

# Hadron Structure and Spectroscopy from Lattice QCD

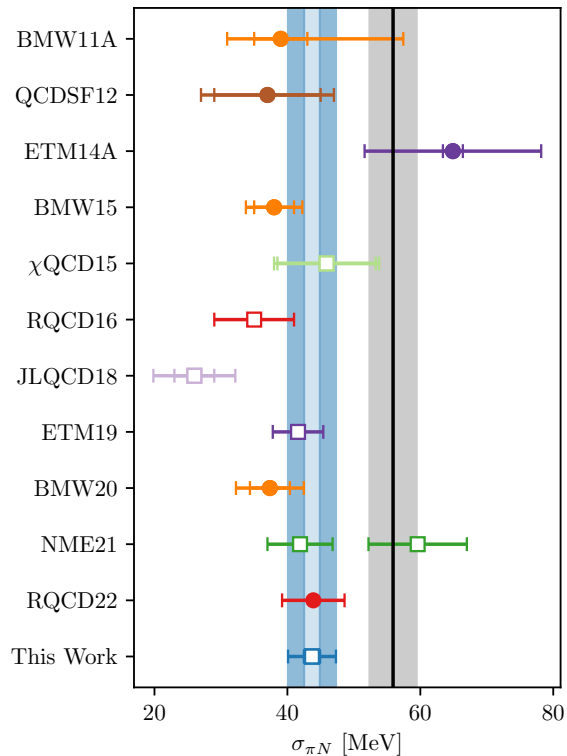
Hartmut Wittig  
CML Retreat 2023  
4–5 October 2023

# Lattice QCD as a first-principles approach

Lattice calculations increasingly competitive:

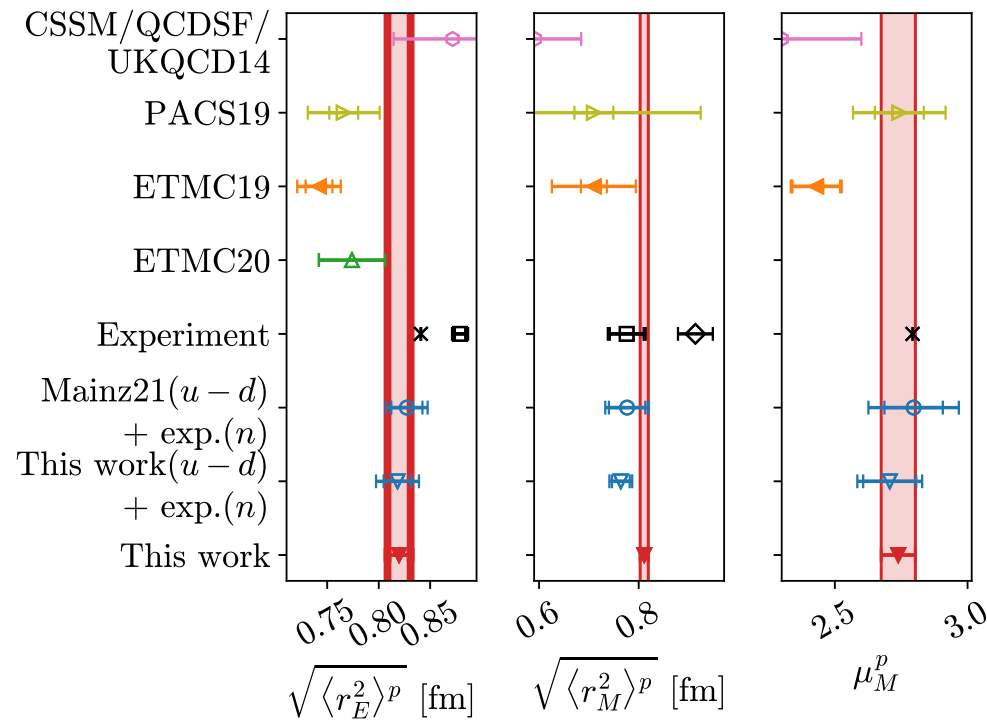
Precise enough to challenge conventional wisdom / phenomenology / experiment

### Pion-nucleon $\sigma$ -term



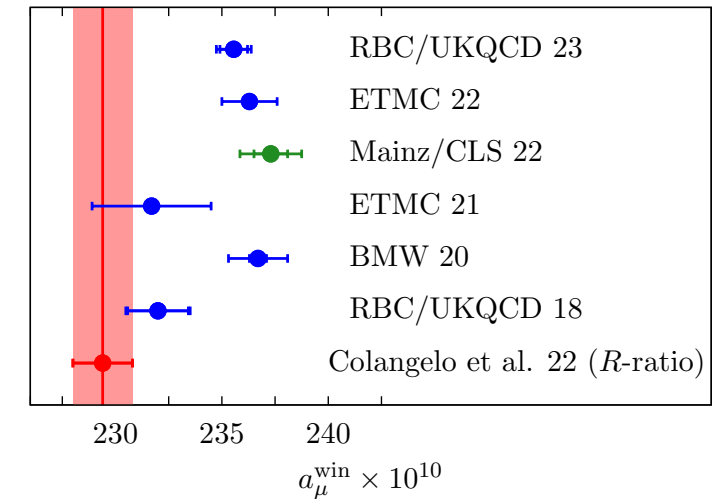
[Djukanovic et al., 2303.08741]

### Proton radii



[Djukanovic et al., 2309.6590, 2309.07491]

### Muon $g-2$



[HW @ Moriond, 2306.04165]

# Hadron spectroscopy and few-body dynamics

49 new conventional and 23 new exotic hadrons discovered at LHC, Belle,... since 2012

Finite-volume quantisation (“Lüscher method”): rigorous formalism amenable to Lattice QCD

$$\det\left(\tilde{\mathcal{K}}^{-1}(E_L) - B(E_L, L)\right) = 0$$

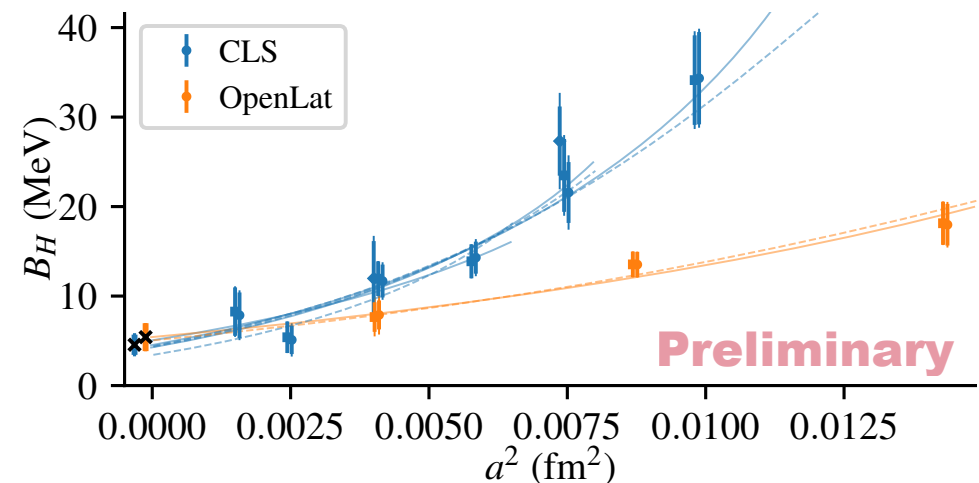
$\tilde{\mathcal{K}}(E_L)$  :  $2 \rightarrow 2$  scattering amplitude

$B(E_L, L)$  : analytically known function

$E_L$  : multi-particle energy levels in finite volume

## Topics / Examples:

- $H$  dibaryon — hyperon-hyperon interactions
- $NN$  scattering and nucleon resonances
- Resonances in charm sector; tetraquarks:  $T_{cc}^+$
- Interpretation of the  $\Lambda(1405)$
- ....

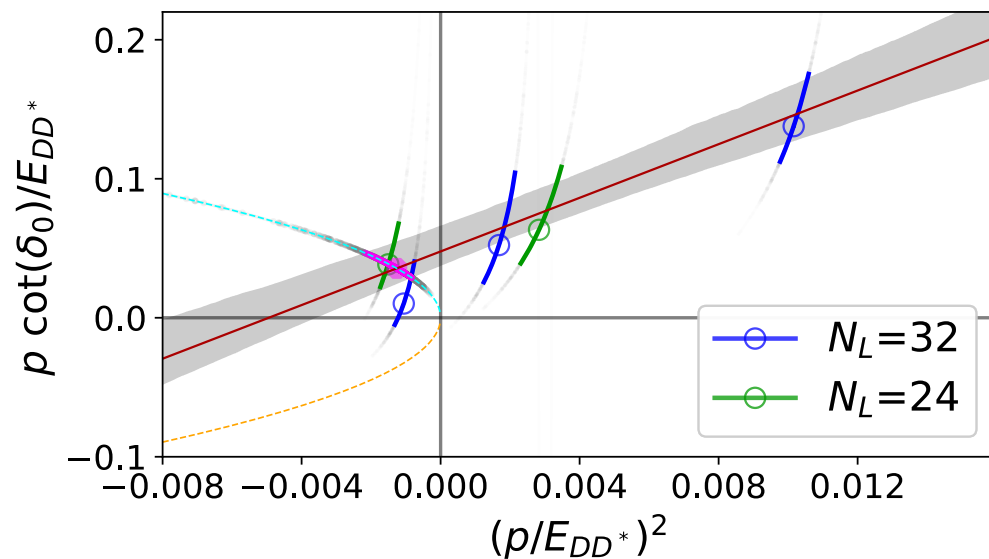


[Green et al., PRL 127 (2021) 242003, and in prep.]

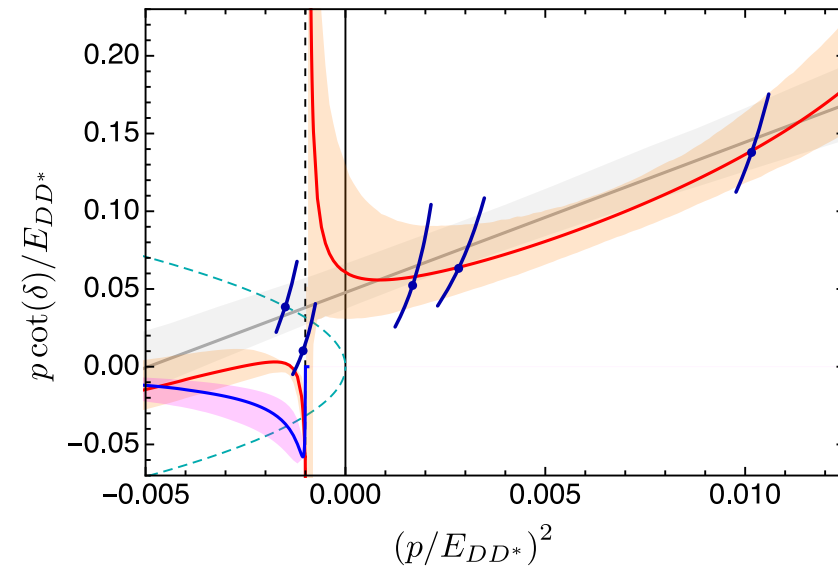
# Hadron spectroscopy and few-body dynamics

## Current challenges:

- Role of the continuum limit — extrapolation to the physical point
- Perform coupled-channel analyses, incorporate higher partial waves
- Two-particle finite-volume quantisation fails above 3- and 4-particle thresholds and below left-hand cut —> employ and implement 3-particle quantisation conditions



[Padmanath & Prelovšek, PRL 129 (2022) 032002]



[Du et al., PRL 131 (2023) 131903]

# Hadron spectroscopy and few-body dynamics

## Future plans:

- Comprehensive programme on baryon-baryon, meson-baryon, meson-meson scattering:
- $H$  dibaryon, nucleon-nucleon channels — approach to the physical point:  $m_\pi = m_\pi^{\text{phys}}$ ,  $a \rightarrow 0$
- Charmed tetraquarks
- Investigation of left-hand cut — precursor to studying 3-particle quantisation condition

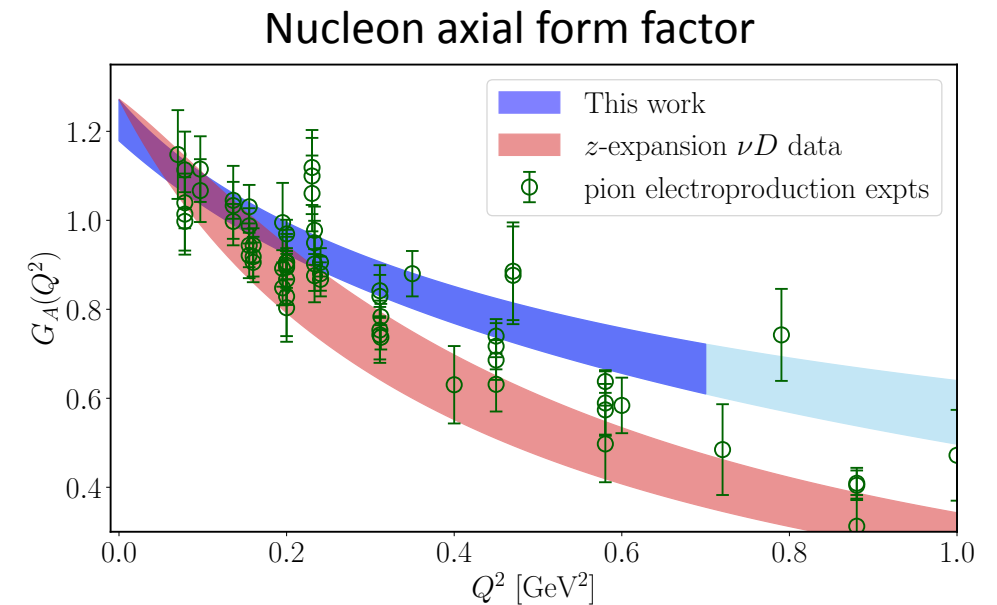
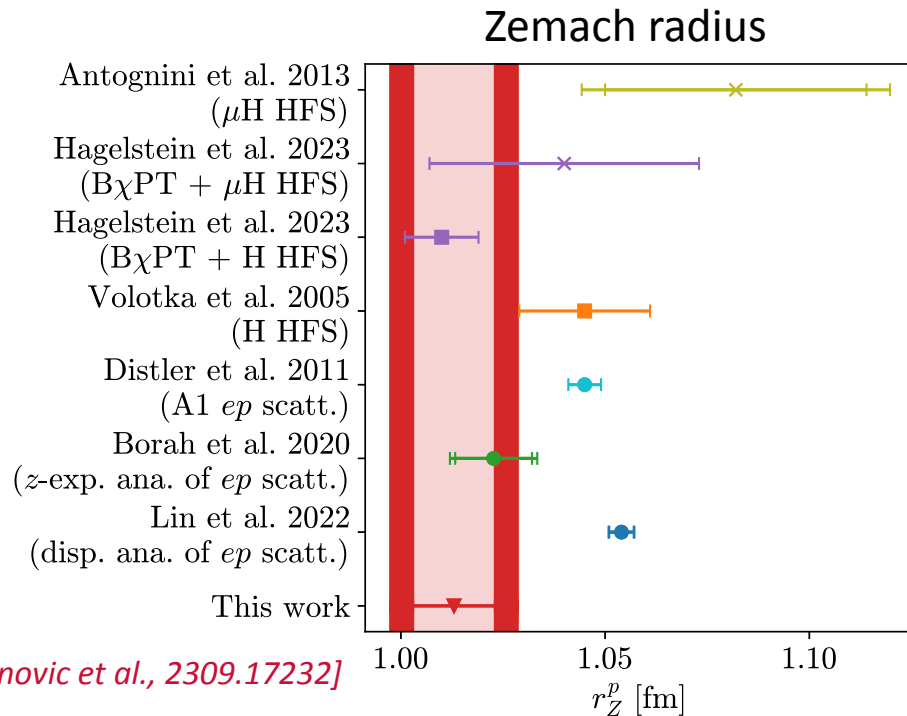
## Collaborators:

- DESY-Zeuthen (Jeremy Green); GSI, TU Darmstadt (Daniel Mohler)
- BaSc Collaboration [“Baryon Scatterers”] (John Bulava, Colin Morningstar, André Walker-Loud)

# Hadron structure observables

## Examples for first-principles determinations of nucleon hadronic matrix elements:

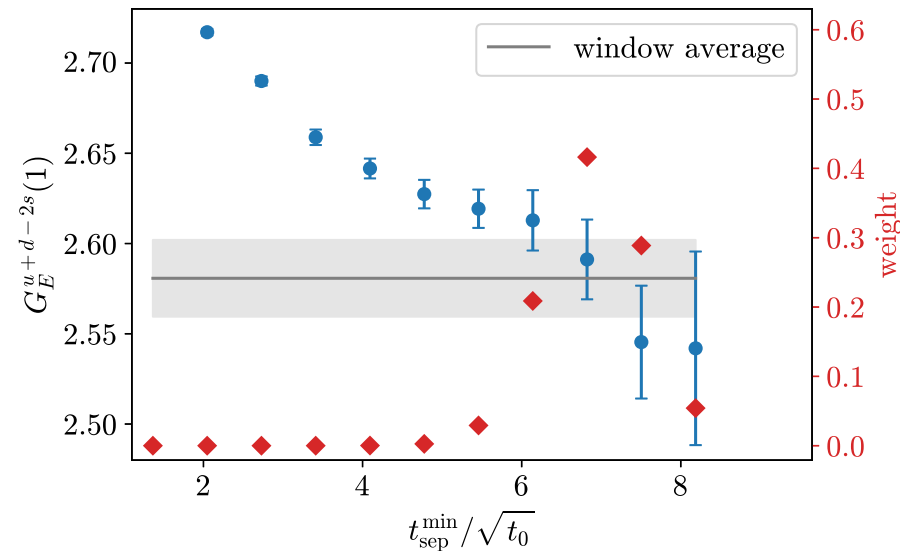
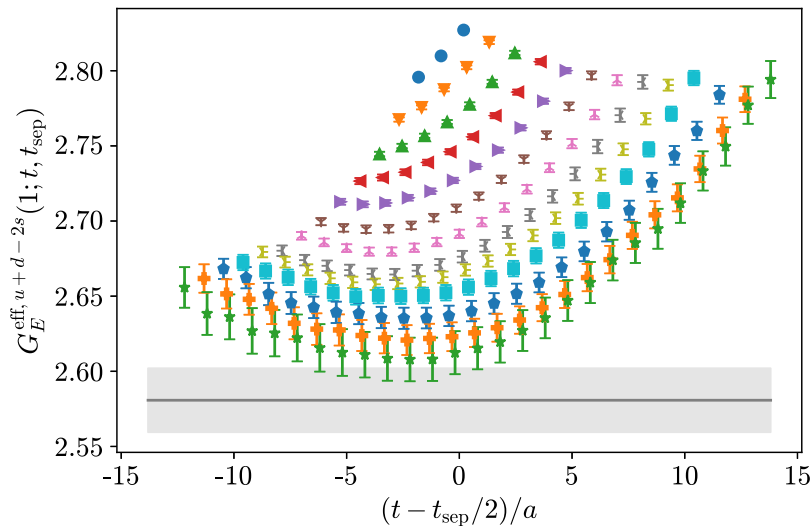
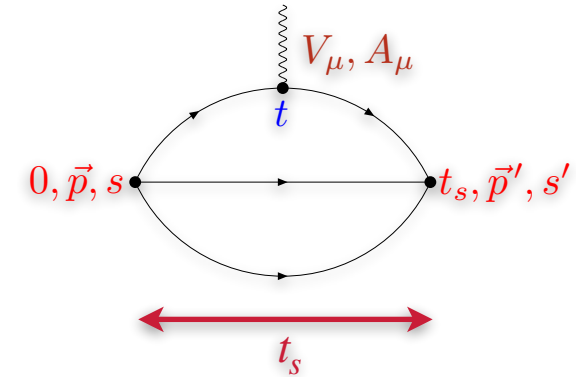
- Axial form factor of the nucleon — input for future neutrino experiments: DUNE, T2HK
- Electric, magnetic and Zemach radii of the proton — final resolution of proton radius puzzle
- PDFs and GPDs — input for EIC



# Hadron structure observables

## Current challenges:

- Noise problem: exponential growth of signal-to-noise ratio in baryonic correlators
- Related problem: unsuppressed contributions from excited states:  $N\pi$ ,  $N\pi\pi$
- GPDs, PDFs: complicated renormalisation of bilocal operators; inverse problem
- Incorporation of isospin-breaking corrections

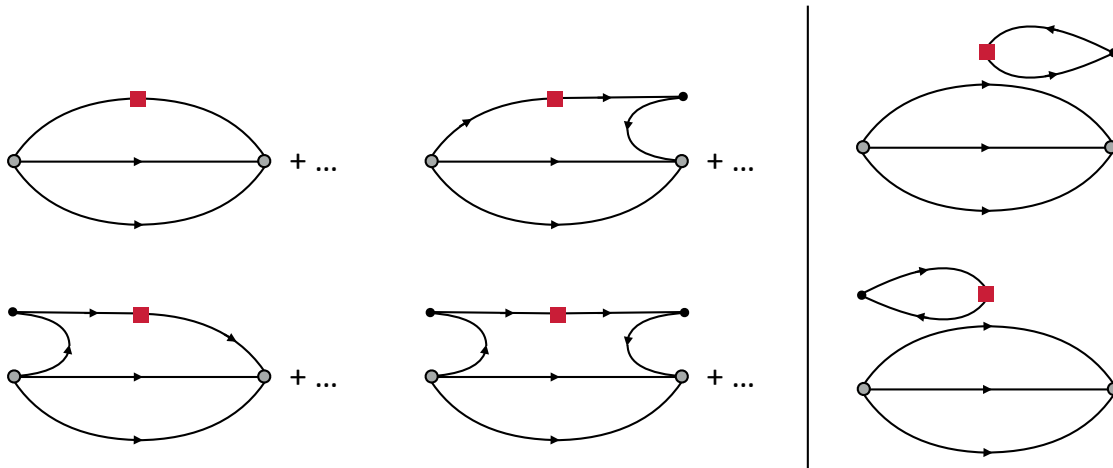


# Hadron structure observables

## Future plans:

- Comprehensive programme to study electromagnetic, axial and strangeness form factors
- Determine nucleon charges ( $g_A, g_S, g_T$ ),  $\sigma$ -terms, and moments of structure functions
- Novel noise-reduction technology: multi-particle interpolating operators, machine-learning techniques

Nucleon 3-point function with explicit  $N\pi$  interpolators:



Cost-efficient observables via trained networks:

$$\langle O \rangle = \langle O \rangle_{\text{approx}} + \langle (O - O_{\text{approx}}) \rangle$$

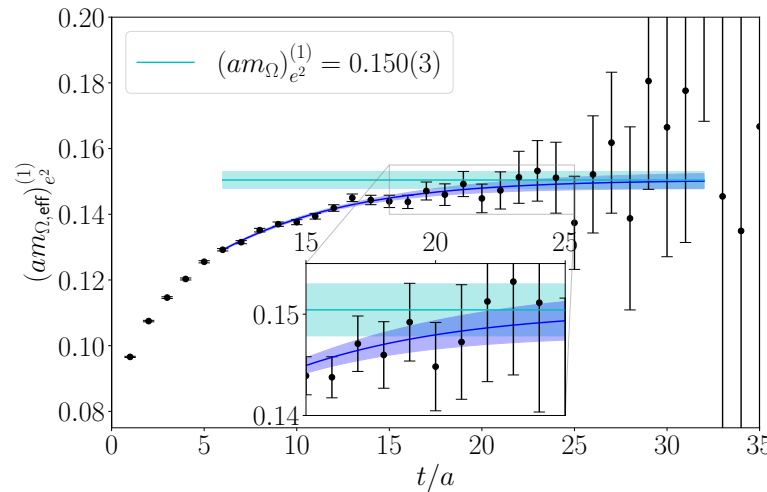
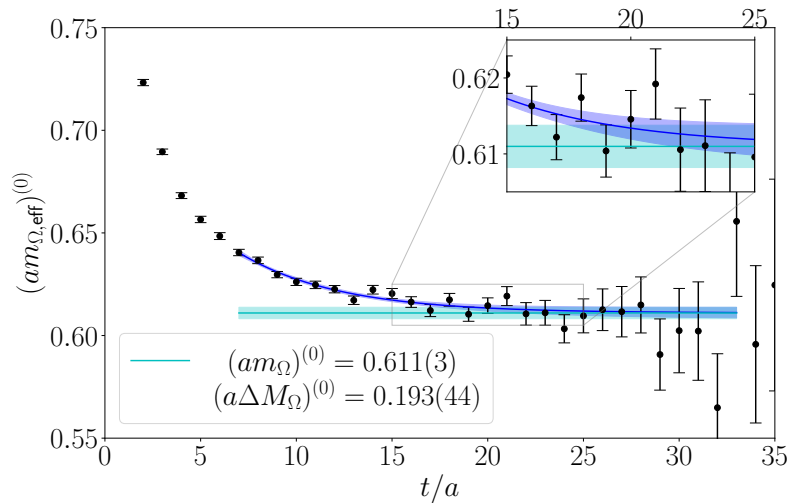
↑                    ↑                    ↑  
exact            "cheap"            correction



# Ancillary calculations

## Precision scale setting

- Lattice scale from lowest-lying octet and decuplet baryon masses including isospin-breaking effects



Precision of  $\lesssim 0.5\%$  in  $\Omega$ -baryon mass;  
Isospin-breaking effects smaller than  
statistical error

[Segner, Hanlon, Risch, HW, arXiv:2212.07176]

## Pion-pion scattering and pion form factor

- Constrain long-distance behaviour of correlator for precision observables (e.g.  $(g-2)_\mu$ ,  $\Delta\alpha_{\text{had}}$ )

## Pion mass-dependence of octet baryons

- Alternative determination of  $\sigma$ -terms via Feynman-Hellmann theorem

# QCD Thermodynamics: Photon emissivity of QGP

Differential photon emission rate per unit volume in hot QCD matter:

$$\frac{d\Gamma^\gamma}{d\omega} = \frac{\alpha_{\text{e.m.}}}{\pi} \frac{2\omega \sigma(\omega)}{e^{\omega/T} - 1} + \mathcal{O}(\alpha_{\text{e.m.}}^2)$$

$\sigma(\omega)$  : in-medium spectral function;

$\omega$  : photon energy

Perform first-principles determination of properties of the thermal medium

Can determine  $\sigma(\omega)$  via dispersion relation without numerically ill-posed inverse problem

*[Cè, Harris, Krasniqi, Meyer, Török, 2309.09884]*

# Summary & PoF V Outlook

Rich research programme in hadron structure & spectroscopy, and QCD thermodynamics

State-of-the-art calculations

Innovative methodology to turn Lattice QCD into a precision tool

## Crucial requirement: High-performance computing

Dedicated resources of  $\approx 500$  Mcore-hours p.a. required

Major new investment at HIM: **HIMSTER-3** — **1.88 M€**

Shared with experimentalists in EMP, SPECIF

Procurement planned in Q4/2024 — Q1/2025

Will need replacement during latter half of PoF V

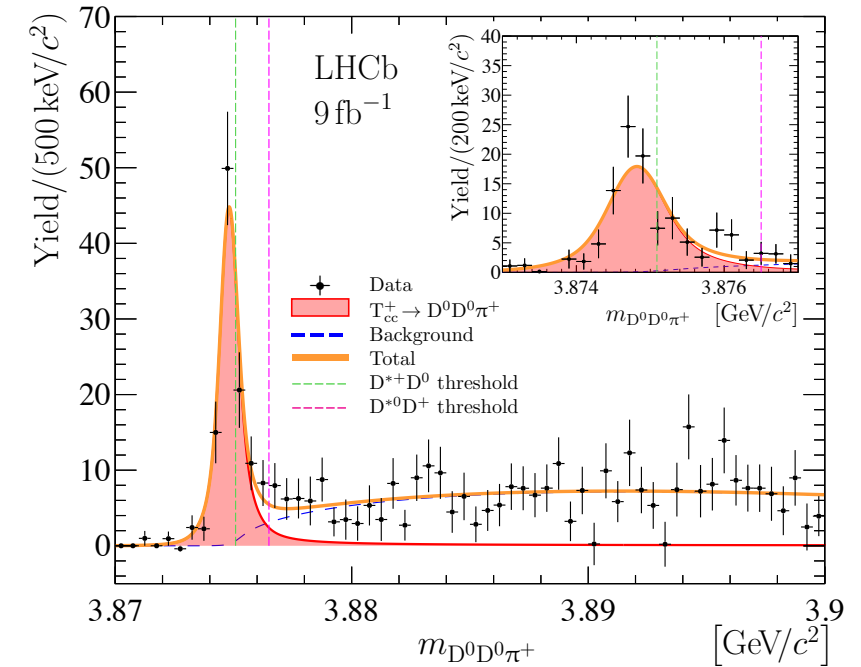
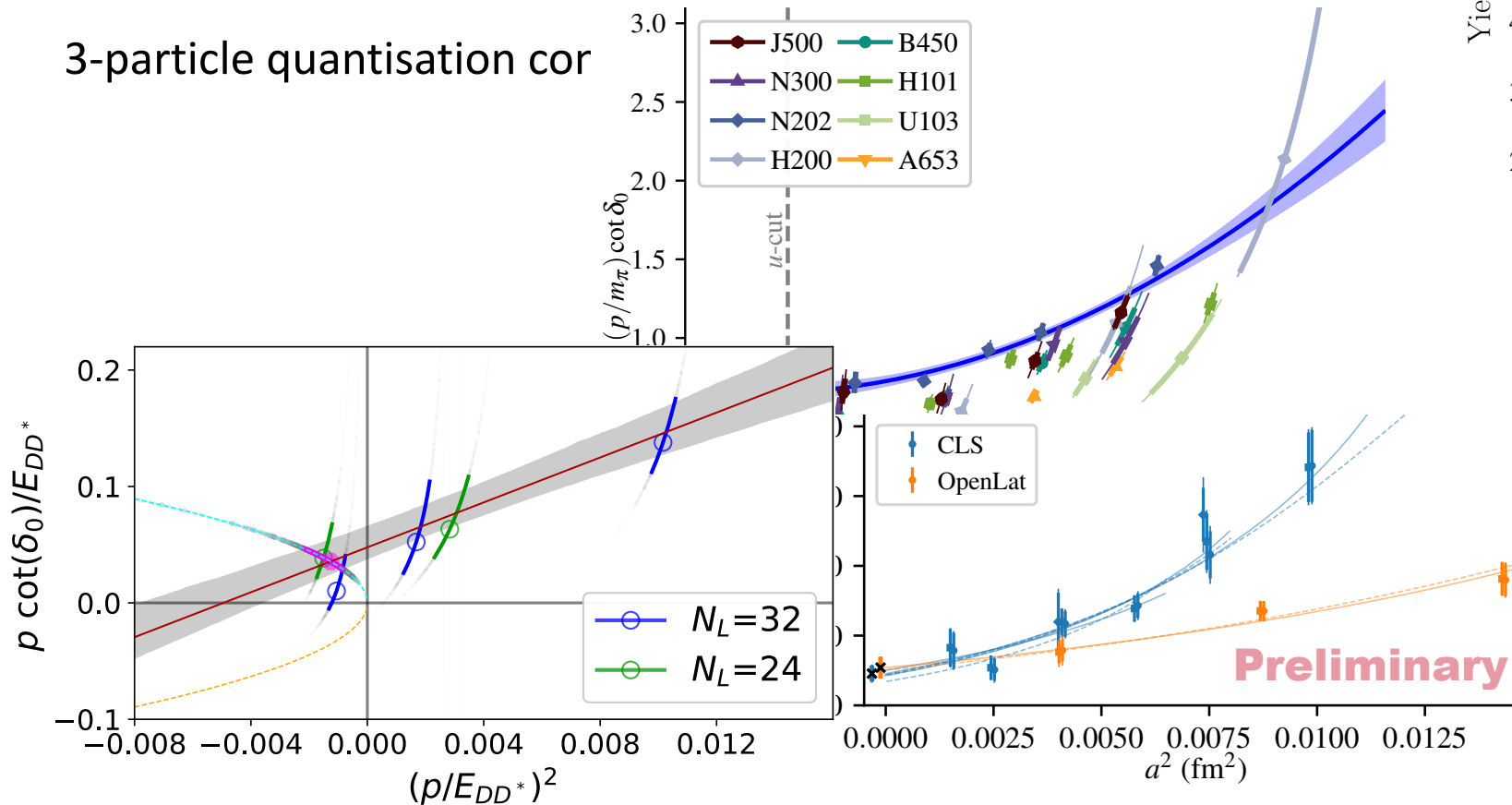
Thank you!

# Hadron spectroscopy and few-body dynamics

Issues:

Role of the continuum limit

3-particle quantisation cor



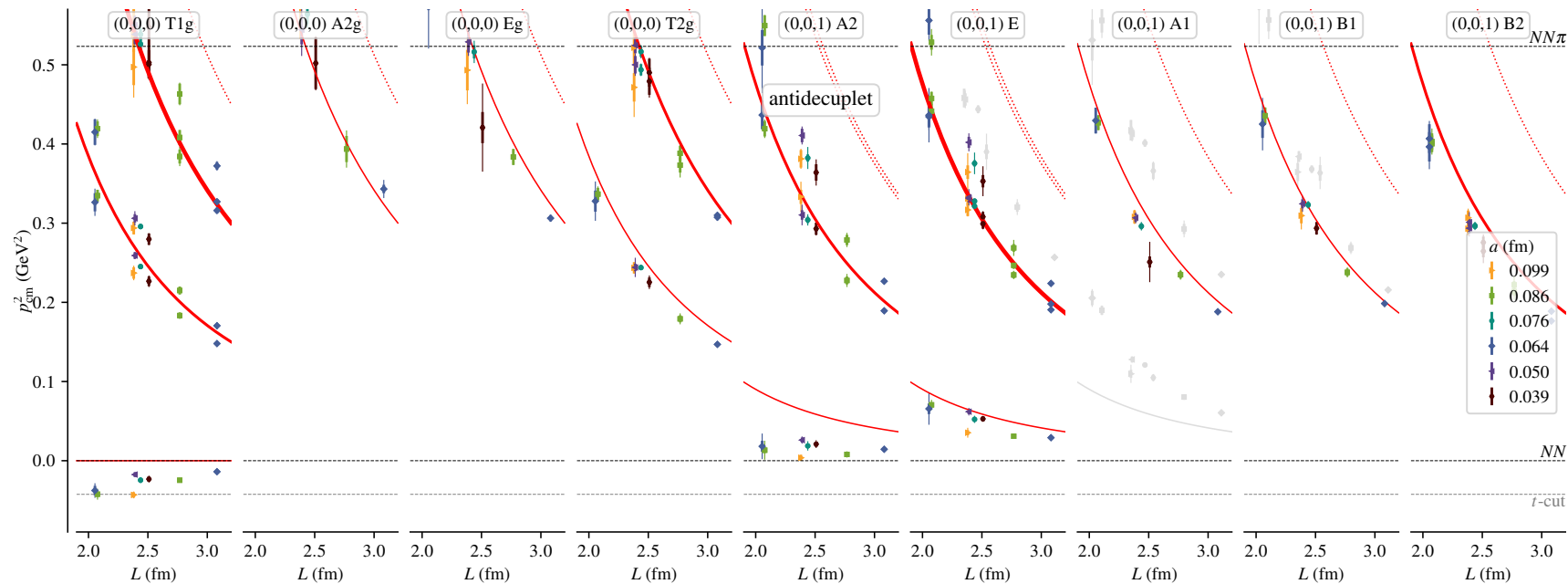
[LHCb, Nature Phys. 18 (2022) 751]

# Hadron spectroscopy: Nucleon-nucleon interactions

[Green, Hanlon, Junnarkar, HW, arXiv:2212.09587]

Disagreement over existence of  $NN$ -bound states at heavier-than-physical pion mass

Perform scaling study for  $NN$ -states at  $m_\pi = m_K \simeq 420 \text{ MeV}$  —  $\overline{10}$  (deuteron) and 27plet (dineutron) Antidecuplet,  $S = 1, I = 0$  (deuteron):



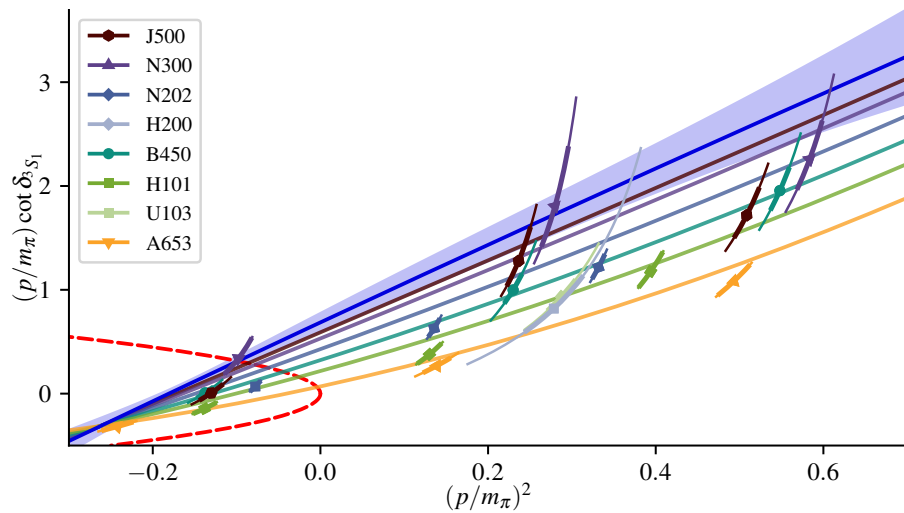
- 300 different energy levels resolved — sensitivity to higher partial waves

# Hadron spectroscopy: Nucleon-nucleon interactions

[Green, Hanlon, Junnarkar, HW, arXiv:2212.09587]

Antidecuplet,  $I = 0$ ,  ${}^3S_1$  phase shift analysis

(mixing with  ${}^3D_1$  neglected)



- $NN$ -interaction weakens as  $a \rightarrow 0$
- Virtual bound state observed
- No bound deuteron at  $m_\pi = m_K \simeq 420 \text{ MeV}$

Include higher partial waves in the fits

