Soft deconfinement & *quark saturation*

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Refs) Baym-Hatsuda-TK-Powell-Song-Takatsuka, "QHC", review on neutron stars (2018)

Fukushima-TK-Weise, "Hard deconfinement and soft-surface delocalization ..." PRD (2020)

TK, "Stiffening of matter in quark-hadron continuity" PRD (2021)

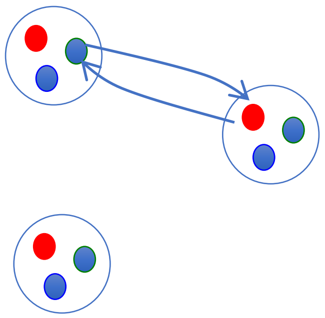
Fujimoto-TK-McLerran, "IdylliQ matter model" arXiv: 2306.04304 [nucl-th]

State of matter: overview

($n_0 = 0.16\ \mathrm{fm}^{-3}$)

[Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

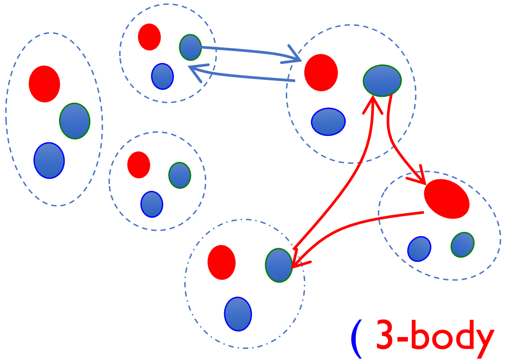


ab-initio nuclear cal.

laboratory experiments

steady progress

- many-quark exchange
- structural change,...
- hyperons, Δ, ...

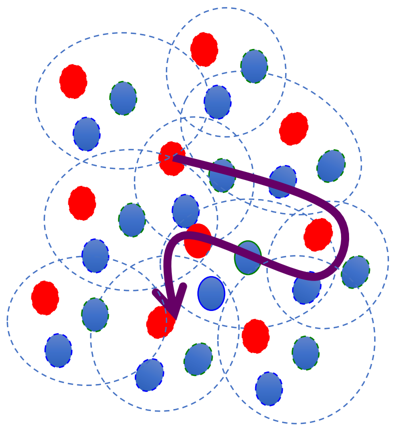


(3-body)

most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

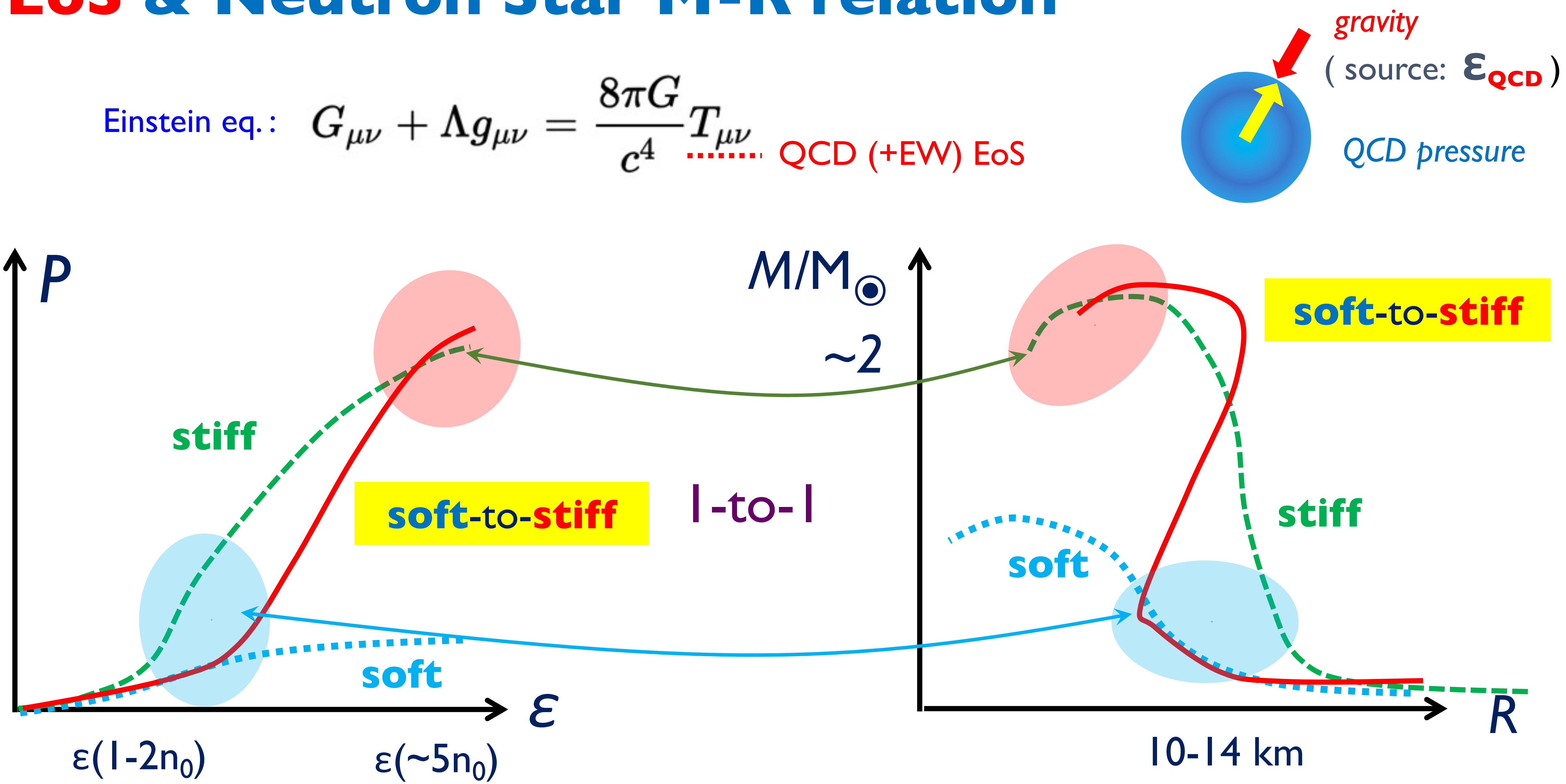
~ 1.4 $M_\odot$ ~ 2 $M_\odot$

n_B

~ $2n_0$ **Hints from NS** ~ $5n_0$ ~ $40n_0$

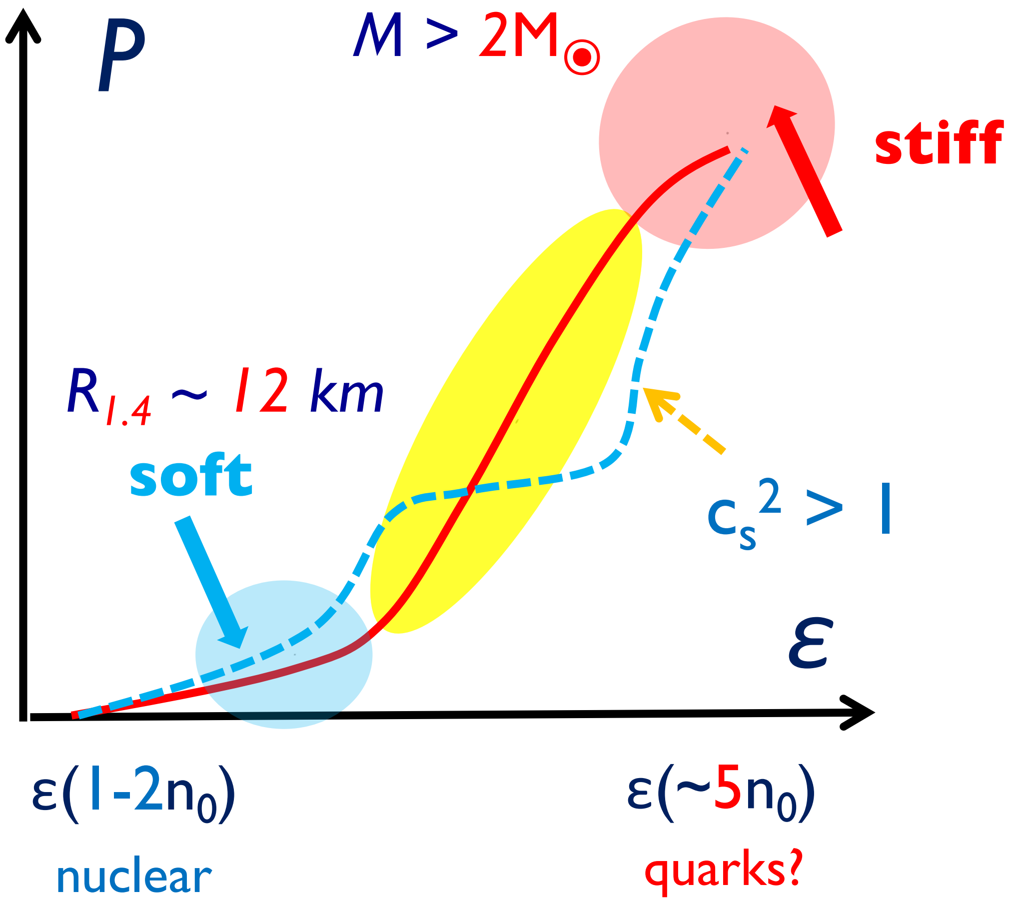
EoS & Neutron Star M-R relation

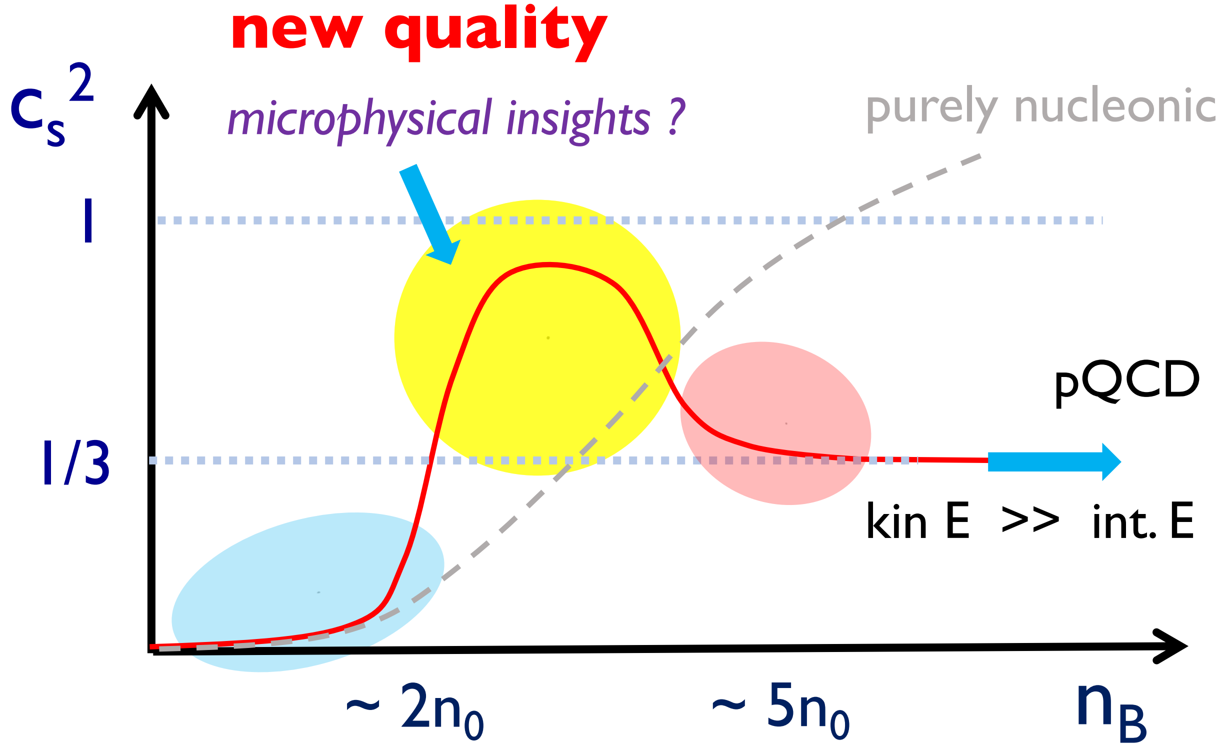
Einstein eq. : $$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$ QCD (+EW) EoS



Soft to stiff is challenging:

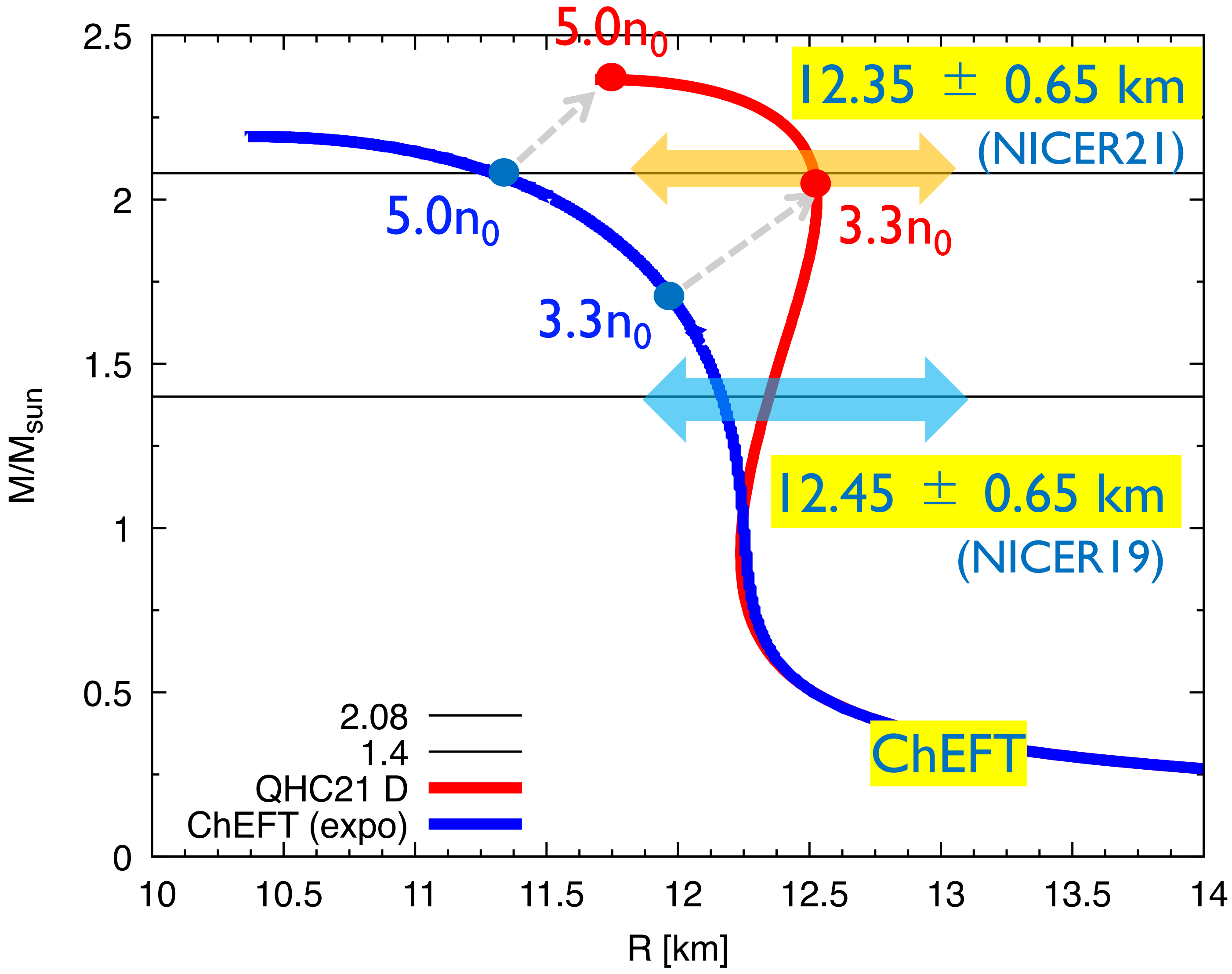
sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*) → *nuclear* & *quark* physics constrain each other

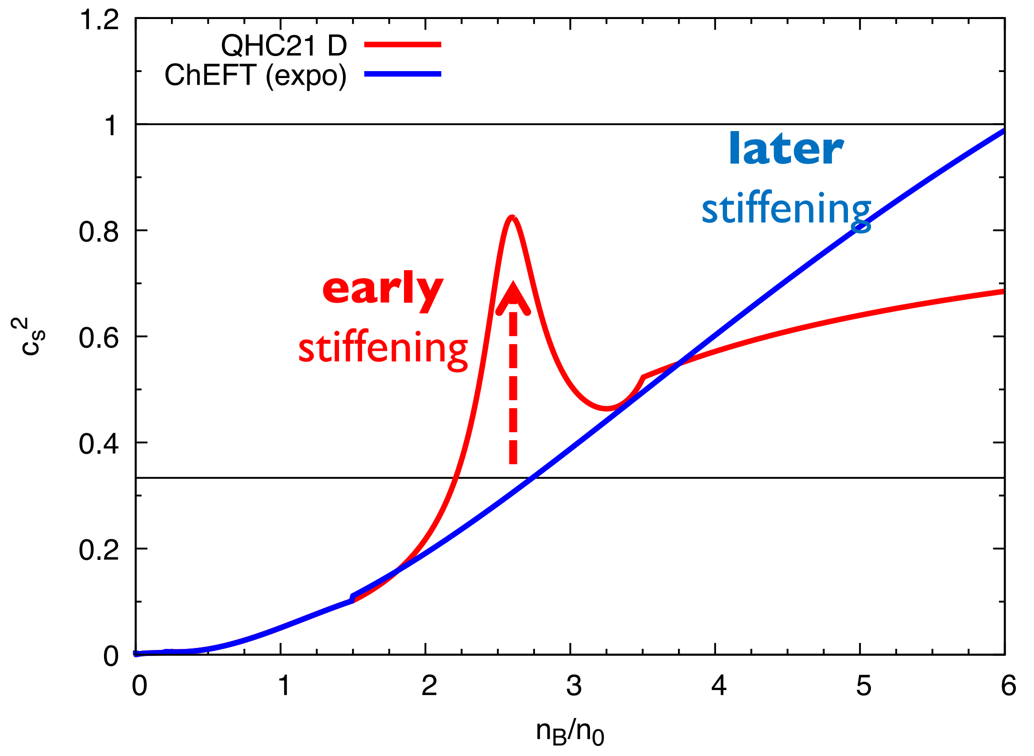




baseline: *quark-hadron continuity* **(QHC)**

Early vs later stiffening $R_{2.1M_\odot} \sim R_{1.4M_\odot}$



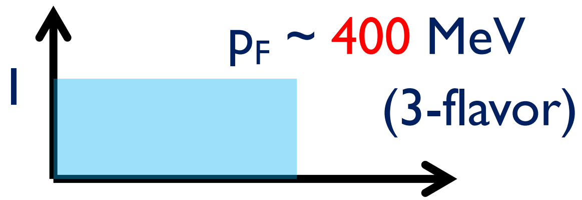


2-3n_0 : already beyond purely nucleonic regime?

Claim

- naive estimate for quark matter formation density: ($R_B \sim 0.5\text{-}0.8$ fm)

$$n_B^{overlap} \sim 1/(\,4\pi\, R_B^3/3\,) \sim 4\text{-}7 n_0$$

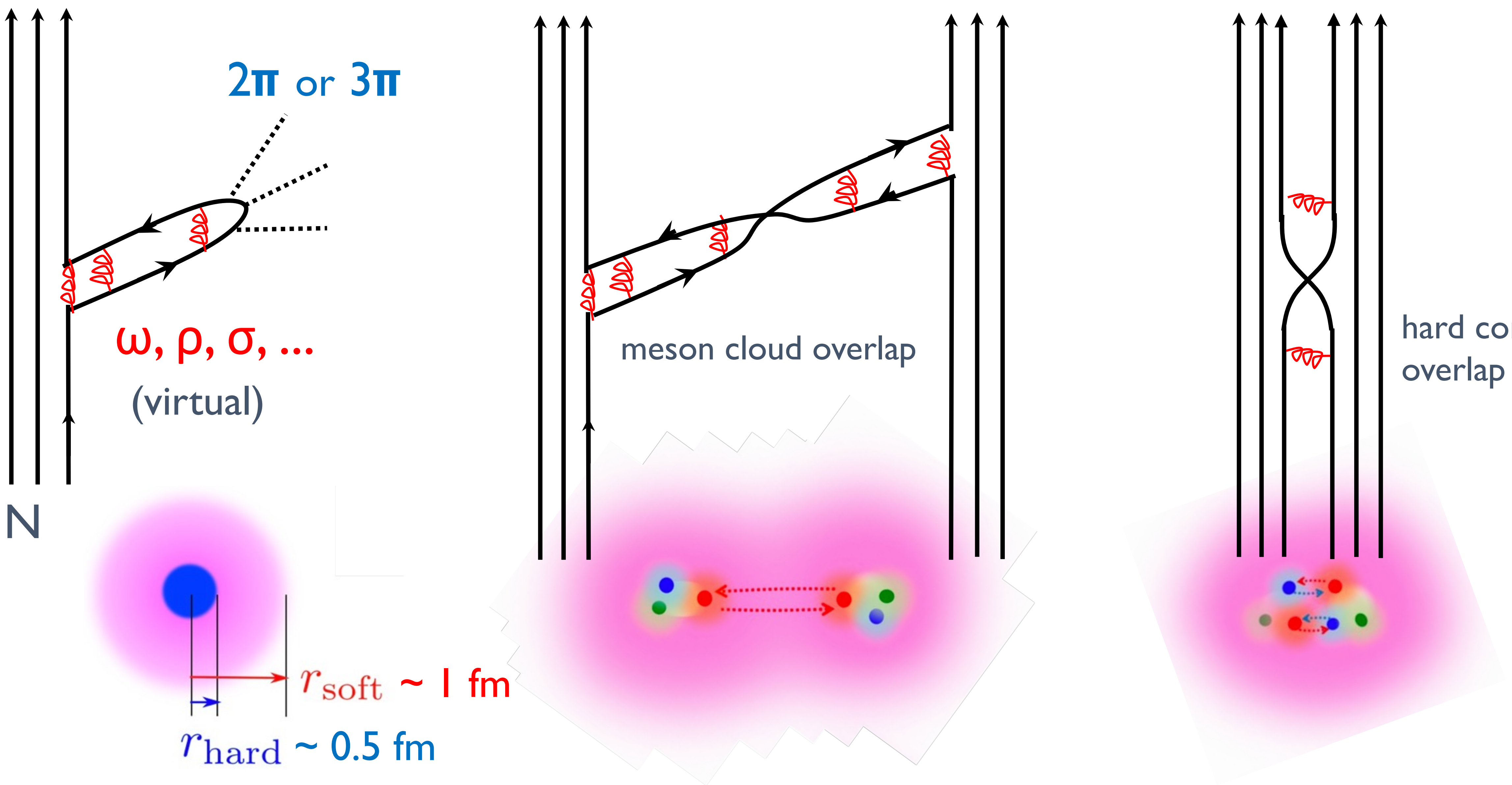


- we claim the existence of **another scale**, characterizing:
  - breakdown of many-body expansion
  - ***soft-deconfinement***
  - ***quark saturation***

$$n_B^{soft} \sim n_B^{q\text{-}sat} \sim 0.5 \times n_B^{overlap} \quad \sim 2\text{-}3 n_0$$

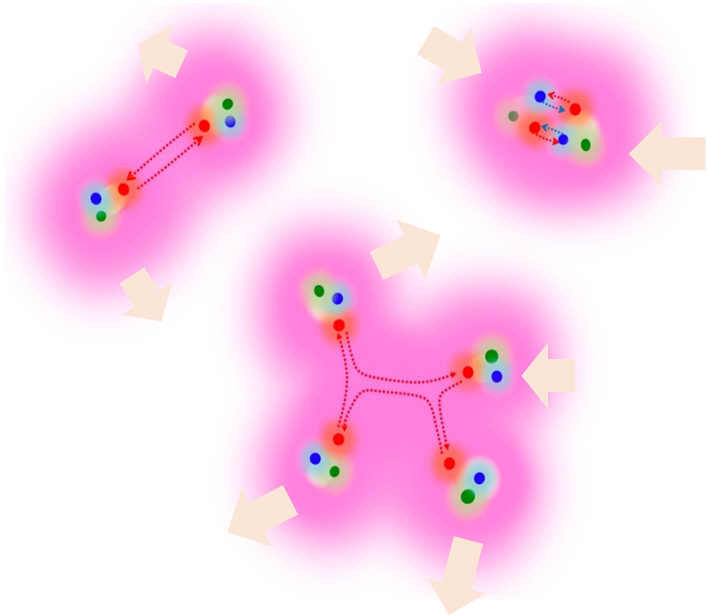
Contents

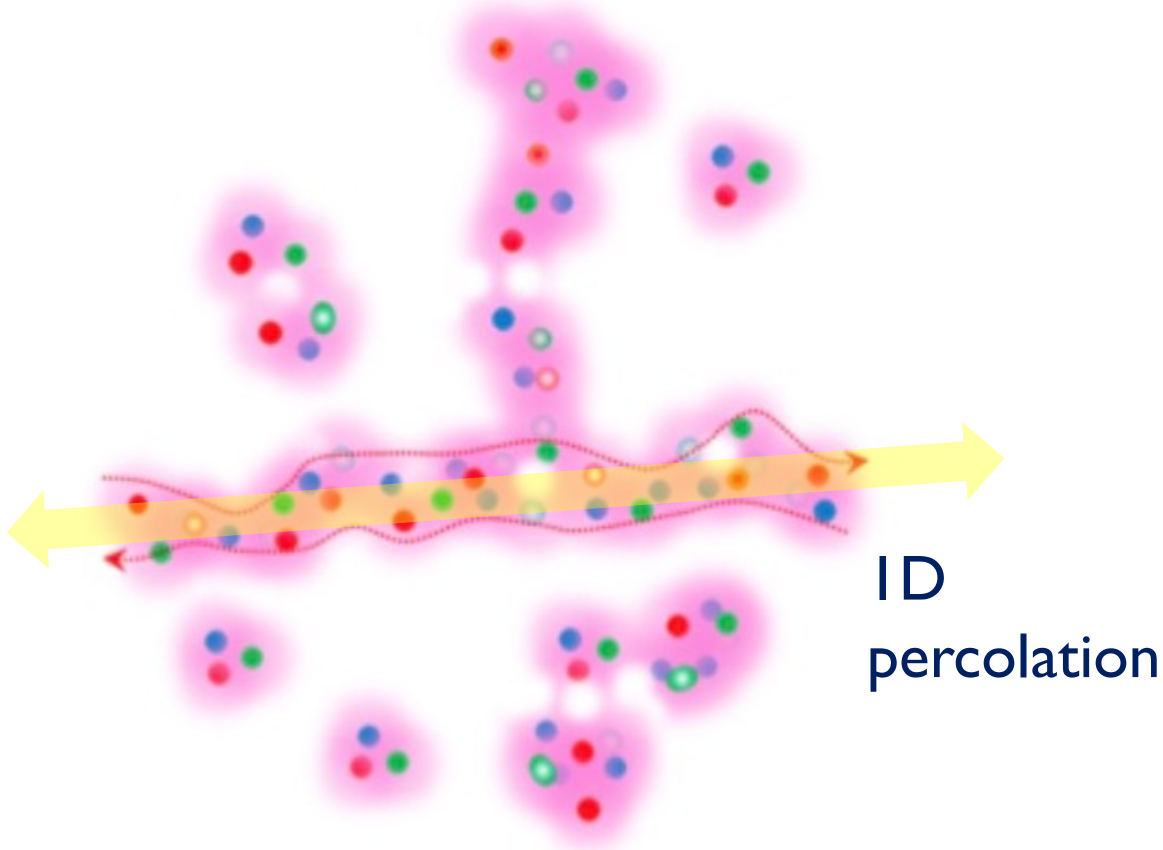
"Soft" & "Hard" scales in a nucleon



Soft Deconfinement

relating "*multi-quark exchanges*" to "*delocalization of quark w.f.*"





how can this transition be described?

Strategy (in dilute regime)

Separate **fast** quark dynamics from **slow** baryon dynamics

=> *Born-Oppenheimer* descriptions

1, The velocity : $k_B/E_B \sim 1/Nc << k_q/E_q \sim 1$ $(k_B \sim k_q \sim n_B{}^{1/3})$

$n_B = n_q{}^R = n_q{}^G = n_q{}^B$

2, Find quark eigenstates for **a given** baryon configuration

3, Take the "**time** average" $\rightarrow$ "**ensemble** average" of baryons

A model of **quantum** percolation

[Kirkpatrick-Eggarter '72,...]

tight-binding Hamiltonian

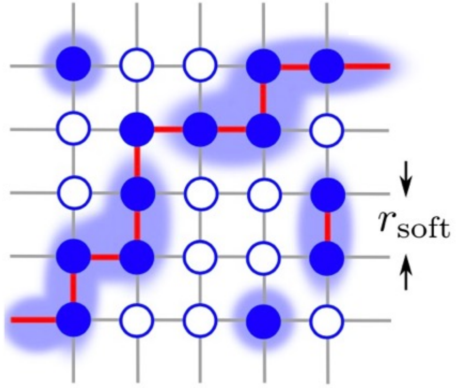
on-site energy / hopping

$$H = \sum_n |n\rangle \varepsilon_n \langle n| + \sum_{n\neq m} |n\rangle V_{nm} \langle m|$$

$|n>$: a quark state exists at a site n

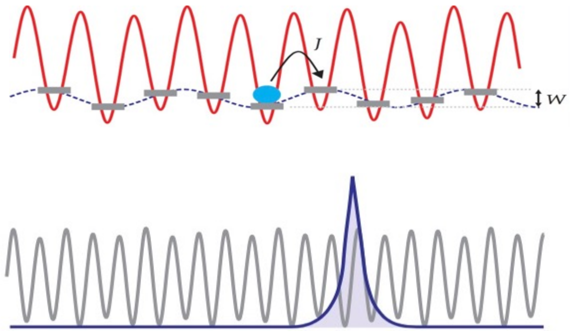
$$V_{nm} = -V \quad (V > 0)$$

nearest-neighbor hopping



- ε,V depends on a given baryon distribution
- ε → ∞ when quarks are out of baryons
- quarks hop only within connected clusters (setup)
  - → geometrical (classic) percolation must occur first

"dirty" potentials



- interference may kill amplitudes (*Anderson localization*)

connected path does not necessarily lead to delocalization

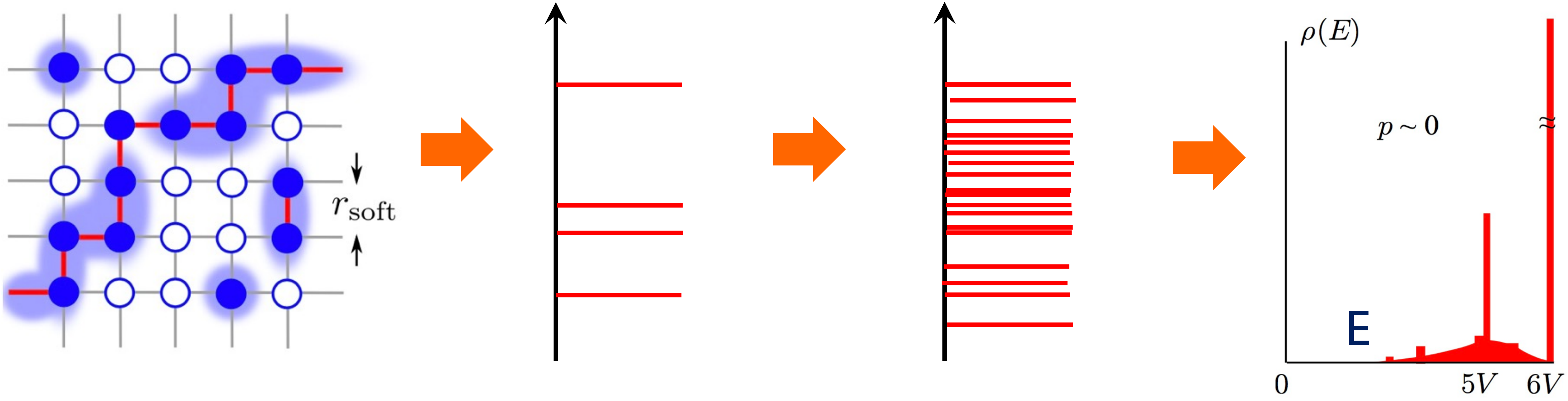
→ ***mode-by-mode*** *percolation*

(some modes delocalize earlier, the others later)

Delineating quark wavefunctions

procedures (e.g. 3D lattice model)

solve a **single** particle problem for **a given geometry** → prepare single particle levels → sum geometries (& normalize) → make histogram (take inf. vol. limit)



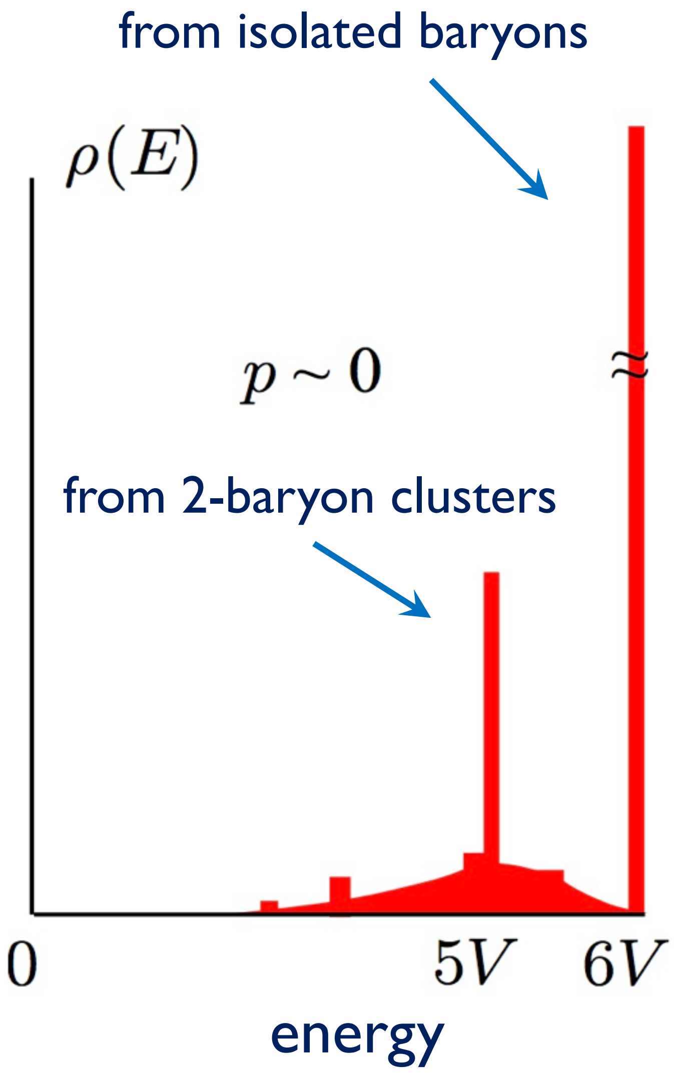
=> we diagnose the **quark contents** of given baryon configurations

quark Fermi sea & mode-by-mode percolation

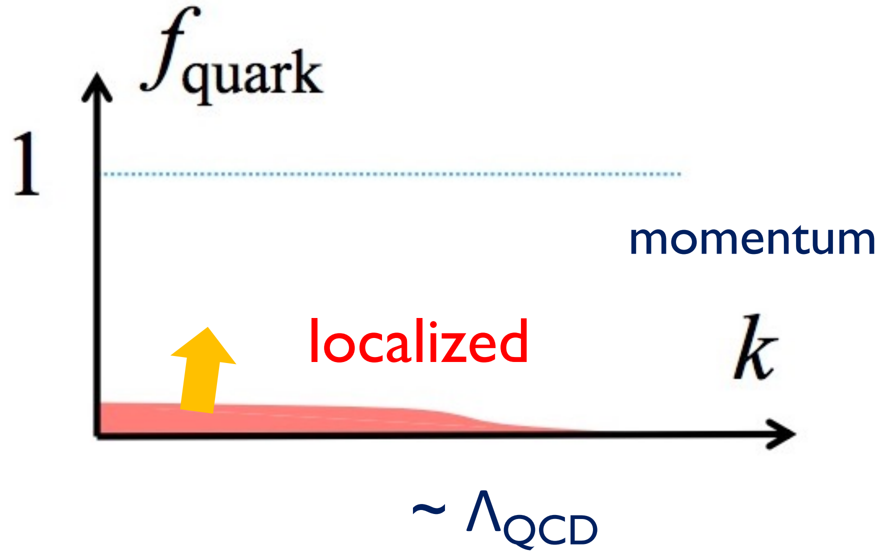
$\int dE \rho(E) = 1$

localized quark

$\rightarrow$ energy ~ R_B^{-1}



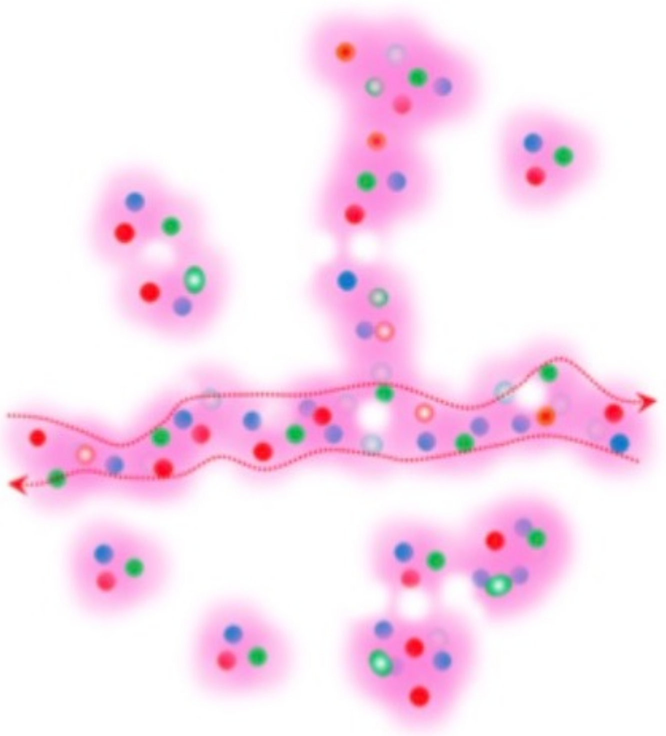
occupation probability

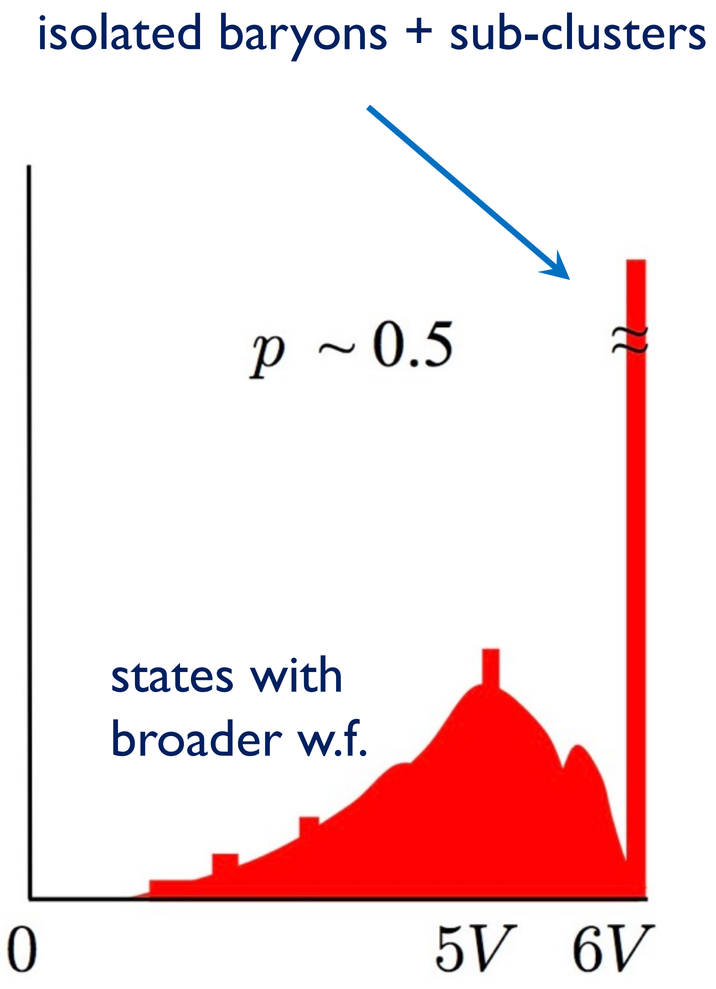


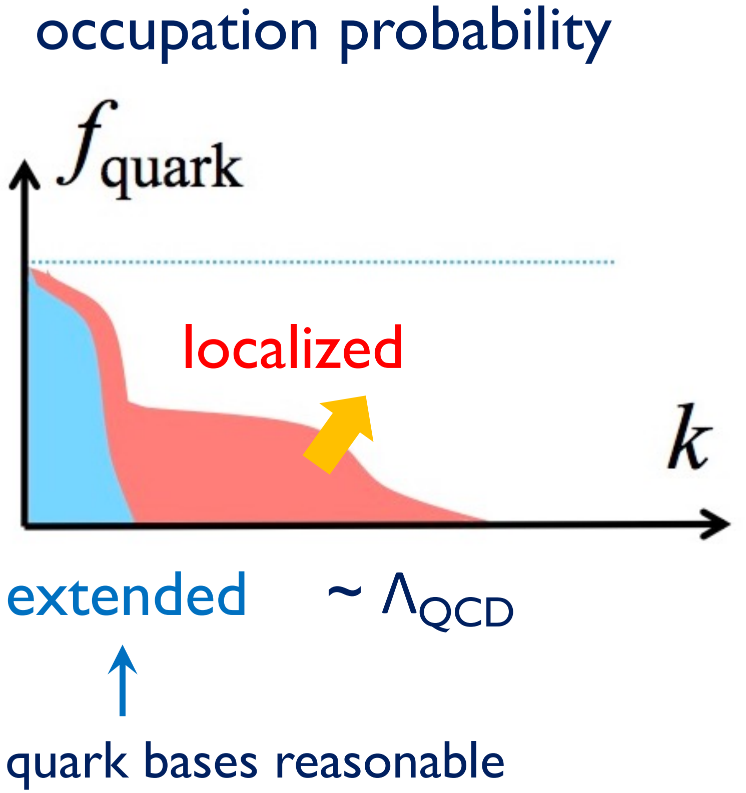
"nuclear"

quark Fermi sea & mode-by-mode percolation

$$\int dE \rho(E) = 1$$



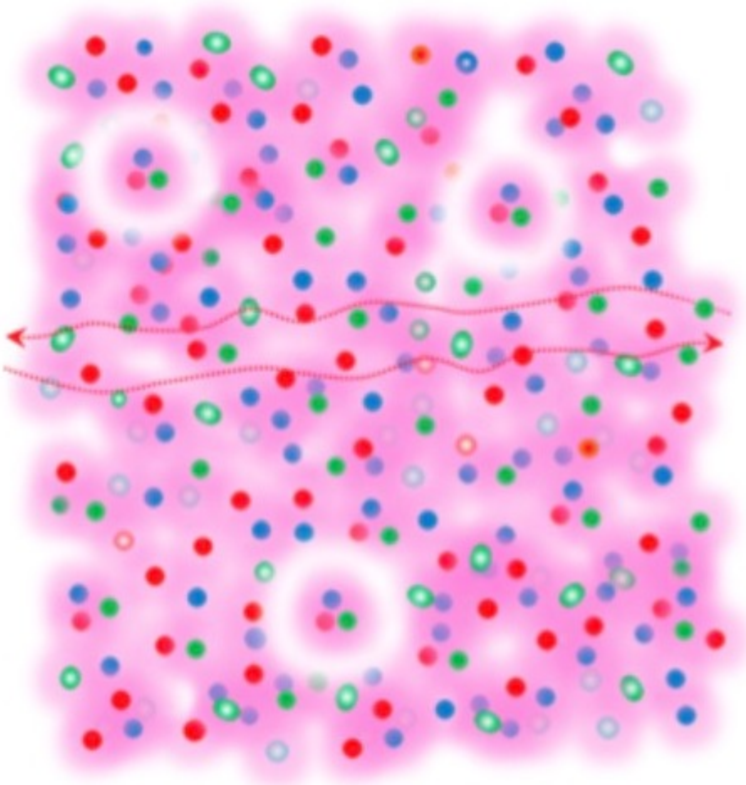


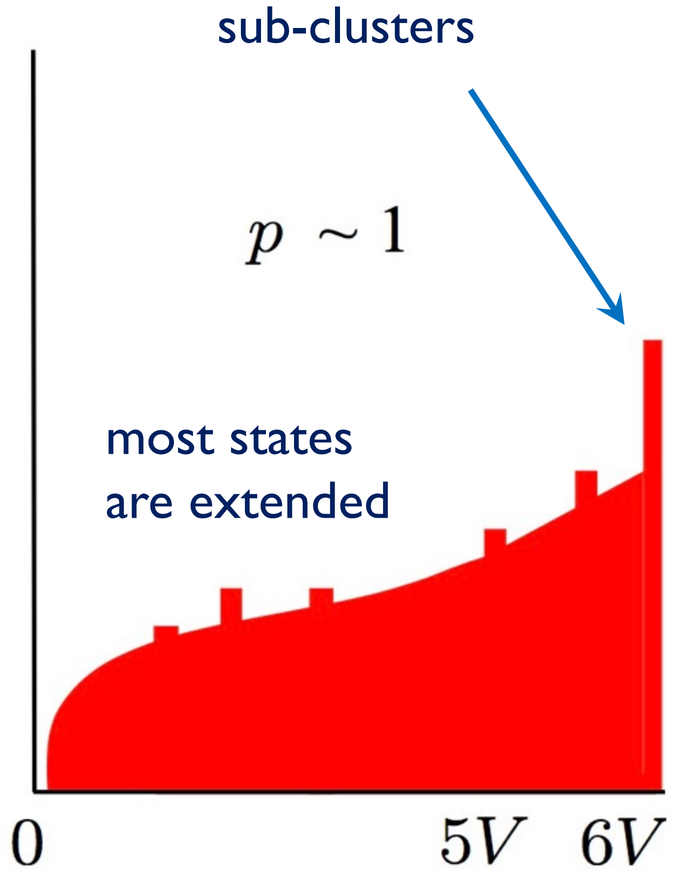


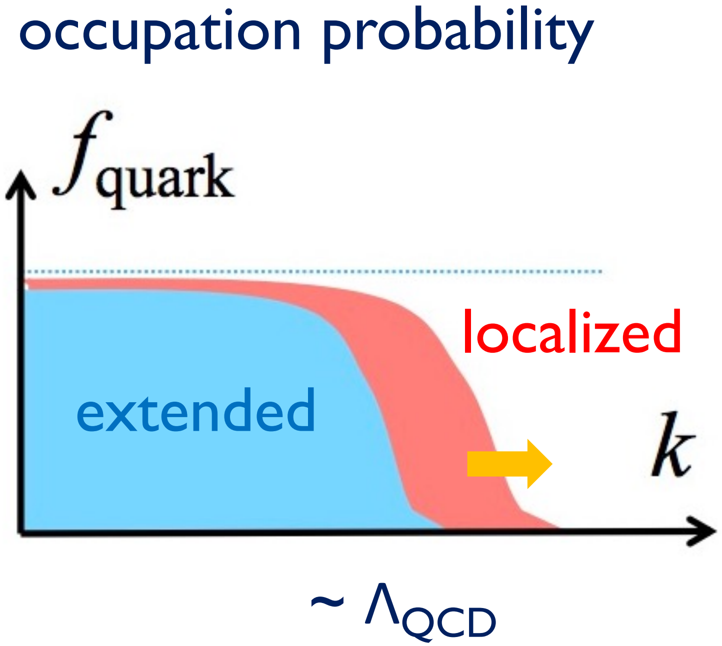
"quark-hadron continuity"

quark Fermi sea & mode-by-mode percolation

$$\int dE \rho(E) = 1$$



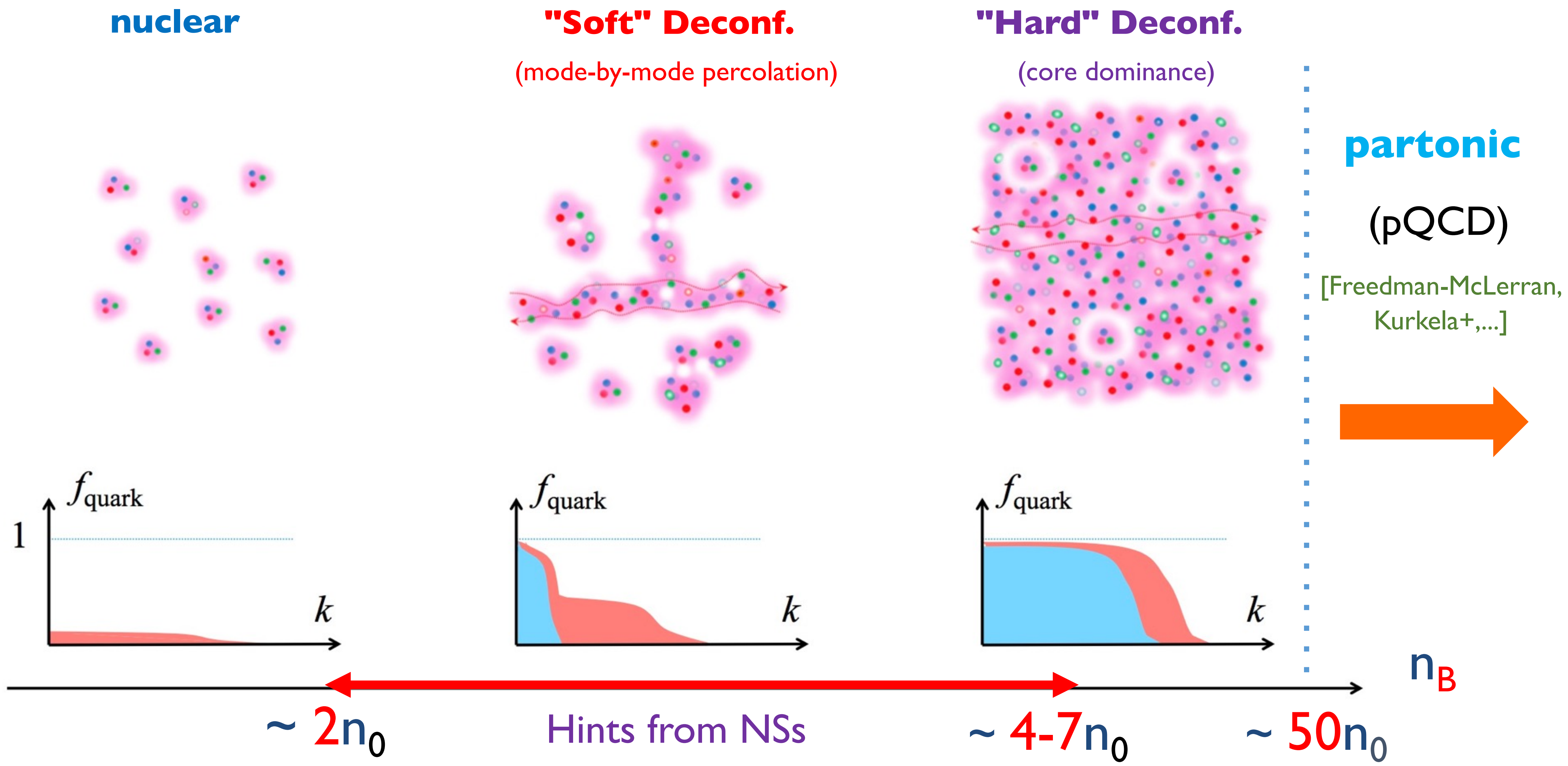




"quarkyonic"

[McLerran-Pisarski ’06,...]

a cartoon


nuclear
"Soft" Deconf.
(mode-by-mode percolation)
"Hard" Deconf.
(core dominance)
partonic
(pQCD)
[Freedman-McLerran,
Kurkela+,...]
f_{quark}
1
k
f_{quark}
k
f_{quark}
k
n_B
$\sim 2n_0$
Hints from NSs
$\sim$ 4-7n_0
$\sim 50n_0$

Contents

Next step ?

how can we go beyond the Born-Oppenheimer picture ?

how can we explain the sound velocity peak?

how can we discuss many hadron (e.g. Δ, Σ, Λ, ..) species?

how can we implement the insights into practical models ?

we propose a simple model of quark-hadron duality

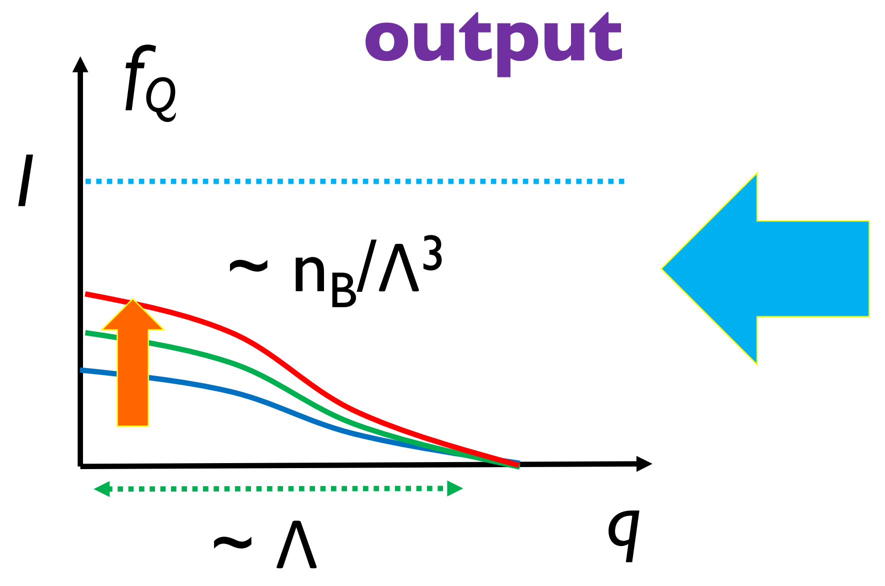
Sum rules for occupation probabilities

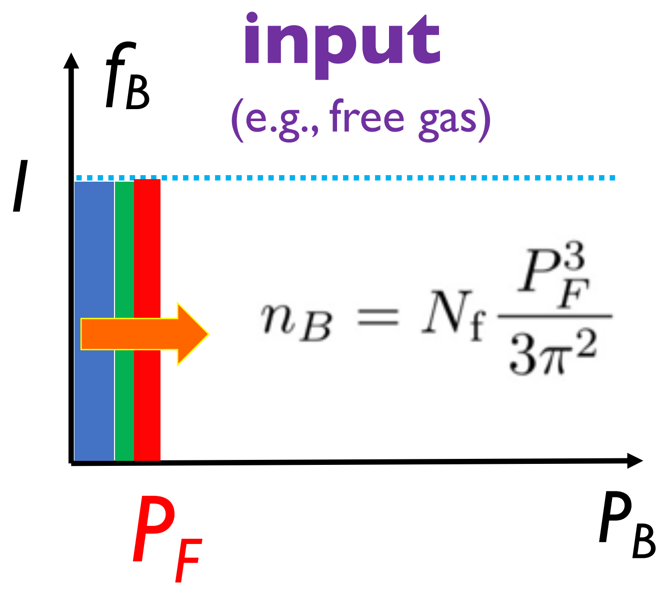
cf) [TK '21, TK-Suenaga '21]

occupation **probability** of **quark** state with p	occupation **probability** of **baryon** state with P_B	**quark** mom. distribution **in a baryon**

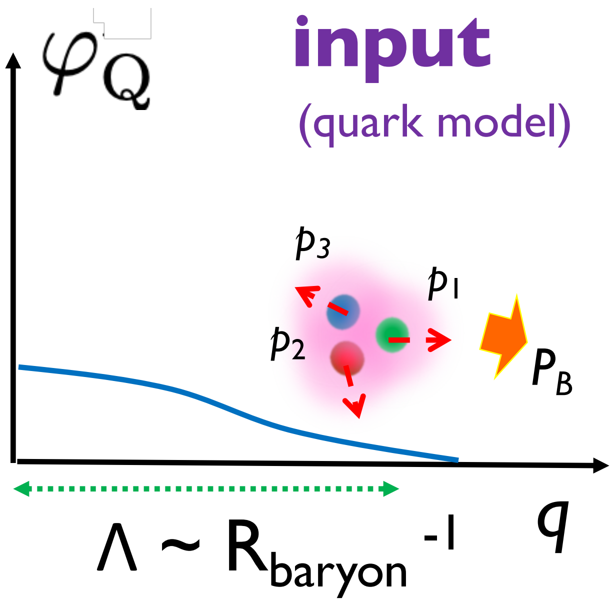
$$f_Q(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B)\,\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_c)$$

e.g.) in **ideal** baryonic matter





$\otimes$



An **ideal** model

[Fujimoto-TK-McLerran, PRL'23]

1) neglect interactions *except* confining forces

isospin, spin

e.g.) 2-flavor hamiltonian: $$\varepsilon_{\mathrm{B}}[f_{\mathrm{B}}] = 4\int_k E_{\mathrm{B}}(k) f_{\mathrm{B}}(k)$$

2) quark distributions in a baryon remains the same (confinement persists)

3) use a special quark distribution → models become analytically *solvable*

$$\varphi_{3\mathrm{d}}(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3}\frac{e^{-q/\Lambda}}{q/\Lambda} \qquad \hat{L} = -\boldsymbol{\nabla}^2 + \frac{1}{\Lambda^2} \qquad \hat{L}[\varphi(\boldsymbol{p}-\boldsymbol{q})] = \frac{(2\pi)^3}{\Lambda^2}\delta(\boldsymbol{p}-\boldsymbol{q})$$

nontrivial output

$$f_{\mathrm{Q}}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\mathrm{B}}(\boldsymbol{P}_B)\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\mathrm{c}})$$

natural at **low** density

nontrivial output

$$f_{\mathrm{B}}(N_{\mathrm{c}}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\mathrm{c}}^3}\hat{L}[f_{\mathrm{Q}}(\boldsymbol{q})]$$

natural at **high** density

useful for studies of the *transient regime* (d.o.f are not clear-cut)

Variational problem with sum rule constraints

[Fujimoto-TK-McLerran, PRL'23]

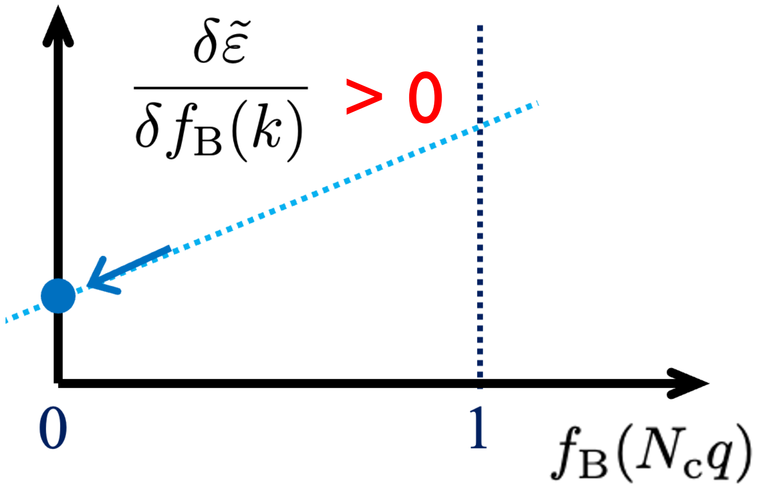
constraint to fix n_B

$$\tilde{\varepsilon} = \varepsilon_{\rm B}[f_{\rm B}] - \lambda_{\rm B} n_{\rm B}$$

$$E_{\rm B}(k) = \sqrt{M_{\rm B}^2 + k^2} \qquad n_{\rm B} = 4\int_k f_{\rm B}(k)$$

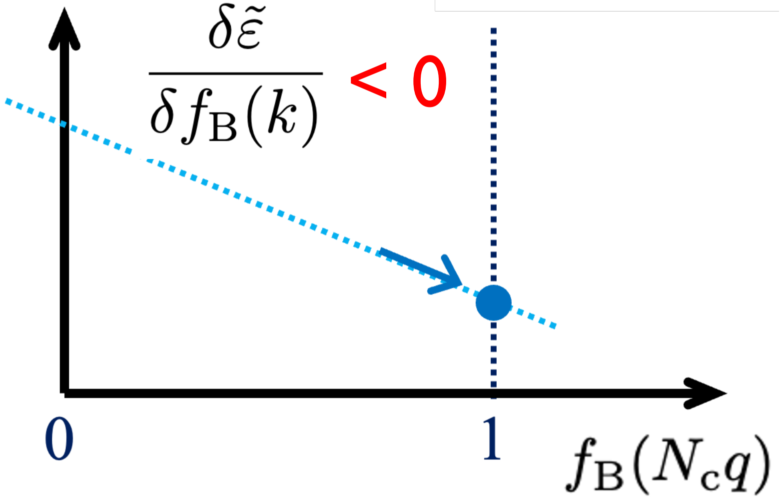
optimization: $$\frac{\delta \tilde{\varepsilon}}{\delta f_{\rm B}(k)} = E_{\rm B}(k) - \lambda_{\rm B}$$ at a given k

$E_B(k) > \lambda_B$

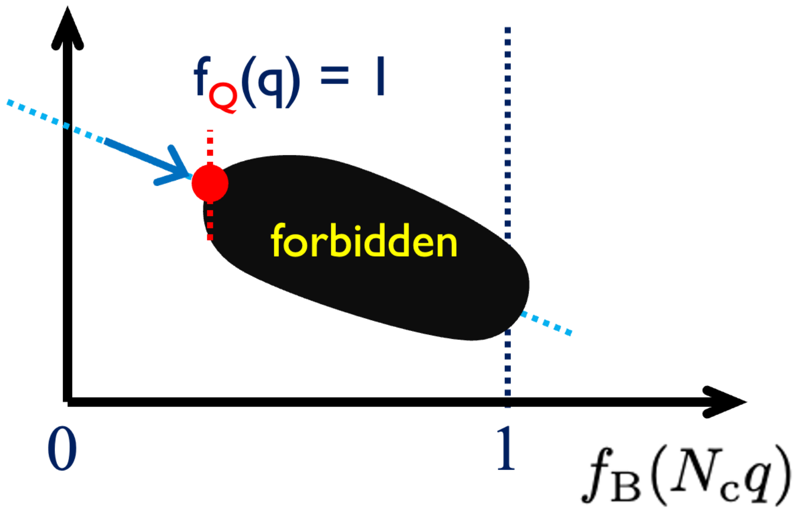


$E_B(k) < \lambda_B$

w.o. saturation

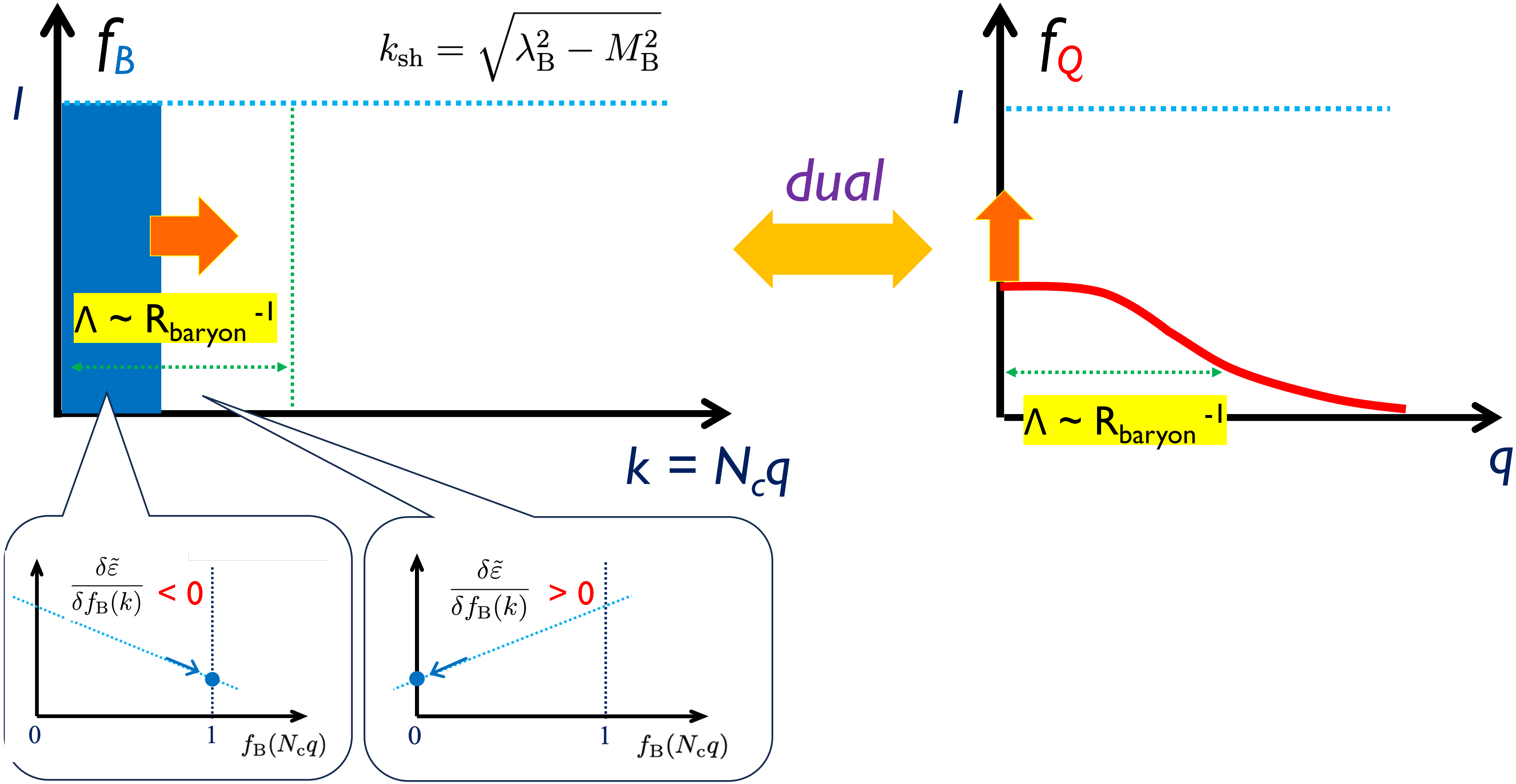


with saturation



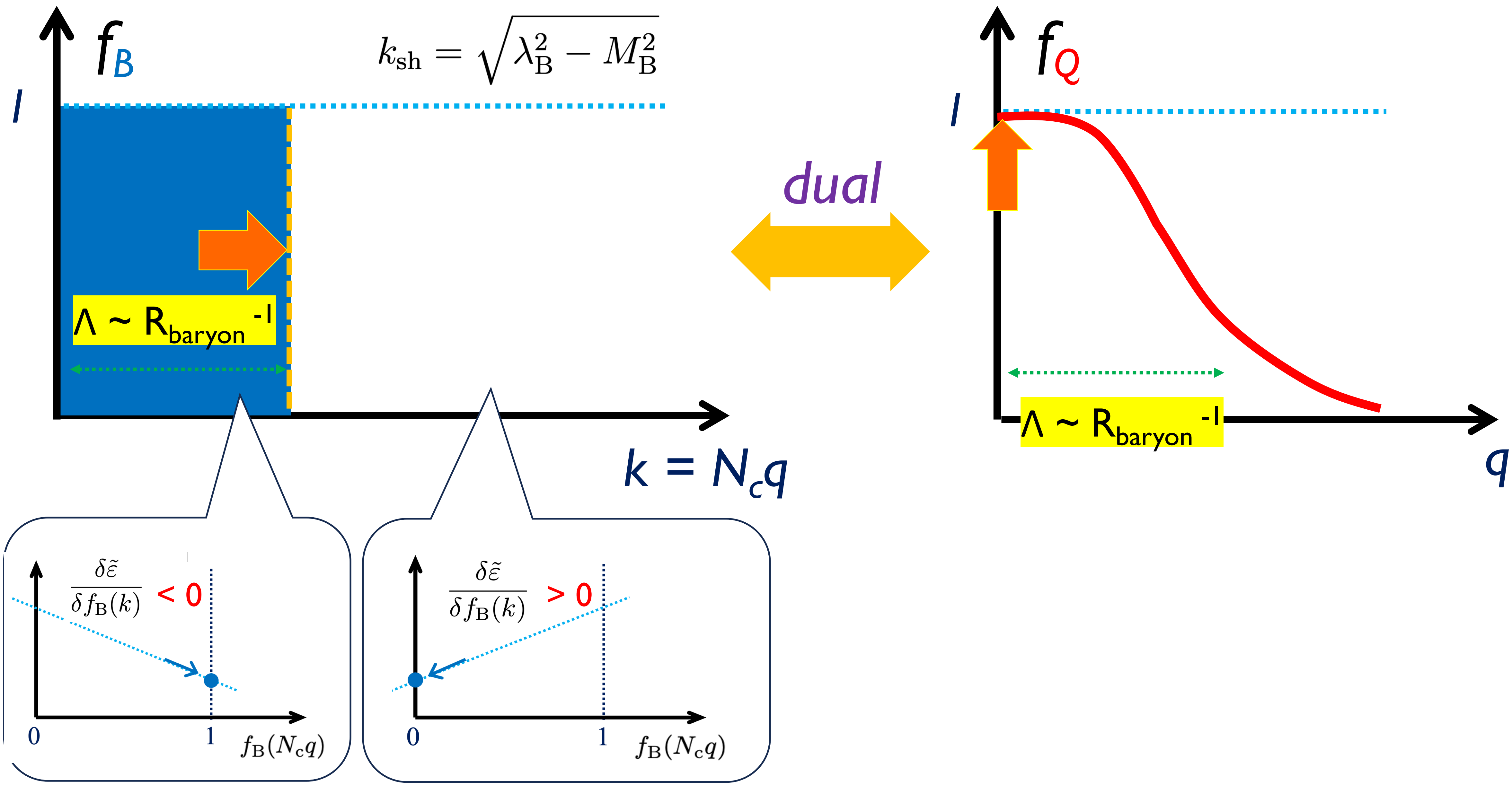
Solution (dilute regime)

[Fujimoto-TK-McLerran, PRL'23]



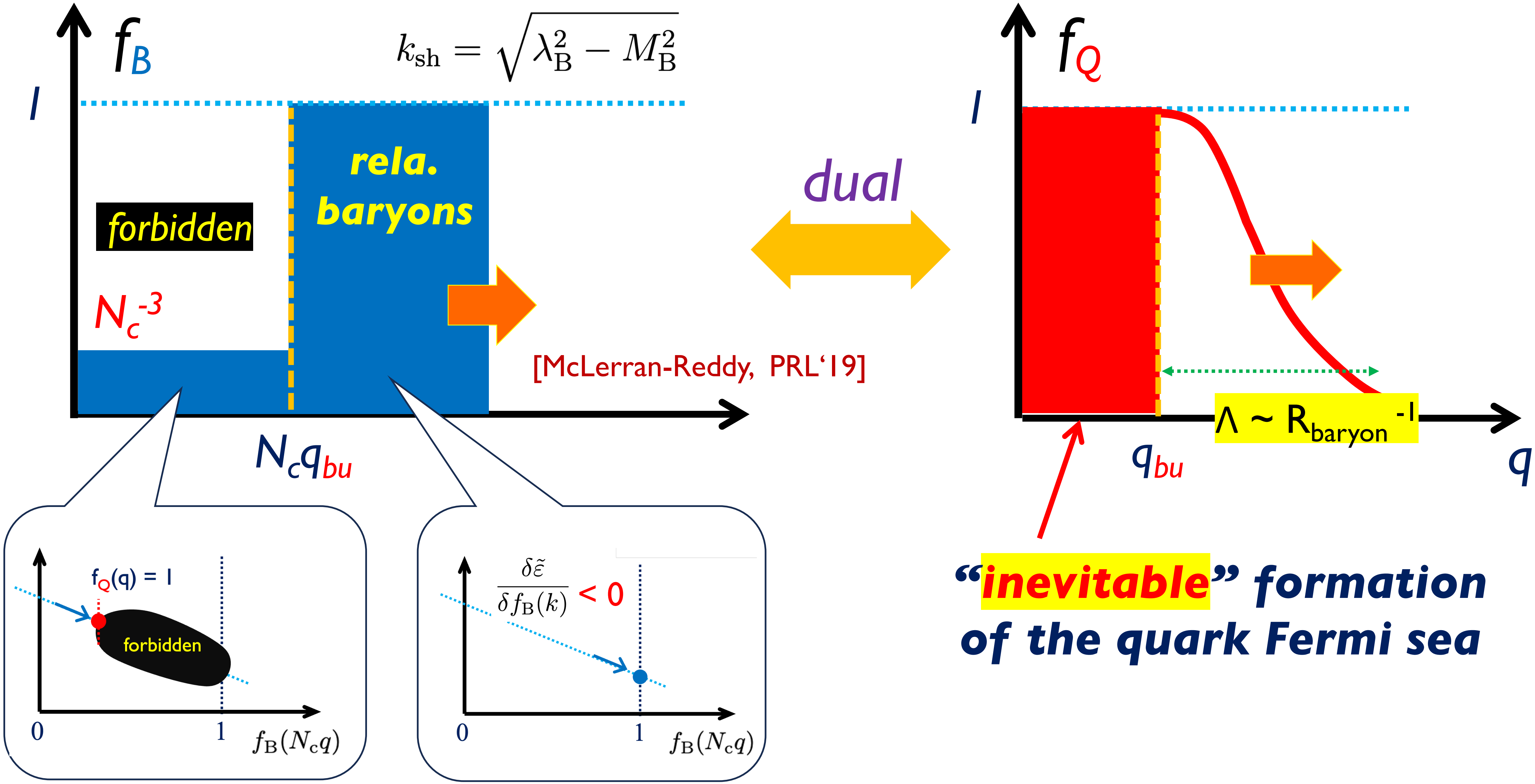
Solution (at saturation)

[Fujimoto-TK-McLerran, PRL'23]



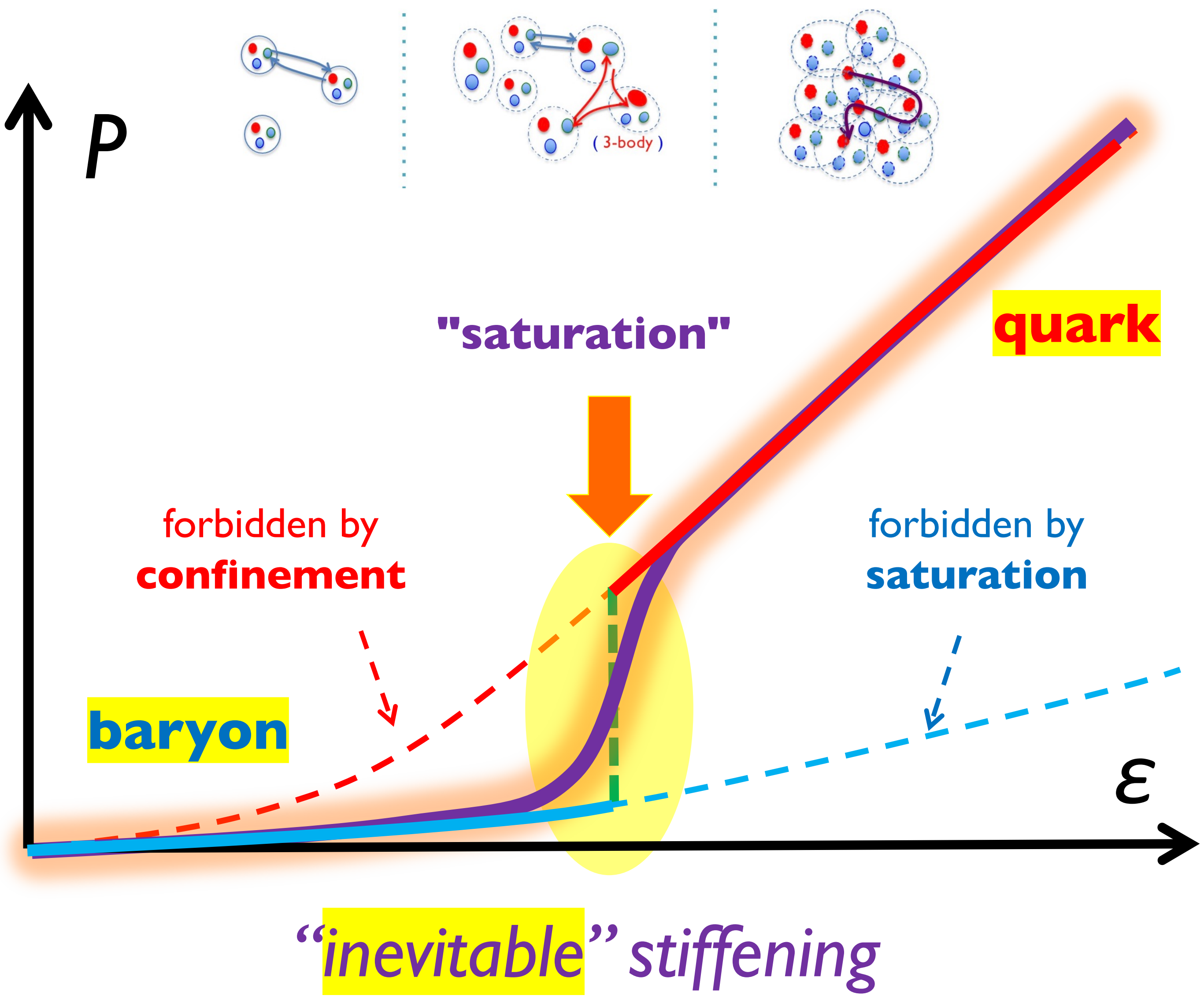
Solution (post saturation)

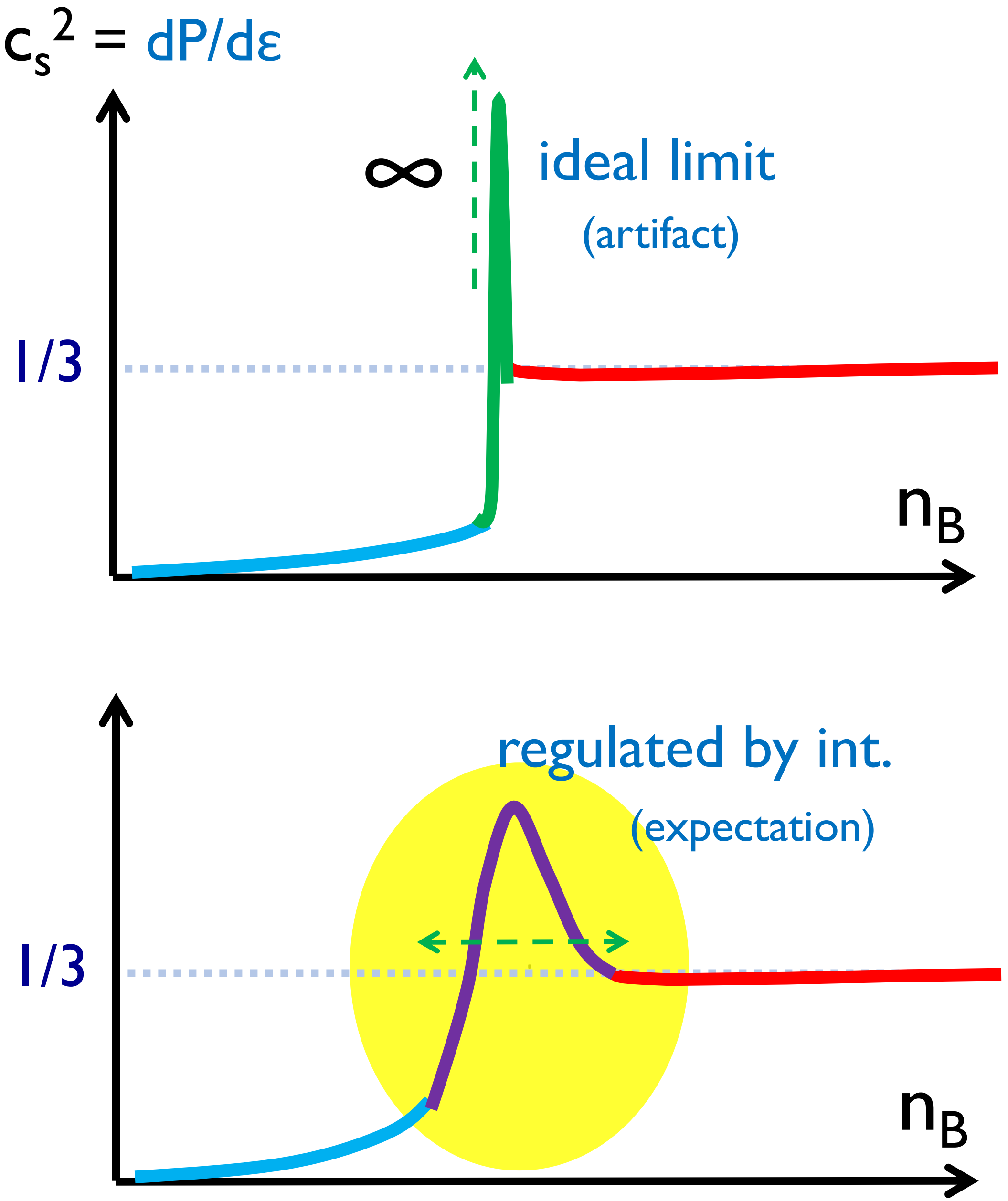
[Fujimoto-TK-McLerran, PRL'23]



"inevitable" formation of the quark Fermi sea

Peak in sound velocity



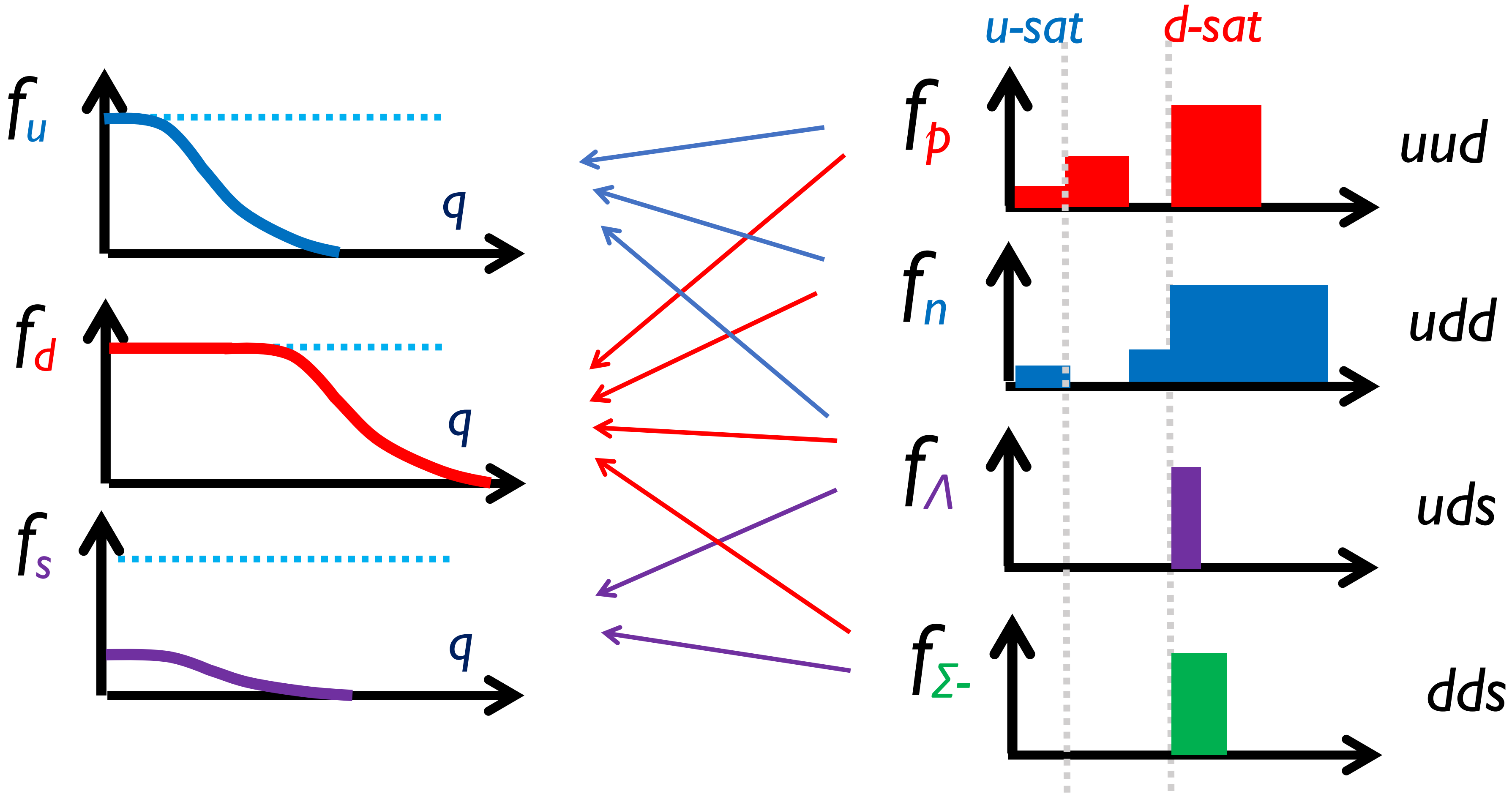


Multi-hadron extension

[Fujimoto-TK-McLerran, in prep.]

$$f_{\mathrm{Q}}(\boldsymbol{q}) = \sum_{B=p,n,\Sigma,\cdots} N_{\mathrm{Q}}^{\mathrm{B}} \int_{\boldsymbol{k}} f_{\mathrm{B}}(\boldsymbol{k})\varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right)$$

Q = u, d, s



and so on...

saturation constraints

flavor correlations

Summary

- Soft-to-stiff EOS
- quark-exchange and soft-deconfinement
- quark saturation effects, can be relevant at 2-3n_0 (!)
- not addressed (please ask in Q&A or personally) :

  relevant interactions at high density

  hyperon puzzle

  finite temperature effects

  proof of concept: isospin QCD in lattice simulations

  ....

Back Up

Quantum numbers ?

quark quantum numbers; N_c, N_f, 2-spins (for a given spatial w.f.)

how many baryon species are needed to saturate quark states?

→ need only $\mathbf{2N_f}$ = **6** species for $N_f = 3$

(full members of singlet, octet, decuplet are NOT necessary)

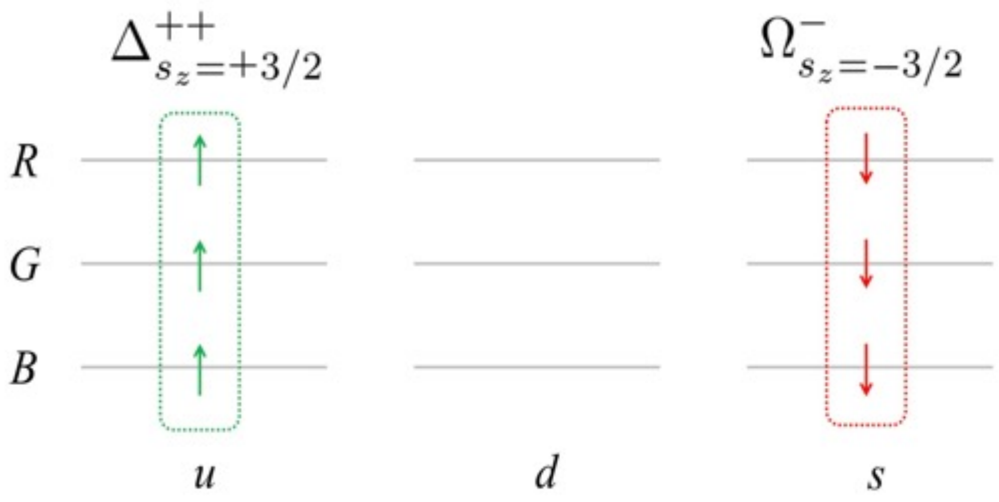
convenient **color-flavor-spin** bases

[neglect N-⊿ splitting etc. for simplicity]

$$\Delta^{++}_{s_z=\pm 3/2} = [u_R\uparrow u_G\uparrow u_B\uparrow], \quad [u_R\downarrow u_G\downarrow u_B\downarrow],$$

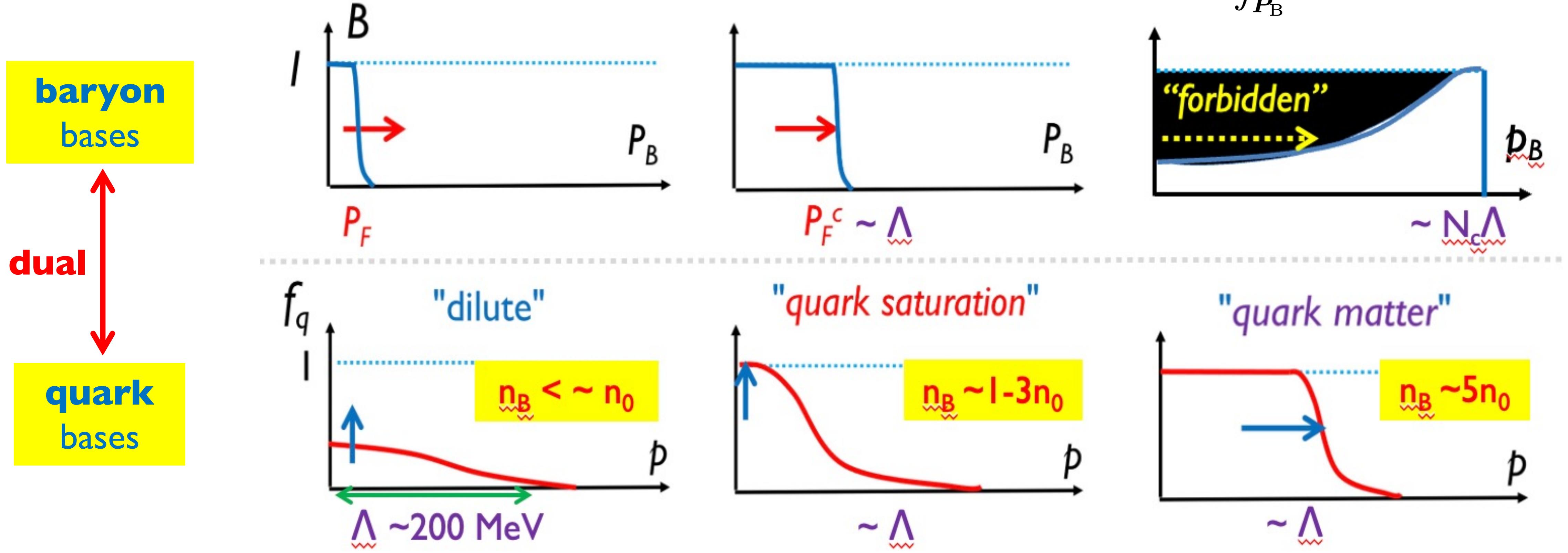
$$\Delta^{-}_{s_z=\pm 3/2} = [d_R\uparrow d_G\uparrow d_B\uparrow], \quad [d_R\downarrow d_G\downarrow d_B\downarrow],$$

$$\Omega^{-}_{s_z=\pm 3/2} = [s_R\uparrow s_G\uparrow s_B\uparrow], \quad [s_R\downarrow s_G\downarrow s_B\downarrow],$$



Evolution of occ. probabilities

$$f_{\rm Q}(\boldsymbol{q}; n_{\rm B}) = \int_{\boldsymbol{P}_{\rm B}} f_{\rm B}(\boldsymbol{P}_{\rm B}; n_{\rm B}) \varphi_{\rm Q}^{\rm B}(\boldsymbol{q}; \boldsymbol{P}_{\rm B})$$



"quark saturation" constraint

→ **relativistic baryons at low density, n_B ~ 1-3n_0 !**

cf) McLerran-Reddy model (2019); microscopic description, TK (2021)

Early vs late stiffening

purely nucleonic descriptions typically lead to late stiffening

$$\varepsilon(n_B) = m_N n_B + a\frac{n_B^{5/3}}{m_N} + bn_B^\alpha \quad \Rightarrow \quad P = \frac{2}{3}a\frac{n_B^{5/3}}{m_N} + b(\alpha-1)n_B^\alpha$$

large (!) small (!) small (!)

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

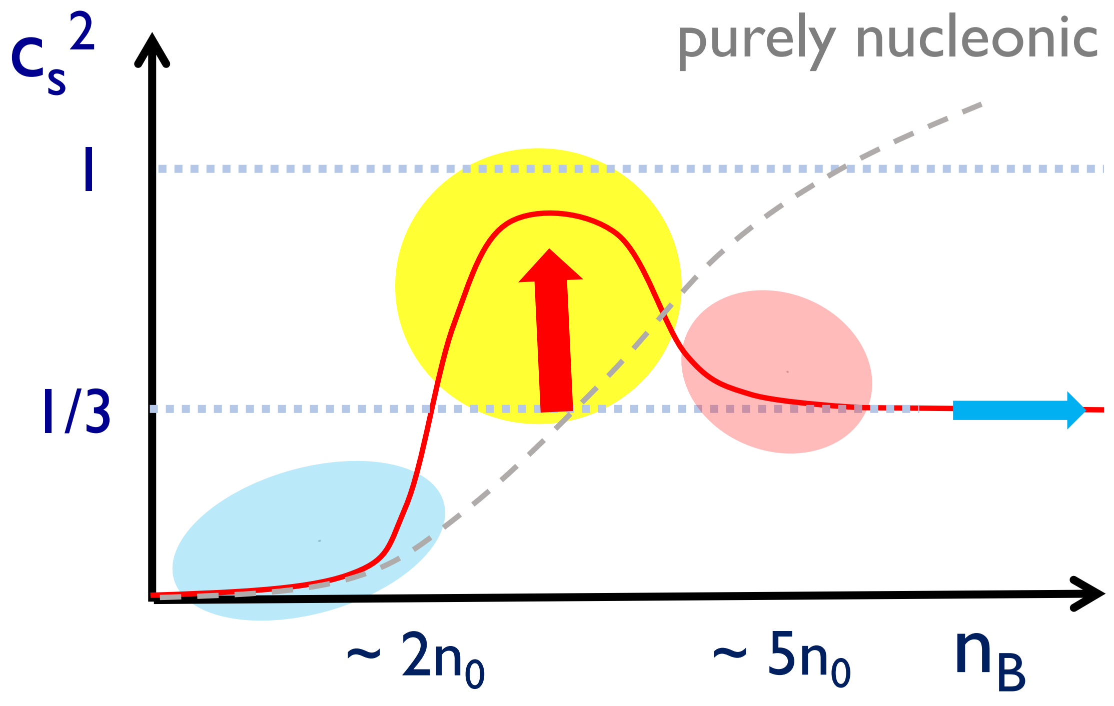
➡ at LO: p << ε (!)

if interactions dominate (at large n_B):

$$P \sim (\alpha - 1)\varepsilon \;\to\; c_s^2 \sim (\alpha - 1)$$

2-body int. → α = 2 3-body int. → α = 3

but power terms grow rather slowly...

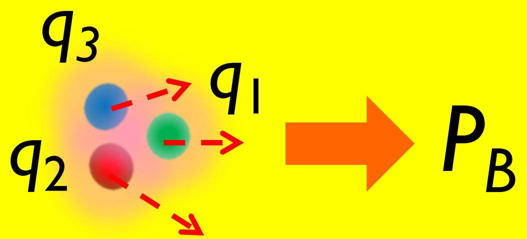


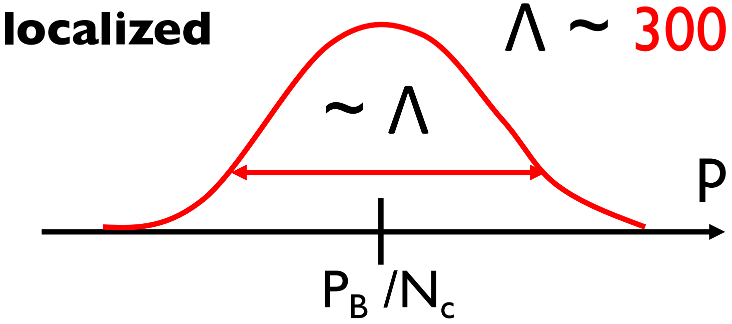
Quarks in a baryon

N_c (=3): number of colors

probability density:

$$\varphi(\boldsymbol{q};\boldsymbol{P}_{\rm B}) = \mathcal{N} e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{q}-\frac{\boldsymbol{P}_{\rm B}}{N_{\rm c}}\right)^2}$$





$\Lambda \sim$ 300 MeV

variance: $\left\langle \left(\boldsymbol{p}-\frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$ *energetic !*

→ large "*mechanical*" pressure

$$\langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B} = \mathcal{N}\int_{\boldsymbol{p}} E_q(\boldsymbol{p}) e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p}-\frac{\boldsymbol{P}_B}{N_c}\right)^2} \simeq \langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B=0} + \frac{1}{6}\left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i}\right\rangle_{\boldsymbol{P}_B=0}\left(\frac{\boldsymbol{P}_B}{N_c}\right)^2 + \cdots$$

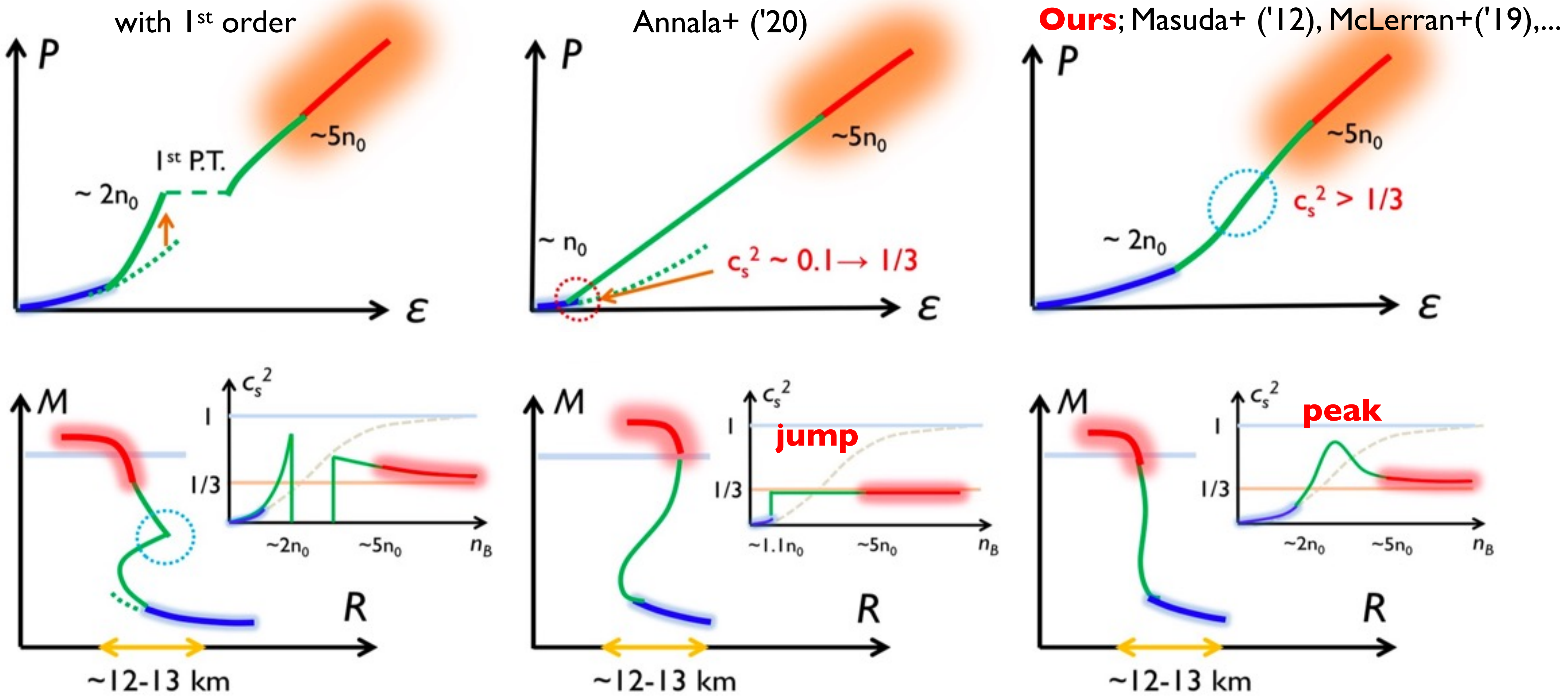
average energy (quark)

x N_c x N_c

$\sim N_c (M_q + E_{kin})$ ≫ $\sim P_B^2 / (N_c E_q)$

baryon mass baryon kin. energy

Three possible scenarios



→ *$R_{1.4}$ and $R_{2.1}$?* → *nuclear physics ?* → *my favorite*

alternative **baseline**: quark EOS

e.g.) free massless quarks

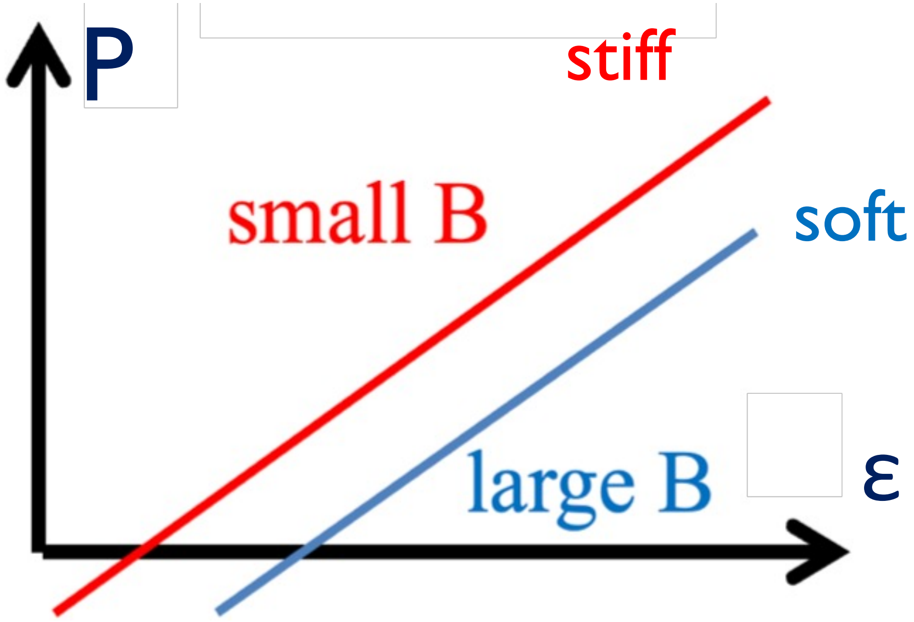
$$P = \frac{\varepsilon}{3} - B'$$

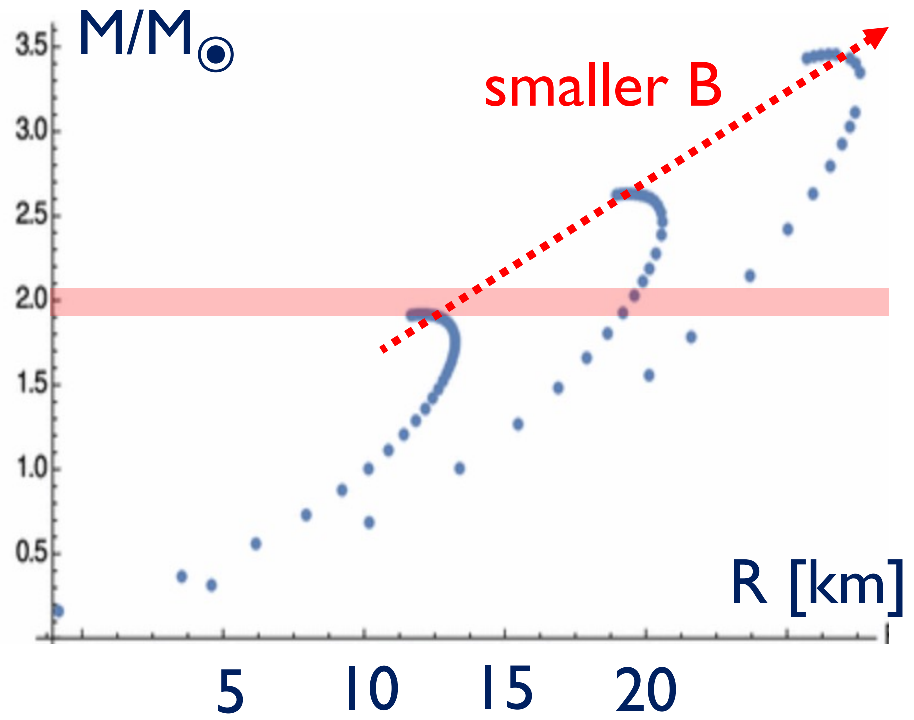
$c_s^2 = 1/3$

normalization

quark kin. E ~ N_c^2 x nucl. kin. E

~ $N_c \times p_F^2/M_q$ ~ $P_F^2/N_c M_q$





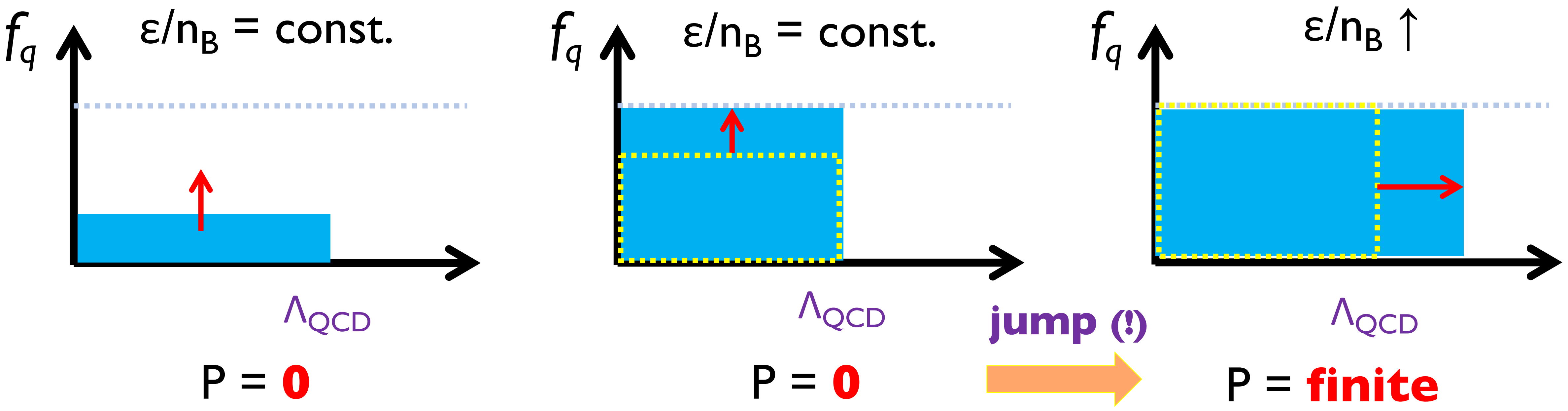
relativistic pressure → stiff EOS

can be a good starting point!?

Jump in pressure : schematic picture

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

energy per particle



f_q continuous → ε, n_B are continuous

Quarks do contribute to ***ε*** *even before saturation; but to P only after the saturation!!*

Stiff quark matter

The appearance of c_s^2 peak is characteristic in the QHC scenarios, but is not sufficient condition for ~ 2.1-2.3$M_\odot$ NS.

Just after the crossover, quarks are not fully relativistic.

Can the chiral restoration makes quarks massless and stiffens EOS?

Unlikely: it adds "*the bag constant*" to the energy density! (look at Dirac sea!)

→ ε increases & P decreases: **significant** softening!

Now, we consider interactions on top of IdylliQ models.

Underlying picture (guess)

- *Gluons remain non-perturbative at 5-10n_0*

  (see, e.g., lattice results for 2-color & isospin QCD)

- *Chiral restoration occurs mildly*

} implicitly included in IdylliQ type models

- *Continuity*: interactions in quark matter should have *natural counterpart* in hadron physics

Short range correlations in a baryon:

my favorite: *color-electric* & *magnetic interactions*

Simple parametric analyses

[TK-Powell-Song-Baym, '14]

rela. kin. energy — interactions

$$\varepsilon(n) = a n^{4/3} + b n^{\alpha}$$

(n: quark density)

→

ideal — interactions

$$P = \frac{\varepsilon}{3} + b\left(\alpha - \frac{4}{3}\right) n^{\alpha}$$

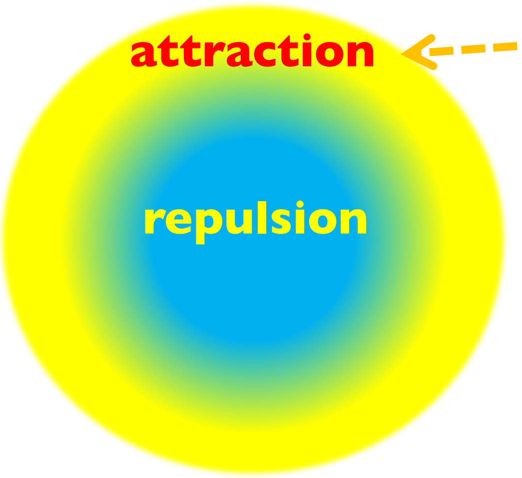
For stiff EOS:

(for large P)

for $\alpha > 4/3$: $b > 0$ (e.g. bulk repulsion, ~ $+ n_B^2/\Lambda^2$)

for $\alpha < 4/3$: $b < 0$ (e.g. surface pairings, ~ $- \Lambda^2 n_B^{2/3}$)

quark
Fermi sea
(ideal combo)



2- or 3-quark correlations

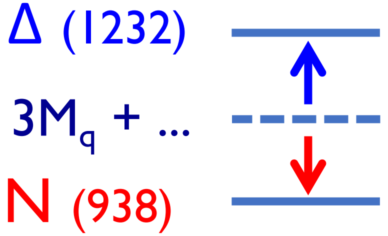
Stiff EOS from **attractive forces**

Color-magnetic interaction play **many** roles

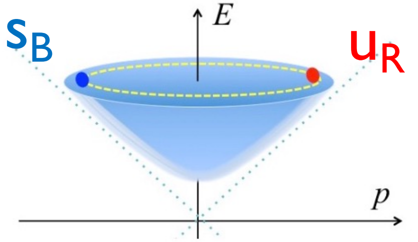
1) **Coupling** ∝ **velocity** ~ **p/E**

become important in relativistic regime & high density

cf)



2) **Pairing** : strongly channel dependent



hadron mass ordering: N-Δ, etc. [DeRujula+ (1975), Isgur-Karl (1978), ...]

color-super-conductivity [Alford, Wilczek, Rajagopal, Schafer,... 1998-]

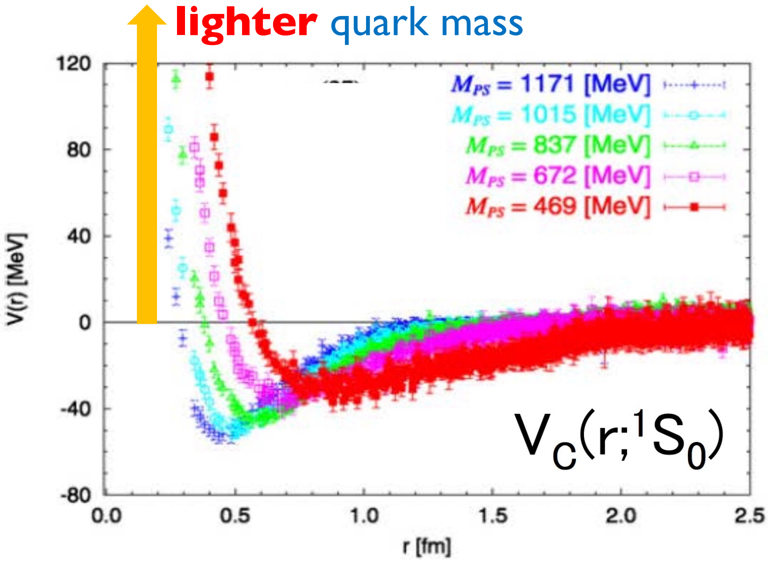
3) **Baryon-Baryon int.** : **short-range** correlation

(**Pauli** + **color-mag.**) [Oka-Yazaki (1980),...]

channel dep. → non-universal hard core (some are attractive!)

mass dep. → stronger hard core in relativistic quarks

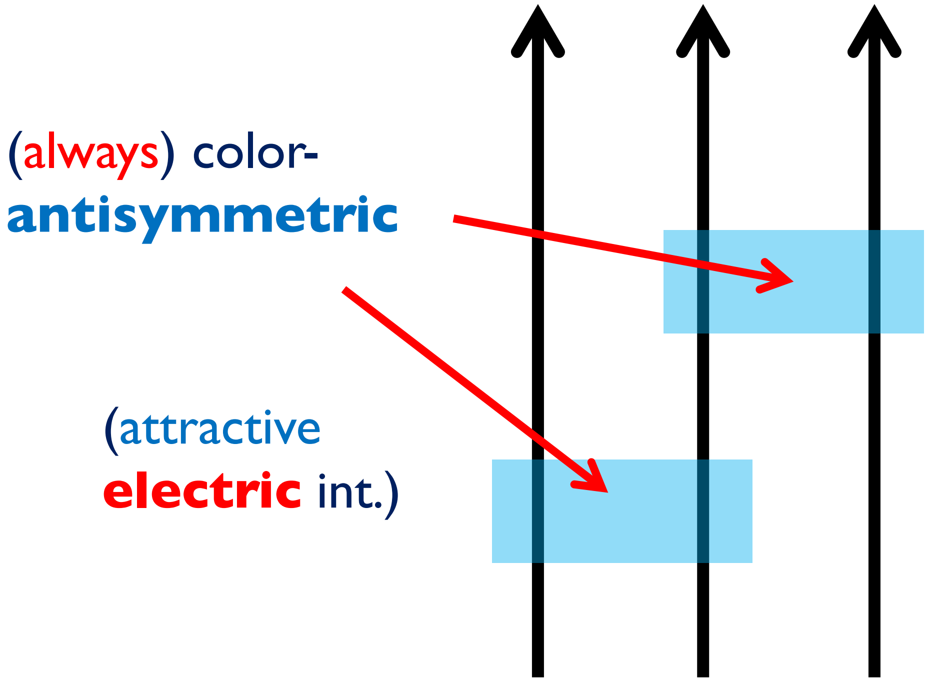
→ **consistent with the lattice QCD** [HAL-collaboration]



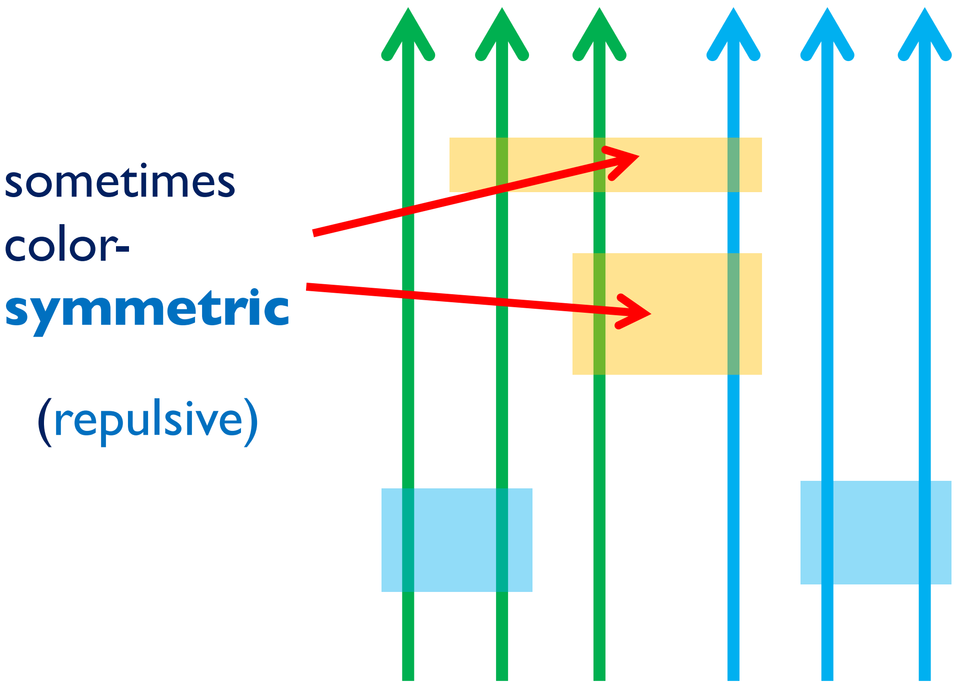
a baryon in dilute regime

in dense regime

(color-singlet)



e.g., **nucleons**



$M_N \sim \underbrace{3M_{q.} + \text{kin.}} + \text{color-EM}$

~ 940MeV ~ 1100MeV ~ −150-200MeV

more chances to feel repulsion

EoS with interactions

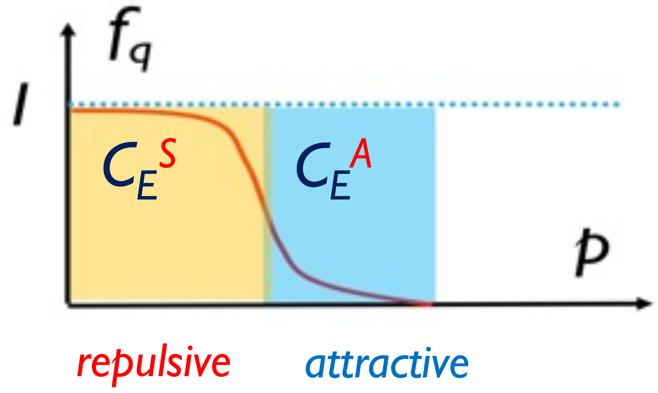
cf) [TK '21, TK-Suenaga '21]

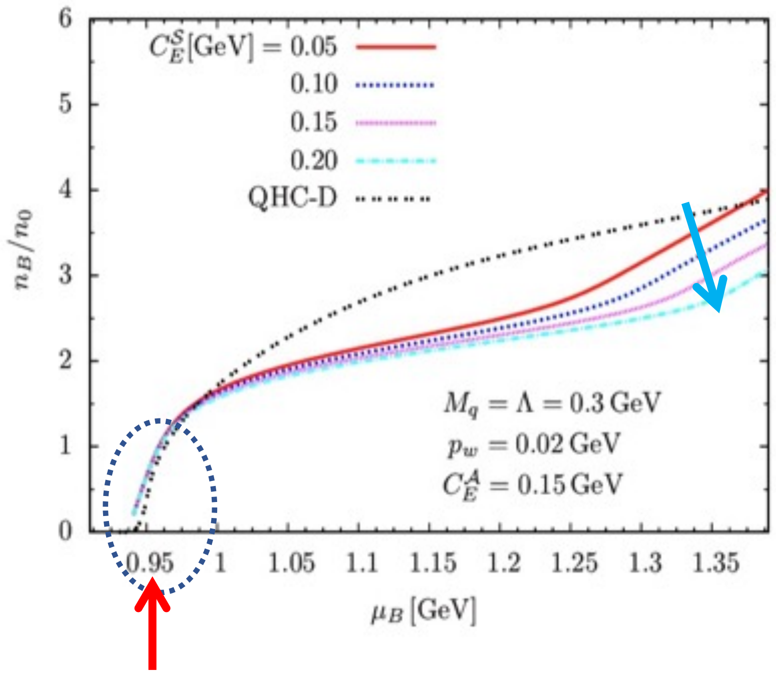
e.g.,

$$\mathcal{V}[f_{\mathrm{Q}}] = -C_E^{\mathcal{A}}\left[1-(f_{\mathrm{Q}})^n\right] + C_E^{\mathcal{S}}(f_{\mathrm{Q}})^n$$

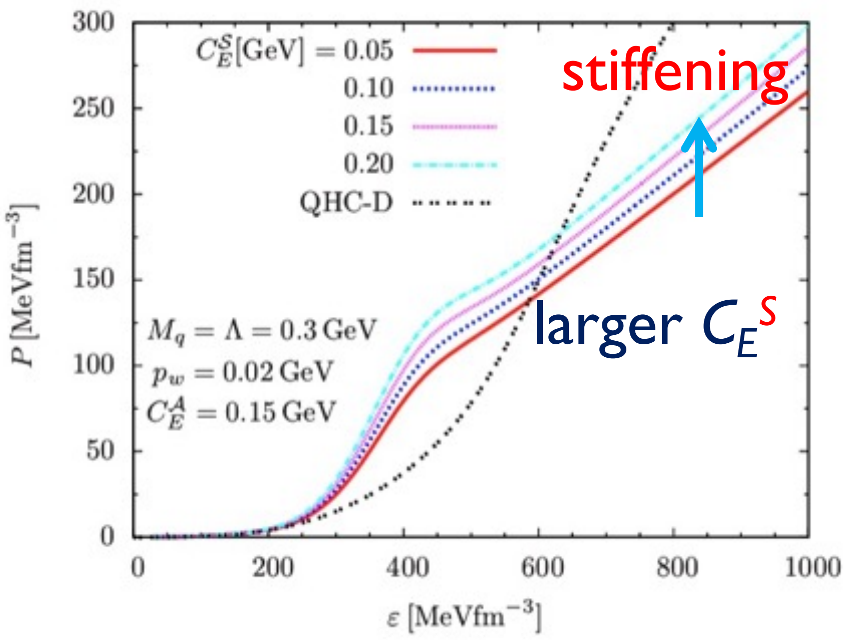
$[1-(f_{\mathrm{Q}})^n]$: → 1 (dilute), → 0 (dense)

$(f_{\mathrm{Q}})^n$: → 0 (dilute), → 1 (dense)

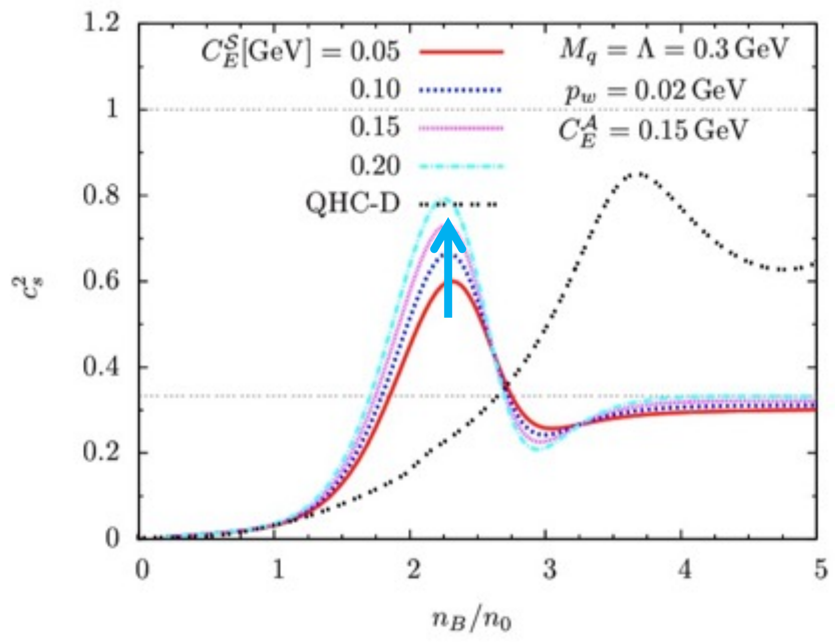




adjust $C_E{}^A$ (fit M_B = 939 MeV)



high density stiffening



stronger peak in c_s

Important relations

sum rule — single baryon contain single R- or G- or B- quark

$$n_q^{R,G,B} = \int_{\mathbf{p}} f_q(p) = \int_{\mathbf{p}} \left(\int_{\mathbf{P_B}} \mathcal{B}(P_B) Q_{\rm in}(\mathbf{p};\mathbf{P_B}) \right) = \int_{\mathbf{P_B}} \mathcal{B}(P_B) = n_B$$

energy density

$$E_B(P_B) \equiv N_c \int_{\mathbf{p}} E_q(\mathbf{p}) Q_{\rm in}(\mathbf{p};\mathbf{P_B})$$

$$\varepsilon = \int_{\mathbf{P_B}} E_B(P_B)\mathcal{B}(P_B) = N_c \int_{\mathbf{P_B}} \left(\int_{\mathbf{p}} E_q(\mathbf{p}) Q_{\rm in}(\mathbf{p};\mathbf{P_B}) \right) \mathcal{B}(P_B) = N_c \int_{\mathbf{p}} E_q(\mathbf{p}) f_q(p)$$

Dual expression: one can freely switch descriptions

No double counting

Finite-T model

Hadron Resonance Gas model for quark distribution

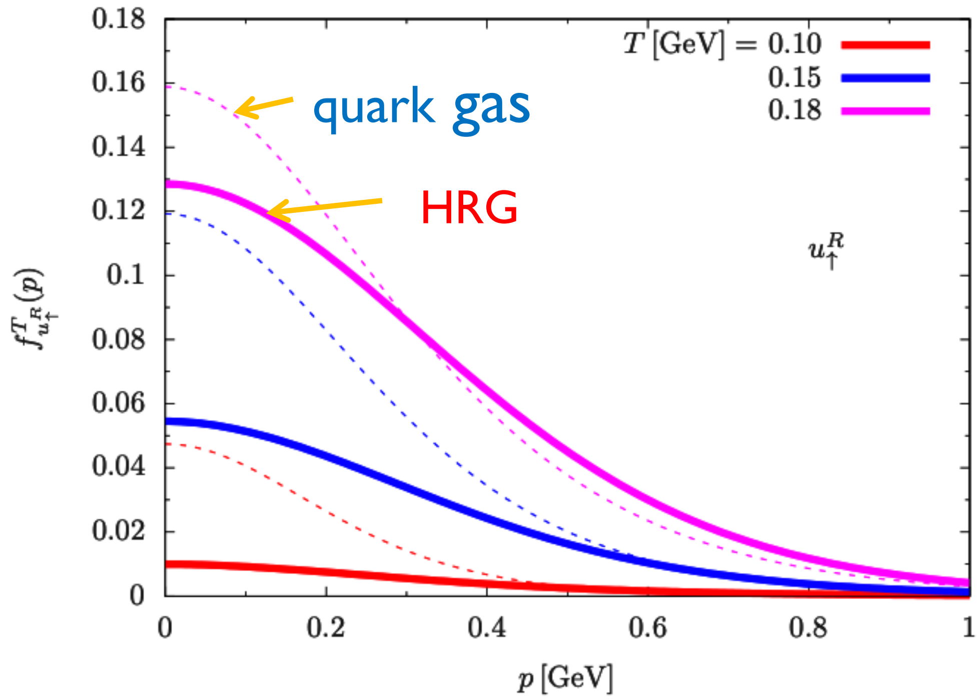
see [TK-Suenaga, '22]

$$f_{\mathbf{q}}^{T}(\boldsymbol{p}) = \sum_{h} \int_{\boldsymbol{P}_h} n_h^T(\boldsymbol{P}_h) Q_{\mathrm{in}}^{h\mathbf{q}}(\boldsymbol{p}; \boldsymbol{P}_h)$$

$$n_h^T(\boldsymbol{P}_h) = [\mathrm{e}^{E_h(\boldsymbol{P}_h)/T} - 1]^{-1}$$

- calculate quark w.f. for mesons up to $L = 3$, $n_r = 4$; $E < \sim 2.5$ GeV

	$n_r\,{}^{2S+1}L_J$	$M_{\rm exp}$	$M_{\rm cal}$	$\bar{\boldsymbol{P}}^2$	$\sqrt{\langle \boldsymbol{r}^2\rangle}$	f_S	α_s
π	1^1S_0	0.14	0.16	0.47	0.50	0.70	0.80
	2^1S_0	1.30	1.28	0.43	0.98		
	3^1S_0	1.81	1.82	0.55	1.38		
	4^1S_0	2.07**	2.22	0.67	1.66		
ρ	1^3S_1	0.78	0.76	0.21	0.66	0.74	0.80
	2^3S_1	1.47	1.44	0.35	1.17		
	3^3S_1	1.91*	1.87	0.48	1.55		
	4^1S_1	2.27**	2.22	0.61	1.83		
K	1^1S_0	0.49	0.49	0.42	0.49	0.72	0.77
	2^1S_0	1.46*	1.46	0.45	0.98		
K^*	1^3S_1	0.89	0.91	0.24	0.63	0.75	0.77
	2^3S_1	1.41	1.54	0.39	1.10		



quark gas ~ HRG

at ~ 0.15-0.18 GeV