The QCD equation of state from Heavy ion collisions and neutron star mergers

Jan Steinheimer-Froschauer

special thanks to

Tom Reichert, M. Omana Kuttan

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FIAS Frankfurt Institute for Advanced Studies

Any EoS in UrQMD

To implement any density dependent EoS in UrQMD:
In UrQMD the real part of the interaction is implemented by a density dependent potential energy $V(n_B)$.
Once the potential energy is known, the change of momentum of each baryon is calculated as:

$$\dot{\mathbf{p}}_i = -\frac{\partial \langle H \rangle}{\partial \mathbf{r}_i} = -\left(\frac{\partial V_i}{\partial n_i} \cdot \frac{\partial n_i}{\partial \mathbf{r}_i}\right) - \left(\sum_{j \neq i} \frac{\partial V_j}{\partial n_j} \cdot \frac{\partial n_j}{\partial \mathbf{r}_i}\right) , \tag{1}$$

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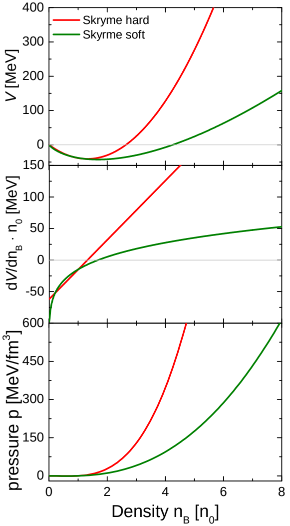
For the potential energy V often a Skyrme model was used that is based on a 2-term expansion in density:

$$U(n_B) = \alpha \cdot n_B + \beta \cdot n_B^{\gamma} \quad \text{with} \quad U(n_B) = \frac{\partial(n_B \cdot V(n_B))}{\partial n_B} \tag{2}$$

Problem: Once saturation density and binding energy is fixed, only 1 d.o.f. left and EoS likely becomes unphysical. No phase transition possible.



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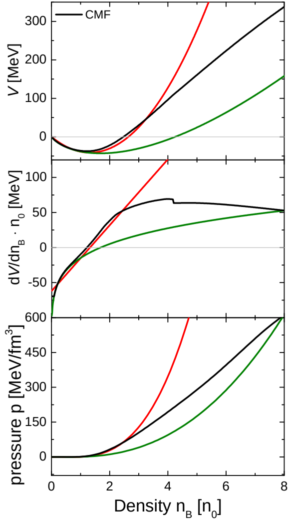
In CMF we can simply use the effective field energy per baryon E_{field}/A calculated from the CMF model:

$$V_{CMF} = E_{\text{field}}/A = E_{\text{CMF}}/A - E_{\text{FFG}}/A \, , \quad (4)$$

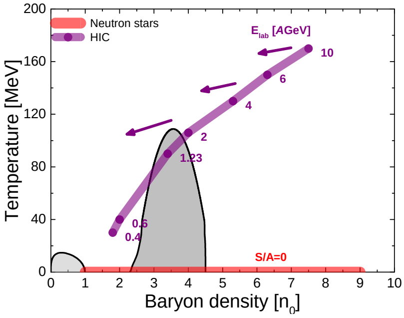
A phase transition can be simply included by adding another minimum in the potential energy: leading to (meta-)stable solutions at high density.

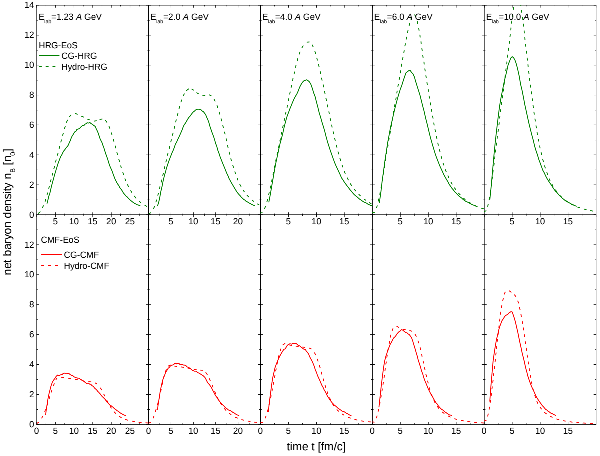


JS , A. Motornenko, A. Sorensen, Y. Nara, V. Koch and M. Bleicher, Eur. Phys. J. C 82, no.10, 911 (2022).

HIC regions of access - SIS range

- Including the CMF EoS in UrQMD vs. a hadron resonance gas baseline.
- Bulk evolution consistent with 3+1D hydro + CMF
- Initial compression from CMF model in UrQMD



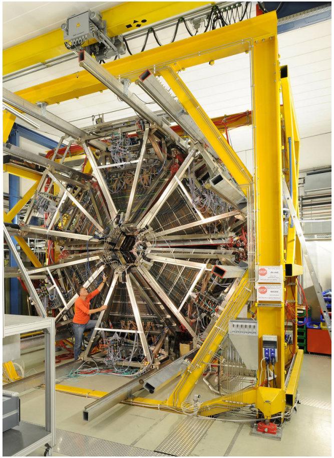


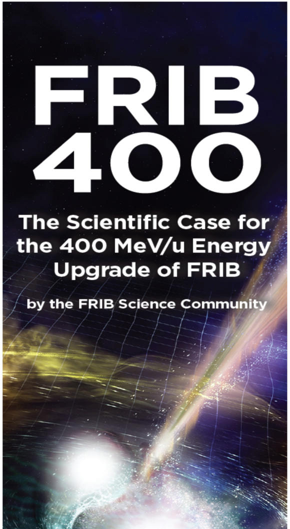
M. Omana Kuttan, A. Motornenko, **JS**, H. Stoecker, Y. Nara and M. Bleicher, Eur. Phys. J. C **82** (2022) no.5, 427

The sensitivity on the EoS in the sub 1GeV beam energy range

Several competing/complimentary facilities will measure in this energy range

HADES@GSI: 400 and 600 MeV/u



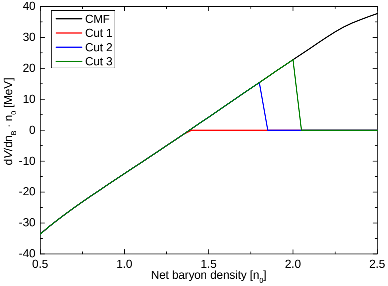


HIAF in china: up to 800 MeV/u from 2027.

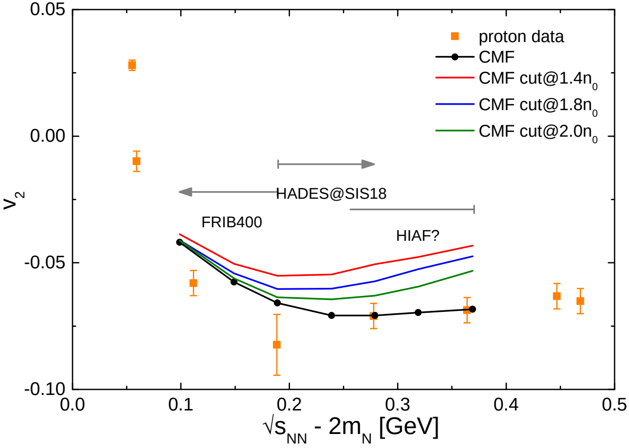
The sensitivity on the EoS in the sub 1GeV beam energy range

Which density ranges can we expect to cover at these beam energies?

Density dependent potential
(actually QMD force)



Elliptic flow of protons

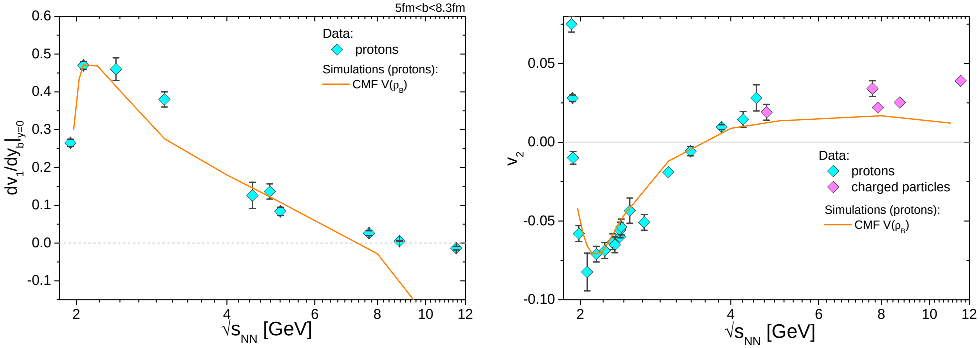


- We can nicely see which ranges beam energy can be used to study which densities.

Results from integrated flow

- The CMF EoS gives good results on flow coefficients.
- Sensitivity up to $\approx 4n_0$.

- $v_1 = p_x/p_T$
- $v_2 = (p_x^2 - p_y^2)/p_T^2$



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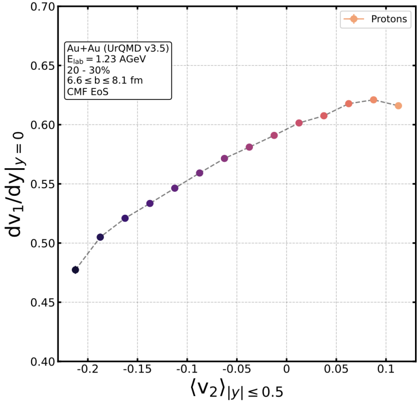
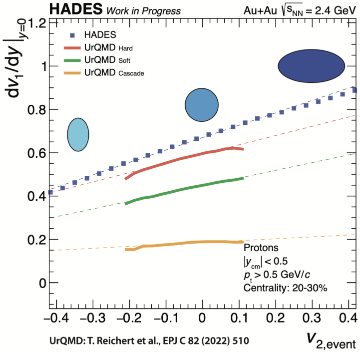
Flow correlations



Reichert et al. Phys.Lett.B 841 (2023) 137947

- HADES (prel!) data prefers hard EoS

- Select events based on integrated final v_2
- Measure dv_1/dy as function of v_2 trigger
- Correlation observed



Kardan, Blume for HADES. QM23 Poster

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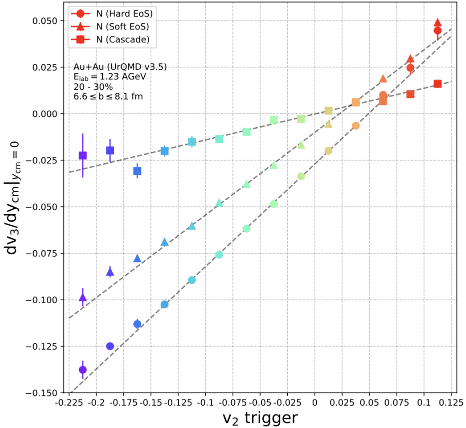
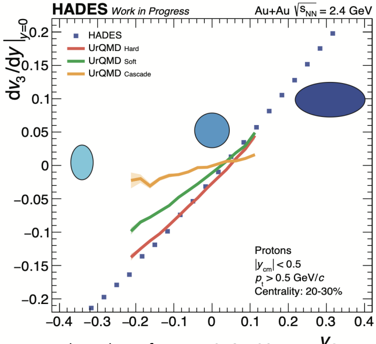
Flow correlations



Reichert et al. Phys.Lett.B 841 (2023) 137947

- HADES (prel!) data prefers hard EoS

- Select events based on integrated final v_2
- Measure dv_3/dy as function of v_2 trigger
- Strong sensitivity to EoS



Kardan, Blume for HADES. QM23 Poster

Other observables

The advantage of using an event generator like UrQMD: we can now study a multitude of observables:

Interferometry

The study of hadron two particle correlations allows for the study of the system size at freeze out which is sensitive to the EoS.

P. Li, JS, T. Reichert, A. Kittiratpattana, M. Bleicher and Q. Li, Sci. China Phys. Mech. Astron. **66** (2023) no.3, 232011

Dileptons

The study of electromagnetic probes (di-electrons) provides direct access to the hot and dense phase and the lifetime of the fireball.

O. Savchuk, A. Motornenko, JS, V. Vovchenko, M. Bleicher, M. Gorenstein and T. Galatyuk, [arXiv:2209.05267 [nucl-th]].

Fluctuations

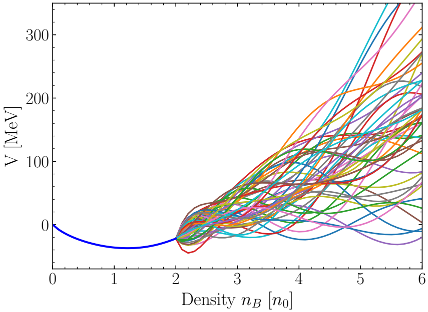
Fluctuations of conserved charges are (not) sensitive to the formation of clusters at a phase transition.

O. Savchuk, R. V. Poberezhnyuk, A. Motornenko, JS, M. I. Gorenstein and V. Vovchenko, [arXiv:2211.13200 [hep-ph]].

All observables indicate a rather stiff EoS.

Statistical analysis of available flow data

- Using Bayesian inference methods we can try to constrain the EoS from flow data
- Use UrQMD as described but parameterize $V(n_B)$ with a seventh order polynomial.



M. Omana Kuttan, JS, K. Zhou and H. Stöcker,Phys. Rev. Lett. 131, no.20, 202303 (2023).

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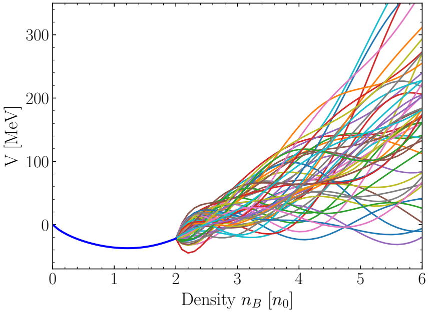
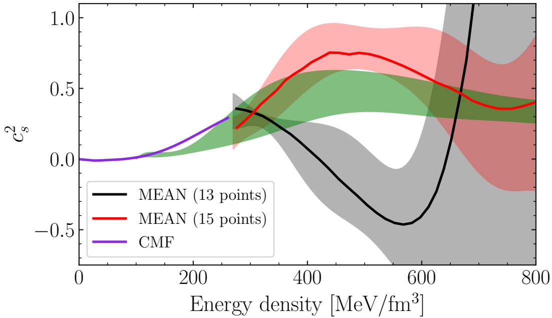


- Results depend strongly on the data used.
- If all data on the mean m_T and v_2 are used, constraints are similar to those from astrophysics (NS and BNSM).



M. Omana Kuttan, JS, K. Zhou and H. Stöcker,Phys. Rev. Lett. 131, no.20, 202303 (2023).

Why suddenly a hard EoS?

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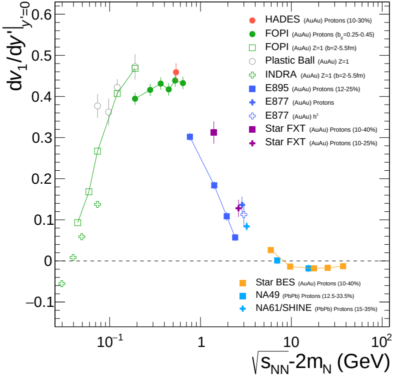
1. Heavy reliance on old AGS data, specifically v_1.



See talk by B Kardan or: J. Adamczewski-Musch *et al.* [HADES], Eur. Phys. J. A **59**, no.4, 80 (2023)

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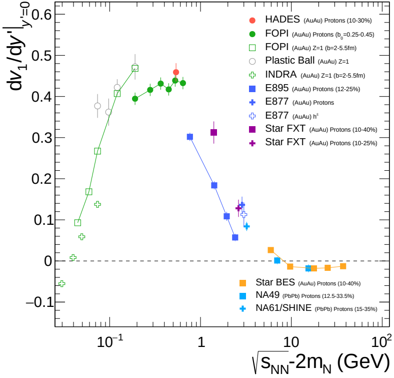
Why suddenly a hard EoS - older results talk about soft EoS

1. Heavy reliance on old AGS data, specifically v_1.
2. The role of momentum dependence?



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(Why and how) Introducing a momentum dependent potential?

- For years the importance of including a momentum dependent potential has been highlighted
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What convinced me that there is at least something

The definition of the single particle potential is:

$$U_i = E_i^* - E_i \tag{5}$$

where E_i is simply the non-interacting single particle energy $E_i = \sqrt{m_i^2 + p_i^2} - \mu_i$.
and E_i^* is the interacting one $E_i^* = \sqrt{m_i^{*2} + p_i^2} - \mu_i^*$.
It is easy to see that U_i will have a momentum dependent part (from the scalar interactions) and a non momentum dependent part (from the vector interactions). [a]

[a] See also recent work by Yasushi Nara.

Momentum dependence from CMF model

- We can use E_i^* from the CMF model in a straight forward way.
- Remember the parity doubling model + hyperons + Δs.

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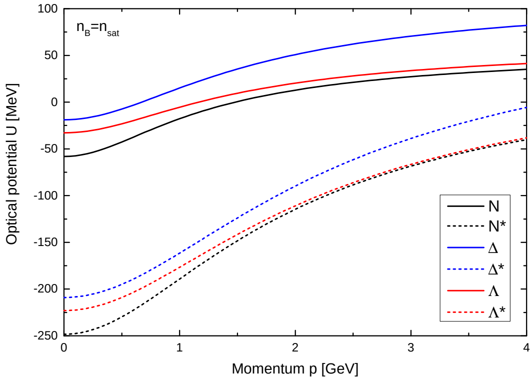
- Parity partners are deeply bound at saturation density. Leads to enhanced correlations and other possibly interesting effects.



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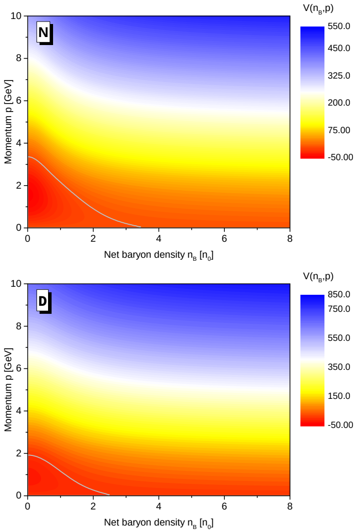
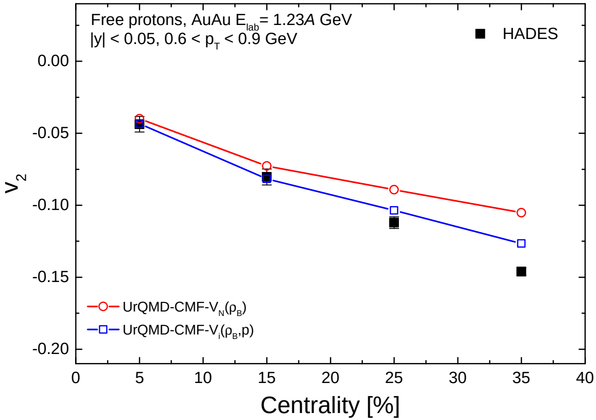
- Momentum dependence for ground state baryons as expected. Delta is more repulsive.
- Parity partners are deeply bound at saturation density. Leads to enhanced correlations and other possibly interesting effects.
- In addition we have all the potentials as function of density + momentum



Momentum dependence effects

- Effects with respect to centrality dependence of v_2 (HADES data)



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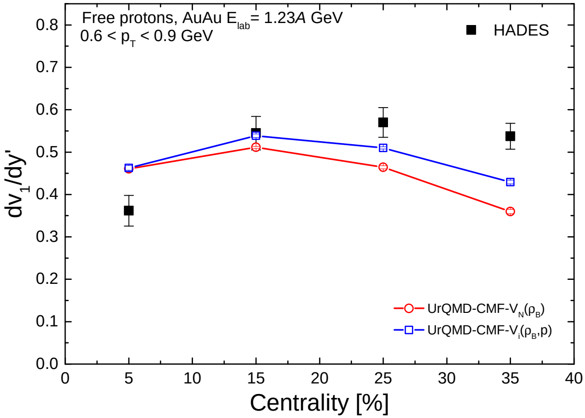

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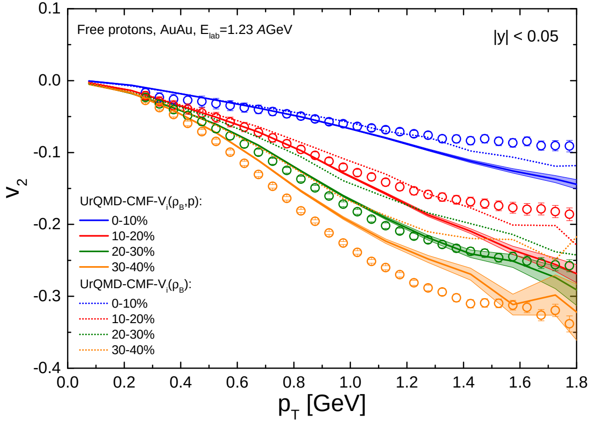
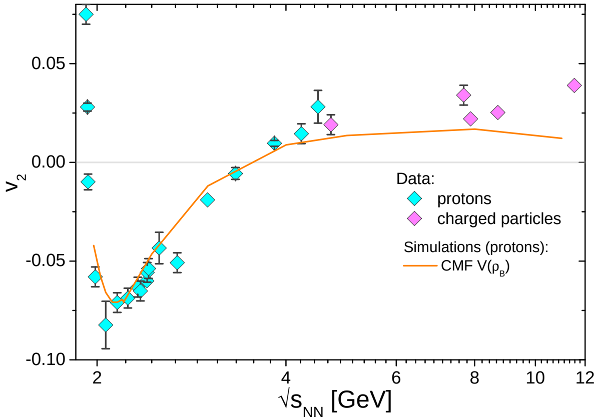
- Momentum dependence



Momentum dependence effects

- The integrated flow without momentum dependence.



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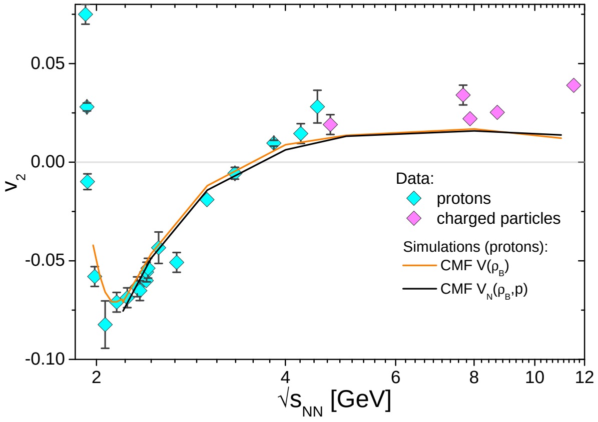
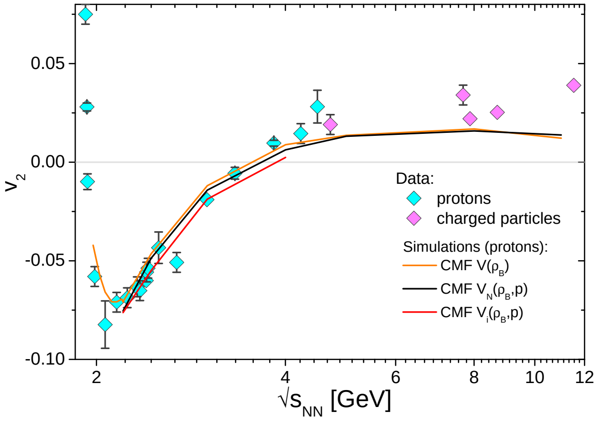


Momentum dependence effects

- The integrated flow without momentum dependence.
- The integrated flow with momentum dependence as for nucleons.
- The integrated flow with momentum dependence for all.
- Effect from Delta potential more important.
- Effect from only nucleon probably small because the momentum dependence is consistent with the overall EoS.
- Picture changes at low beam energies. Also keep in mind effect on pions and isospin.



Summary and conclusions

- Can use HIC in SIS regime to scan the high density QCD PD.
- Especially for HIC in the low to intermediate beam energies new ideas/methods for old and new models are necessary.
- Effects of the EoS don't occur at the same beam energy: **Need consistent modeling!!**
- Showed an example on how statistical analyses of large datasets available now and in the future can be constructed.
- Still much room and need for further development on how to connect the different regimes and observables.
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Future challenges and ways to approach them

1. Momentum dependence of potentials and the value of a good CMF.
2. The role of the Δ at finite temperature and in the isospin dependence.
3. Relativistic effects.
4. How to combine all that in a quantitative statistical analysis (or inference).

More open challenges

- Sensitivity to fluctuations and correlations in the nucleus - nuclear structure.
- Which observables should be used to connect the isospin dependence in HIC to GW observables?
- Pions depend on Δ-interaction which do not appear in cold NS.
- We use classical Hamiltonian dynamics. Clearly wrong. But how wrong?
- Proper relativistic QMD description is difficult to achieve (no interaction theorem).
- How can the finite T EoS be implemented?
- Interaction length scale at high density? Density dependence of the QMD-range parameter?
- Can we even think about changing d.o.f. at the phase transition?

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- Fortunately we have so many experiments and observables to come.

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- Good thing: Enough to do for a several year research program.

HBT

- Hanbury-Brown-Twiss (HBT) correlations for charged pions are a tool to measure the freezeout volume and time.
- Pions that are emitted close in coordinate space are correlated in momentum space.
- Simulation with a PT show a clear maximum.
- 'Old' data seem inconclusive, newest STAR data have much smaller error and favor the no-PT scenario.
- Sensitivity only up to $\approx 4n_0$.

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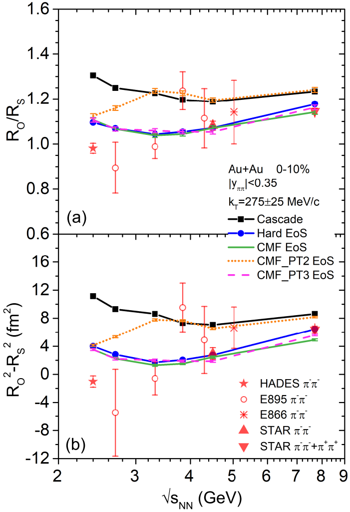

Dileptons

- Hydro simulations have suggested a strong increase (of factor 2) of the dilepton yield for a phase transition:
  F. Seck, T. Galatyuk, A. Mukherjee, R. Rapp, JS and J. Stroth, [arXiv:2010.04614 [nucl-th]].
- A significant increase of the low mass dilepton yield is observed when a phase transition is included in the UrQMD-CMF model.

O. Savchuk, A Motornenko, JS, V. Vovchenko, M. Bleicher, M. Gorenstein, T. Galatyuk, [arXiv:2209.05267 [nucl-th]].