Curvature masses in local potential approximation

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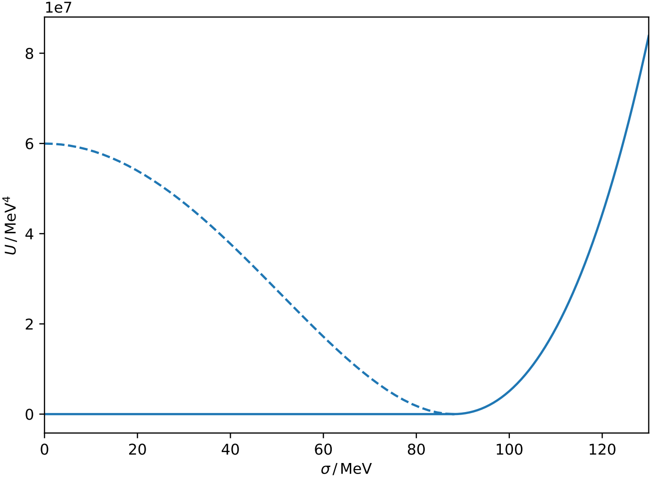
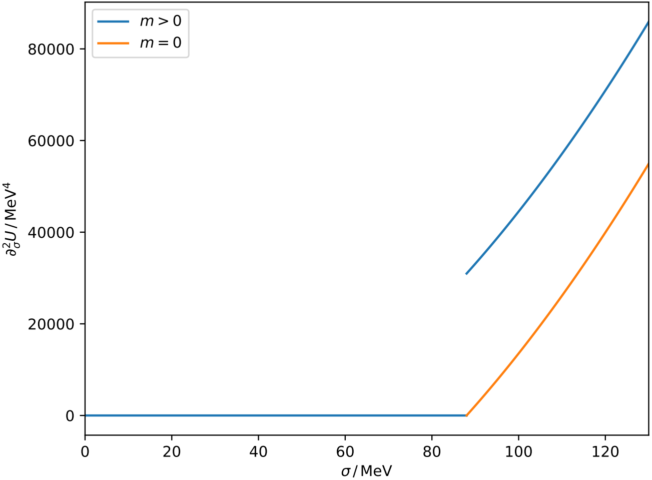
Motivation

Potential of chiral model in symmetry broken phase



Motivation

Second derivative @ min $=$ curvature mass^2



This talk: sometimes in LPA $m = 0$ (not m_π, not at CP)

Introduction

The quark-meson model (QMM)

$$S = \int \mathrm{d}^4 x \bigg(\bar{\psi}(Z^{\mathrm{F}} \partial\!\!\!/ + g\sigma + g\mathrm{i}\gamma_5 \tau \cdot \pi)\psi + \frac{Z^{\mathrm{B}}}{2}(\partial_\mu \sigma)(\partial^\mu \sigma) + \frac{Z^{\mathrm{B}}}{2}(\partial_\mu \pi)\cdot(\partial^\mu \pi) + \frac{\kappa}{2}(\sigma^2 + \pi^2) + \frac{\lambda}{4}(\sigma^2+\pi^2)^2 \bigg)$$

- 4 Bosons: σ, π and $2N_{\mathrm{c}}$ Fermions: $\bar{\psi}, \psi$
- 3 couplings: g, κ, λ and 2 wave function renormalizations: Z^{F}, Z^{B}
  - UV couplings $\rightarrow$ no k dependence
- Renormalizable
- This talk: only chiral limit and vacuum

The functional renormalization group (FRG)

- is exact
- transforms path integral into ∞-dim. PDE
- requires/allows for uncontrolled/non-perturbative approximations

$$Z[J] = \int \mathcal{D}\phi \exp\left(-S[\phi] + \text{sources}\right)$$

$\downarrow$ FRG

"UV" "IR"

$$S = \Gamma_\Lambda \xrightarrow[\text{Wetterich equation}]{\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{STr}\left(\left(\Gamma_k^{(2)}[\phi] + R_k^{\mathrm{T}}\right)^{-1} \partial_k R_k^{\mathrm{T}} - \text{norm.}\right)} \Gamma_0 = \Gamma$$

classical action quantum effective action

Local potential approximation (LPA)

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- To use FRG we need an ansatz

$$\Gamma_k^{\mathsf{LPA}}[\phi] = \int \mathrm{d}^4x \Big(\bar{\psi}(Z^{\mathrm{F}} \not{\partial} + g\sigma + g\mathrm{i}\gamma_5 \tau \cdot \pi)\psi + \frac{Z^{\mathrm{B}}}{2}(\partial_\mu \sigma)(\partial^\mu \sigma) + \frac{Z^{\mathrm{B}}}{2}(\partial_\mu \pi) \cdot (\partial^\mu \pi) + U_k\big(\sigma^2 + \pi^2\big) \Big)$$

- Only U can change during the flow
- $Z^{\mathrm{F}} = Z^{\mathrm{B}} = 1$ in LPA
- Enforce Wetterich equation for homogeneous field configurations

Regulator

- (full-d) Litim regulator

$$R^{\mathsf{Litim}} = \begin{pmatrix} R^{\mathsf{B}} & 0 & 0 & 0 \\ 0 & \mathbb{1}_\pi R^{\mathsf{B}} & 0 & 0 \\ 0 & 0 & 0 & \mathbb{1}_{\mathsf{c}}\mathbb{1}_{\mathsf{f}} R^{\mathsf{F}} \\ 0 & 0 & -\left(\mathbb{1}_{\mathsf{c}}\mathbb{1}_{\mathsf{f}} R^{\mathsf{F}}\right)^{\mathrm{T}} & 0 \end{pmatrix}$$

$$R^{\mathsf{B}} = (k^2 - p^2)\,\Theta(k^2 - p^2)$$

$$R^{\mathsf{F}} = \not{p}\left(\frac{k}{|p|} - 1\right)\Theta(k^2 - p^2)$$

- Integrals can be evaluated analytically
- Preserves Lorentz symmetry, but breaks causality

Deriving the PDE

Wetterich equation

$$\Gamma^{\mathsf{LPA}} \quad \downarrow \quad R^{\mathsf{Litim}}$$

$$\partial_k \Gamma_k^{\mathsf{LPA}} = \frac{1}{2}\,\mathrm{STr}\left(\left(\Gamma_k^{\mathsf{LPA}\,(2)}[\sigma] + (R^{\mathsf{Litim}})_k^{\mathrm{T}}\right)^{-1}\partial_k (R^{\mathsf{Litim}})_k^{\mathrm{T}} - \mathrm{norm.}\right)$$

$$\sigma(x) = \sigma \geq 0, \pi = 0 \quad \downarrow \quad \bar{\psi} = 0, \psi = 0$$

$$\partial_k U = \frac{k^5}{32\pi^2(k^2 + \partial_\sigma^2 U)} + \frac{3k^5}{32\pi^2\left(k^2 + \frac{1}{\sigma}\partial_\sigma U\right)} - \frac{N_c k^5}{4\pi^2(k^2 + g^2\sigma^2)} + \mathrm{norm.}$$

- Non-linear 2-dim. scalar 2nd order partial differential equation

Reformulation

- Flow in RG time $t = \ln\left(\frac{\Lambda}{k}\right)$: $\quad k = \Lambda \leftrightarrow t = 0, \quad k = 0 \leftrightarrow t = \infty$
- Use $\rho = \frac{1}{2}\sigma^2$ instead of σ
- $v = \partial_\rho U$ as dependent quantity instead of U

$$\partial_t v = \underbrace{\frac{k^6(3\partial_\rho v + 2\rho\partial_\rho^2 v)}{32\pi^2(k^2 + v + 2\rho\partial_\rho v)^2}}_{\substack{\sigma \\ \text{diffusion}}} + \underbrace{\frac{3k^6\partial_\rho v}{32\pi^2\left(k^2 + v\right)^2}}_{\substack{\pi \\ \text{advection}}} - \underbrace{\frac{N_c g^2 k^6}{2\pi^2(k^2 + g^2\rho)^2}}_{\substack{\psi \\ \text{source}}}$$

- Conservative form:

$$\partial_t v = \partial_\rho\left(- \frac{k^6}{32\pi^2(k^2 + v + 2\rho\partial_\rho v)} - \frac{3k^6}{32\pi^2\left(k^2 + v\right)} + \frac{N_c k^6}{2\pi^2(k^2 + g^2\rho)}\right)$$

- Initial condition linear: $v(t = 0) = \kappa + 2\lambda\rho$

MFA

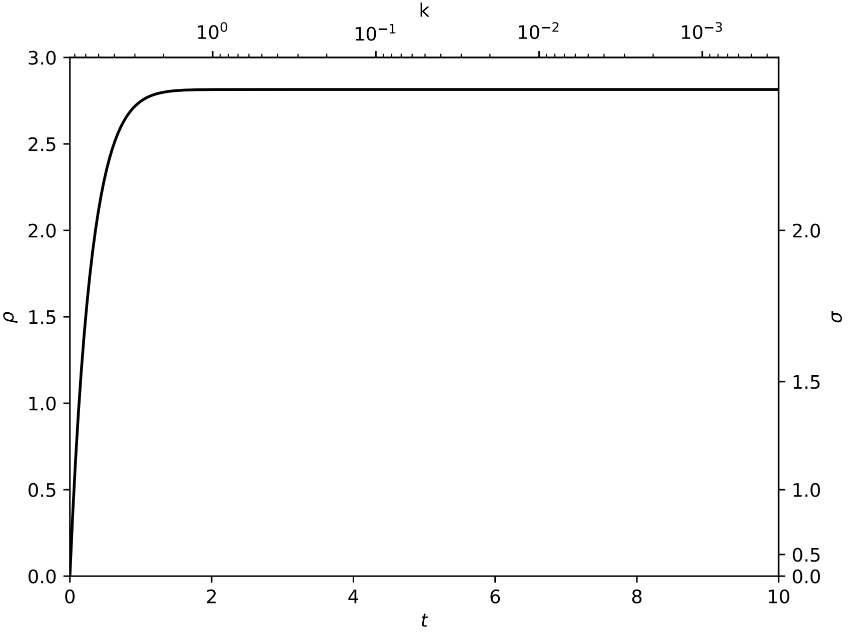
- Only quarks $\implies$ mean field approximation (MFA)

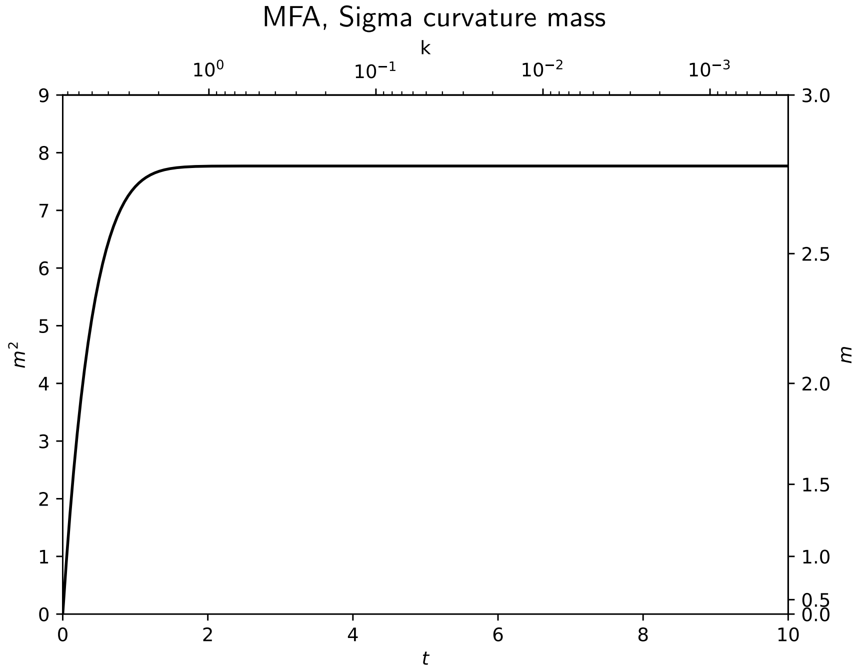
$$\partial_t v = \frac{k^6(3\partial_\rho v + 2\rho\partial_\rho^2 v)}{32\pi^2(k^2+v+2\rho\partial_\rho v)^2} + \frac{3k^6\partial_\rho v}{32\pi^2\left(k^2+v\right)^2} - \frac{N_c g^2 k^6}{2\pi^2(k^2+g^2\rho)^2}$$

σ — diffusion; π — advection; ψ — source

- Potential symbolically solvable
- Vacuum expectation value and mass quasi-exact (numerical root finding via bisection)
- The following example has $\Lambda = \frac{15}{2}\,\text{a.u.},\ \kappa = 0,\ \lambda = \frac{1}{2},\ g = \frac{3}{\sqrt{8}}$

MFA, Vacuum expectation value





MFA, Sigma curvature mass
k
10^0
10^{-1}
10^{-2}
10^{-3}
m^2
m
t
0
1
2
3
4
5
6
7
8
9
2
4
6
8
10
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Truncations

Truncations

- Consider other truncations to gain insight
- Of the 7 possible combinations

$$\sigma,\ \pi,\ \psi,\ \sigma\pi,\ \sigma\psi,\ \pi\psi,\ \sigma\pi\psi$$

there is one more besides "ψ" that is integrable, namely "π"

$$\partial_t v = \underbrace{\cancel{\frac{k^6(3\partial_\rho v + 2\rho\partial_\rho^2 v)}{32\pi^2(k^2 + v + 2\rho\partial_\rho v)^2}}}_{\substack{\sigma \\ \text{diffusion}}} + \underbrace{\boxed{\frac{3k^6\partial_\rho v}{32\pi^2\left(k^2 + v\right)^2}}}_{\substack{\pi \\ \text{advection}}} - \underbrace{\cancel{\frac{N_c g^2 k^6}{2\pi^2(k^2 + g^2\rho)^2}}}_{\substack{\psi \\ \text{source}}}$$

- This is just the large N O(N) model

Large N O(N) model

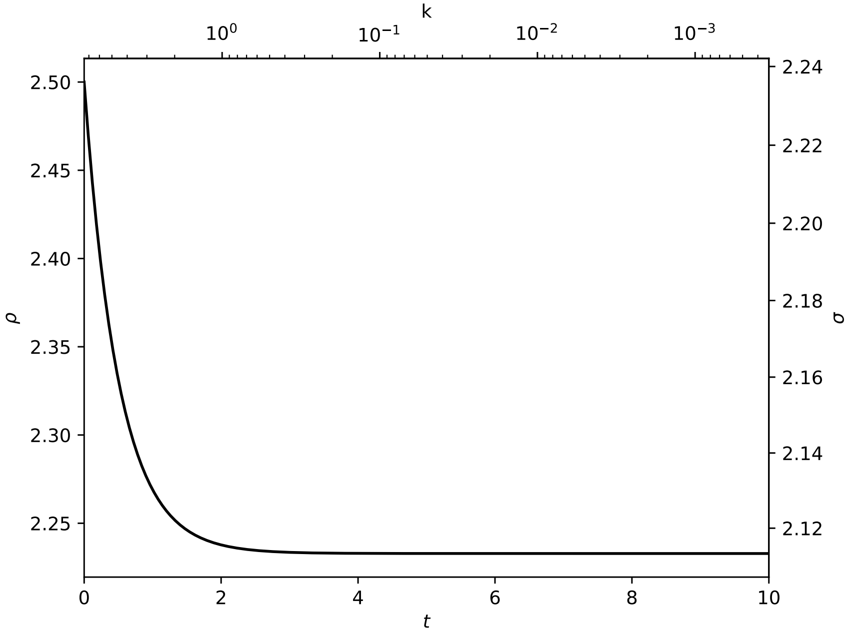
- Related with MFA via Legedre transformation
- Potential quasi-exact
  (numerical root finding via bisection)
- Vacuum expectation value and mass symbolically solvable

$$\rho(t) = -\frac{\kappa}{2\lambda} - \frac{3\Lambda^2}{64\pi^2}\big(1 - \exp(-2t)\big)$$

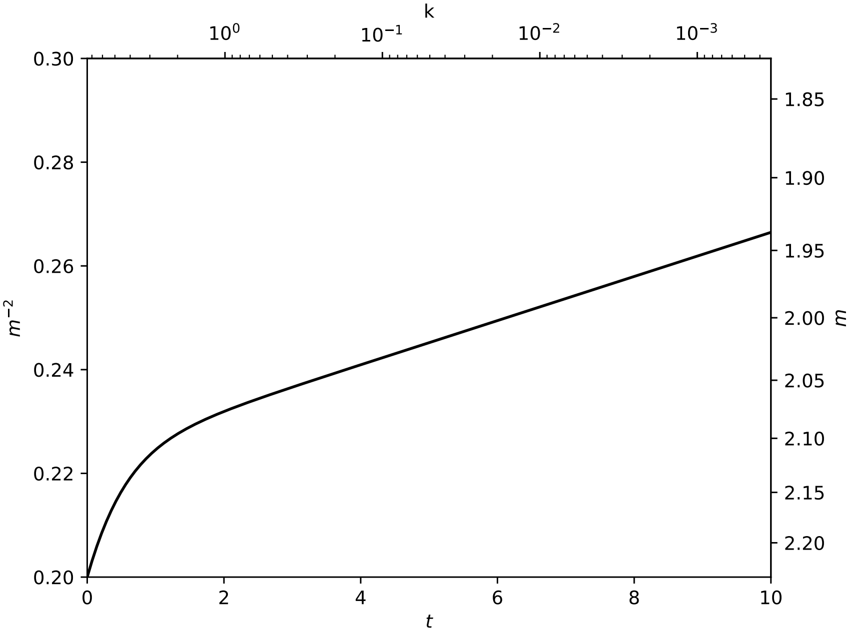
$$m^2(t) = \frac{-32\pi^2\kappa - 3\lambda\Lambda^2\big(1 - \exp(-2t)\big)}{16\pi^2 + 6\lambda t}$$

- $\lim_{t\to\infty} m^2 = 0$
- The following example has $\Lambda = \frac{15}{2}\,\mathrm{a.u.},\ \kappa = -\frac{5}{2}\,\mathrm{a.u.}^2,\ \lambda = \frac{1}{2}$

Large N O(N), Vacuum expectation value



Large N O(N), Sigma curvature mass



Possible reasons

- Vanishing mass is perhaps
  - a Large N phenomenon, mass non-zero at finite N
  - due to triviality of scalar theories in $d \geq 4$ dimensions
- Impact of dimension is easily tested, flow equations are very similar
- $d = 3$:

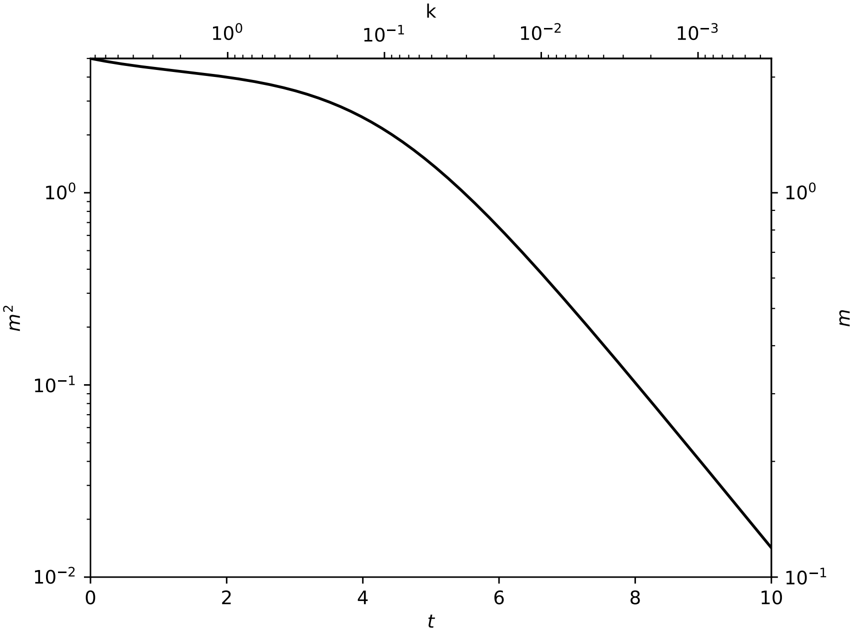
$$m^2(t) = \frac{-6\pi^2\Lambda\kappa - 6\lambda\Lambda^2\big(1 - \exp(-t)\big)}{3\pi^2\Lambda + 6\lambda\big(\exp(t) - 1\big)}$$

- $d = 5$:

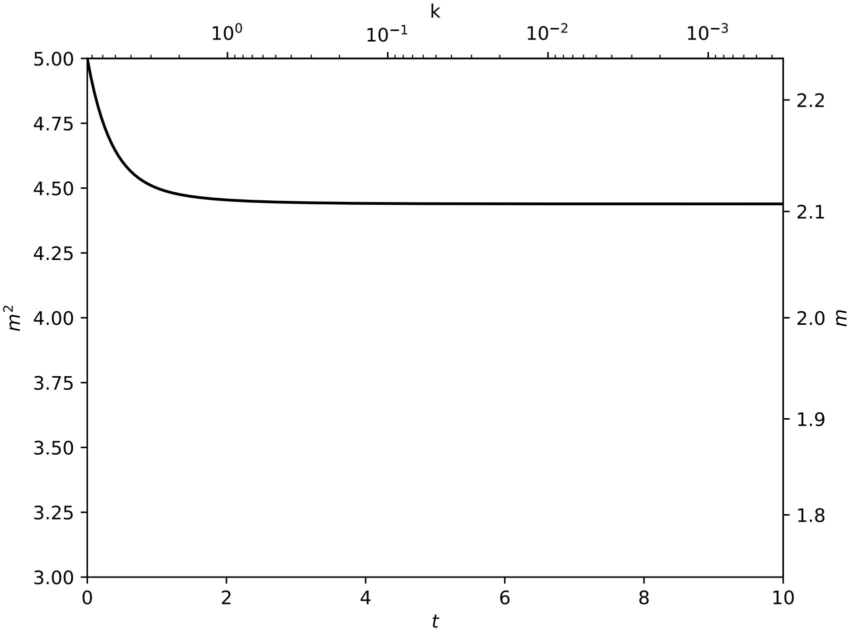
$$m^2(t) = \frac{-90\pi^3\kappa - 3\lambda\Lambda^3\big(1 - \exp(-3t)\big)}{45\pi^3 + 9\lambda\Lambda\big(1 - \exp(-t)\big)}$$

- Mass vanishes (fast) for $d = 3$ but stays non-zero for $d = 5$
  $\Longrightarrow$ Dimension relevant, but not due to triviality

Sigma curvature mass, Large N O(N), $d = 3$



Sigma curvature mass, Large N O(N), $d = 5$



Large N_f quark-meson model

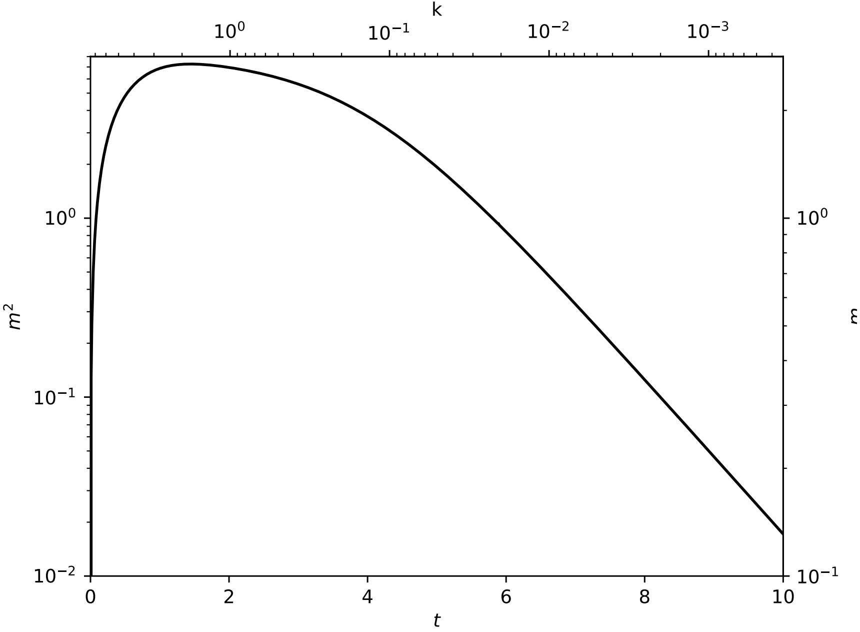
- Impact of finite N difficult to analyze due to diffusion (remember $\sigma \leftrightarrow$ diffusion)
- One more truncation without σ:

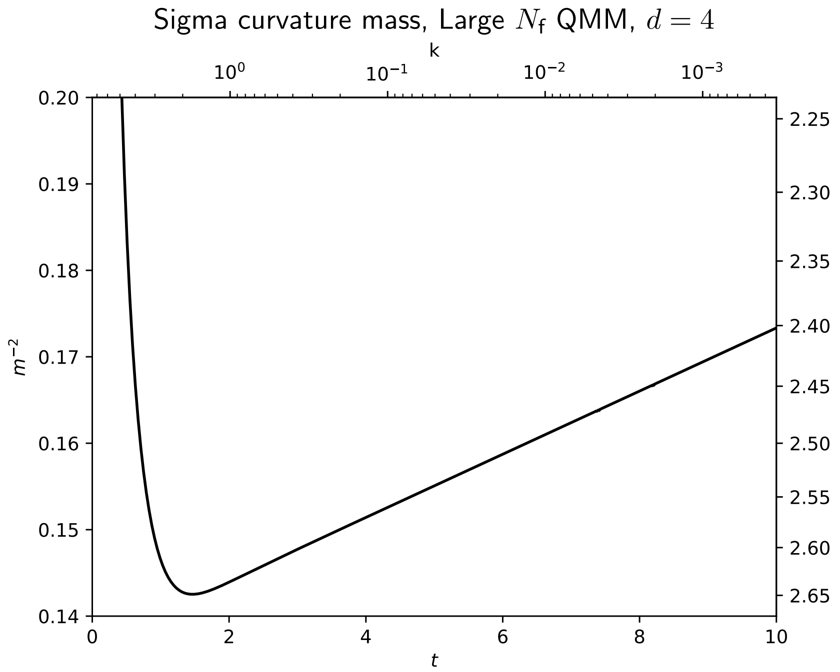
$$\partial_t v = \frac{k^6(3\partial_\rho v + 2\rho\partial_\rho^2 v)}{32\pi^2(k^2 + v + 2\rho\partial_\rho v)^2} + \frac{3k^6\partial_\rho v}{32\pi^2\left(k^2+v\right)^2} - \frac{N_\mathrm{c} g^2 k^6}{2\pi^2(k^2+g^2\rho)^2}$$

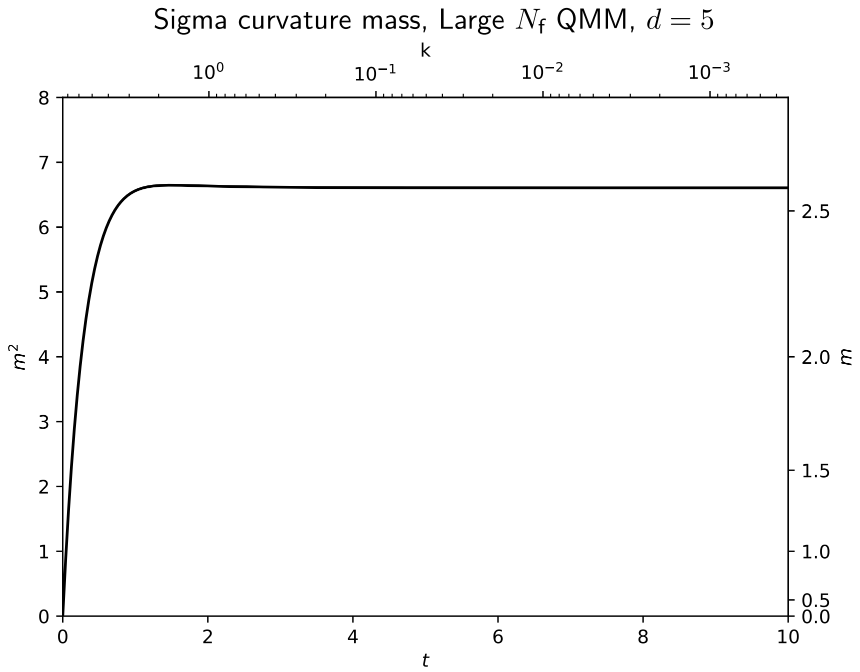
σ diffusion | π advection | ψ source

- Not analytically solvable
- But, can be reduced to ODE via method of characteristics $\longrightarrow$ much easier to solve

Sigma curvature mass, Large N_f QMM, $d = 3$




Sigma curvature mass, Large N_f QMM, $d = 4$
k
10^0
10^{-1}
10^{-2}
10^{-3}
0.20
0.19
0.18
0.17
0.16
0.15
0.14
m^{-2}
2.25
2.30
2.35
2.40
2.45
2.50
2.55
2.60
2.65
m
0
2
4
6
8
10
t



Sigma curvature mass, Large N_f QMM, $d = 5$
k
10^0
10^{-1}
10^{-2}
10^{-3}
m^2
m
t
0
1
2
3
4
5
6
7
8
2
4
6
8
10
0.0
0.5
1.0
1.5
2.0
2.5

Summary of results without diffusion

- In $d = 3, 4$ sigma mass still vanishes as $t \to \infty$
  - Asymptotic behavior unchanged
  - "pions win over quarks"
- Let's collect the results obtained so far

fields		$d = 3$	$d = 4$	$d = 5$	
π	large N O(N)	$m^2 \sim \exp(-t)$	$m^2 \sim t^{-1}$	$m^2 \sim 1$	exactly solvable
ψ	MFA	$m^2 \sim 1$	$m^2 \sim 1$	$m^2 \sim 1$	exactly solvable
$\pi\psi$	large N_f QMM	$m^2 \sim \exp(-t)$	$m^2 \sim t^{-1}$	$m^2 \sim 1$	reduces to ODE

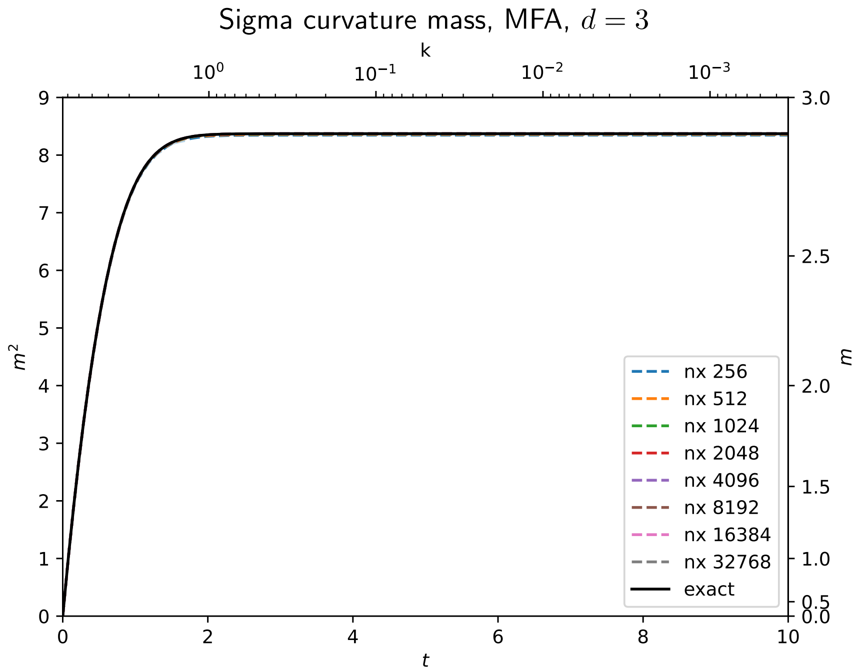
Numerical setup

Numerics

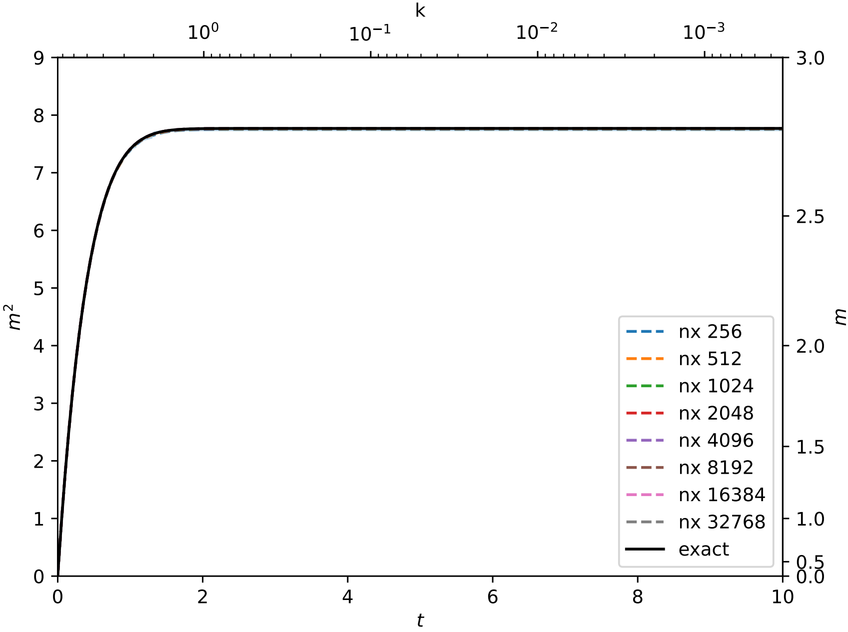
- (Semi)analytical methods exhausted
- From now on solve PDEs via method of lines
  - $\rho_{\text{min}} = 0$
  - $\rho_{\text{max}} = \frac{15}{2}$ a.u.^{2}
  - Conservative first order upwind discretization
  - TR-BDF2 time stepper
  - naive addaptive step size

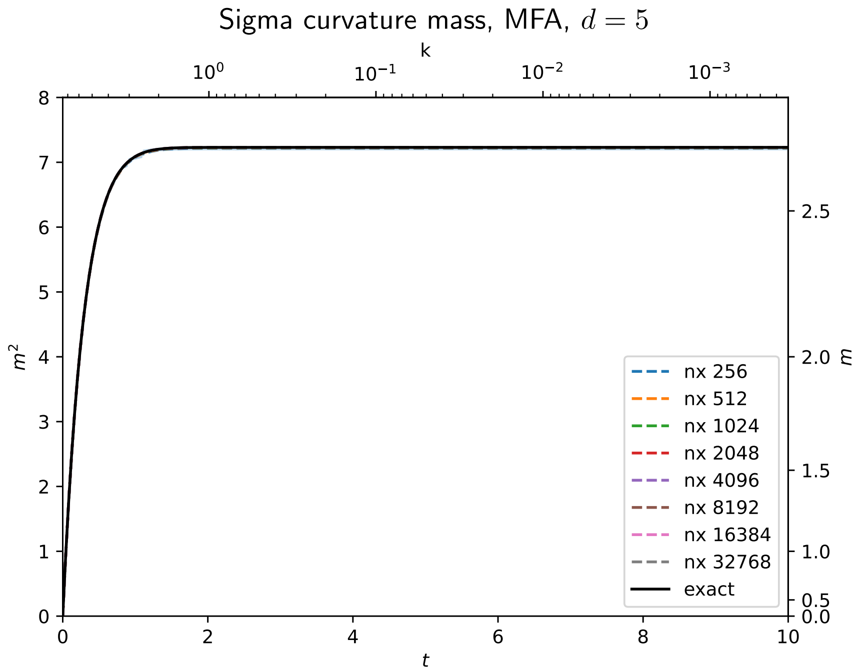
$\longrightarrow$ Not perfect, but good enough for now

- Extract ρ and m^2 by linear interpolation of discretized solution
- Increase spatial resolution to check for (apparent) convergence
- We use the ψ parameter set if the truncation includes fermions and the π parameter set otherwise
- Next: reproduce the exact results numerically


Sigma curvature mass, MFA, $d = 3$
k
10^{0}
10^{-1}
10^{-2}
10^{-3}
m^2
m
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact

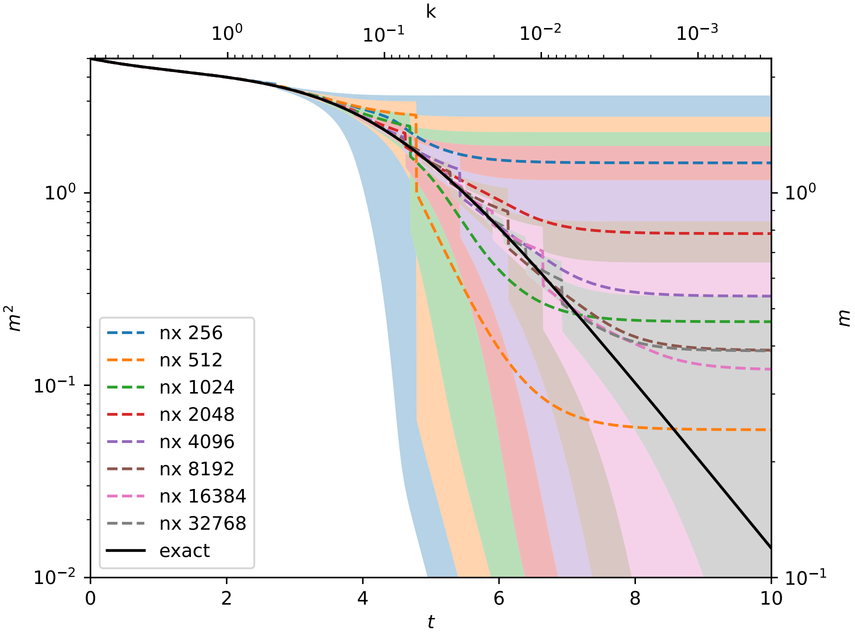
Sigma curvature mass, MFA, $d = 4$

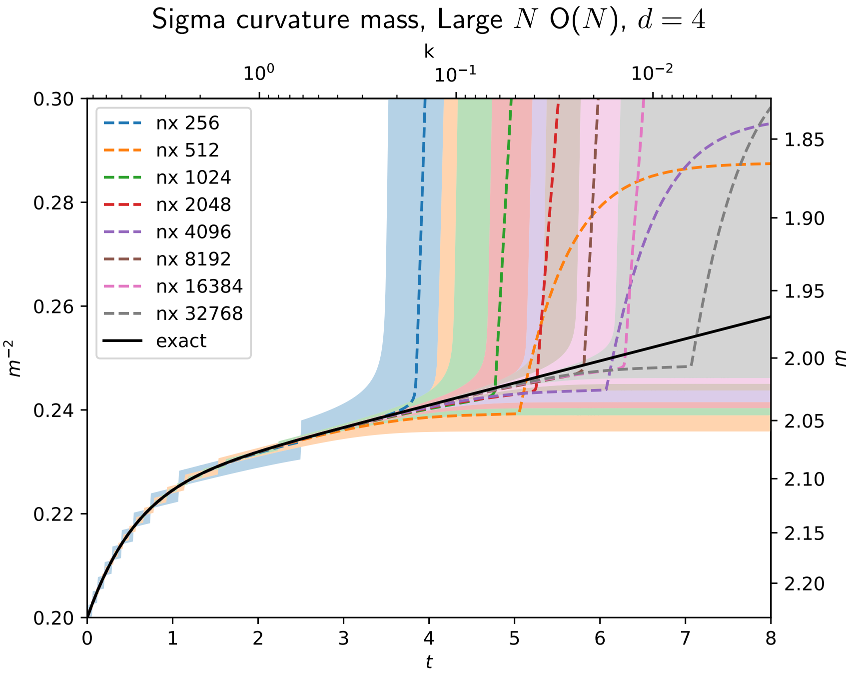




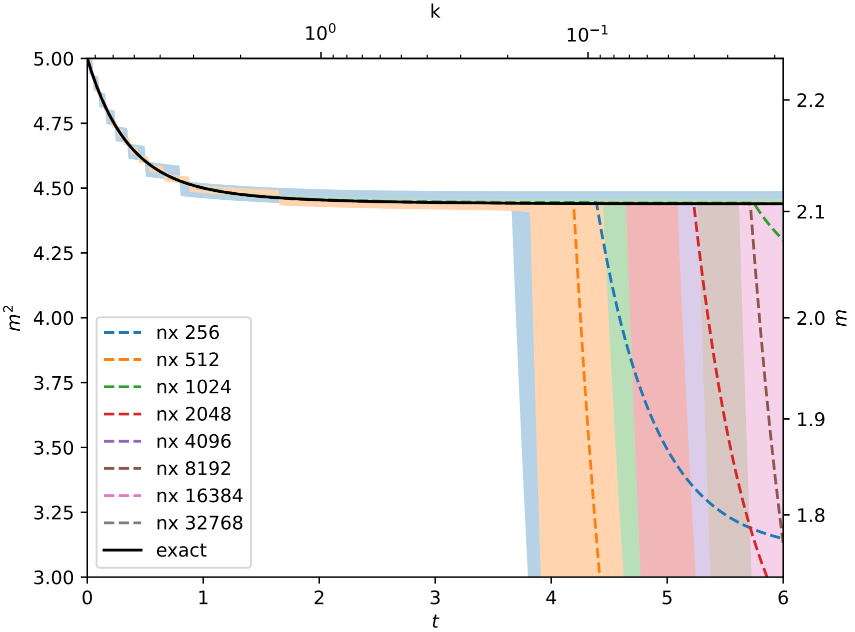
Sigma curvature mass, MFA, $d = 5$
k
10^0
10^{-1}
10^{-2}
10^{-3}
m^2
m
t
0
1
2
3
4
5
6
7
8
0.0
0.5
1.0
1.5
2.0
2.5
0
2
4
6
8
10
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact

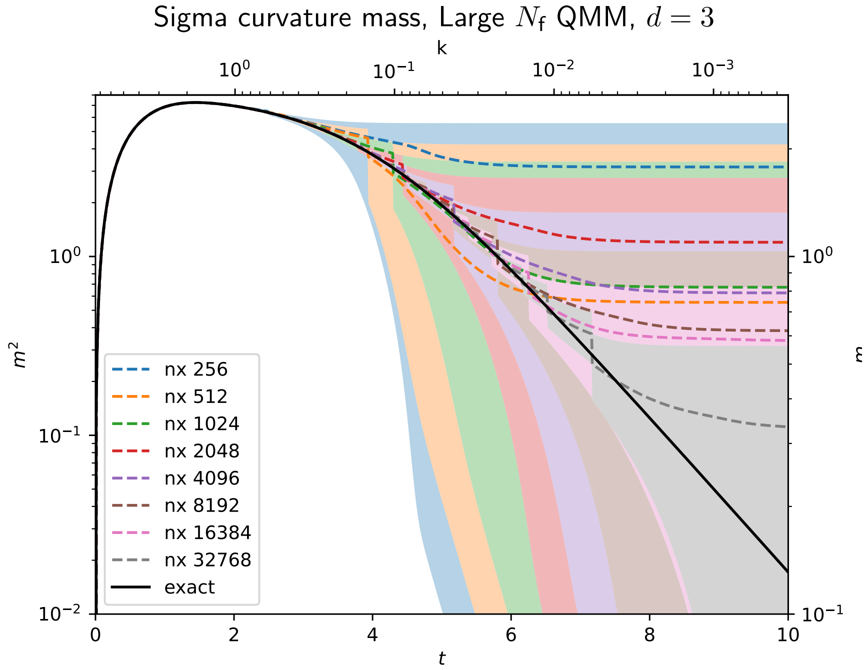
Sigma curvature mass, Large N O(N), $d = 3$



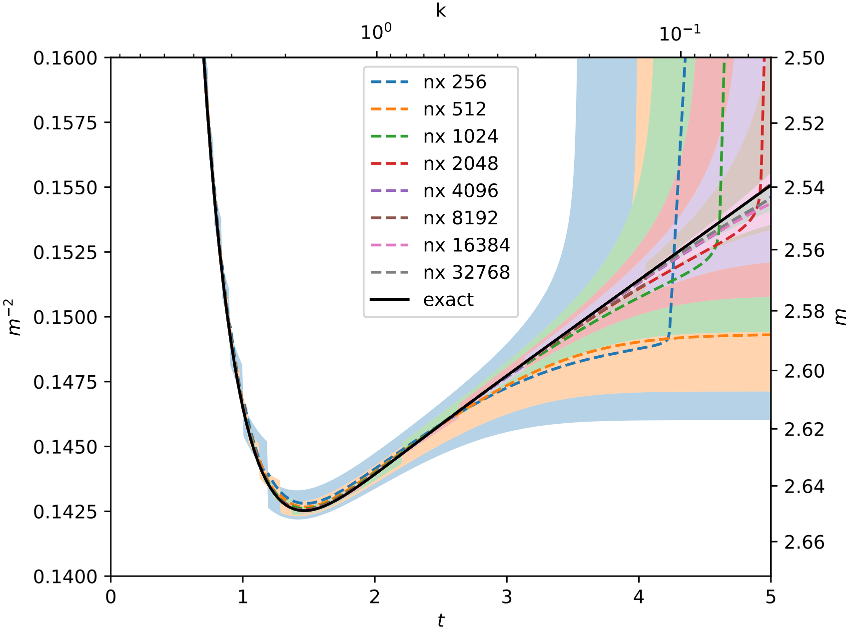

Sigma curvature mass, Large N O(N), $d = 4$
k
10^0
10^{-1}
10^{-2}
0.30
0.28
0.26
0.24
0.22
0.20
m^{-2}
1.85
1.90
1.95
2.00
2.05
2.10
2.15
2.20
m
0
1
2
3
4
5
6
7
8
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact

Sigma curvature mass, Large N O(N), $d = 5$

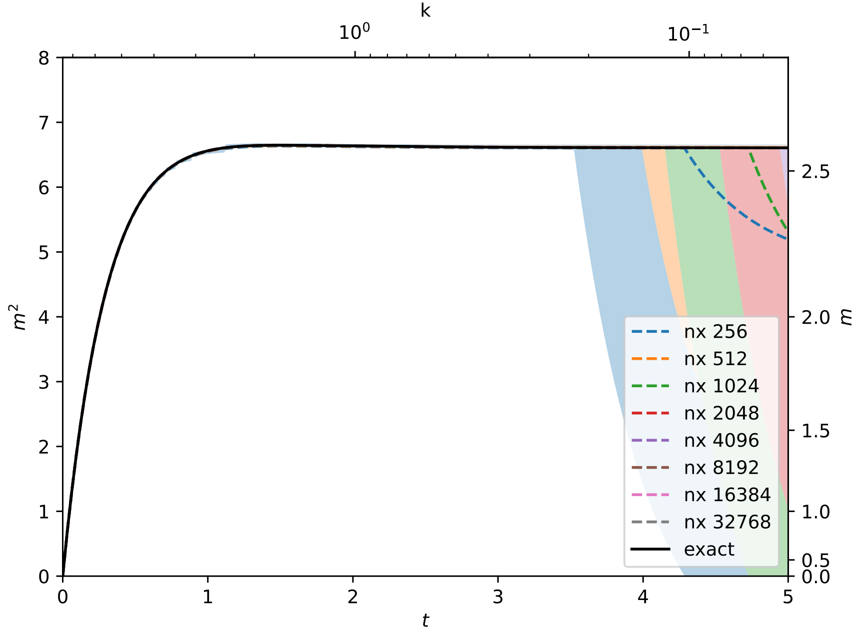



Sigma curvature mass, Large N_f QMM, $d = 3$
k
10^0
10^{-1}
10^{-2}
10^{-3}
m^2
m
10^0
10^{-1}
10^{-2}
t
0
2
4
6
8
10
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact

Sigma curvature mass, Large N_f QMM, $d = 4$



Sigma curvature mass, Large N_f QMM, $d = 5$



Results with diffusion

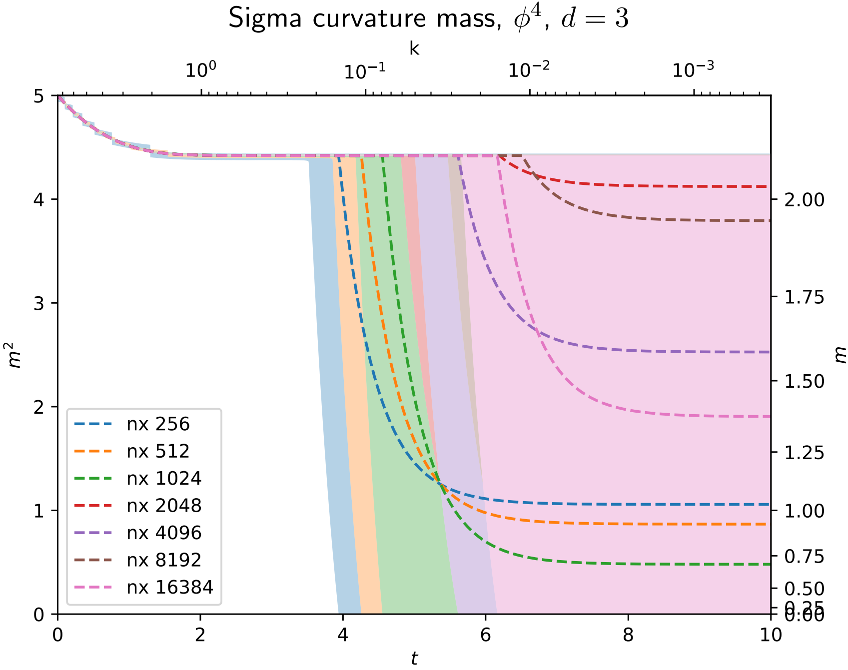
ϕ^4 model

- We start with the truncation containing only σ $= \phi^4$ model = O(1) model

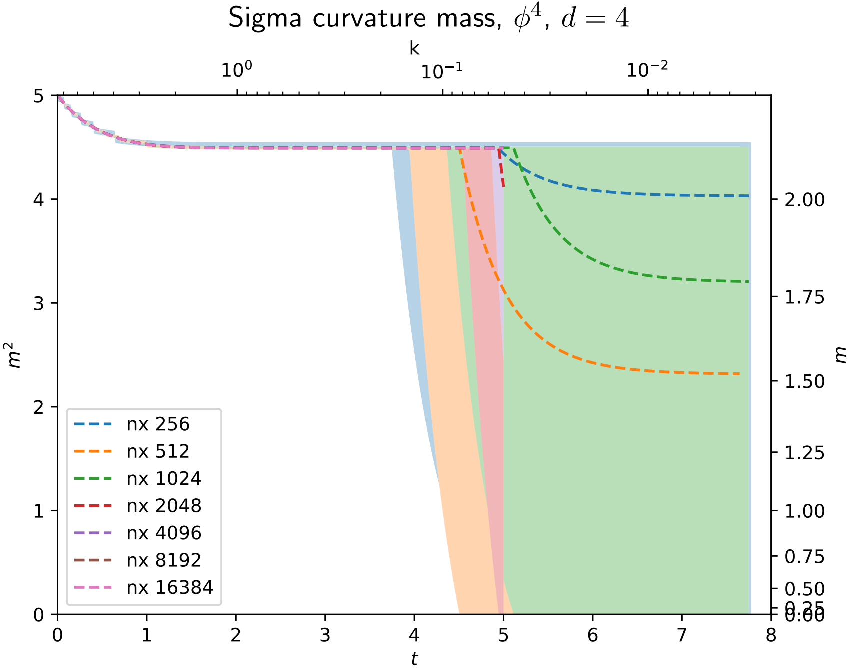
$$\partial_t v = \frac{k^6(3\partial_\rho v + 2\rho\partial_\rho^2 v)}{32\pi^2(k^2 + v + 2\rho\partial_\rho v)^2} + \frac{3k^6\partial_\rho v}{32\pi^2\left(k^2 + v\right)^2} - \frac{N_c g^2 k^6}{2\pi^2(k^2 + g^2\rho)^2}$$

σ	π	ψ
diffusion	advection	source

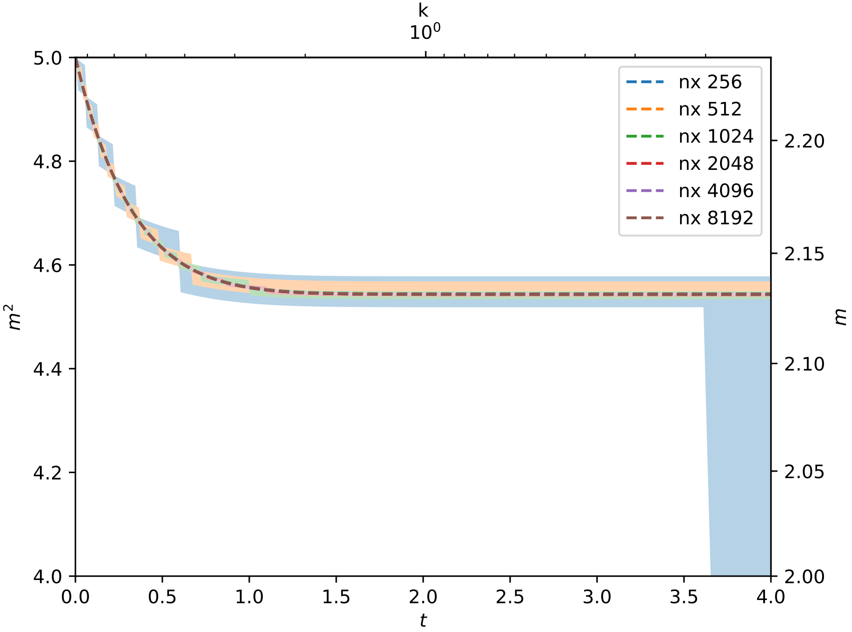
- Turns out to be numerically the second hardest
- If this has vanishing mass it's not the pions fault



Sigma curvature mass, ϕ^4, $d = 3$
k
10^0
10^{-1}
10^{-2}
10^{-3}
m^2
m
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384


Sigma curvature mass, ϕ^4, $d = 4$
k
10^0
10^{-1}
10^{-2}
5
4
3
2
1
0
m^2
0
1
2
3
4
5
6
7
8
t
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
m
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384

Sigma curvature mass, ϕ^4, $d = 5$



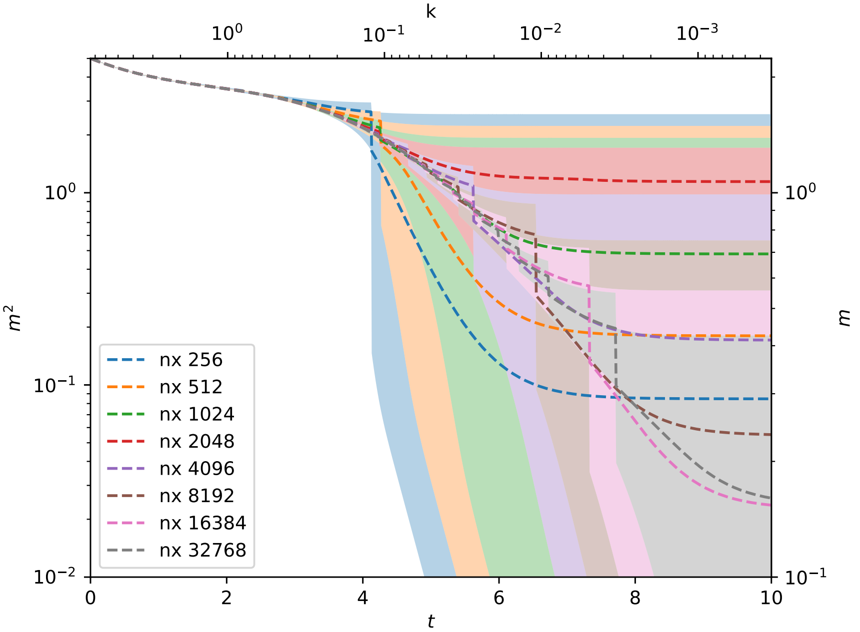
- A finite mass in $d = 3, 4$ with bosonic fluctuations is possible!
- Next comes the full finite N O(N) model (for $N = 4$)

$$\partial_t v = \frac{k^6(3\partial_\rho v + 2\rho\partial_\rho^2 v)}{32\pi^2(k^2 + v + 2\rho\partial_\rho v)^2} + \frac{3k^6\partial_\rho v}{32\pi^2\left(k^2 + v\right)^2} - \frac{N_c g^2 k^6}{2\pi^2(k^2 + g^2\rho)^2}$$

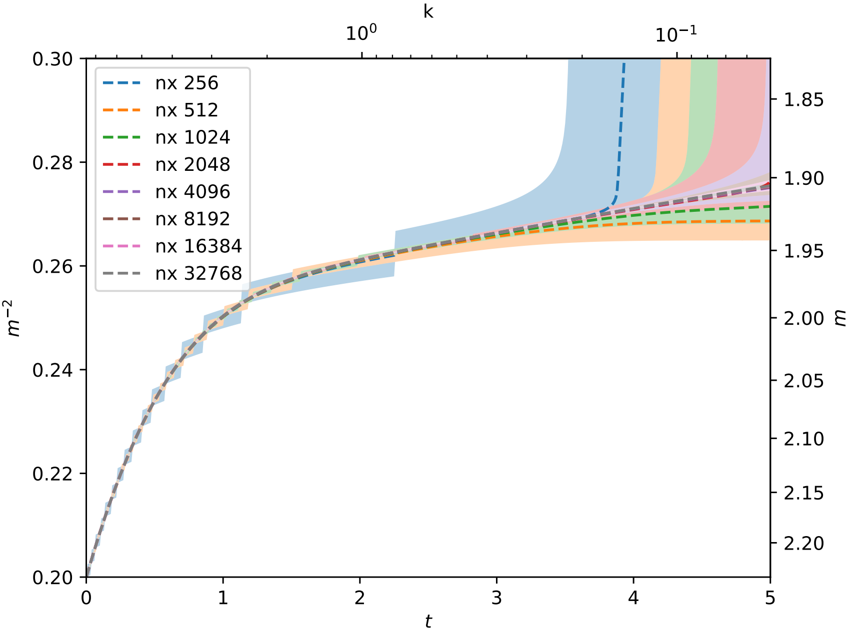
σ	π	ψ
diffusion	advection	source

- This is the deciding test: Do the pions lead to a vanishing sigma mass or can the diffusion rescue it?

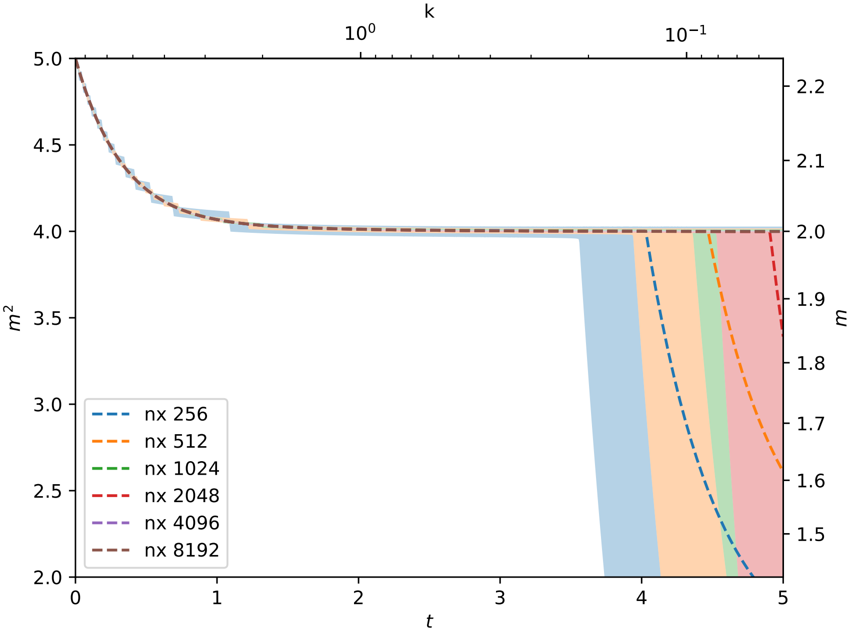
Sigma curvature mass, O(N), $d = 3$



Sigma curvature mass, O(N), $d = 4$



Sigma curvature mass, O(N), $d = 5$



Yukawa model

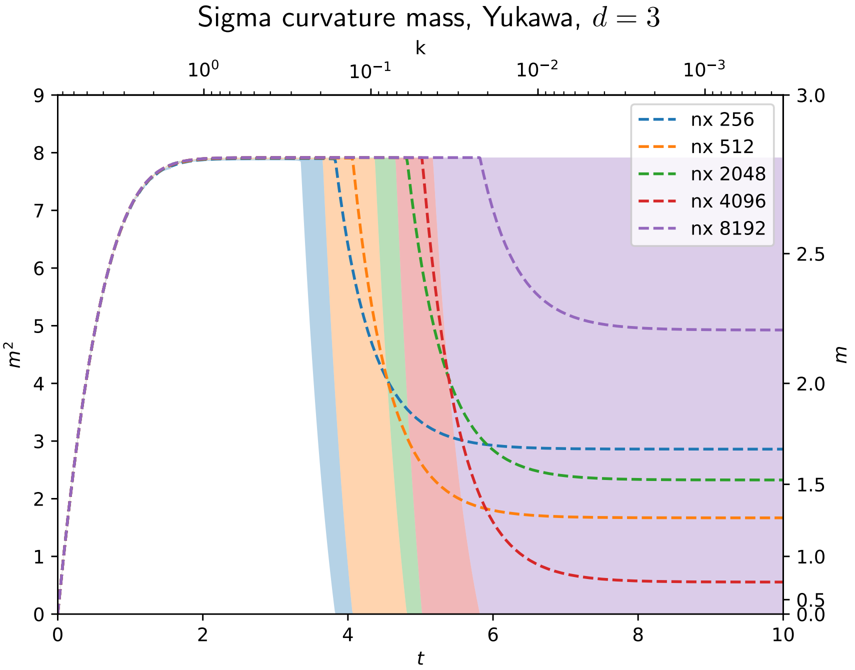
- The pions won!
- The sigma mass of the O(N) model in LPA vanishes in $d = 3, 4$!
- With this unexpected result in mind we continue the investigation

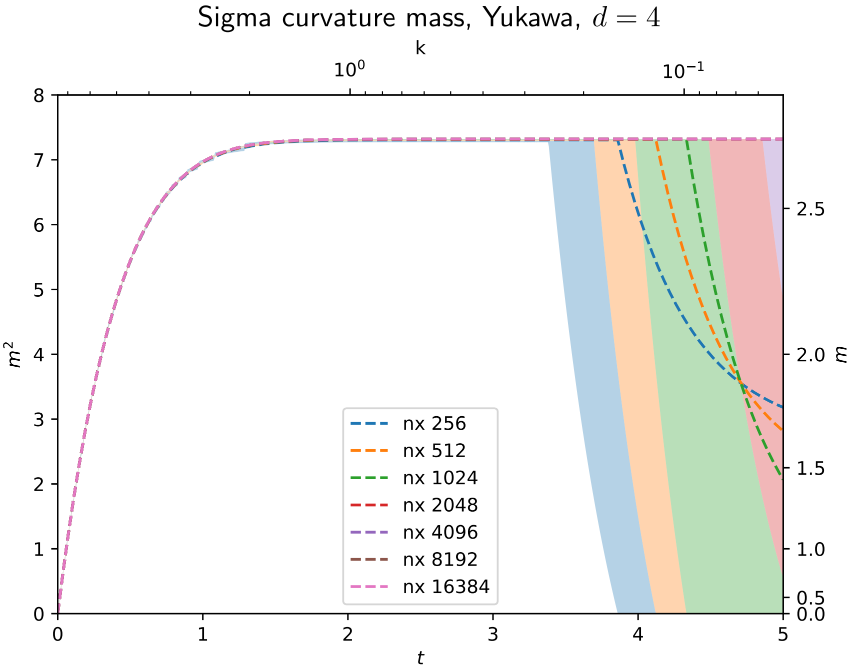
$$\partial_t v = \frac{k^6(3\partial_\rho v + 2\rho\partial_\rho^2 v)}{32\pi^2(k^2 + v + 2\rho\partial_\rho v)^2} + \frac{3k^6\partial_\rho v}{32\pi^2\left(k^2 + v\right)^2} - \frac{N_c g^2 k^6}{2\pi^2(k^2 + g^2\rho)^2}$$

$$\sigma \qquad \pi \qquad \psi$$

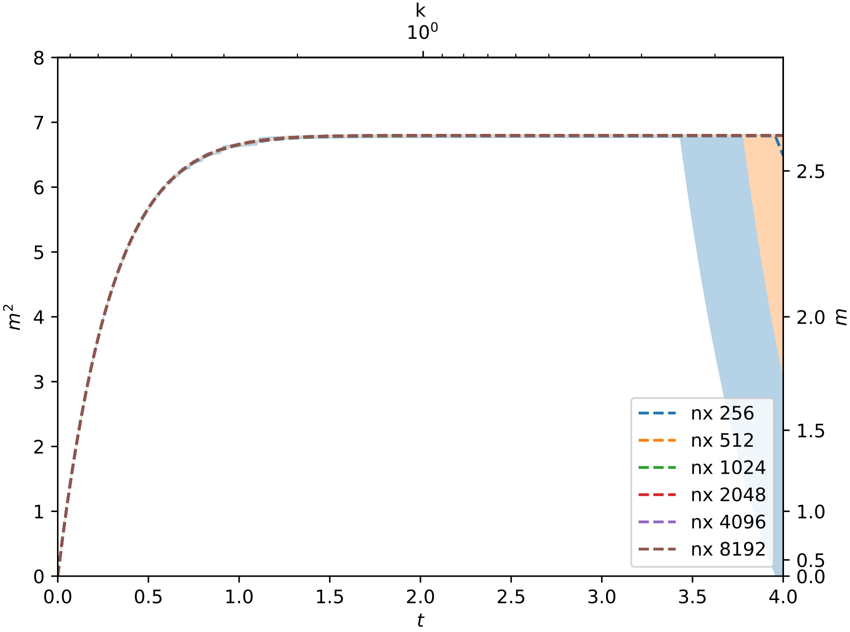
$$\text{diffusion} \qquad \text{advection} \qquad \text{source}$$

- In this Yukawa-like model there are no pions so we expect a finite sigma mass in all dimensions
- Turns out to be numerically the hardest


Sigma curvature mass, Yukawa, $d = 3$
k
10^0
10^{-1}
10^{-2}
10^{-3}
nx 256
nx 512
nx 2048
nx 4096
nx 8192
m^2
m
t
0
1
2
3
4
5
6
7
8
9
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
2
4
6
8
10


Sigma curvature mass, Yukawa, $d = 4$
k
10^0
10^{-1}
m^2
m
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384

Sigma curvature mass, Yukawa, $d = 5$



Quark-meson model

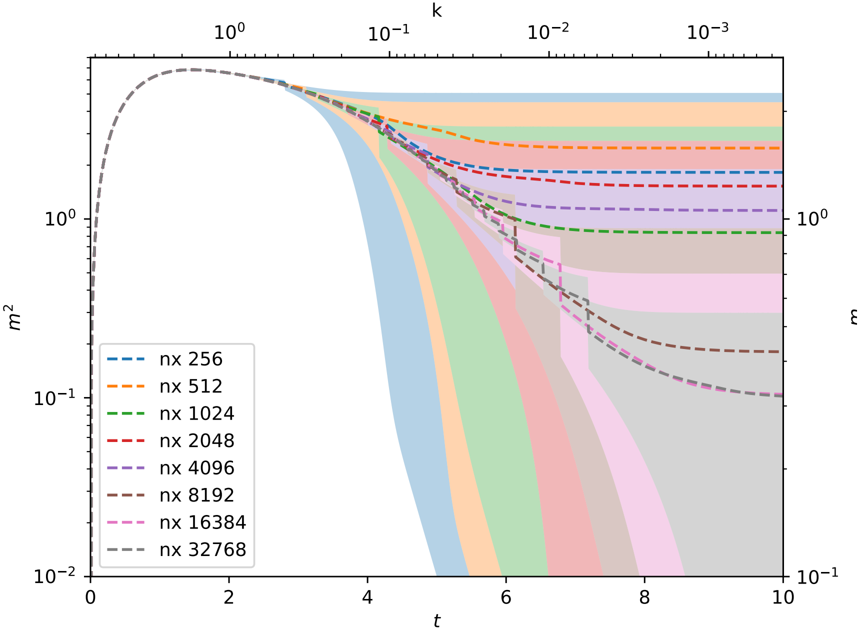
- Finally, the full quark meson model in LPA

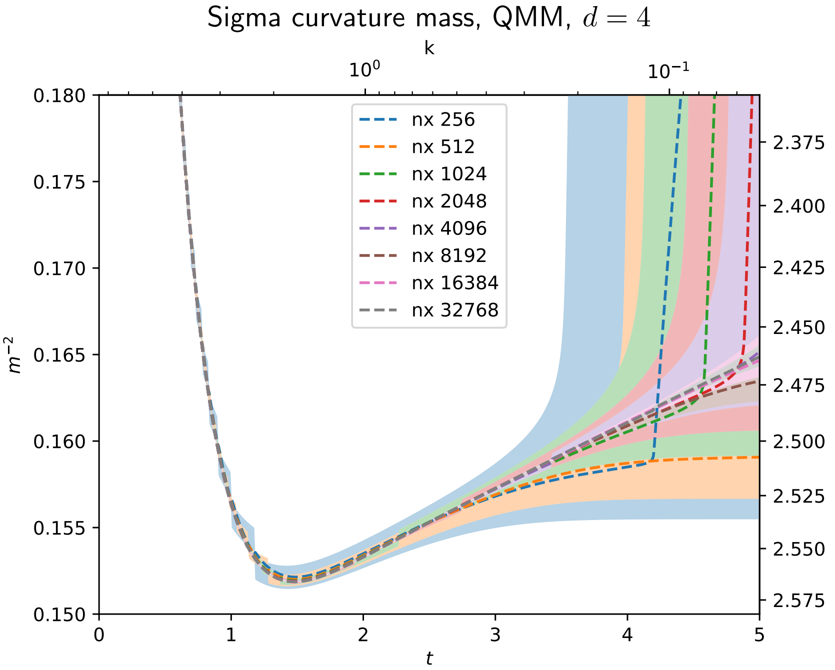
$$\partial_t v = \frac{k^6(3\partial_\rho v + 2\rho\partial_\rho^2 v)}{32\pi^2(k^2 + v + 2\rho\partial_\rho v)^2} + \frac{3k^6\partial_\rho v}{32\pi^2\left(k^2+v\right)^2} - \frac{N_c g^2 k^6}{2\pi^2(k^2+g^2\rho)^2}$$

σ diffusion — π advection — ψ source

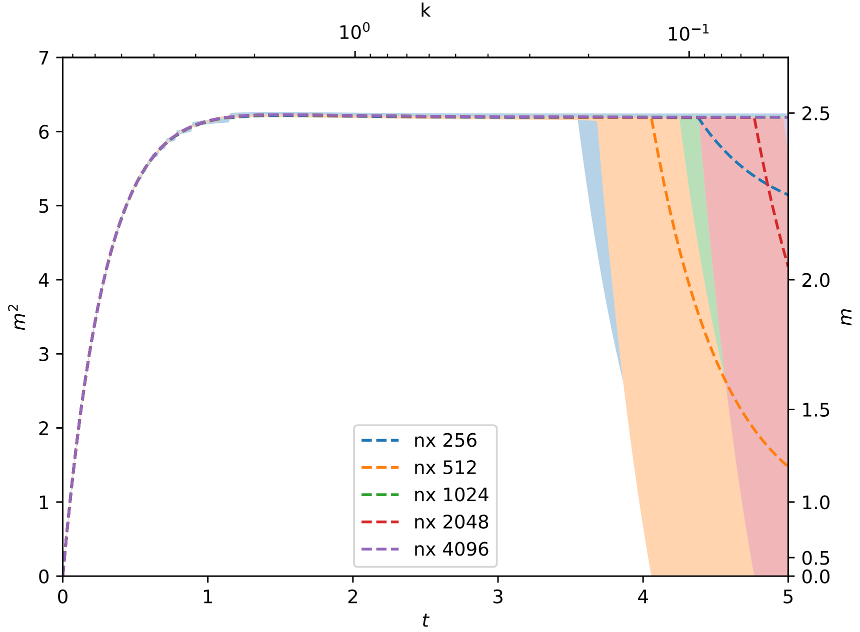
- If the sigma and the quarks do not conspire we will again find a vanishing sigma mass

Sigma curvature mass, QMM, $d = 3$




Sigma curvature mass, QMM, $d = 4$
k
10^0
10^{-1}
0.180
0.175
0.170
0.165
0.160
0.155
0.150
m^{-2}
2.375
2.400
2.425
2.450
2.475
2.500
2.525
2.550
2.575
m
0
1
2
3
4
5
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768

Sigma curvature mass, QMM, $d = 5$



Summary

fields		$d=3$	$d=4$	$d=5$	
σ	ϕ^4	$m^2 \sim 1$	$m^2 \sim 1$	$m^2 \sim 1$	
π	large N O(N)	$m^2 \sim \exp(-t)$	$m^2 \sim t^{-1}$	$m^2 \sim 1$	exactly solvable
ψ	MFA	$m^2 \sim 1$	$m^2 \sim 1$	$m^2 \sim 1$	exactly solvable
$\sigma\pi$	O(N)	$m^2 \sim \exp(-t)$	$m^2 \sim t^{-1}$	$m^2 \sim 1$	
σ ψ	Yukawa	$m^2 \sim 1$	$m^2 \sim 1$	$m^2 \sim 1$	
$\pi\psi$	large N_f QMM	$m^2 \sim \exp(-t)$	$m^2 \sim t^{-1}$	$m^2 \sim 1$	reduces to ODE
$\sigma\pi\psi$	QMM	$m^2 \sim \exp(-t)$	$m^2 \sim t^{-1}$	$m^2 \sim 1$	

Open questions

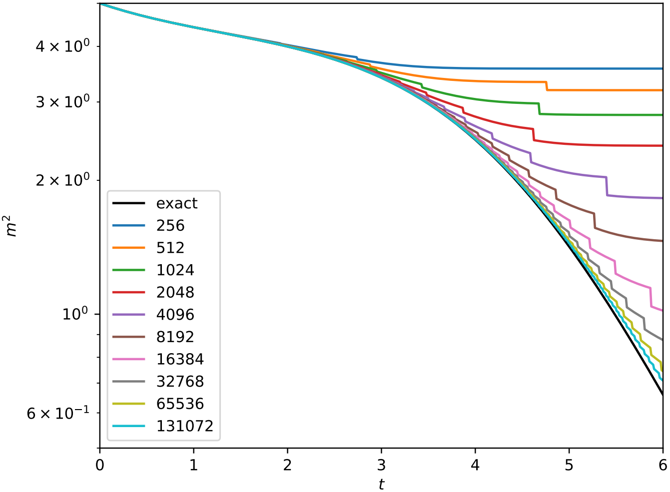
- What about other regulators?
  - $(d-1)$ Litim shows same behavior
  - Unknown in general
- What about explicit chiral symmetry breaking?
  - Mass will be finite
  - Size of splitting between m_σ and m_π unknown
- How to get numerically further into the IR
  - for $d = 4$
  - and small Δx

Main question

How to get extraordinary evidence for the numerical results?

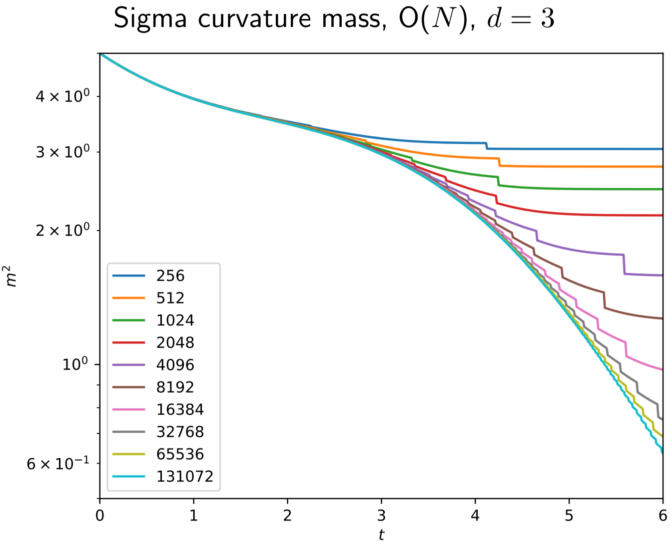
Towards extraordinary evidence

Sigma curvature mass, Large N O(N), $d = 3$



Point of evaluation shifted by three grid cells to the right

Towards extraordinary evidence



Point of evaluation shifted by three grid cells to the right

Conclusion

Result

$$d = 3, 4$$

&

LPA

&

in symmetry broken phase

&

in chiral limit

&

pion fluctuations included

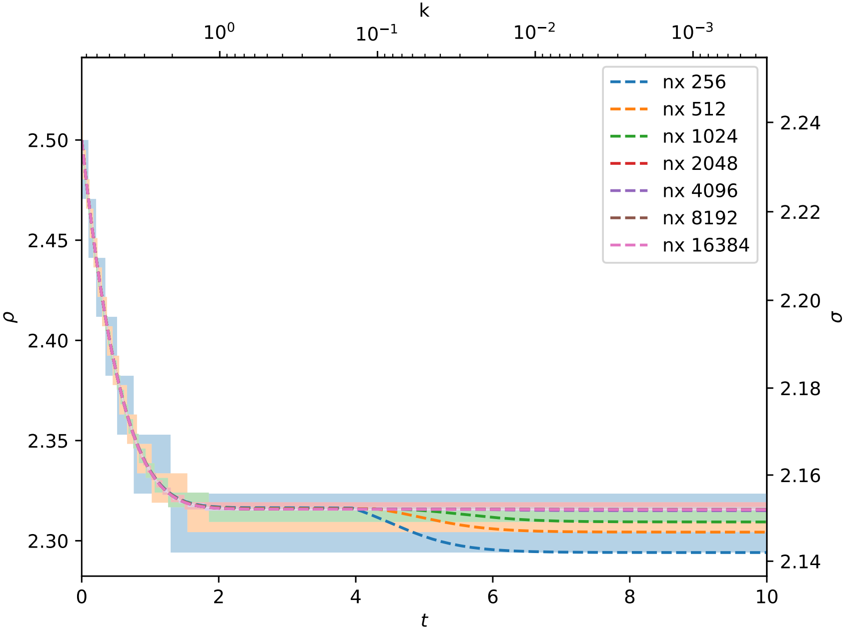
$$\implies m(t \to \infty) = 0$$

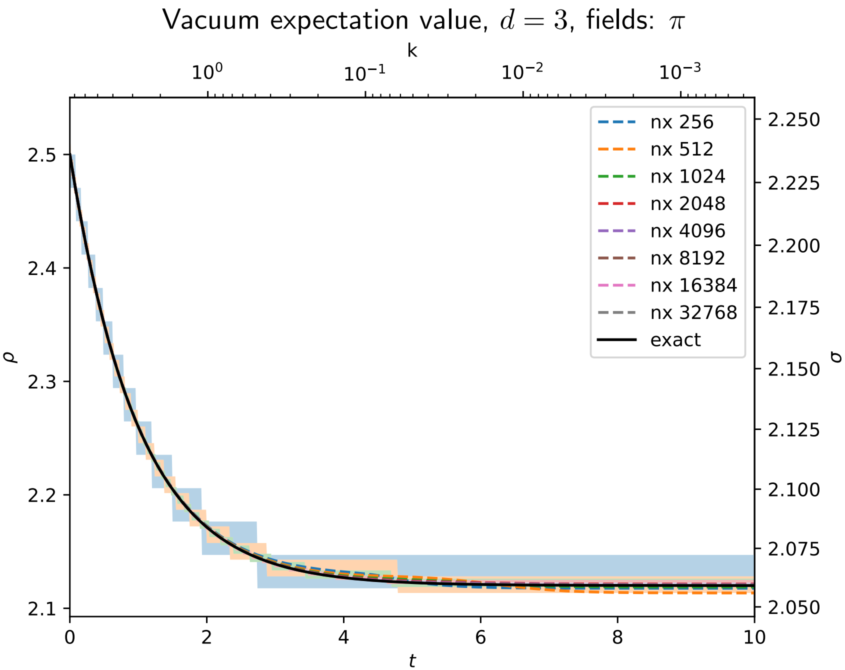
Outlook

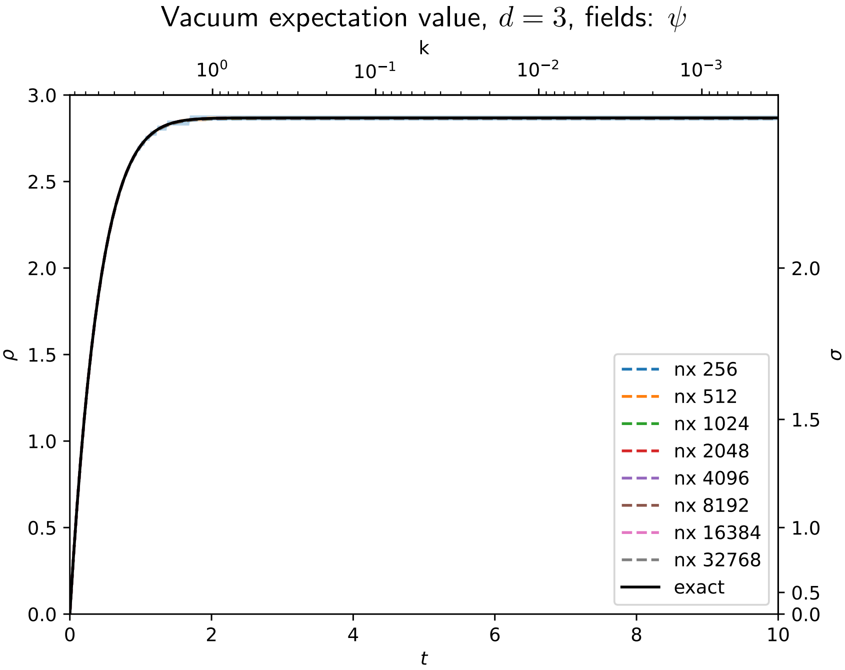
Analyse convergence for algebraic scaling

Appendix

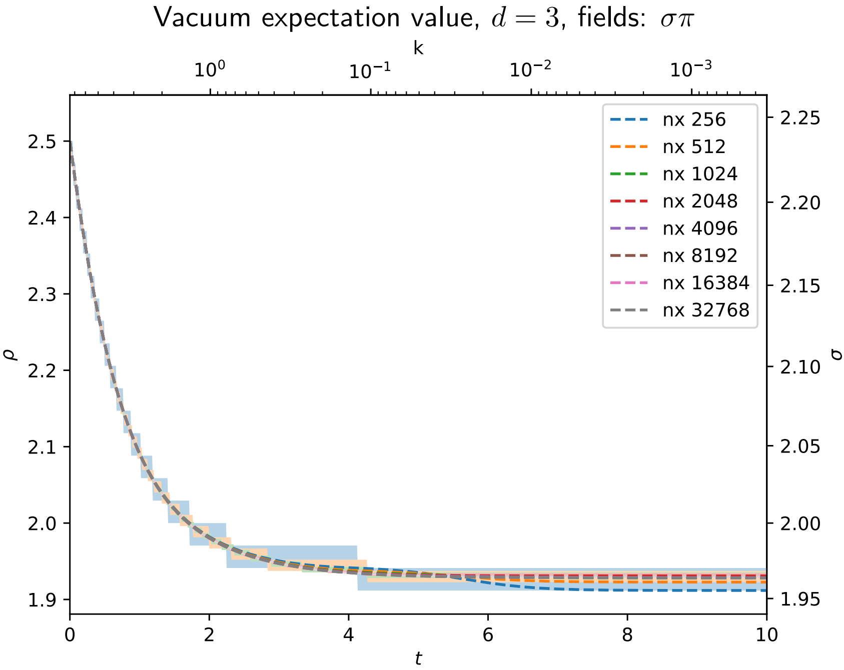
Vacuum expectation value, $d = 3$, fields: σ

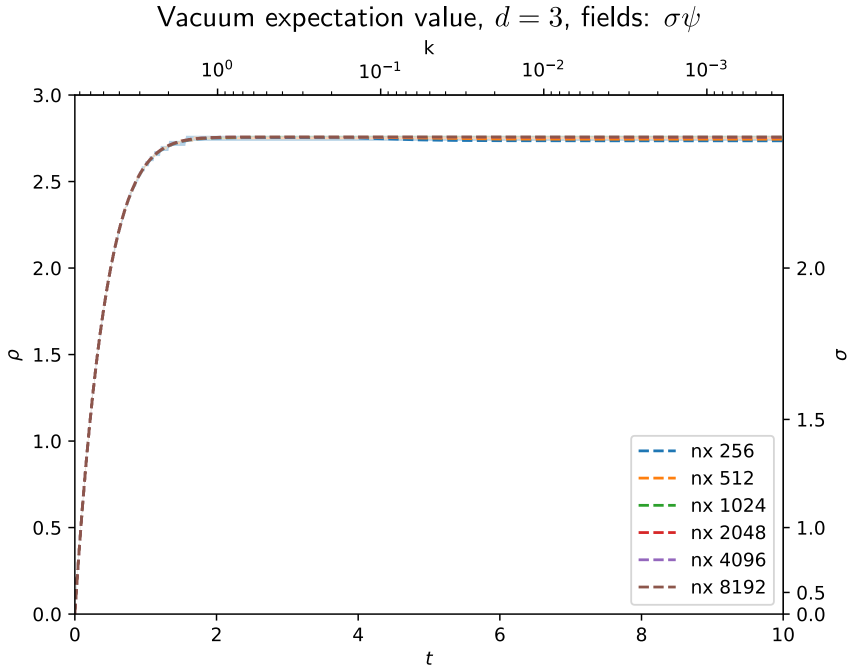



Vacuum expectation value, $d = 3$, fields: π
k
10^0
10^{-1}
10^{-2}
10^{-3}
ρ
σ
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact

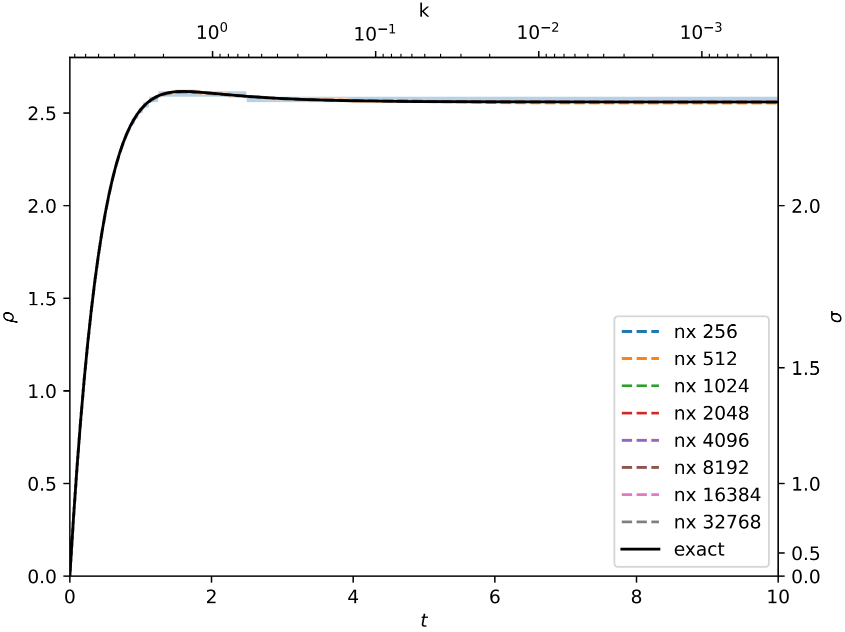


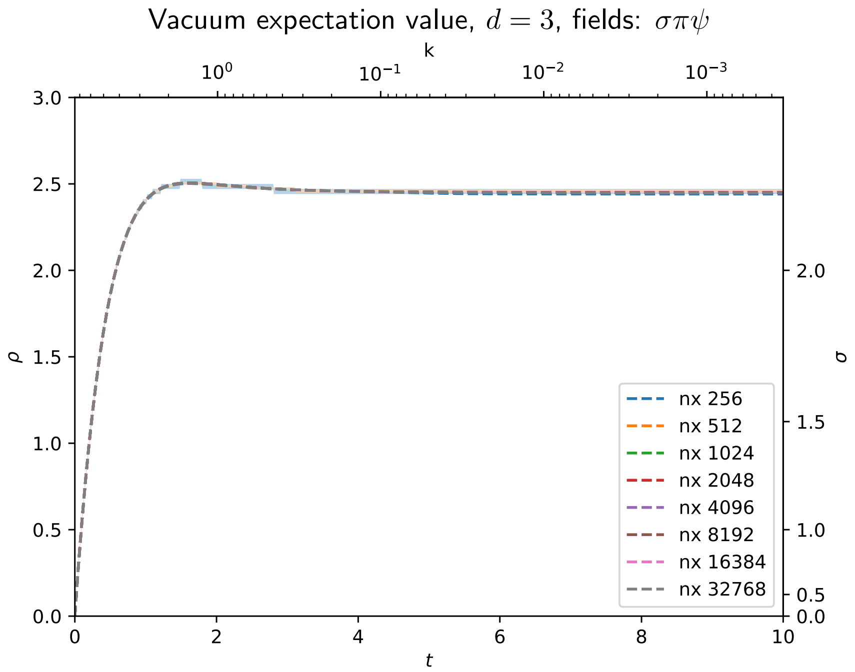
Vacuum expectation value, $d = 3$, fields: ψ
k
10^{0}
10^{-1}
10^{-2}
10^{-3}
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ρ
σ
2.0
1.5
1.0
0.5
0.0
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact
0
2
4
6
8
10
t


Vacuum expectation value, $d = 3$, fields: $\sigma\pi$
k
10^{0}
10^{-1}
10^{-2}
10^{-3}
ρ
σ
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
2.5
2.4
2.3
2.2
2.1
2.0
1.9
2.25
2.20
2.15
2.10
2.05
2.00
1.95
0
2
4
6
8
10

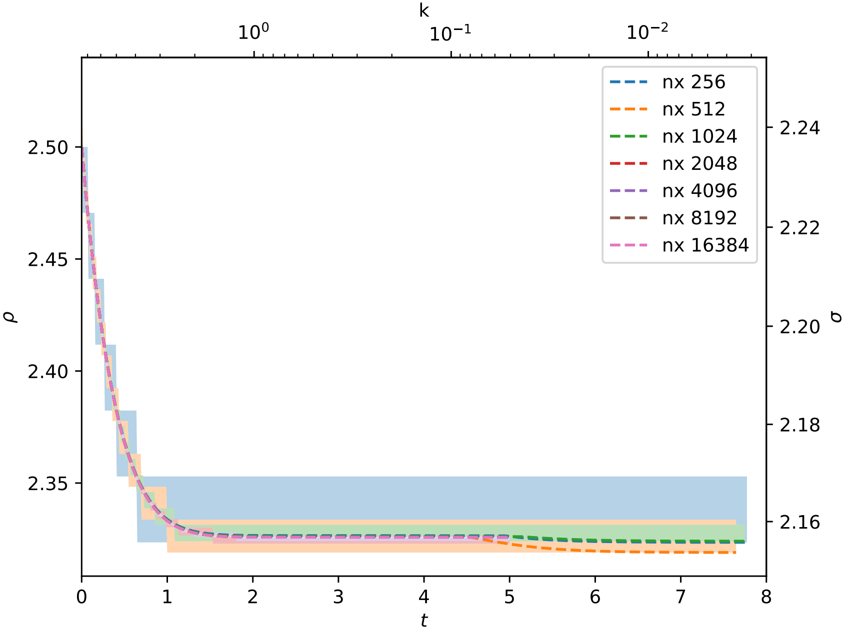

Vacuum expectation value, $d = 3$, fields: $\sigma\psi$
k
10^{0}
10^{-1}
10^{-2}
10^{-3}
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ρ
σ
2.0
1.5
1.0
0.5
0.0
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
0
2
4
6
8
10
t

Vacuum expectation value, $d = 3$, fields: $\pi\psi$

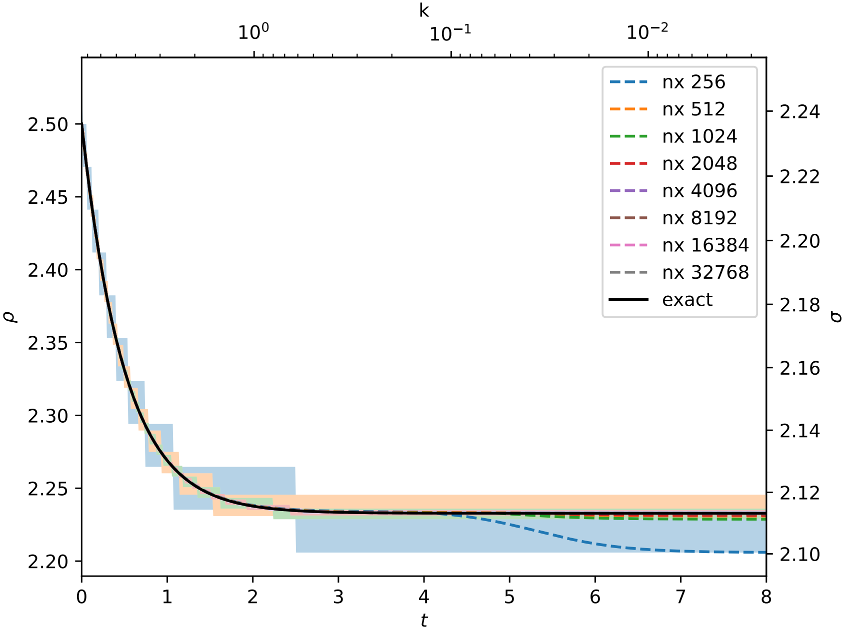


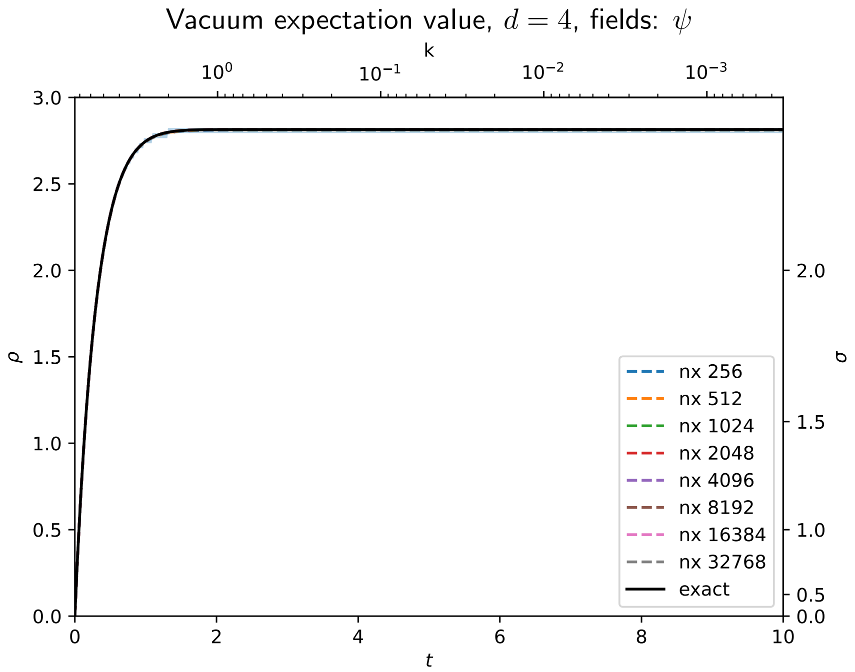

Vacuum expectation value, $d = 3$, fields: $\sigma\pi\psi$
k
10^0
10^{-1}
10^{-2}
10^{-3}
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ρ
σ
2.0
1.5
1.0
0.5
0.0
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
0
2
4
6
8
10
t

Vacuum expectation value, $d = 4$, fields: σ

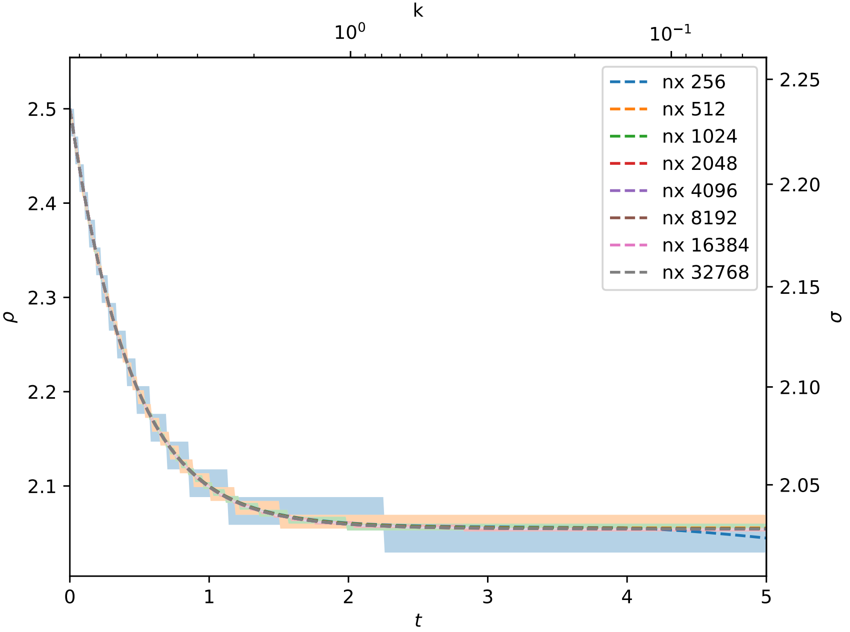


Vacuum expectation value, $d = 4$, fields: π

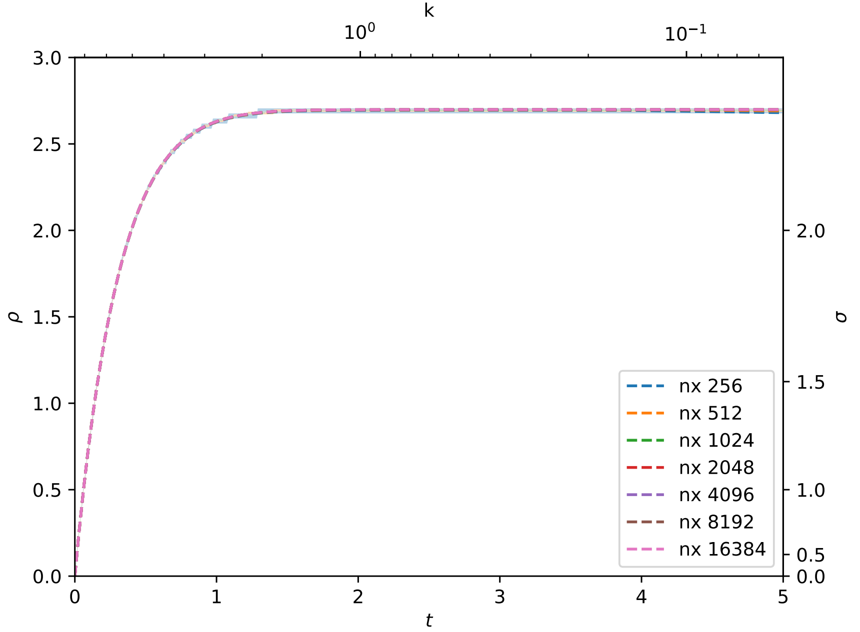


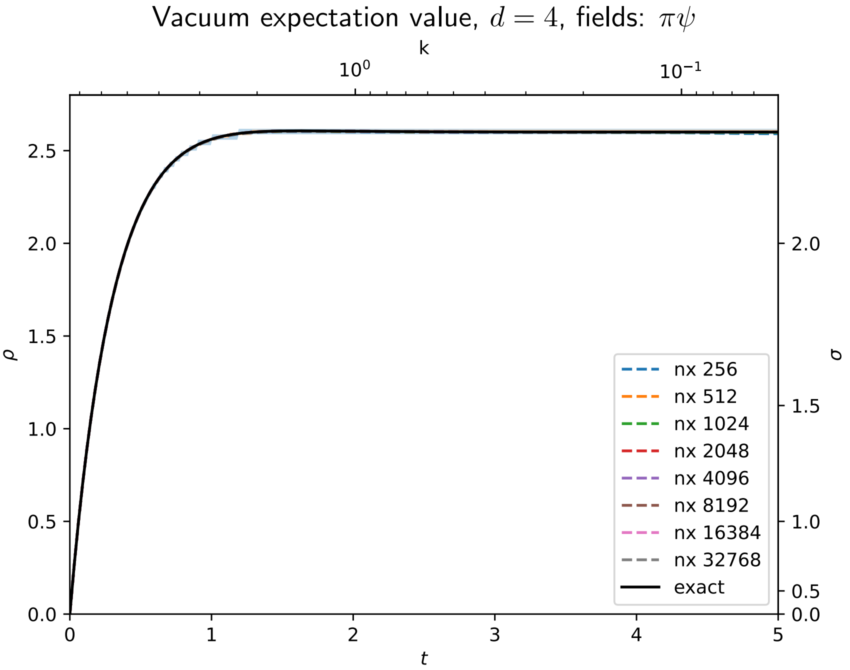

Vacuum expectation value, $d = 4$, fields: ψ
k
10^0
10^{-1}
10^{-2}
10^{-3}
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ρ
σ
2.0
1.5
1.0
0.5
0.0
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact
0
2
4
6
8
10
t

Vacuum expectation value, $d = 4$, fields: $\sigma\pi$



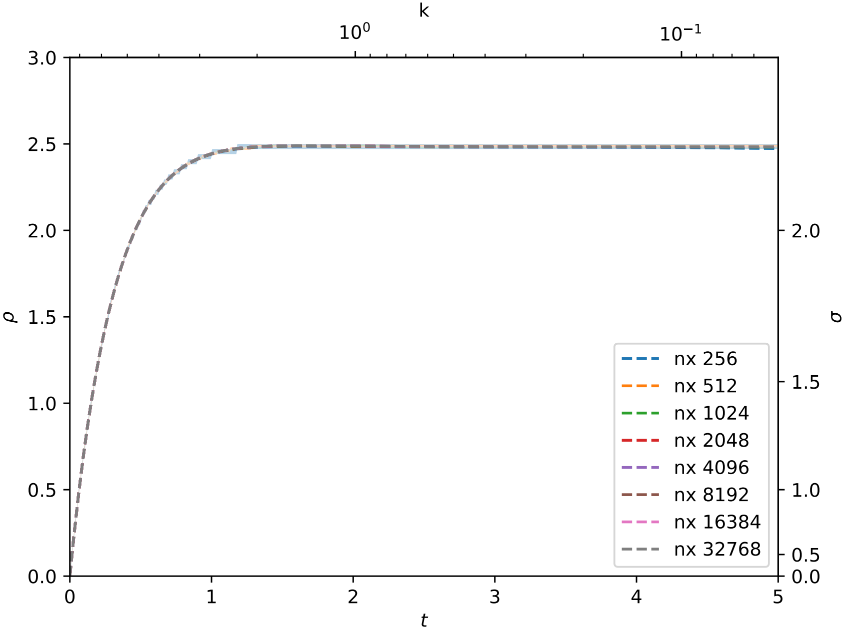
Vacuum expectation value, $d = 4$, fields: $\sigma\psi$



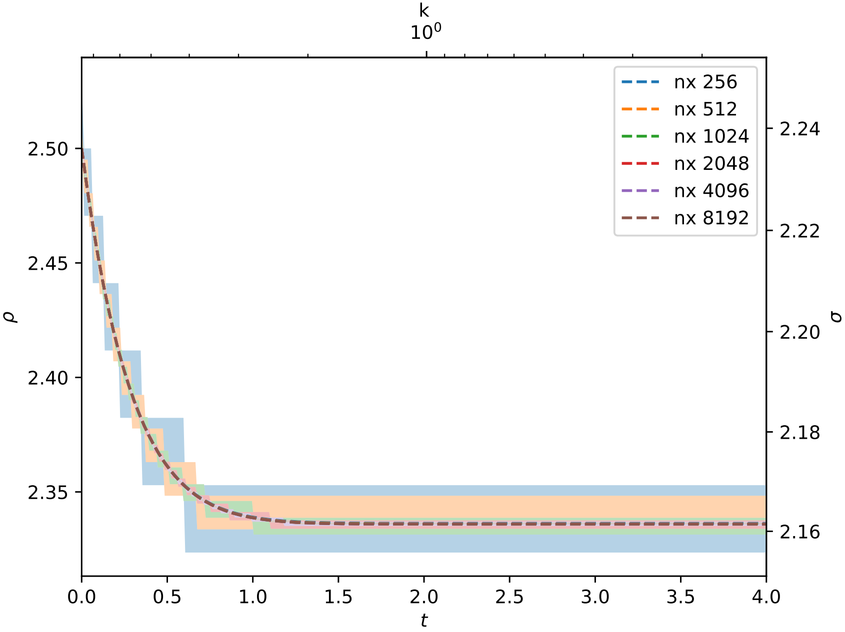


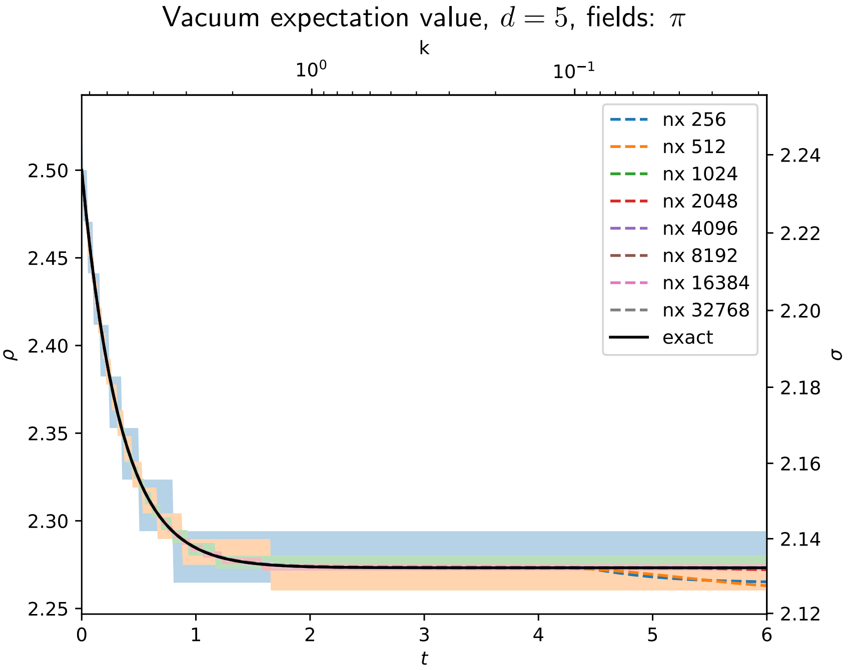
Vacuum expectation value, $d = 4$, fields: $\pi\psi$
k
10^0
10^{-1}
2.5
2.0
1.5
1.0
0.5
0.0
ρ
σ
2.0
1.5
1.0
0.5
0.0
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact
0
1
2
3
4
5
t

Vacuum expectation value, $d = 4$, fields: $\sigma\pi\psi$

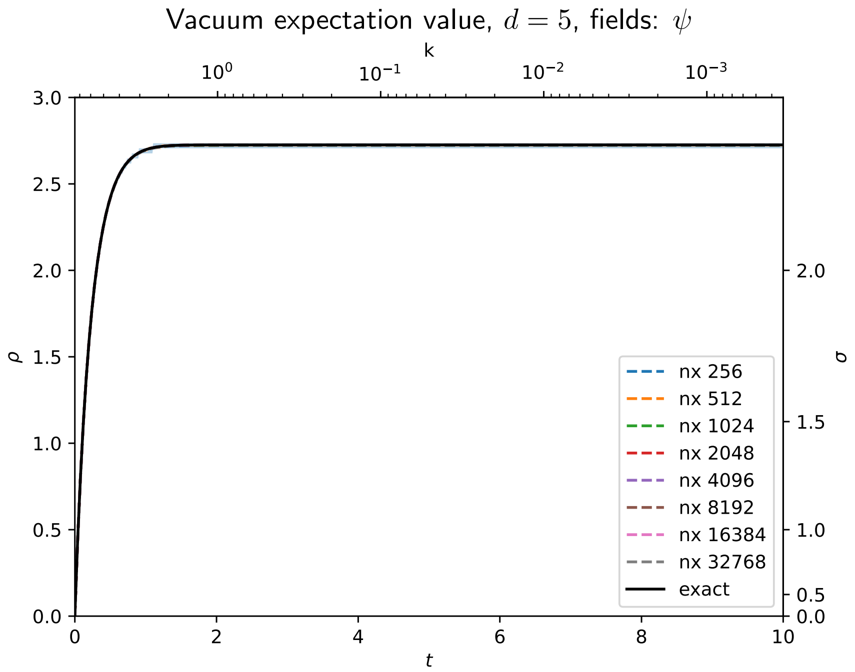


Vacuum expectation value, $d = 5$, fields: σ

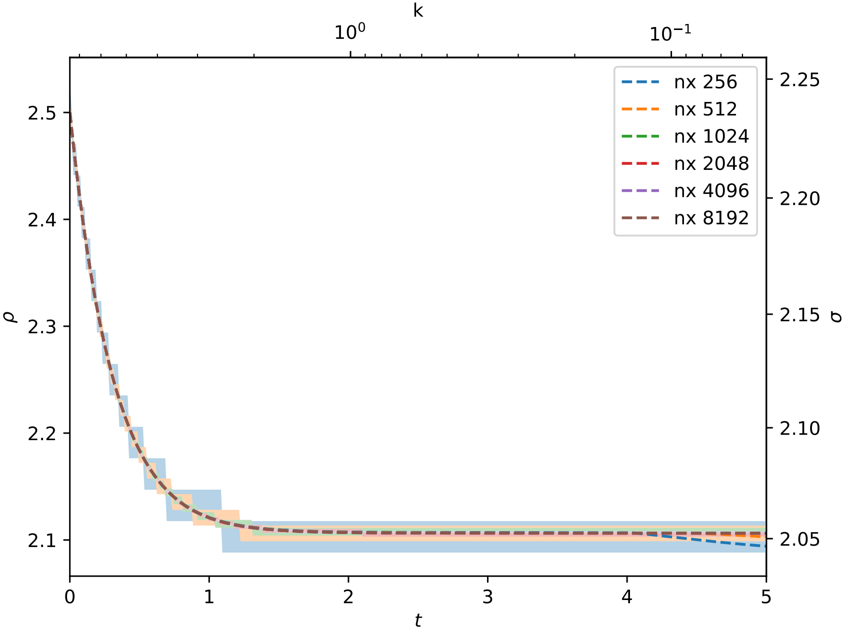




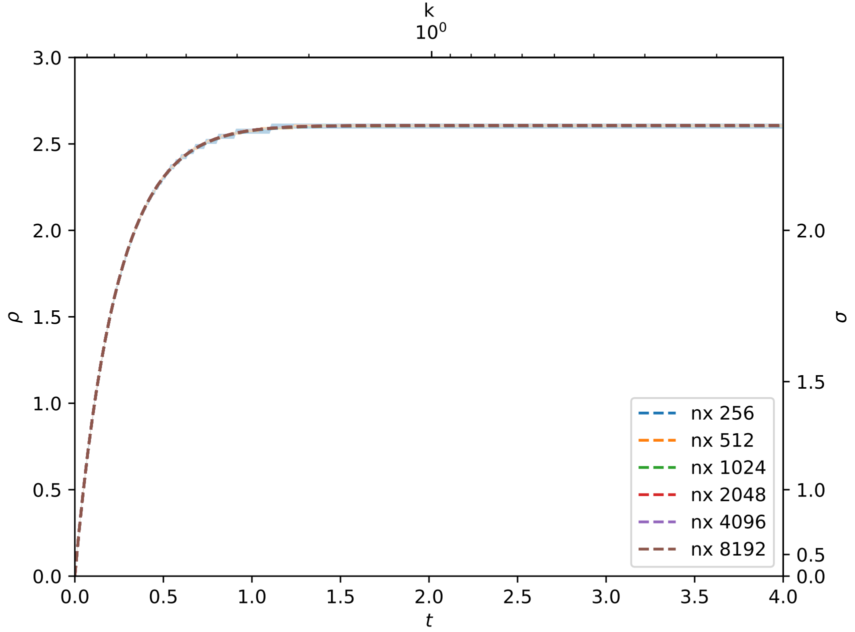
Vacuum expectation value, $d = 5$, fields: π
k
10^0
10^{-1}
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact
ρ
2.50
2.45
2.40
2.35
2.30
2.25
σ
2.24
2.22
2.20
2.18
2.16
2.14
2.12
0
1
2
3
4
5
6
t


Vacuum expectation value, $d = 5$, fields: ψ
k
10^{0}
10^{-1}
10^{-2}
10^{-3}
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ρ
σ
2.0
1.5
1.0
0.5
0.0
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact
0
2
4
6
8
10
t

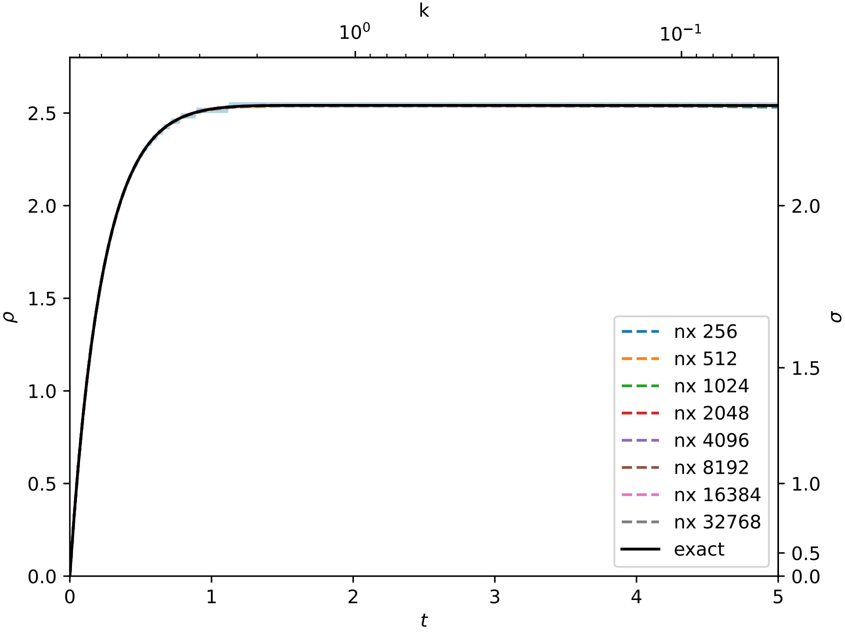
Vacuum expectation value, $d = 5$, fields: $\sigma\pi$



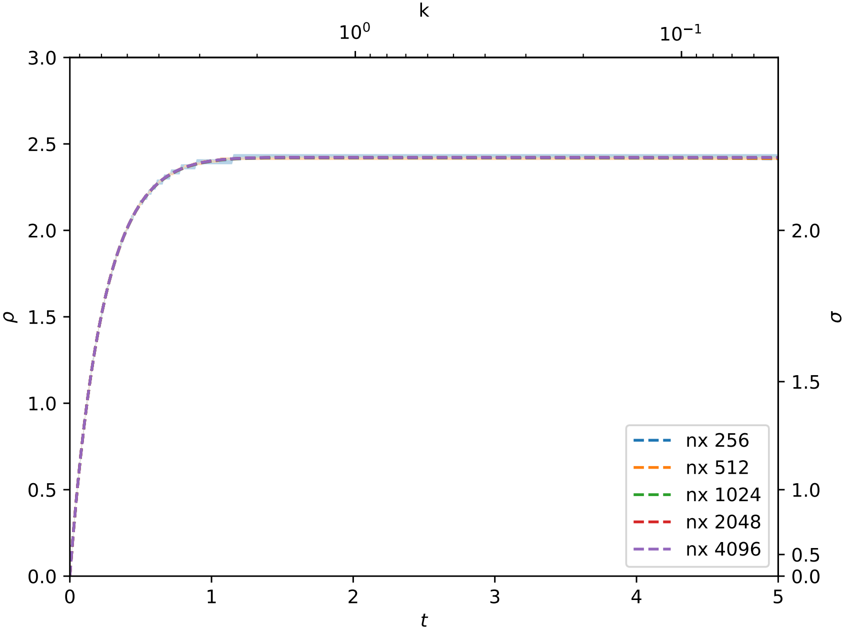
Vacuum expectation value, $d = 5$, fields: $\sigma\psi$

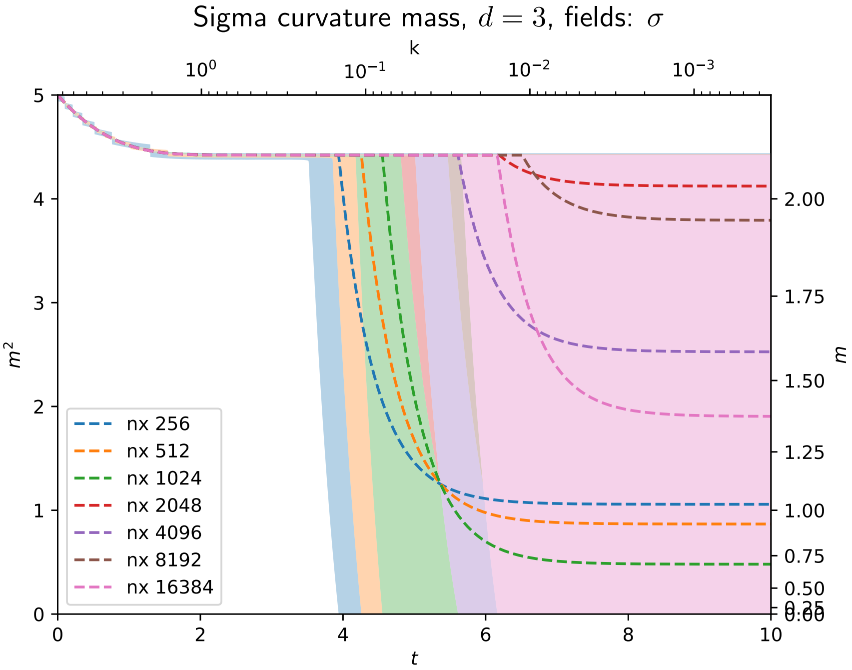


Vacuum expectation value, $d = 5$, fields: $\pi\psi$

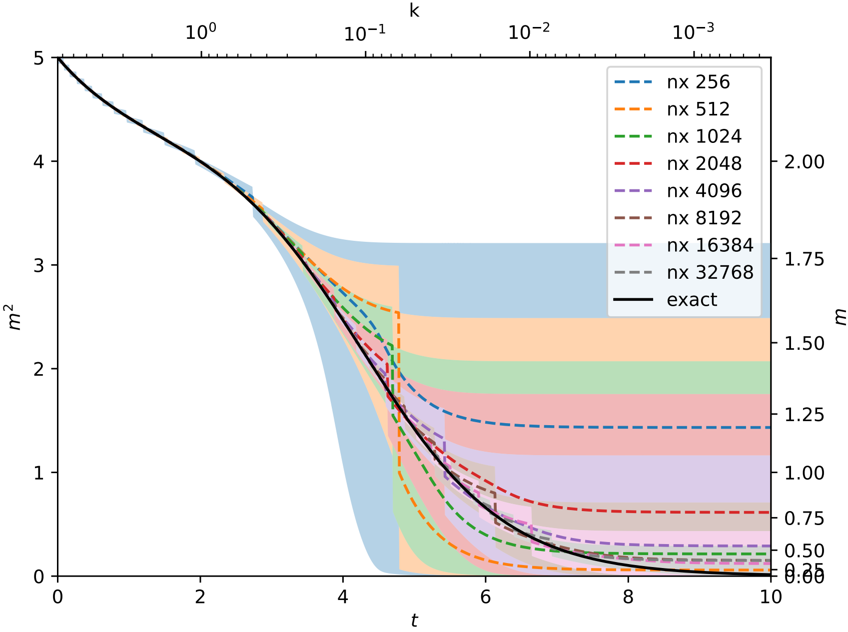


Vacuum expectation value, $d = 5$, fields: $\sigma\pi\psi$

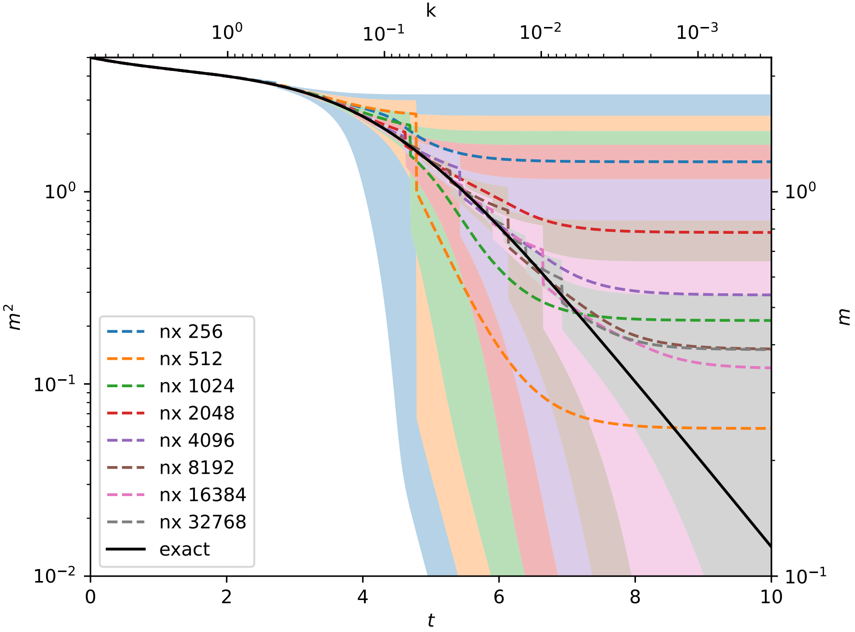


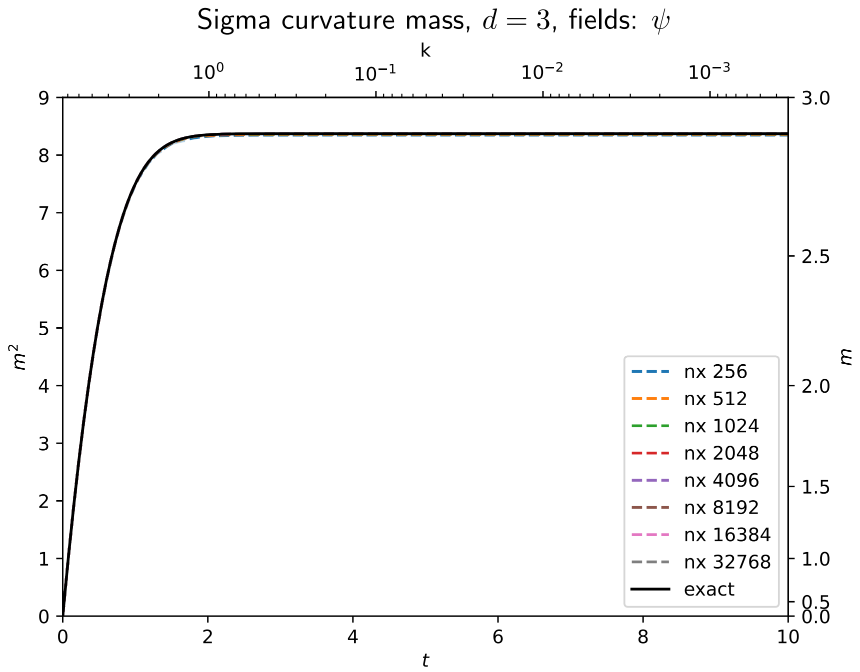

Sigma curvature mass, $d = 3$, fields: σ
k
10^0
10^{-1}
10^{-2}
10^{-3}
m^2
m
t
0
1
2
3
4
5
0
2
4
6
8
10
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384

Sigma curvature mass, $d = 3$, fields: π



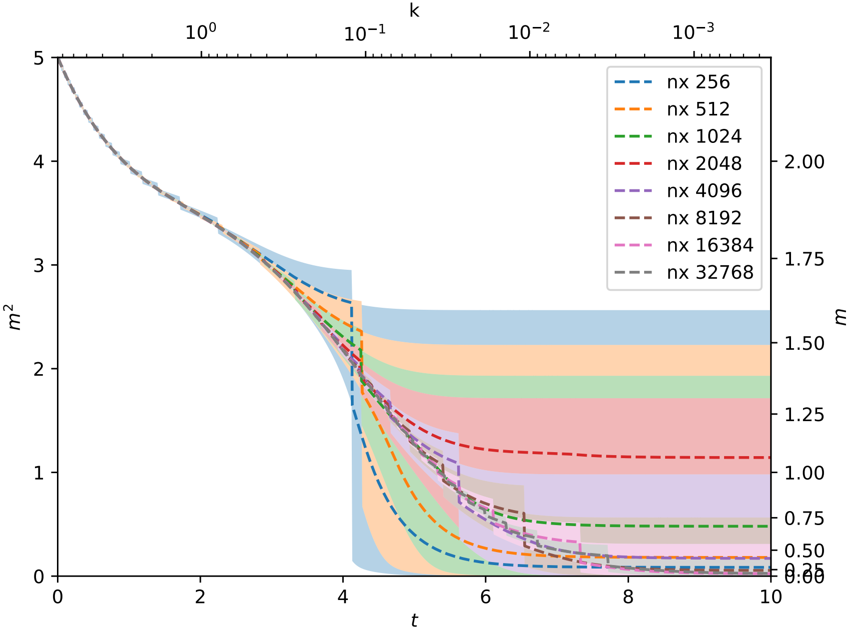
Sigma curvature mass, $d = 3$, fields: π

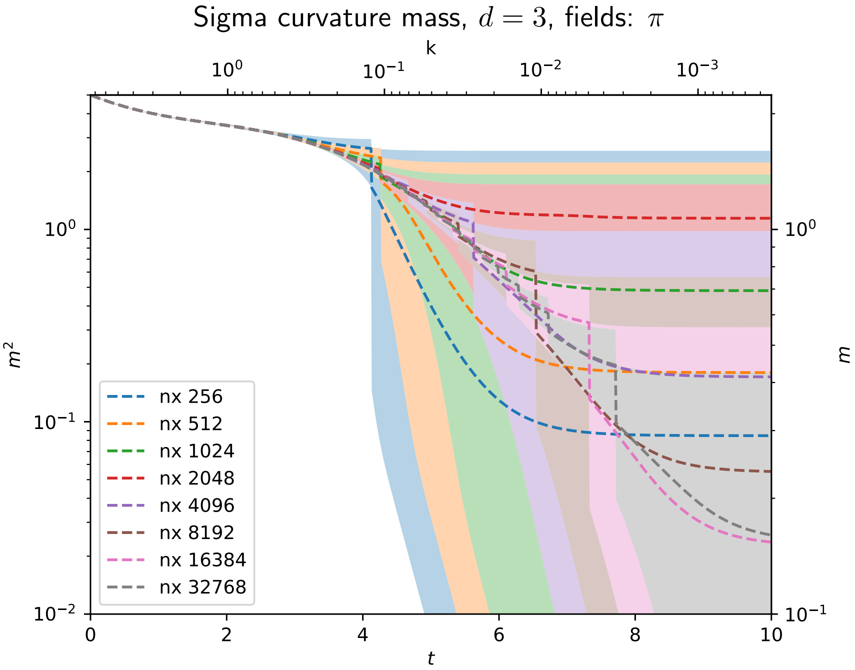


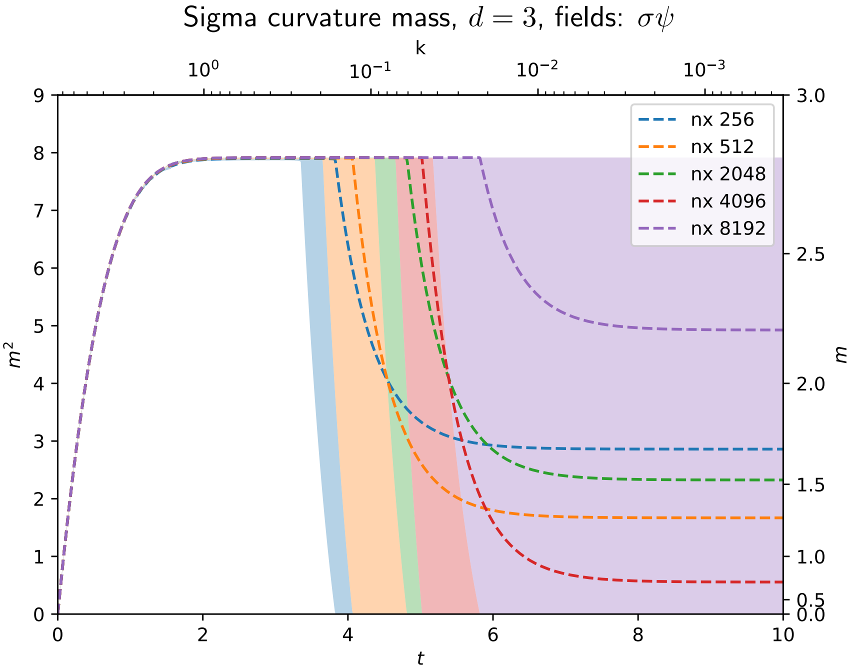


Sigma curvature mass, $d = 3$, fields: ψ
k
10^0
10^{-1}
10^{-2}
10^{-3}
m^2
m
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact

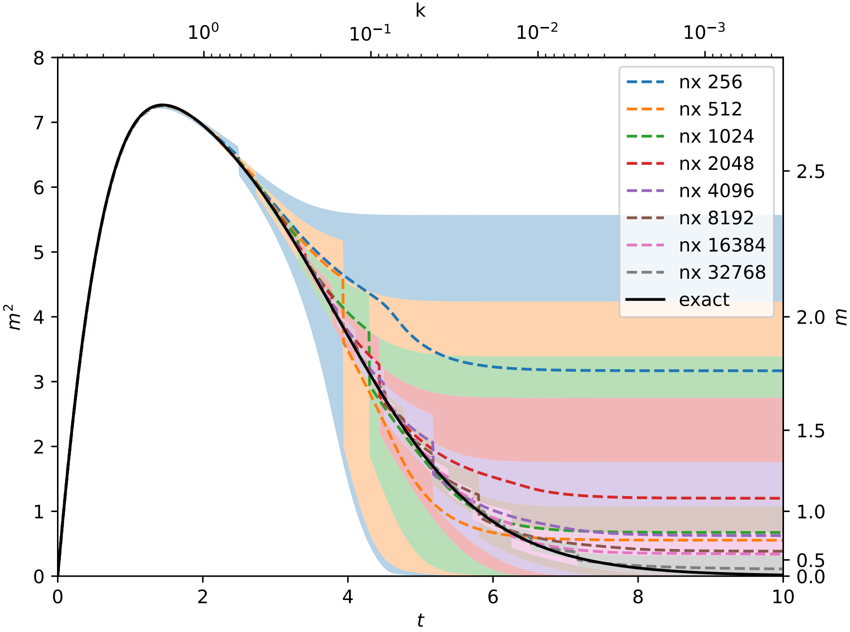
Sigma curvature mass, $d = 3$, fields: $\sigma\pi$



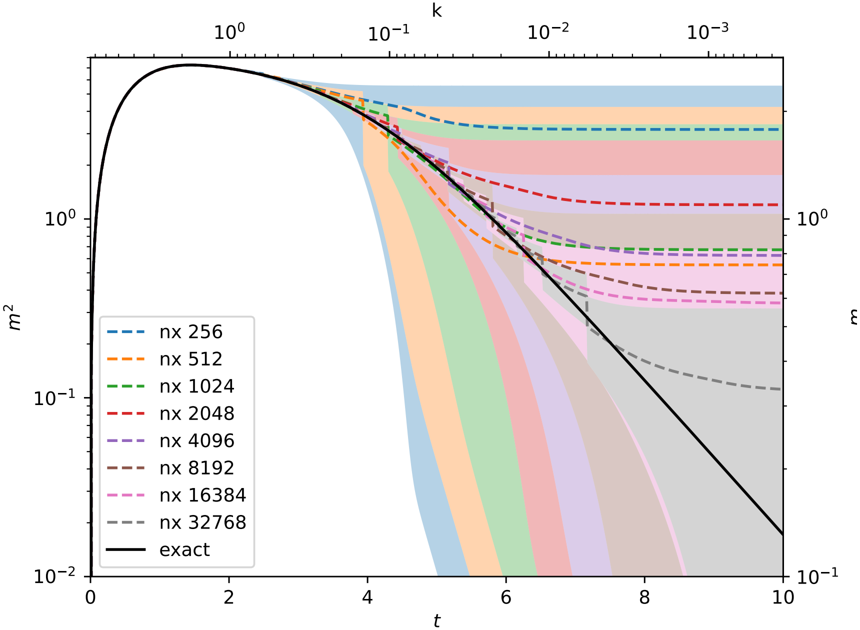

Sigma curvature mass, $d = 3$, fields: π
k
10^0
10^{-1}
10^{-2}
10^{-3}
m^2
m
10^0
10^{-1}
10^{-2}
10^0
10^{-1}
0
2
4
6
8
10
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768


Sigma curvature mass, $d = 3$, fields: $\sigma\psi$
k
10^0
10^{-1}
10^{-2}
10^{-3}
nx 256
nx 512
nx 2048
nx 4096
nx 8192
m^2
m
t
0
1
2
3
4
5
6
7
8
9
0
2
4
6
8
10
3.0
2.5
2.0
1.5
1.0
0.5
0.0

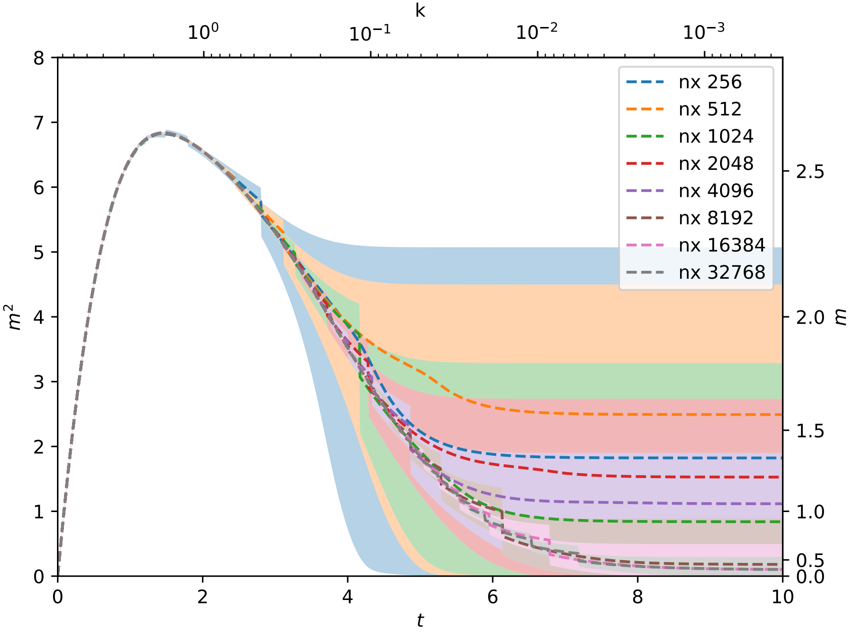
Sigma curvature mass, $d = 3$, fields: $\pi\psi$



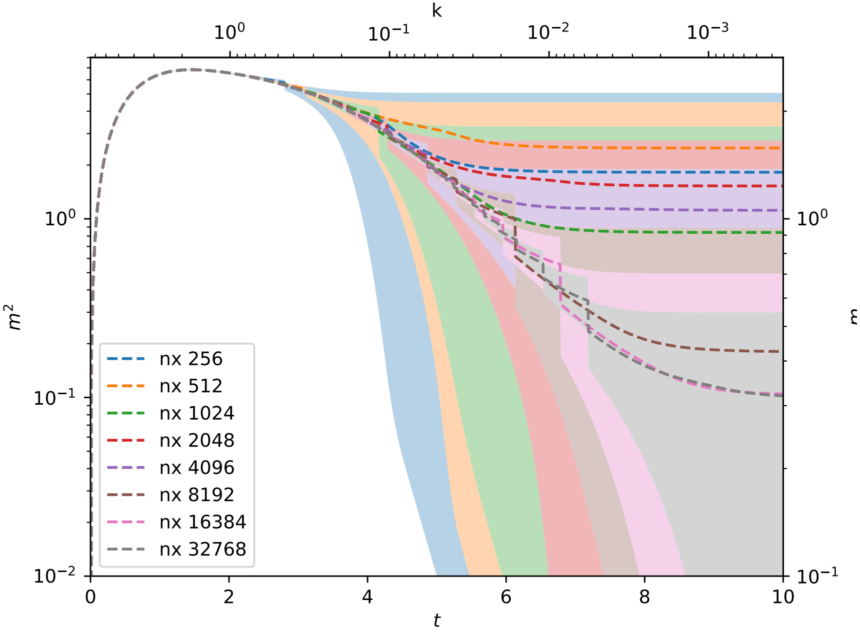
Sigma curvature mass, $d = 3$, fields: π

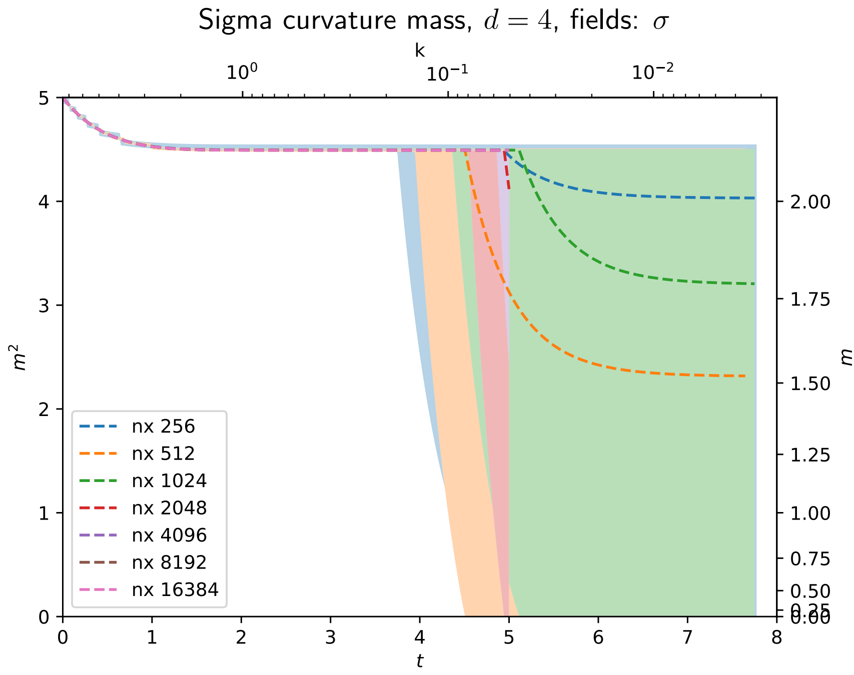


Sigma curvature mass, $d = 3$, fields: $\sigma\pi\psi$



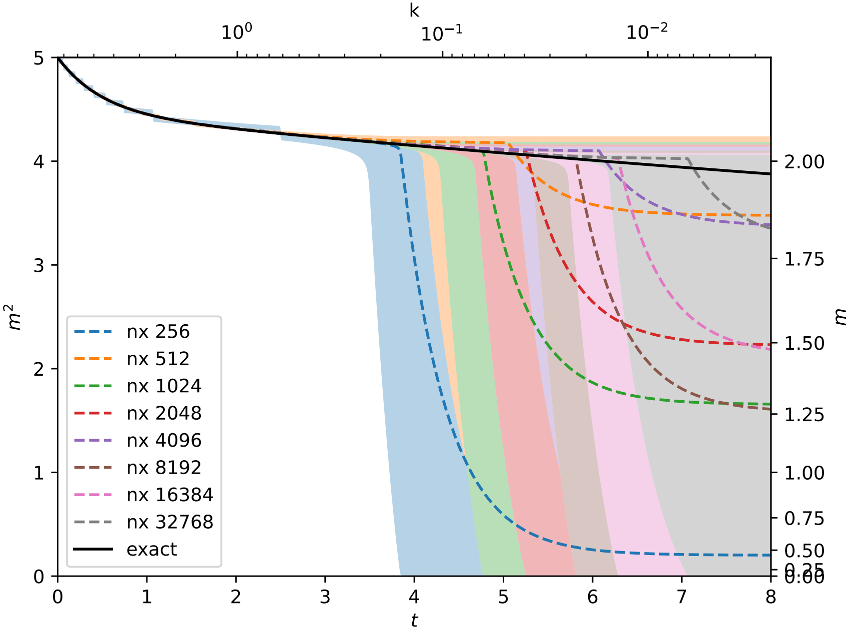
Sigma curvature mass, $d = 3$, fields: π



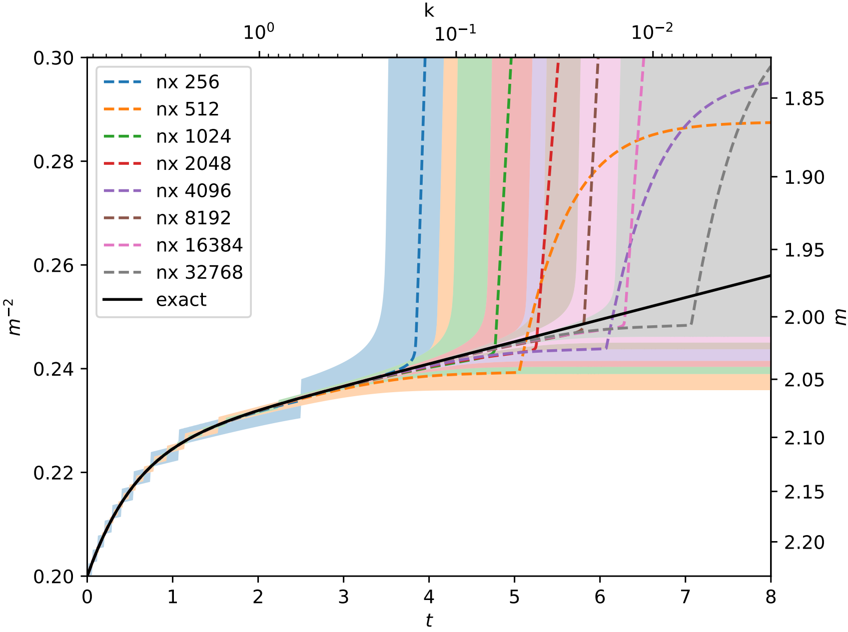


Sigma curvature mass, $d = 4$, fields: σ
k
10^0
10^{-1}
10^{-2}
m^2
m
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384

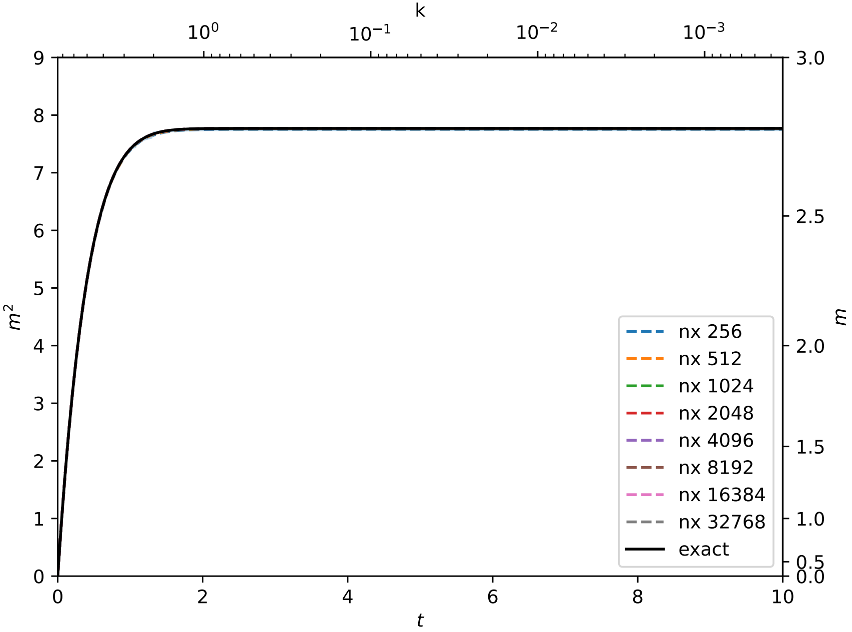
Sigma curvature mass, $d = 4$, fields: π

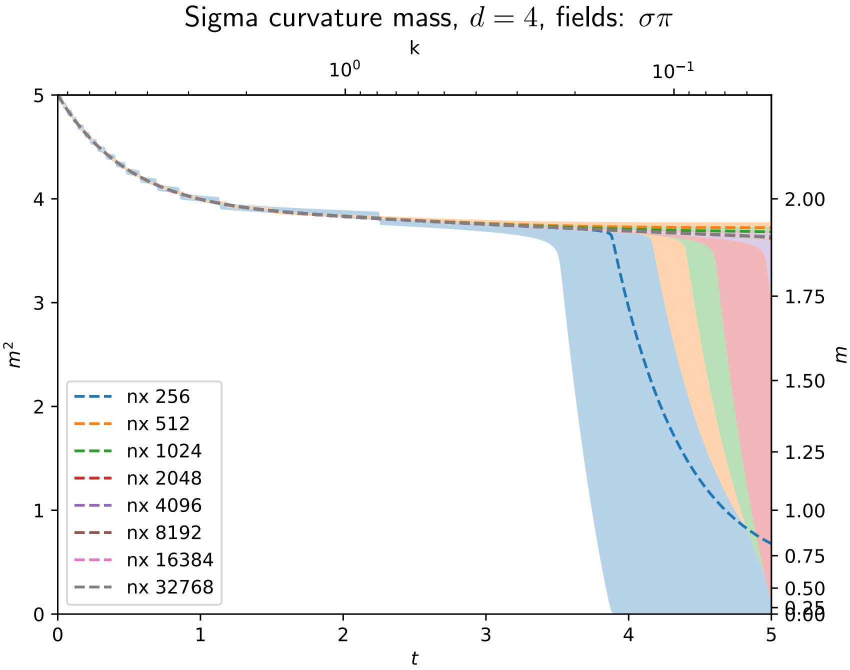


Sigma curvature mass, $d = 4$, fields: π

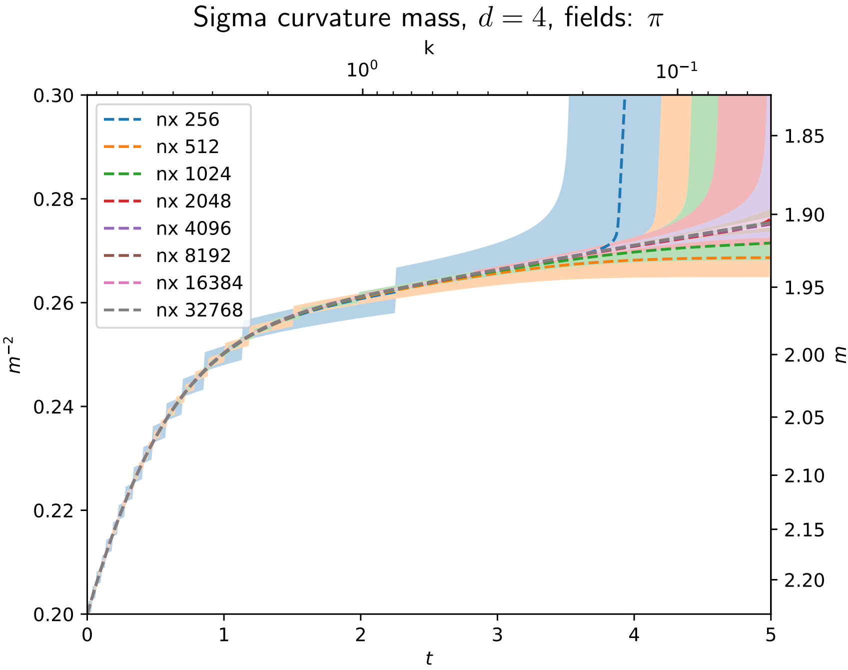


Sigma curvature mass, $d = 4$, fields: ψ



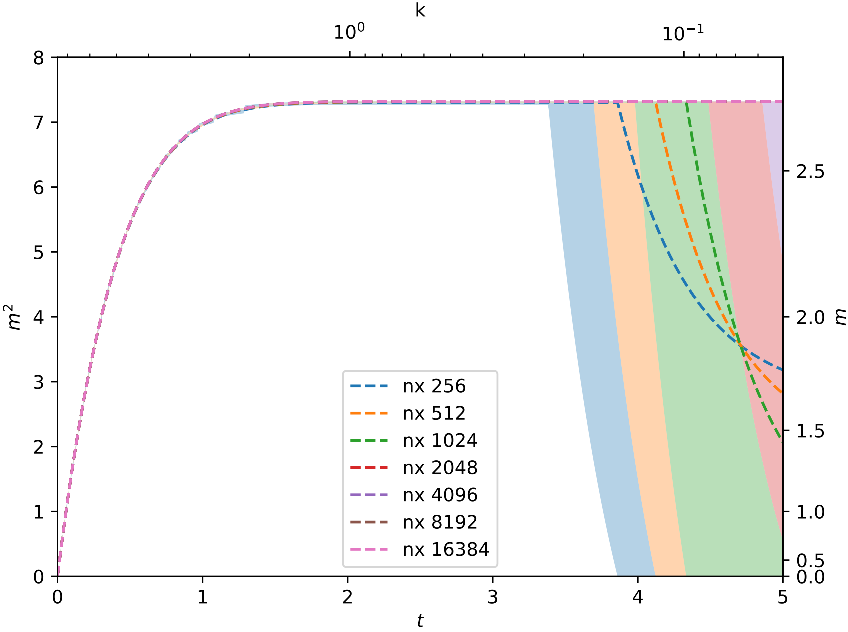


Sigma curvature mass, $d = 4$, fields: $\sigma\pi$
k
10^0
10^{-1}
m^2
m
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768

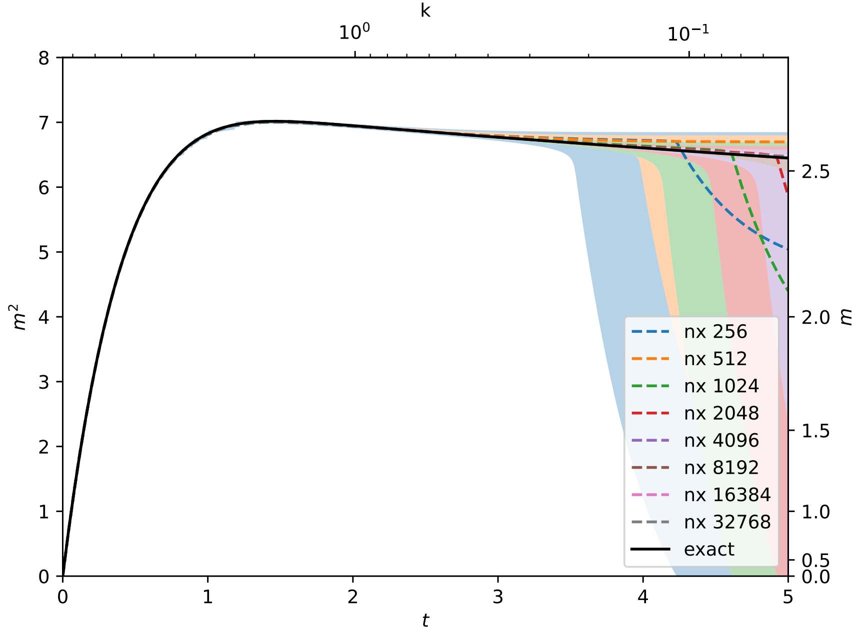


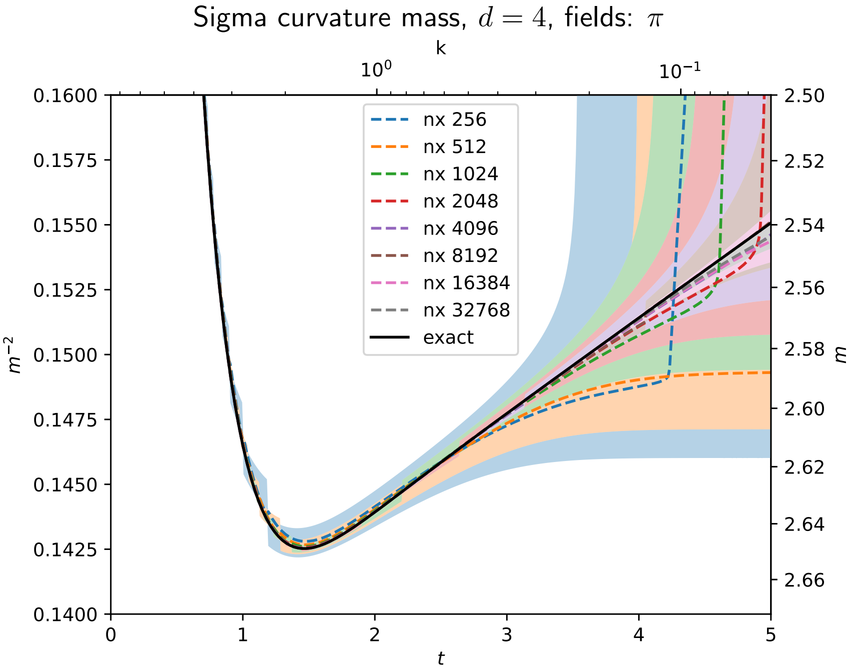
Sigma curvature mass, $d = 4$, fields: π
k
10^0
10^{-1}
0.30
0.28
0.26
0.24
0.22
0.20
m^{-2}
1.85
1.90
1.95
2.00
2.05
2.10
2.15
2.20
m
0
1
2
3
4
5
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768

Sigma curvature mass, $d = 4$, fields: $\sigma\psi$

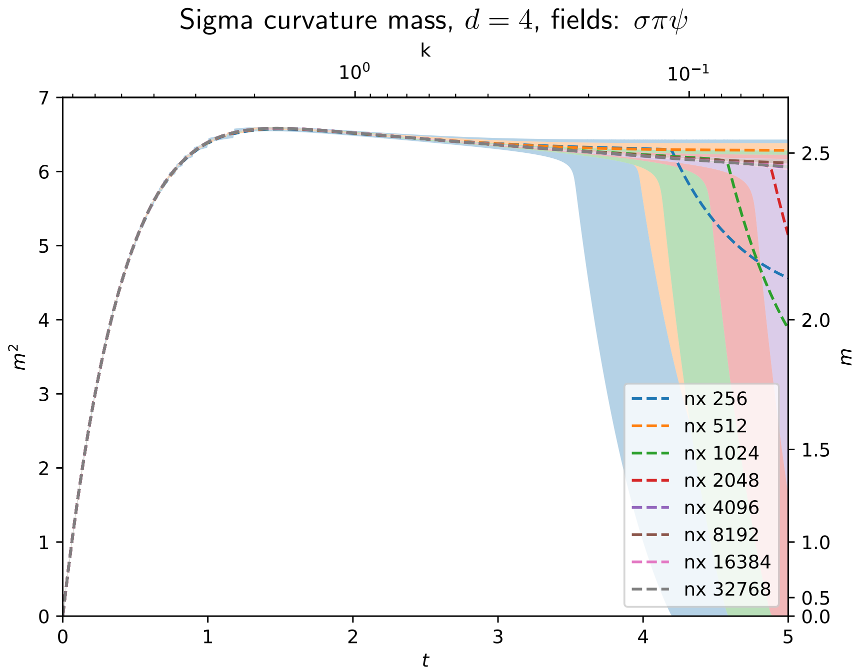


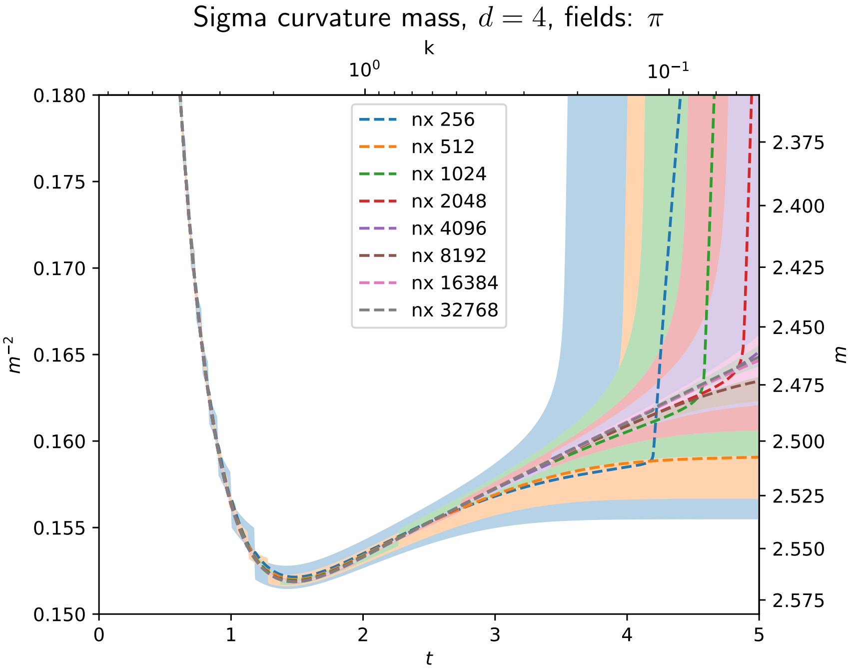
Sigma curvature mass, $d = 4$, fields: $\pi\psi$

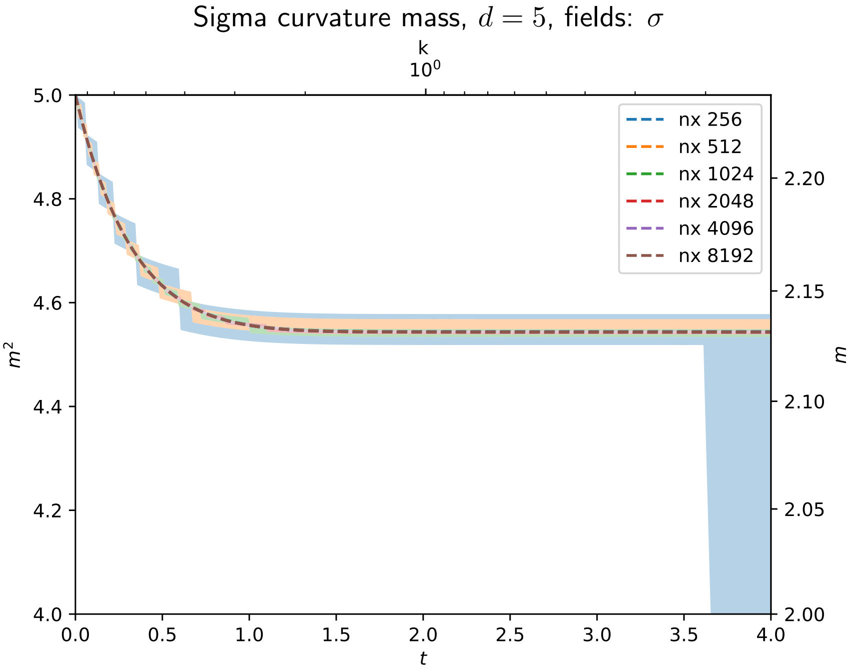




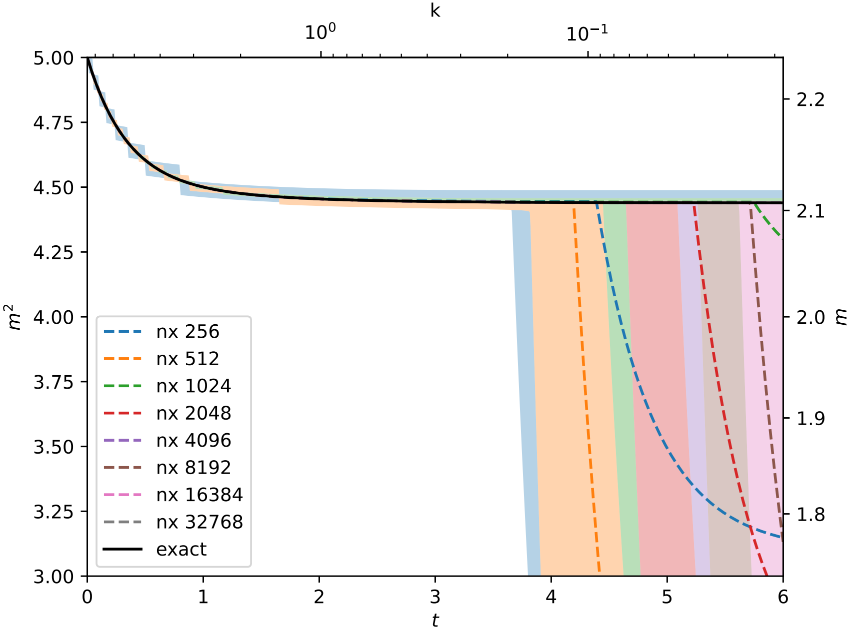
Sigma curvature mass, $d = 4$, fields: π
k
10^0
10^{-1}
0.1600
0.1575
0.1550
0.1525
0.1500
0.1475
0.1450
0.1425
0.1400
m^{-2}
2.50
2.52
2.54
2.56
2.58
2.60
2.62
2.64
2.66
m
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact
0
1
2
3
4
5
t

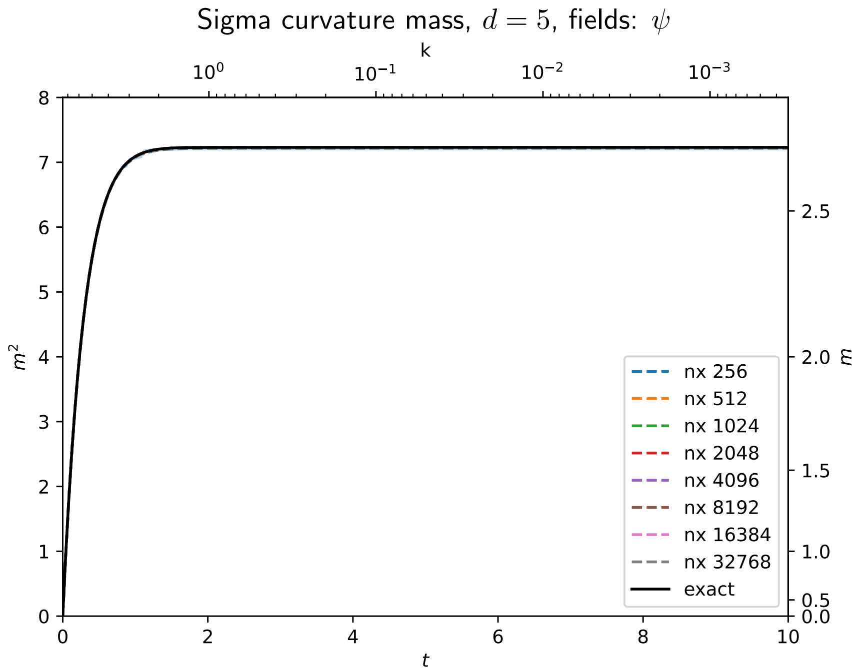

Sigma curvature mass, $d = 4$, fields: $\sigma\pi\psi$
k
10^0
10^{-1}
m^2
m
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768


Sigma curvature mass, $d = 4$, fields: π
k
10^0
10^{-1}
0.180
0.175
0.170
0.165
0.160
0.155
0.150
m^{-2}
0
1
2
3
4
5
t
2.375
2.400
2.425
2.450
2.475
2.500
2.525
2.550
2.575
m
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768

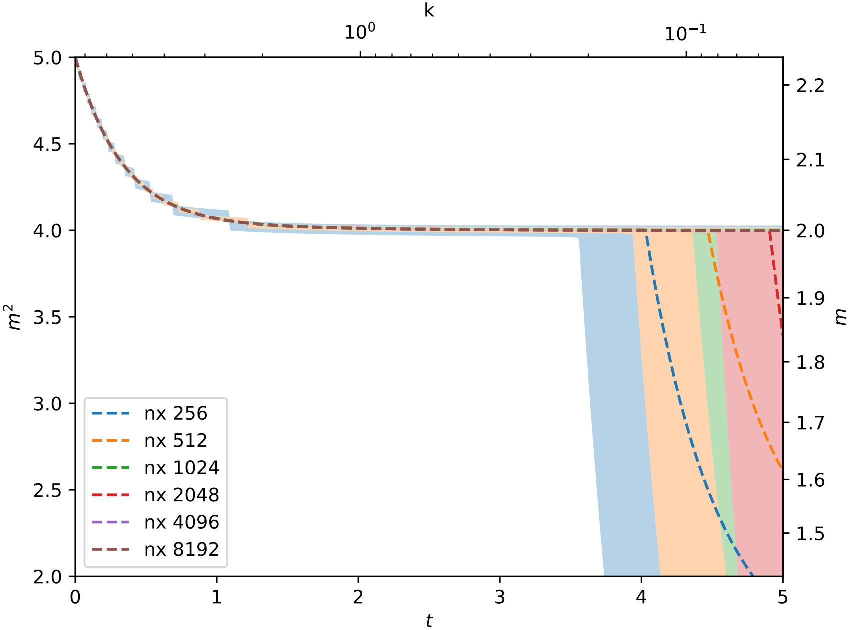

Sigma curvature mass, $d = 5$, fields: σ
k
10^0
5.0
4.8
4.6
4.4
4.2
4.0
m^2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
t
2.20
2.15
2.10
2.05
2.00
m
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192

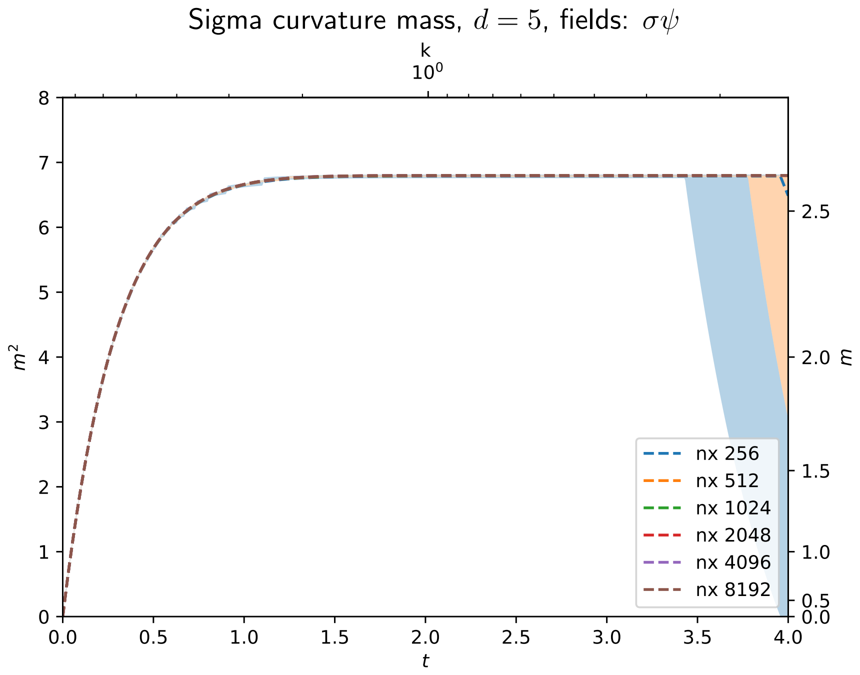
Sigma curvature mass, $d = 5$, fields: π

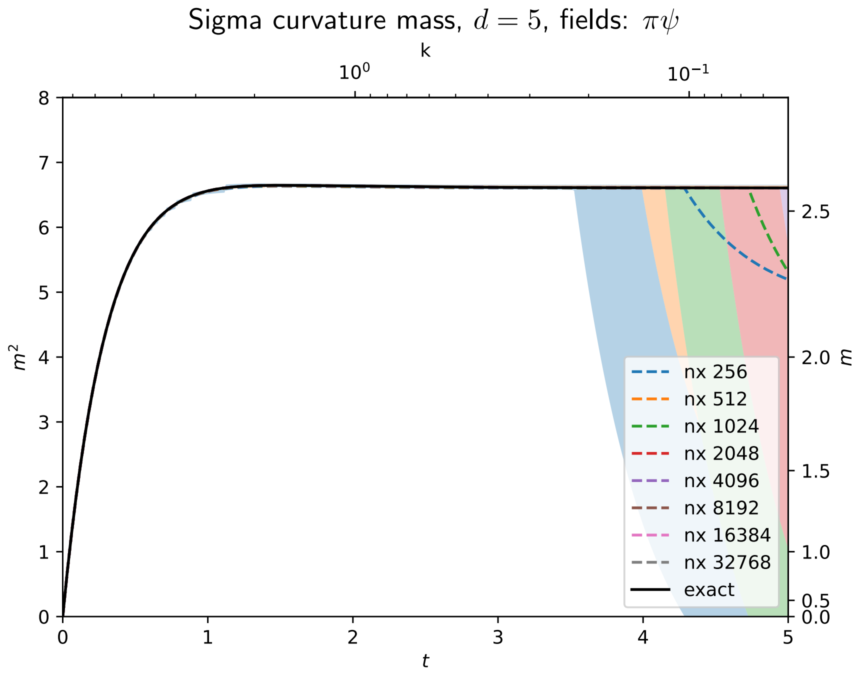



Sigma curvature mass, $d = 5$, fields: ψ
k
10^0
10^{-1}
10^{-2}
10^{-3}
m^2
m
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact

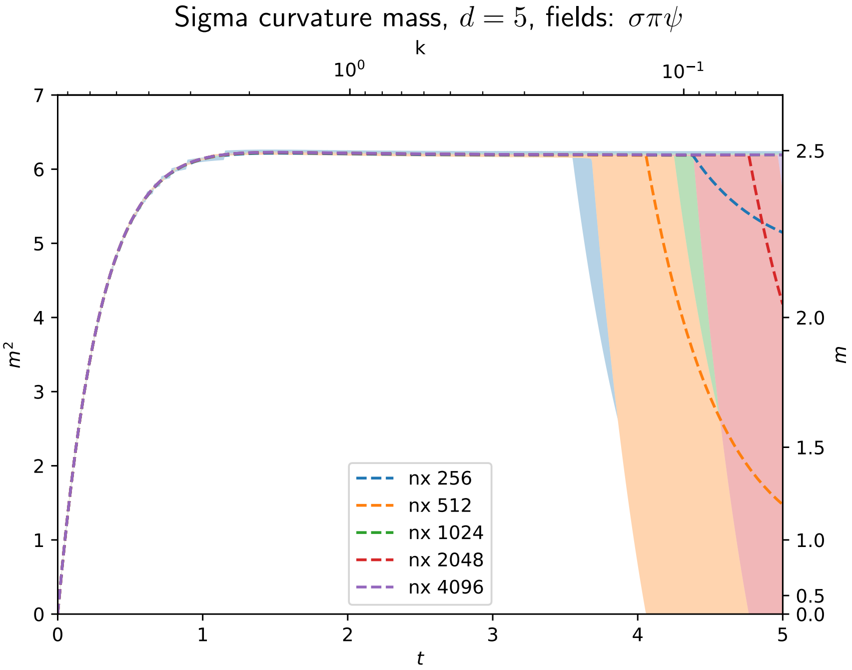
Sigma curvature mass, $d = 5$, fields: $\sigma\pi$




Sigma curvature mass, $d = 5$, fields: $\sigma\psi$
k
10^0
8
7
6
5
4
3
2
1
0
m^2
2.5
2.0
1.5
1.0
0.5
0.0
m
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192



Sigma curvature mass, $d = 5$, fields: $\pi\psi$
k
10^0
10^{-1}
m^2
m
t
0
1
2
3
4
5
6
7
8
2.5
2.0
1.5
1.0
0.5
0.0
nx 256
nx 512
nx 1024
nx 2048
nx 4096
nx 8192
nx 16384
nx 32768
exact


Sigma curvature mass, $d = 5$, fields: $\sigma\pi\psi$
k
10^0
10^{-1}
m^2
m
t
nx 256
nx 512
nx 1024
nx 2048
nx 4096