NUCLEAR MATTER PROPERTIES AT HIGH μ_B

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CRC-TR 211 Strong-interaction matter under extreme conditions

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

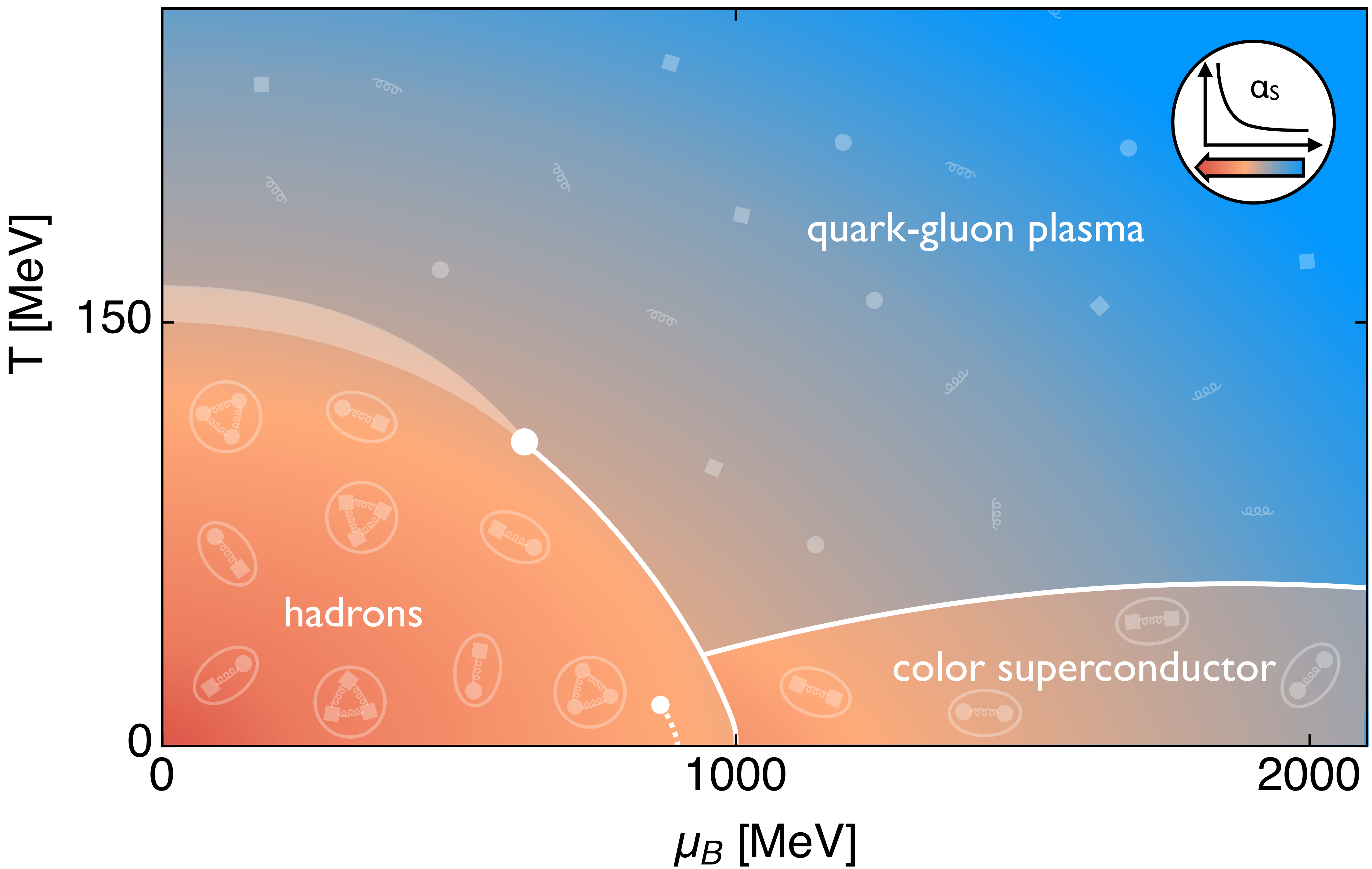
HFHF

PROBING DENSE BARYONIC MATTER WITH HADRONS II: FAIR PHASE-0

GSI - 21/02/2024

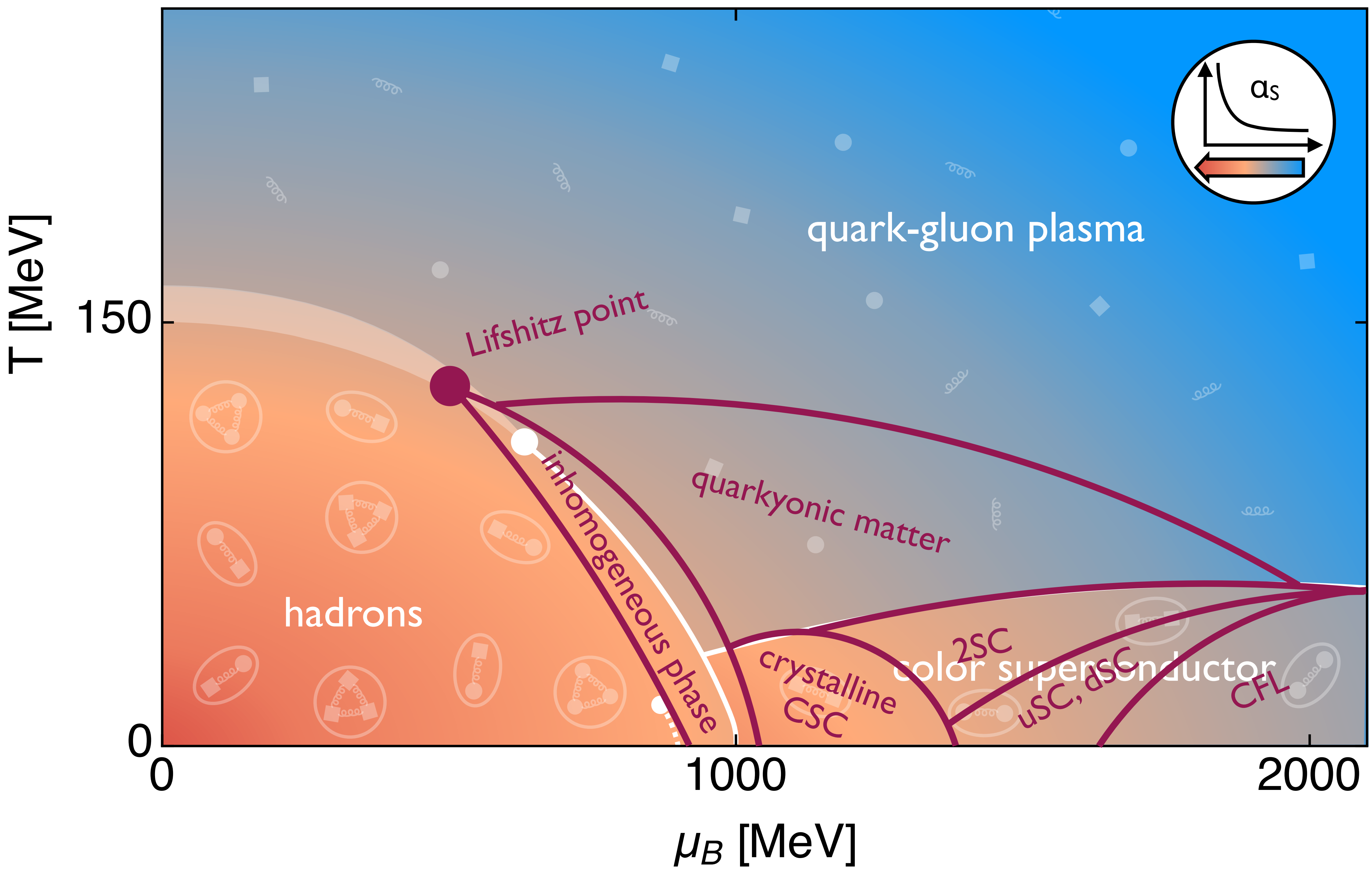
QCD PHASE DIAGRAM

in theory:



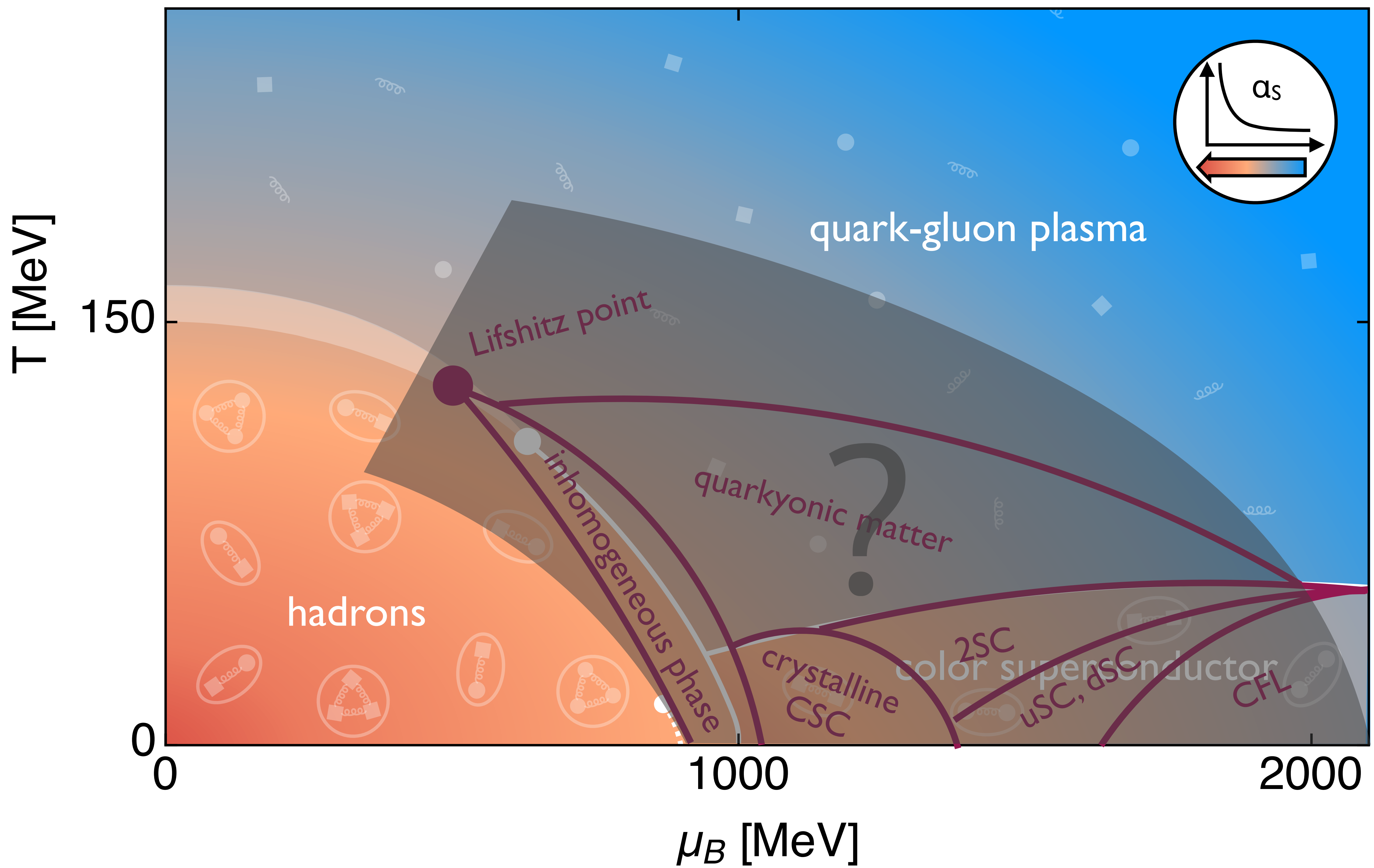
QCD PHASE DIAGRAM

in theory:



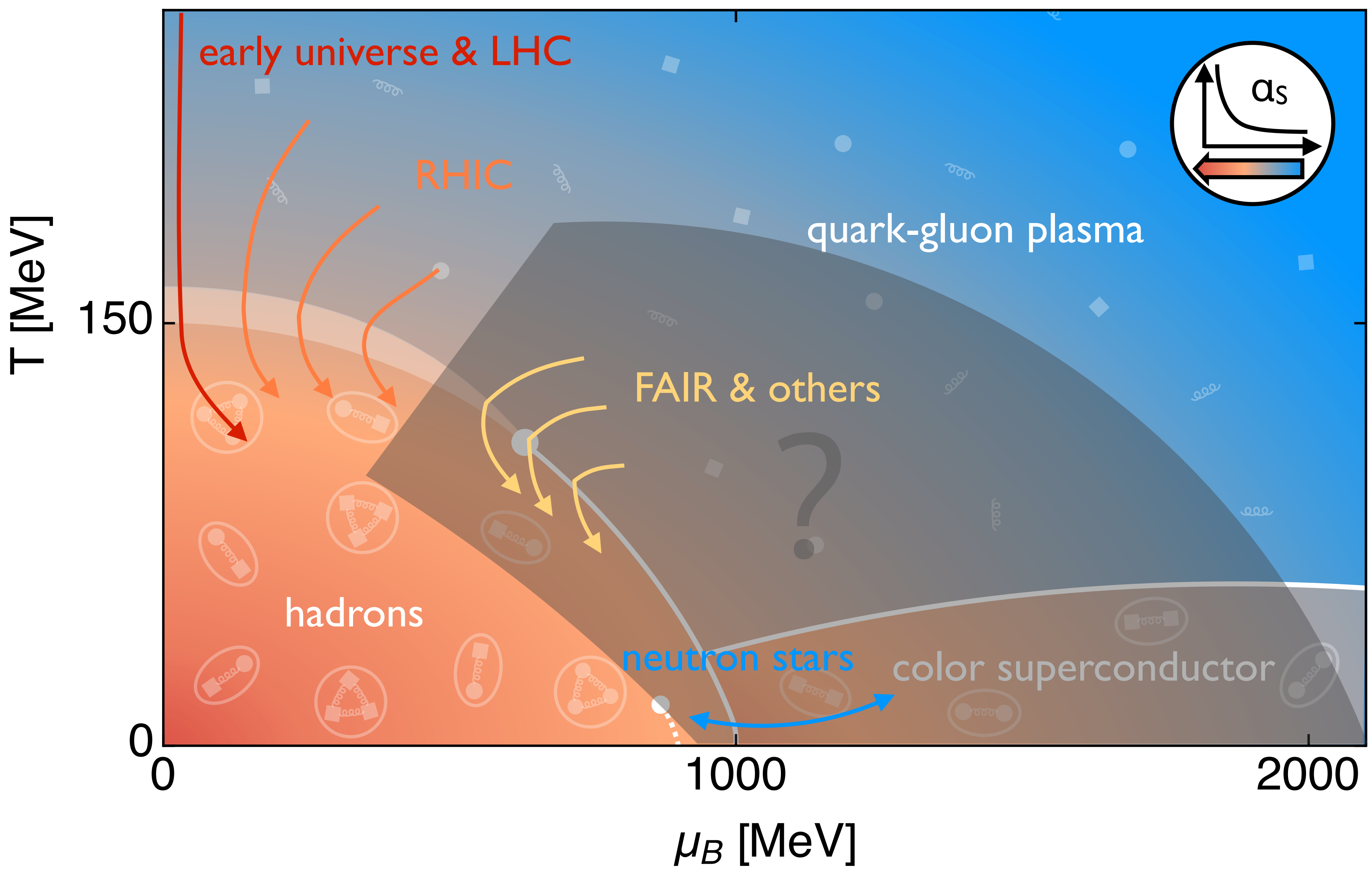
QCD PHASE DIAGRAM

in theory:



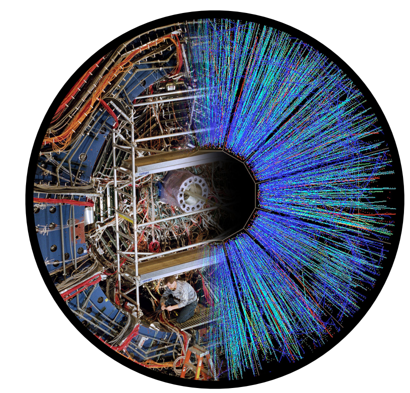
QCD PHASE DIAGRAM

in nature/experiment:

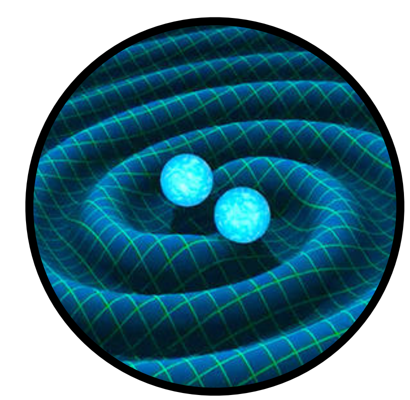


Experiments:

heavy-ion collisions



e.g. gravitational waves



MIXING, MOATS AND MODULATIONS

QCD AT FINITE DENSITY

Lagrangian at $\mu = 0$

$$\mathscr{L}_{\text{QCD}} = \bar{q}\left(i\gamma^\mu D_\mu - m_q\right) q - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a$$

$$D_\mu = \partial_\mu - igA_\mu^a T^a$$
$$F_{\mu\nu}^a T^a = -\frac{i}{g}[D_\mu, D_\nu]$$

- various symmetries, among them charge conjugation C

$$C: \begin{array}{rcl} \bar{q}q & \longrightarrow & \bar{q}q \\ \bar{q}\gamma^\mu q & \longrightarrow & -\bar{q}\gamma^\mu q \\ A^\mu & \longrightarrow & -A^\mu \end{array}$$

QCD at finite density

$$\mathscr{L}_{\text{QCD}} = \bar{q}\left(i\gamma^\mu D_\mu - m_q\right) q - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + i\mu\, \bar{q}\gamma^0 q$$

- breaks C (and Lorentz invariance)
- symmetric under CK

"complex conjugation" $K : i \longrightarrow -i$

This is the QCD analogue of PT symmetric quantum theories

[Bender, Boettcher (1998)] [Medina, Meisinger, Nishimura, Ogilvie, Pangeni, M. Schindler, S. Schindler]

A SIMPLE MODEL: CK-EXTENDED ϕ^4

Gain intuition from Euclidean scalar field theory with a vector coupling

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2\phi^2 + \lambda\phi^4 - h\phi + \frac{1}{2}(\partial_\mu \omega^0)^2 + \frac{1}{2}m_\omega(\omega^0)^2 - ig\phi\omega^0$$

- scalar field ϕ, vector field ω^μ
- linear coupling between ϕ and ω^0: mixing
- imaginary coupling ig: repulsion
- possesses CK symmetry

Since ω^0 enters quadratically, it can be integrated out

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2\phi^2 + \lambda\phi^4 - h\phi + \frac{1}{2}\phi \frac{g^2}{-\partial_\mu^2 + m_\omega^2} \phi$$

So CK-symmetric mixing ...

... modifies the dispersion of ϕ

$$E^2(p^2) = p^2 + m^2 + \frac{g^2}{p^2 + m_\omega^2}$$

... leads to a non-Hermitian Hessian

$$H = \begin{pmatrix} p^2 + m^2 & -ig \\ -ig & p^2 + m_\omega^2 \end{pmatrix}$$

[Schindler, Schindler, Medina, Ogilvie (2019)]

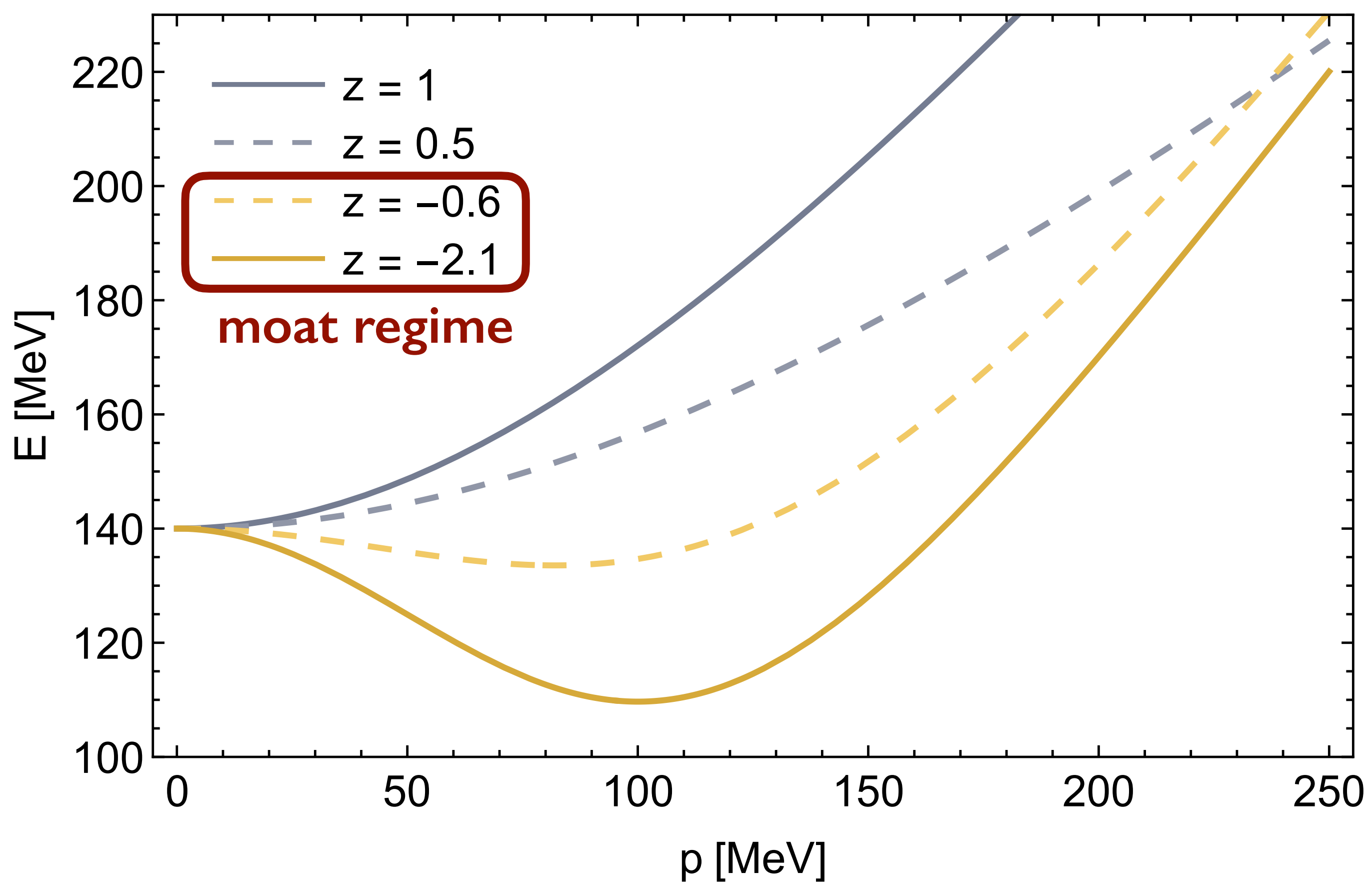
MODIFIED DISPERSION

in the small-momentum regime:

$$E^2(p^2) = p^2 + m^2 + \frac{g^2}{p^2 + m_\omega^2} = \underbrace{\left(1 - \frac{g^2}{m_\omega^4}\right)}_{z} p^2 + \underbrace{\frac{g^2}{m_\omega^6}}_{w} p^4 + \underbrace{m^2 + \frac{g^2}{m_\chi^2}}_{m_\phi^2} + \mathcal{O}(p^6)$$

→ $z < 0$ for strong-enough mixing: **moat regime**

[Pisarski, FR (2021)]



[Caerlaverock Castle, Scotland (source: Wikipedia)]

→ particles are favored to have nonzero momentum

"gain energy by going faster"

MODIFIED HESSIAN

Eigenvalues of the modified Hessian determine masses of physical particles,

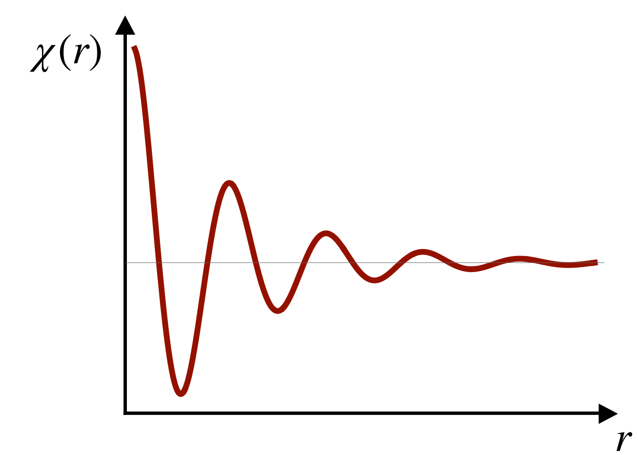
$$H = \begin{pmatrix} p^2 + m^2 & -ig \\ -ig & p^2 + m_\omega^2 \end{pmatrix} \longrightarrow \begin{pmatrix} p^2 + M_+^2 & 0 \\ 0 & p^2 + M_-^2 \end{pmatrix}, \quad M_\pm = \frac{m^2 + m_\omega^2}{2} \pm \frac{1}{2}\sqrt{(m^2 - m_\omega^2)^2 - 4g^2}$$

→ strong-enough mixing: masses come in complex conjugate pairs

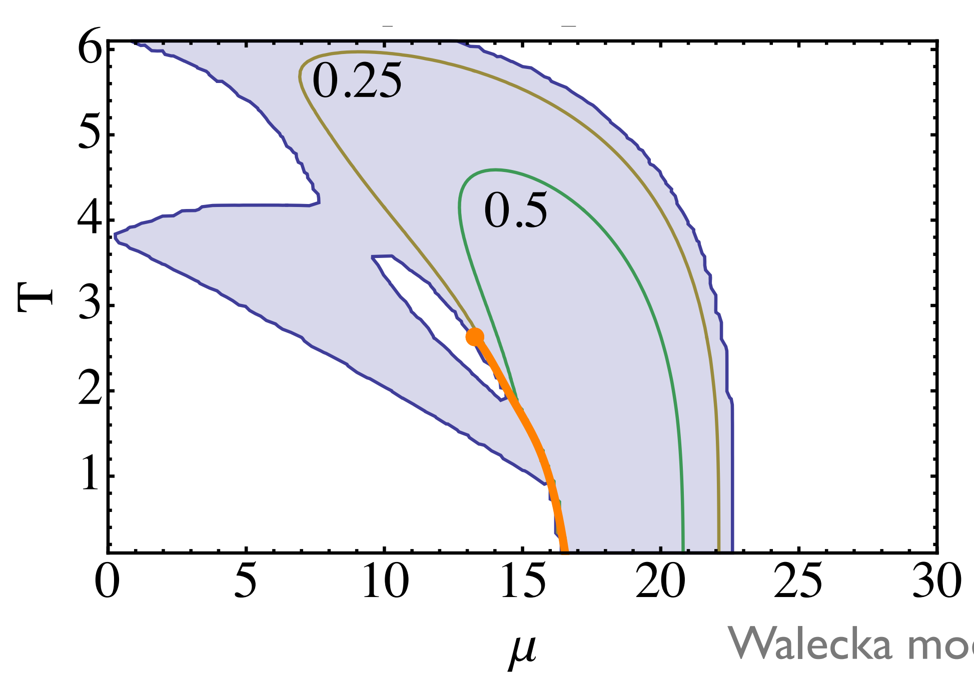
(happens before a moat appears unless $m^2 \geq 3m_\chi^2$)

Masses determine screening properties of the physical fields

$$\lim_{r\to\infty} \langle \chi(r)\chi(0) \rangle \sim e^{-Mr}$$



- real M: ordinary exponential decay of disordered fields
- complex M: **spatial modulations** $\sim e^{-\mathrm{Re}[M]\,r} \sin\left(\mathrm{Im}[M]\,r\right)$



→ spatial modulations in the phase diagram

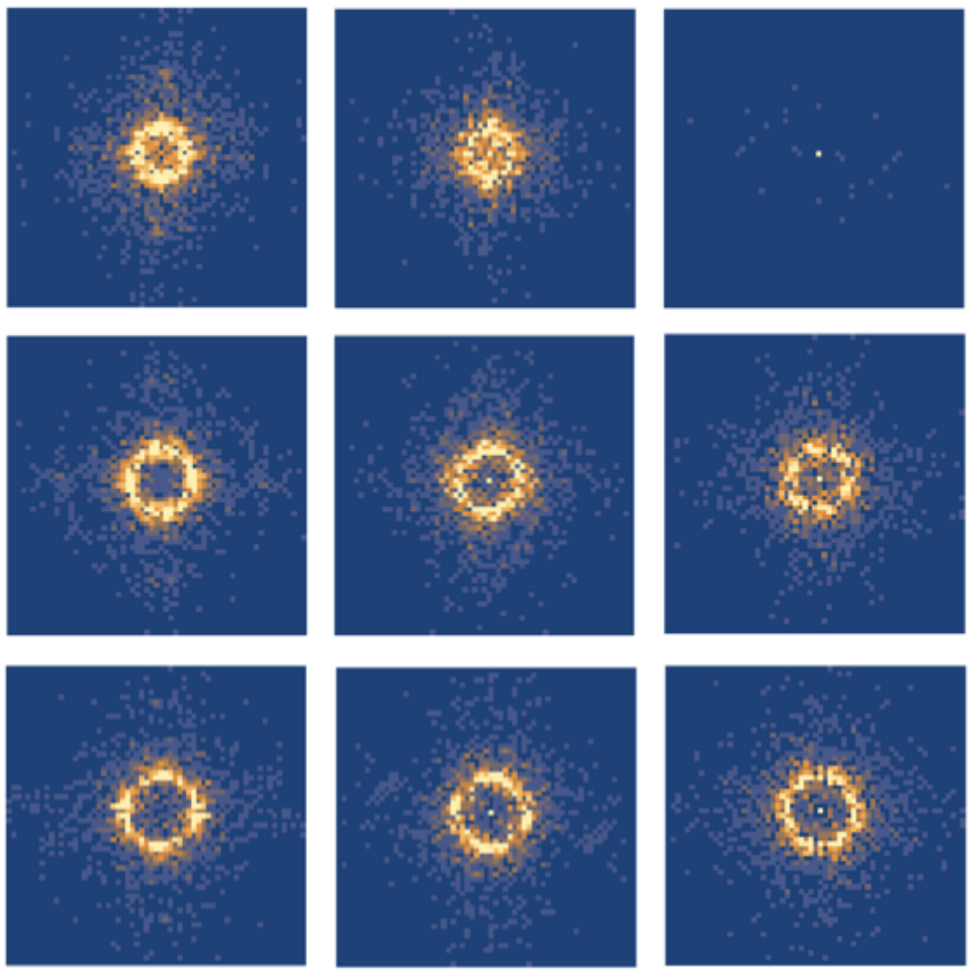
Walecka model [Nishimura, Ogilvie, Pangeni (2016)]

MIXING, MOATS AND MODULATIONS

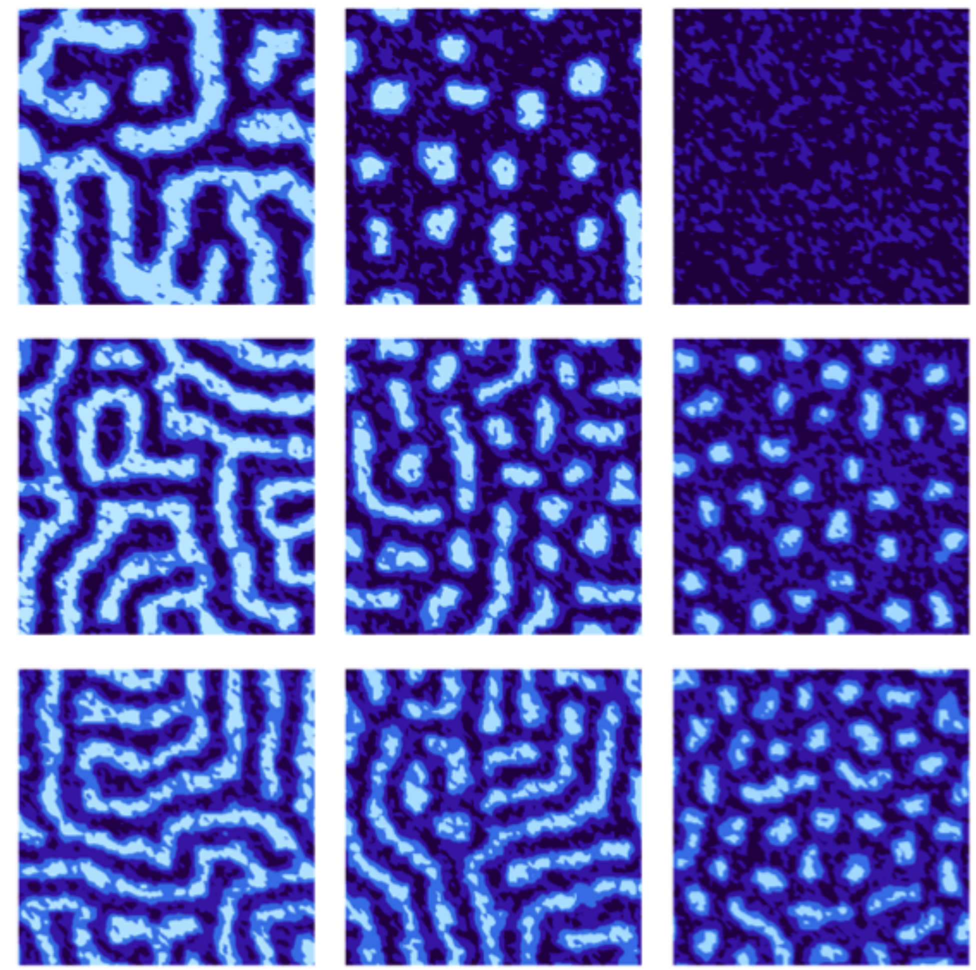
Snapshots of field configurations of the CK-extended scalar theory:

[Schindler, Schindler, Medina, Ogilvie (2019)]

- momentum space: $\phi(p)$



- position space: $\phi(x)$



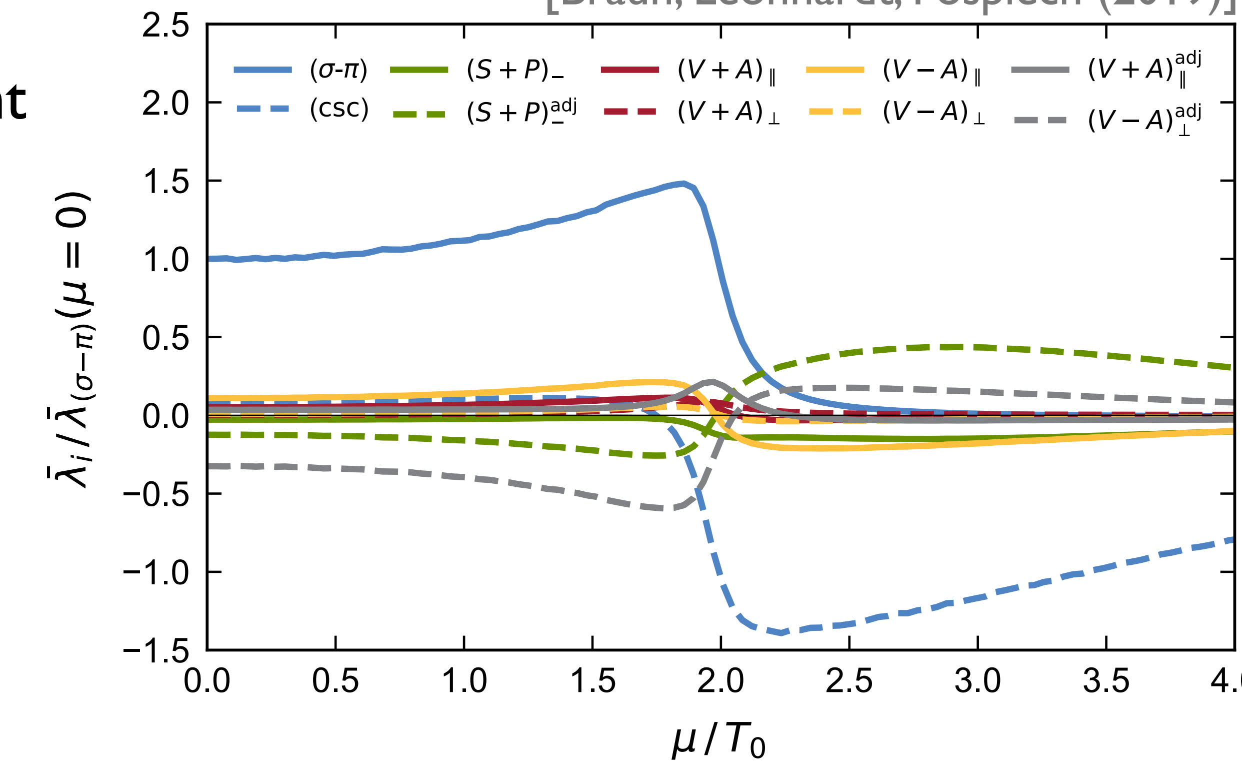
moat regime ⟷ spatial modulations/"patterns"

- I will argue that this structure is generic, also for dense nuclear matter
- indicates the existence of inhomogeneous phases, liquid crystals, quantum pion liquids, ...

DENSE NUCLEAR MATTER

important QCD effects for the ground state:

[Braun, Leonhardt, Pospiech (2019)]



- formation of a chiral condensate through resonant quark scattering

$$\bar{\sigma} \sim \langle \bar{q} q \rangle$$

→ chiral symmetry breaking

- finite density n equivalent to vector condensate

$$-ih_\omega \bar{\omega}^0 \sim \langle \bar{q} \gamma^0 q \rangle = n$$

→ short-distance repulsion: imaginary quark-omega coupling ih_ω

- Polyakov loop: ~order parameter for confinement

$$L = \frac{1}{N_c} \langle \mathrm{tr}\, P \rangle \,, \quad \bar{L} = \frac{1}{N_c} \langle \mathrm{tr}\, P^\dagger \rangle$$

temporal Wilson line

$$P(\vec{x}) = \mathscr{P} \exp\left[ig \int_0^\beta dx_0\, A_0(x_0, \vec{x}) \right]$$

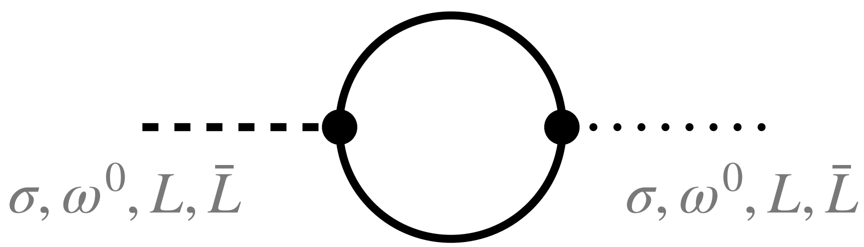
  - $\ln L/\bar{L}$ measure the free energy of a single static quark/antiquark: $L = 0 \leftrightarrow$ confinement
  - C-symmetry breaking: $L \neq \bar{L}$

[McLerran, Svetitsky (1981)]

MIXING IN DENSE NUCLEAR MATTER

All these effects mix due to fundamental quark interactions at finite μ!

[Haensch, FR, von Smekal (2023)]



$$H = \begin{pmatrix} H_{\sigma\sigma} & H_{\sigma\omega^0} & H_{\sigma\bar{L}} & H_{\sigma L} \\ H_{\sigma\omega^0} & H_{\omega^0\omega^0} & H_{\omega^0\bar{L}} & H_{\omega^0 L} \\ H_{\sigma L} & H_{\omega^0 L} & H_{L\bar{L}} & H_{LL} \\ H_{\sigma\bar{L}} & H_{\omega^0\bar{L}} & H_{\bar{L}\bar{L}} & H_{L\bar{L}} \end{pmatrix}$$

- σ-ω^0 mixing well known [Kunihiro (1991); Wolf, Friman, Soyeur (1998)]; crucial for dynamic universality [Son, Stephanov (2004)]
- repulsive vector interaction: ω^0-mixing purely imaginary
- C-symmetry breaking: L-mixing $\neq$ $\bar{L}$-mixing
- complex conjugation K: $L \to \bar{L}$

→ non-Hermitian (but CK-symmetric) Hessian/mass matrix of dense nuclear matter

Intuition from CK-extended scalar theory is applicable to QCD

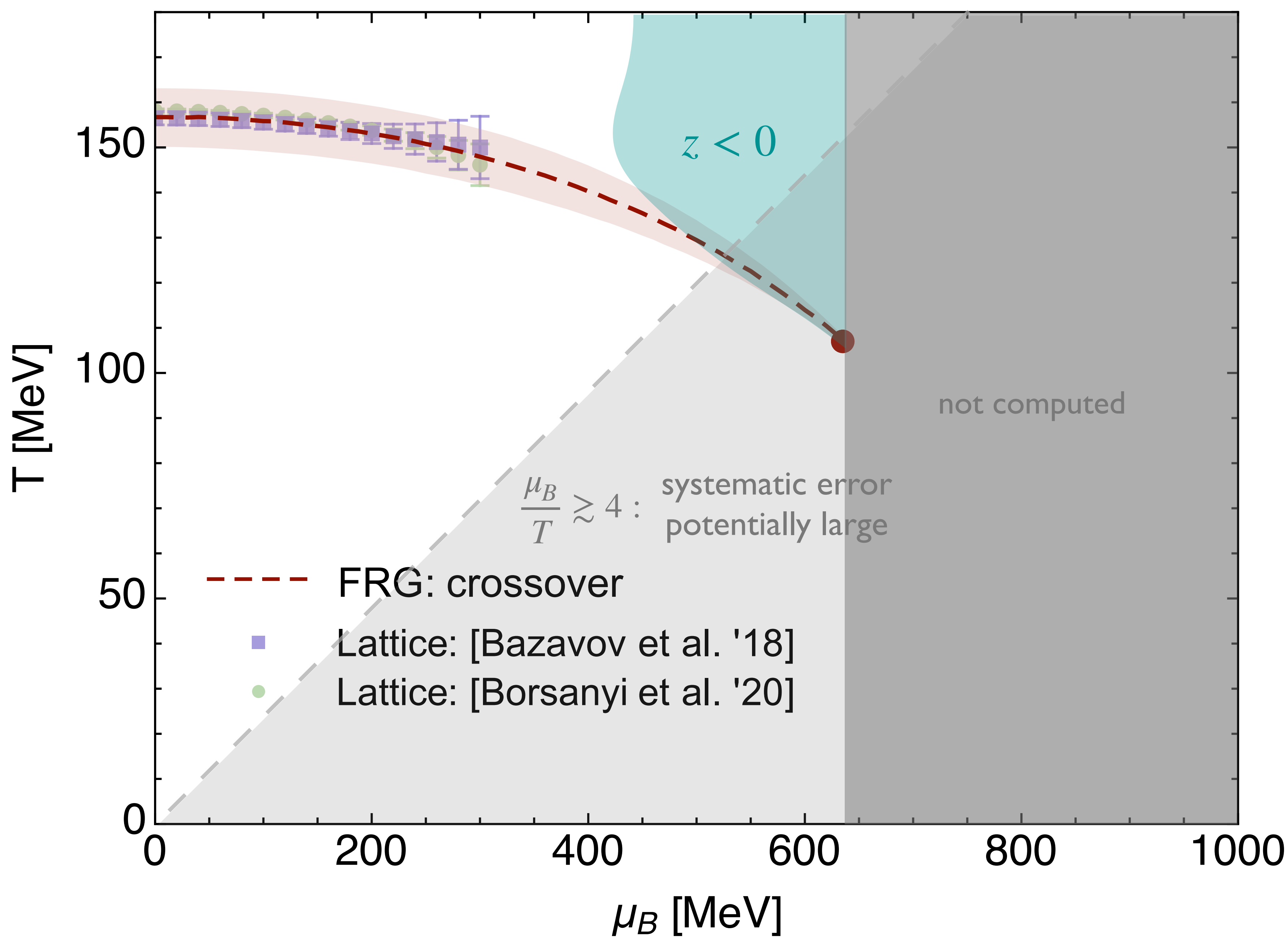
→ **mixing, moats and modulations to be expected in nuclear matter!**

WHERE CAN THIS APPEAR?

- some examples in low-energy models at large μ
- first results also in QCD:

Basar, Buballa, Carignano, Dunne, Koenigstein, Nussinov, Ogilvie, Pannullo, Pisarski, FR, Thies, Tsvelik, Valgushev, Winstel, ...

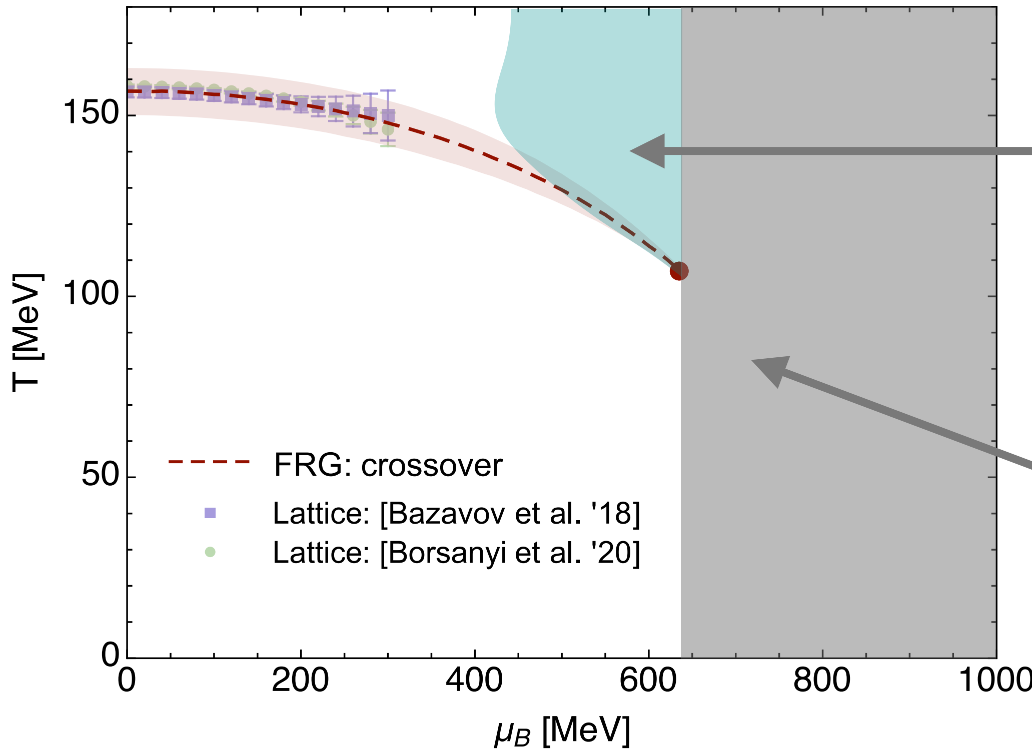
[Fu, Pawlowski, FR 2019)]

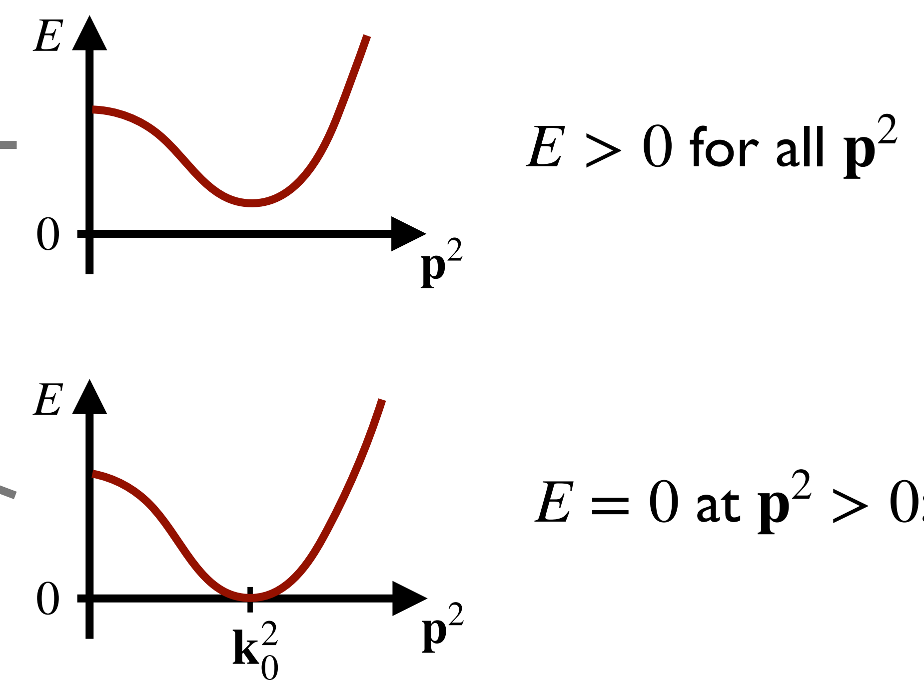


→ indication for extended region with $z < 0$ in QCD: **moat regime**

IMPLICATIONS OF THE MOAT

The energy gap might close at lower T and larger μ_B :

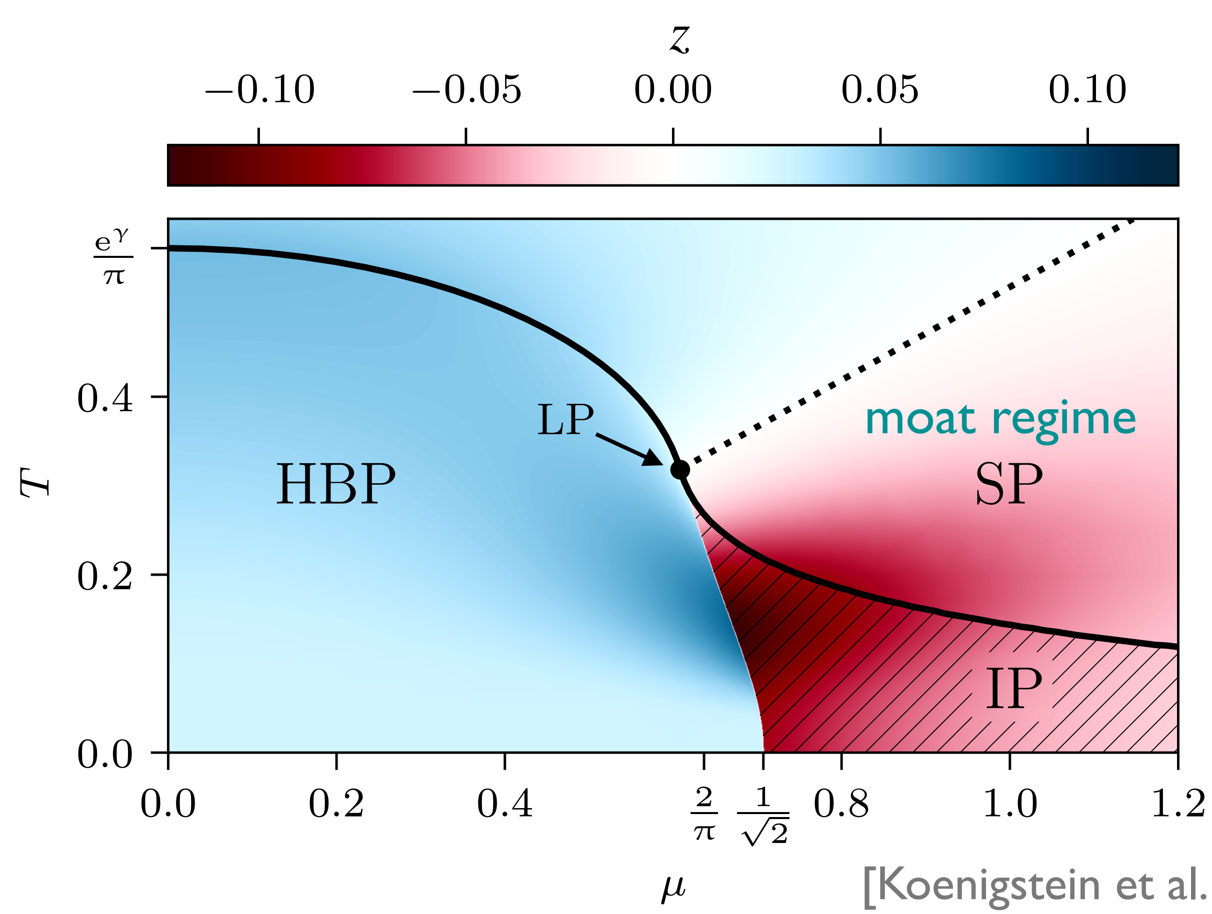




Zero energy cost to condense particles with nonzero momentum k_0

→ instability towards formation of an **inhomogeneous condensate**

- Example: Gross-Neveu Model in 1+1 dim. at large N_f



[Koenigstein et al. (2021)]

IMPLICATIONS OF THE MOAT

BUT: formation of inhomogeneous phases depends on dynamics of soft (massless) modes.

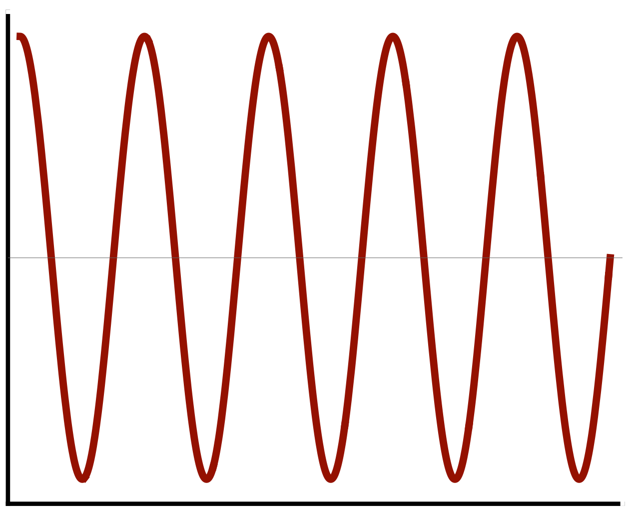
→ fluctuation-induced instabilities of inhomogeneous phases

→ other types of phases possible (possibly without long-range order!)

inhom. phase

no instability
(typical in mean-field)

$$\langle \phi(x)\phi(0)\rangle \sim \sin(k_0\, x)$$

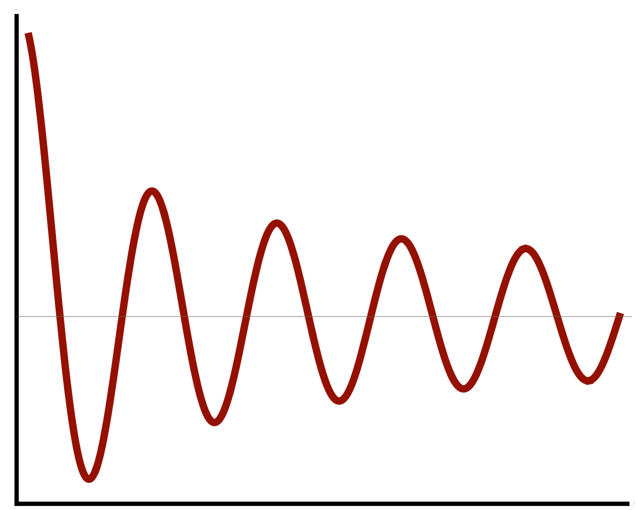


[Fukushima, Hatsuda, RPP 74 (2010)]
[Buballa, Carignano, PPNP 81 (2014)]

liquid crystal

Landau-Peierls instability
(Goldstones from spatial SB)

$$\langle \phi(x)\phi(0)\rangle \sim \sin(k_0 x)\, x^{-\alpha}$$

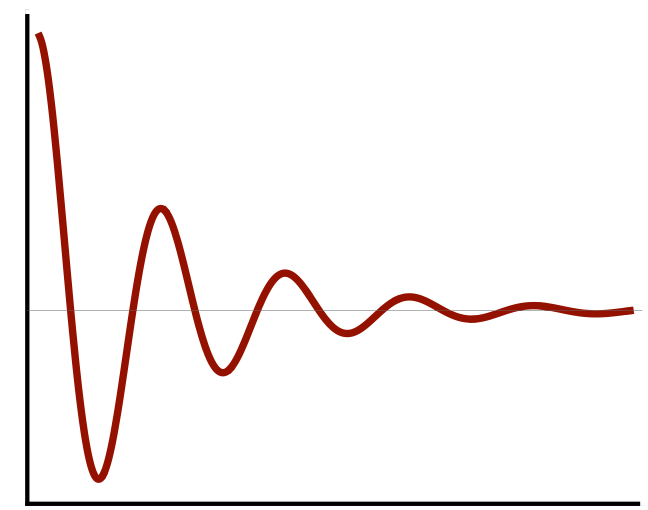


[Landau, Lifshitz, Stat. Phys. I, §137]
[Lee et al., PRD 92 (2015)]
[Hidaka et al., PRD 92 (2015)]

quantum pion liquid

PTV instability
(Goldstones from flavor SB)

$$\langle \phi(x)\phi(0)\rangle \sim \sin(k_0 x)\, e^{-mx}$$

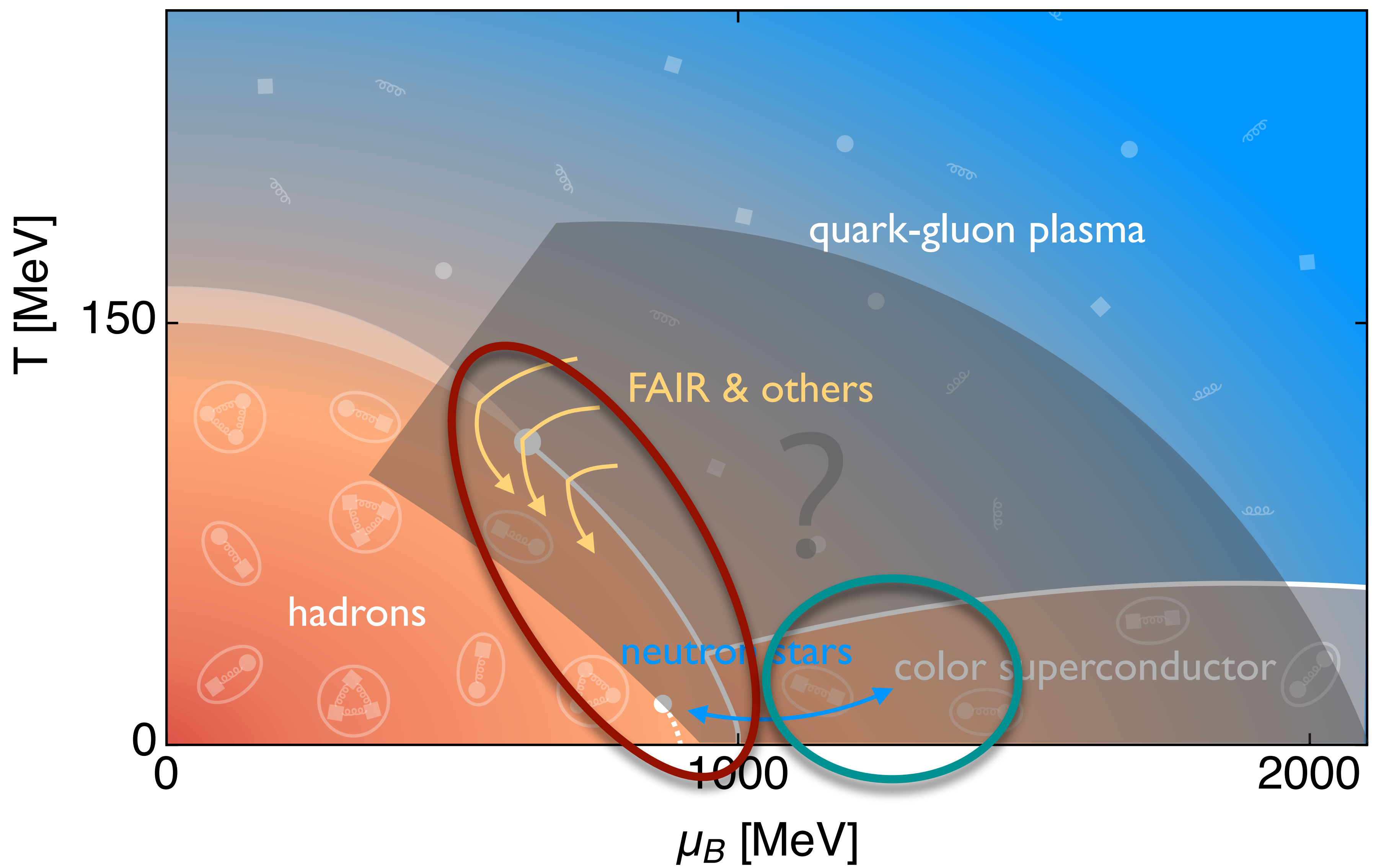


[Pisarski, Tsvelik, Valgushev, PRD 102 (2020)]
[Pisarski, PRD 103 (2021)]
[Schindler, Schindler, Ogilvie (2021)]

either way...

- the moat is a common feature of regimes with spatial modulations
- and seems to be a generic feature of dense nuclear matter

WHERE DO WE EXPECT NEW PHASES?



modulated phases are expected in the "unknown" region of the phase diagram

this is/will be covered by FAIR and other fixed target experiments

→ **search for moats/modulations in heavy-ion collisions!**

SEARCH FOR MOAT REGIMES

intuitive idea: [Pisarski, FR (2021)]

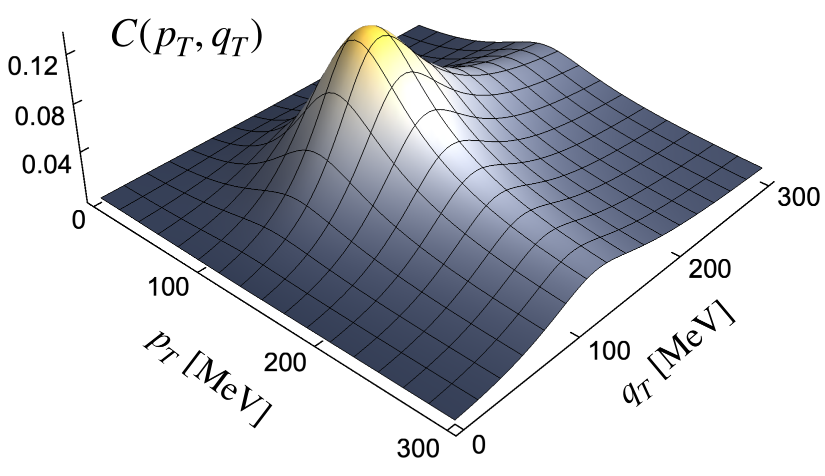
Characteristic feature of a moat regime: minimal energy at nonzero momentum

$\Rightarrow$ **enhanced particle production at nonzero momentum**

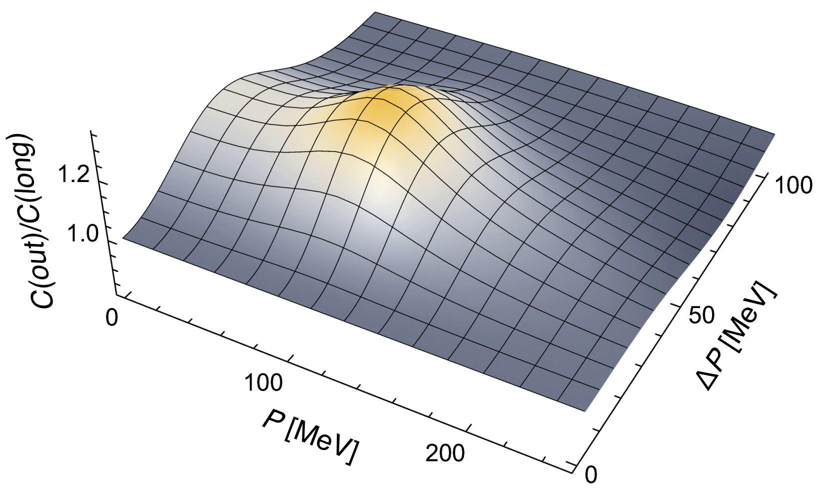
⟶ look for signatures in the **momentum dependence** of particle correlations

- example: two-particle correlations

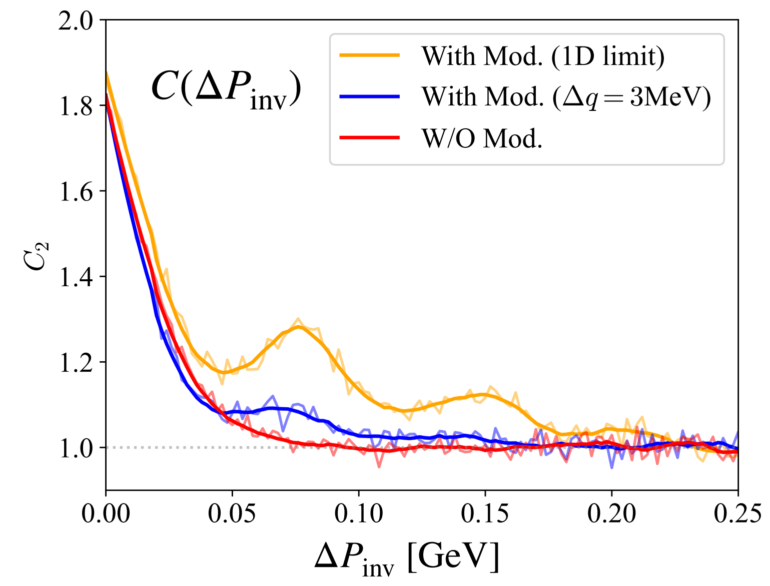
thermodynamic fluctuations in a moat regime
[Pisarski, FR (2021)]



HBT in a moat regime
[FR, Pisarski, Rischke (2023)]



HBT from primordial inhomogeneity
[Fukushima et al. (2023)]

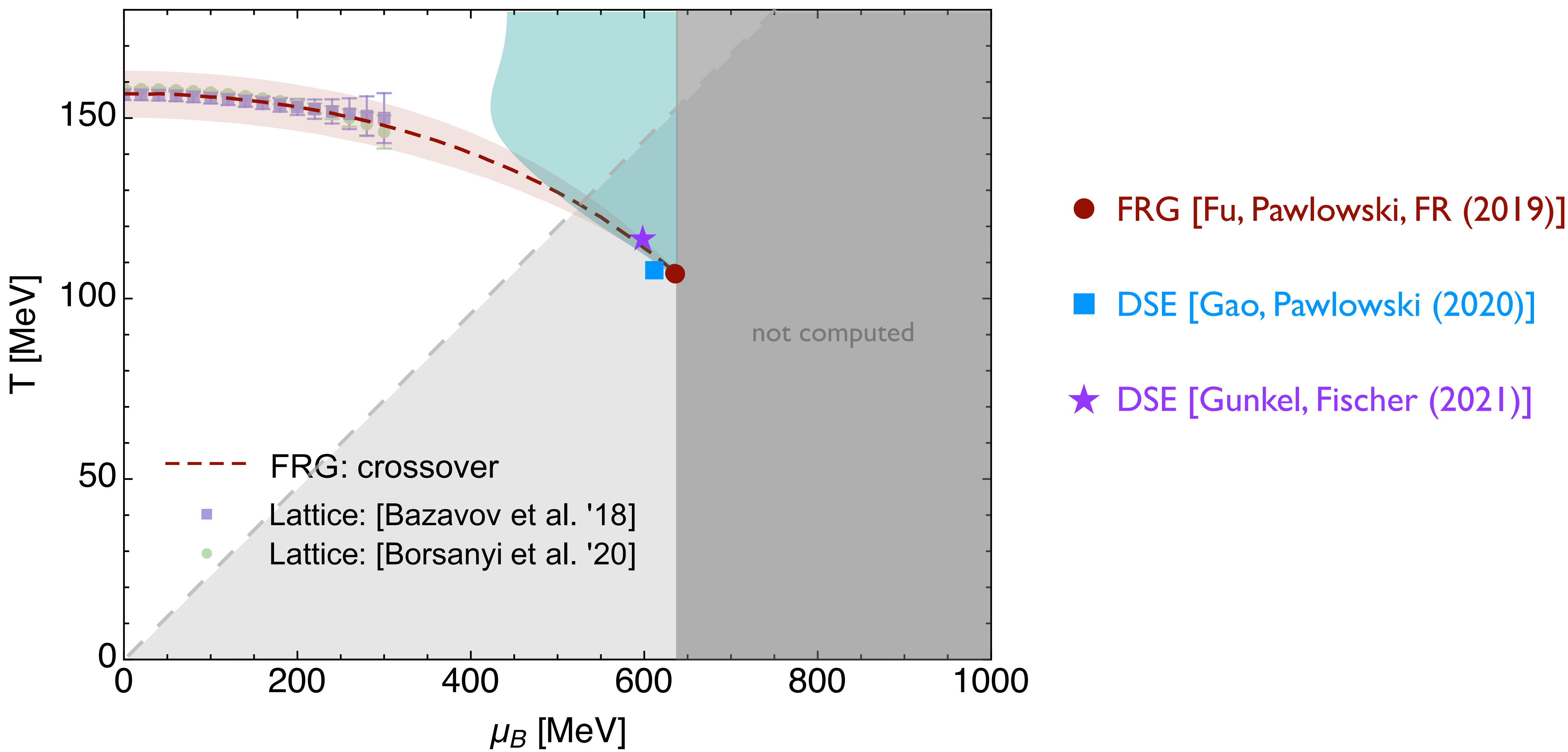


- work in progress: dilepton production [Nussinov, Ogilvie, Pannullo, Pisarski, FR, Schindler, Winstel]

⟶ **peak position related to wavenumber of underlying spatial modulation**

WHAT ABOUT THE CEP?

predictions from QCD that agree with available lattice data:



- QCD results from functional methods place CEP at $\sqrt{s} \approx 3.6 - 4.1\,\mathrm{GeV}$
- based on the assumption of homogeneous chiral transition

Either CEP is around this region, or something supersedes CEP, like a spatially modulated phase

→ **something is cooking, but what exactly?**

SUMMARY

Mixing, moats and modulations likely in dense nuclear matter

- generic feature of systems with C-symmetry breaking and competing attractive and repulsive interactions
- details not well understood (a lot of mean-field studies, but they miss crucial physics)
- not necessarily with long-range correlations
- simple underlying "mean-field mixing" mechanism

→ suitable for transport simulations?

... expected to occur in FAIR range

- need to understand (and find more) possible experimental signatures
- phase itself could be detected, not only the transition to it: "easier" to find?

... relevant for neutron stars

e.g., nuclear pasta [Caplan, Horowith (2017)]