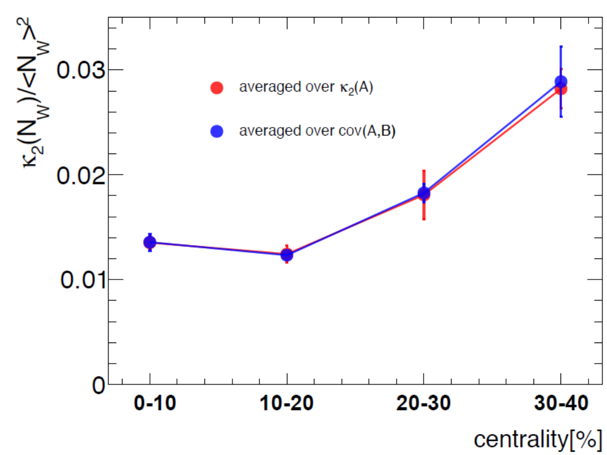
2) Correct the un-mixed particle cumulants $\kappa_n[N]$ for volume effects

Basic assumptions: 1) independent-source production and 2) binomial efficiencies

Event-mixing scheme:



2nd-order volume cumulant reconstructed from mixed evts in a toy model simulation:



Refinements and extension to higher orders

V. Koch, A. Rustamov, R. Holzmann & J. Stroth

With Volker working out the full math & combinatorics, we

- could set the method on a firm ground
- went from using cumulants to using factorial cumulants
- generalized the formalism to higher orders
- realized that the mixing is (formally) not mandatory (but may help)

➔ An extended paper on these matters will soon appear in arXiv

Using factorial cumulants to obtain $\kappa_j[N_w]$

Volume-affected factorial cumulants are expressed as:

$$C_1[N] = \langle N_w \rangle C_1[n] = \langle N_w \rangle \langle n \rangle = \langle N \rangle$$

$$C_2[N] = \bar{C}_2[N] + \langle N \rangle^2 \frac{\kappa_2[N_w]}{\langle N_w \rangle^2}$$

$$C_3[N] = \bar{C}_3[N] + 3 \langle N \rangle \bar{C}_2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + \langle N \rangle^3 \frac{\kappa_3[N_w]}{\langle N_w \rangle^3}$$

$$C_4[N] = \bar{C}_4[N] + 4 \langle N \rangle \bar{C}_3[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 3\bar{C}_2^2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 6 \langle N \rangle^2 \bar{C}_2[N] \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} + \langle N \rangle^4 \frac{\kappa_4[N_w]}{\langle N_w \rangle^4}$$

$\bar{C}_k[N]$ are the factorial cumulants at <u>fixed volume</u>, with $\bar{C}_k[N] = \langle N_w \rangle C_k[n]$

In general, the volume cumulants $\kappa_j[N_w]$ can then be obtained from

$$\frac{\kappa_2[N_w]}{\langle N_w \rangle^2} = \frac{C_2[M] - \bar{C}_2[M]}{\langle M \rangle^2}$$

$$\frac{\kappa_3[N_w]}{\langle N_w \rangle^3} = -3 \frac{\bar{C}_2[M]}{\langle M \rangle^2} \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + \frac{C_3[M] - \bar{C}_3[M]}{\langle M \rangle^3}$$

$$\frac{\kappa_4[N_w]}{\langle N_w \rangle^4} = -6 \frac{\bar{C}_2[M]}{\langle M \rangle^2} \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} - \frac{4\bar{C}_3[M]\langle M \rangle + 3\bar{C}_2[M]^2}{\langle M \rangle^4} \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + \frac{C_4[M] - \bar{C}_4[M]}{\langle M \rangle^4}$$

→ Note that these expressions are invariant w.r.t. efficiency/acceptance effects

Calculate your own cumulants with Python

Code developed by Anar Rustamov

Formulae for participant/volume fluctuations and their corrections

Select cumulant order: 6

Derive formulae for: Volume fluctuations

Create a .cc file — Derive

Pure cumulants

```
Deriving formulas for volume fluctuations for order: 6
_______________________________________
k_1[A] = <W>*<a>

k_2[A] = <W>*k_2[a] + <a>**2*k_2[W]

k_3[A] = <W>*k_3[a] + <a>**3*k_3[W] + 3*<a>*k_2[W]*k_2[a]

k_4[A] = <W>*k_4[a] + <a>**4*k_4[W] + 6*<a>**2*k_2[a]*k_3[W] + 4*<a>*k_2[W]*k_3[a] +
3*k_2[W]*k_2[a]**2

k_5[A] = <W>*k_5[a] + <a>**5*k_5[W] + 10*<a>**3*k_2[a]*k_4[W] + 10*<a>**2*k_3[W]*k_3[a] +
5*<a>*k_2[W]*k_4[a] + 15*<a>*k_2[a]**2*k_3[W] + 10*k_2[W]*k_2[a]*k_3[a]

k_6[A] = <W>*k_6[a] + <a>**6*k_6[W] + 15*<a>**4*k_2[a]*k_5[W] + 20*<a>**3*k_3[a]*k_4[W] +
45*<a>**2*k_2[a]**2*k_4[W] + 15*<a>**2*k_3[W]*k_4[a] + 6*<a>*k_2[W]*k_5[a] +
60*<a>*k_2[a]*k_3[W]*k_3[a] + 15*k_2[W]*k_2[a]*k_4[a] + 10*k_2[W]*k_3[a]**2 +
```

Mixed cumulants

```
Deriving formulas for volume fluctuations for order: 6
_______________________________________
cov11[AB] = <W>*cov11[ab] + <a>*<b>*k_2[W]

cov12[AB] = <W>*cov12[ab] + <a>*<b>**2*k_3[W] + <a>*k_2[W]*k_2[b] + 2*<b>*cov11[ab]*k_2[W]

cov13[AB] = <W>*cov13[ab] + <a>*<b>**3*k_4[W] + 3*<a>*<b>*k_2[b]*k_3[W] + <a>*k_2[W]*k_3[b]
+ 3*<b>**2*cov11[ab]*k_3[W] + 3*<b>*cov12[ab]*k_2[W] + 3*cov11[ab]*k_2[W]*k_2[b]

cov14[AB] = <W>*cov14[ab] + <a>*<b>**4*k_5[W] + 6*<a>*<b>**2*k_2[b]*k_4[W] +
4*<a>*<b>*k_3[W]*k_3[b] + <a>*k_2[W]*k_4[b] + 3*<a>*k_2[b]**2*k_3[W] +
4*<b>**3*cov11[ab]*k_4[W] + 6*<b>**2*cov12[ab]*k_3[W] + 12*<b>*cov11[ab]*k_2[b]*k_3[W] +
4*<b>*cov13[ab]*k_2[W] + 4*cov11[ab]*k_2[W]*k_3[b] + 6*cov12[ab]*k_2[W]*k_2[b]

cov15[AB] = <W>*cov15[ab] + <a>*<b>**5*k_6[W] + 10*<a>*<b>**3*k_2[b]*k_5[W] +
10*<a>*<b>**2*k_3[b]*k_4[W] + 15*<a>*<b>*k_2[b]**2*k_4[W] + 5*<a>*<b>*k_3[W]*k_4[b] +
<a>*k_2[W]*k_5[b] + 10*<a>*k_2[b]*k_3[W]*k_3[b] + 5*<b>**4*cov11[ab]*k_5[W] +
```

```
Description
_______________________________________
Contributions from volume/participant fluctuations to cumulants of multiplicity distributions.
Formulas are derived within the model of independent particle sources with the following notations:

kappa_n[N] -> nth order cumulant or multiplicity distribution.
kappa_n[n] -> nth order cumulant or multiplicity distribution per single source
<W> -> mean number of sources
<n> -> mean number of particles per source
```

A. Rustamov, R. Holzmann, V. Koch, J. Stroth

Toy model: Glauber MC + Negative Binomial

Events are generated in a toy model using Glauber MC for centrality sampling and a Negative Binomial distribution (NBD) for particle generation

$$P(n;\mu,k) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \left(\frac{\mu}{k}\right)^n \left(\frac{\mu}{k}+1\right)^{-(n+k)}$$

$$n_s = fN_w + (1-f)N_{coll}$$

$$\kappa_1^{NBD} = \mu,$$
$$\kappa_2^{NBD} = \frac{\mu(k+\mu)}{k},$$
$$\kappa_3^{NBD} = \frac{\mu(k+\mu)(k+2\mu)}{k^2},$$
$$\kappa_4^{NBD} = \frac{\mu(k+\mu)(k^2+6k\mu+6\mu^2)}{k^3}$$

Parameters from NBD fits to charged track distributions:

experiments	μ	k	f
HADES	0.24	20.34	1
STAR	0.31	5.66	0.94
ALICE	29.3	1.6	0.8

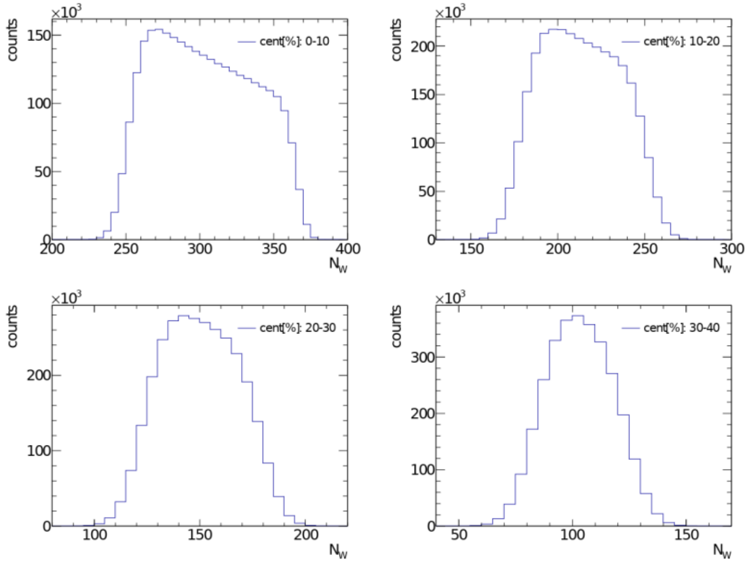
$$\kappa_1^{NBD}(HADES) = 0.24$$
$$\kappa_2^{NBD}(HADES) = 0.2428$$
$$\kappa_3^{NBD}(HADES) = 0.2486$$
$$\kappa_4^{NBD}(HADES) = 0.2602$$

→ HADES distribution is close to Poisson

Toy model calculations for HADES

realized by A. Rustamov

N_w distributions generated for 4 centrality selections:

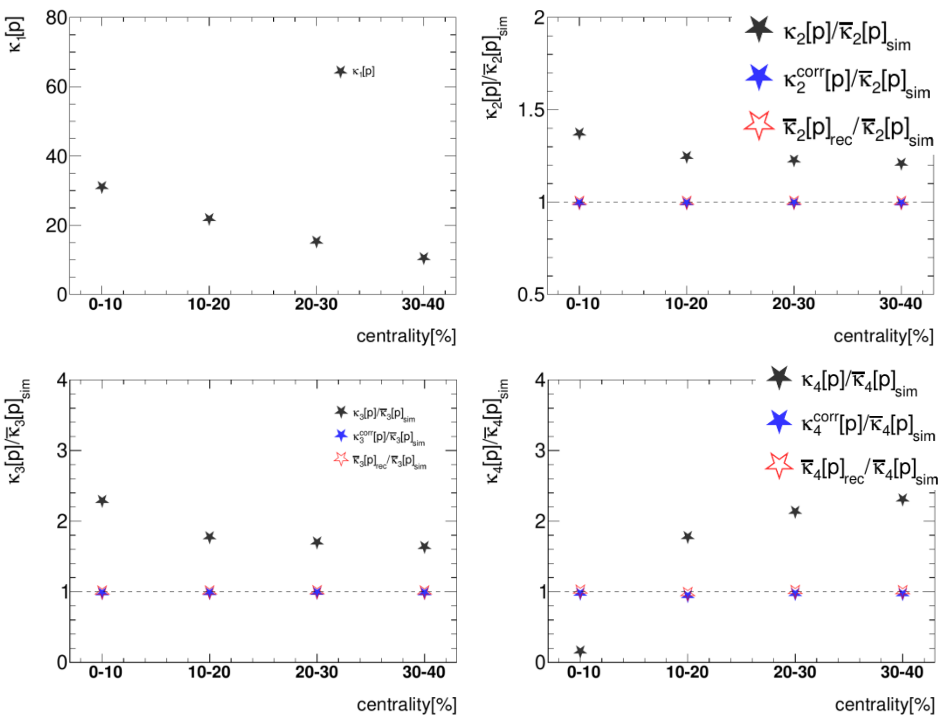


$$\kappa_1^{NBD}(HADES) = 0.24$$
$$\kappa_2^{NBD}(HADES) = 0.2428$$
$$\kappa_3^{NBD}(HADES) = 0.2486$$
$$\kappa_4^{NBD}(HADES) = 0.2602$$

→ particle production (encoded in NBD) is close to Poisson and intrinsic correlations are small in this example!

Extract the volume cumulants from the track nb cumulants $\kappa_j[M]$ and correct the observed proton cumulants κ_j *[N] to find*

$$\bar{\kappa}_j[N] = \kappa_j^{corr}[N] + \Delta_j$$

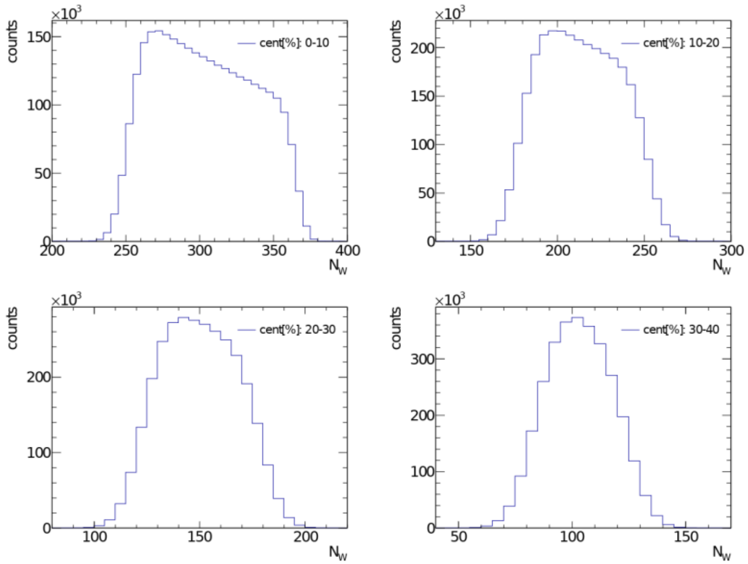


→ Proton cumulants are correctly reconstructed up to 4th order

Toy model calculations for HADES

realized by A. Rustamov

N_w distributions generated for 4 centrality selections:



$$\kappa_1^{NBD}(HADES) = 0.24$$
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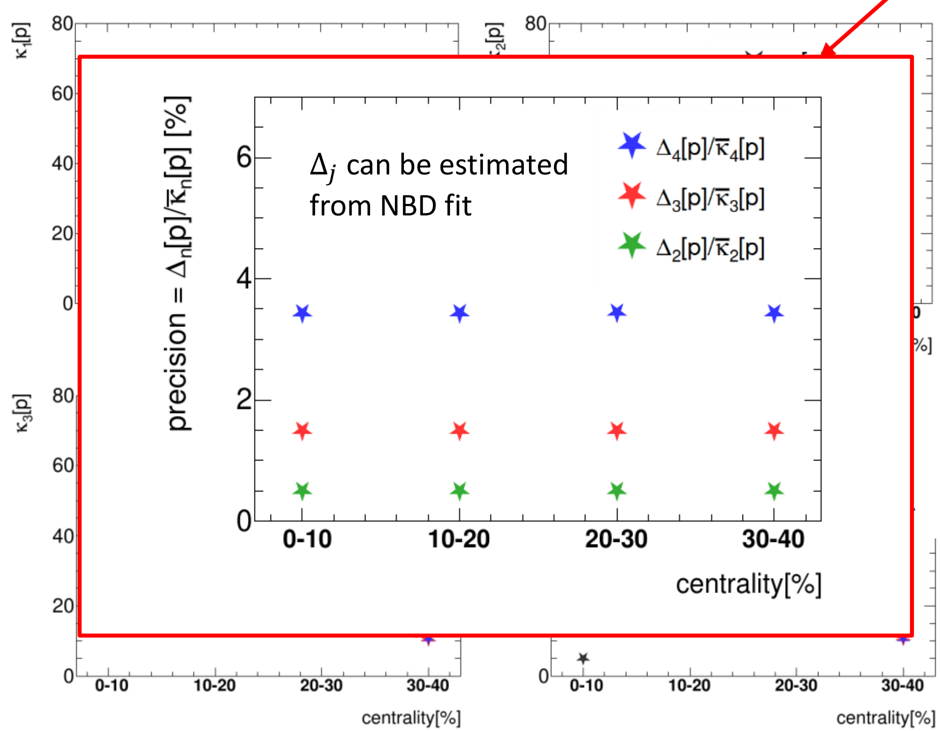


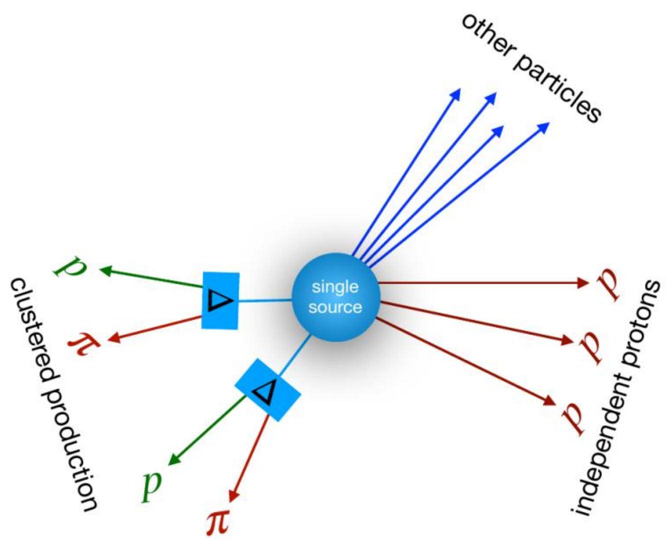
→ particle production (encoded in NBD) is close to Poisson and intrinsic correlations are small in this example!

→ In data, Δ_j is quite small and can furthermore be estimated from NBD fits to track distribution!

A more sophisticated model with resonances

realized by A. Rustamov

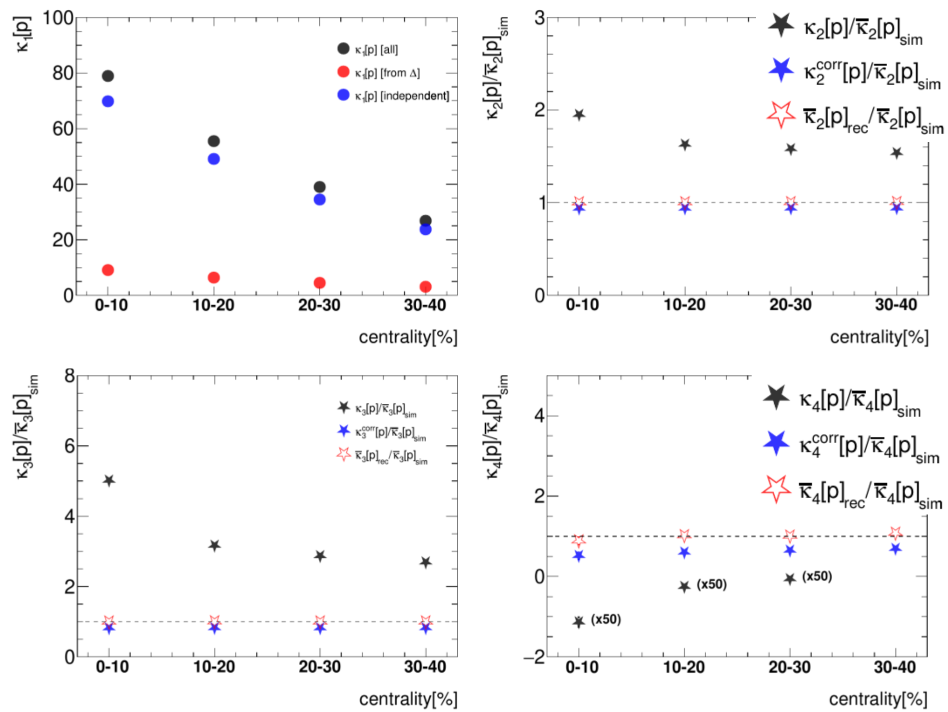
Pions and protons from Δ decays:



→ proton production is a mix of Δ decay and direct

Extract the volume cumulants from the track nb cumulants $\kappa_j[M]$ and correct the observed proton cumulants κ_j *[N] to find*

$$\bar{\kappa}_j[N] = \kappa_j^{corr}[N] + \Delta_j$$

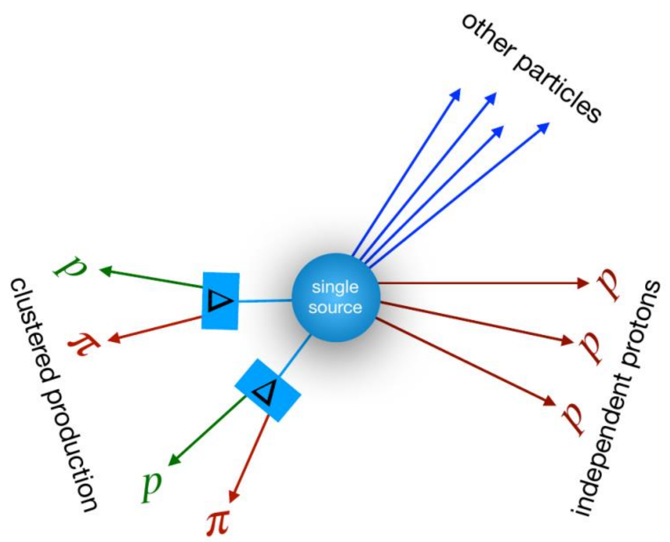


→ Proton cumulants can still be reconstructed within ≈10%

A more sophisticated model with resonances

realized by A. Rustamov

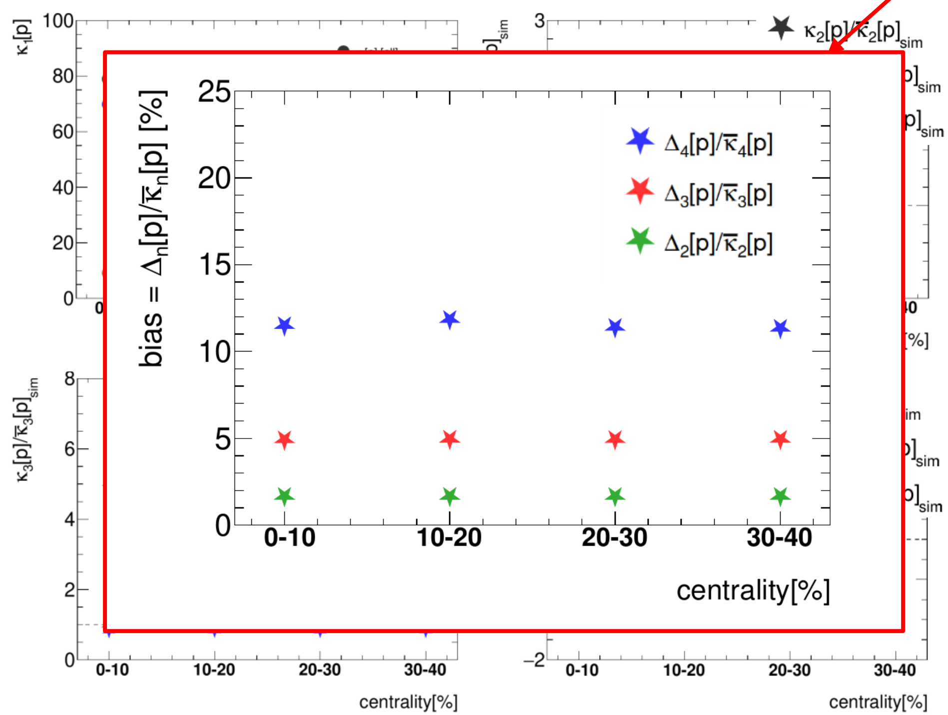
Pions and protons from Δ decays:



→ proton production is a mix of Δ decay and direct

Extract the volume cumulants from the track nb cumulants $\kappa_j[M]$ and correct the observed proton cumulants κ_j *[N] to find*

$$\bar{\kappa}_j[N] = \kappa_j^{corr}[N] + \Delta_j$$



→ In data, Δ_j can also be estimated from NBD fits to track distribution!

Open questions / Possible extensions

- Is the independent-source assumption always correct?
  Anyhow, what is the nature of these sources at low energies? N_{part}?
- Can we also handle non-binomial efficiencies?
- We need to validate the method on experimental data now.
  For HADES, this is work in progress by M. Nabroth, A. Rustamov et al.
- Could be interesting for other expts as well

Extra slides

Proton fluctuation signal purity

within same evt

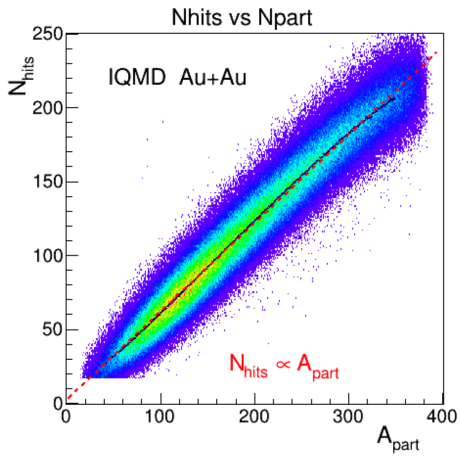
- Proton pid impurities $\leq \mathbf{10^{-3}}$
- Weak decays, e.g. $\Lambda \to p + \pi^-$, $\Sigma \to p + \pi^-$ $\leq \mathbf{6.5 \cdot 10^{-4}}$
- Knock-out (spallation) protons $\leq \mathbf{3 \cdot 10^{-3}}$
  - from secondary reactions in target / target holder
  - 50% pp, 45% np, <5% πp (Geant3 + GCalor)

different evt classes

- Au + C reactions on target holder (8μm kapton) foils $\leq \mathbf{10^{-3}}$
  - suppressed by trigger & centrality selection
  - asymmetric rapidity distribution $y > y_0$
- Event pile-up (central evt + min. bias evt) $\leq \mathbf{3 \cdot 10^{-5}}$

relative contribution

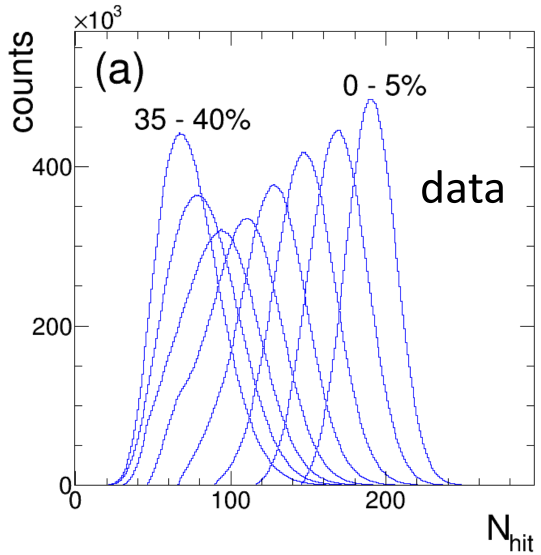
HADES ad-hoc approach: N_{hit} as a proxy for N_{part}



IQMD simulation shows that N_{hit} is proportional to N_{part}

→ use N_{hit} as proxy for vol. flucs.
i.e. rescale & adjust the v_n

Observed N_{hit} distributions (selected on FW)



fit IQMD cumulants to adjust scaled v_n

morph N_{hit} into N_{part}

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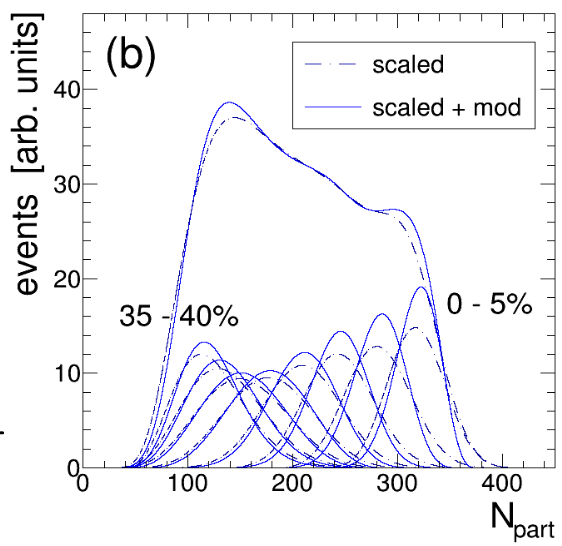
Reconstructed N_{part} distributions



True Poissonian process: N_{hit} = Poisson(λN_{part})

Applying **total cumulance** to
X = N_{hit} = Poisson($X|_{Z=N_{part}}$)
we obtain a relation between the cumulants of N_{hit} and N_{part}:

See also
Broniowski & Olszewski, PRC95 (2017)

$$\kappa_1[N_{hit}] = \lambda\,\langle N_{part}\rangle = \lambda\,\kappa_1[N_{part}]$$

$$\kappa_2[N_{hit}] = \lambda\,\kappa_1[N_{part}] + \lambda^2\,\kappa_2[N_{part}]$$

$$\kappa_3[N_{hit}] = \lambda\,\kappa_1[N_{part}] + 3\lambda^2\,\kappa_2[N_{part}] + \lambda^3\,\kappa_3[N_{part}]$$

$$\kappa_4[N_{hit}] = \lambda\,\kappa_1[N_{part}] + 7\lambda^2\,\kappa_2[N_{part}] + 6\lambda^3\,\kappa_3[N_{part}] + \lambda^4\,\kappa_4[N_{part}]$$

.
.
.

In general one has:

$$\kappa_n[N_{hit}] = \sum_{i=1}^{n} \lambda^i\, S_2(n,i)\,\kappa_i[N_{part}] \qquad (1)$$

With the inverse:

$$\kappa_n[N_{part}] = \sum_{i=1}^{n} \lambda^{-i}\, S_1(n,i)\,\kappa_i[N_{hit}] \qquad (2)$$