Advancing Heavy-Ion Collisions in Theory & Experiment: Model Evaluation & 3D

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Probing dense baryonic matter with hadrons II: FAIR Phase-0

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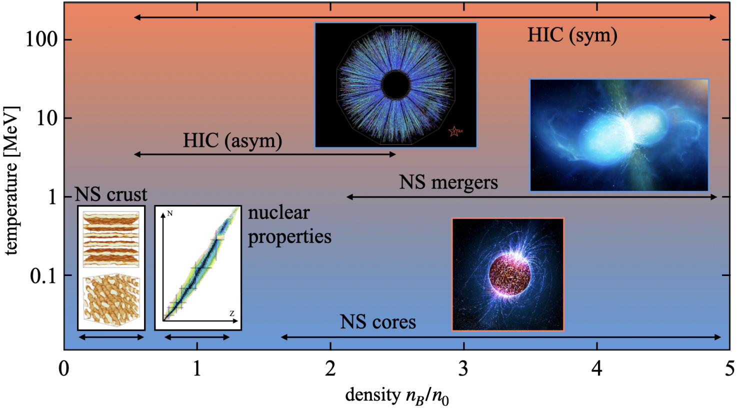
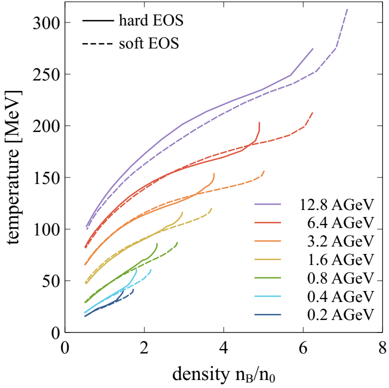
Heavy-Ion Collisions Contribution to EOS Assessment

$1 \lesssim \rho/\rho_0 \lesssim 4.5$ symmetric matter

$1 \lesssim \rho/\rho_0 \lesssim 2.5?$ symmetry energy





Oleh Savchuk

Sorensen *et al.* PPNP134(24)104080

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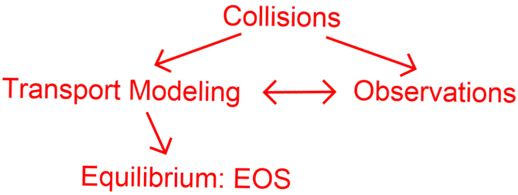
Transport & Observables

Transport modeling needed because equilibrium not reached in collisions



Conclusions usually reached on basis of 1-2 models

Variations btw model predictions may be stronger than EOS sensitivity

$\Rightarrow$ Transport Model Evaluation Project (TMEP)

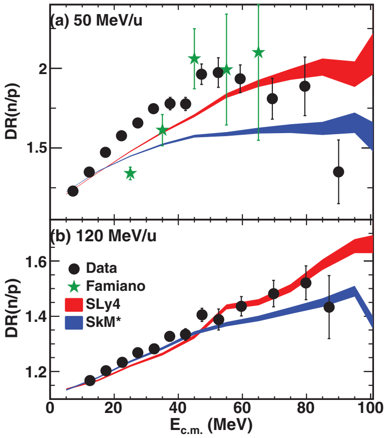
EOS effects weak - need to concentrate on observables particularly sensitive to EOS

Symmetry-energy effects weak $\leftrightarrow$ asymmetry variations ~ 0.1

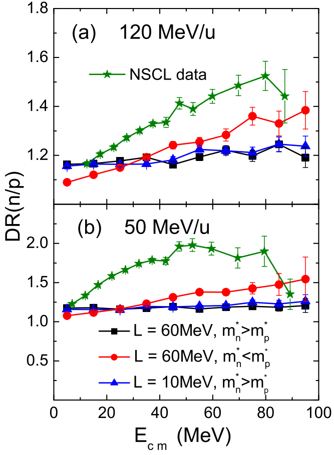
$\Rightarrow$ Isospin observables

Opposing Conclusions May Emerge

E.g., on sensitivity to ordering of effective masses



Coupland *et al.* PRC94(16)011601



Kong *et al.* PRC91(15)047601

Theoretical Background f/Transport

Many-body derivations: BBGKY hierarchy or Green's functions, principally exact

At some point truncation, typically losing impact of long-term correlations/fluctuations, $G_0^{-1} G = 1 + \Sigma G \rightarrow \Sigma \simeq VGVGG$

In parallel, separation of scales: short de Broglie wavelength, collision range/duration

Loss of long-term correlations irrelevant at high densities when short-term correlations dominate, but hurts at low when long-term correlations persist

Two groups of models:

- Boltzmann-equation models, short-term correlations only
- molecular dynamics & stochastic models attempt to capture long-term correlations, borrow treatment f/short-range from other group

Mean fields $\leftrightarrow$ EOS, momentum dependence, in-medium rates, fluctuations, cluster formation...

Transport Model Evaluation Project

TMEP: Models evaluated under controlled conditions
Review: Wolter *et al.* PPNP122(22)103962

History

- 2009/2014, Au + Au at 100 & 400 MeV/nucl Xu *et al.* PRC93(16)044609
  $\rho(\mathbf{r})$-evolution & nucleonic observables (stopping, flow)
  differences hard to understand $\rightarrow$ switch to simplified conditions
- 2018-21, Box w/periodic boundaries, close to equilibrium, analytic limits
  Mean field, collision term, π production in cascade mode
- 2023, Again HIC: Sn + Sn at 270 MeV/nucl
  Subthreshold π production for different symmetry energies in the context of SπRIT measurements

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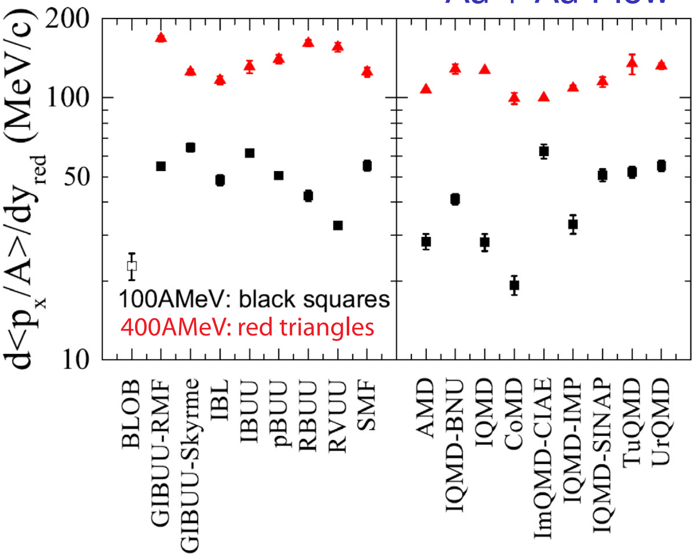
Au + Au Flow

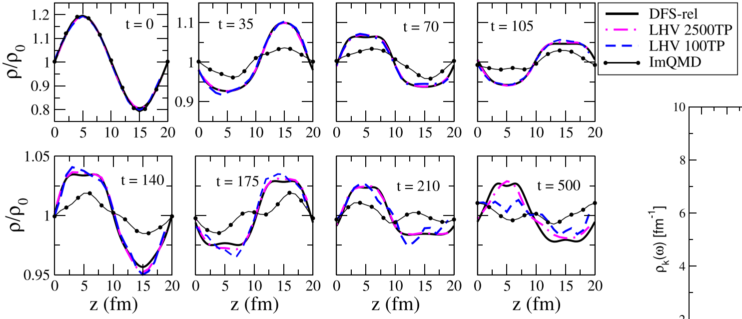


Xu *et al.* PRC93(16)044609

Mean Field

Standing-wave initialization in a box

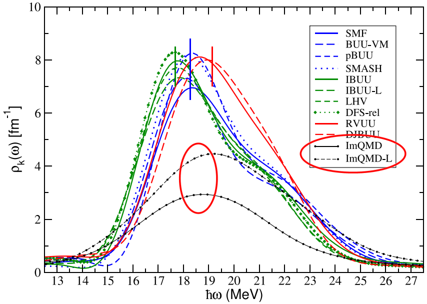
Colonna *et al.* PRC104(21)044609



DFS - exact; LHV - Boltzmann model

ImQMD-L - better treatment of nonlinear ρ^γ

Response function: Fourier transform of density in position & time:



Vertical lines: Landau theory

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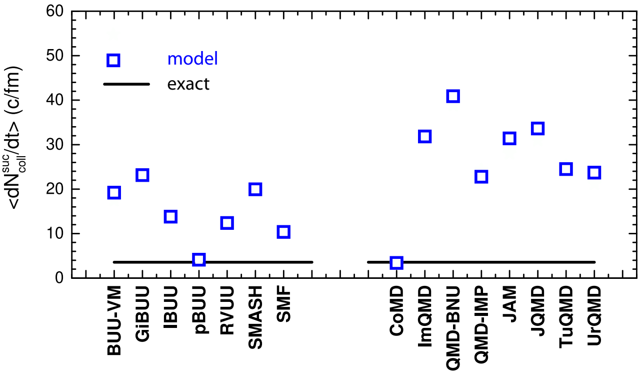
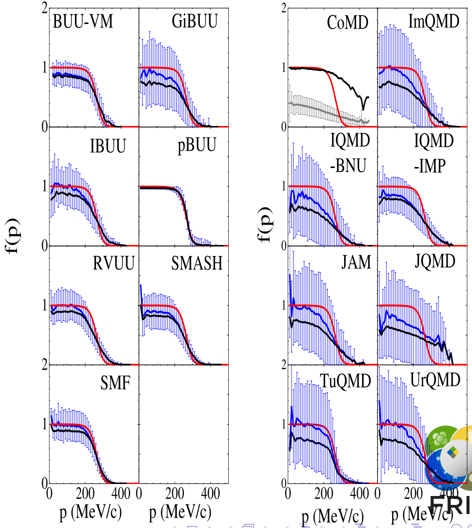
Collision Integral in a Box

System of nucleons only, initialized @$T = 5\,\mathrm{MeV}$, w/Pauli pcple Zhang *et al.* PRC97(18)034625





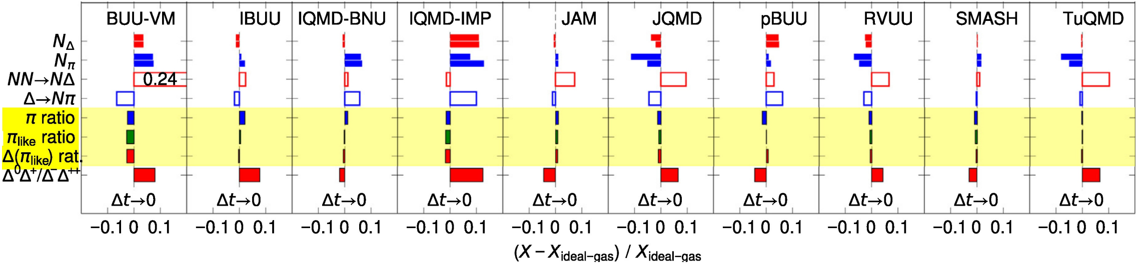
Differences tied to Pauli pcple:

red - exact, blue - average

Pion Production in a Box

Asymmetric system initialized at $T = 60$ MeV w/nucleons only, no Pauli blocking

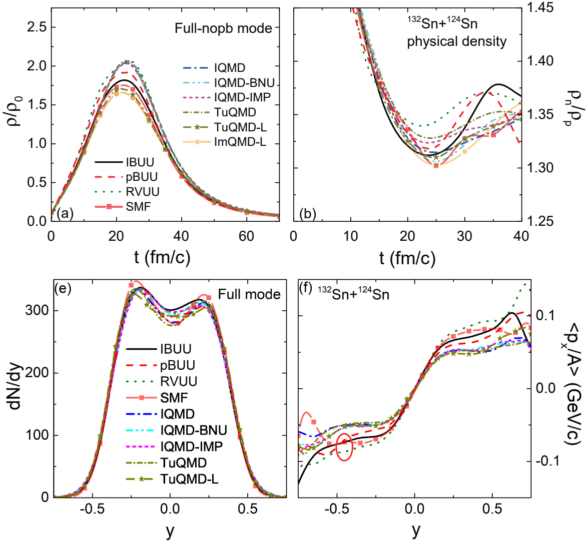
Ono *et al.* PRC100(19)044617



Differences due to correlations btw collisions/collision strategies

$\rightarrow$ cancel in π^-/π^+-like ratios

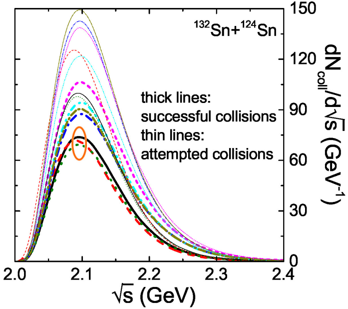
Pion Production in Sn + Sn at 270 MeV/nucl



Common initialization, no mo-dep

same $\sigma_{el,inel}$, Γ_Δ Xu *et al.* arXiv:2308.05347

Differences already at nucleon level: BUU vs QMD, Pauli



$dNN \rightarrow N\Delta$

BUU "more elastic":
more flow & lower inelastic rates

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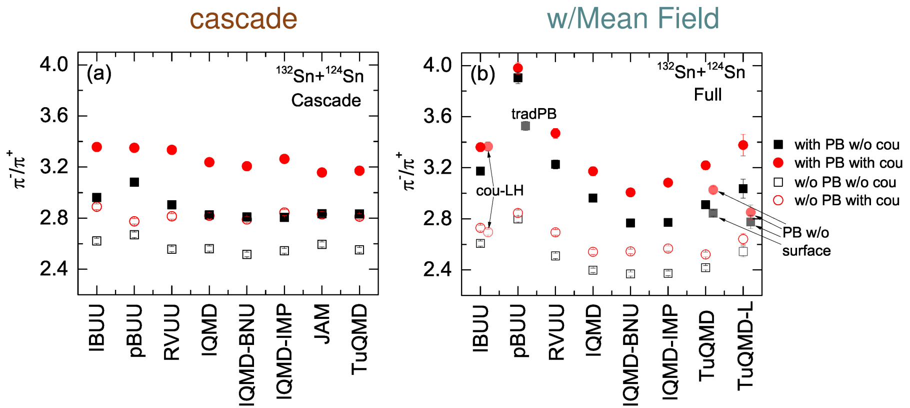
Charged Pion Ratio in Sn + Sn at 270 MeV/nucl



Good agreement w/o mean field, but not so good with, due to differences in nucleon evolution

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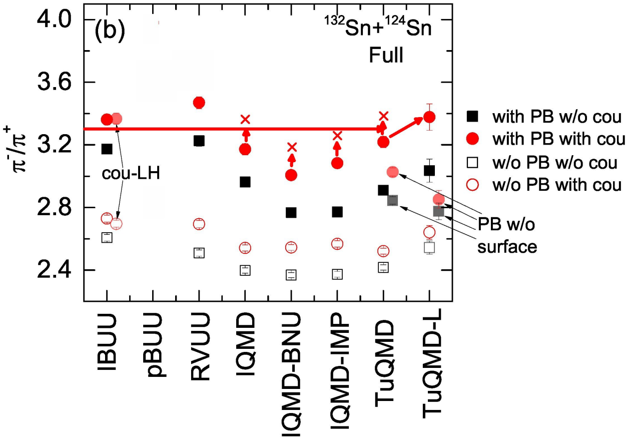
Charged Pion Ratio in Sn + Sn at 270 MeV/nucl



More adjustment w/mean field: pBUU not fully adhering to mean-field specifications + more radical Pauli

Correction for treatment of nonlinear term in QMD: TQMD test, TQMD & TQMD-L

If similar effect in other QMD models, much better agreement? $\lesssim$ 10%

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Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

What is it?!

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

some control, v_n

Still not clear what the system is...

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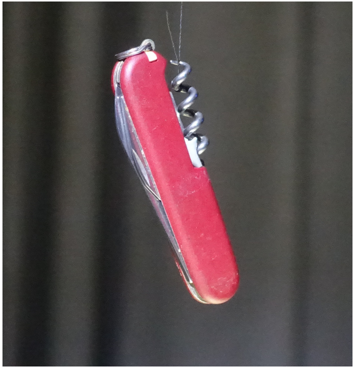
Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

some control, v_n



full control, $\frac{d^3N}{dp^3}$

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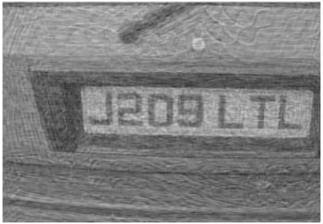
Claim: You can go from center to right panel through deblurring

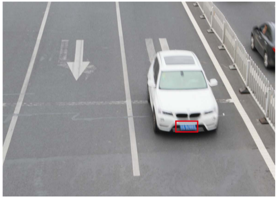
Deblurring by Example

Budd, *Crime Fighting Math*, plus.maths.org magazine

Blurred Photo of Moving Car | Deblurred | Photo of Parked Car

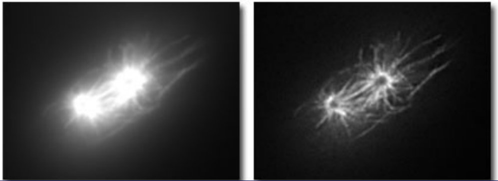


Fast Moving



Deblurring in Optical Microscopy

Before and After Nearest Neighbor Deconvolution Analysis



Correcting f/Distortions Due to Apparatus or Method

Detector efficiency ϵ, n measured ptcle number, N actual number

$$N \simeq \frac{1}{\epsilon} n$$

Typical energy loss in thick target $\overline{\Delta E}$ for detected particle

$$E_{\mathrm{prod}} \simeq E_{\mathrm{det}} + \overline{\Delta E}$$

General problem stated probabilistically, with $P(\zeta|\xi)$ - probability to measure ptcle characteristic to be ζ when it is actually ξ

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi) \, N(\xi)$$

For small distortions, P finite only when ζ little different from ξ. Optical terminology: P - blurring or transfer function.

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Bayesian Deblurring

Distorted $n(\zeta)$ measured, while pristine $N(\xi)$ sought:

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi) \, N(\xi)$$

$P(\zeta|\xi)$ - probability that ptcle with ζ detected while it really has characteristic ξ, understood given the method/apparatus, can be simulated (Geant4) & can depend on N

$Q(\xi|\zeta)$ - unknown complementary probability that ptcle has characteristic ξ while measured at ζ

Bayesian relation: number of times ptcle has characteristic in $\mathrm{d}\xi$ while measured in $\mathrm{d}\zeta$ is

$$P(\zeta|\xi) \, N(\xi) \, \mathrm{d}\xi \, \mathrm{d}\zeta = Q(\xi|\zeta) \, n(\zeta) \, \mathrm{d}\xi \, \mathrm{d}\zeta$$

Hence $$N(\xi) = \frac{\int \mathrm{d}\zeta \, Q(\xi|\zeta) \, n(\zeta)}{\int \mathrm{d}\zeta' \, P(\zeta'|\xi)} \, , \quad Q(\xi|\zeta) = \frac{P(\zeta|\xi) \, N(\xi)}{\int \mathrm{d}\xi' \, P(\zeta|\xi') \, N(\xi')}$$

Richardson-Lucy method solves eqs iteratively till stabilization

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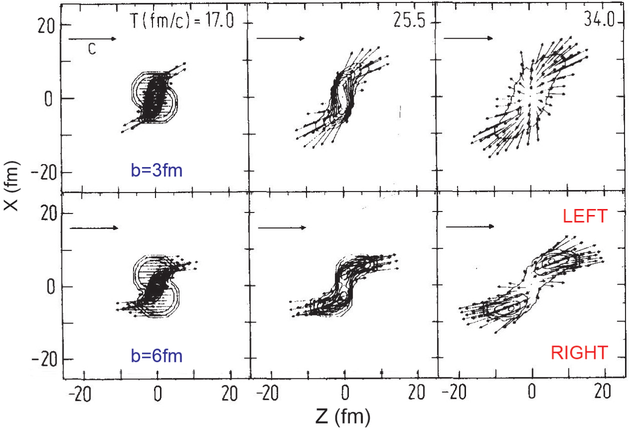
Side Focus in Hydrodynamic Calculations

Matter dispersed in the final stage, but most likely direction of motion away from the beam, e.g., in the calculations by Buchwald for Nb + Nb at 400 MeV/nucl

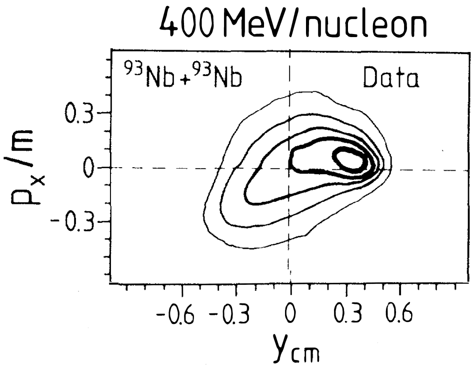
Stöcker&Greiner Phys.Rep. 137(86)277



Can this be seen experimentally??

1984 Claim

Gustafsson *et al.* PRL 18(84)1590 Plastic Ball Group claims to see preferential emission away from the beam axis, in $d^3N_{ch}/dy\, d^2p^\perp$ for 400 MeV/nucl Nb + Nb collisions, when determining reaction plane from flow tensor, $\mathbf{S}^{\perp z} = \sum_\nu \mathbf{p}_\nu^\perp\, p^z/2m_\nu$



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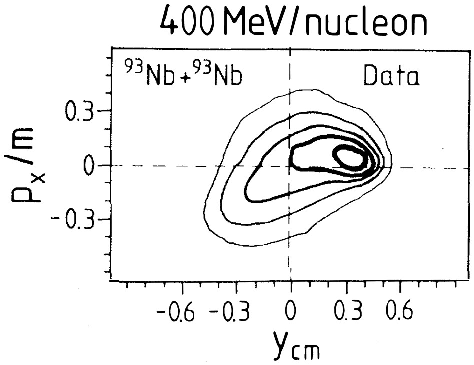
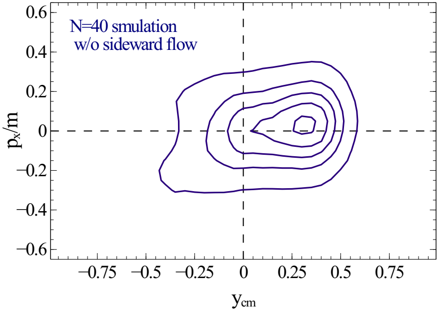
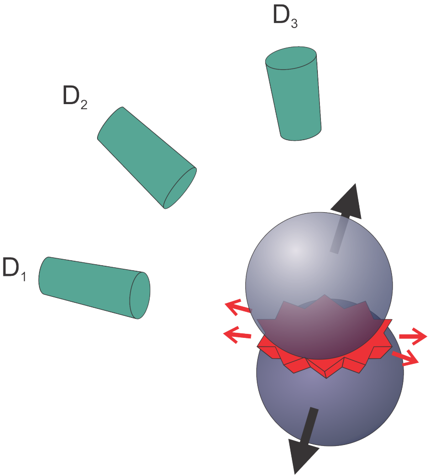




The observation can be explained by particle self-correlation, w/o invoking transverse collective movement

Estimating Reaction-Plane Direction w/o Self-Correlation



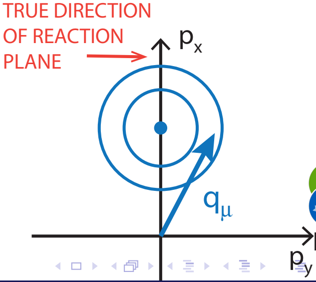
Plane direction f/particle μ estimated with

$$\mathbf{q}_\mu = \frac{1}{N} \sum_{\nu \neq \mu} \omega_\nu \, \mathbf{p}_\nu^\perp \qquad \omega_\nu = \begin{cases} +1, & \text{if } p_\nu^z > 0 \\ -1, & \text{if } p_\nu^z < 0 \end{cases}$$

N - measured particle multiplicity; other ptcles in the event used as reference for μ

PD&Odyniec PLB157(85)146

Problem: Reference vector $\mathbf{q}_\mu$ Gaussian fluctuates around true plane direction, blurring features



Current Solution: Angular Moments of Distributions

Solution: average angular moments (azimuthal Fourier coefficients)

$$v_n = \langle \cos n\phi \rangle$$

ϕ - angle relative to true reaction plane

Voloshin&Zhang ZfPhC70(1996)665

v_n derived from average scalar products/contractions, e.g.,

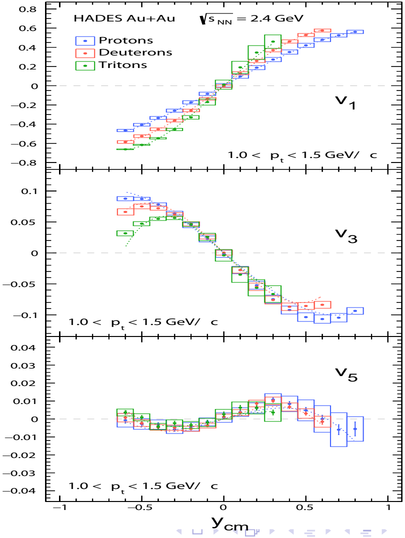
$$\langle \mathbf{p}_\mu^\perp \cdot \mathbf{q}_\mu \rangle \simeq p^\perp \langle q^x \rangle \langle \cos\phi \rangle$$

for different $p^\perp$, y and ptcle ID

Problem: unclear physics in v_n especially for higher n

1.23 GeV/nucl Au + Au $b \simeq 6$ fm

HADES PRL125(2020)262301

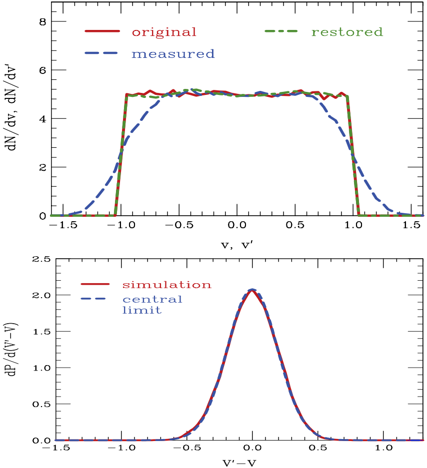


Schematic 1D Model

Proposition: Carry out as good determination of 3D info as you can & refine with deblurring. ~~v_n~~?

First 1D deblurring test. Projectile at unknown velocity V deexcites emitting $N = 10$ ptcles distributed with box-like dN/dv in projectile cm. Task: Measuring ptcles in lab, determine dN/dv. Cm velocity V' estimated from remaining ptcles, so V' & dN/dv' smeared:

$$\frac{dN}{dv'} = \int dV' \, \frac{dP}{dV'} \, \frac{dN}{dv}$$

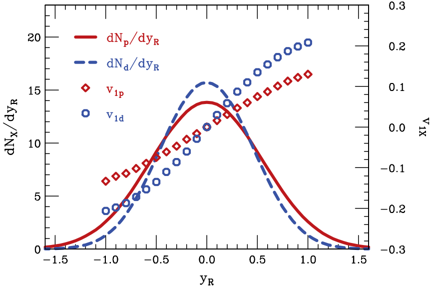


→Central-limit smear + RL deblur

PD&Kurata-Nishimura PRC105(2022)034608

3D Model for Collisions

Customary thermal model with flow, N, d, t, ^{3}He, ^{4}He. $\langle Z_{Tot} \rangle = 50$
Rapidity dstr, temperature & flow typical for semicentral collisions at 300 MeV/nucl



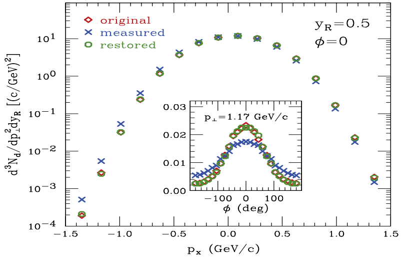
$$\frac{dN}{d\phi'} = \int d\Phi' \, \frac{dP}{d\Phi'} \, \frac{dN}{d\phi}$$

$$\phi' + \Phi' = \phi + \Phi$$

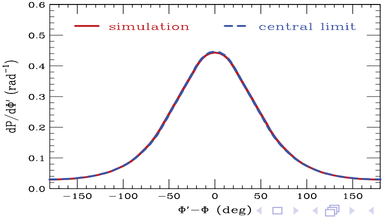
RL deblur + central-limit

Strong anisotropies restored!

Triple differential spectrum in reaction plane:



Uncertainty in reaction plane:

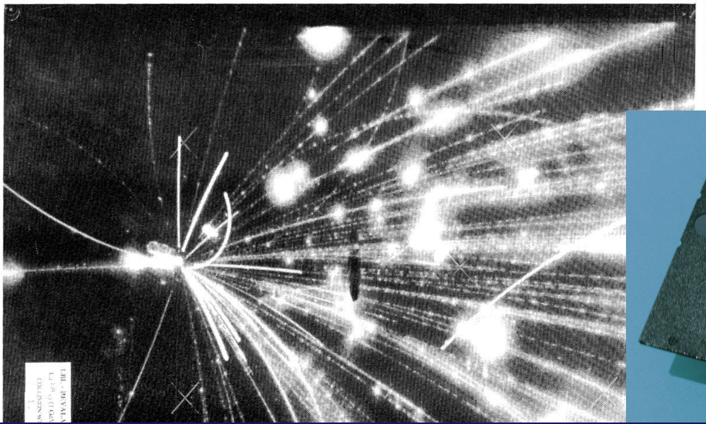


Ar + KCl @ 1.8 GeV/nucl

Ströbele PRC 27(83)1349

495 events from Streamer Chamber, $b \lesssim 2.4\,\mathrm{fm}$

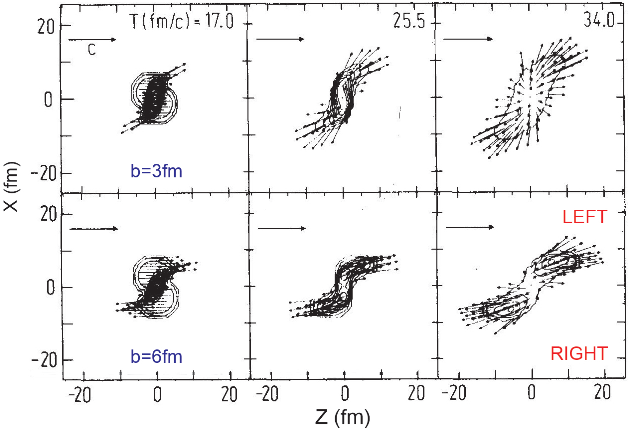
PD&Odyniec PLB 157(85)146



Reminder: Hydrodynamic Calculations

Matter dispersed in the final stage, but most likely direction of motion away from the beam, e.g., in the calculations by Buchwald for Nb + Nb at 400 MeV/nucl

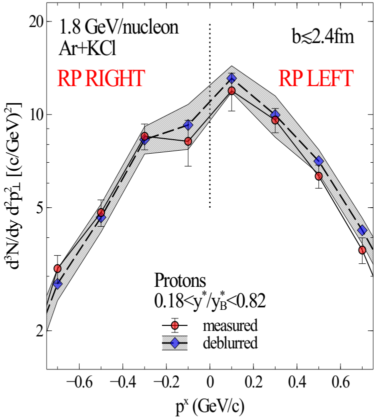
Stöcker&Greiner Phys.Rep. 137(86)277



Can this be seen experimentally??

Side-Focus in Ar + KCl 1.8 GeV/nucl?

protons

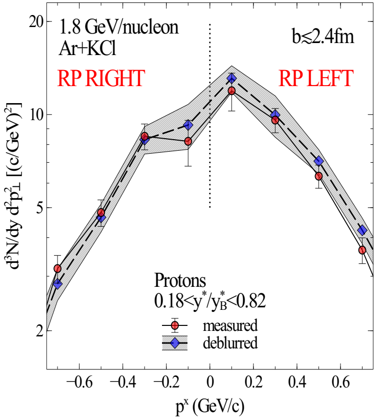


Particles in the forward hemisphere, $y^* \sim 0.5 y_B^*$

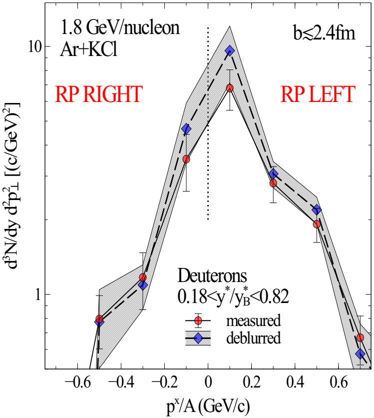
PD, Ströbele, Nzabahimana PRC108(23)L051603

Side-Focus in Ar + KCl 1.8 GeV/nucl?

protons



deuterons



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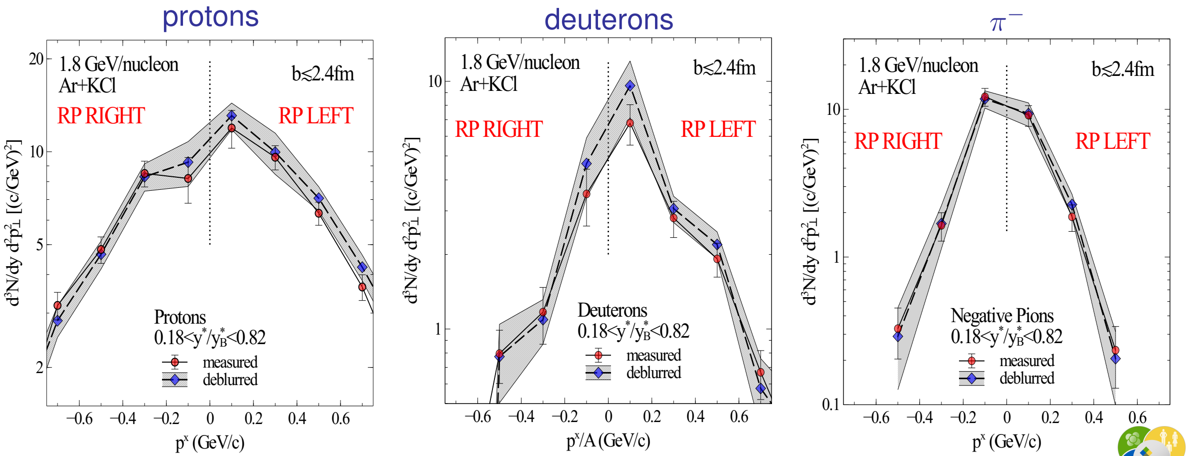

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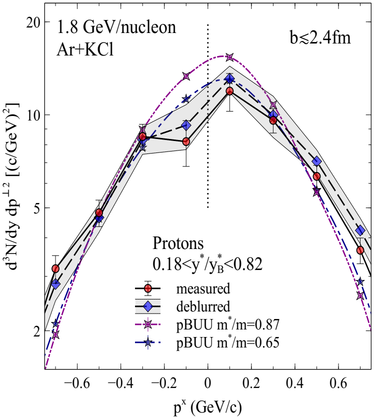
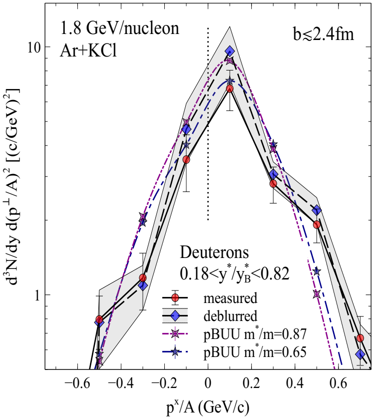
PD, Ströbele, Nzabahimana PRC108(23)L051603

Side-Focus: Experiment vs Theory

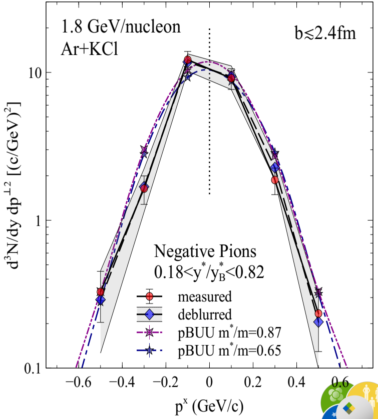
protons



deuterons



π^-



Transport Calculations: sensitivity to mean-field *p*-dependence in Ar+KCl

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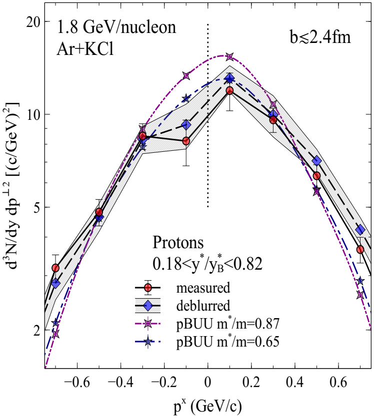
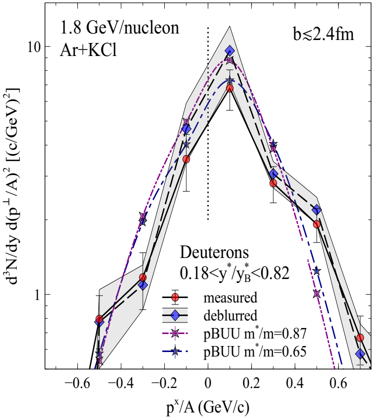
Side-Focus: Experiment & Theory

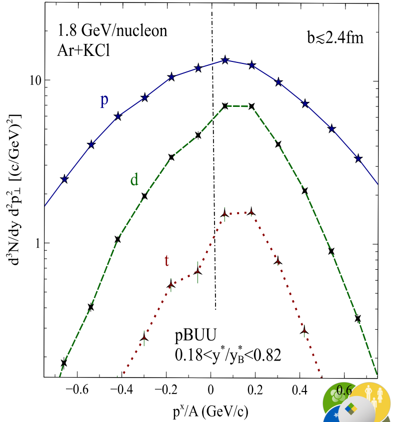
protons



deuterons



H Isotopes



Transverse boost $v^x \sim 0.1\,c$

What's Behind Deblurring's Success?

Singular value decomposition f/forward conditional probability:

$$P_{ij} = \sum_{\mathrm{n}} \sigma_{\mathrm{n}}\, U_{\mathrm{n}i}\, V_{\mathrm{n}j} \qquad \Rightarrow \qquad Q_{ji} \overset{?}{=} \sum_{\mathrm{n}} \sigma_{\mathrm{n}}^{-1}\, V_{\mathrm{n}j}\, U_{\mathrm{n}i}$$

i - measurement, j - reality, Q - backward conditional probability. Instability??

Plain Reaction-Plane Deblurring:

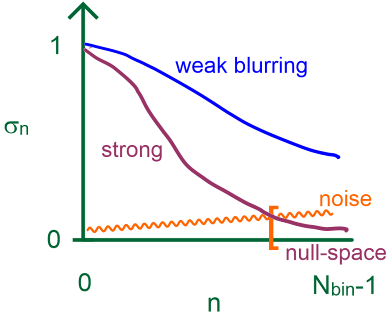
$$U_{\mathrm{n}}(\varphi) = V_{\mathrm{n}}(\varphi) = \begin{cases} \frac{1}{\sqrt{2\pi}}, & \mathrm{n} = 0 \\ \frac{\cos(n\varphi)}{\sqrt{\pi}}, & \mathrm{n} > 0 \end{cases}$$

$$\sigma_{\mathrm{n}} = \langle \cos(n\Delta\Phi) \rangle$$

with $\Delta\Phi$ estimated-true reaction plane deviation

Detector effects yield more complicated vectors

Positivity + regularization stabilize restoration!



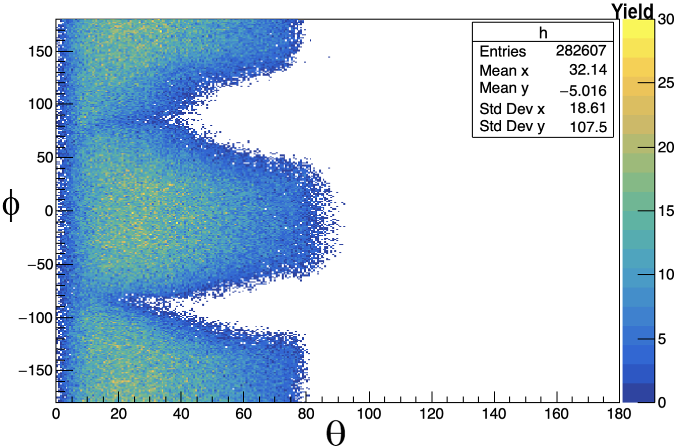
Hansen *et al. Deblurring Images* 2006; Sinethemba Mamba, PD *to be submitted*

Restoration with Inefficiencies

SΠRIT@RIKEN Time-Projection Chamber

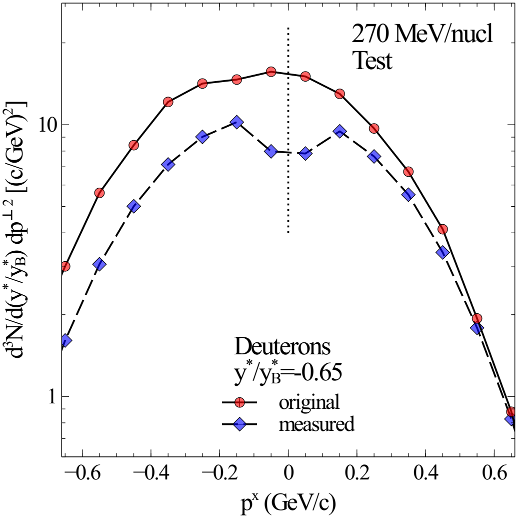
Sn + Sn @ 270 MeV/nucl

Proton distribution in lab angles



Strong azimuthally-asymmetric inefficiencies for slow particles and at small polar angles

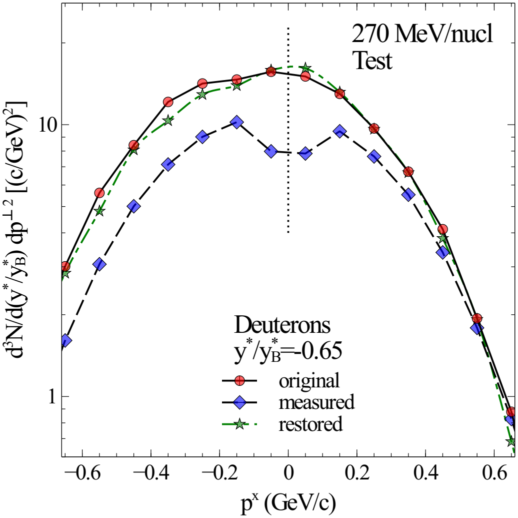
Simulated Restoration f/SΠRIT TPC in Backward CM Hemisphere



Preliminary (minimal statistics)

Flow model ran forward through efficiency simulator for the SΠRIT TPC: not only particles lost but also reaction-plane effects

Simulated Restoration f/SΠRIT TPC in Backward CM Hemisphere



Preliminary (minimal statistics)

Flow model ran forward through efficiency simulator for the SΠRIT TPC: not only particles lost but also reaction-plane effects - restored through deblurring

Conclusions

- TMEP has promoted tests for codes to meet
- Many code weaknesses were identified, motivating authors to improve them
- In parallel to the improvements, expectations are rising, such as in the context of the symmetry energy
- On the observables front, deblurring can give access to 3-differential distributions associated with the reaction plane, completely circumventing v_n
- Side focus in Ar + KCl collisions at 1.8 GeV/nucl with $v^x \sim 0.1\,c$, visible with just $\sim$ 500 collision events, is just an example of what may be achieved!
- Other nuclear problems where deblurring started producing results: $^{26}\text{O} \rightarrow ^{24}\text{O} + n + n$ decay, source-imaging from 2-particle correlations in HIC

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FRIB