ML & RG — A Lattice Field Theory Application

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STRUCTURES CLUSTER OF EXCELLENCE

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ML & RG

Connection (machine) learning ⟷ renormalisation group?

Many mainly conceptional investigations recently, e.g.:

- Bayesian renormalisation group [Berman, Heckmann, Klinger, 2204.12939] [Berman, Heckmann, Klinger, 2212.11379] [Berman, Heckmann, Klinger, 2305.10491]
- Inverse renormalisation group, ML & QFT [Bachtis, Aarts, Lucini, 2010.00054] [Bachtis, Aarts, Lucini, 2102.09449] [Bachtis, Aarts, Di Renzo, Lucini, 2107.00466]
- Large width limit of NNs, (non-)Gaussian processes and RG [Yaida, 1910.00019]
- Renormalisation group flows as optimal transport [Cotler, Rezchikov, 2202.11737] [Cotler, Rezchikov, 2308.12355]

RG and block spinning

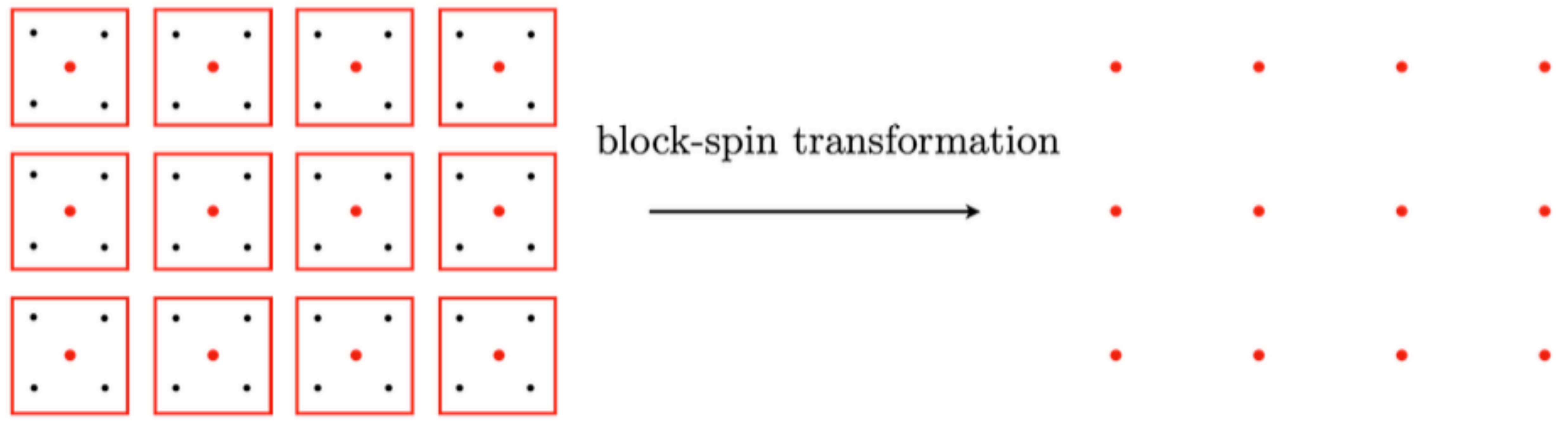
- **Renormalisation group**: Change of a system under scale transformations

- Wilsonian RG: **effective description** by integrating out modes above certain momentum scale

$$Z = \int \mathscr{D}\phi\, e^{-S[\phi]} \longrightarrow \int \left[\mathscr{D}\phi\right]_{k\leq\Lambda} e^{-S_{\text{eff}}[\phi]}$$

- Block spinning RG

[Kadanoff, 1965]



Bayesian renormalisation

Bayesian inference: change of distribution under incorporation of new information

$$p(\theta|X) = \frac{p(X|\theta)}{p(X)} p(\theta)$$

θ : parametrising distribution
X : new information

Bayesian renormalisation: inverse to inference, coarse graining of model parameters

[Berman, Heckmann, Klinger, 2204.12939]
[Berman, Heckmann, Klinger, 2212.11379]
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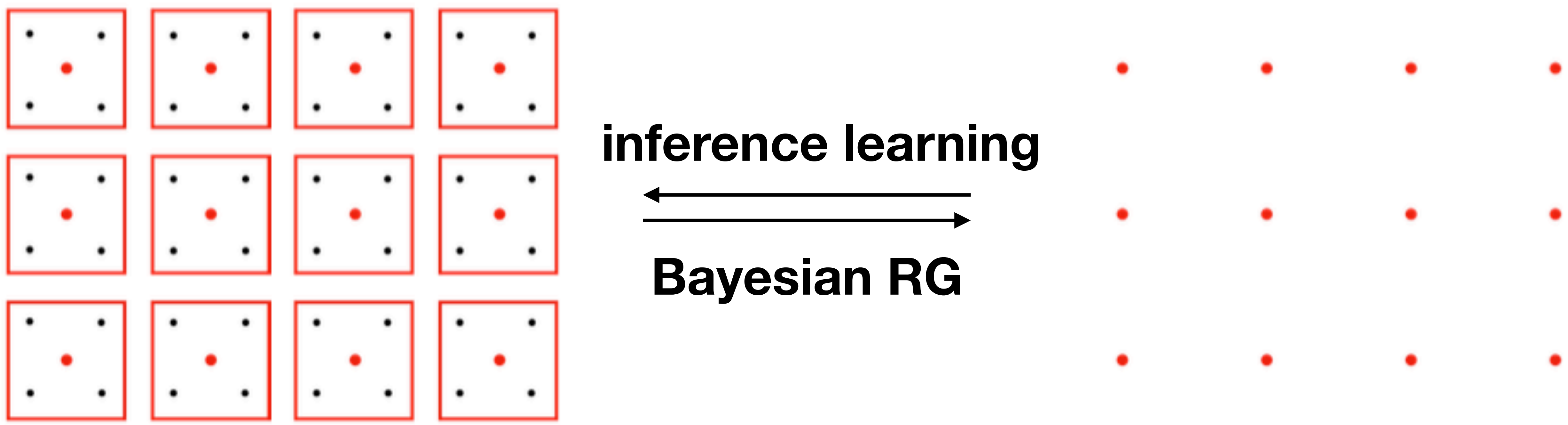
- Exact RG flow with $p(\theta) = e^{-S_\theta[\phi]}/Z$
- RG scale: Fisher metric



Neural block spinning

General, non-linear block spinning transformations?

Universal Approximation Theorem: Neural networks (NN) can represent any continuous function

⇝ represent block spinning transformations by NN

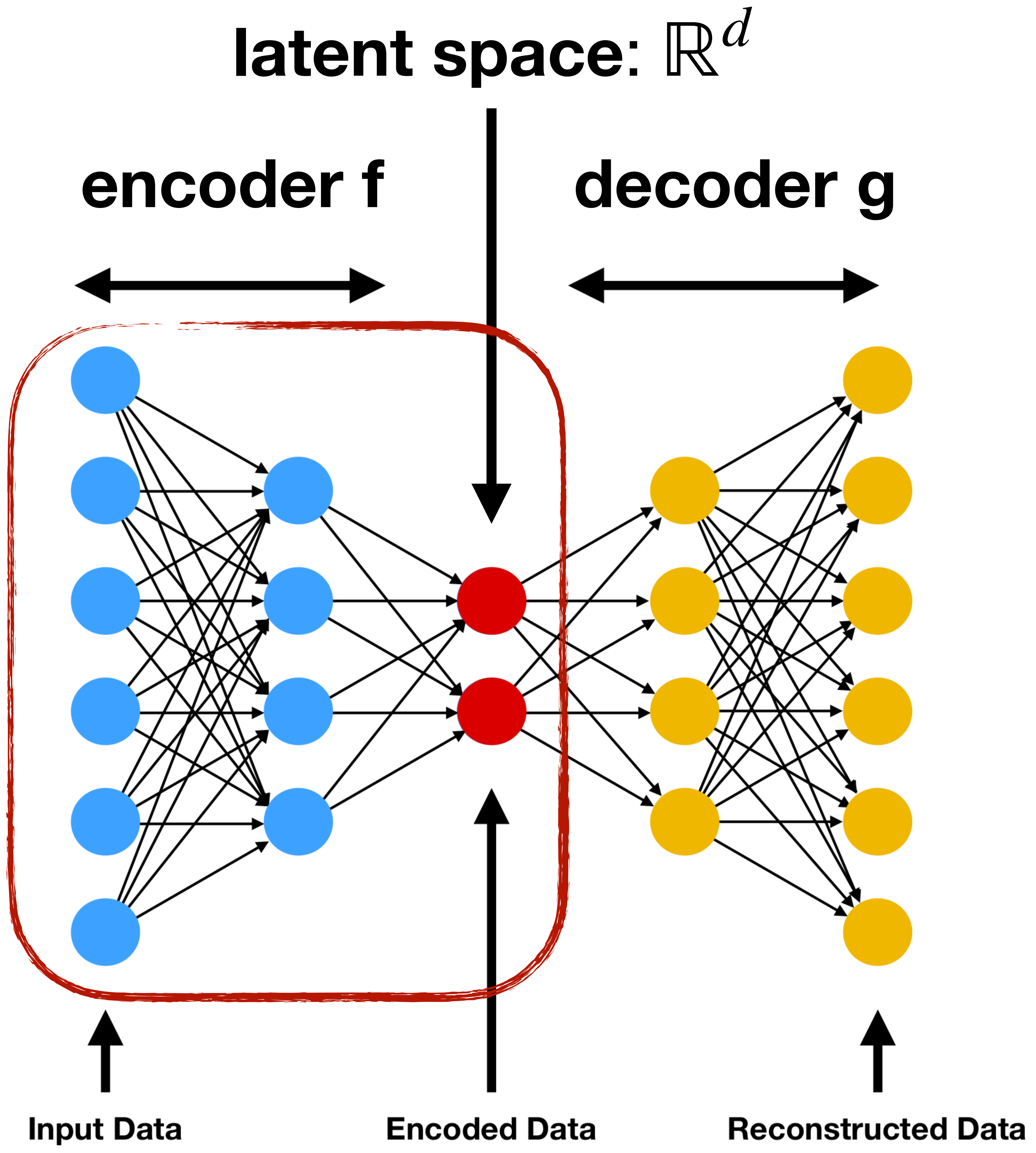
Optimise NN params by minimising reconstruction loss ⇝ **'ideal block spinning'**

Autoencoders

- **Autoencoder** $A = g \circ f : \mathbb{R}^D \to \mathbb{R}^D, x \mapsto \hat{x}$
  - Encoder $f : \mathbb{R}^D \to \mathbb{R}^d, x \mapsto y$
  - Decoder $g : \mathbb{R}^d \to \mathbb{R}^D, y \mapsto \hat{x}$

- **RG step** ⇝ What is (not) learned?

→ Analyse **latent space**



[https://www.compthree.com/blog/autoencoder/]

Latent space analysis with Jacobians

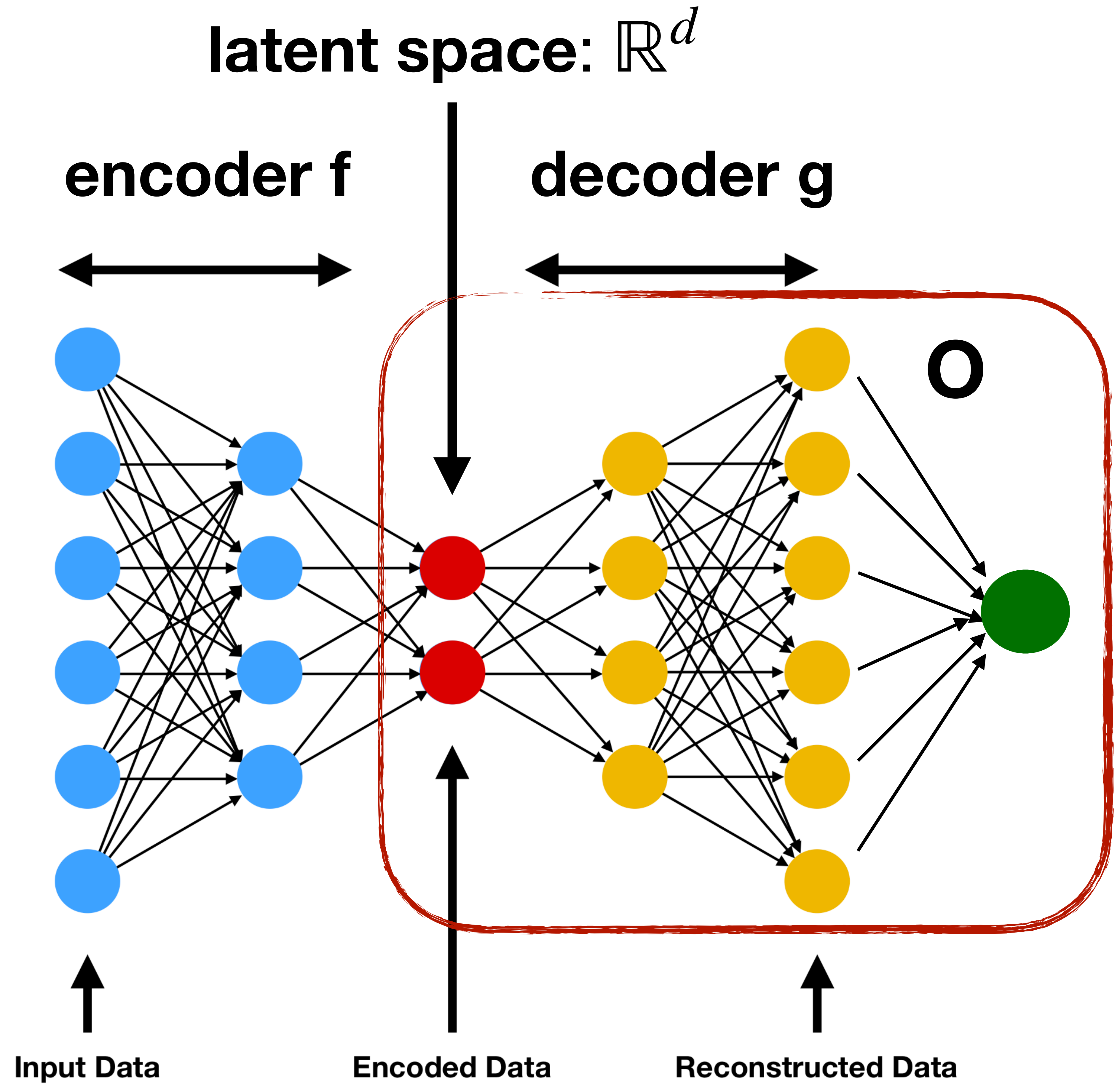
- Consider **Jacobians**, e.g. $(\partial g)_{ij} := \frac{\partial g_i}{\partial y_j}$

- Compose decoder g with **observable** $O : \mathbb{R}^D \to \mathbb{R}$

$$h_O := O \circ g$$

→ Jacobian ∂h_O encodes correlation of observable with latent space

- Rotate latent space into **'eigenbasis'** (singular value decomposition)



[https://www.compthree.com/blog/autoencoder/]

ϕ^4-theory on the lattice

- Scalar lattice ϕ^4-theory in d=2: $$S = \sum_x \left\{ -2\kappa \sum_{\mu=1}^{2} \phi(x)\phi(x+\hat{\mu}) + (1-2\lambda)\phi(x)^2 + \lambda\phi(x)^4 \right\}$$
- Z_2-symmetry spontaneously broken

Magnetisation

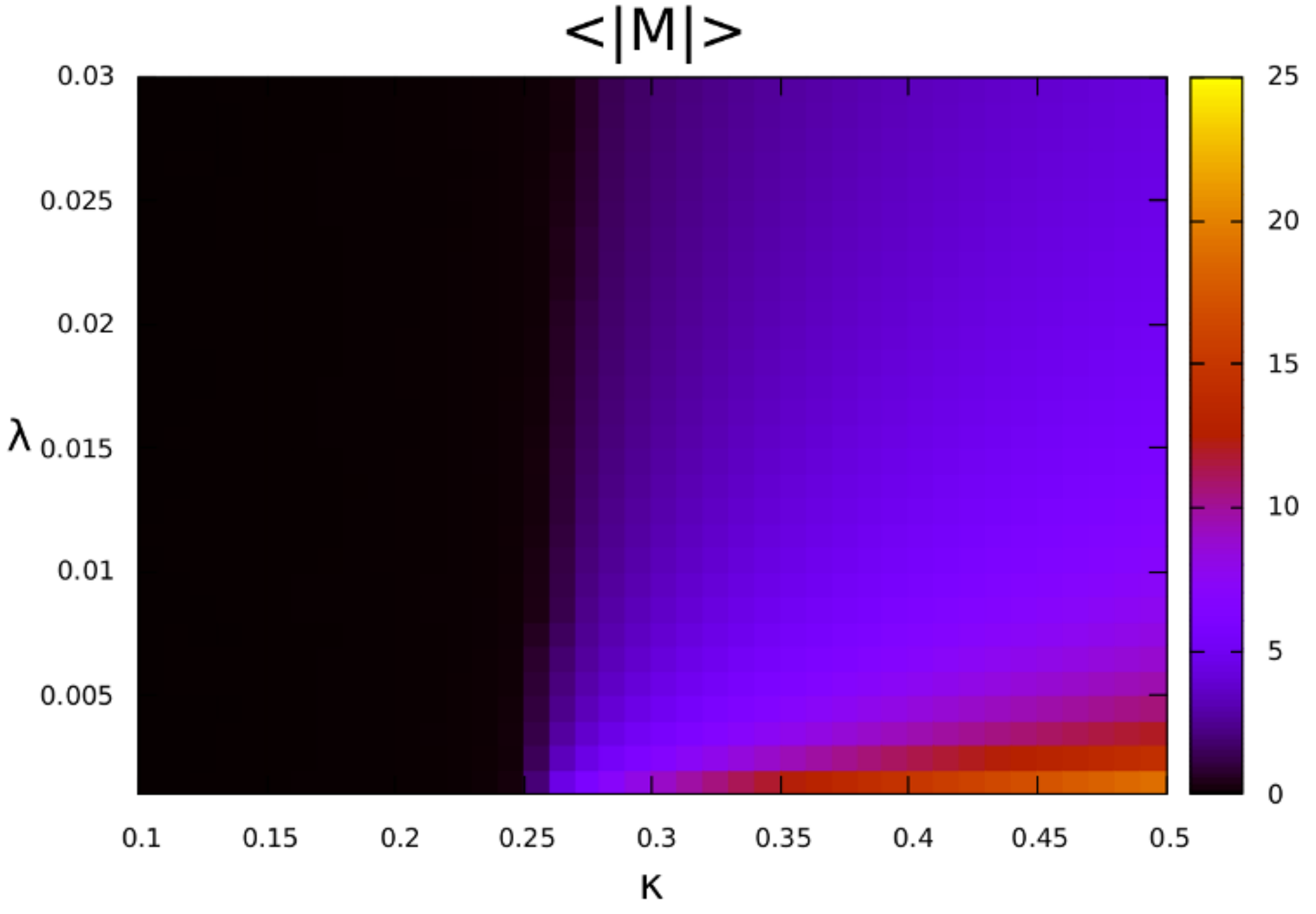
$$\langle M \rangle = \left\langle \frac{1}{\Omega} \sum_x \phi(x) \right\rangle$$

Susceptibility

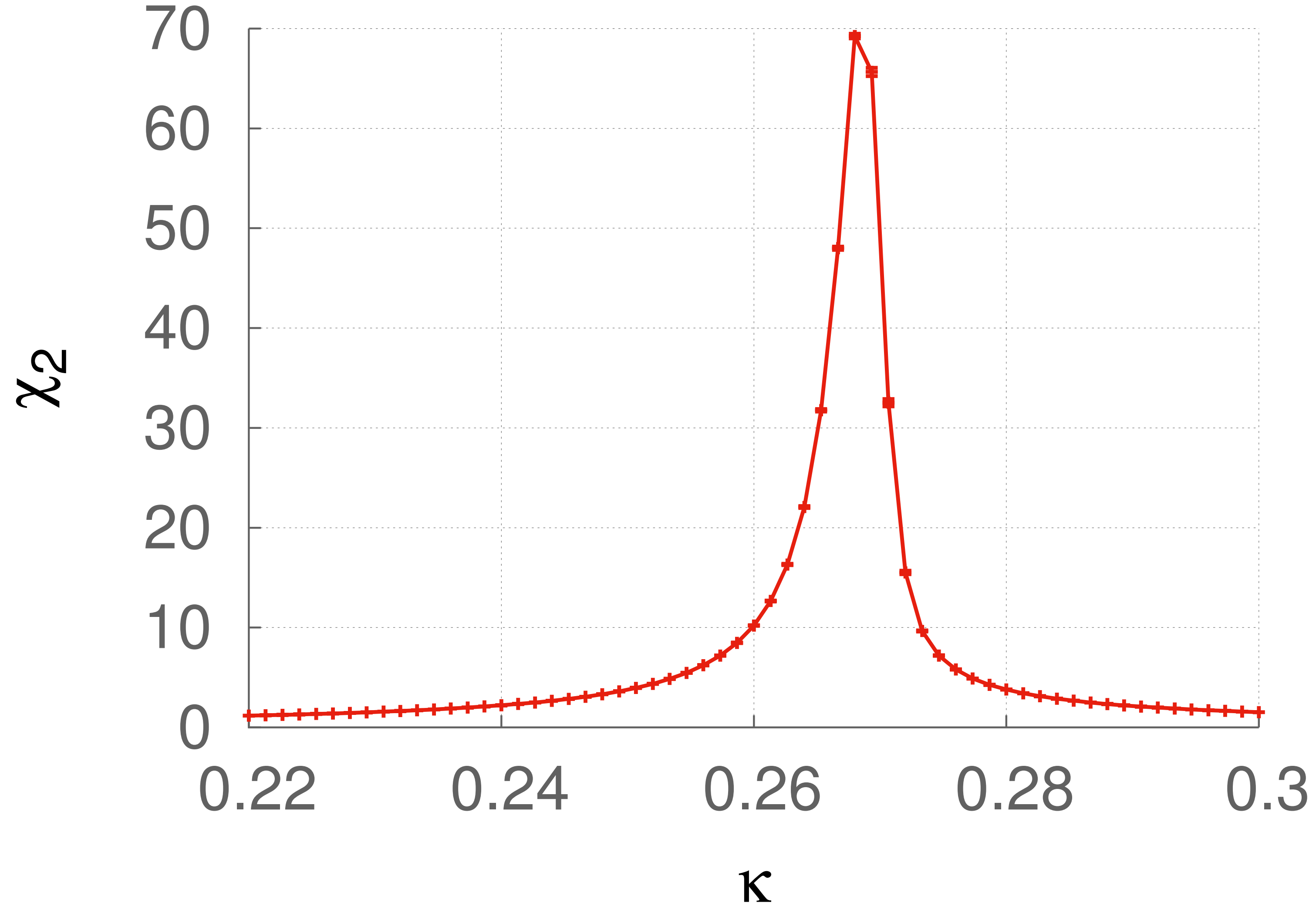
$$\chi = \Omega\left(\langle M^2 \rangle - \langle M \rangle^2\right)$$

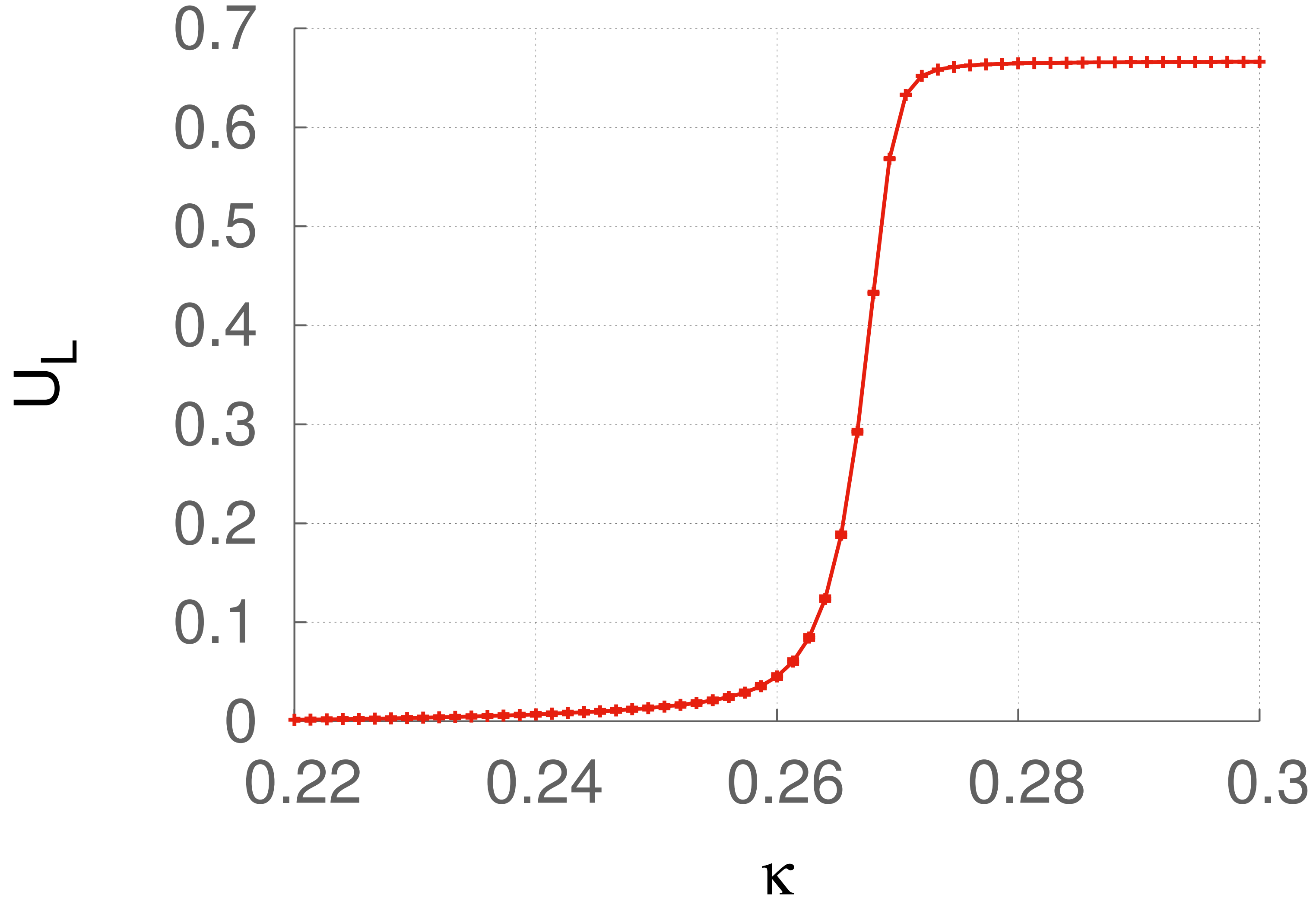
Binder cumulant

$$U_L = 1 - \frac{\langle M^4 \rangle}{3\langle M^2 \rangle^2}$$

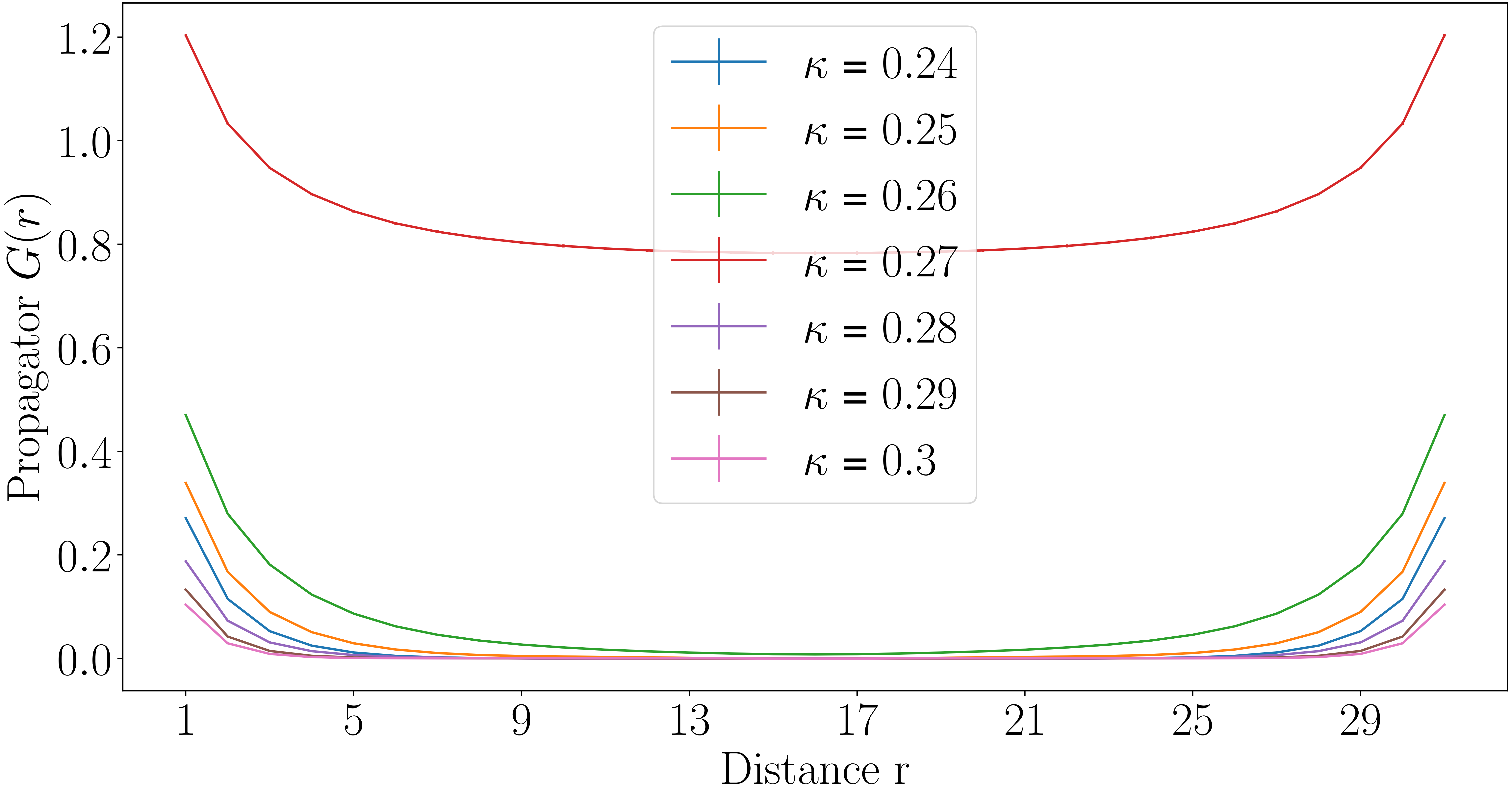


[Pawlowski, Stamatescu, Ziegler, 1705.06231]



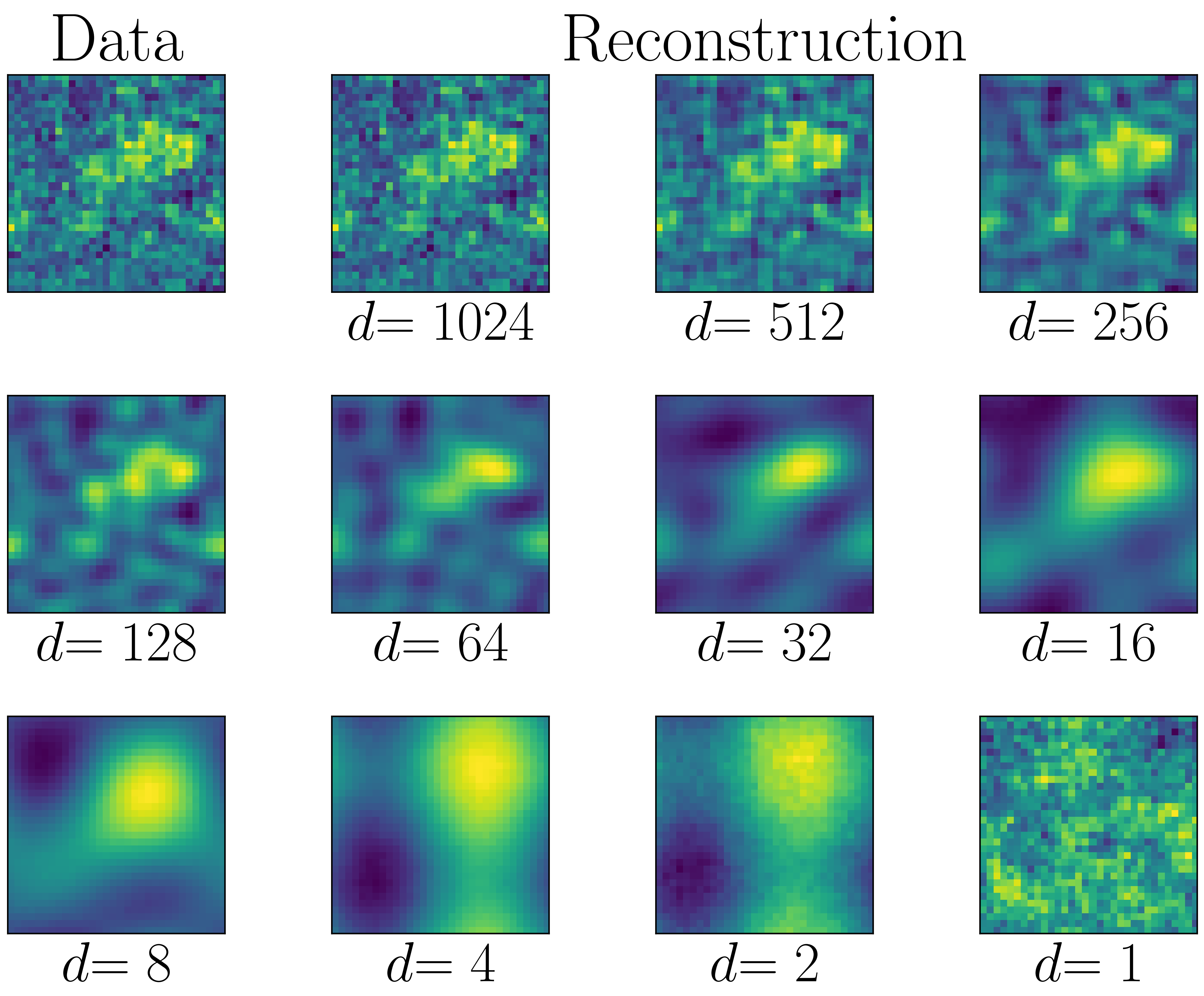


Propagators for different hoppings

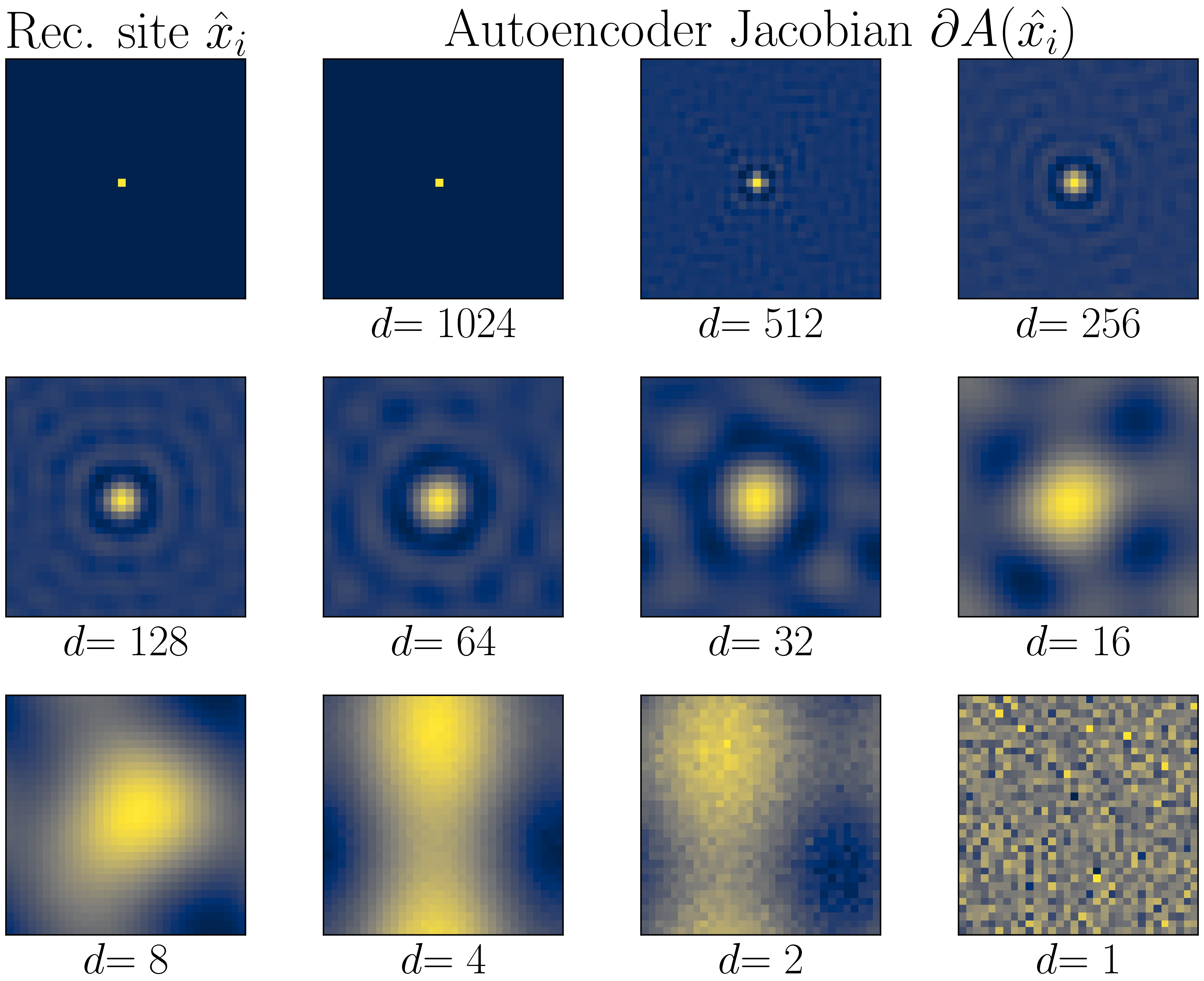


Reconstruction of configurations

$\kappa = 0.269$

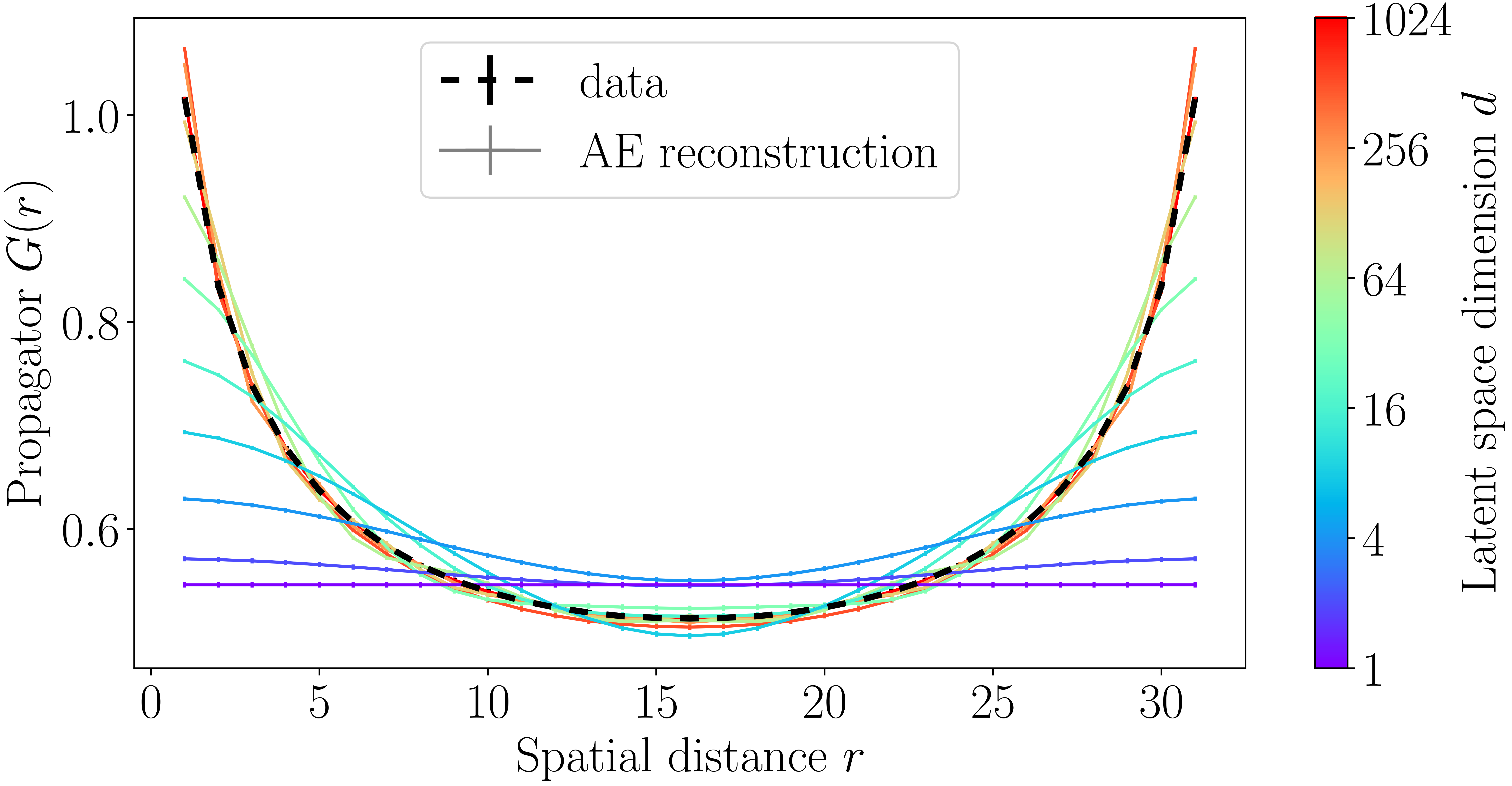


AE Jacobian

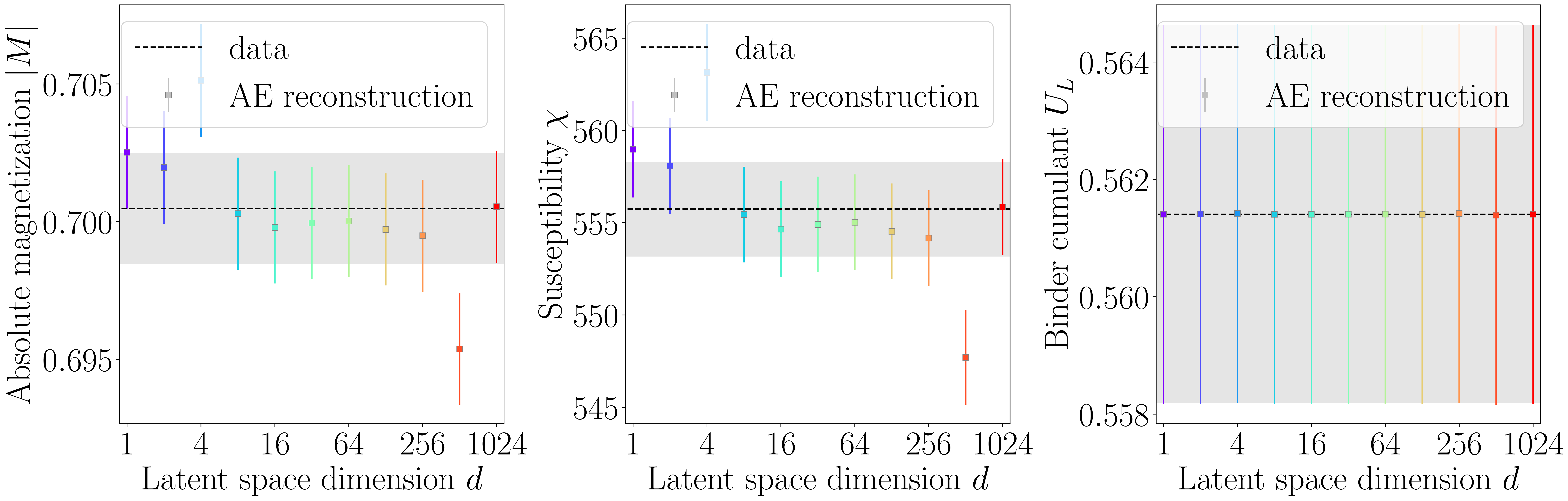


Rec. site $\hat{x}_i$
Autoencoder Jacobian $\partial A(\hat{x}_i)$
$d = 1024$
$d = 512$
$d = 256$
$d = 128$
$d = 64$
$d = 32$
$d = 16$
$d = 8$
$d = 4$
$d = 2$
$d = 1$

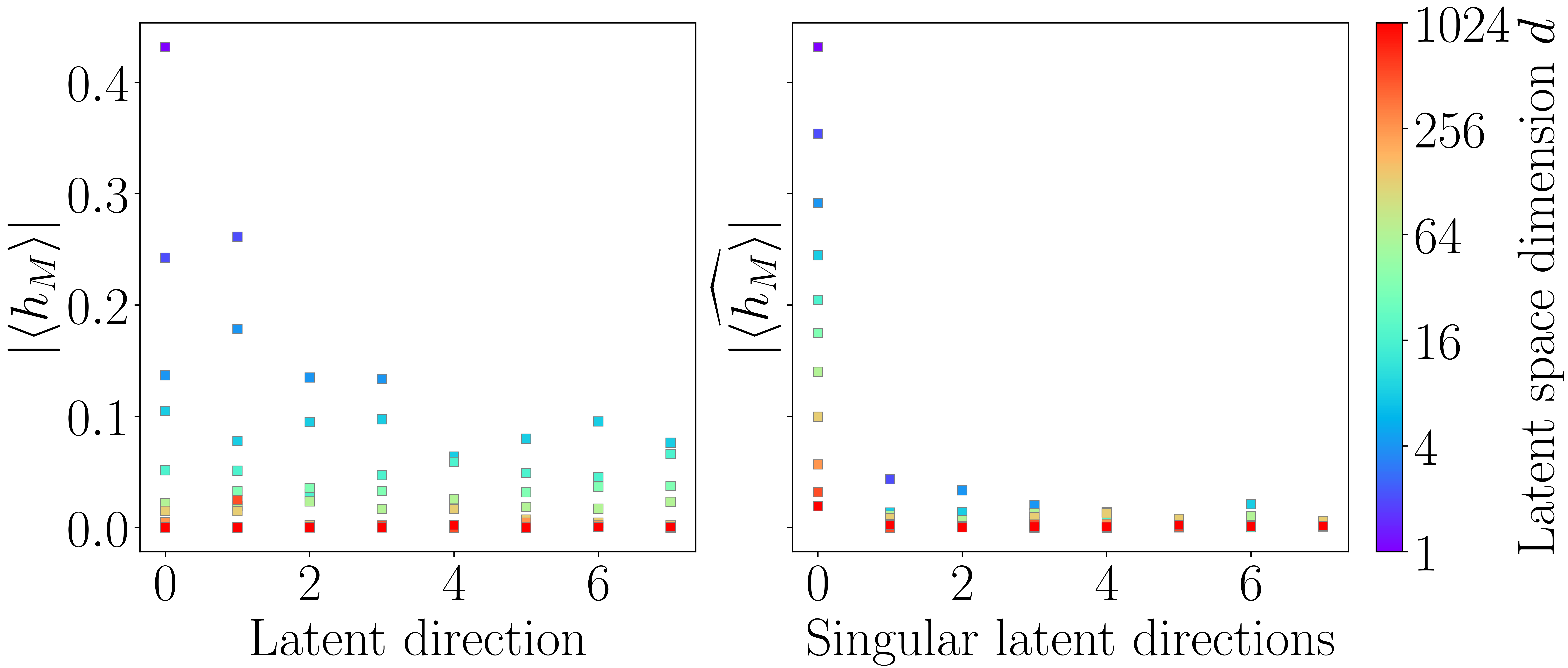
Propagator reconstruction



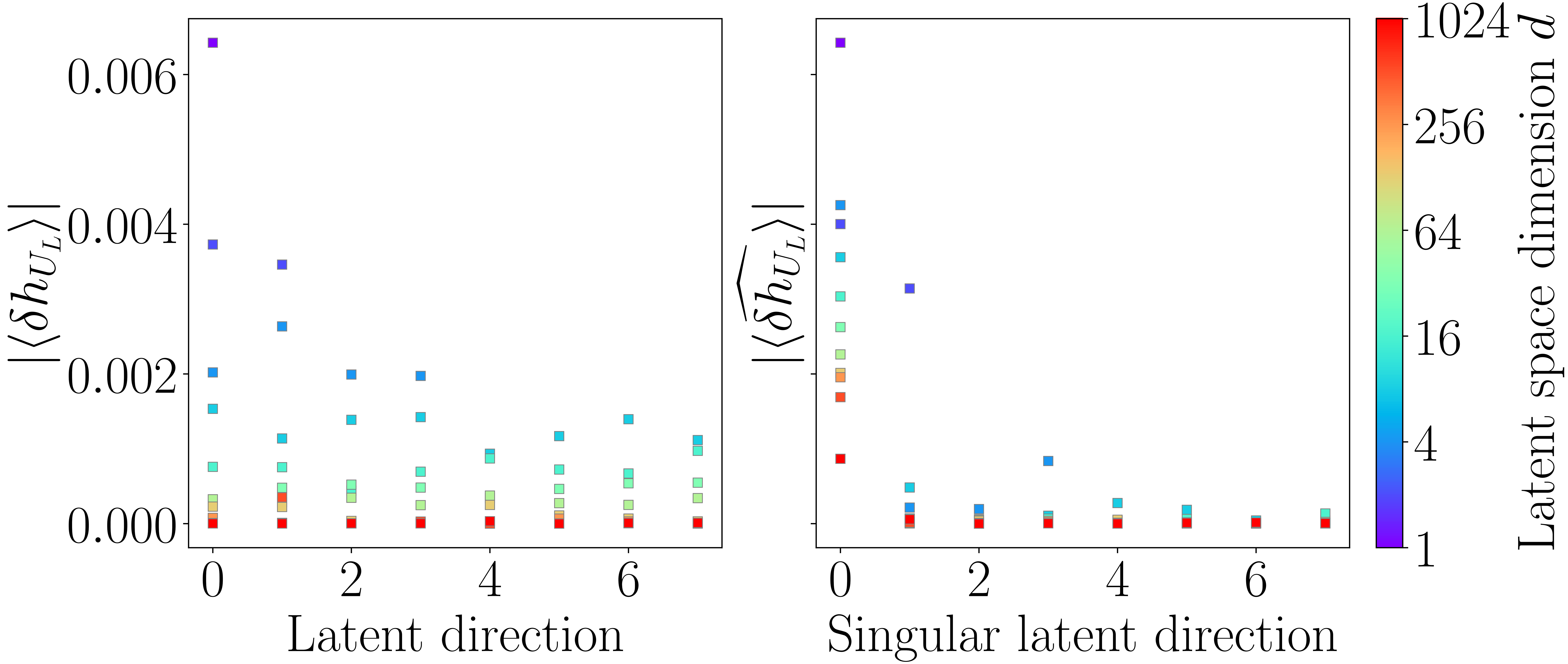
Cumulant reconstruction



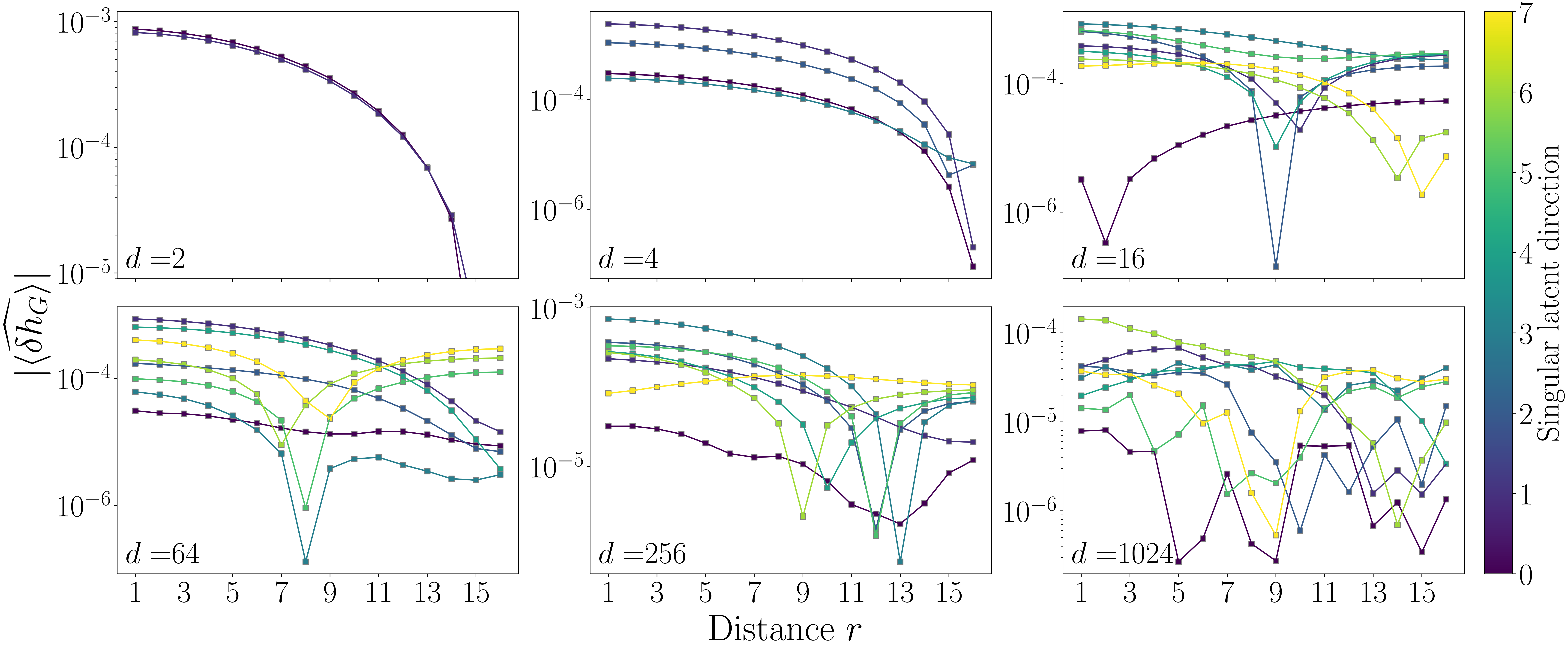
Jacobian decoder ∘ magnetisation



Jacobian decoder ∘ Binder cumulant



Jacobian decoder ∘ propagator



Conclusions

Summary

- RG trafos with generalised block spinning using NNs
- Scalar lattice ϕ^4-theory
- Analyse information compressions by Jacobians of block spinning trafos and singular value decomposition

Possible future directions

- Other observables & cost functions
- More general block spinning patterns by e.g. convolutional NNs