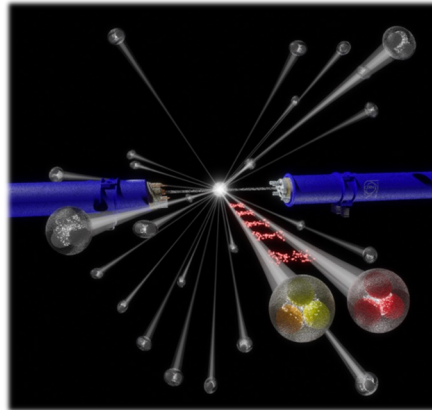




# Hadronic interaction studies in many-body systems at the LHC

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20<sup>th</sup> February 2024, EMMI Workshop, GSI, Germany

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# Hadronic interactions in many-body systems

- Properties of nuclei and hypernuclei cannot be described satisfactorily with two-body forces only

*H. Hammer, A. Nogga, A. Schwenk, RMP 85, 197 (2013)*  
*L.E. Marcucci et al., Front. Phys. 8:69 (2020)*

- NNN interaction contributes ~10% to the binding energies of  $^3\text{H}$  and  $^4\text{He}$

- Many-body scattering requires three-body calculations (e.g. **neutron-deuteron**)

*L. Girlanda et al., PRC 102, 064003 (2020)*  
*E. Epelbaum et al., PRC 99, 024313 (2019)*

## $^3\text{H}$ and $^4\text{He}$ binding energies and n-d scattering length

Potential(NN)	$^3\text{H}$ [MeV]	$^4\text{He}$ [MeV]	$^2a_{nd}$ [fm]
AV18	7.624	24.22	1.258
CDBonn	7.998	26.13	
N3LO-Idaho	7.854	25.38	1.100

Potential(NN+NNN)	$^3\text{H}$ [MeV]	$^4\text{He}$ [MeV]	$^2a_{nd}$ [fm]
AV18/UIX	8.479	28.47	0.590
CDBonn/TM	8.474	29.00	
N3LO-Idaho/N2LO	8.474	28.37	0.675
<b>Exp.</b>	<b>8.48</b>	<b>28.30</b>	<b>0.645±0.010</b>

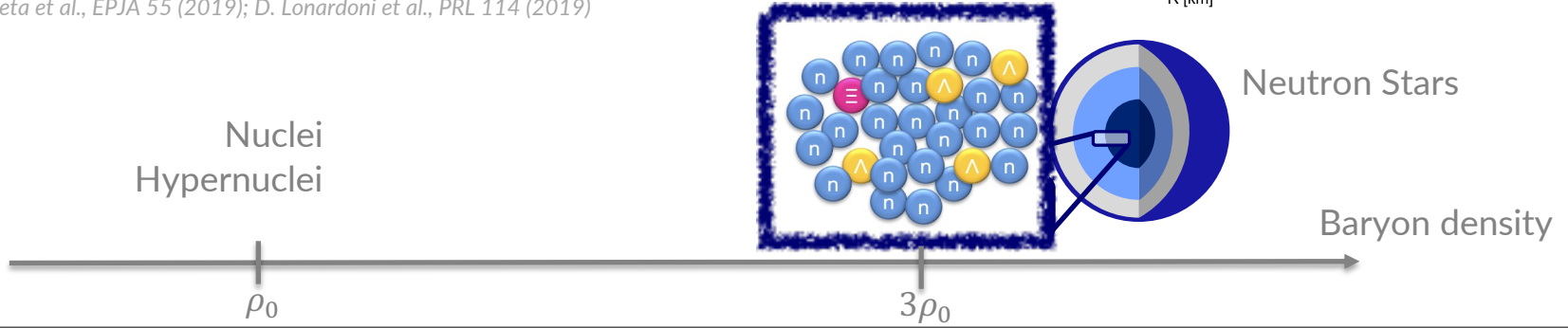
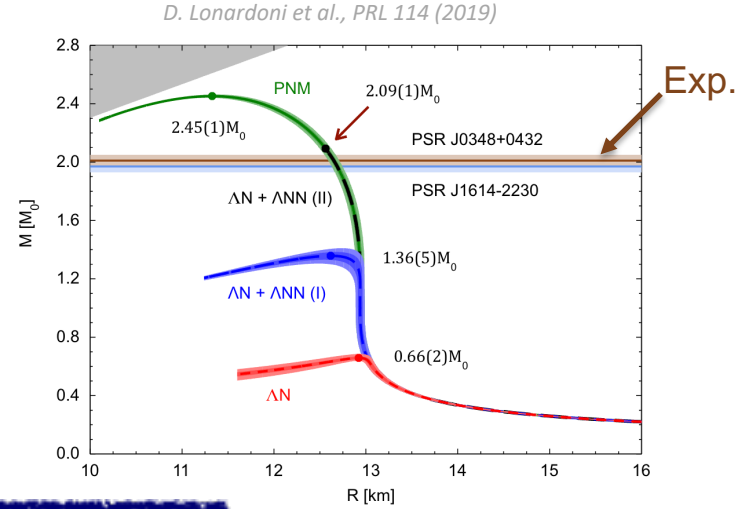
*L.E. Marcucci et al., Front. Phys. 8:69 (2020)*



# Hadronic interactions in many-body systems

- Production of hyperons energetically favourable in neutron stars (NS) around 2-3  $\rho_0$
- Only two-body  $\Lambda N$ 
  - Too soft equation of state (EoS)
- Introduction of three-body  $\Lambda NN$  forces
  - Stiffens EoS, model-dependent
  - Need for additional experimental constraints

*D. Logoteta et al., EPJA 55 (2019); D. Lonardoni et al., PRL 114 (2019)*

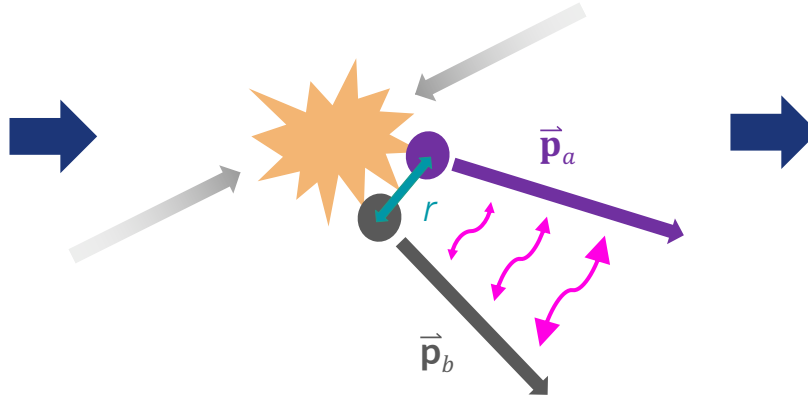


# The femtoscopy technique at the LHC

ALICE at the LHC



Hadronic interaction

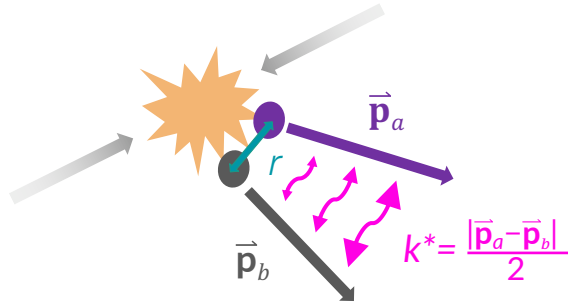


Femtoscopy technique

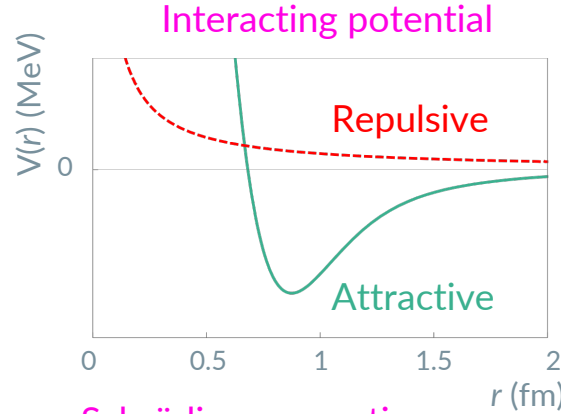
Correlation function

$$C(\vec{p}_a, \vec{p}_b) \equiv \frac{P(\vec{p}_a, \vec{p}_b)}{P(\vec{p}_a) P(\vec{p}_b)}$$

# The femtoscopy technique at the LHC



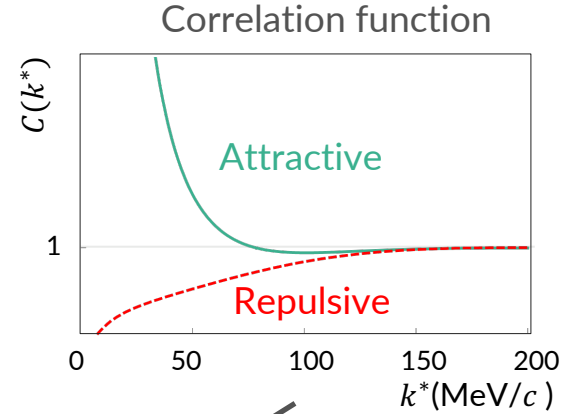
Emission source  $S(\vec{r})$



Interacting potential

Repulsive

Attractive



Correlation function

Attractive

Repulsive

Schrödinger equation

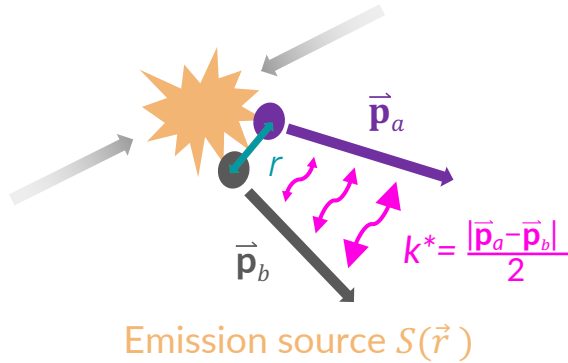
CATS Framework: D. Mihaylov et al., Eur. Phys. J. C78 (2018) 394

Two-particle wave function

$$C(k^*) = \int S(\vec{r}) |\psi(\vec{k}^*, \vec{r})|^2 d^3\vec{r} = \mathcal{N}(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

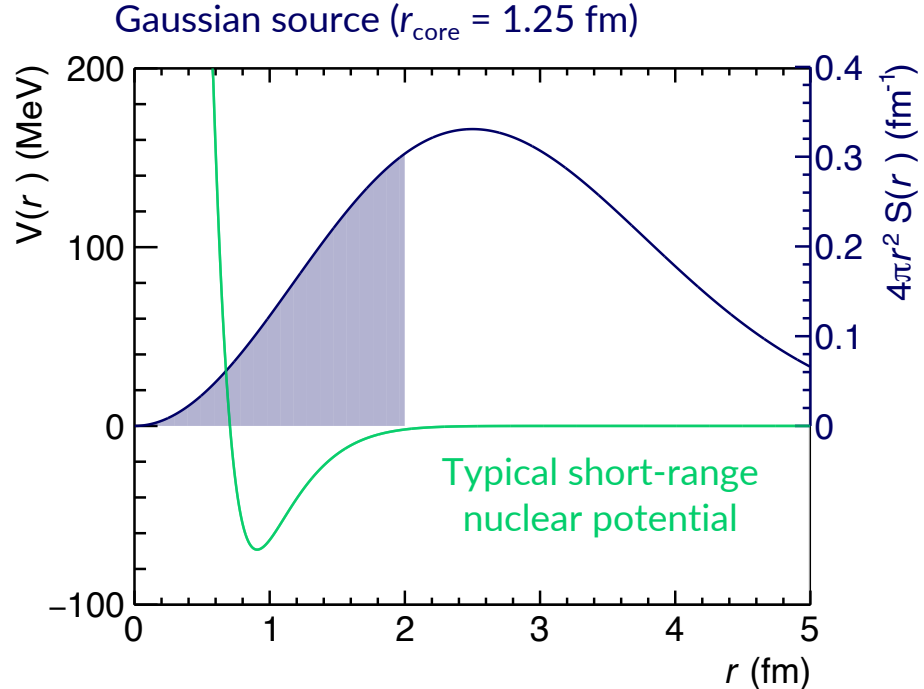
Measuring  $C(k^*)$ , fixing the source  $S(\vec{r})$ , study the interaction

# The femtoscopy technique at the LHC



Small particle-emitting source created in pp and p-Pb collisions at the LHC:

- **Essential ingredient for detailed studies of the strong interaction**
- Source function determined using p-p correlation functions



# Source determination

Femtoscscopy used in a “traditional” way: known interaction → determine the source size

**p-p interaction:**

Coulomb + Argonne v18

**p- $\Lambda$  interaction:**

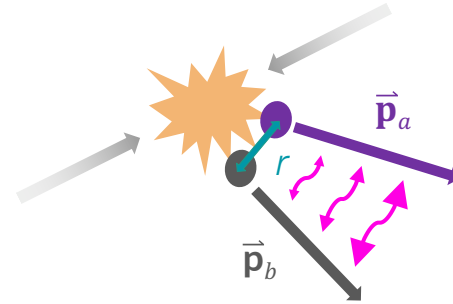
$\chi$ EFT **NLO** and **LO**

$$C(k^*) = \int S(\vec{r}) |\psi(\vec{k}^*, \vec{r})|^2 d^3\vec{r}$$

- Fit of the data as function of  $m_T$
- Gaussian source

$$S(r) = G[r, r_{core}(m_T)]$$

$$= \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right)$$



# Source determination

Femtoscopy used in a “traditional” way: known interaction → determine the source size

**p-p interaction:**

Coulomb + Argonne v18

**p-Λ interaction:**

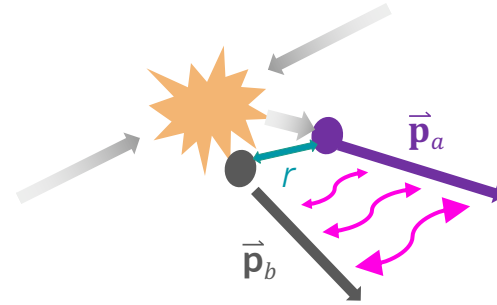
χEFT **NLO** and **LO**

$$C(k^*) = \int S(\vec{r}) |\psi(\vec{k}^*, \vec{r})|^2 d^3\vec{r}$$

- Fit of the data as function of  $m_T$
- Gaussian source

$$S(r) = G[r, r_{core}(m_T)] = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right)$$

+ Resonances with  $c\tau \sim r_{core} \sim 1\text{fm}$  ( $\Delta^{++}, N^*, \Sigma^*$ )



Particle	Primordial fraction	Resonances $\langle c\tau \rangle$
Proton	33 %	1.6 fm
Lambda	34 %	4.7 fm

*U. Wiedemann and U. Heinz PRC 56 (1997)*

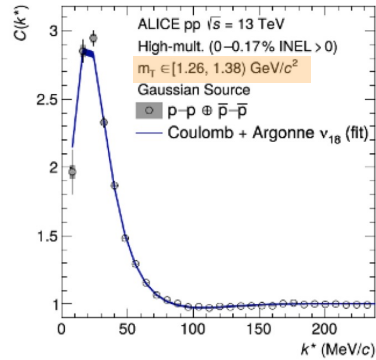


# Source determination

Femtoscopy used in a “traditional” way: known interaction → determine the source size

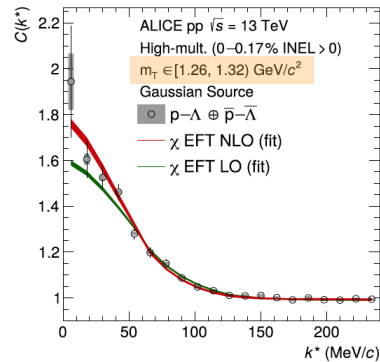
## p-p interaction:

Coulomb + Argonne v18

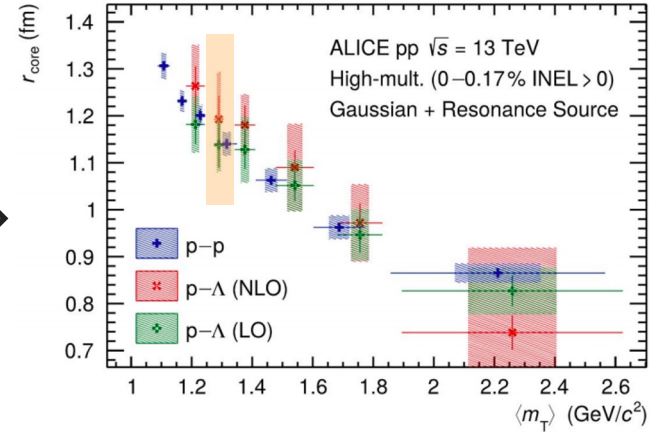


## p-Λ interaction:

χEFT NLO and LO



ALICE Coll., PLB, 811 (2020)



- Gaussian source

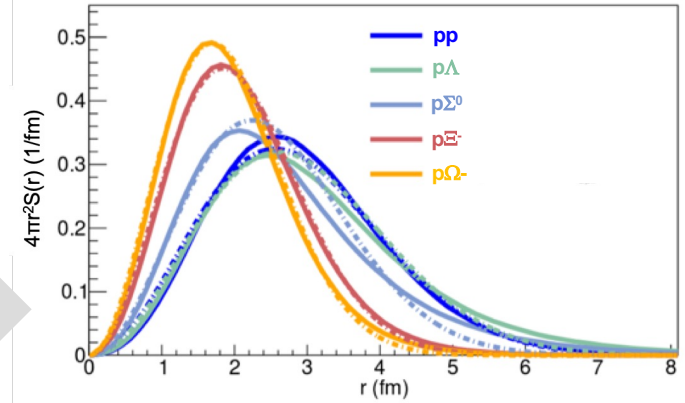
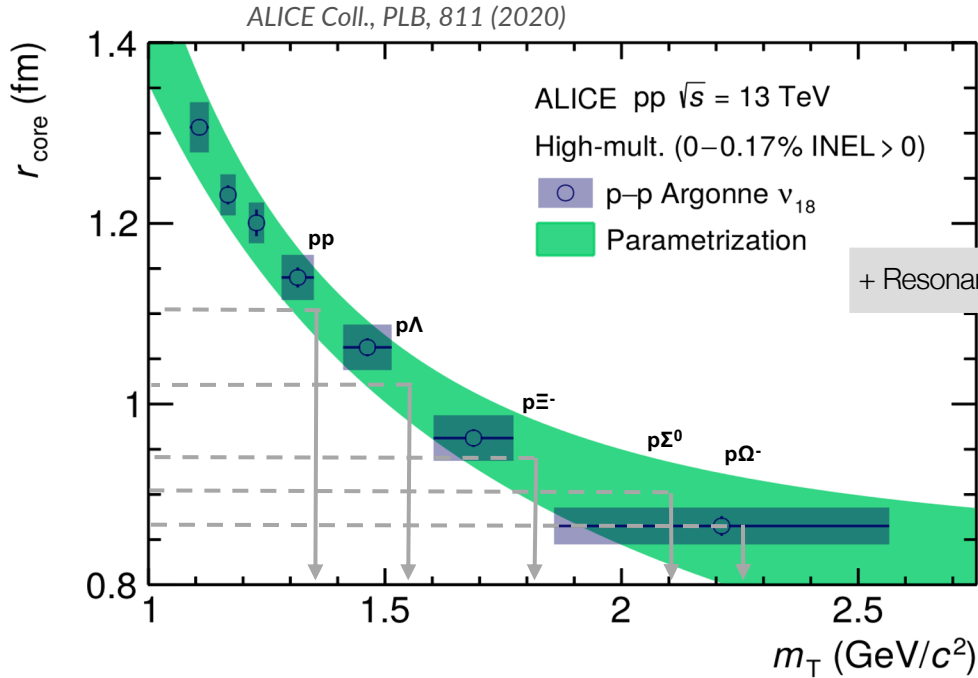
$$S(r) = G[r, r_{core}(m_T)] = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right)$$

+ Resonances with  $\tau \sim r_{core} \sim 1\text{fm}$  ( $\Delta^{++}, N^*, \Sigma^*$ )

One universal source for all hadrons

Similar results in  $K^+p, \pi\pi$

# Source determination



Pair	$r_{\text{Core}}$ [fm]	$r_{\text{Eff}}$ [fm]
p-p	1.1	1.2
p- $\Lambda$	1.0	1.3
p- $\Sigma^0$	0.87	1.02
p- $\Xi^-$	0.93	1.02
p- $\Omega^-$	0.86	0.95

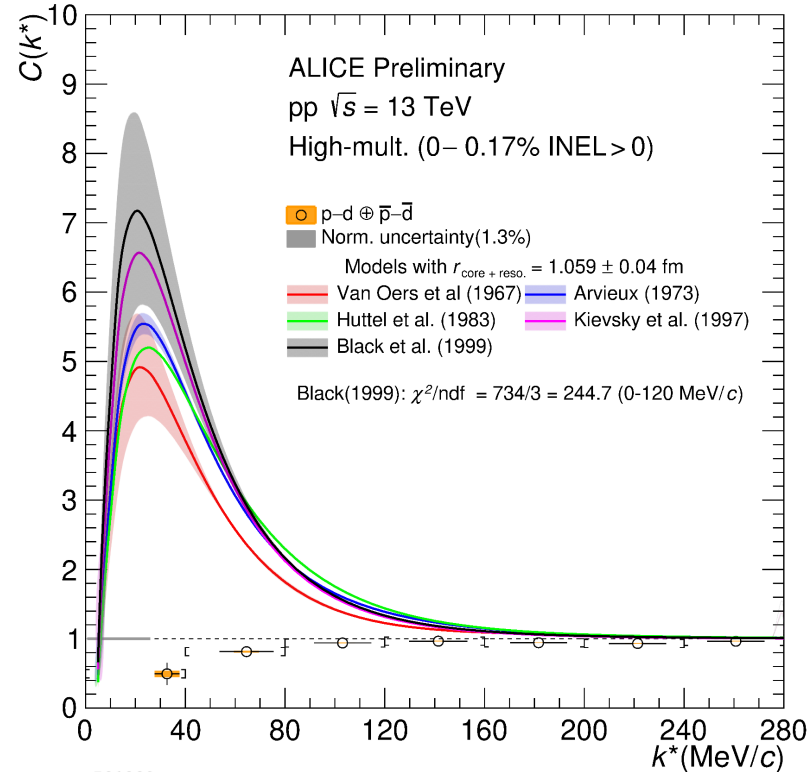
- Point-like particle models anchored to scattering experiments

	$S = 1/2$		$S = 3/2$	
	$f_0(\text{fm})$	$d_0(\text{fm})$	$f_0(\text{fm})$	$d_0(\text{fm})$
<span style="color: red;">—</span> Van Oers et al. (1967)	$-1.30^{+0.20}_{-0.20}$	—	$-11.40^{+1.20}_{-1.80}$	$2.05^{+0.25}_{-0.25}$
<span style="color: blue;">—</span> Arvieux (1973)	$-2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$-11.88^{+0.10}_{-0.40}$	$2.63^{+0.01}_{-0.02}$
<span style="color: green;">—</span> Huttel et al. (1983)	-4.0	—	-11.1	—
<span style="color: magenta;">—</span> Kievsky et al. (1997)	-0.024	—	-13.7	—
<span style="color: gray;">—</span> Black et al. (1999)	$0.13^{+0.04}_{-0.04}$	—	$-14.70^{+2.30}_{-2.30}$	—

W. T. H. Van Oers, & K. W. Brockman Jr, *NPA* 561 (1967);  
 J. Arvieux et al., *NPA* 221 (1973); E. Huttel et al., *NPA* 406 (1983);  
 A. Kievsky et al., *PLB* 406 (1997); T. C. Black et al., *PLB* 471 (1999);

- Coulomb + strong interaction using the Lednický model
- Only s-wave interaction
- Source radius evaluated using the hadron-hadron universal mT scaling

**Point-like particle description doesn't work for p-d**



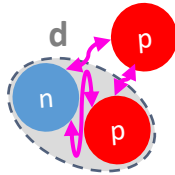
ALI-PREL-501009

# Proton-deuteron correlation

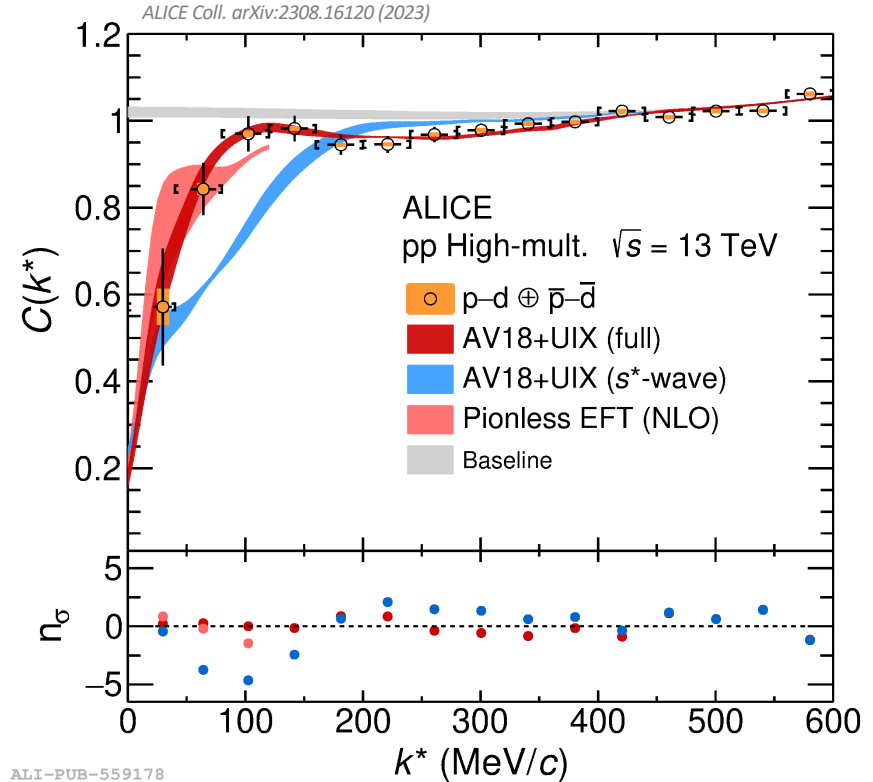
The measured p-d correlation function reflects the full three-nucleon dynamics:

Coulomb + strong interaction (NN and NNN) +  
Quantum Statistics

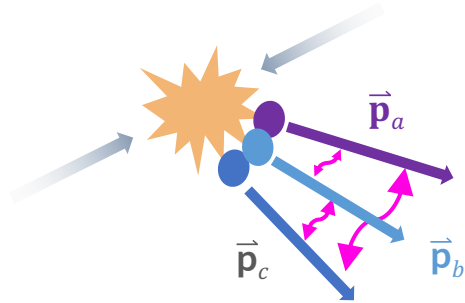
*M. Viviani et al., PRC 108 (2023) 6, 064002*



- Sensitivity to the short inter-particle distances
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics



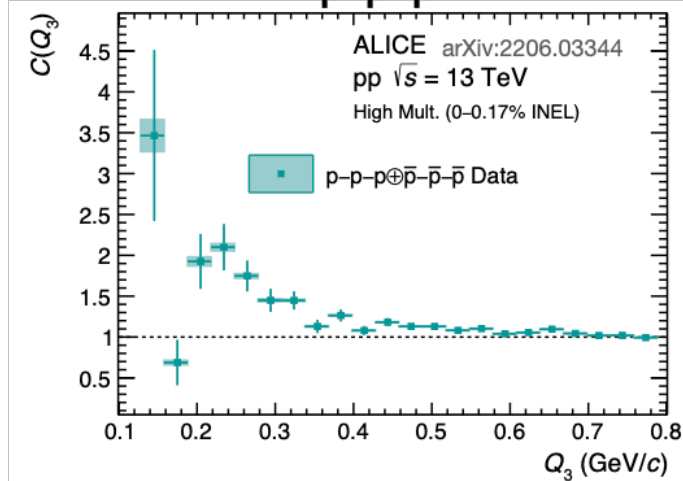
# Three-particle femtoscopy



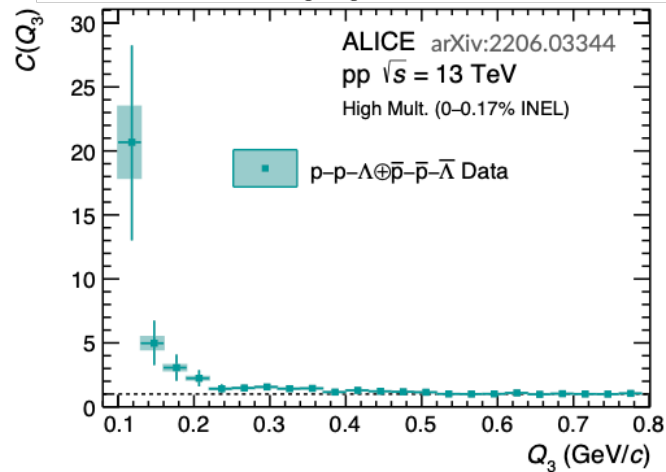
$$C(\vec{p}_a, \vec{p}_b, \vec{p}_c) = \int S(\vec{x}_a, \vec{x}_b, \vec{x}_c) |\psi_{\vec{p}_a, \vec{p}_b, \vec{p}_c}(\vec{x}_a, \vec{x}_b, \vec{x}_c)|^2 d^3\vec{x}_a d^3\vec{x}_b d^3\vec{x}_c$$

Three-body scattering  
wave function

**p-p-p** ALICE Coll., EPJ A 59, 145 (2023)



**p-p- $\Lambda$**



# Interpretation of the measurements

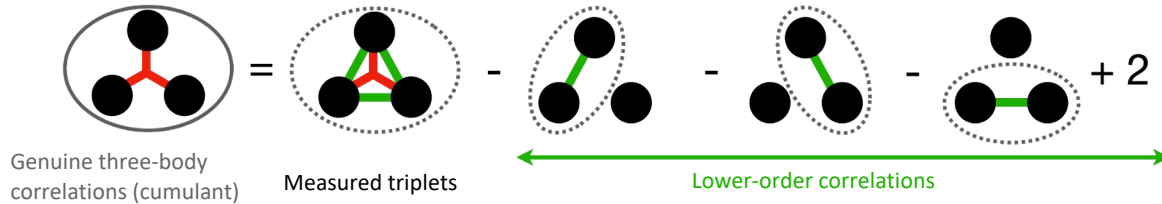
- Step 0: Cumulant expansion method, to answer the question:  
“Is there any genuine three-body effect in the measured correlation function?”
- Step 1: Calculate the three-particle scattering wave function and compute the correlation function

# Step 0: Cumulants

# Cumulants in femtoscopy

Genuine three-body effects can be isolated using the Kubo's cumulant expansion method

*J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)*

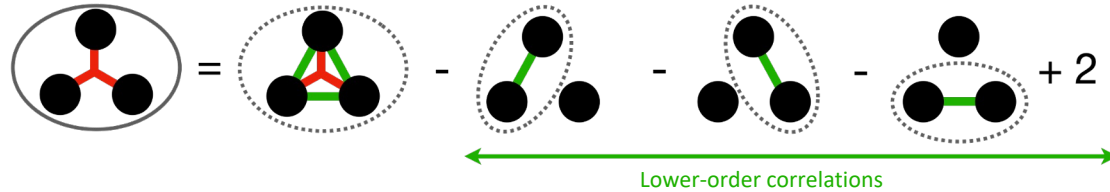




# Cumulants in femtoscopy

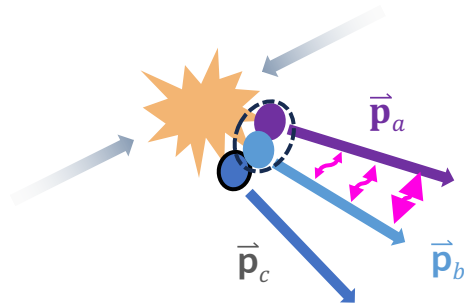
Genuine three-body effects can be isolated using the Kubo's cumulant expansion method

*J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)*



## Data-driven method:

One particle from another collision



## Projector method:

Each term of the lower-order correlations can be calculated using two-particle correlation functions

*R. Del Grande, L. Serksnyte et al, EPJC 82 (2022)*

$$\psi_{\vec{p}_a, \vec{p}_b, \vec{p}_c}(\vec{x}_a, \vec{x}_b, \vec{x}_c) = \psi_{\vec{p}_a, \vec{p}_b}(\vec{x}_a, \vec{x}_b) e^{i \vec{p}_c \cdot \vec{x}_c}$$

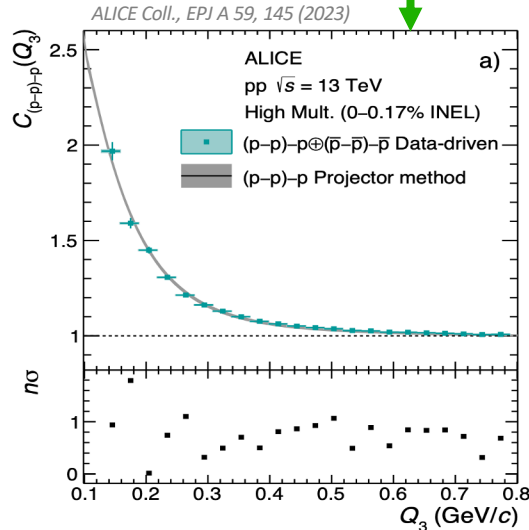
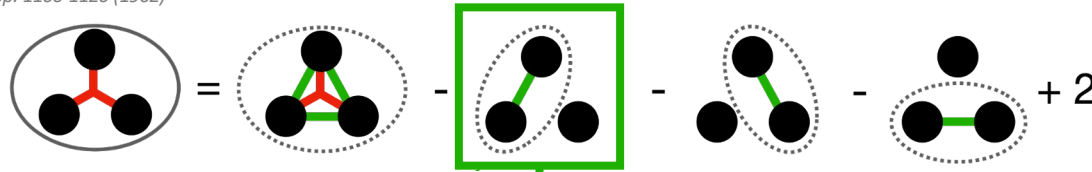
Correlated pair

Uncorrelated particle

# Cumulants in femtoscopy

Genuine three-body effects can be isolated using the Kubo's cumulant expansion method

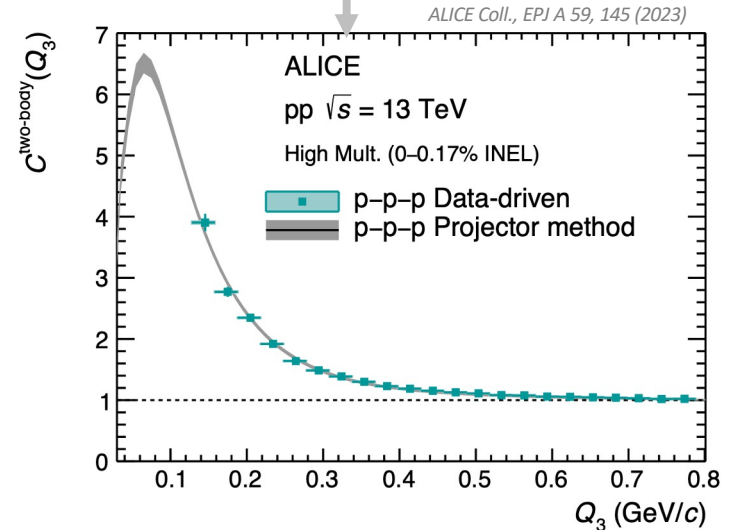
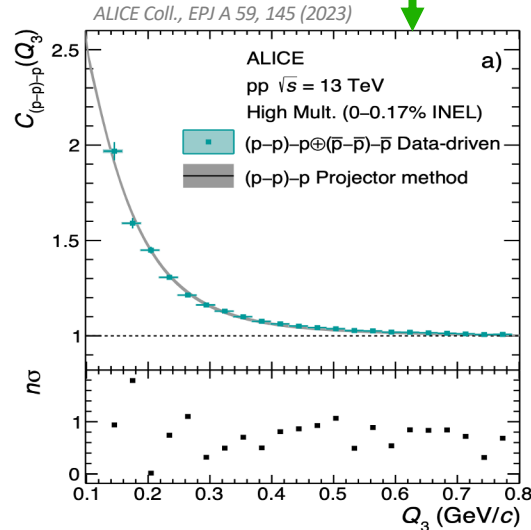
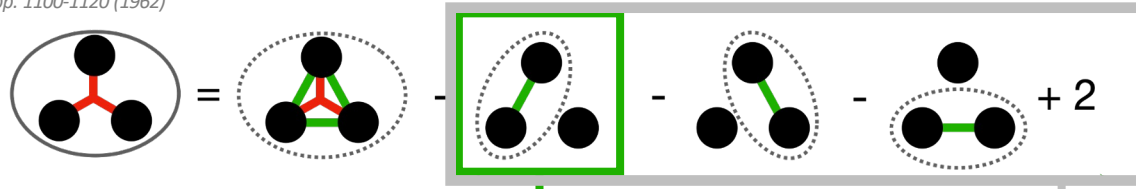
*J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)*



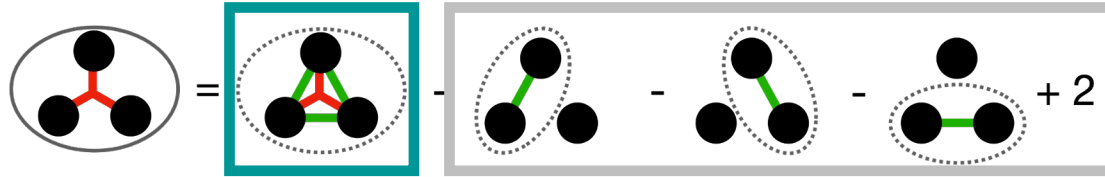
# Cumulants in femtoscopy

Genuine three-body effects can be isolated using the Kubo's cumulant expansion method

*J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)*

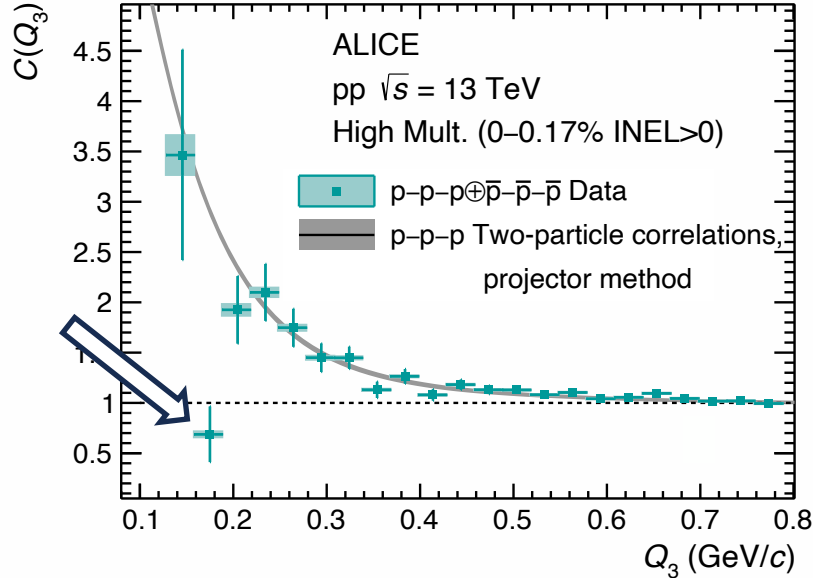


# p-p-p and p-p- $\Lambda$ correlations



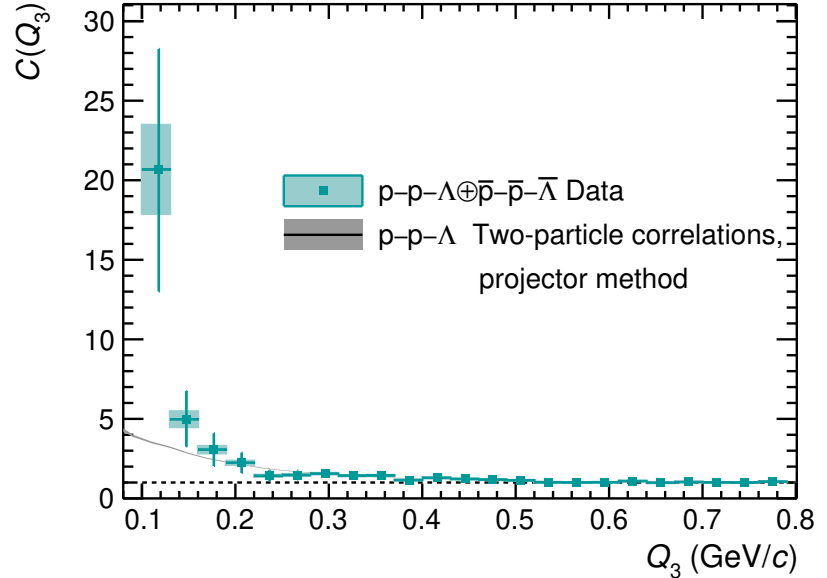
## p-p-p

ALICE Coll., EPJ A 59, 145 (2023)

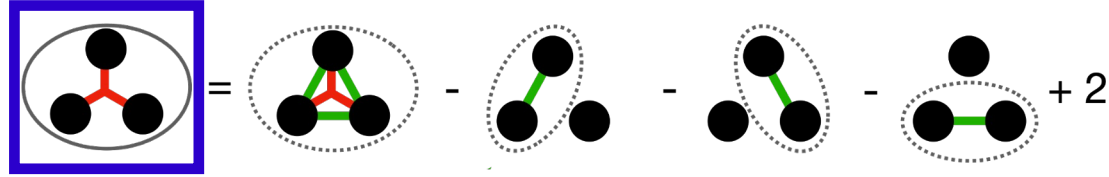


## p-p- $\Lambda$

ALICE Coll., EPJ A 59, 145 (2023)

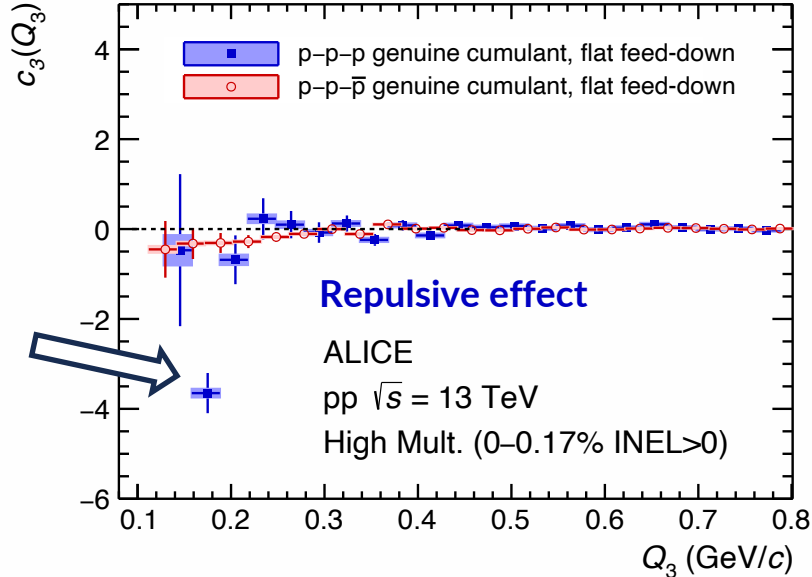


# p-p-p and p-p- $\Lambda$ cumulants



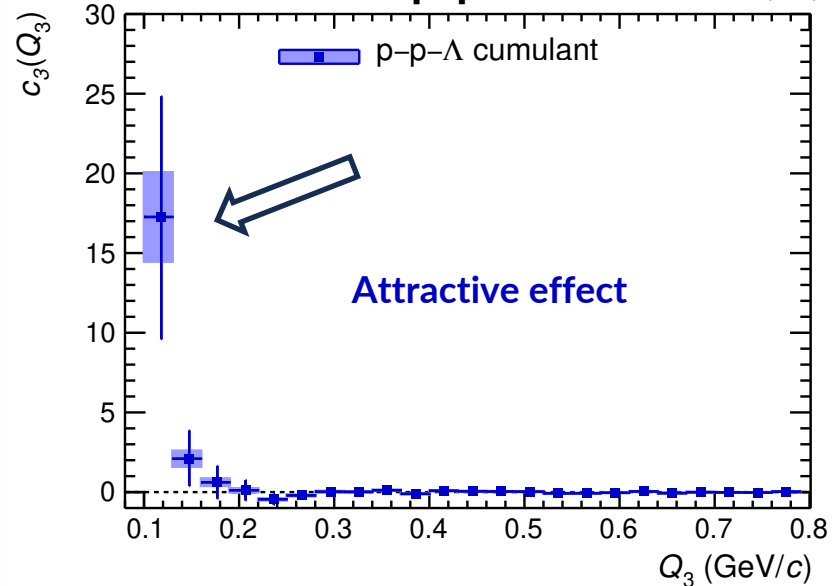
**p-p-p**

ALICE Coll., EPJ A 59, 145 (2023)



**p-p- $\Lambda$**

ALICE Coll., EPJ A 59, 145 (2023)



# Step 1: Calculation of the three-body wave function

# Hyperspherical coordinates

- Decomposition of the wave function in hyperspherical harmonics:

$$\psi = \sum_{[K]} \psi_{[K]} = \sum_{[K]} \rho^{-5/2} u_{[K]}(\rho) Y_{[K]}(\Omega)$$

- Schrödinger equation with the interaction:

$$-\frac{\hbar^2}{2m} \left( \frac{\partial^2 u_{[K]}(\rho)}{\partial \rho^2} - \frac{(K + 3/2)(K + 5/2)}{\rho^2} u_{[K]}(\rho) \right) + \sum_{[K']} U_{[K][K']}(\rho) u_{[K']}(\rho) = E u_{[K]}(\rho)$$

- The hypercentral potential is obtained as

$$U_{[K][K']}(\rho) = \int d\Omega Y_{[K]}^*(\Omega) [V_{12} + V_{23} + V_{31} + V_{123}] Y_{[K']}(\Omega)$$

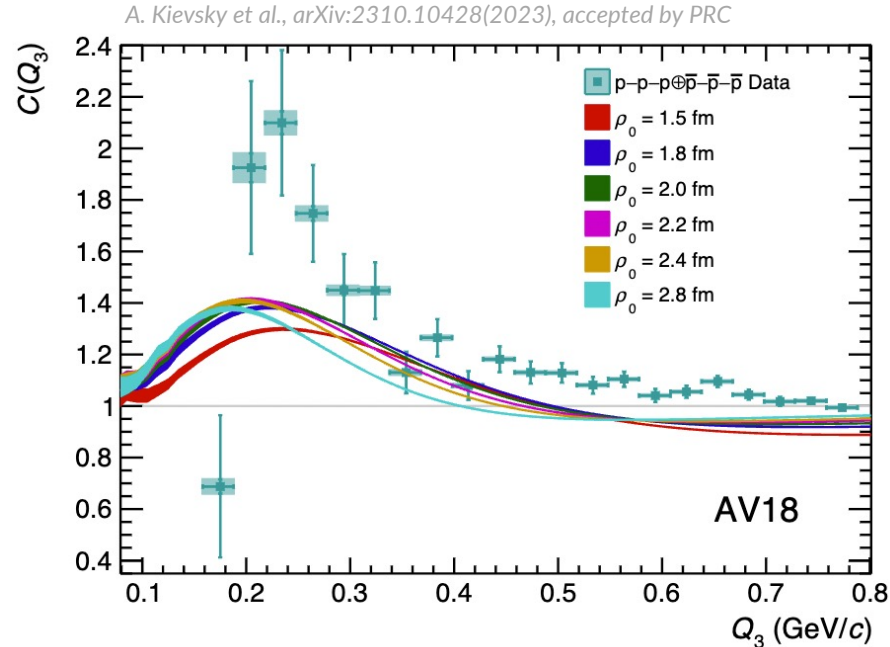
- Three-body correlation function:

$$C(Q_3) = \int S(\rho) |\psi(Q_3, \rho)|^2 \rho^5 d\rho$$

in collaboration with A. Kievsky, M. Viviani, L. Marcucci and E. Garrido.

*A. Kievsky et al., arXiv:2310.10428(2023), accepted by PRC*

- Hyper-Source radius:  $\rho_0 = 2 R_M$
- Wave function:  
AV18 + Coulomb + Antisymmetrisation
- Interaction at  $K \leq 2$ , free solution for  $K > 2$

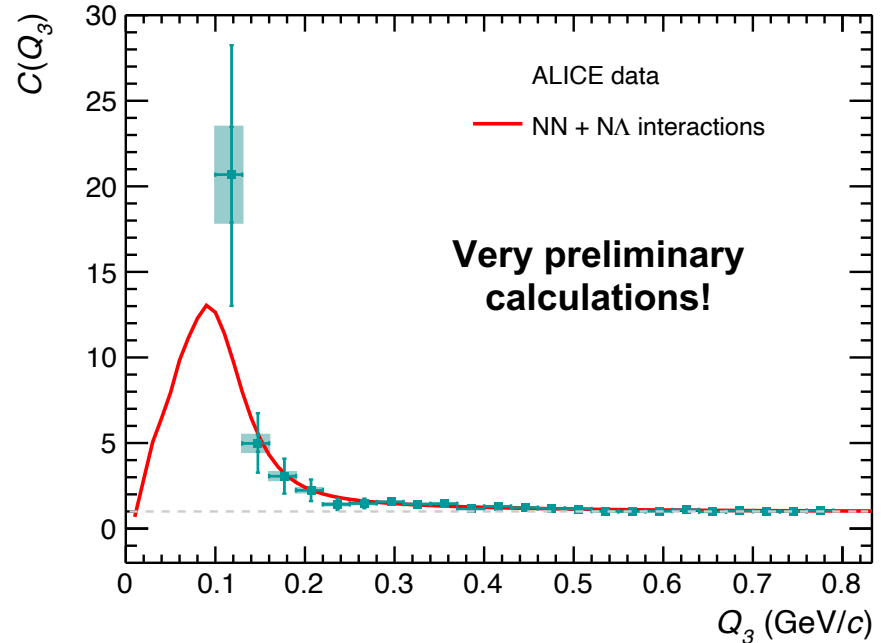




- Three-body correlation function:

$$C(Q_3) = \int S(\rho) |\psi(Q_3, \rho)|^2 \rho^5 d\rho$$

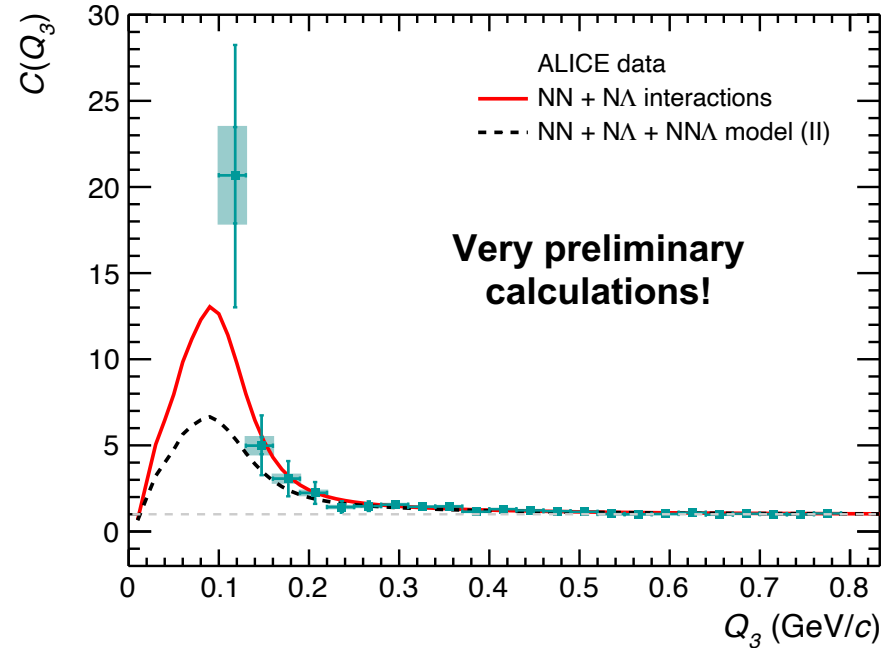
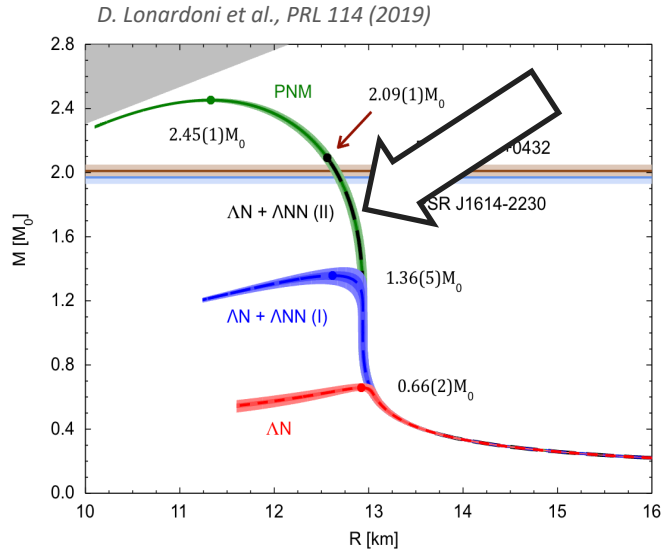
- Two-body interactions + Coulomb + Antisymmetrisation
- Hyper-source radius determined from the two-body p-p and p- $\Lambda$  sources
- Calculations include the interaction for  $K \leq 10$



- Three-body correlation function:

$$C(Q_3) = \int S(\rho) |\psi(Q_3, \rho)|^2 \rho^5 d\rho$$

- Inclusion of the  $\Lambda$ NN (model II)



Larger statistics is needed to test three-body interaction models

Femtoscopy in pp collisions at the LHC

- small particle source size (about 1 fm)
- alternative method to study many-body systems

Results achieved by ALICE:

- p-d correlation function shows sensitivity to the full three-nucleon dynamics
- Free scattering of three hadrons measured for the first time:
  - p-p-p correlation function: evidence of antisymmetrisation effects
  - p-p- $\Lambda$  correlation function: no strong evidence of a three-body repulsion (modelling has to be improved)

# Backup

- $X_i$  denotes the general  $i$ -th stochastic variable
- The most general decomposition of 2-particle correlation is:

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2<sup>nd</sup> term on the right is the 2-particle cumulant
- Cumulants cannot be measured directly, however:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

- The most general decomposition of 3-particle correlation is:

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle &= \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \\ &+ \langle X_1 X_2 \rangle_c \langle X_3 \rangle + \langle X_1 X_3 \rangle_c \langle X_2 \rangle + \langle X_2 X_3 \rangle_c \langle X_1 \rangle \\ &+ \langle X_1 X_2 X_3 \rangle_c \end{aligned}$$

- Using the 2-particle cumulant:  $\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$
- Working recursively from higher to lower orders, we have 3-particle cumulant expressed in terms of the measured 3-, 2-, and 1-particle averages:

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \end{aligned}$$

In terms of the probability distribution functions we have:

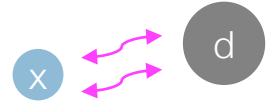
$$\begin{aligned}
 \langle X_1 X_2 \rangle_c &= \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle = \\
 &= \int X_1 X_2 f(X_1, X_2) dX_1 dX_2 - \int X_1 f_1(X_1) dX_1 \int X_2 f_1(X_2) dX_2 \\
 &= \int X_1 X_2 \left[ f(X_1, X_2) - f_1(X_1) f_1(X_2) \right] dX_1 dX_2 \\
 &= \int X_1 X_2 f_c(X_1, X_2) dX_1 dX_2
 \end{aligned}$$

with

$$f_c(X_1, X_2) = f(X_1, X_2) - f_1(X_1) f_1(X_2)$$

For three particles:

$$f_c(X_1, X_2, X_3) = f(X_1, X_2, X_3) - f(X_1, X_2) f(X_3) - f(X_1, X_3) f(X_2) - f(X_3, X_2) f(X_1) + 2f(X_1) f(X_2) f(X_3)$$



- Coulomb + strong interaction using the Lednický model

$$\psi(\vec{k}^*, \vec{r}^*) = e^{i\delta_c} \sqrt{A_c(\eta)} \left[ e^{-i\vec{k}^* \cdot \vec{r}^*} F(-i\eta, 1, i\xi) + f_C(k^*) \frac{\tilde{G}(\rho, \eta)}{r^*} \right]$$

$$f_C(k^*) = \left( \frac{1}{f_0} + \frac{d_0 \cdot k^{*2}}{2} - \frac{2h(k^*)}{a_c} - ik^* A_C(k^*) \right)^{-1}$$

- Point-like particle models anchored to scattering experiments

System	Spin averaged		$S = 1/2$		$S = 3/2$	
	$f_0(\text{fm})$	$d_0(\text{fm})$	$f_0(\text{fm})$	$d_0(\text{fm})$	$f_0(\text{fm})$	$d_0(\text{fm})$
p-d			$-1.30^{+0.20}_{-0.20}$	—	$-11.40^{+1.20}_{-1.80}$	$2.05^{+0.25}_{-0.25}$
			$-2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$-11.88^{+0.10}_{-0.40}$	$2.63^{+0.01}_{-0.02}$
			-4.0	—	-11.1	—
			-0.024	—	-13.7	—
			$0.13^{+0.04}_{-0.04}$	—	$-14.70^{+2.30}_{-2.30}$	—
$K^+$ -d	-0.470	1.75				
	-0.540	0.0				

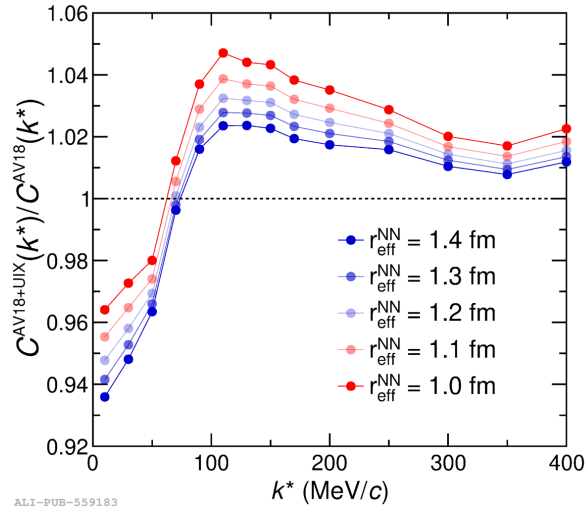
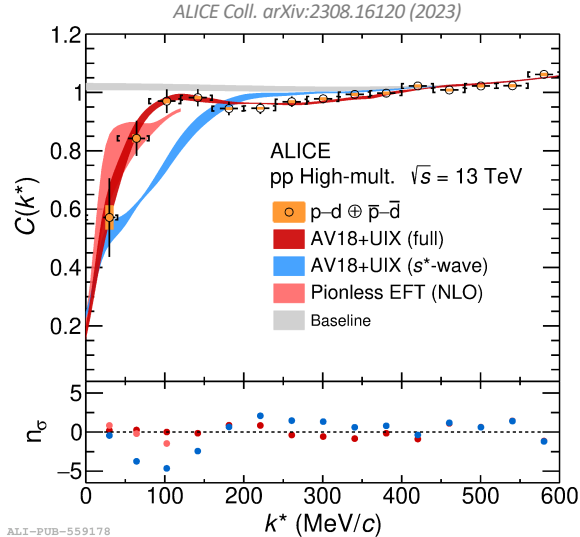
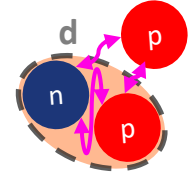
- Only s-wave interaction
- Source radius evaluated using the hadron-hadron universal  $m_T$  scaling



# Proton-deuteron correlation

The measured p-d correlation function reflects the full three-nucleon dynamics:  
**Coulomb + strong interaction (NN and NNN) + Quantum Statistics**

*M. Viviani et al., arXiv:2306.02478 (2023)*



- Sensitivity to the short inter-particle distances
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics

