

# Radiative corrections in nucleon time-like form factors measurements

J. Van de Wiele and S. Ong

PANDA XLII .Collaboration Meeting,  
10-14 September ( 2012 ), Paris, FRANCE

# I. Introduction

The fully general radiative corrections to lowest order, including the final and initial states radiations, are studied in proton-antiproton annihilation to an electron positron pair.

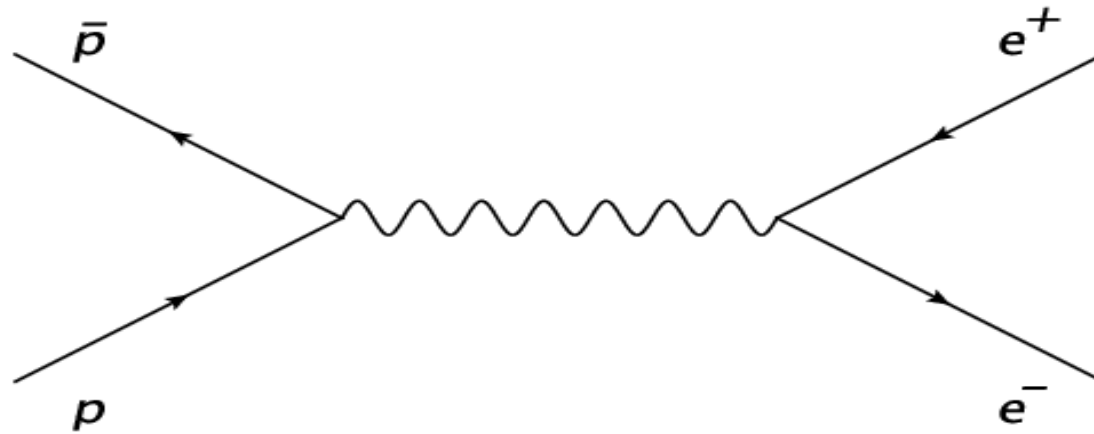
Among the recent paper devoted to this subject, one should refer to the Phys. Rev. D82, 094016 (2010) where the possibility to measure the charge asymmetry is presented.

Radiative corrections depend on the details of the target and detector configurations and also the kinematic cuts in event selection. They also depend in a complicated way on the kinematics and dynamics of the scattering

A Monte Carlo simulation is needed to take into account the radiative corrections together with efficiency and acceptance corrections

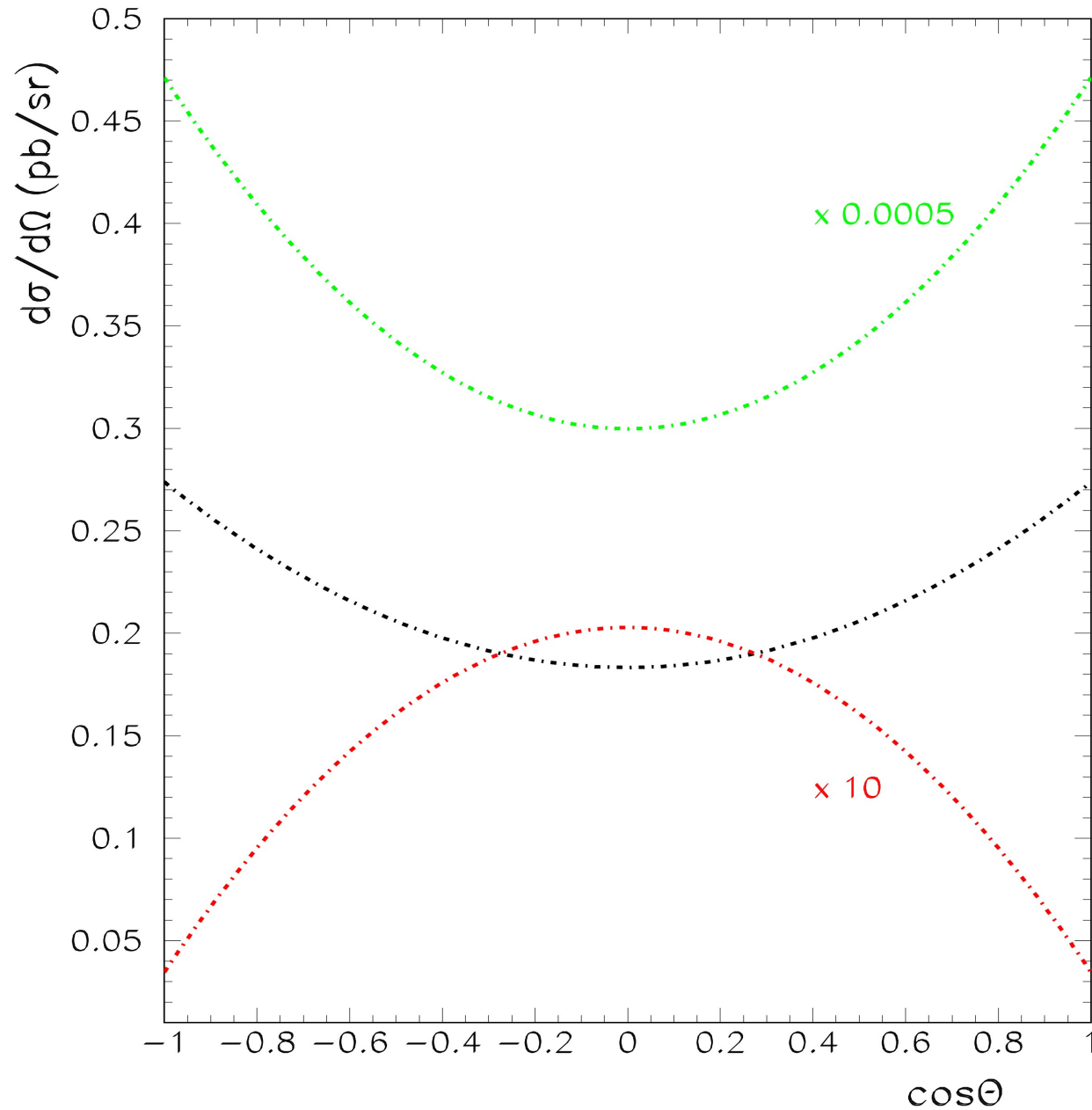
- Only the **internal radiative** corrections are investigated in this present work. The different models of the proton form factors are introduced explicitly in the hard photon (Bremsstrahlung photon) correction.
- This study is a first step towards a correct extraction of the form factors from angular distribution measurements in the data analysis stage.

Born diagram in proton-antiproton annihilation to an electron-positron pair (one photon exchange diagram). The scattering angle of the lepton is defined in the CM system



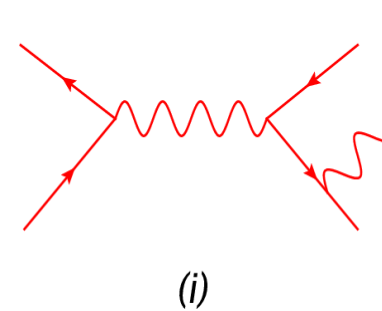
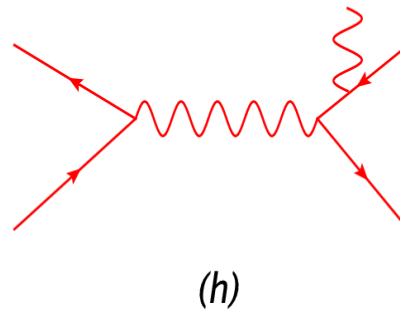
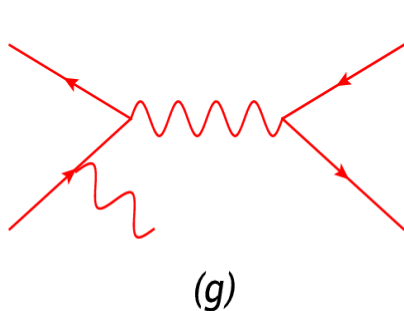
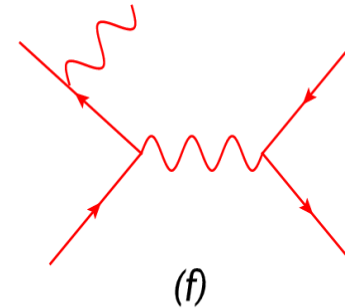
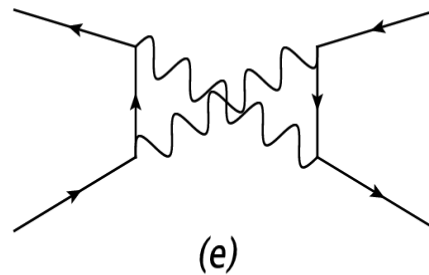
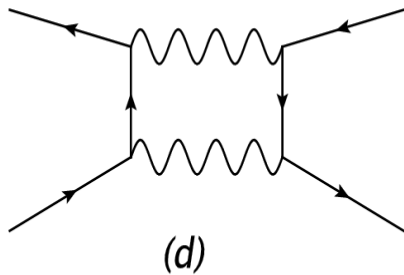
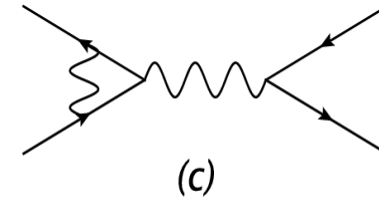
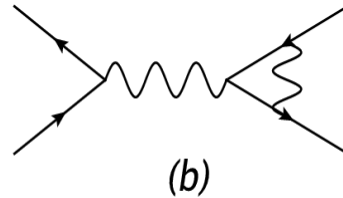
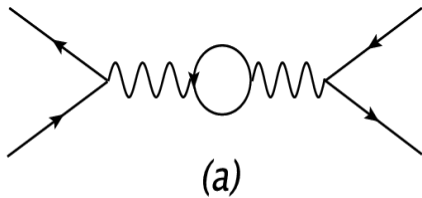
$$\left[ \frac{d\sigma}{d\Omega} \right]_B = \frac{\alpha^2}{4s\beta_p} \left\{ |G_M(s)|^2 (1 + \cos^2 \theta) + (1 - \beta_p^2) |G_E(s)|^2 \sin^2 \theta \right\}$$

Born cross section for  $s=12.9 \text{ GeV}^2$  and for different model assumption of the form factors. Black curve : model 1, from the Babar data fit (Phys. Rev. D73,012005 (2006)); Red curve : model 2, based on the VDM model of Iachello et al. (Phys. Rev. C69, 055204 (2004)); Green curve : the point-like model of the proton



## II QED radiative corrections to first order.

The virtual correction comes from the interference between the Born diagram and the set of diagrams in black. The Bremsstrahlung photon correction is represented by the second set of diagrams in red.



The subscripts « R and B » in the cross sections correspond to the measured and Born cross sections respectively.

« SV » : soft and virtual corrections ; « H » : hard photon correction.

$$\left[ \frac{d\sigma}{d\Omega} \right]_R = \left[ \frac{d\sigma}{d\Omega} \right]_B (1 + \delta)$$

$$\delta = \delta_{SV} + \delta_H$$

We write down the soft and virtual correction together to remove the infrared singularities

$$\delta_{SV}(\omega) = \delta_{vac} + (\delta_{vertex}^e + \delta_R^e(\omega)) + (\delta_{vertex}^p + \delta_R^p(\omega)) + (\delta_{box} + \delta_R^{e-p}(\omega))$$

$\omega \rightarrow$  infrared cut - off parameter

$$\delta_H = \frac{\iint \frac{d^5\sigma}{d\Omega_e d\Omega_\gamma dE_\gamma} d\Omega_\gamma dE_\gamma}{[d\sigma / d\Omega]_B}$$



The Bremsstrahlung corrections ((f)-(i)) lead to infrared singularities, when the energy of the real photon emitted is going to zero. These singularities are cancelled order by order by virtual corrections.

We adopt the standard treatment, separating the soft photon contribution with the emitted photon energy up to an **infrared cut-off** parameter, where the **soft photon approximation** holds, and the hard photon contribution from this infrared cut-off up to an experimental cut (event selection criteria etc...) depending on the energy resolution of the detector.

We have checked that the total correction (virtual+soft+hard) does not depend on this infrared cut-off

The « cut » on the photon energy is determined after a full simulation in a realistic detector configuration. We use the two models (1 and 2) of the form factors to evaluate numerically the hard photon correction.

$$\left[ \frac{d\sigma}{d\Omega_e} \right]_R (E_\gamma^{cut}) = \left[ \frac{d\sigma}{d\Omega_e} \right]_B (1 + \delta_{SV}(\omega)) + \int_{\omega}^{E_\gamma^{cut}} \int \frac{d^5\sigma}{d\Omega_e dE_\gamma d\Omega_\gamma} d\Omega_\gamma dE_\gamma$$

# Virtual and soft photon corrections For $\bar{p} p \rightarrow e^+ e^-$

$s = 5.4 \text{ GeV}^2$

$\theta_{e^+} \setminus \delta$	$\delta_{vacuum}$	$\delta_{vertex}^e$	$\delta_R^e$	$\delta_{vertex}^p$	$\delta_R^p$	$\delta_{box}$	$\delta_R^{e-p}$	$\delta_{SV}^e$	$\delta_{SV}^{e-p}$
30.	0.0103	-0.2602	-0.0147	0.0160	-0.0127	-0.0033	-0.0697	-0.2646	-0.3309
60.	0.0103	-0.2602	-0.0147	0.0147	-0.0127	-0.0020	-0.0365	-0.2646	-0.2991
90.	0.0103	-0.2602	-0.0147	0.0132	-0.0127	0.0000	0.0000	-0.2646	-0.2641
120.	0.0103	-0.2602	-0.0147	0.0147	-0.0127	0.0020	0.0365	-0.2646	-0.2261
150.	0.0103	-0.2602	-0.0147	0.0160	-0.0127	0.0033	0.0697	-0.2646	-0.1915

Table 1:  $T_{\bar{p}} = 1 \text{ GeV}$   $s = 5.4 (\text{GeV}/c)^2$   $\omega = 12 \text{ MeV}$   $\omega/E_{e^+} \approx 1\%$

Dominant term

Electron + proton

$s = 12.9 \text{ GeV}^2$

$\theta_{e^+} \setminus \delta$	$\delta_{vacuum}$	$\delta_{vertex}^e$	$\delta_R^e$	$\delta_{vertex}^p$	$\delta_R^p$	$\delta_{box}$	$\delta_R^{e-p}$	$\delta_{SV}^e$	$\delta_{SV}^{e-p}$
30.	0.0135	-0.2991	-0.0006	0.0113	-0.0314	-0.0112	-0.1169	-0.2792	-0.4162
60.	0.0135	-0.2991	-0.0006	0.0095	-0.0314	-0.0059	-0.0558	-0.2792	-0.3668
90.	0.0135	-0.2991	-0.0006	0.0062	-0.0314	0.0000	0.0000	-0.2792	-0.3043
120.	0.0135	-0.2991	-0.0006	0.0095	-0.0314	0.0059	0.0558	-0.2792	-0.2452
150.	0.0135	-0.2991	-0.0006	0.0113	-0.0314	0.0112	0.1169	-0.2792	-0.1824

Table 2:  $T_{\bar{p}} = 5 \text{ GeV}$   $s = 12.9 (\text{GeV}/c)^2$   $\omega = 18 \text{ MeV}$   $\omega/E_{e^+} \approx 1\%$

$$\bar{p}p \rightarrow e^+e^-.$$

**Total radiative corrections**

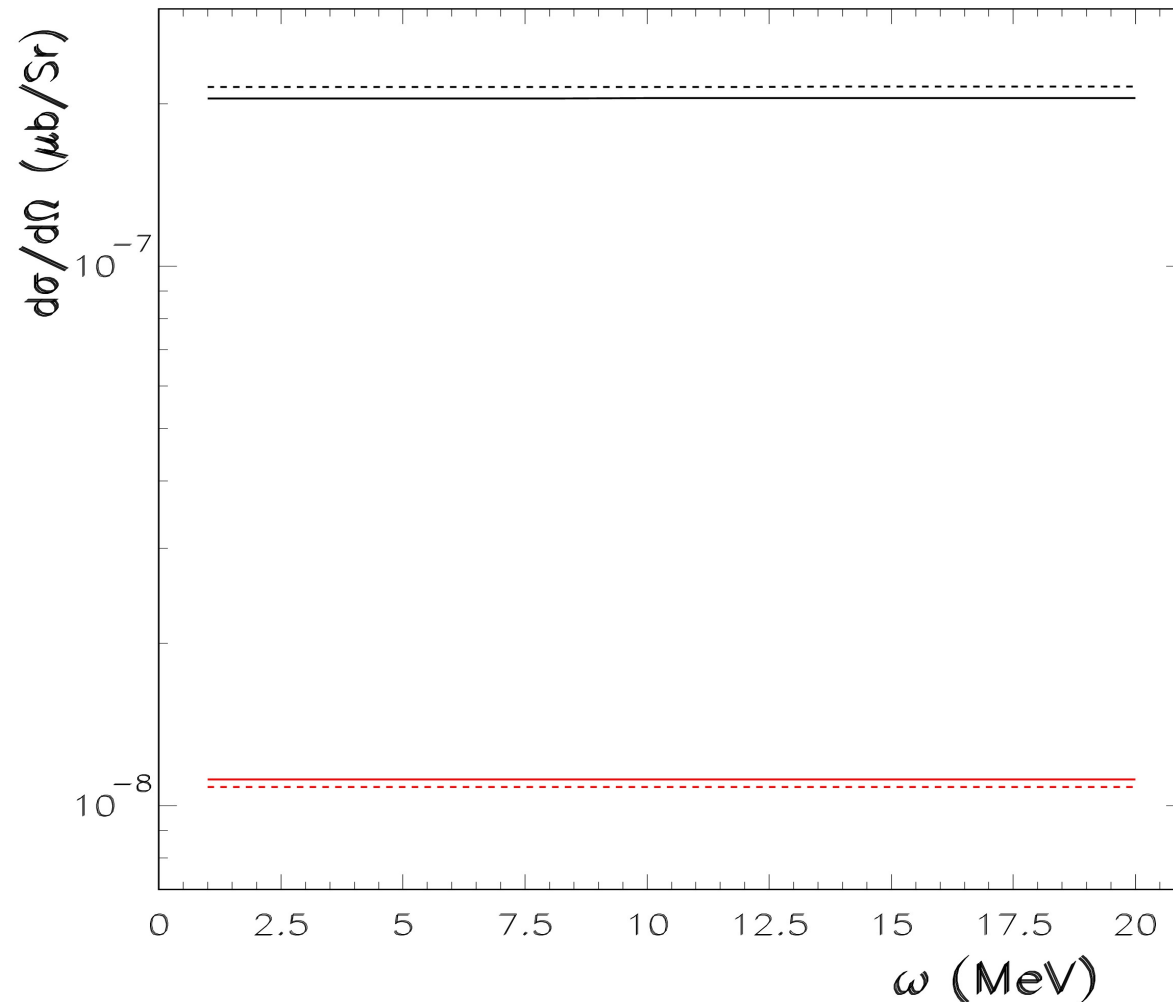
$$\delta_{tot} = \delta_{SV} + \delta_{hard}$$

$\theta_{e^+} \setminus \delta$	$\delta_{tot}^e$	$\delta_{tot}^{e-p}$	$\delta_{tot}^e$	$\delta_{tot}^{e-p}$
30.	-0.1154	-0.1502	-0.1517	-0.2388
60.	-0.1154	-0.1298	-0.1517	-0.1980
90.	-0.1154	-0.1080	-0.1517	-0.1616
120.	-0.1154	-0.0831	-0.1517	-0.1183
150.	-0.1154	-0.0598	-0.1517	-0.0725

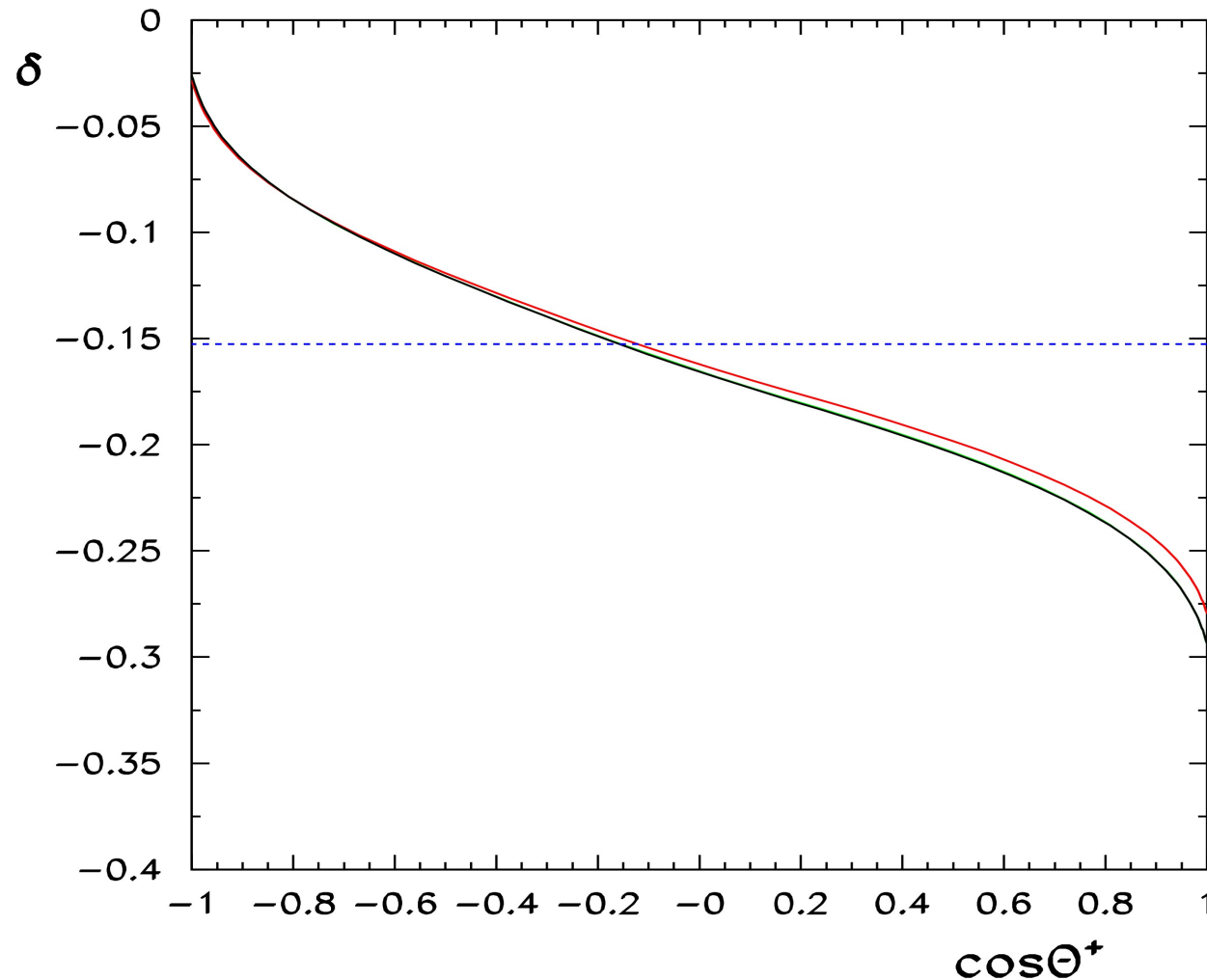
Table 1: Total radiative corrections for  $s = 5.4 \, (GeV/c)^2$  (two left columns) and for  $s = 12.9 \, (GeV/c)^2$  (two right columns), assuming the energy of the hard photon emission up to  $100 \, MeV$

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Numerical check for the cross section versus the infrared cut-off parameter ( $s=12.9 \text{ GeV}^2$  and a fixed scattering angle ( $30^\circ$ ). The black and red curves correspond to the models 1 and 2 respectively (the dotted curves : only the final state radiation is taken into account)



Total radiative correction for  $S=12.9 \text{ GeV}^2$  versus the scattering angle of the positron in the CM system, for different models of the form factors: model 1 (black line); model 2 (red line). The cut on the real photon energy : 100 MeV

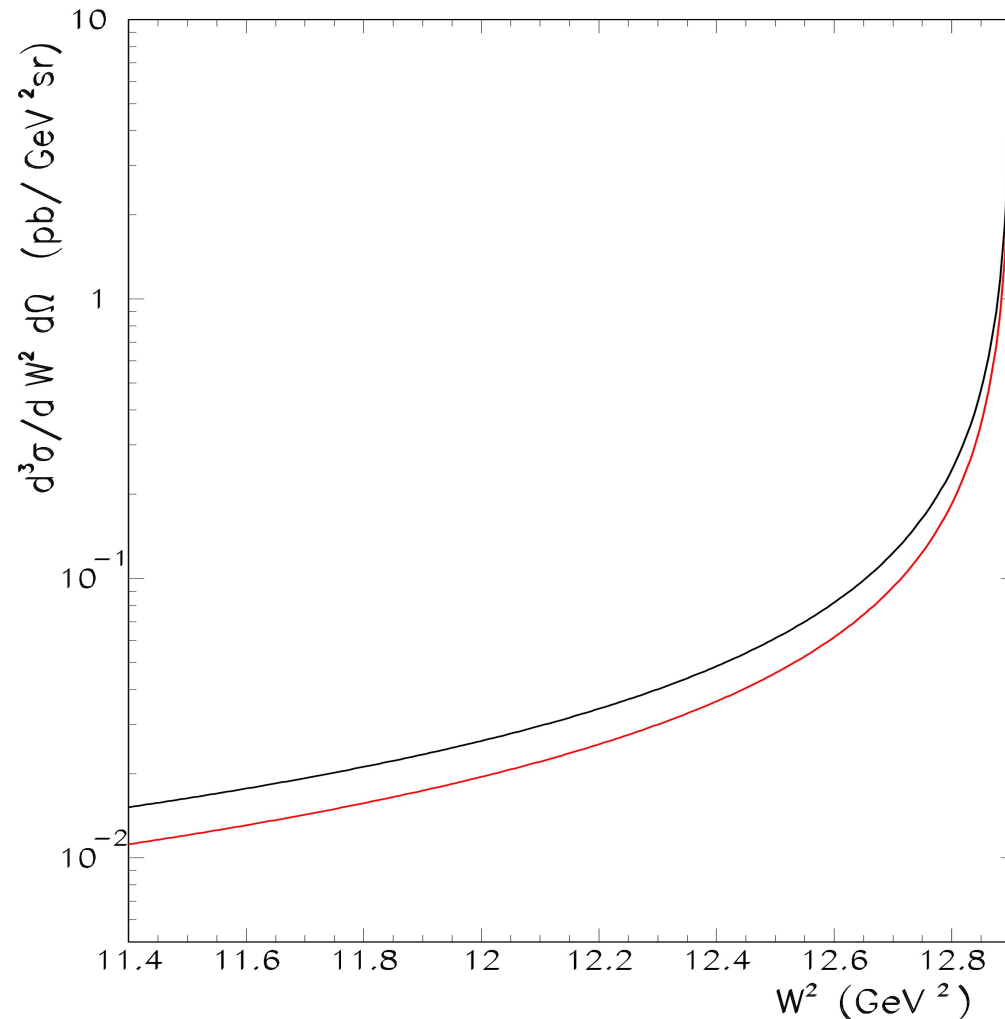


Model 1: black curve ; model 2 : red curve, for  $s=12.9 \text{ GeV}^2$  and a fixed scattering angle ( $30^\circ$ ).

$W$  is the invariant mass of the lepton pair system, the condition used to select the ( $e^+e^-$ ) events should be :

$$W^2 \geq W_{cut}^2$$

$$E_\gamma \leq E_\gamma^{cut} = \frac{s - W_{cut}^2}{2\sqrt{s}}$$



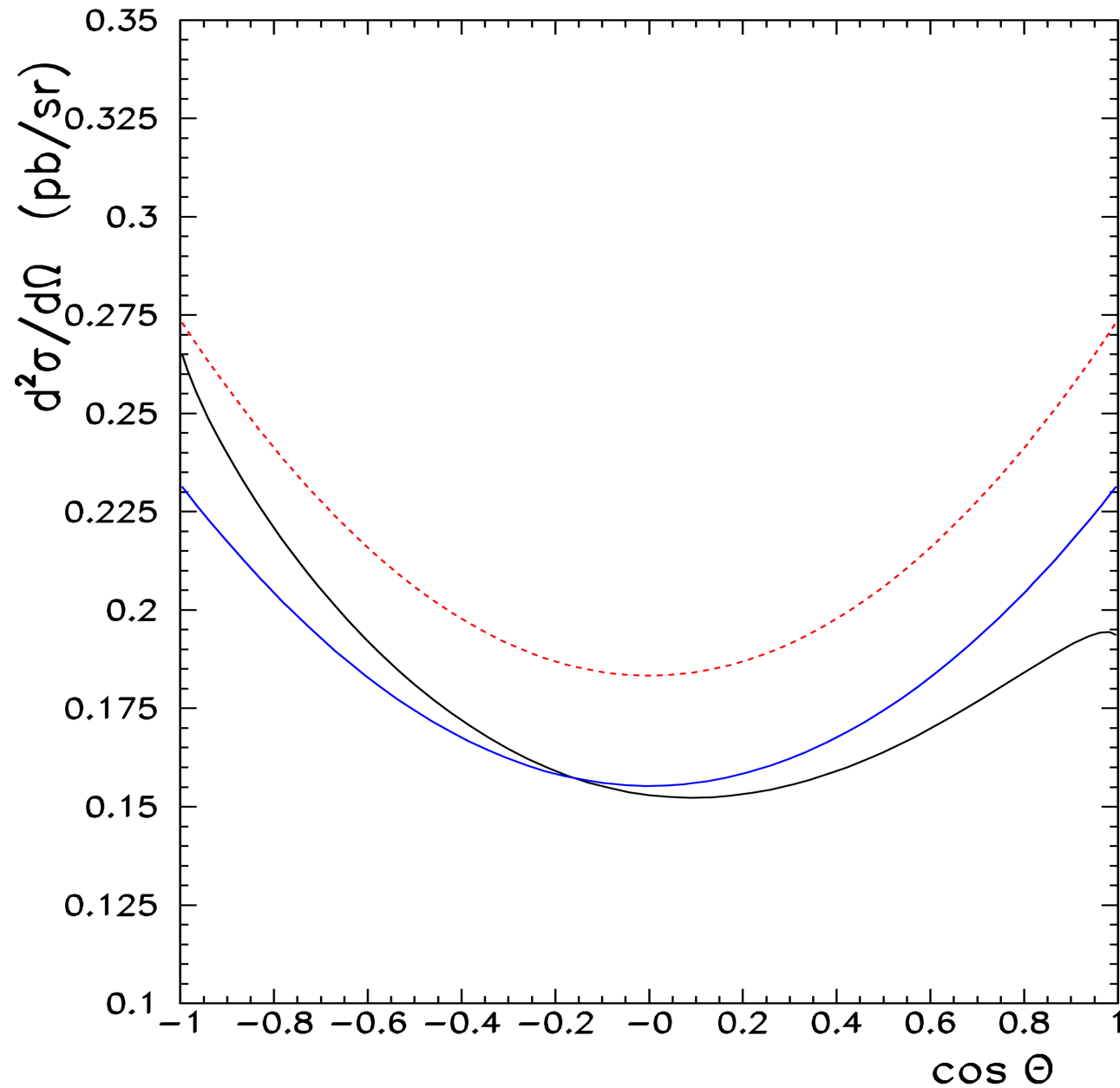
Let us define two interesting observables  $S$  and  $A$ .  
 $S$  contains the charge-even terms and is the corrected Born cross section for form factors extraction.  $A$  is the charge asymmetry due to the odd part.

$$S = \frac{1}{2} \left( \frac{d\sigma}{d\Omega_{e+}} + \frac{d\sigma}{d\Omega_{e-}} \right)$$

$$A = \frac{\left( \frac{d\sigma}{d\Omega_{e+}} - \frac{d\sigma}{d\Omega_{e-}} \right)}{\left( \frac{d\sigma}{d\Omega_{e+}} + \frac{d\sigma}{d\Omega_{e-}} \right)}$$



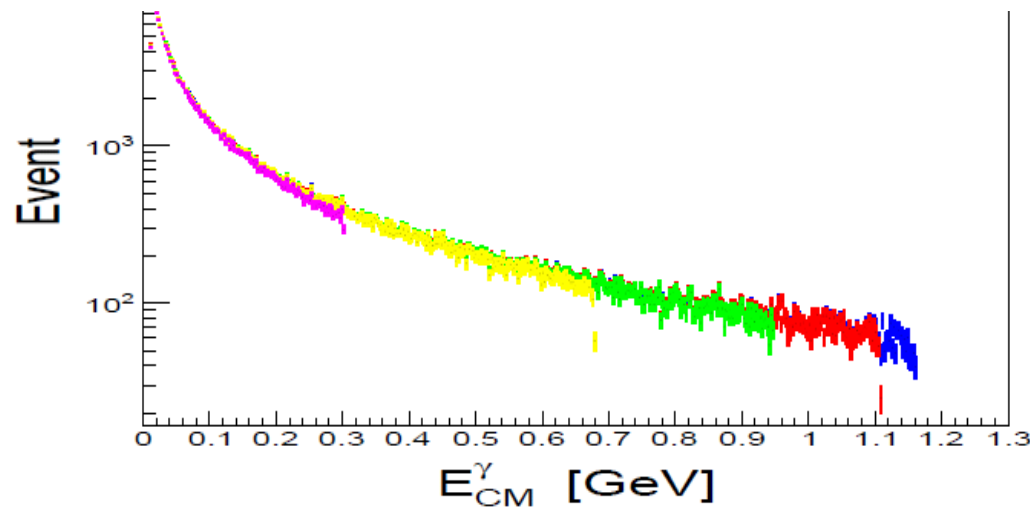
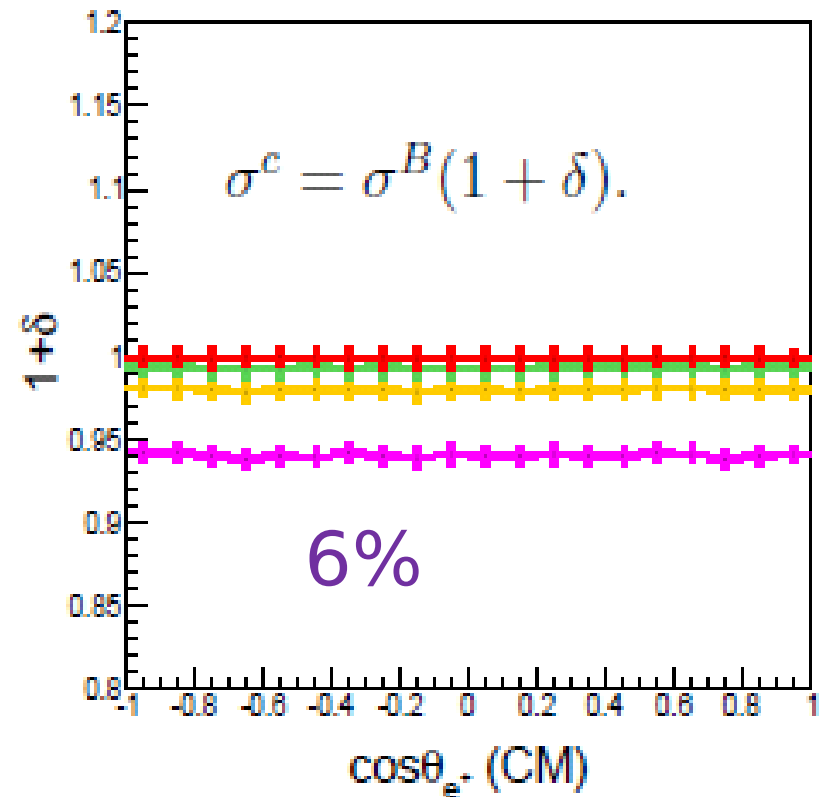
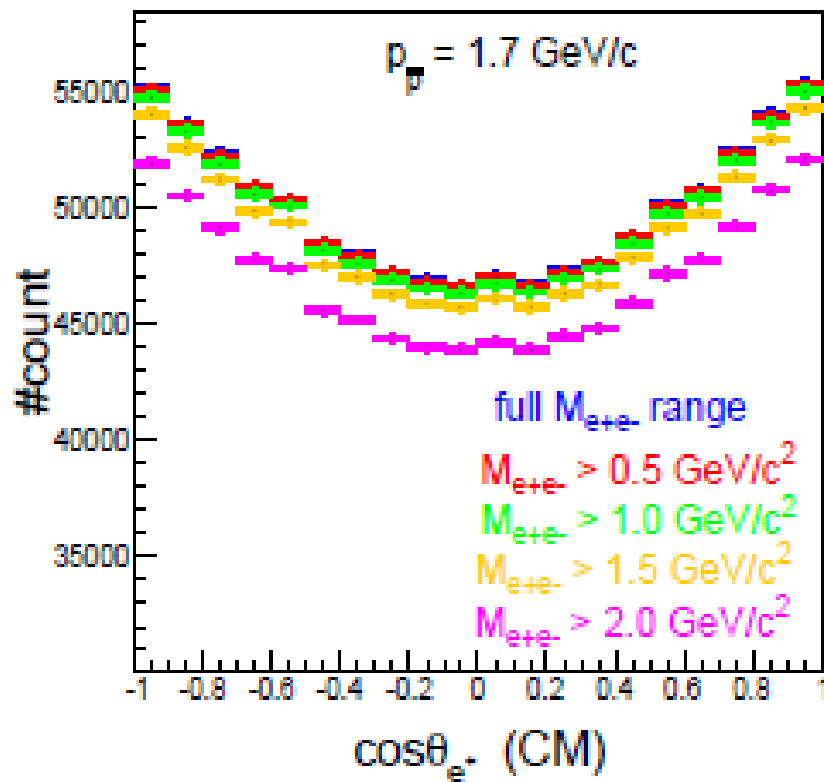
Differential cross section versus the scattering angle of the positron for  $s=12.9 \text{ GeV}^2$ . Red line : Born term; blue line (only the final state radiation) and black line (full final and initial state radiations)



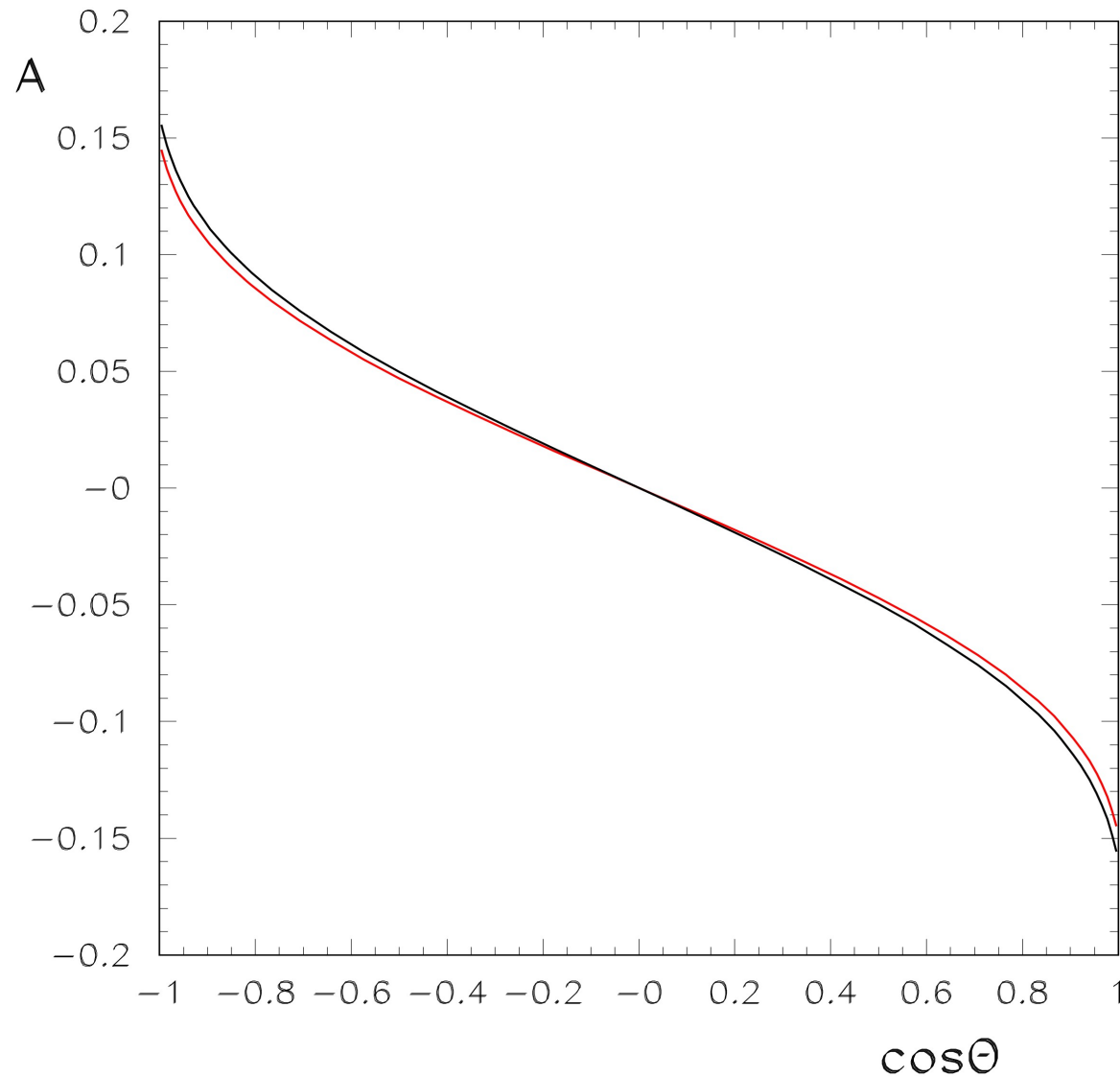
Comparison between our results and « PHOTOS » simulations (only the final state radiation is taken into account for radiative correction calculation)

Total radiative correction	PHOTOS	Present work
$S=5.4 \text{ GeV}^2$ $E_{\text{photon(cut)}}=300 \text{ MeV}$	-6 %	-4.86 %

# Angular distribution with hard photon cut-off (Dbeyssi, Sudol, Tomasi-Gustafsson)



Asymmetry term for  $s=12.9 \text{ GeV}^2$  with the models 1 and 2 of the form factors ( $E_{\text{photon}}(\text{cut})=100 \text{ MeV}$ ). Black curve : model 1 ; red curve : model 2.



### III. Conclusion

- 1) If the precision of the cross section measurement is in order of 3- 5%, The total model independent radiative correction obtained in the present work must be used to correct the measured Born cross section.
- 2) In contrast with the elastic electron-proton scattering, we can use the differential cross sections of the  $e^+$  and  $e^-$  separately, to exhibit the odd-charge asymmetry (interference between the  $C=+1$  and  $C=-1$  final state).
- 3) We have shown that the interference between the initial and final state radiations is the main contribution to the charge asymmetry.
- 4) The finite contribution of the box diagrams (two-photon exchange) is negligible, less than 1 %.

5) The event generator (TFFIPNO) for the nucleon time-like form factors measurement including radiative corrections is now available.

We would like to emphasize that the actual PHOTOS Code is not able to take into account the initial state radiation; the interference between the initial and final state radiations breaks down the factorization scheme in Photos.

## References

- 1) L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41, 205 (1969)
- 2) L. C. Maximon and J. A. Tjon, Phys. Rev. C62, 054320 (2000)
- 3) G. 't Hooft and M. Veltman, Nucl. Phys. B153, 365 (1979)
- 4) A. I. Ahmadvov, V. V. Bytev, E. A. Kuraev and E. Tomasi-Gustafsson, Phys. Rev. D82, 094016 (2010)
- 5) E. Barberio, B. Van Eijk and Z. Was, Comput. Phys. Commun. 66, 115 (1991)
- 6) J. Van de Wiele and S. Ong, arXiv : 1202-1114 [nucl-th] ; a new version of the manuscript is submitted for publication in EPJ A.