Strange nuclear matter from first-principles scattering amplitudes

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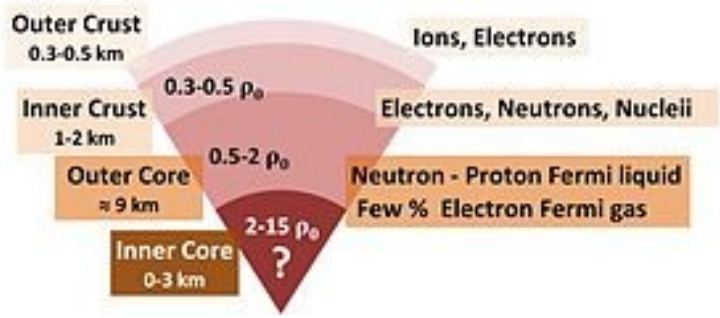
KhuK Jahrestagung, Physikzentrum Bad Honnef Dec. 5-6th, 2024

Neutron Stars:

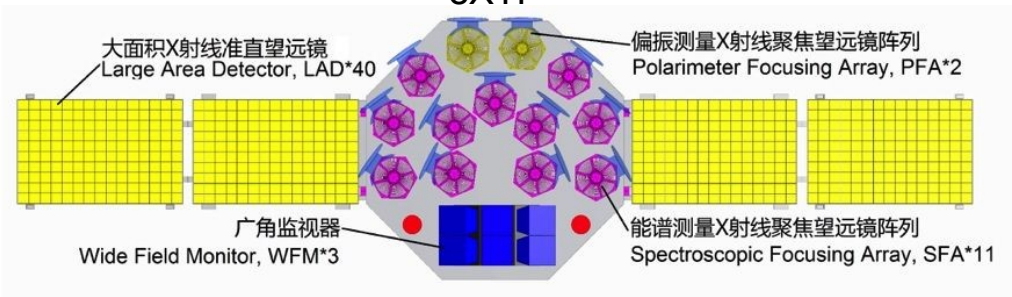
- High-density astrophysical laboratories!



commons.wikimedia.org

- Ongoing/Planned observatories:

eXTP



- Synergy with earth-based experiments:

The Hyperon Puzzle:

- Hyperons: baryons with strange quarks

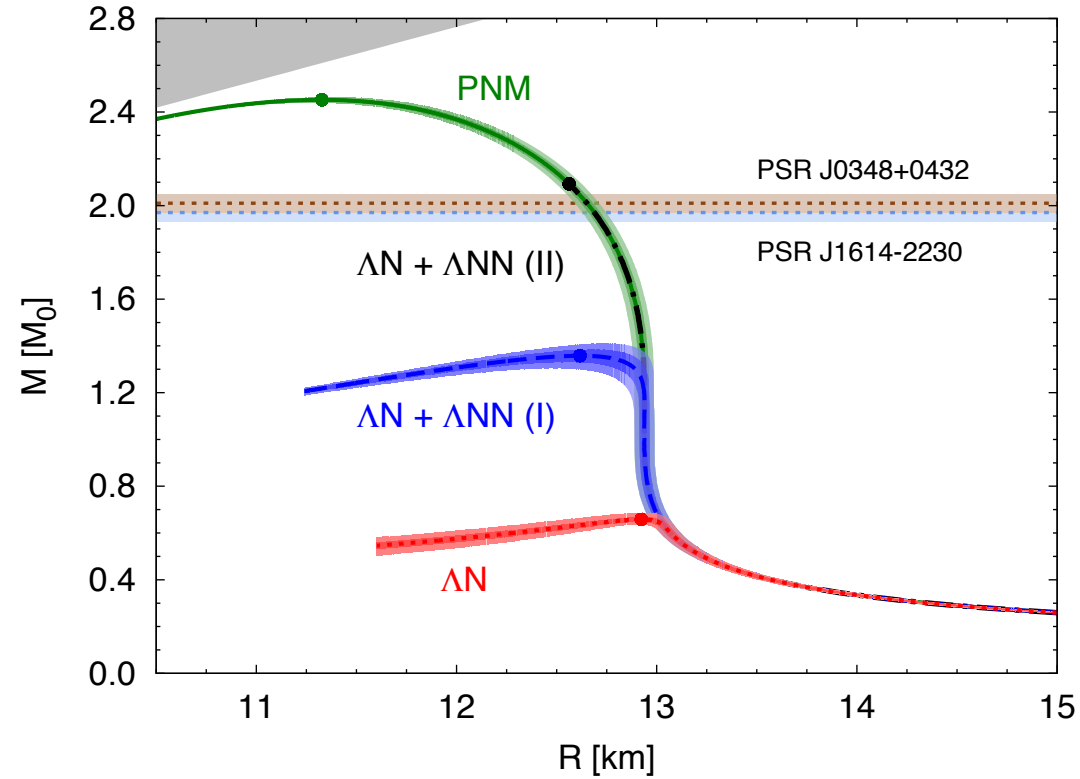
$$\Lambda,\ \Sigma,\ \Xi,\ \Omega$$

- High neutron star (NS) density => hyperons are produced:

$$e^- + n \rightarrow \Sigma^- + \nu_{\mathrm{e}}$$

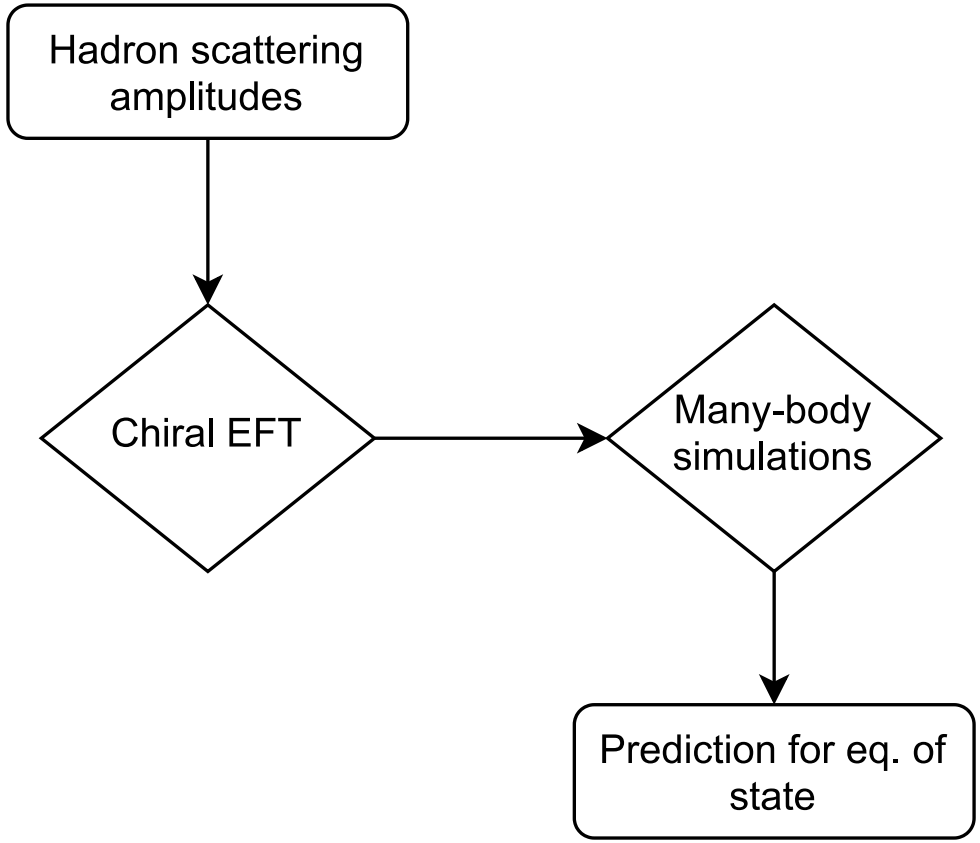
- Hyperon interactions 'soften' NS Equation of State

- Challenge: hyperon interactions not well understood.



Plot from Lonardoni et al, Phys. Rev. Lett. 2015

From amplitudes to stars:



- Many body simulations compute Eq. Of State

- Chiral EFT computes potentials for many body simulations

- LEC's of chiral EFT set from experiment and/or **Lattice QCD**

- 'Light' quark masses are required:

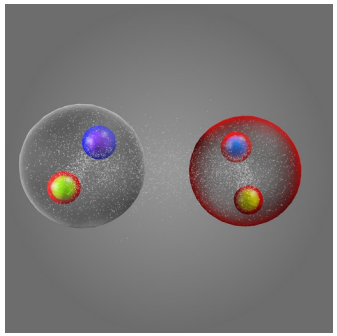
$$m_\pi^{\rm phys} \leq m_\pi < 300{\rm MeV}$$

Related goal: bound states

- Few-body hypernuclei
- Exotic Hadrons:
  - Boson with two charm quarks: tetraquark/molecule

LHCb Collaboration, Nature Physics (2022)

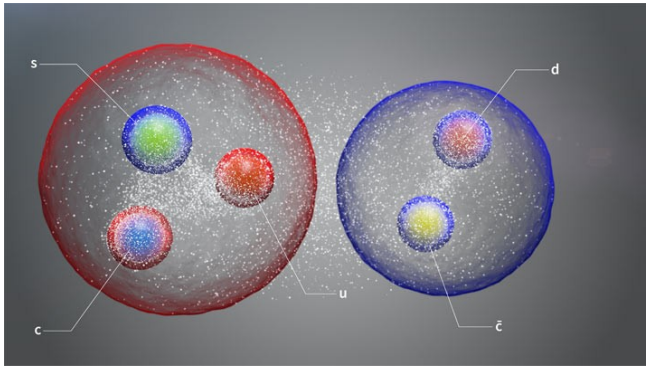
$T_{cc}^{+}(3875)$



  - Fermion with charm-anticharm pair: pentaquark

LHCb Collaboration, Phys. Rev. Lett. 131, 031901 (2023)

$P_{\psi s}^{\Lambda}(4338)$



  - ***Extra/missing state compared to quark model multiplet

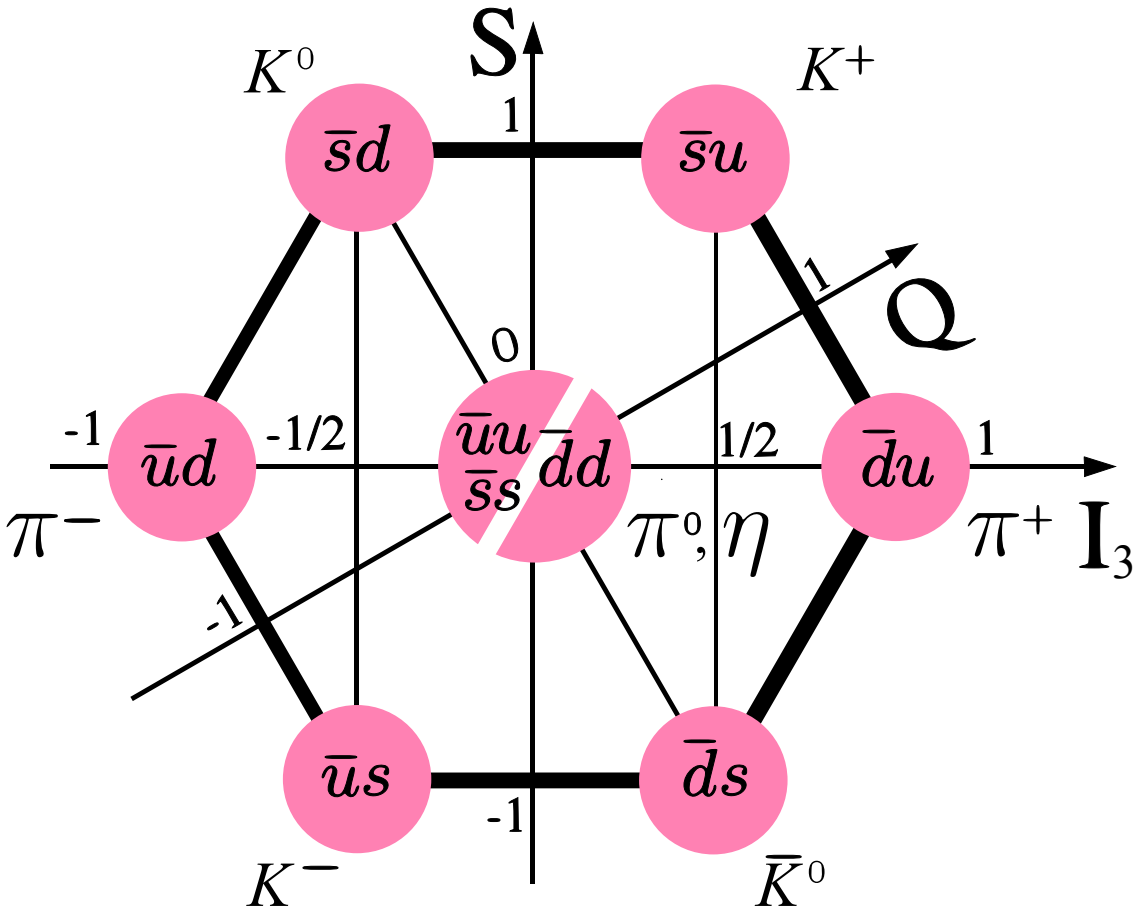
The Eightfold Way: meson singlet and octet

M. Gell-Mann, Y. Ne'emann '61

$$3 \otimes \bar{3} = 1 \oplus 8$$

Isospin analogy: addition of angular momentum for two spin-half states

$$2 \otimes 2 = 1 \oplus 3$$

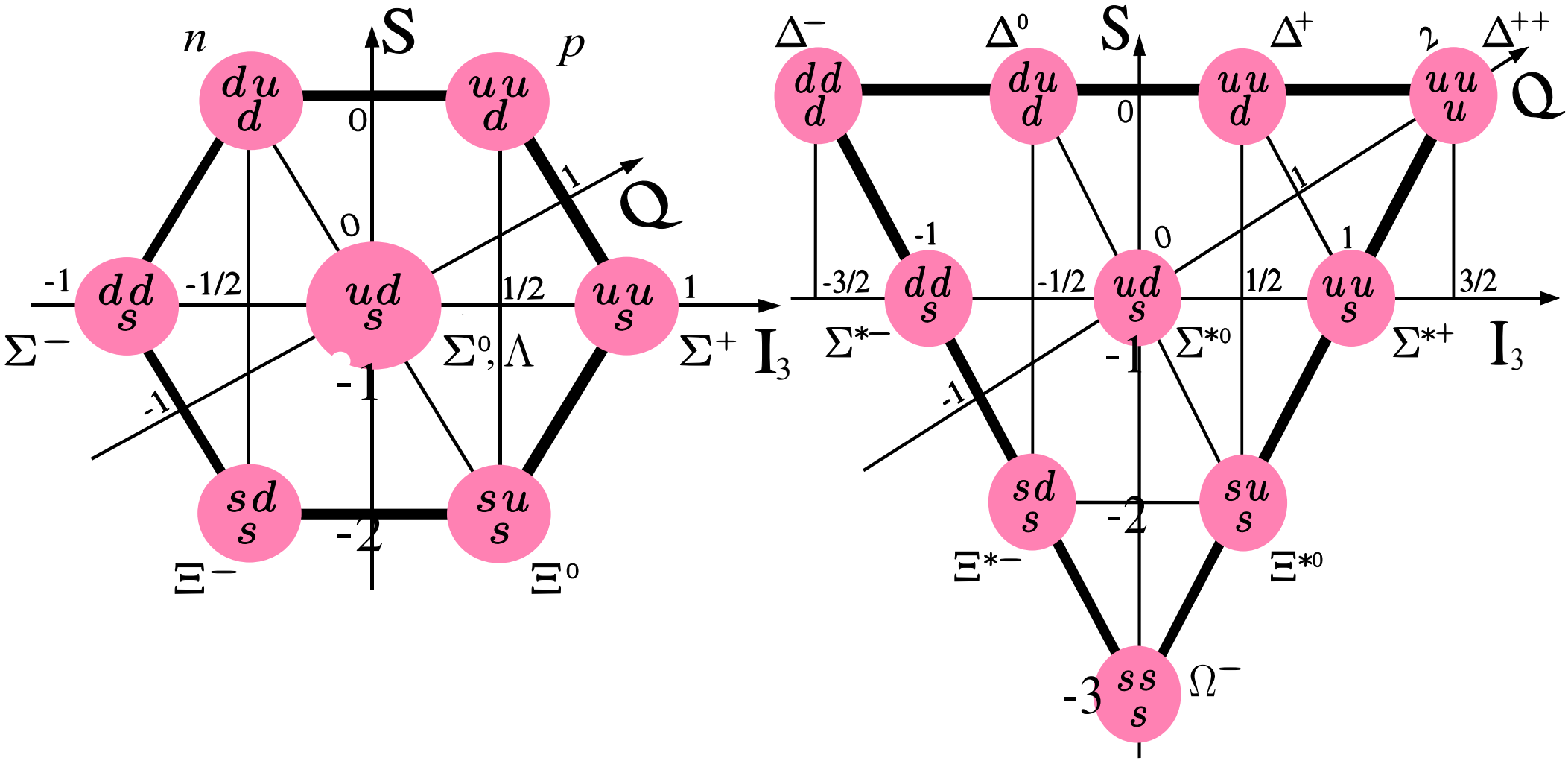


$$(\bar{u}u,\ \bar{d}d,\ \bar{s}s)$$

$$\eta'$$

The Eightfold Way: Baryon octet and decuplet:

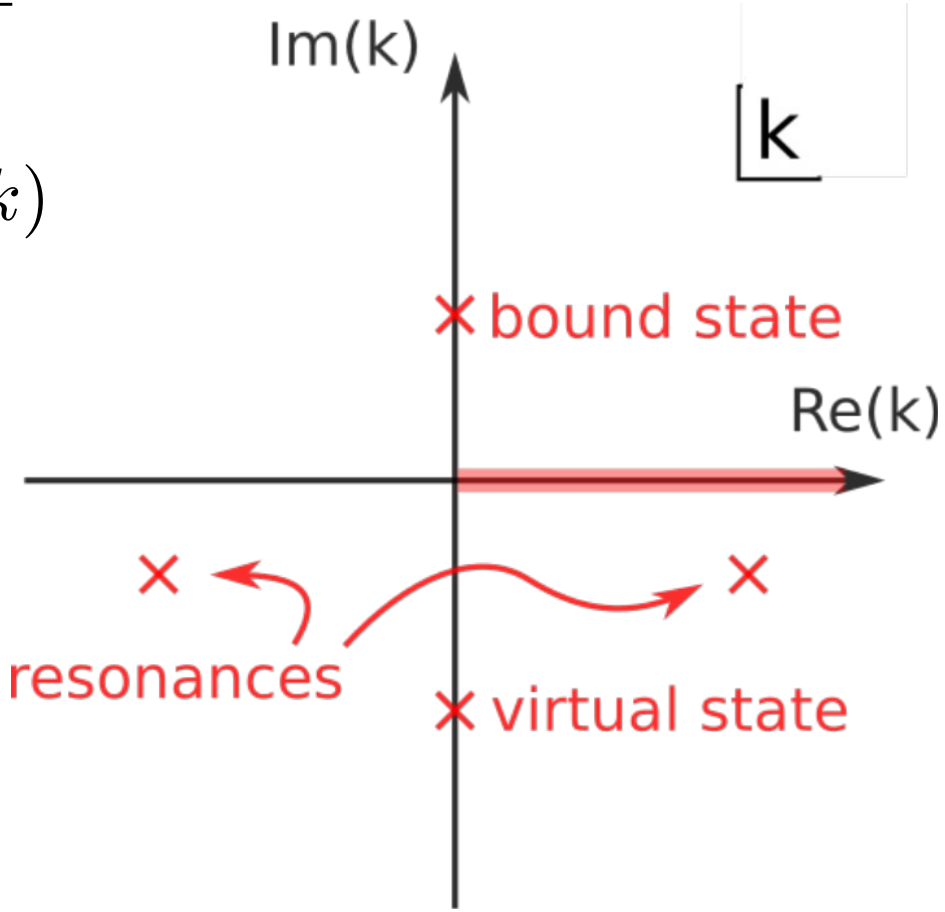
$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus \cancel{1}$$



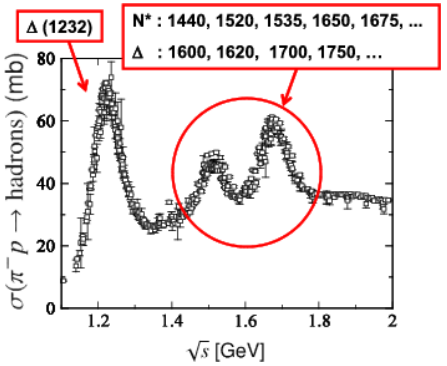
Question: does this work for hadron interactions?

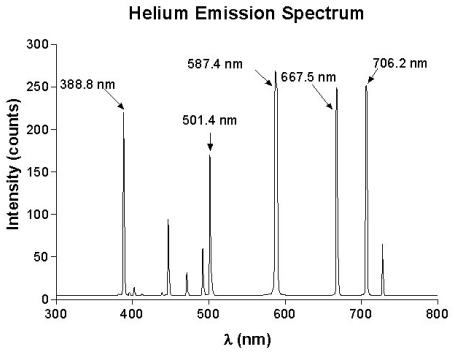
What are bound states and resonances?

- Poles of (2-2) scattering amplitude $T(k)$ in the complex plane
- data on the positive real axis used for analytic continuation
- Bound states occur below threshold
- Resonance poles come in pairs above threshold.
- Nearby resonances cause narrow 'peaks'



from I. Matuschek, V. Baru, F.-K. Guo, C. Hanhart
Eur.Phys.J.A 57 (2021) 3, 101





The Curious Case of the $\Lambda(1405)$: the first 'exotic' hadron?

- Low-energy K+p interactions → sub-threshold S=-1 resonance.

  R. Dalitz. S.F. Tuan '59

- Now well-established: $\Lambda(1405)$ with $I(J^P) = 0(1/2^-)$

- Three-quark interpretation difficult:
  - $1/2^-$ state should be ~500 MeV above $\Lambda(1115)$
  - Already have the $\Lambda(1670)$

- Is the $\Lambda(1405)$ an exotic meson-baryon bound state?

Meson-baryon interactions in SU(3) flavor:

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$

- Low-energy interaction governed by chiral symmetry. At LO:

Weinberg, Tomozawa '66

  - attraction in 8 and 1
  - repulsion in 27, no interaction in (anti-)decuplet

→ Should be two states!

U. Meissner, Symmetry 12 (2020) 6, 981
D. Jido, et al., Nucl.Phys.A 725 (2003) 181

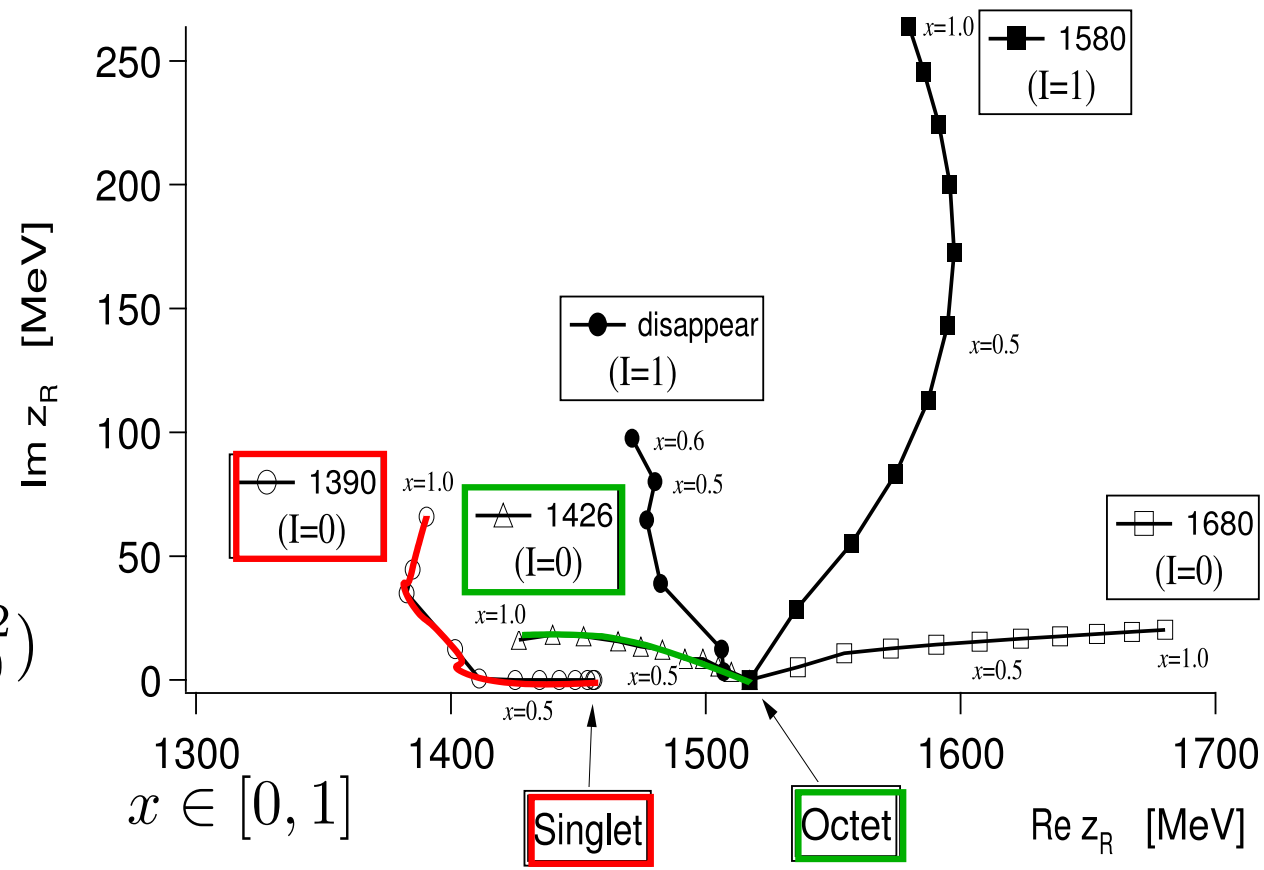
- Pole trajectories from chiral EFT + (model dependent) unitarization:

Octet Baryons:

$$m_i(x) = m_0 + x(m_i - m_0)$$
$$m_0 = 1151\,\mathrm{MeV}$$

Octet mesons:

$$M_i^2(x) = M_0^2 + x(M_i^2 - M_0^2)$$
$$M_0 = 368\,\mathrm{MeV}$$

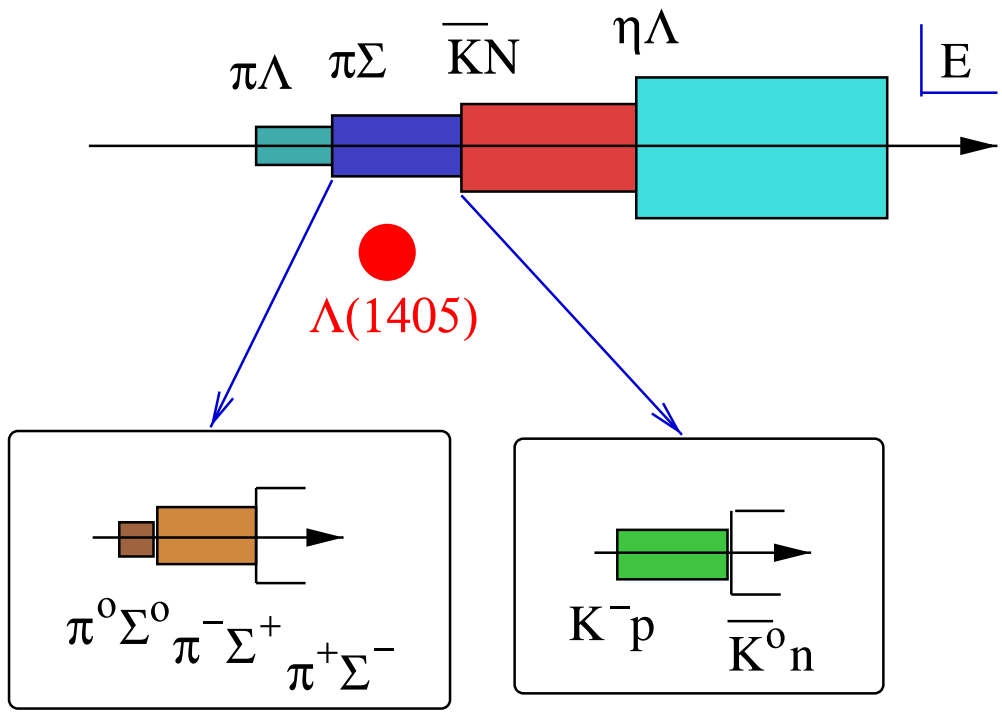


$x \in [0, 1]$

Quo Vadis?

- If three-quark: one state, but should be heavier
- If meson-baryon 'molecule': two states
- Particle Data Group lists $\Lambda(1405) : ****$ $\Lambda(1380) : **$
- Analytic continuation difficult. Many thresholds.

pole 1 [MeV]	pole 2 [MeV]
$1424^{+7}_{-23} - i\,26^{+3}_{-14}$	$1381^{+18}_{-6} - i\,81^{+19}_{-8}$
$1421^{+3}_{-2} - i\,19^{+8}_{-5}$	$1388^{+9}_{-9} - i\,114^{+24}_{-25}$
$1434^{+2}_{-2} - i\,10^{+2}_{-1}$	$1330^{+4}_{-5} - i\,56^{+17}_{-11}$
$1429^{+8}_{-7} - i\,12^{+2}_{-3}$	$1325^{+15}_{-15} - i\,90^{+12}_{-18}$



from T. Hyodo, U. Meissner, PDG '21

from U. Meissner, Symmetry 12 (2020) 6, 981

Scattering in lattice QCD:

Asymptotic limit of $\langle 0|\hat{\mathcal{O}}'(\tau_f)\hat{\mathcal{O}}^\dagger(\tau_i)|0\rangle$ contains no info about on-shell amplitudes.
→ ~~Haag-Ruelle~~

L. Maiani, M. Testa, *Phys. Lett.* **B245** (1990) 585
M. Bruno, M. T. Hansen, JHEP 06 (2021) 043

- Finite volume method: two-hadron energies below $n \geq 3$ hadron thresholds:

$$\det[K^{-1}(E^L_{\rm cm}) - B(E^L_{\rm cm})] = 0$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531

For single channel:
$$K^{-1}_{\ell\ell'} = \delta_{\ell\ell'}\,\cot\delta_\ell$$

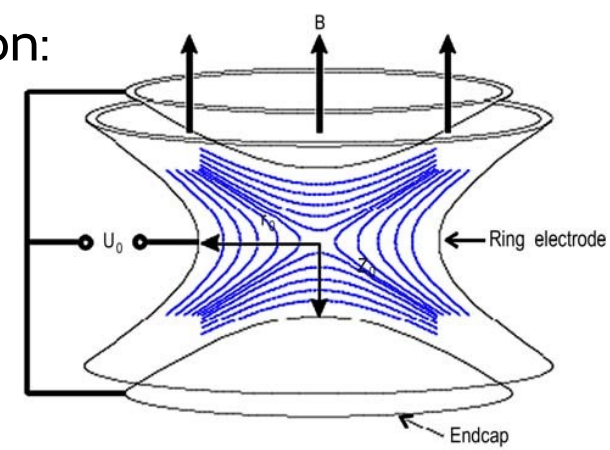
- Determinant over all partial waves and channels
  - Truncation at some $\ell_{\rm max}$ → systematic error

- Signal comes from interaction shift. Large-*L* threshold expansion:

$$\Delta E = E^{I=2}_{2\pi} - 2m_\pi = -\frac{4\pi a_0^{I=2}}{m_\pi L^3} + \mathrm{O}(L^{-4})$$

- For attractive *s*-wave interaction $E^L_{\rm cm} < E_{\rm thresh}$

  → direct constraints in complex plane!

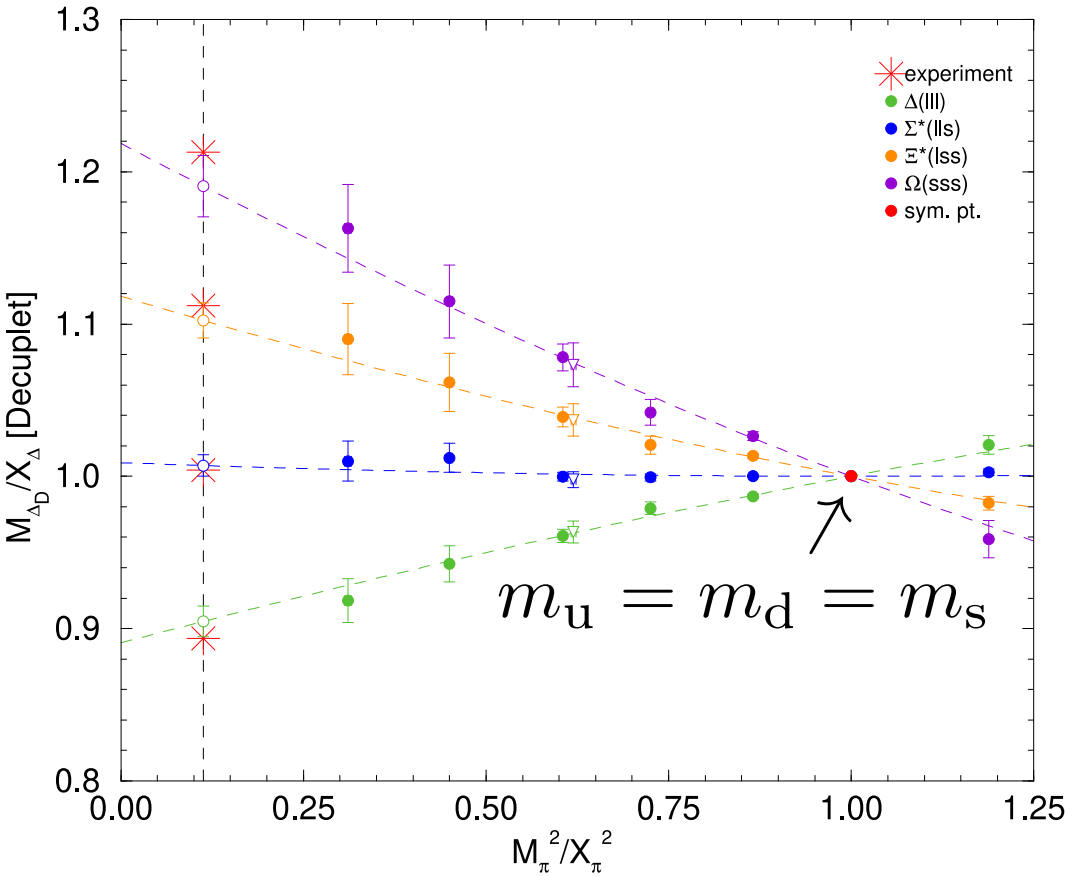


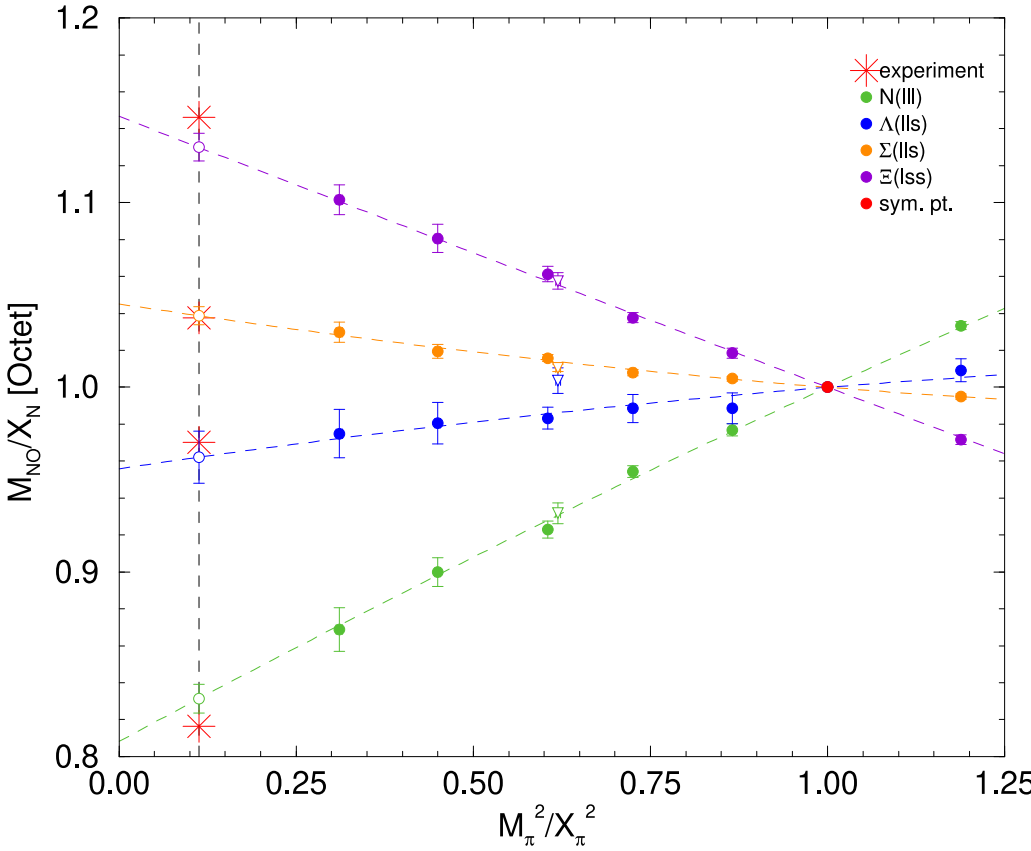
Analogy: two cold atoms in a trap
T. Busch, B.-G. Englert, K. Rzazewski, M. Wilkens, Found. Phys., 28 (1998)

Varying quark masses: connection to The Eightfold Way

Lattice QCD results for baryon masses:

$$m_{\mathrm{u}} = m_{\mathrm{d}}, \qquad m_{\mathrm{u}} + m_{\mathrm{d}} + m_{\mathrm{s}} = \mathrm{phys.}$$





from QCDSF Collaboration, Phys.Rev.D 84 (2011) 054509

See also RQCD Collaboration, JHEP 05 (2023) 035

Recent lattice computation of the $\pi\Sigma - \overline{K}N$ amplitude:

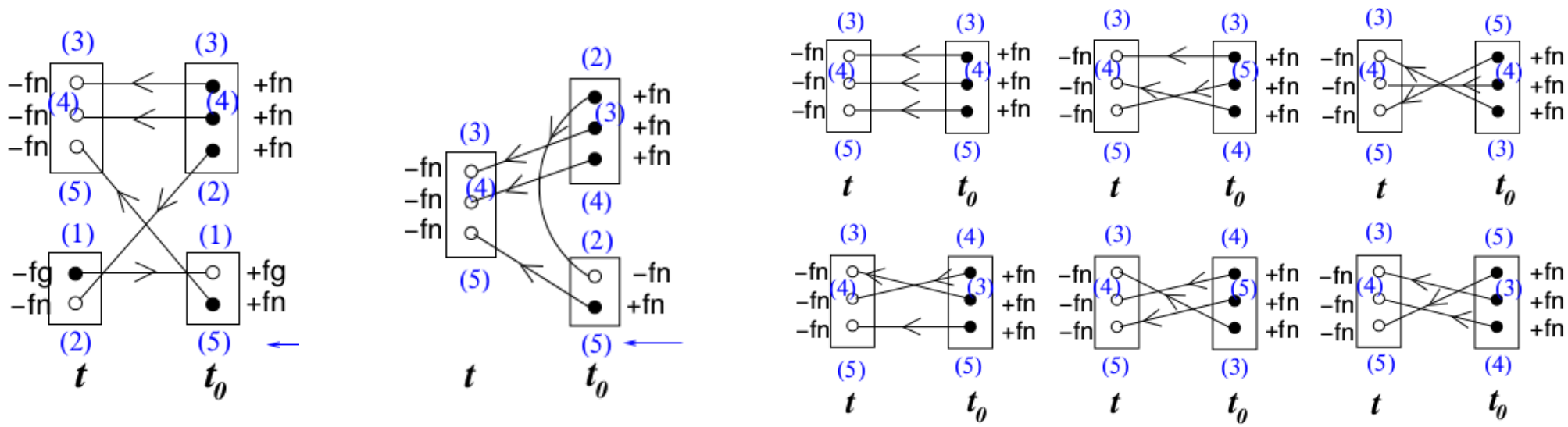
JB, B. Cid-Mora, A. Hanlon, B. Hoerz, D. Mohler, C. Morningstar, J. Moscoso, A. Nicholson, F. Romero-Lopez, A. Walker-Loud
(For the Baryon Scattering (BaSc) Collaboration) Phys.Rev.Lett. 132 (2024) 5, 051901 (Editor's Suggestion)

- CLS (D200) lattice:

$$64^3 \times 128,\ a = 0.064\text{fm},\ m_\pi = 200\text{MeV},\ \frac{m_\pi^2}{m_X^2} = 0.23$$

$$m_\pi + m_\Sigma \approx 1380\,\text{MeV}$$

- Correlation functions from tensor contraction: inverse of large sparse Dirac matrix



- Factorization enabled by the distillation/stochastic LapH algorithms for quark propagation

M. Peardon et al. Phys.Rev.D 80 (2009) 054506; C. Morningstar et al. Phys.Rev.D 83 (2011) 114505

Results: coupled-channel amplitude

- Pole 1

$$E_1 = [1455(13)(2)(17) - i\,11.5(4.4)(4)(0.1)]\,\mathrm{MeV}$$

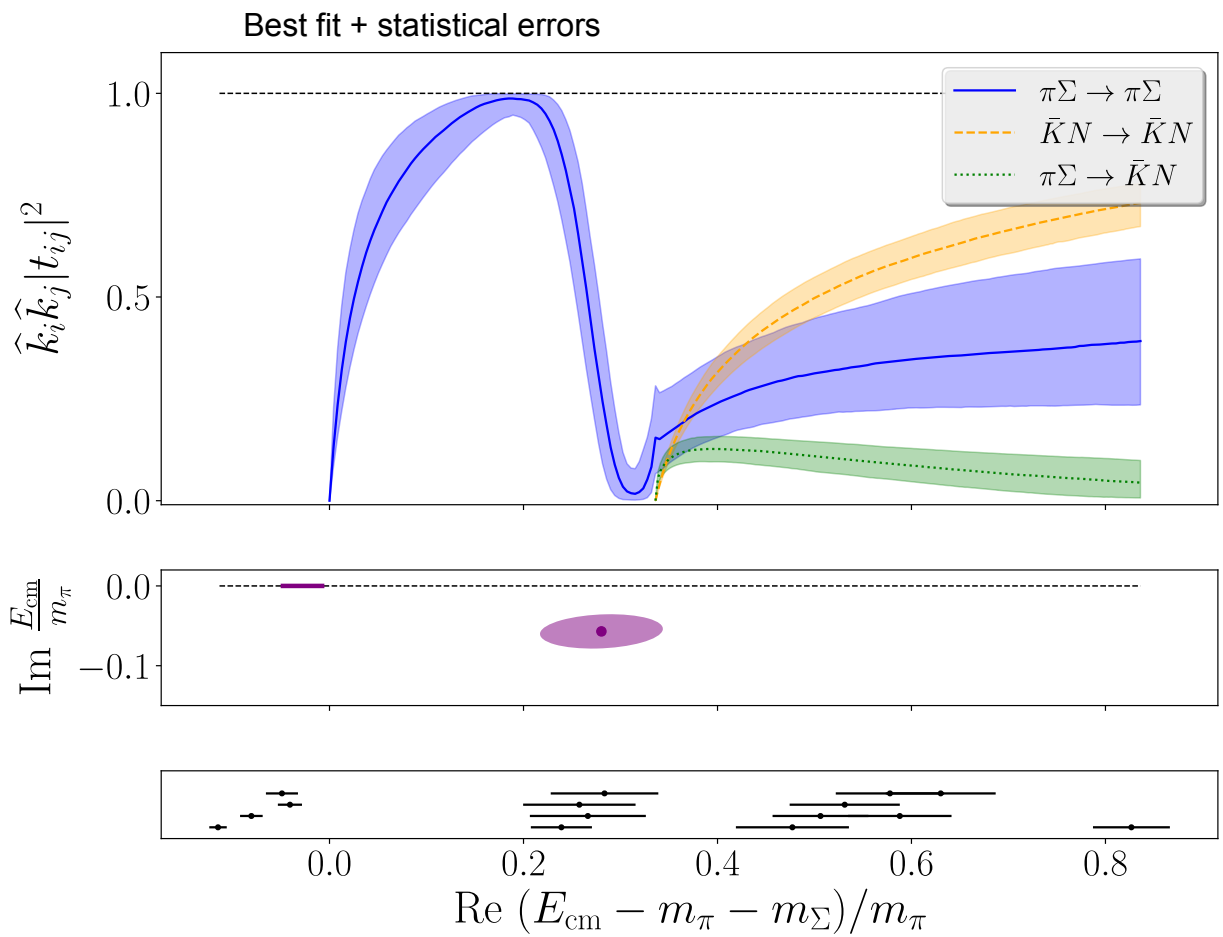
- Pole 2

$$E_2 = 1392(9)(2)(16)\,\mathrm{MeV}$$

- Residues:

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 1.9(4)(6)$$

$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 0.53(9)(10)$$



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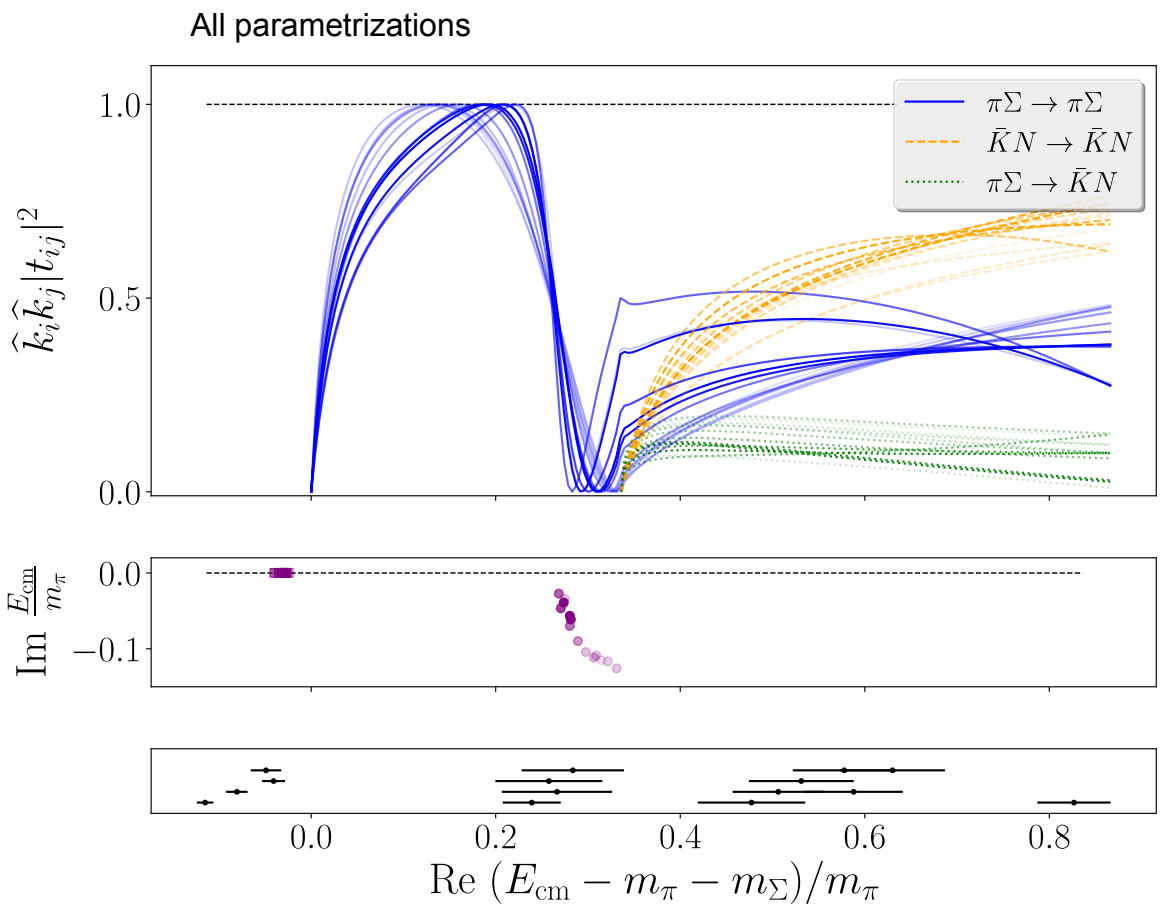
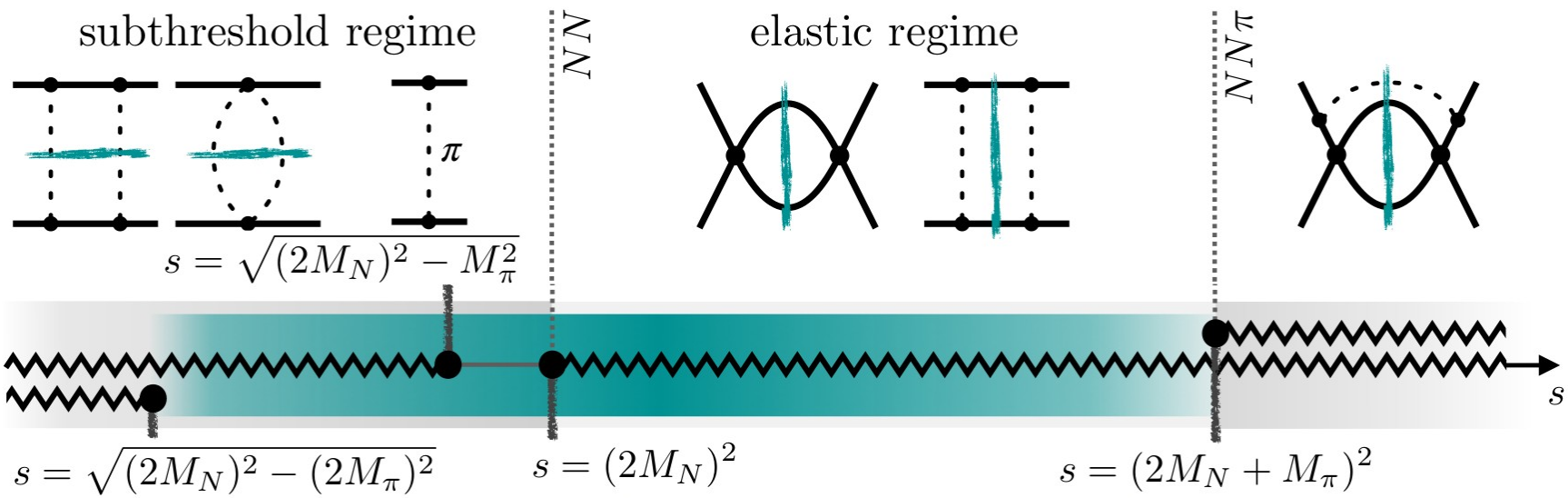


What about hyperon-hyperon?

Challenge: nearby non-analyticities in analytic continuation:

- Right-hand (threshold) cuts
- Left-hand (cross-channel) cuts
- ...

A. Baião Raposo, Lattice '23



$NN \to NN$ angular-momentum-projected amplitude

- Must be accounted for twice:
  - Infinite volume: parametrization/analytic continuation
  - Finite volume: new formalism required

Parametrization/analytic continuation:

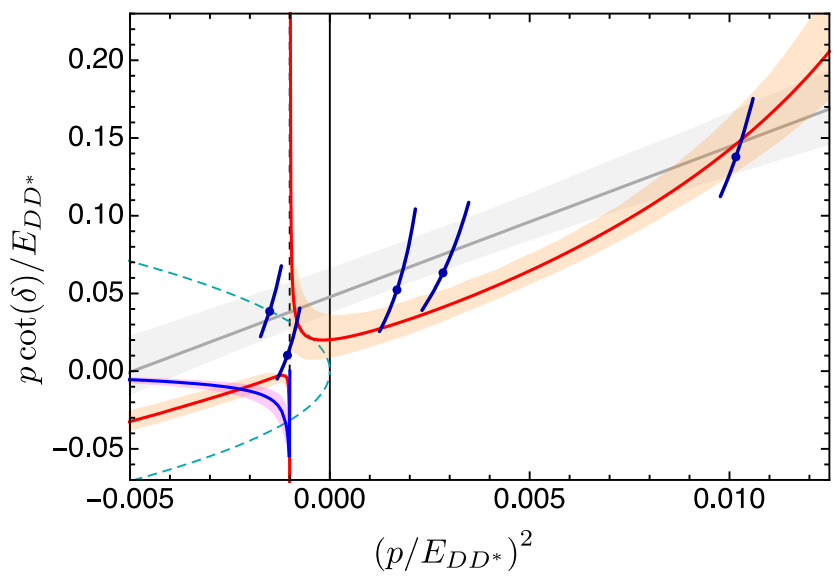
- Typically a variant of the effective range expansion (ERE):

$$p_{\rm cm}^{2\ell+1}\cot\delta_\ell = \frac{1}{a} + \frac{r}{2}p_{\rm cm}^2 + \ldots$$

s-wave pole occurs if

$$p_{\rm cm}\cot\delta_0 - ip_{\rm cm} = 0$$

- Radius of convergence limited by nearest cut
  - Ex: $T_{cc}(3875)^+$ in *DD**-scattering
    M.-L. Du, et al., Phys.Rev.Lett. 131 (2023) 13
  - Points from lattice QCD at $m_\pi = 280\text{MeV}$, gray band is ERE fit
    S. Prelovsek, M. Padmanath, Phys. Rev. Lett. 129, 032002 (2022);
    See also: S. Chen et al., PLB 833, 137391 (2022); Y. Lyu et al., 2302.04505
  - Left hand cut invalidates naive FV formalism
    J. R. Green, et al., Phys.Rev.Lett. 127 (2021) 24, 242003



from M.-L. Du, et al., 2303.09441 [hep-ph]

  - Near left-hand cuts, pole positions from ERE not trustworthy

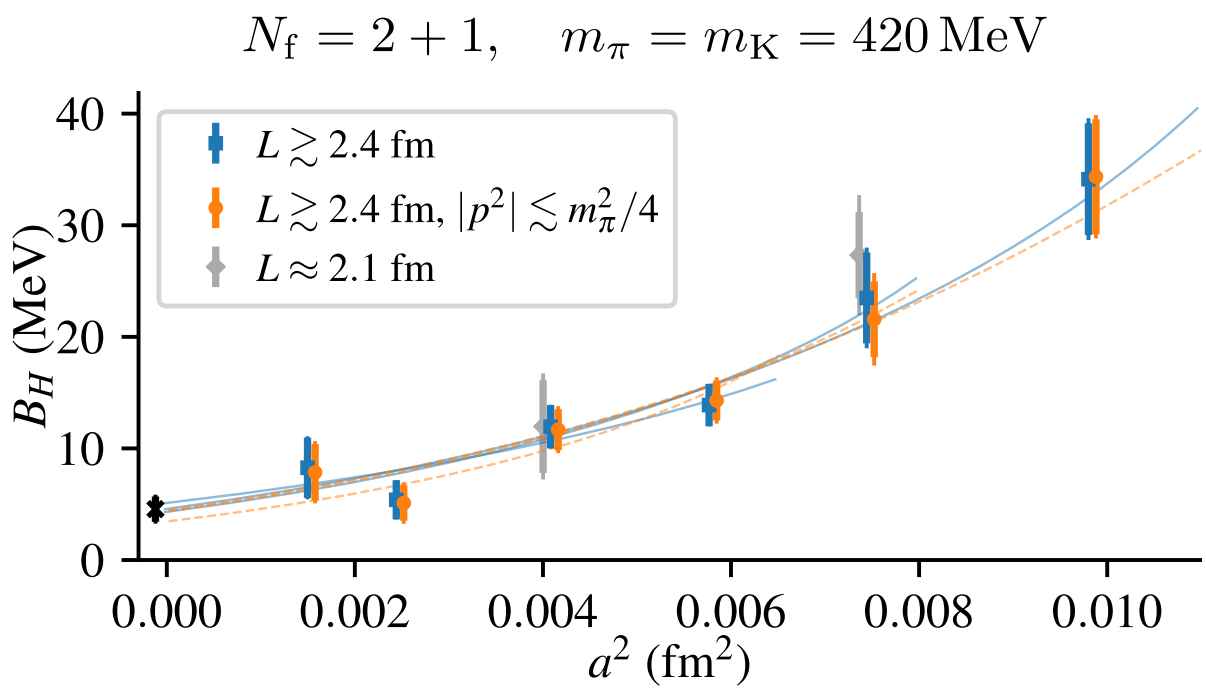
Another challenge: cutoff effects

- Finite-volume energies have asymptotic $O(a^2)$ cutoff effects
  (log corrections may be significant)

N. Husung, Eur.Phys.J.C 83 (2023) 2, 142

- Significant cutoff effects observed for $\Lambda\Lambda$-scattering (*H*-dibaryon)

J. Green, A. Hanlon, P. Junnarkar, H. Wittig, Phys.Rev.Lett. 127 (2021) 24, 242003



- Also observed for DD* scattering (preliminary)

J. Green, A. Hanlon, H. Wittig, M. Padmanath, R. J. Hudspith, S. Paul, Lattice '23

Conclusions

- Lattice QCD compliments experiment in hyperon physics:
  - ➔ Compute processes difficult to realize experimentally
  - ➔ Vary the quark masses → connect to SU(3) flavor symmetry

- Opportunities for low-lying meson-baryon resonances at the physical point are good:
  - ➔ Tractable statistical errors
  - ➔ Several coupled channels: octet-octet (Lambda(1405)) and octet-decouplet
  - ➔ Three-body right hand cuts

- Computations of hyperon-baryon amplitudes must address cutoff effects, left-hand cuts