Lattice QCD results on conserved charge fluctuations

Frithjof Karsch
Bielefeld University

UNIVERSITÄT BIELEFELD
Faculty of Physics

with a lot of input from 'HotQCD,

in particular,
Sipaz Sharma
Jishnu Goswami
Mugdha Sarkar

- QCD phase diagram – constraining the location of a critical endpoint
- Precision results for 2nd order cumulants
- Higher order cumulants: Skewness, kurtosis and beyond
- Charmed cumulants: Towards continuum extrapolated results

CRC-TR 211 Strong-interaction matter under extreme conditions
DFG Deutsche Forschungsgemeinschaft

Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular — regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

$$t \sim \frac{T - T_c}{T_c} + \kappa_2 \left(\frac{\mu_l}{T}\right)^2$$

$$h \sim \frac{m_l}{T_c}$$

Pseudo-critical temperatures

response functions
2nd order cumulants

- magnetic — mixed — thermal

$$\frac{\partial^2 \ln Z}{\partial h^2} \qquad \frac{\partial^2 \ln Z}{\partial h\, \partial t} \qquad \frac{\partial^2 \ln Z}{\partial t^2}$$

$$\sim \left(\frac{m_l}{T_c}\right)^{1/\delta-1} \qquad \sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta} \qquad \sim \left(\frac{m_l}{T_c}\right)^{-\alpha/\beta\delta}$$

$$\sim -0.79 \qquad \sim -0.34 \qquad \sim +0.11$$

divergence: strong — moderate — none

O(4) critical exponents

$\alpha = -0.21$

$\beta = 0.38$

$\delta = 4.82$

Fluctuation observables in QCD

– chiral condensate: $\langle\bar\psi\psi\rangle_l = \dfrac{\partial P/T}{\partial m_l/T}$, $\langle\bar\psi\psi\rangle_l = (\langle\bar\psi\psi\rangle_u + \langle\bar\psi\psi\rangle_d)/2$

– chiral order parameter: $M = \dfrac{2}{f_K^4}\left[m_s\langle\bar\psi\psi\rangle_l - m_l\langle\bar\psi\psi\rangle_s\right]$

$$m_l = (m_u + m_d)/2$$

– chiral susceptibility: $\chi_M = m_s\left(\dfrac{\partial M}{\partial m_u} + \dfrac{\partial M}{\partial m_d}\right)$ **magnetic**

– mixed chiral susceptibility: $\chi_t = T\dfrac{\partial M}{\partial T}$ **mixed**

– conserved charge fluctuations: $\chi_x = T^4\left.\dfrac{\partial^4 P/T^4}{\partial \mu_X^4}\right|_{\mu_X=0}$ **thermal**

$$X = B,\ Q,\ S, \ldots$$

Higher order cumulants
– Taylor expansion of QCD EoS and the HRG –

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0} , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

cumulants at vanishing chemical potential provide information on the equation of state as well as freeze-out conditions at small non-zero chemical potential

HotQCD data collection for (2+1)-flavor QCD-EoS

EoS:2017:
arXiv:1701.04325

$N_\tau = 6$				$N_\tau = 8$				$N_\tau = 12$			
β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.
5.980	0.00435	135.29	81200	6.245	0.00307	134.64	180320	6.640	0.00196	134.94	5834
6.010	0.00416	139.71	120790	6.285	0.00293	140.45	172110	6.680	0.00187	140.44	5833
6.045	0.00397	145.05	120770	6.315	0.00281	144.95	138150	6.712	0.00181	144.97	13846
6.080	0.00387	150.59	79390	6.354	0.00270	151.00	107510	6.754	0.00173	151.10	14200
6.120	0.00359	157.17	66180	6.390	0.00257	156.78	135730	6.794	0.00167	157.13	15476
6.150	0.00345	162.28	79660	6.423	0.00248	162.25	115850	6.825	0.00161	161.94	16772
6.170	0.00336	165.98	49760	6.445	0.00241	165.98	120270	6.850	0.00157	165.91	19542
6.200	0.00324	171.15	122700	6.474	0.00234	171.02	139980	6.880	0.00153	170.77	21220
6.225	0.00314	175.76	122730	6.500	0.00228	175.64	133070	6.910	0.00148	175.76	12303

added O(10) times new confs.

added O(20) times new confs.

all new

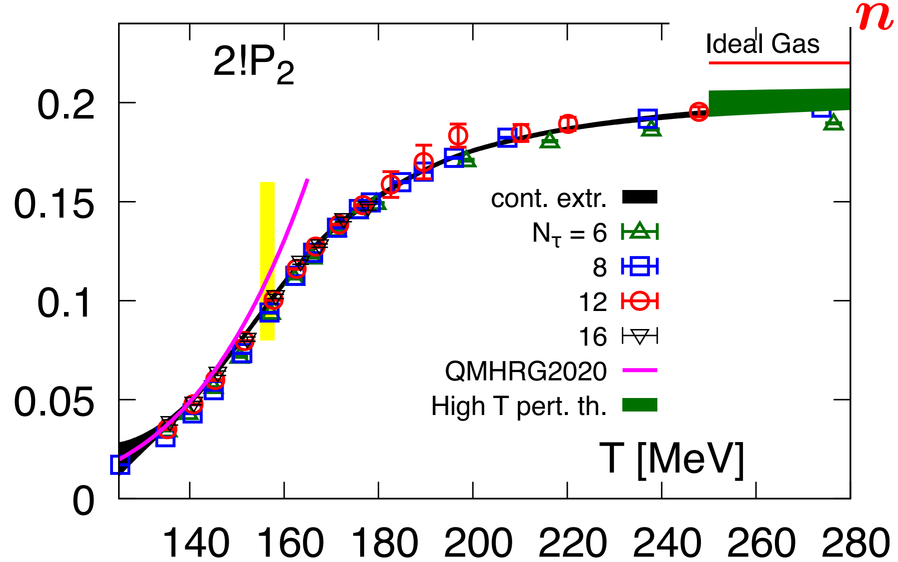
EoS 2022:
arXiv:2202.09184

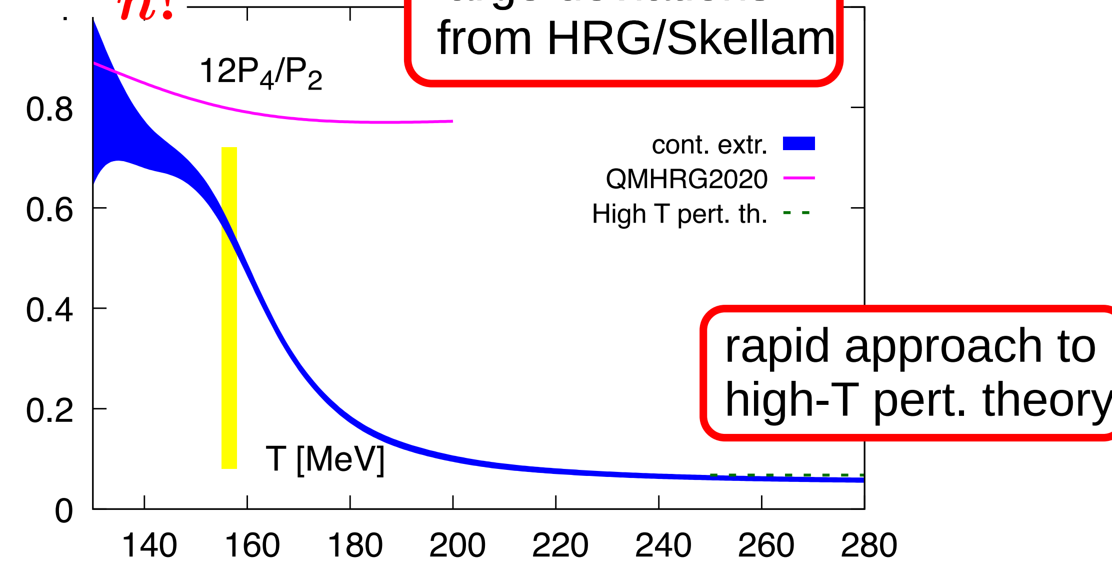
$N_\tau = 8$				$N_\tau = 12$				$N_\tau = 16$			
β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.
6.175	0.003307	125.28	2,200,000								
6.245	0.00307	134.84	1,275,380	6.640	0.00196	135.24	330,447	6.935	0.00145	135.80	17671
6.285	0.00293	140.62	1,598,555	6.680	0.00187	140.80	441,115	6.973	0.00139	140.86	23855
6.315	0.00281	145.11	1,559,003	6.712	0.00181	145.40	416,703	7.010	0.00132	145.95	26122
6.354	0.00270	151.14	1,286,603	6.754	0.00173	151.62	323,738	7.054	0.00129	152.19	26965
6.390	0.00257	156.92	1,602,684	6.794	0.00167	157.75	299,029	7.095	0.00124	158.21	21656
6.423	0.00248	162.39	1,437,436	6.825	0.00161	162.65	214,671	7.130	0.00119	163.50	18173
6.445	0.00241	166.14	1,186,523	6.850	0.00157	166.69	156,111	7.156	0.00116	167.53	19926
6.474	0.00234	171.19	373,644	6.880	0.00153	171.65	144,633	7.188	0.00113	172.60	17163
6.500	0.00228	175.84	294,311	6.910	0.00148	176.73	131,248	7.220	0.00110	177.80	3282

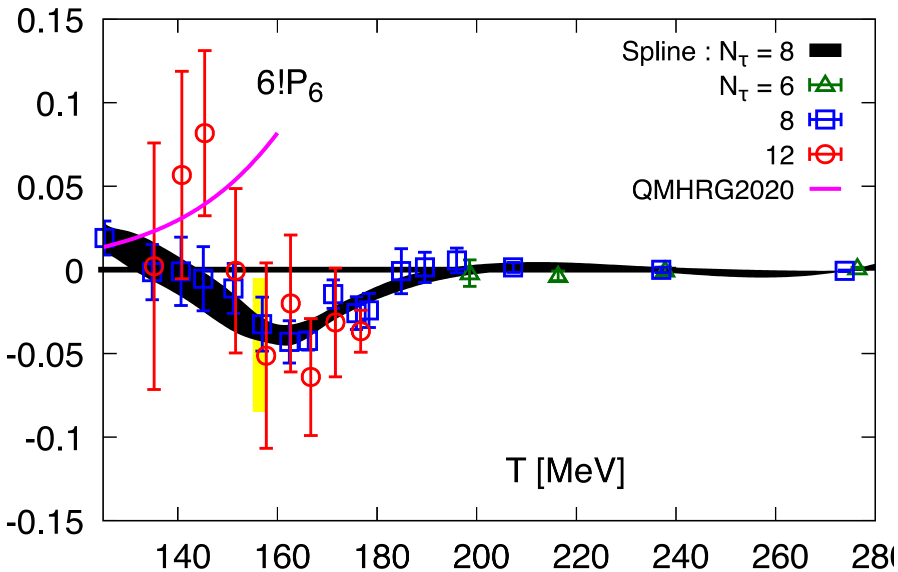
A. Bazavov et al. (HotQCD), Phys. Rev. D 105 (2022) 074511, arXiv:2202.09184 & arXiv:1701.04325

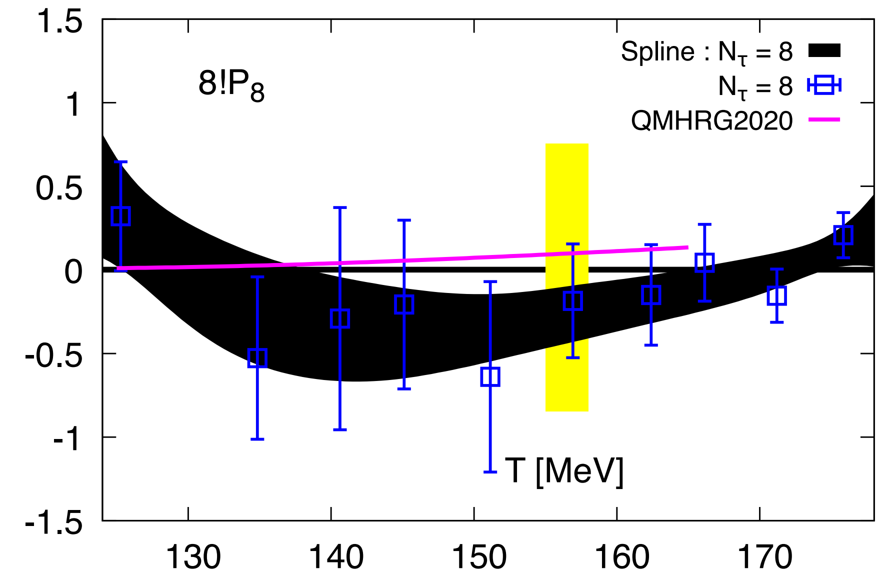
Up to 8th order Taylor expansion for pressure

$$P_n \equiv \frac{\tilde{\chi}_n^B}{n!} \sim \frac{\chi_n^B}{n!}$$





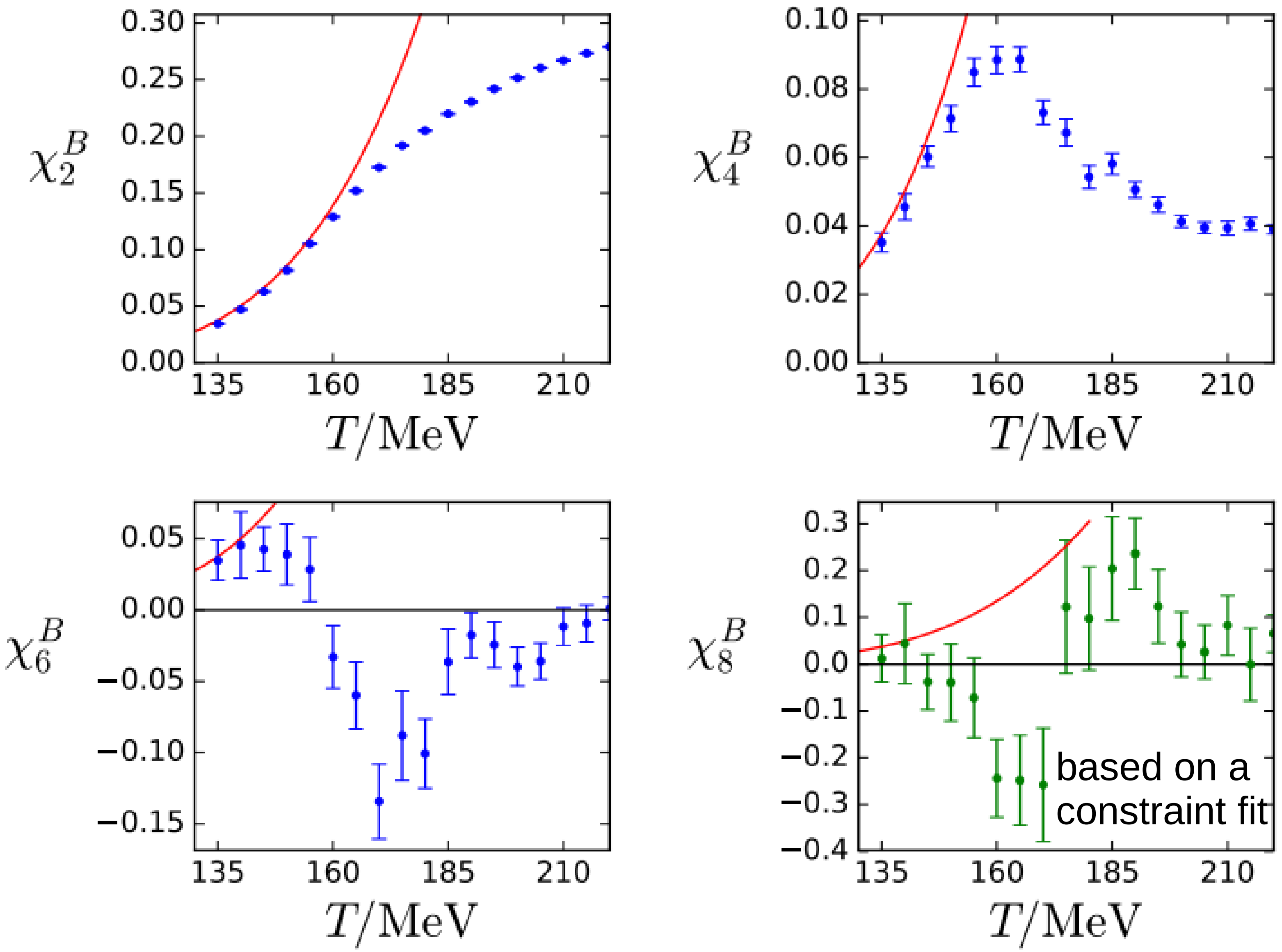




A. Bazavov et al. (HotQCD), Phys. Rev. D 105 (2022) 074511, arXiv:2202.09184 & arXiv:1701.04325

Up to 8th order cumulants are used frequently
– imag. chem. pot. extrapolations –

$48^3 \times 12$



S. Borsanyi et al. , JHEP 10 (2018) 205, arXiv:1805.04445

Comparing Taylor series and Pade resummation

Taylor series

$$\frac{\Delta p}{T^4} \equiv \frac{p(T,\mu_B)}{T^4} - \frac{p(T,0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T) \left(\frac{\mu_B}{T}\right)^{2k} \quad , \; P_{2k} \sim \chi_{2k}^B$$

$$= \frac{P_2^2}{P_4} \left(\bar{x}^2 + \bar{x}^4 + c_{6,2}\bar{x}^6 + c_{8,2}\bar{x}^8 + ...\right)$$

use $P_2 > 0, \; P_4 > 0$

$$\bar{x} \equiv \sqrt{P_4/P_2}\,(\mu_B/T)$$

$$c_{6,2} = \frac{P_6 P_2}{P_4^2} \quad , \quad c_{8,2} = \frac{P_8 P_2^2}{P_4^3}$$

Pade approximation

pressure: $$\left.\frac{p(T,\mu_B)}{T^4}\right|_{[nm]} = \frac{P_2^2}{P_4} P_{[nm]}$$

$$P_{[4,4]} = \frac{(1-c_{6,2})\bar{x}^2 + (1 - 2c_{6,2} + c_{8,2})\,\bar{x}^4}{(1-c_{6,2}) + (c_{8,2} - c_{6,2})\bar{x}^2 + (c_{6,2}^2 - c_{8,2})\bar{x}^4}$$

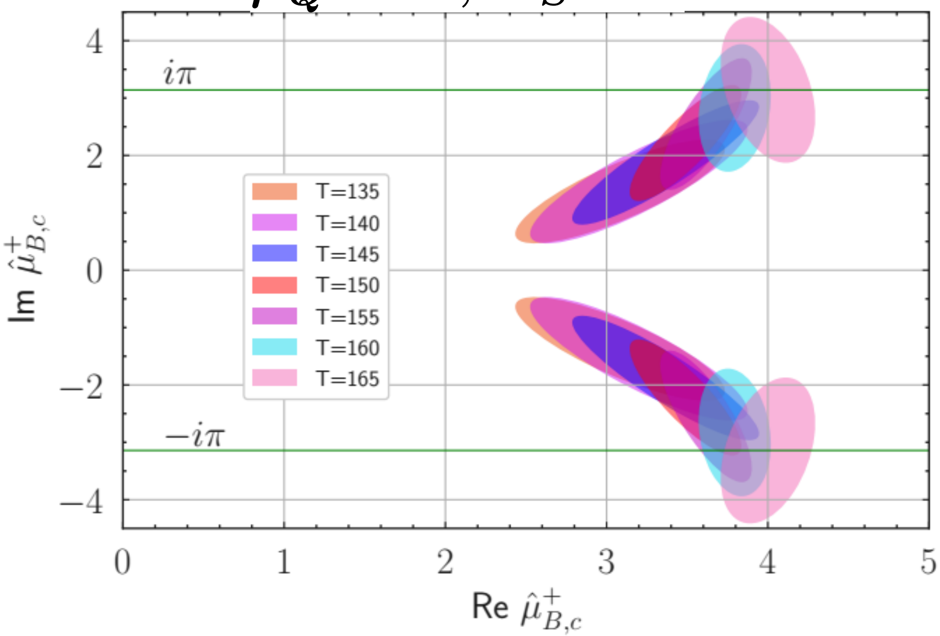
to this order: poles are obtained from a quadratic eq. in $\bar{x}^2 \sim \chi_4^B/\chi_2^B \cdot \hat{\mu}_B^2$

Poles of [n,n] Pade approximants in QCD

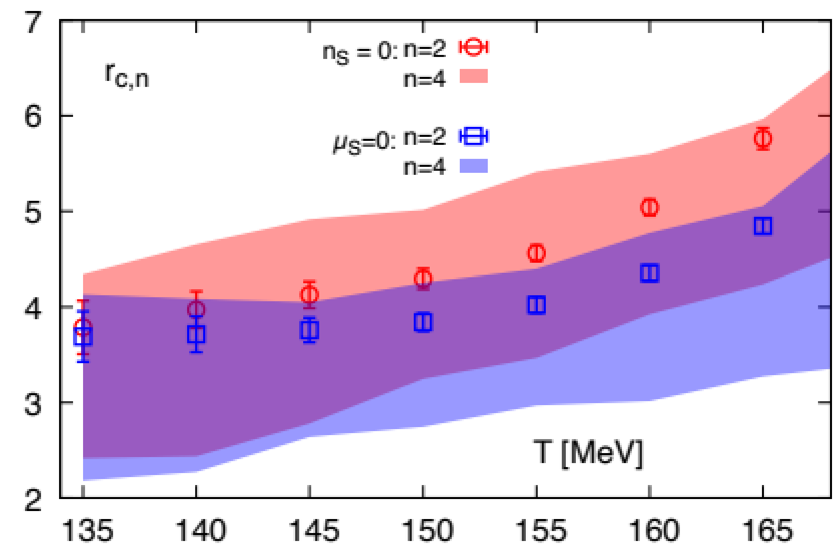
$$\hat{\mu}^{\pm}_{B,c} = \pm r_{c,4} e^{\pm i\Theta_{c,4}} \ , \ \ r_{c,4} = \sqrt{\frac{12\tilde{\chi}_0^{B,2}}{\tilde{\chi}_0^{B,4}}} \left| \frac{1-c_{6,2}}{c_{6,2}^2 - c_{8,2}} \right|^{1/4} \ , \ \ c_{2k,2} = \frac{2\tilde{\chi}_0^{B,2}}{(2k)!\tilde{\chi}_0^{B,2k}} \left(\frac{12\tilde{\chi}_0^{B,2}}{\tilde{\chi}_0^{B,4}} \right)^{k-1}$$

complex poles move to real axis as temperature decreases

$\mu_Q = 0 \ , \ n_S = 0$



distance of complex poles from the origin is given by the **Mercer-Roberts estimator** for the radius of convergence

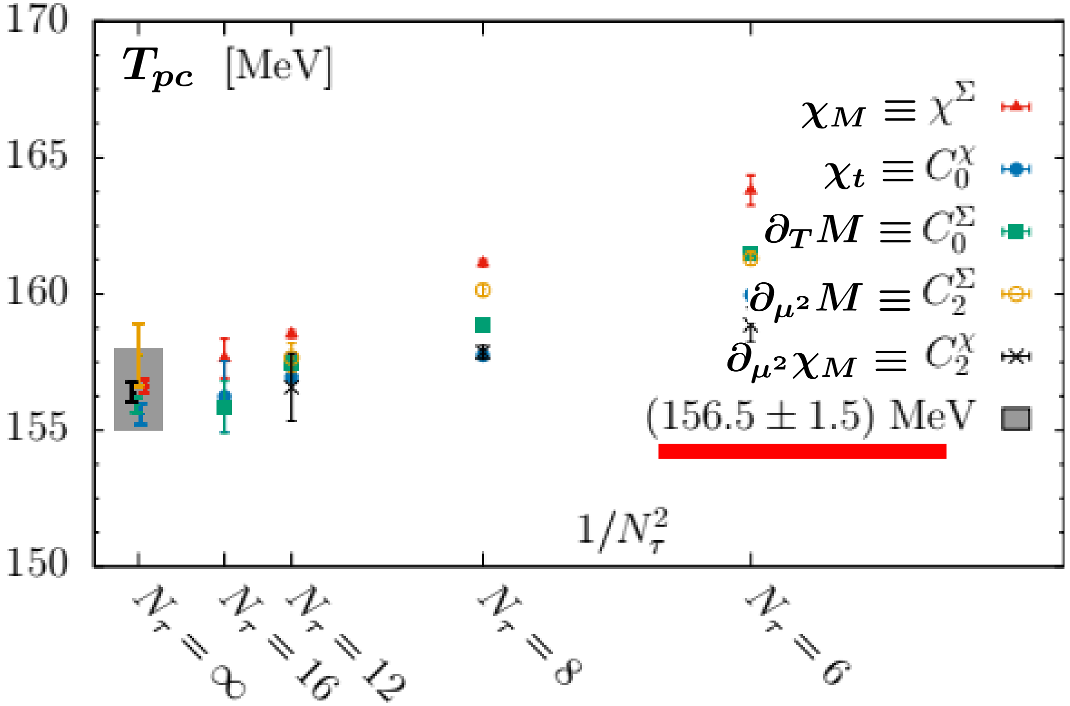


HotQCD, arXiv:2202.09184

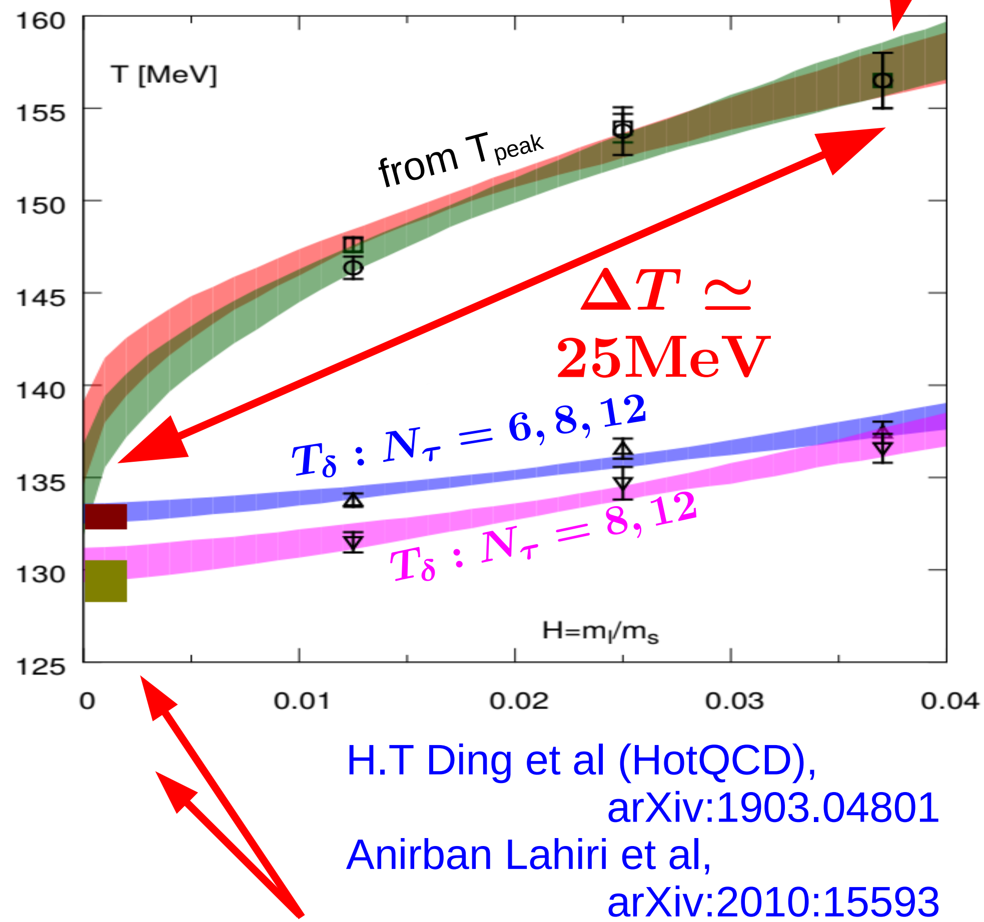
within current errors poles on the real axis (critical point) are possible only for $T \leq 135\text{MeV} \ , \ \mu_B/T > 2.5$

higher statistics will sharpen the constraint

Pseudo-critical and critical temperatures



A. Bazavov et al (HotQCD), arXiv:1812.08235



H.T Ding et al (HotQCD), arXiv:1903.04801
Anirban Lahiri et al, arXiv:2010:15593

physical masses

$$T_{pc}^{phys} = (156.5 \pm 1.5)\ \mathrm{MeV}$$

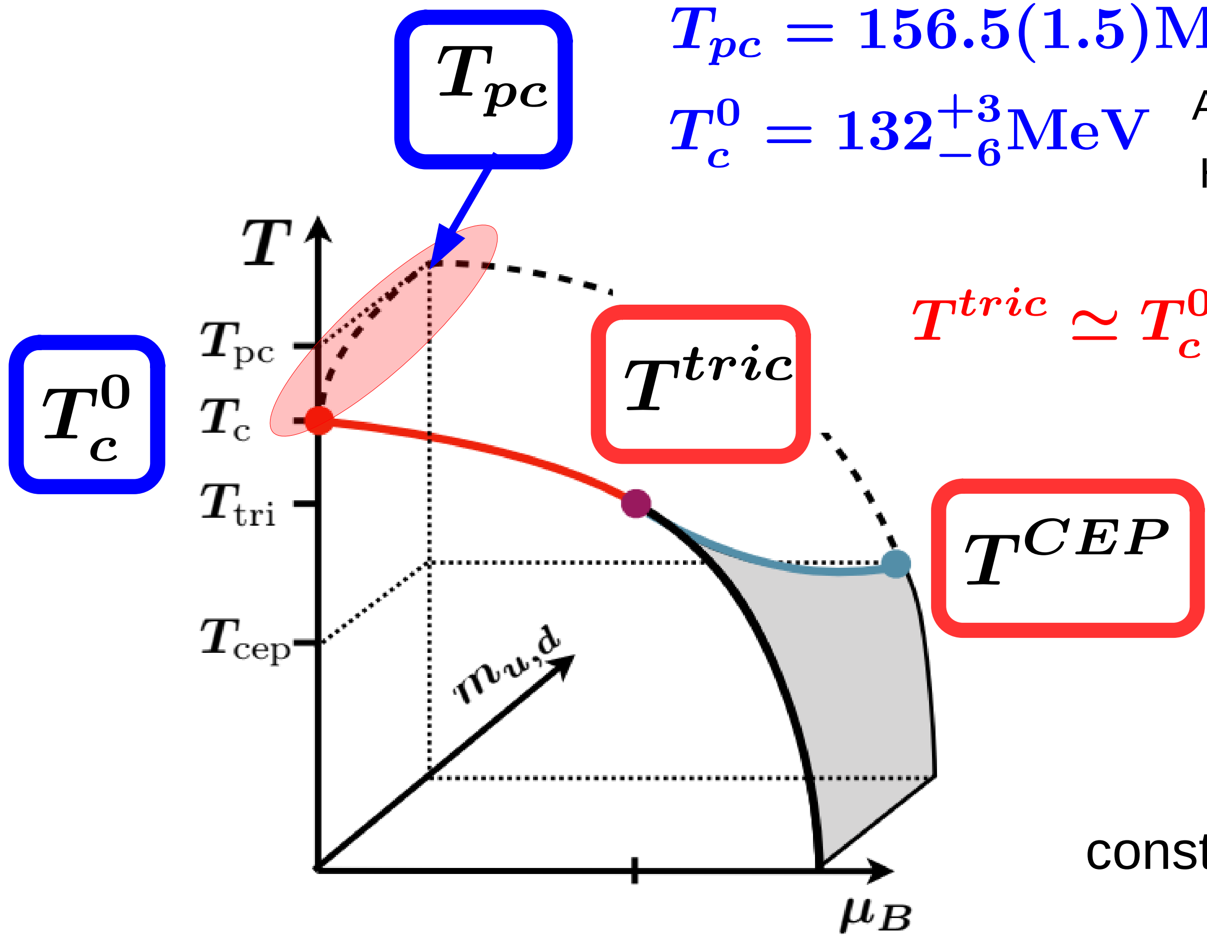
chiral limit extrapolations

$$T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$

also: A. Y. Kotov et al., arXiv: 2105.09842

Summary: Phases of strong-interaction matter

determination of T_c^0 puts an upper limit on T^{CEP}



$T_{pc} = 156.5(1.5)\text{MeV}$

$T_c^0 = 132^{+3}_{-6}\text{MeV}$

A. Bazavov et al (HotQCD), arXiv:1812.08235

H.T. Ding et al (HotQCD), arXiv:1903.04801

$$T^{tric} \simeq T_c^0 - 2.2(5)\ \text{MeV} \left(\frac{\mu_B}{T_c}\right)^2_{tric}$$

HotQCD@Lattice2021 (M. Sarkar) arXiv:2112.15398

constraint on the location of the CEP

$$T^{CEP} < T^{tric} < T_c^0$$

$$T^{CEP} < 135\ \text{MeV}$$

the temperature range below T_{pc} is most important for getting information on a possible CEP

Summary: Phases of strong-interaction matter

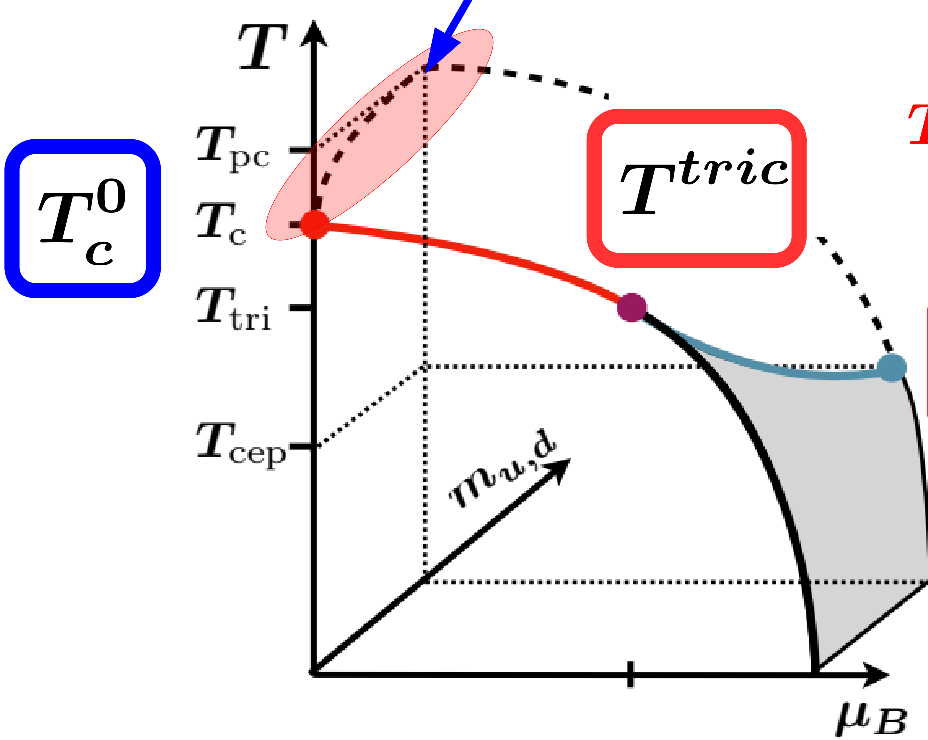
determination of T_c^0 puts an upper limit on T^{CEP}

$$T_{pc} = 156.5(1.5)\mathrm{MeV}$$

$$T_c^0 = 132^{+3}_{-6}\mathrm{MeV}$$

A. Bazavov et al (HotQCD), arXiv:1812.08235

H.T. Ding et al (HotQCD), arXiv:1903.04801



$$T^{tric} \simeq T_c^0 - 2.2(5)\ \mathrm{MeV} \left(\frac{\mu_B}{T_c}\right)^2_{tric}$$

HotQCD@Lattice2021 (M. Sarkar)
arXiv:2112.15398

more likely, if

$$(\mu_B/T)_{tric} \geq 2$$

$$T^{CEP} < T^{tric} < T_c^0$$

$$T^{CEP} < 125\ \mathrm{MeV}$$

the temperature range below T_{pc} is most important for getting information on a possible CEP

Baryon-Strangeness Correlations: A Diagnostic of Strongly Interacting Matter

V. Koch, A. Majumder, and J. Randrup

Nuclear Science Division, Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 94720, USA

(Received 23 May 2005; published 24 October 2005)

The correlation between baryon number and strangeness elucidates the nature of strongly interacting matter, such as that formed transiently in high-energy nuclear collisions. This diagnostic can be extracted theoretically from lattice QCD calculations and experimentally from event-by-event fluctuations. The analysis of present lattice results above the critical temperature severely limits the presence of $q\bar{q}$ bound states, thus supporting a picture of independent (quasi)quarks.

DOI: 10.1103/PhysRevLett.95.182301 PACS numbers: 25.75.Gz, 12.38.Gc, 12.38.Mh, 25.75.Nq

The principal goal of high-energy heavy-ion collisions is the creation and exploration of a novel state of matter in which the quarks and gluons are deconfined over distances considerably larger than that of a hadron [1]. It had originally been assumed that asymptotic freedom would cause such matter to behave as a plasma of massless quarks and gluons interacting with a relatively weak screened chromodynamic Coulomb force. While this picture is supported qualitatively by the rapid rise in the entropy density at a temperature of about $T_c \simeq 170$ MeV, as obtained by lattice QCD calculations, the fact that the high-T behavior falls somewhat below that of an ideal gas of massless quarks and gluons [2] indicates that the chromodynamic plasma has a more complex structure.

Indeed, recent results from lattice QCD calculation on spectral functions [3–6] suggest the presence of bound, color-neutral states well above T_c. This has led to the suggestion that at moderate temperatures, $T \simeq 1 - 2T_c$, the system is composed of medium-modified (massive) quarks and gluons together with their (many and possibly colored) bound states [7–10].

Furthermore, jet quenching observed at the Relativistic Heavy-Ion Collider (RHIC) suggests that the matter formed is not hadronic [11]. Moreover, the RHIC collisions exhibit strong elliptic and radial collective flows that are consistent with predictions by ideal fluid dynamics [12,13]. If ideal hydrodynamics is indeed the correct framework, rapid thermalization must occur and this in turn would seem to rule out a weakly interacting plasma. Thus, the nature of the matter being created at RHIC is still not clarified and the purpose of this Letter is to offer a novel diagnostic tool for elucidating this issue by probing the relevant degrees of freedom and their correlations.

In particular, we will argue that the correlation between the strangeness S and the baryon number B provides a useful diagnostic for the presence of strong correlations between quarks and antiquarks. In order to understand this, consider first a situation in which the basic degrees of freedom are weakly interacting quarks and gluons. Then strangeness is carried exclusively by the s and $\bar{s}$ quarks which in turn carry baryon number in strict proportion to their strangeness, $B_s = -\frac{1}{3}S_s$, thus rendering strangeness and baryon number strongly correlated. This feature is in stark contrast to a hadron gas in which the relation between strangeness and baryon number is less intimate. For example, at small baryon chemical potential the strangeness is carried primarily by kaons, which have no baryon number.

These elementary considerations lead us to introduce the following correlation coefficient,

$$C_{BS} \equiv -3\frac{\sigma_{BS}}{\sigma_S^2} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = -3\frac{\langle BS\rangle}{\langle S^2\rangle}. \quad (1)$$

The average $\langle\cdot\rangle$ is taken over a suitable ensemble of events and the last expression uses the fact that $\langle S\rangle$ vanishes. When the active degrees of freedom are individual quarks, the total strangeness is $S = \nu_{\bar{s}} - \nu_s$, while the baryon number can be expressed as $B = \frac{1}{3}(U + D - S)$, where $U = \nu_u - \nu_{\bar{u}}$ is the upness and $D = \nu_d - \nu_{\bar{d}}$ is the downness. Thus, if the flavors are uncorrelated, we have $\sigma_{BS} = -\frac{1}{3}\sigma_S^2$ and C_{BS} is unity.

In a gas of hadron resonances, the total baryon number is $B = \sum_k n_k B_k$ and its total strangeness is $S = \sum_k n_k S_k$, where the species k has baryon number B_k and strangeness S_k. If the multiplicities $\{n_k\}$ are uncorrelated, as is generally assumed in the standard statistical treatment [14], C_{BS} may be expressed in terms of the associated multiplicity variances $\sigma_k^2 \equiv \langle n_k^2\rangle - \langle n_k\rangle^2 \approx \langle n_k\rangle$,

$$C_{BS} = -3\frac{\sum_k \sigma_k^2 B_k S_k}{\sum_k \sigma_k^2 S_k^2} \approx -3\frac{\sum_k \langle n_k\rangle B_k S_k}{\sum_k \langle n_k\rangle S_k^2}, \quad (2)$$

where the approximate expression holds for Poisson statistics. Thus, in a gas of uncorrelated hadrons, the numerator receives contributions from only (strange) baryons (and antibaryons), while the denominator receives contributions also from (strange) mesons,

$$C_{BS} \approx 3\frac{\langle\Lambda\rangle + \langle\bar{\Lambda}\rangle + \cdots + 3\langle\Omega^-\rangle + 3\langle\bar{\Omega}^+\rangle}{\langle K^0\rangle + \langle\bar{K}^0\rangle + \cdots + 9\langle\Omega^-\rangle + 9\langle\bar{\Omega}^+\rangle}. \quad (3)$$

While Eq. (1) [or its more explicit version (6)] holds

Conserved charge correlations

baryon number – strangeness correlations

$$\frac{\chi_{11}^{BS}(T,\mu_B)}{\chi_2^S(T,\mu_B)} \equiv \frac{\langle BS\rangle}{\langle S^2\rangle}$$

$$C_{BS} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = -3\frac{\langle BS\rangle}{\langle S^2\rangle}$$

Free Quark Gas

$$C_{BS} = -3\frac{\langle BS\rangle}{\langle S^2\rangle} = -3\frac{\frac{\pm 1}{3}\cdot(\mp 1)}{(\pm 1)^2} = 1$$

Hadron Resonance Gas (HRG)

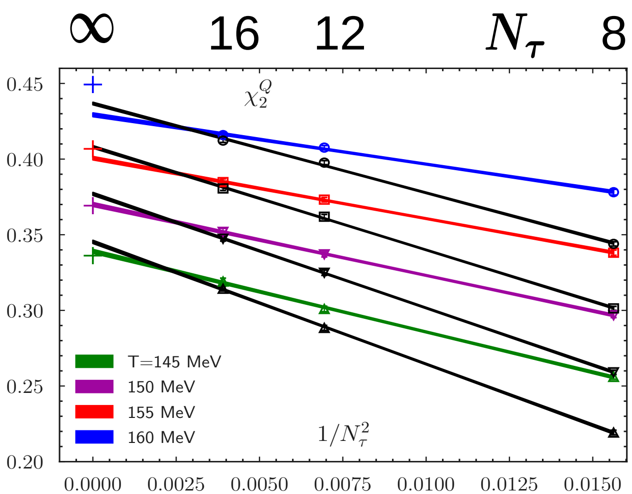
$$C_{BS} \approx 3\,\frac{\langle\Lambda\rangle + \langle\bar{\Lambda}\rangle + \ldots + 3\langle\Omega^-\rangle + 3\langle\bar{\Omega}^+\rangle}{K^0 + \bar{K}^0 + \ldots + 9\langle\Omega^-\rangle + 9\langle\bar{\Omega}^+\rangle}$$

Fluctuations and Correlations
– precision calculation of all 2nd order cumulants –

two constraints: only 4 out of 6
2nd order observables are independent

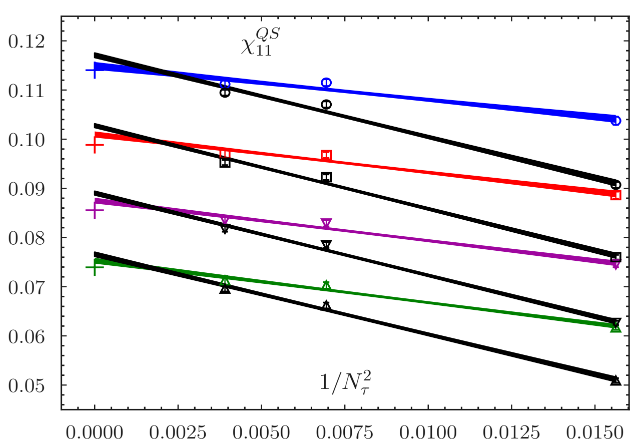
$$\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS} \qquad \chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS}$$

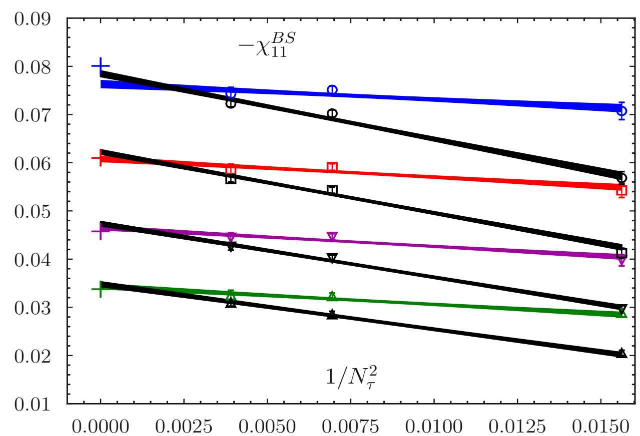
– exact relations in QCD for $\mu_X = 0,\ X = B, Q, S$
– satisfied in HRG models to better than 1%

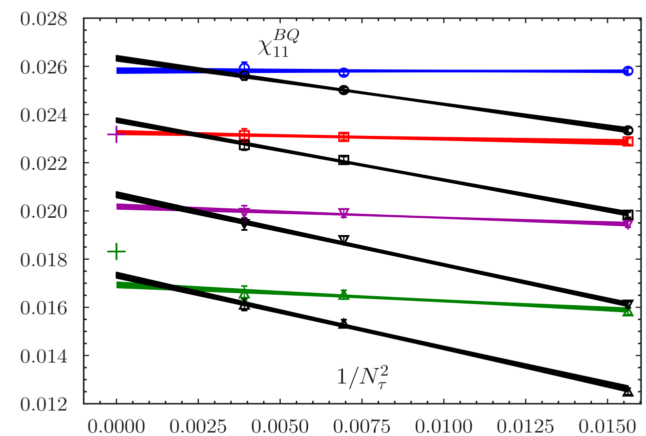


– well controlled continuum extrapolation

scale setting used: colored af_K, black: r1/a







– continuum extrapolated results for all 2nd order cumulants in (2+1)-flavor QCD

D. Bollweg et al (HotQCD), Phys. rev. D104 (2021) 7, arXiv:2107.10011

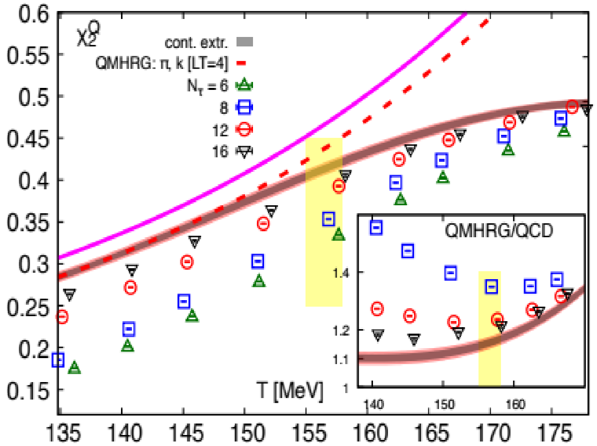
2^{nd} order cumulants: fluctuations and correlations

two constraints: only 4 out of 6
2^{nd} order observables are independent

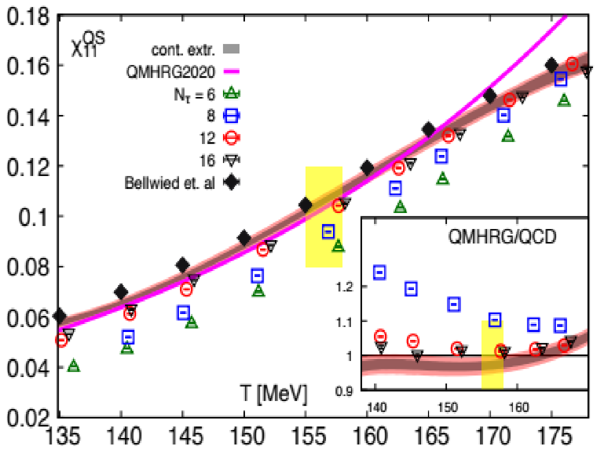
$$T_c = (156.5 \pm 1.5)\mathrm{MeV}$$

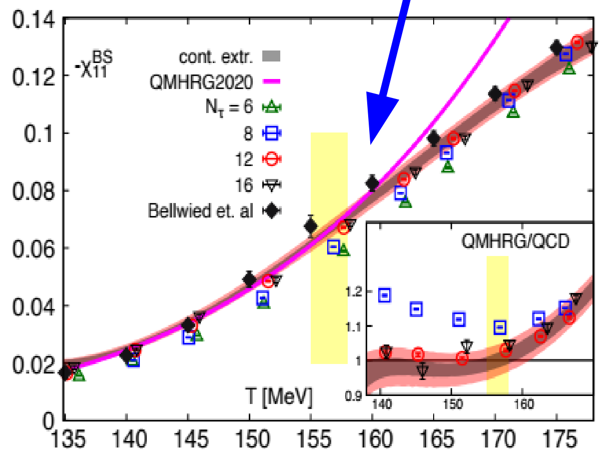
$$\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS} \qquad \chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS}$$

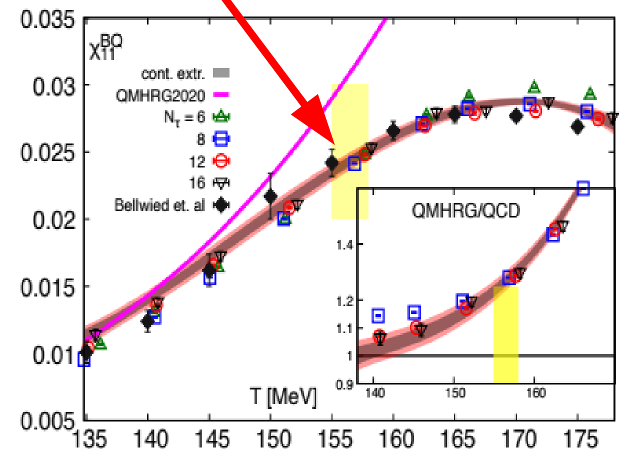
– exact relations in QCD for $\mu_X = 0,\ X = B, Q, S$
– satisfied in HRG models to better than 1%



significant difference in
BS and BQ
correlations





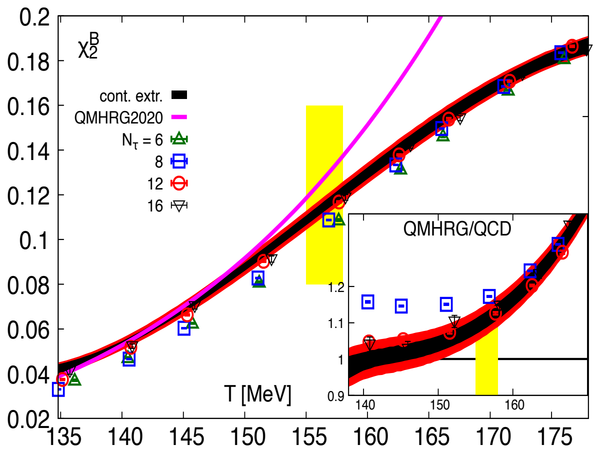


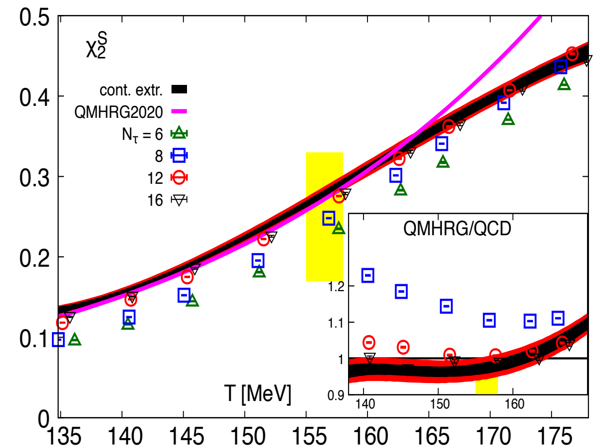
– continuum extrapolated results for all 2^{nd} order cumulants in (2+1)-flavor QCD

D. Bollweg et al (HotQCD), Phys. rev. D104 (2021) 7, arXiv:2107.10011

2nd order cumulants: fluctuations and correlations

at T_{pc}	χ_{11}^{BQ}	χ_{11}^{BS}	χ_{11}^{QS}	$(\chi_2^Q)_{LT=4}$	χ_2^S	χ_2^B	χ_{11}^{BS}/χ_2^S	χ_{11}^{BQ}/χ_2^B
QCD[this work]	0.0243(7)(9)	-0.066(4)(5)	0.106(3)(5)	0.413(8)(9)	0.279(9)(12)	0.115(5)(7)	-0.236(5)(6)	0.212(4)(5)
QMHRG2020[HotQCD]	0.031(3)	-0.066(6)	0.103(5)	0.437(14)	0.272(14)	0.127(10)	-0.243(8)	0.244(3)
QMHRG2016+ [Houston]	0.031(3)	-0.071(7)	0.104(5)	0.444(15)	0.277(16)	0.132(10)	-0.256(7)	0.235(2)
PDGHRG	0.030(2)	-0.046(4)	0.094(4)	0.419(12)	0.234(11)	0.106(8)	-0.197(6)	0.283(2)
EVHRG2020[$b = 1$ fm^3]	0.027(2)	-0.059(5)	0.103(5)	0.431(13)	0.264(13)	0.113(8)	-0.223(5)	0.243(2)
S-matrix [Friman et al,2015]	0.020(1)	-0.062(5)	0.107(4)	–	–	–	–	–





– largest deviations from HRG seen for χ_{11}^{BQ} : ~25%

$$\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS}$$

$$-\chi_{11}^{BS}/\chi_2^B = 0.57(6)$$

- **strangeness fluctuations and correlations are well described by QMHRG2020 at T_{pc}**

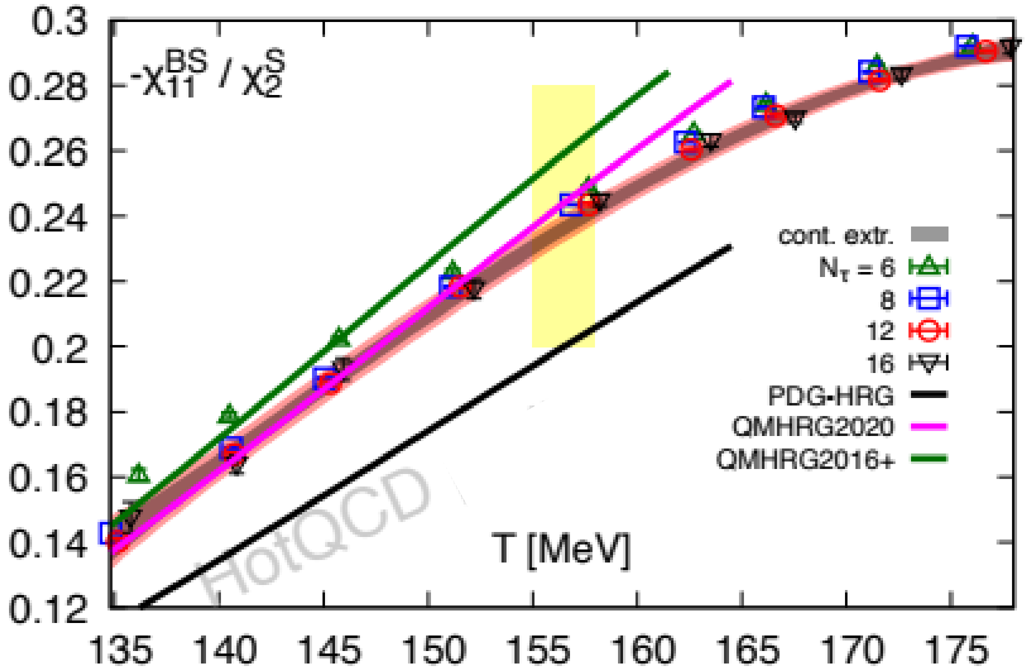
- **electric charge fluctuations and correlations deviate from QMHRG2020 already at about 145 MeV**

- baryon number fluctuations are sensitive to BS-correlations

$$\chi_2^B(T_{pc}) = 2\chi_{11}^{BQ}\left(1 - \frac{\chi_{11}^{BS}}{2\chi_{11}^{BQ}}\right)_{T_{pc}} \simeq 2.6 \cdot (2\chi_{11}^{BQ})$$

Ratio of baryon number – strangeness correlation and net strangeness fluctuations

$$\frac{\mu_S}{\mu_B} \equiv -\frac{\chi_{11}^{BS}}{\chi_2^S} - q_1 \frac{\chi_{11}^{QS}}{\chi_2^S} + \mathcal{O}(\mu_B^2)$$



- evidence for experimentally not yet observed strange baryons?
- evidence for strong flavor correlations

$$-\frac{\chi_{11}^{BS}}{\chi_2^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{11}^{us}}{\chi_2^s}$$

D. Bollweg et al. (HotQCD), arXiv:2107.10011

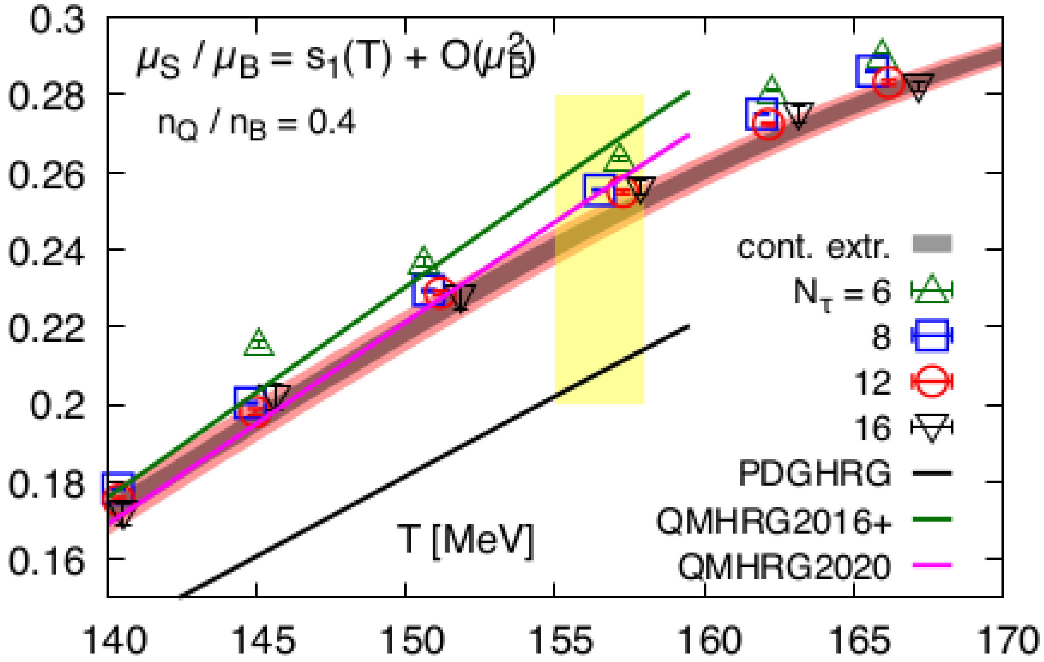
PDG-HRG: uses experimentally known hadron spectrum listed by the Particle Data Group
QM-HRG: uses additional hadrons predicted to exist in Quark Model calculations

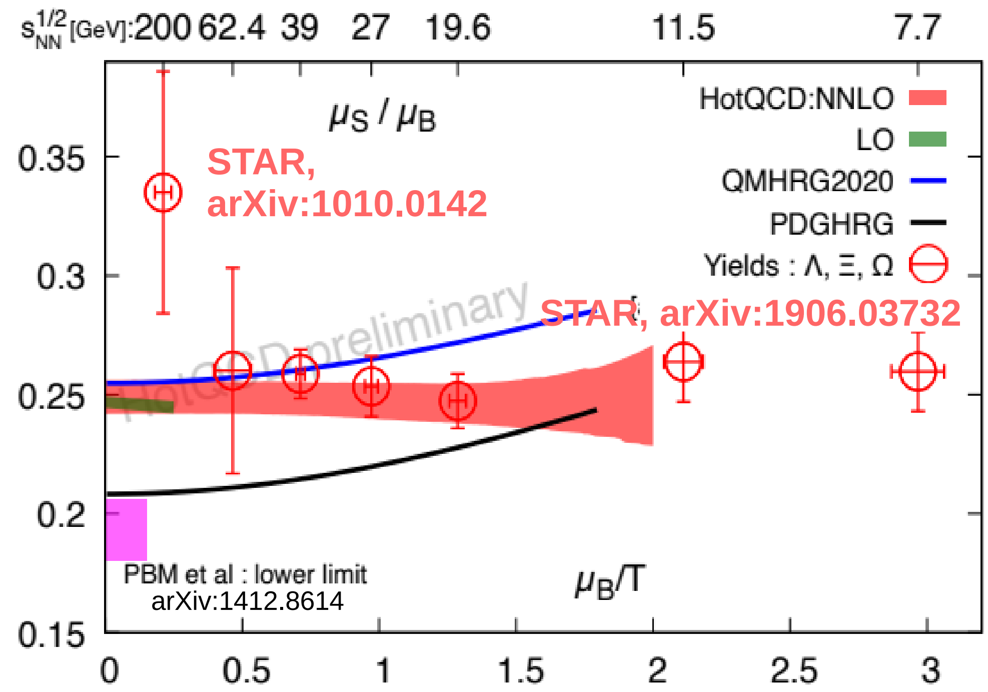
– description of BS-correlations in HRG models is particularly sensitive to "missing resonances" in the strangeness sector

Baryon number – strangeness chemical potentials at freeze-out from strange baryon yields

BS ratios probe strangeness content in an HRG

$$\frac{\mu_S}{\mu_B} \equiv -\frac{\chi_{11}^{BS}}{\chi_2^S} - q_1 \frac{\chi_{11}^{QS}}{\chi_2^S} + \mathcal{O}(\mu_B^2)$$





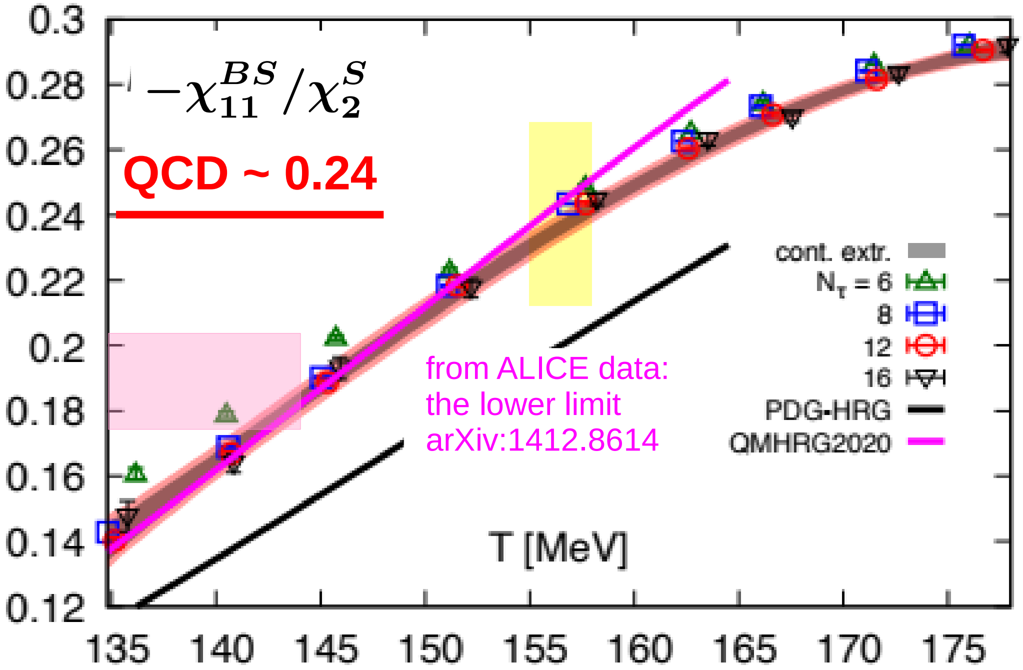
STAR multi-strange baryon yields are consistent with freeze-out at T_{pc} and a μ_S/μ_B that reflects contributions from additional strange baryons

at T_{pc}

QCD: $\frac{\mu_S}{\mu_B} \simeq 0.24$

PDG-HRG: $\frac{\mu_S}{\mu_B} \simeq 0.21$

Experimental access to baryon number – strangeness correlation ?



for Skellam distributions mean values and second order cumulants as well as correlations are related:

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(\langle N_q\rangle + \langle N_{-q}\rangle)$$

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}\sum_{n=1}^{|q|} n^2(\langle N_n\rangle + \langle N_{-n}\rangle)$$

$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3}\sum_{n=-q_N}^{q_N}\sum_{m=-q_M}^{q_M} nm\langle N_{n,m}\rangle$$

$$|q| = 1\ (B),\ 2\ (Q),\ 3\ (S)$$

testing assumption of Skellam distribution:

$$-\frac{\chi_{BS}}{T^2} > \frac{1}{VT^3}[2\langle\Lambda+\Sigma^0\rangle + 4\langle\Sigma^+\rangle + 8\langle\Xi\rangle + 6\langle\Omega^-\rangle] = 97.4 \pm 5.8.$$

$$\chi_{11}^{QS}/T^2 = (178.5 \pm \pm 17.14)$$

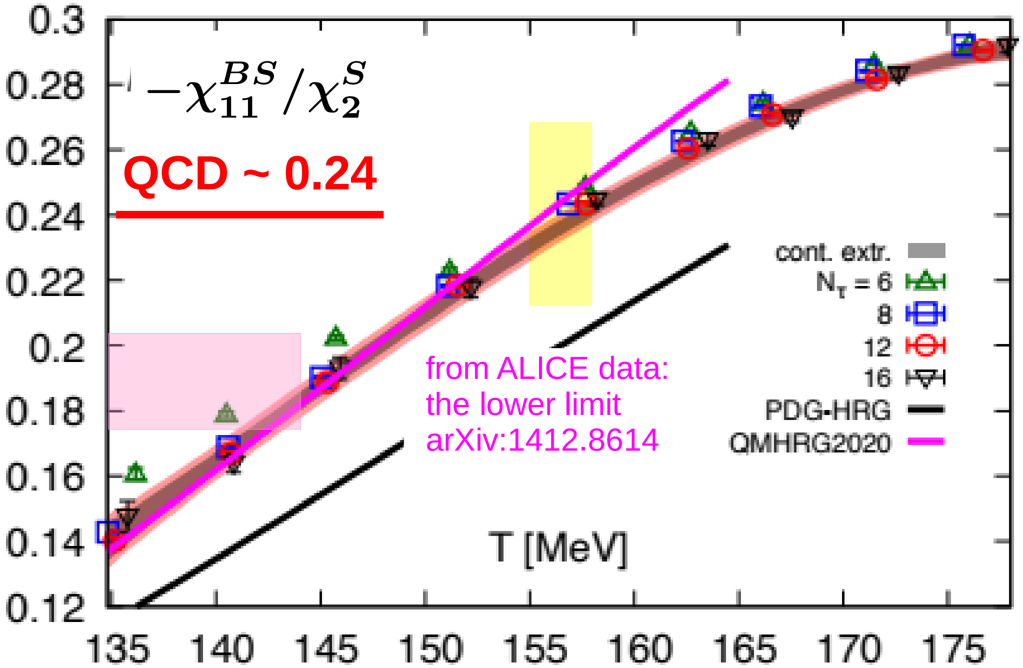
$$\frac{\chi_S}{T^2} \simeq \frac{1}{VT^3}[(\langle K^+\rangle + \langle K^0\rangle + \langle\Lambda+\Sigma^0\rangle + \langle\Sigma^+\rangle + \langle\Sigma^-\rangle + 4\langle\Xi^-\rangle + 4\langle\Xi^0\rangle + 9\langle\Omega^-\rangle + \text{antiparticles}) - (\Gamma_{\phi\to K^+} + \Gamma_{\phi\to K^-} + \Gamma_{\phi\to K^0} + \Gamma_{\phi\to\bar{K}^0})\langle\phi\rangle] = (504 \pm 24).$$

P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel, Phys. Lett. B747 (2015) 292, arXiv:1412.8614

getting control over B rather than P fluctuations is important!!!

$$\Rightarrow \quad -\frac{\chi_{BS}}{\chi_S} \geq 0.193(22)$$

Experimental access to baryon number – strangeness correlation ?



for Skellam distributions mean values and second order cumulants as well as correlations are related:

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}\left(\langle N_q\rangle + \langle N_{-q}\rangle\right)$$

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}\sum_{n=1}^{|q|} n^2\left(\langle N_n\rangle + \langle N_{-n}\rangle\right)$$

$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3}\sum_{n=-q_N}^{q_N}\sum_{m=-q_M}^{q_M} nm\langle N_{n,m}\rangle$$

$$|q| = 1\ (B),\ 2\ (Q),\ 3\ (S)$$

testing assumption of Skellam distribution:

$$-\frac{\chi_{BS}}{T^2} > \frac{1}{VT^3}[2\langle\Lambda+\Sigma^0\rangle + 4\langle\Sigma^+\rangle + 8\langle\Xi\rangle + 6\langle\Omega^-\rangle] = 97.4 \pm 5.8.$$

$$\chi_{QS}/T^2 = (178.5 \pm \pm 17.14)$$

$$\frac{\chi_S}{T^2} \simeq \frac{1}{VT^3}[(\langle K^+\rangle + \langle K^0\rangle + \langle\Lambda+\Sigma^0\rangle + \langle\Sigma^+\rangle + \langle\Sigma^-\rangle + 4\langle\Xi^-\rangle + 4\langle\Xi^0\rangle + 9\langle\Omega^-\rangle + \text{antiparticles}) - (\Gamma_{\phi\to K^+} + \Gamma_{\phi\to K^-} + \Gamma_{\phi\to K^0} + \Gamma_{\phi\to\bar{K}^0})\langle\phi\rangle] = (504 \pm 24).$$

P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel, Phys. Lett. B747 (2015) 292, arXiv:1412.8614

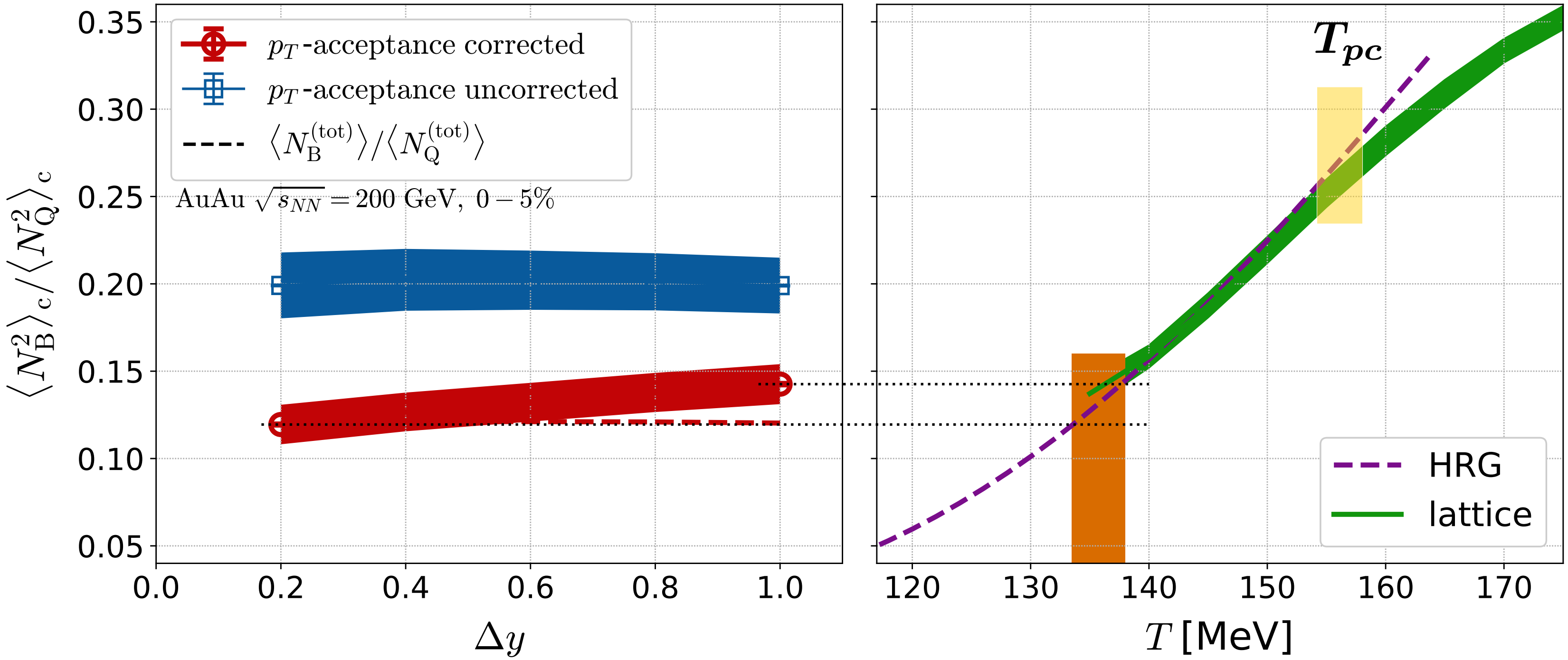
$$\frac{2\chi_{11}^{QS} - \chi_{11}^{BS}}{\chi_2^S} = 0.90 \pm 0.13$$

consistent with QCD constraint

$$\chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS}$$

A puzzle ?

χ_2^B/χ_2^Q from STAR data using proton number distribution as proxy and implementing a "simple treatment for strange baryon contributions" to obtain χ_2^B



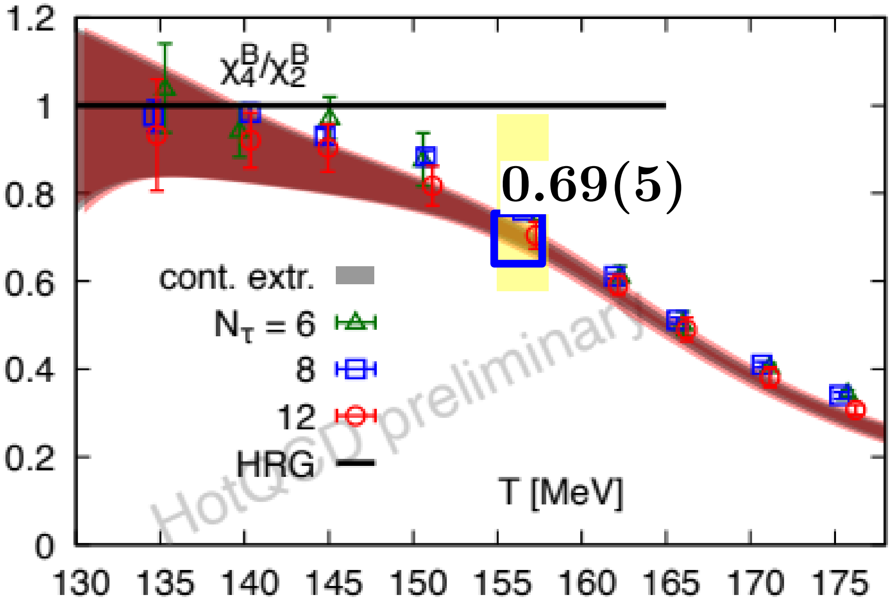
M. Kitazawa et al, arXiv:2205.10030

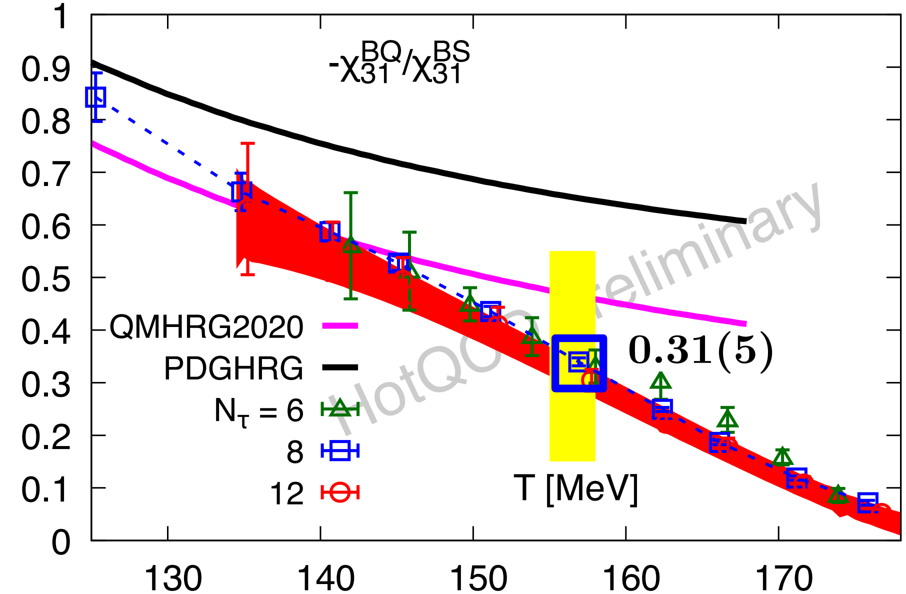
– result is about a factor 2 smaller than expected

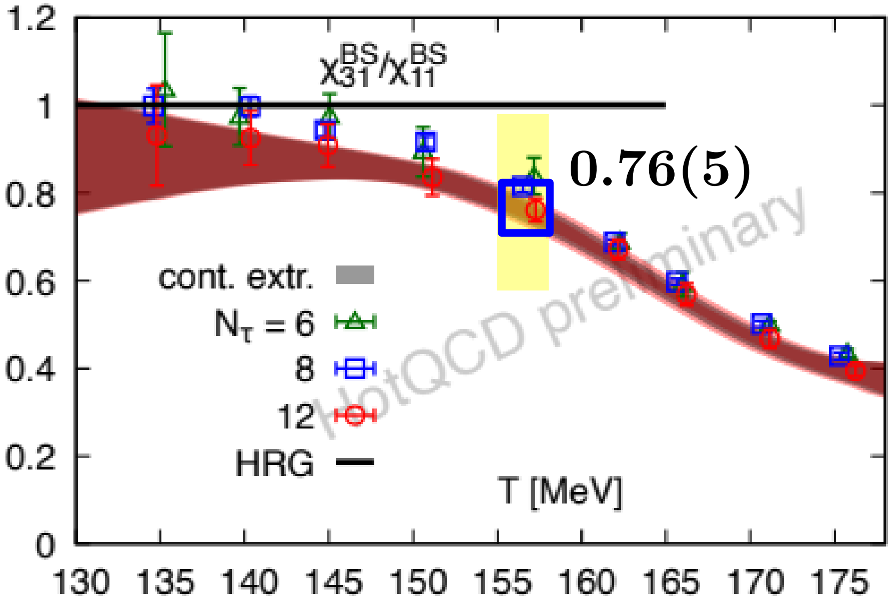
– consistent with the expected, missing strange baryon contribution?

Ratios of 4th and 2nd order cumulants

– large deviations from Skellam –







– ratios of 4th and 2nd order cumulants differ from non-inter. HRG for T>145 MeV

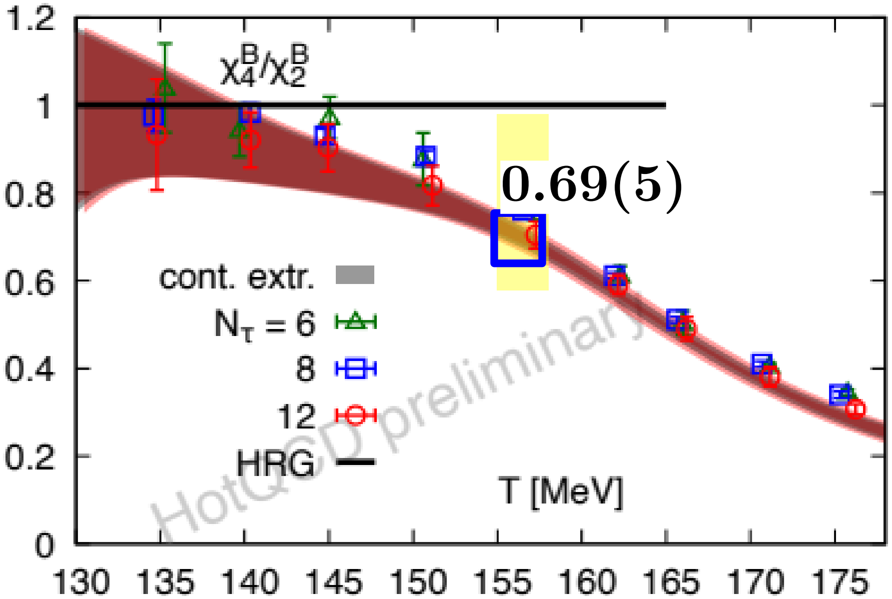
– they change by ~(20-40)% in the crossover region

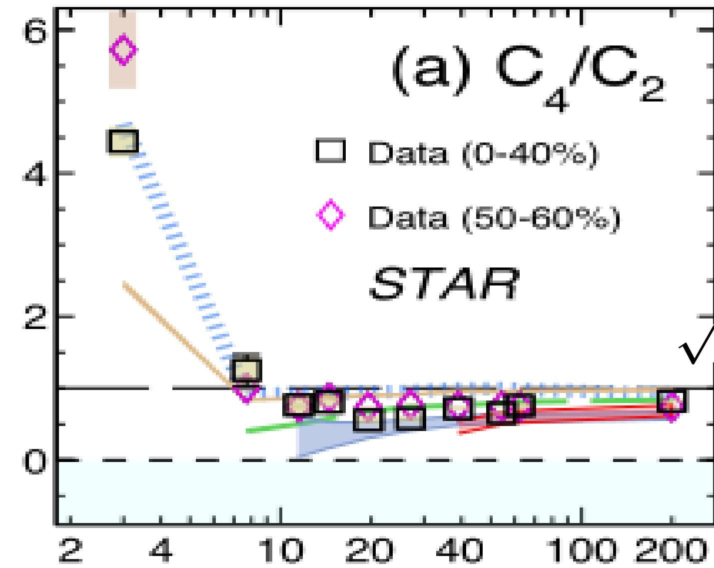
sensitive probes for freeze-out

at ALICE freeze-out temperature
$T_{fo} = 156.5(1.5)\mathrm{MeV}$

Ratios of 4th and 2nd order cumulants

– large deviations from Skellam –





$\sqrt{s_{NN}} = 200GeV$

$0.823(24)(16)$

STAR PRL 2023, arXiv:2207.09837

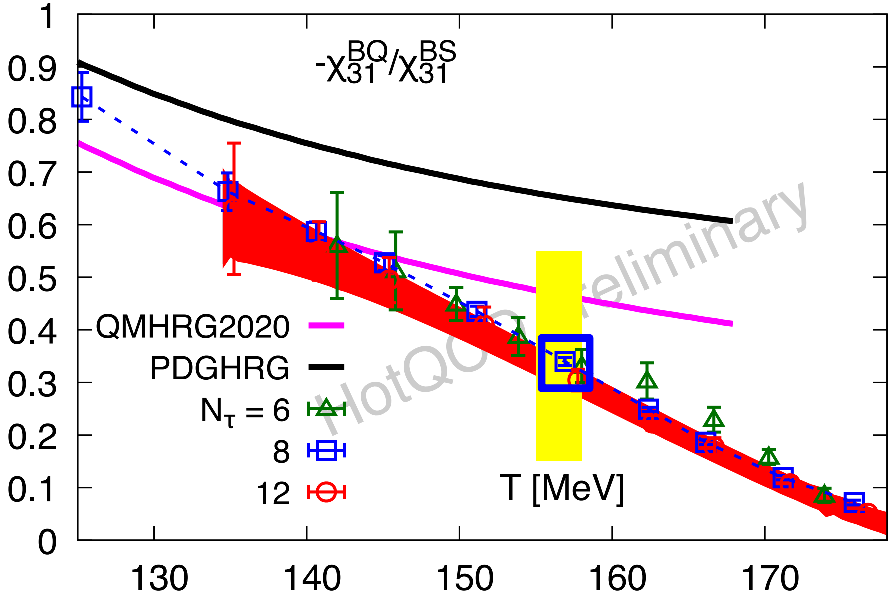
– Why are cumulants of proton number fluctuations a good proxy for ratios of baryon number cumulants, although they may not be good proxies for the individual cumulants ?

constraint:

$$\chi_{n+1}^B = 2\chi_{n1}^{BQ} - \chi_{n1}^{BS}$$

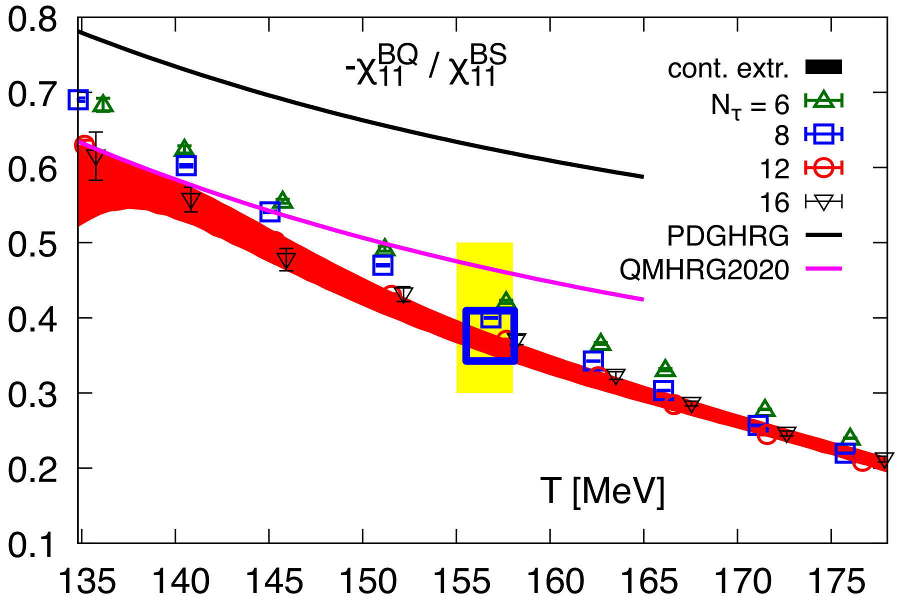
$$\Rightarrow \quad \frac{\chi_4^B}{\chi_2^B} = \frac{1-\frac{\chi_{31}^{BS}}{2\chi_{31}^{BQ}}}{1-\frac{\chi_{11}^{BS}}{2\chi_{11}^{BQ}}}\ \frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}} \equiv R(T)\frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}}$$

Ratios of 4th and 2nd order cumulants



$$\frac{\chi_4^B}{\chi_2^B} = \frac{1 - \frac{\chi_{31}^{BS}}{2\chi_{31}^{BQ}}}{1 - \frac{\chi_{11}^{BS}}{2\chi_{11}^{BQ}}} \frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}} \equiv R(T)\frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}}$$

$$R(T_{pc}) \simeq 1.15 \; \frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}}$$

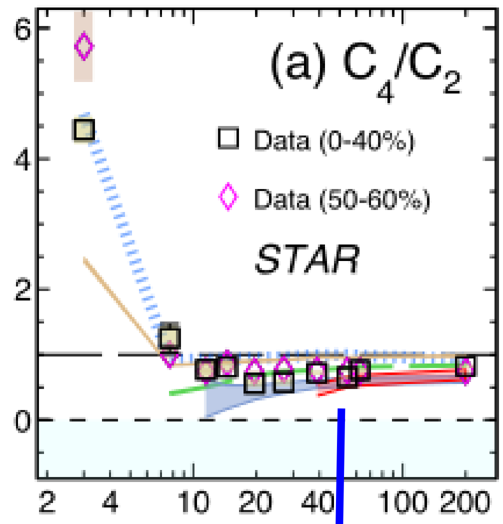


reminder:

$$\chi_2^B(T_{pc}) = 2\chi_{11}^{BQ}\left(1 - \frac{\chi_{11}^{BS}}{2\chi_{11}^{BQ}}\right)_{T_{pc}} \simeq 2.6(2\chi_{11}^{BQ})$$

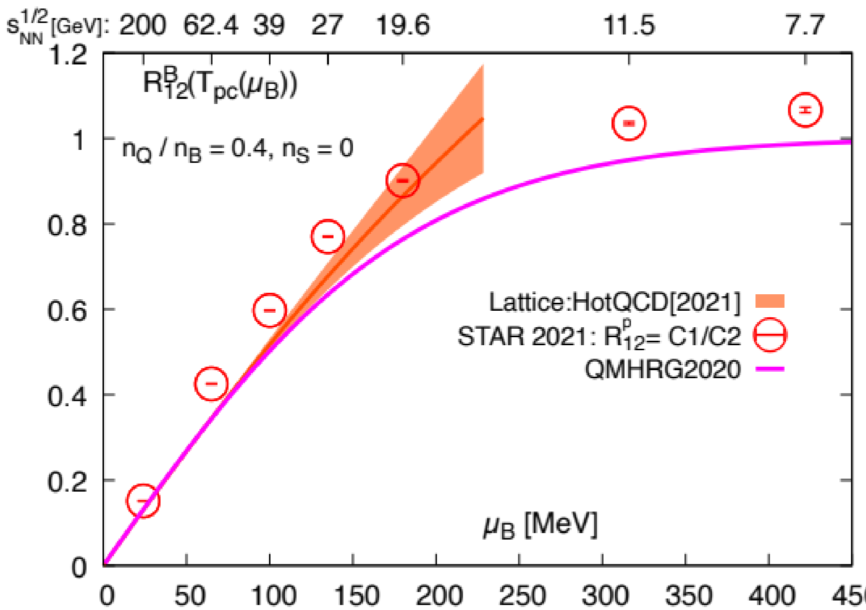
unlike χ_2^B , χ_4^B themselves, their ratio is much less sensitive to BS-correlations

Ratio of 4th and 2nd order cumulants

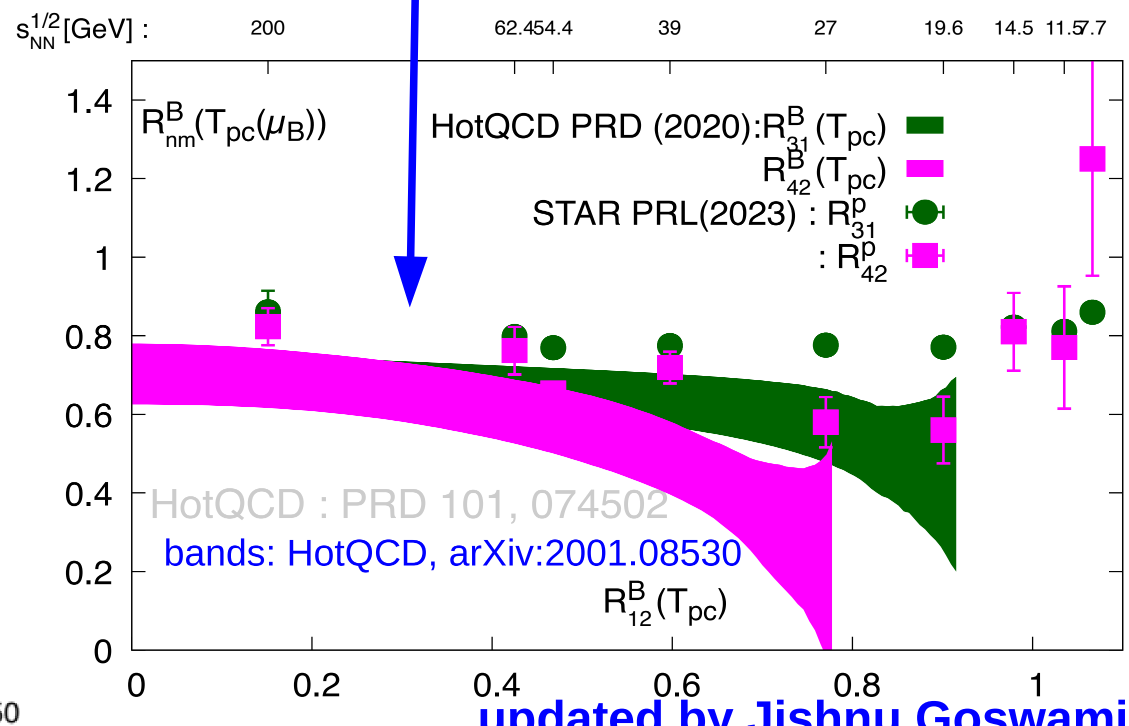


mean over variance

$$R_{12}^X = (M/\sigma^2)_X = \frac{\chi_1^X}{\chi_2^X}$$



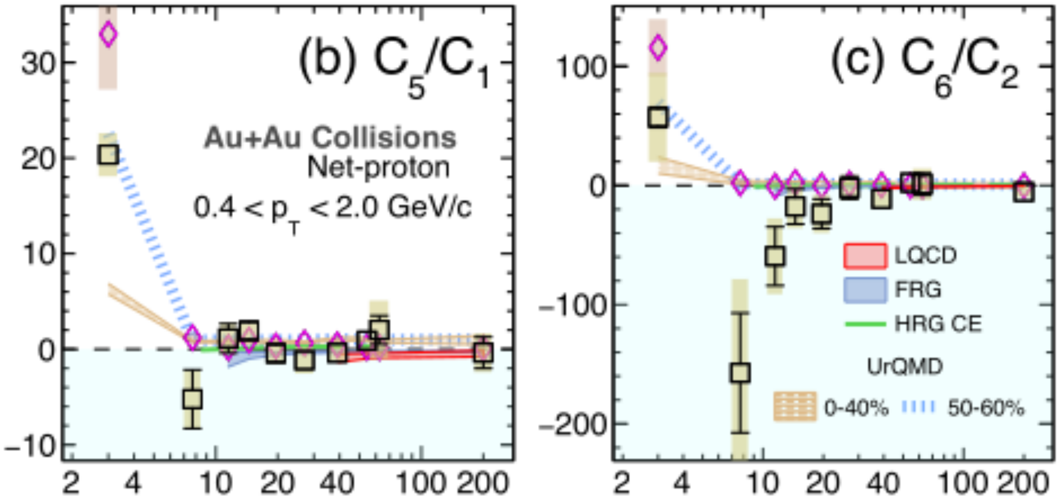
STAR PRL 2023, arXiv:2207.09837



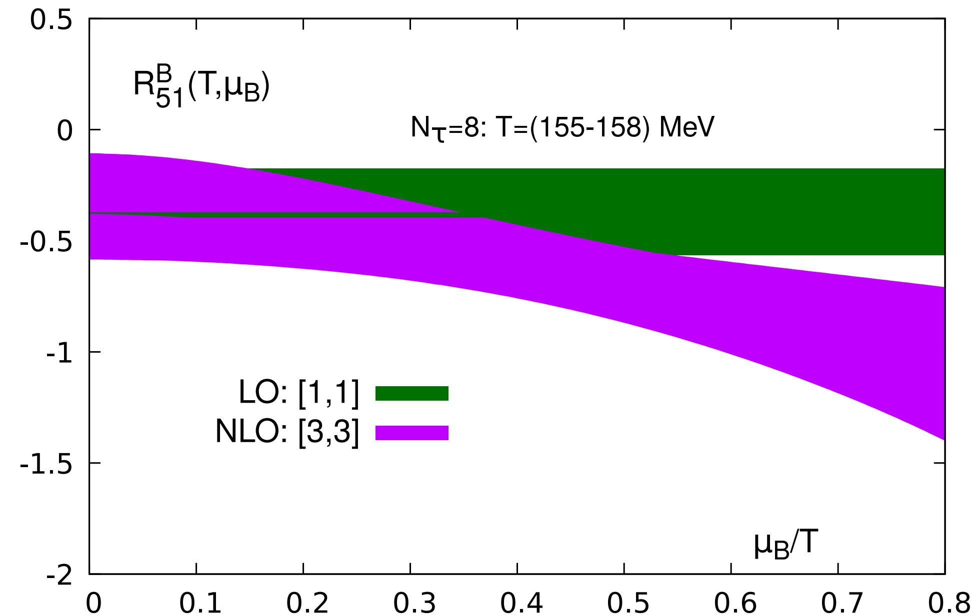
updated by Jishnu Goswami

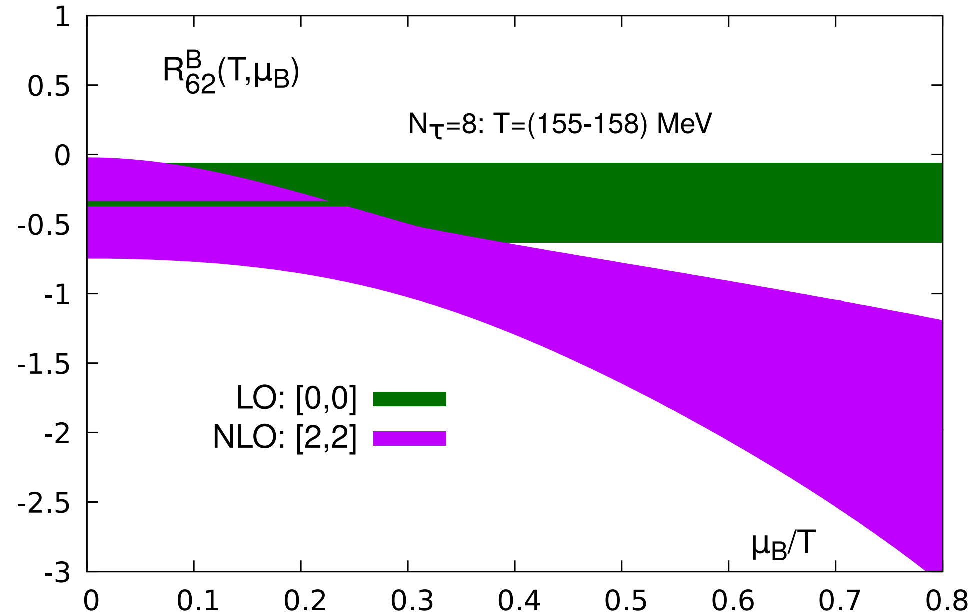
Higher order cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101, 074502 (2020), arXiv:2001.08530



STAR PRL 2023, arXiv:2207.09837





new STAR data are consistent with the expected QCD-pattern

$$R_{31} > R_{42} > R_{51} > R_{62}$$

Charm fluctuations and correlations

– calculating cumulants involving charm is difficult because

a) the large charm quark mass cumulants are strongly, exponentially suppressed

$$\chi_{1,1}^{X,C} \sim \mathrm{e}^{-M/T} \sim O(\mathrm{e}^{-10})$$

b) as a consequence the quark mass needs to be well tuned to get accurate charm hadron masses: 5% error on charmed hadron mass changes cumulants by a factor 2

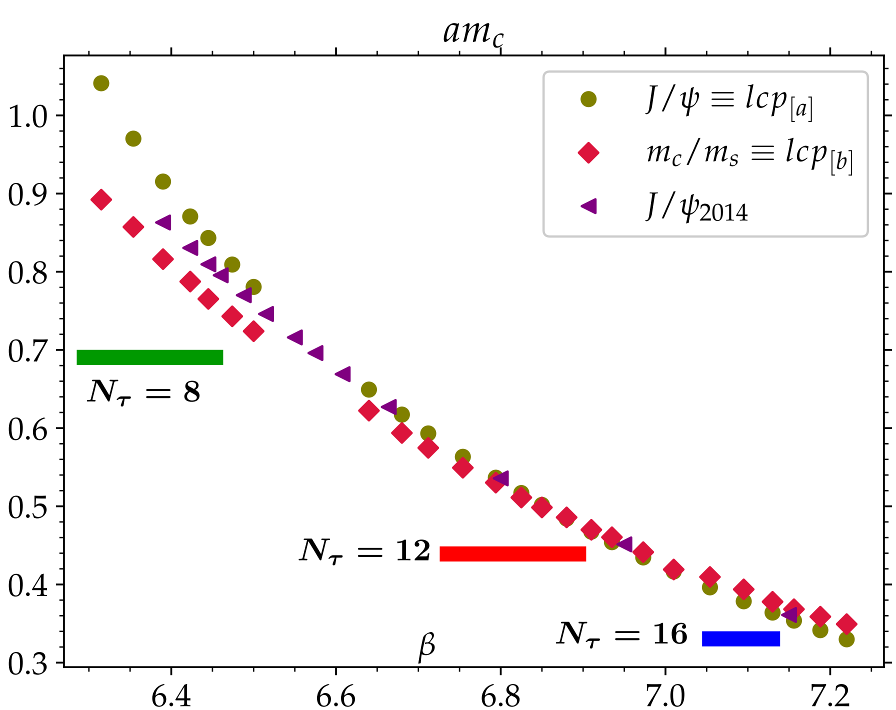
– lattice cut-off effects need to be controlled well to allow for a unique definition of a “line of constant physics”

c) charmed hadron spectrum less well known (in particular in the baryon sector): influence of missing states in the charm sector is much larger than in the light and strange quark sector

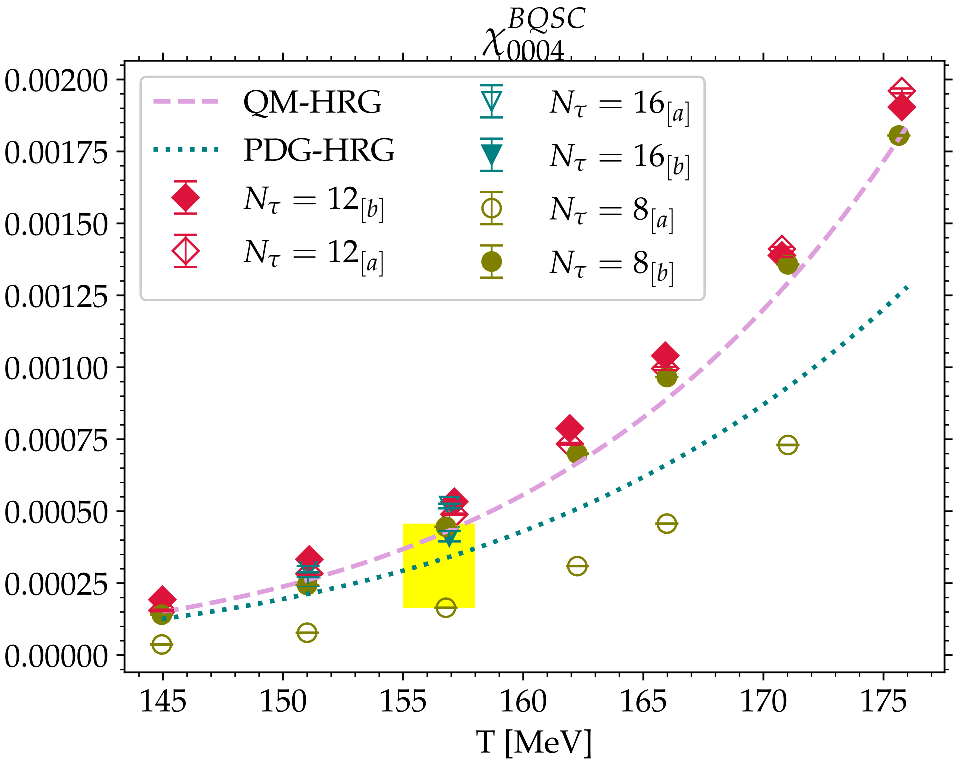
Charmed cumulants: towards a controlled continuum limit extrapolation

– tuning the charm quark mass on the lattice using different criteria introduces cut-off effects
– approaching the continuum limit on different trajectories

the choice of a trajectory is irrelevant in the continuum limit, results will be unique



$$\beta \equiv 10/g^2 \to \infty \;\Rightarrow\; a \to 0$$



all currently published data on charm cumulants are based on data from $N_\tau = 8$ lattices

A. Bazavov et al, arXiv:1404.4043

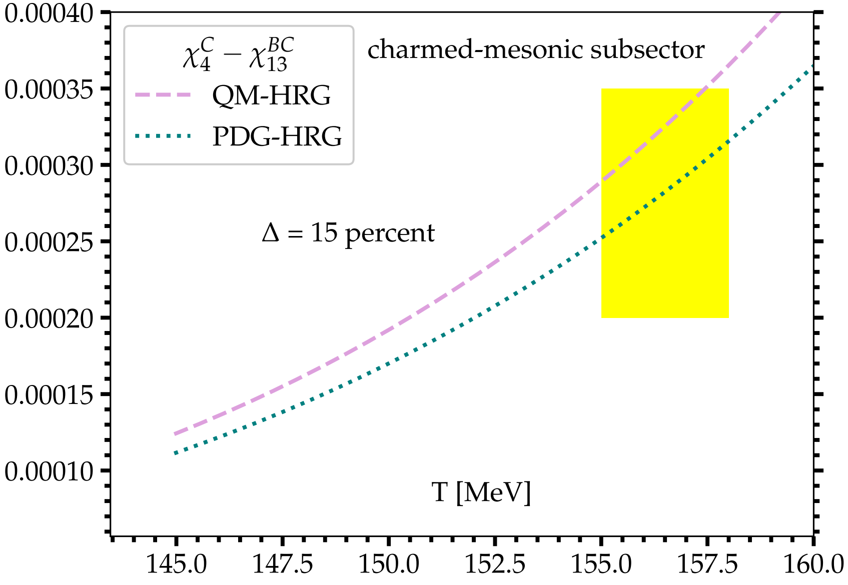
Sipaz Sharma, arXiv:2212.11148

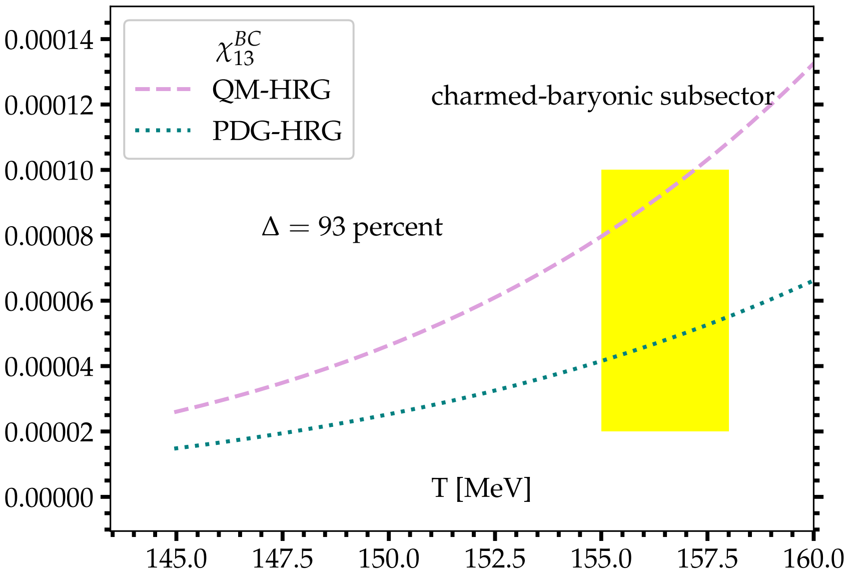
first continuum extrapolations using $N_\tau = 8, 12, 16$ data: Sipaz Sharma @ Lattice2023

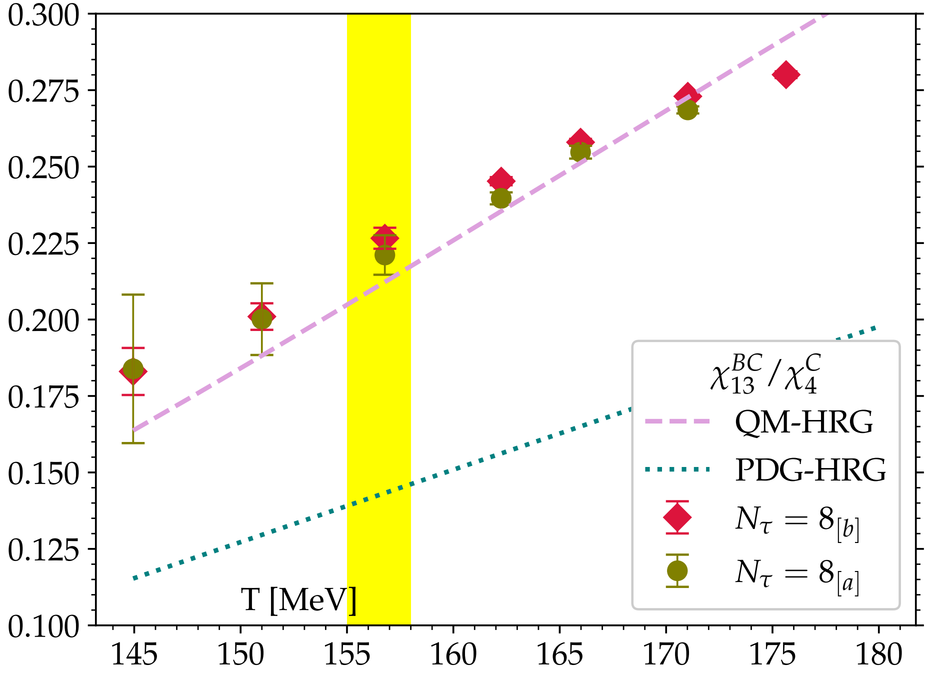
Charmed cumulants: the influence of missing charmed resonances

– **largest in baryon sector**: using Quark Model calculations amounts to a change of **almost a factor 2** compared to PDG-HRG

(compared to about 15% in the strange hadron sector)

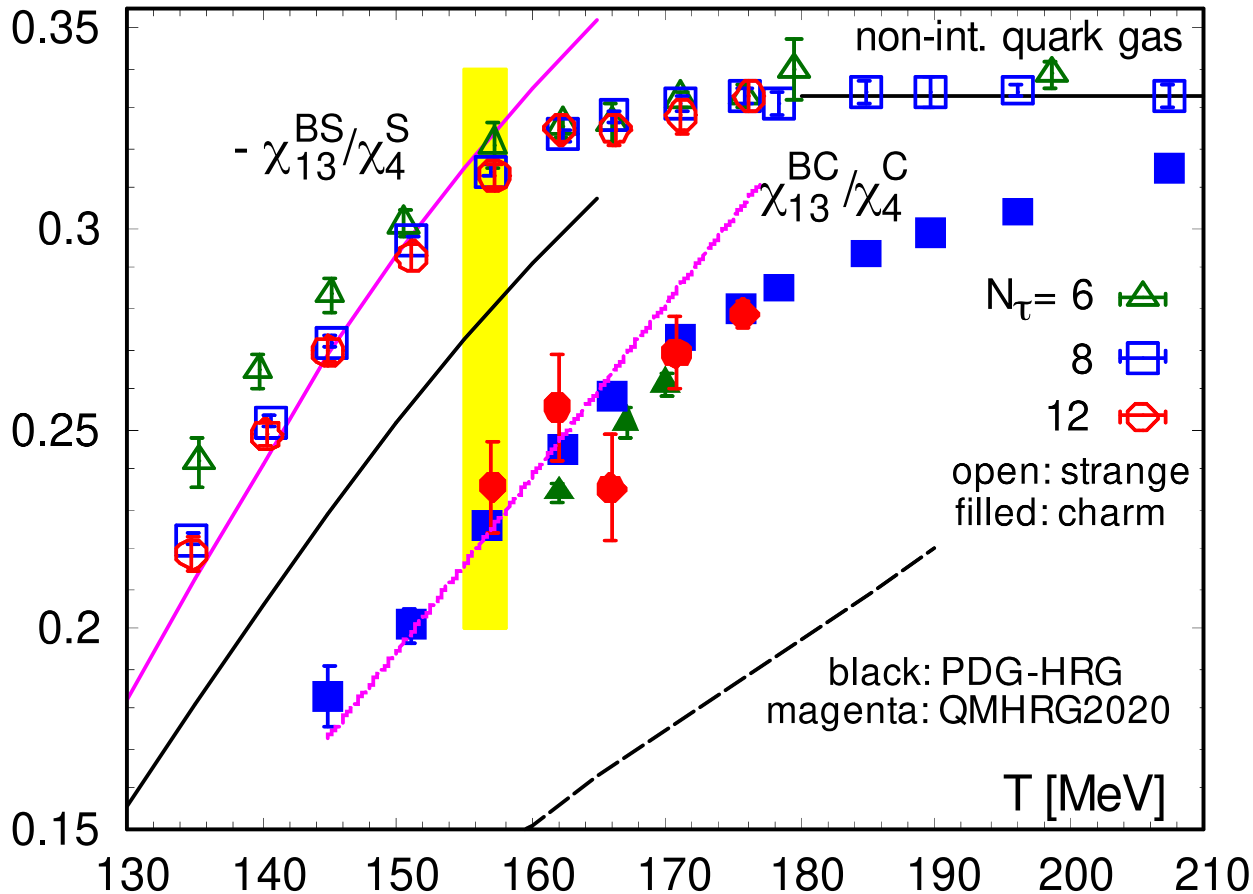






Sipaz Sharma, HotQCD preliminary

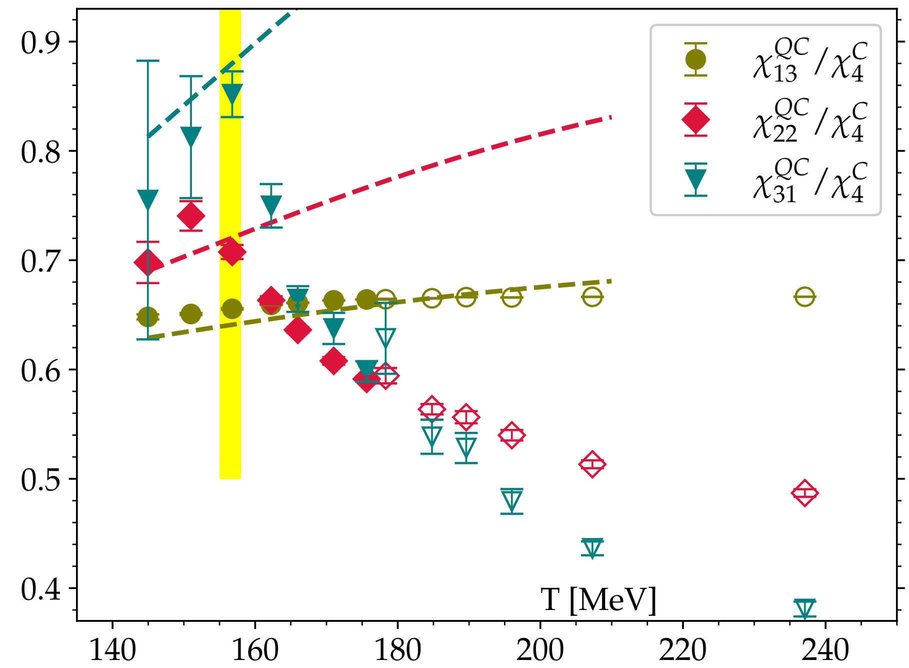
The charmed Koch-ratio



note: contributions from doubly-charmed hadrons (in HRG/Boltzmann approximation) are suppressed by a factor $\mathcal{O}(10^{-4})$

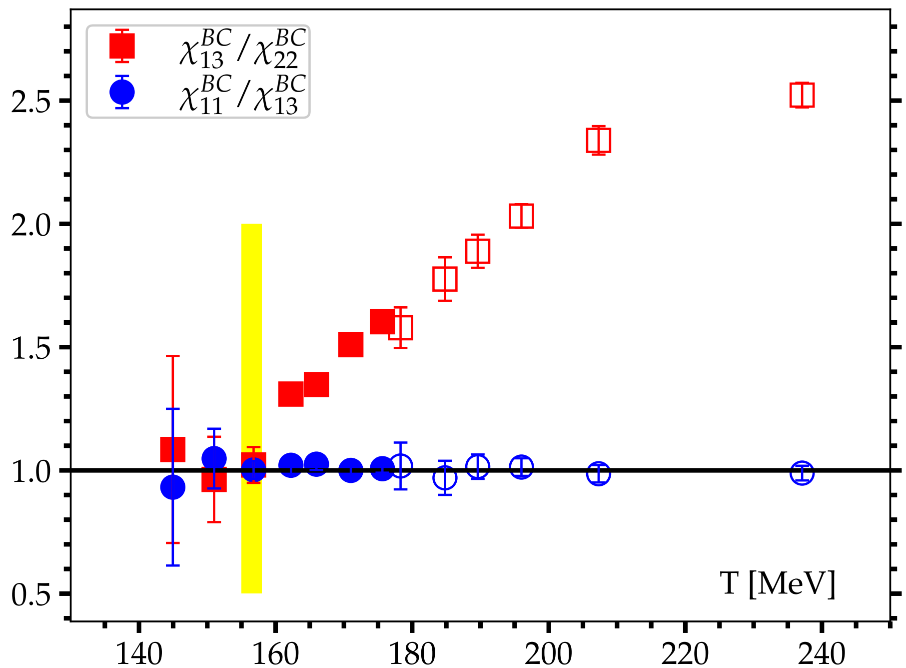
$$\frac{\chi_{13}^{BC}}{\chi_4^C} = \frac{\chi_{11}^{BC}}{\chi_2^C} + \mathcal{O}(10^{-4})$$

Sensitivity to different sectors of the charmed hadron spectrum



– contribution of multiply charged charmed baryons can be separated

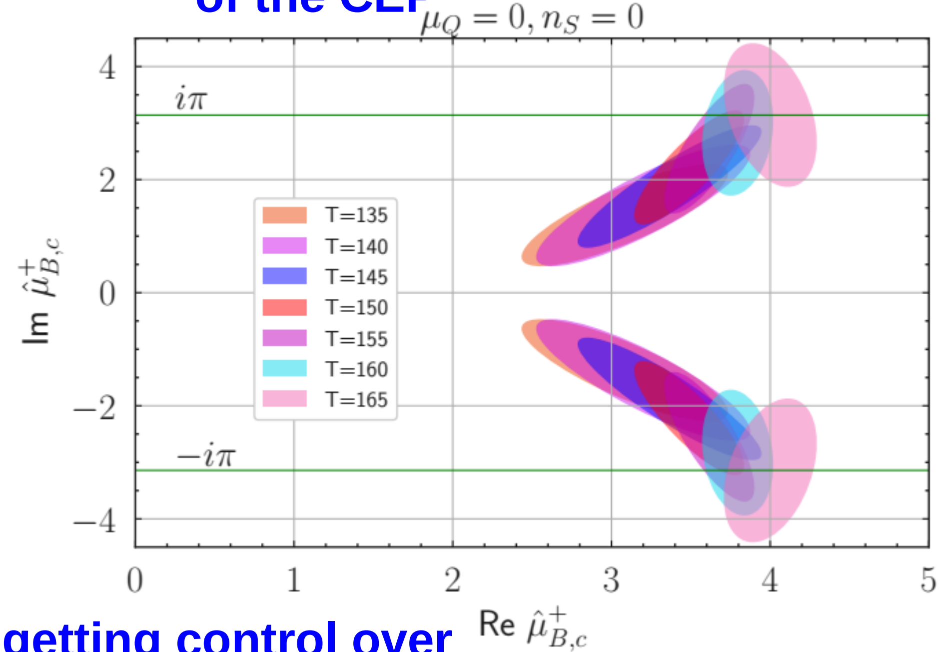
– influence of missing states is more prominent for states with |Q|=2



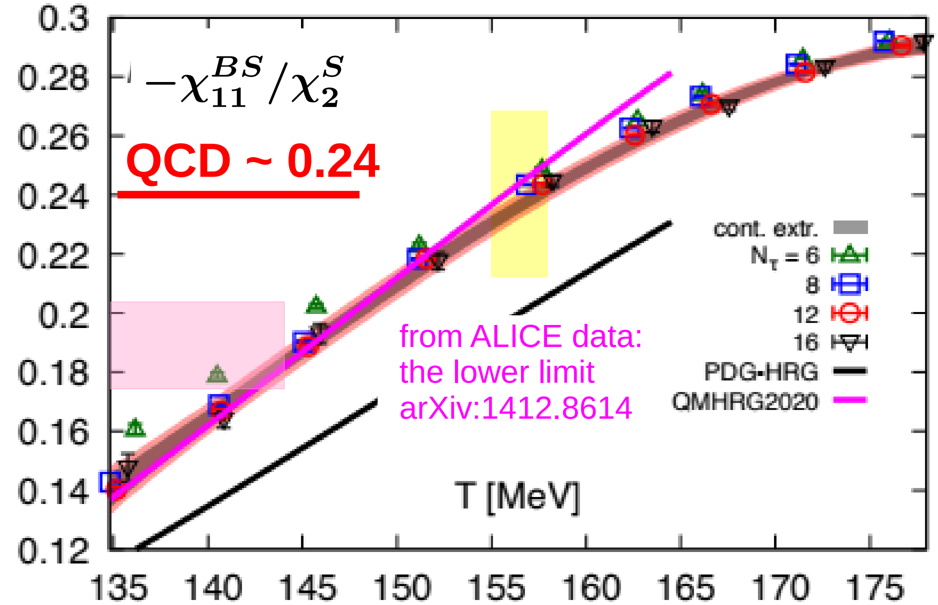
– charm quark degrees of freedom start showing up at T_{pc}

May be discussed further

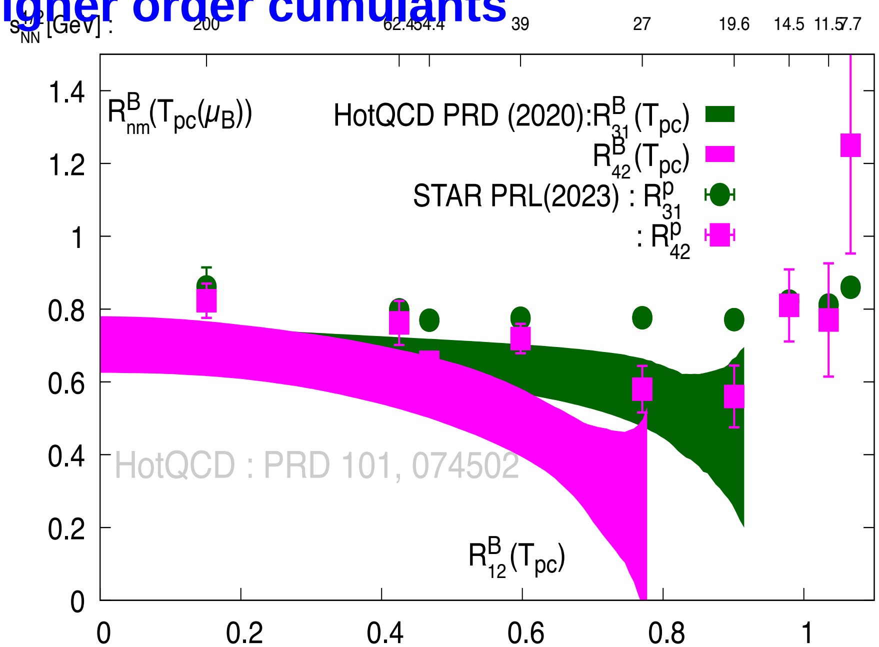
constraining the location of the CEP



the influence of BS-corr. B-fluctuations



getting control over higher order cumulants



influence of poor knowledge of charm spectrum on cumulants (and yields)