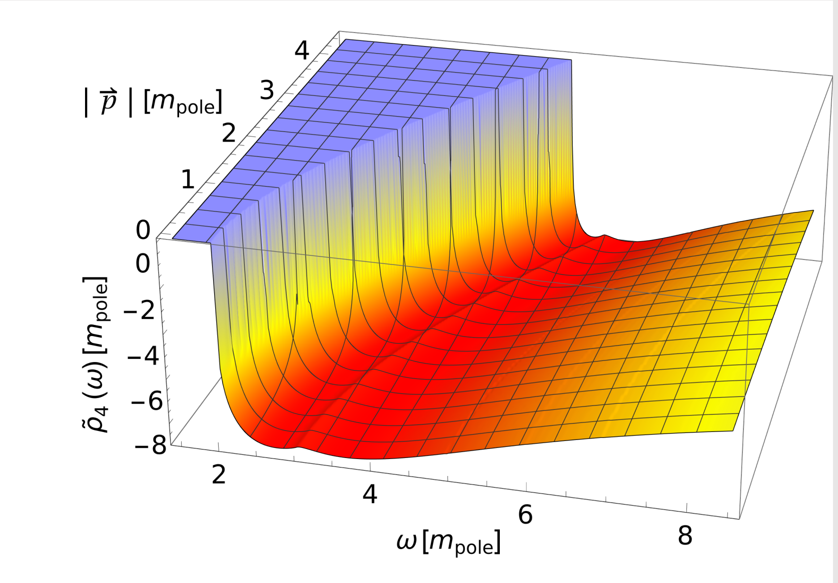
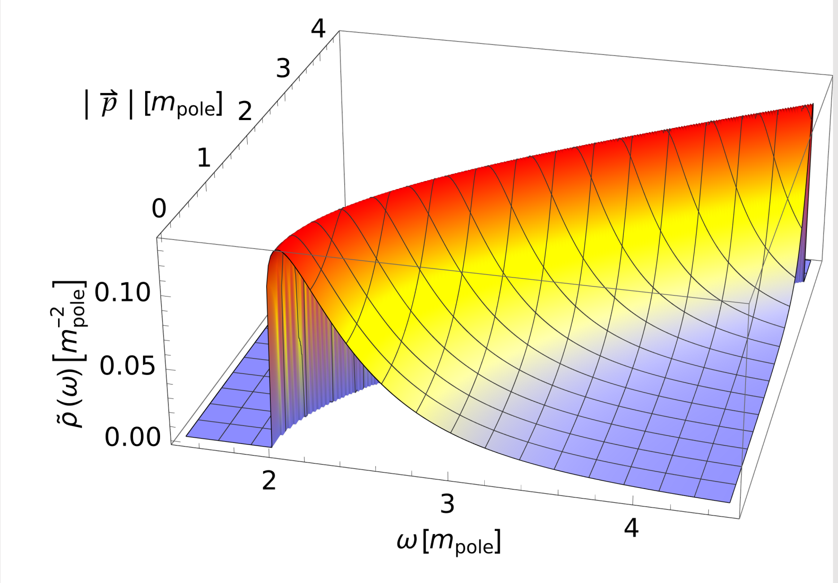
Spectral functions from spectral flows

Jonas Wessely

Functional Methods in Strongly Correlated Systems
(FUNSCS2023)
Hirschegg 13.09.2023





In collaboration with **Jan Horak, Friederike Ihssen, Jan M. Pawlowski, Nicolas Wink**
- arXiv:2303.16719

Outline

- Real time correlators with *spectral* functional methods

- Spectral fRG and the Callan-Symanzik cut-off

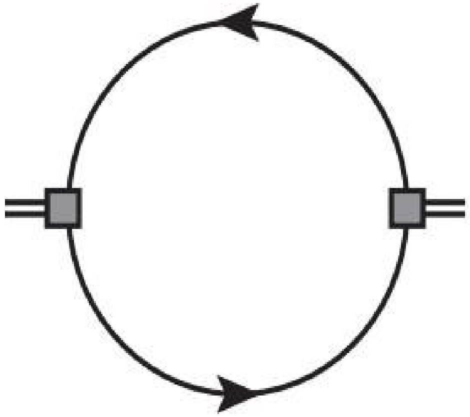
- Results for real scalar fields in (2+1) dimensions

Real time correlators with spectral functional methods

Transport coefficients

$$\mathscr{D}_s = \lim_{\omega \to 0} \frac{\sigma(\omega, \overrightarrow{p} = 0)}{\omega \chi_q \pi}$$

$$\sigma(\omega, \mathbf{p}) \propto \ \mathrm{Im}$$

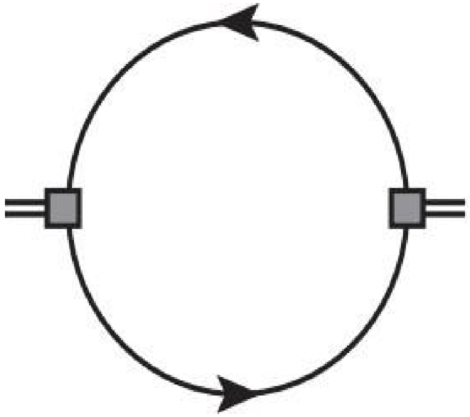


Real time correlators with spectral functional methods

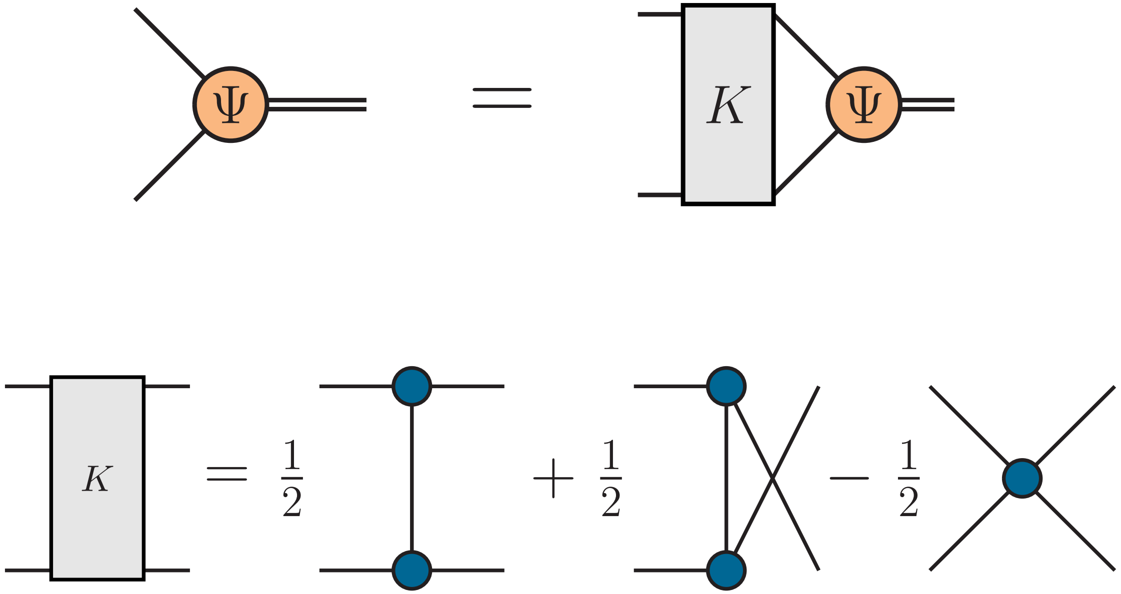
Transport coefficients

$$\mathscr{D}_s = \lim_{\omega\to 0} \frac{\sigma(\omega, \vec{p} = 0)}{\omega \chi_q \pi}$$

$$\boldsymbol{\sigma}(\omega, \mathbf{p}) \propto \mathrm{Im}$$



Bound states

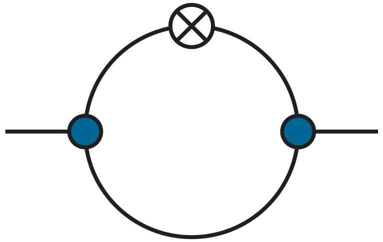


Need for real time correlation functions

Spectral functional equations

Spectral diagrams and spectral renormalisation

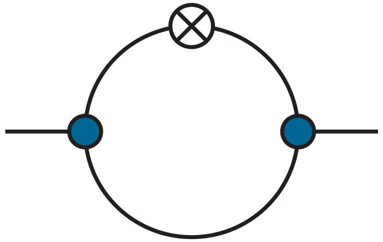
(Horak, Pawlowski, Wink arXiv: 2006.09778)



$$\propto \int_q G(q)^2 G(p+q)$$

Spectral functional equations

Spectral diagrams and spectral renormalisation
(Horak, Pawlowski, Wink arXiv: 2006.09778)



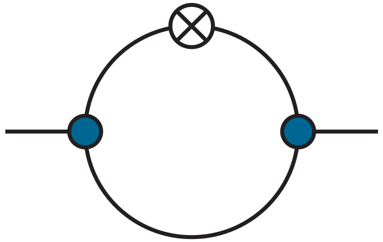
$$\propto \int_q G(q)^2 G(p+q)$$

$$G(p) = \int_\lambda \frac{\rho(\lambda)}{p^2 + \lambda^2}$$

$$= \int_{\lambda_1,\lambda_2,\lambda_3} \rho(\lambda_1)\rho(\lambda_2)\rho(\lambda_2) \int_q \frac{1}{(q^2+\lambda_1^2)(q^2+\lambda_2)((p+q)^2+\lambda_3^2)}$$

Spectral functional equations

Spectral diagrams and spectral renormalisation
(Horak, Pawlowski, Wink arXiv: 2006.09778)



$$\propto \int_q G(q)^2 G(p+q)$$

$$G(p) = \int_\lambda \frac{\rho(\lambda)}{p^2+\lambda^2}$$

$$= \int_{\lambda_1,\lambda_2,\lambda_3} \rho(\lambda_1)\rho(\lambda_2)\rho(\lambda_2) \int_q \frac{1}{(q^2+\lambda_1^2)(q^2+\lambda_2)((p+q)^2+\lambda_3^2)}$$

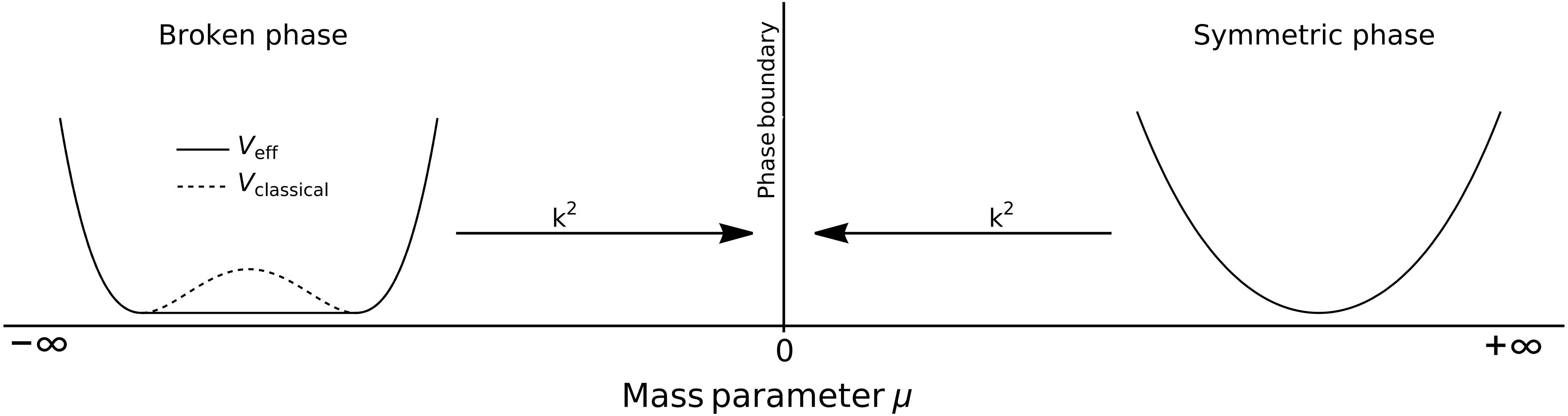
- Loop integrals can be calculated in dimReg
- Access to the full complex plane

- But: additional spectral integrals
- Spectral renormalisation for diverging diagrams

Spectral fRG and the Callan-Symanzik Cutoff

The flowing mass parameter

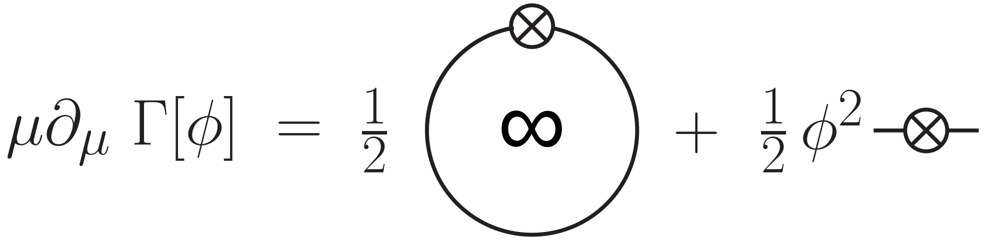
$$S[\phi] = \int \mathrm{d}^3x \left\{ \frac{1}{2}\phi\Big(-\partial^2 + \mu\Big)\phi + \frac{\lambda_\phi}{4!}\phi^4 \right\}$$



Spectral fRG and the Callan-Symanzik Cutoff

The flowing mass parameter

$$S[\phi] = \int \mathrm{d}^3x \left\{ \frac{1}{2}\phi\Big(-\partial^2 + Z_\phi \mu\Big)\phi + \frac{\lambda_\phi}{4!}\phi^4 \right\}$$

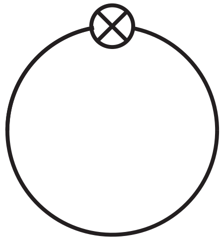


- Without UV-regularisation, divergent diagram!

Spectral fRG and the Callan-Symanzik Cutoff

The flowing mass parameter

$$S[\phi] = \int \mathrm{d}^3x \left\{ \frac{1}{2}\phi\Big(-\partial^2 + Z_\phi \mu\Big)\phi + \frac{\lambda_\phi}{4!}\phi^4 \right\}$$

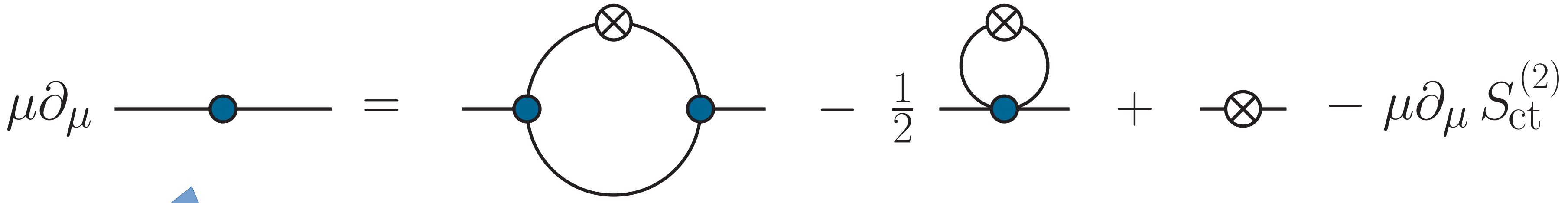
$\mu\partial_\mu \, \Gamma[\phi] \;=\; \frac{1}{2}$  $+ \; \frac{1}{2}\phi^2$  $- \; \mu\partial_\mu \, S_{\mathrm{ct}}[\phi]$

- Without UV-regularisation divergent diagram!
- Introduce counter-term flow via limiting prozedure over UV-finite regulators
- Counter-term flow determined by flowing renormalisation condition

fQCD collaboration: arXiv:2206.10232

Spectral fRG and the Callan-Symanzik cut-off

flowing renormalisation conditions



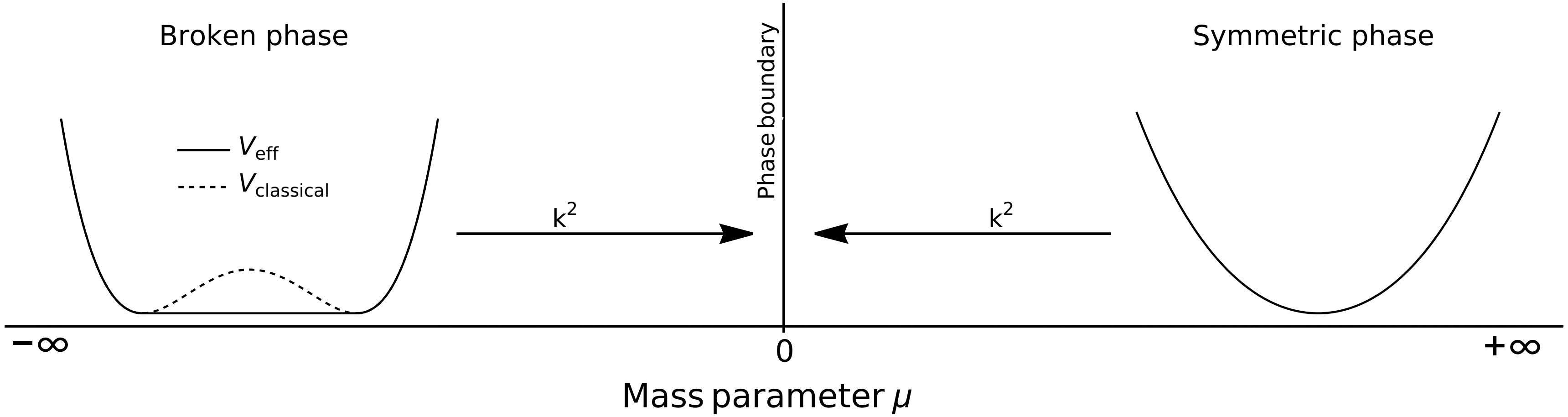
$$\rho(\omega) = \frac{\mathrm{Im}\Gamma^{(2)}}{||\Gamma^{(2)}||^2}\Big|_{p^2=-(\omega+i\,0^+)}$$

- Diagrams in the flow are finite in (2+1) dimensions since the insertion of the cut-off lowers the degree of divergence by 2
- But: initial condition implicitly sets a renormalisation condition
- Exploiting the counter-term gives us the opportunity to control the flow in theory space and eliminates fine-tuning

J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – arXiv:2303.16719

Real scalar field in 3 dimension

flowing on-shell renormalisation



Classical pole mass on the physical minimum ϕ_0

$$m^2_{\text{pole}} = 2|\mu| = 2k^2$$

Flowing on-shell condition in the broken phase

$$\Gamma^{(2)}[\phi_0]\Big|_{p^2=-2k^2} = 0$$

Classical pole mass at $\phi_0 = 0$

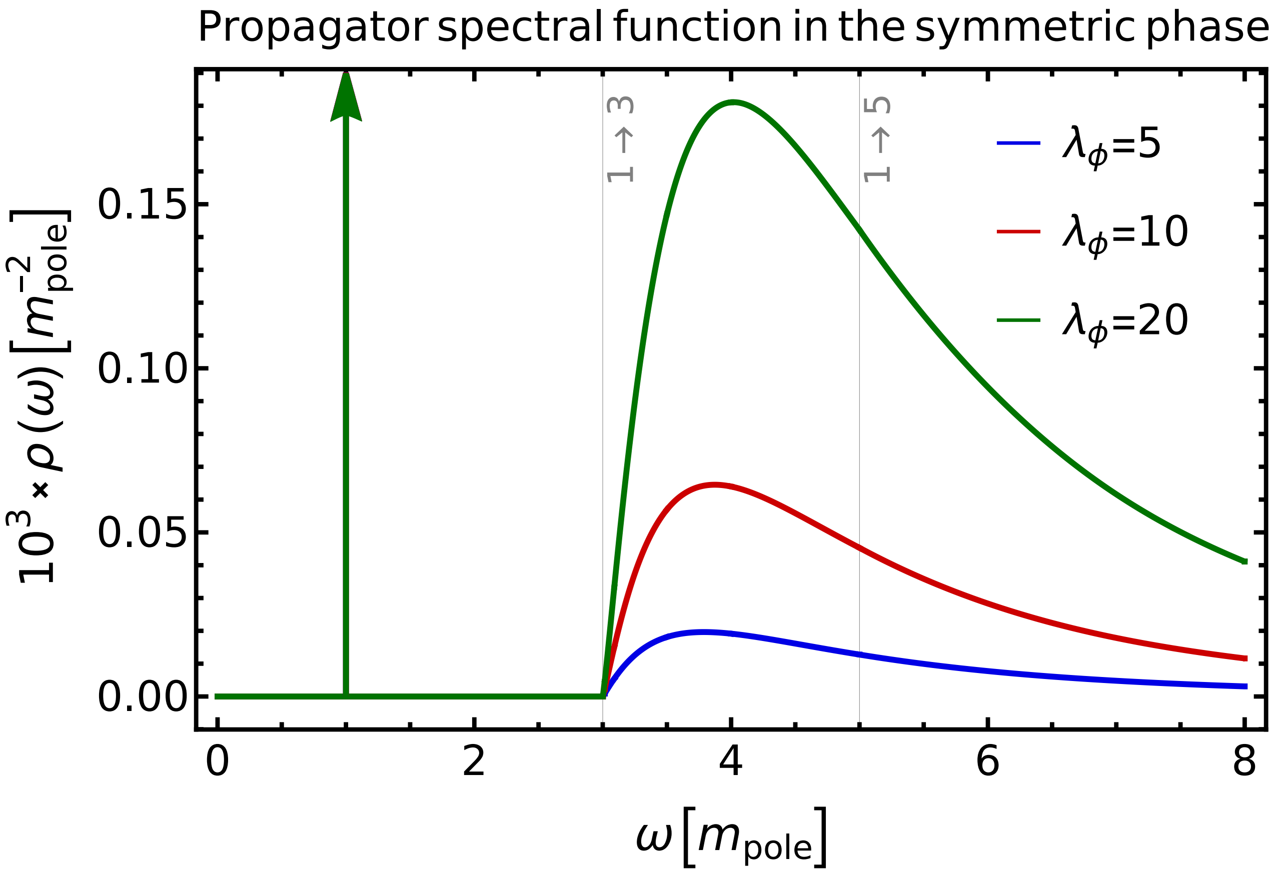
$$m^2_{\text{pole}} = \mu = k^2$$

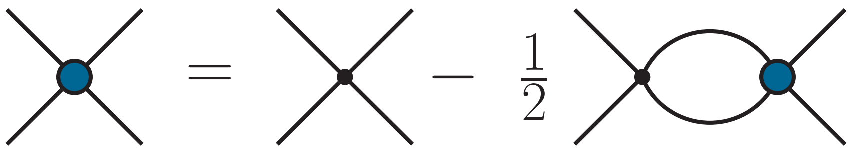
Flowing on-shell condition in the symmetric phase

$$\Gamma^{(2)}[\phi_0 = 0]\Big|_{p^2=-k^2} = 0$$

J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – arXiv:2303.16719

Results in the symmetric phase

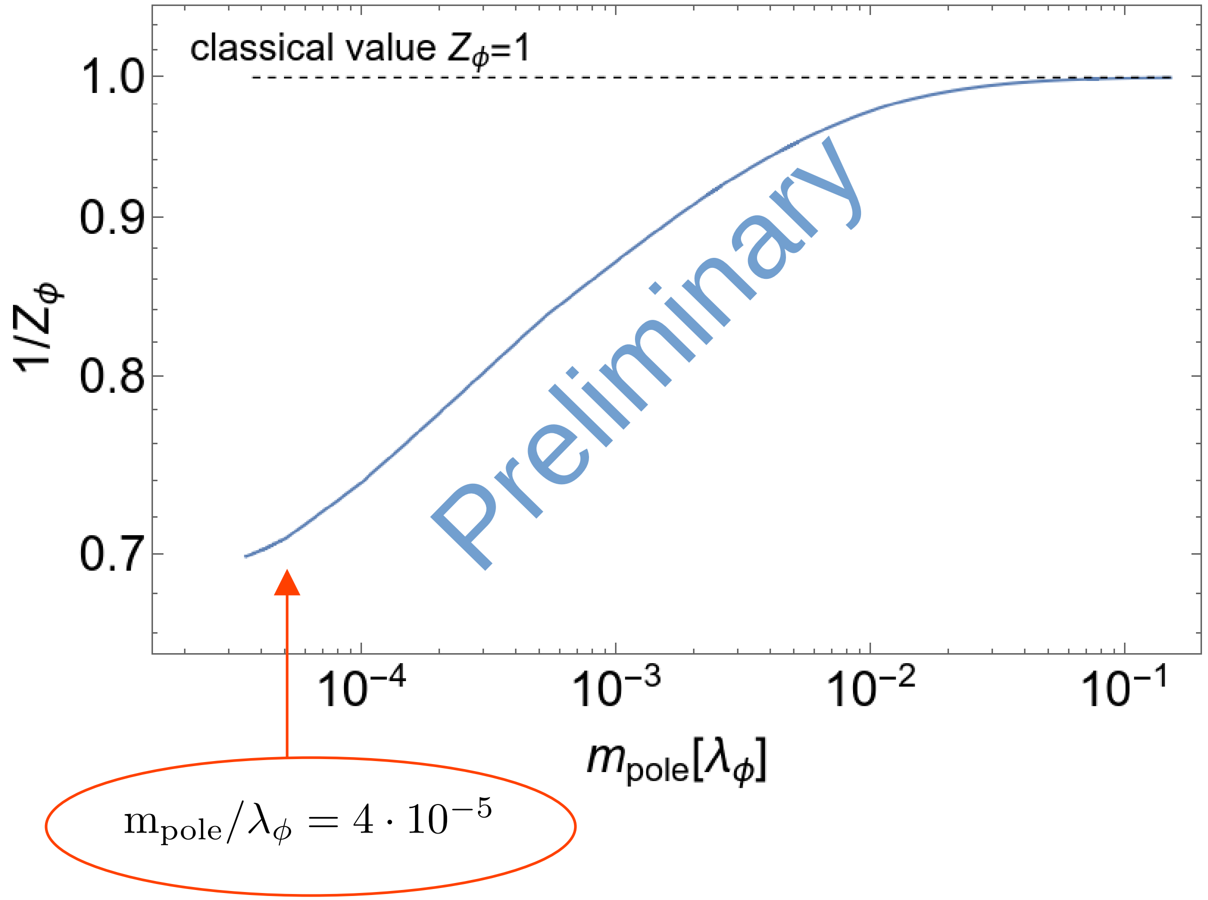


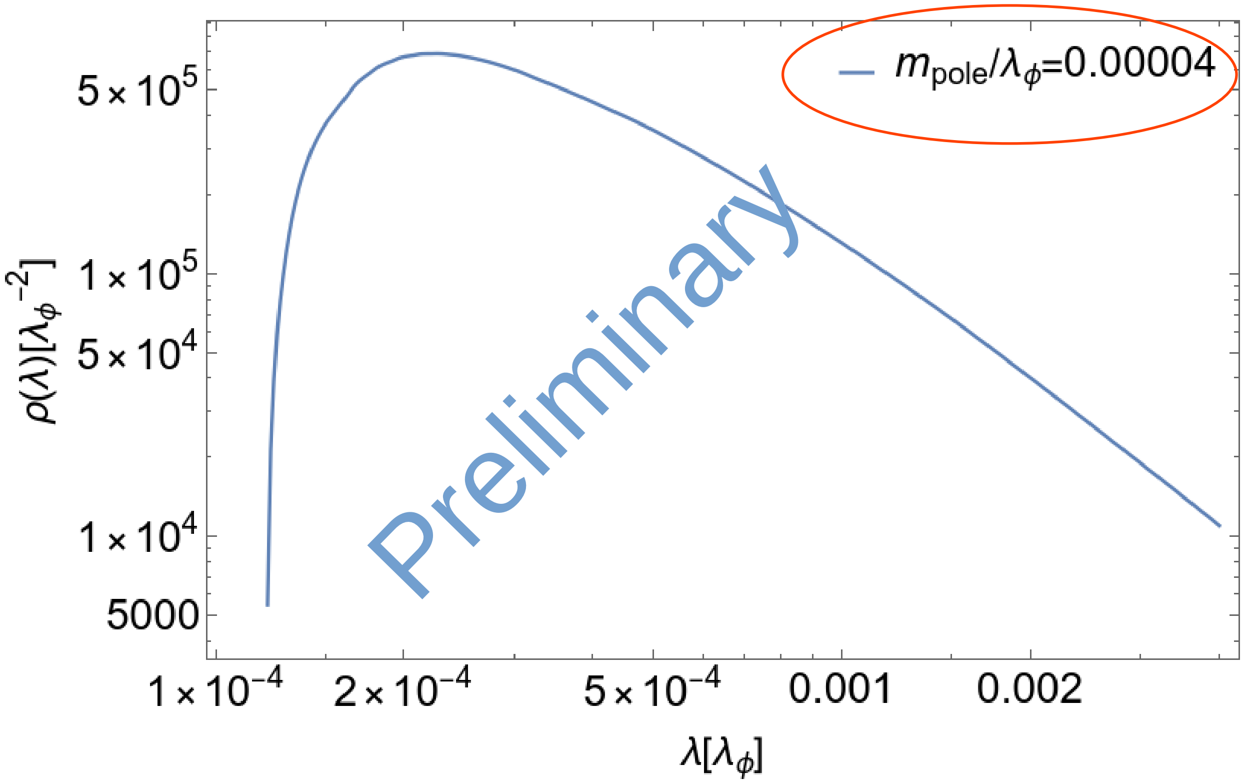


$$\Gamma^{(4)}(p^2 = s) = \lambda_\phi + \int_\lambda \frac{\rho_4(\lambda)}{\lambda^2 + p^2}$$

J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – arXiv:2303.16719

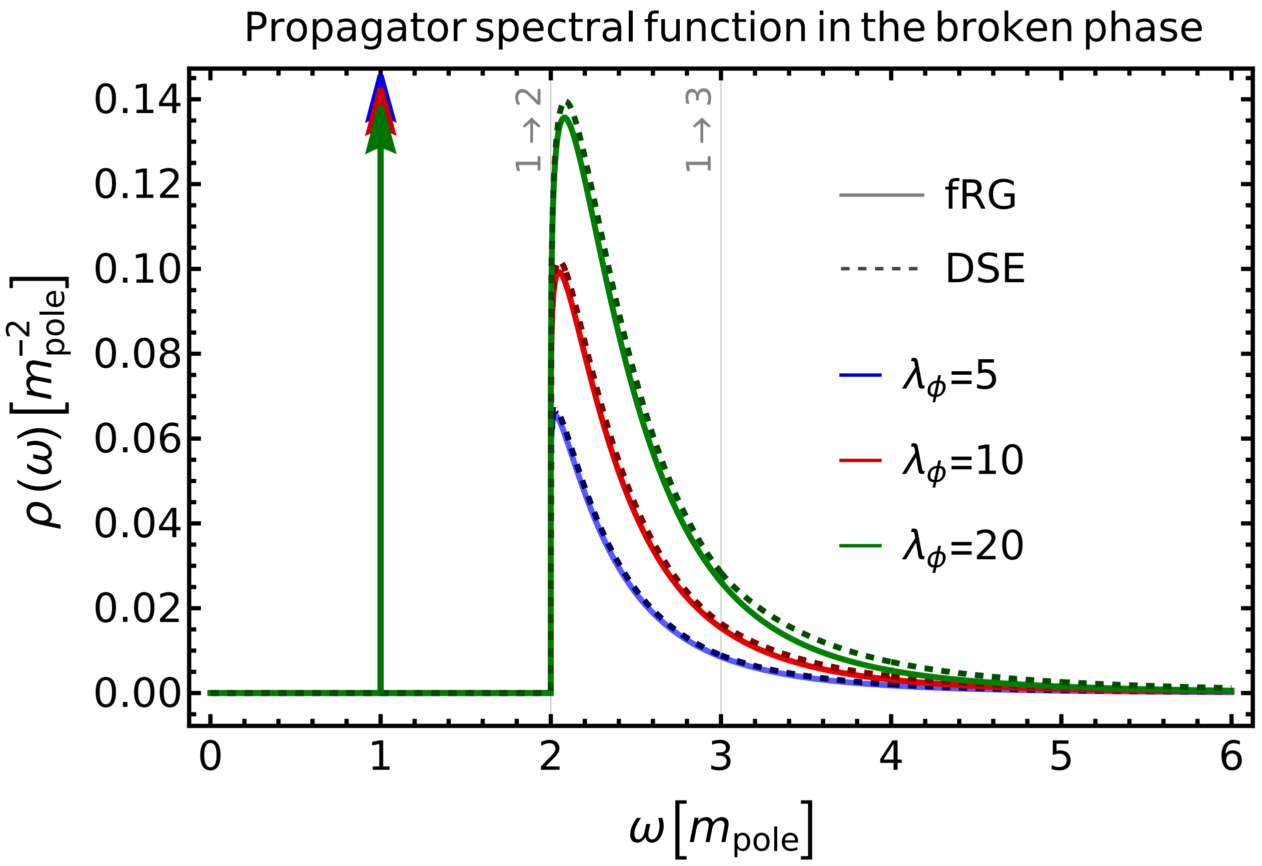
Results in the symmetric phase

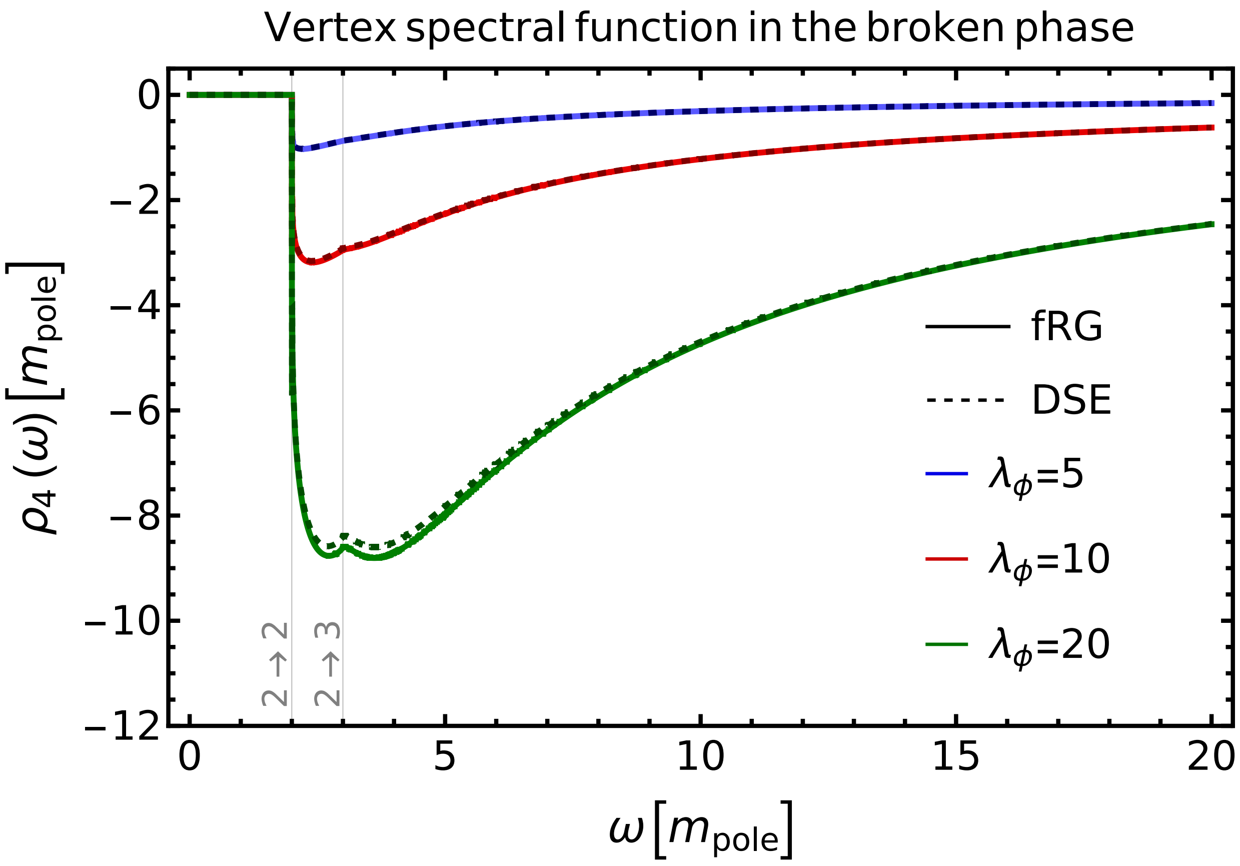




J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – arXiv:2303.16719
K. Kockler, J. M. Pawlowski, JW – in preparation

Results in the broken phase

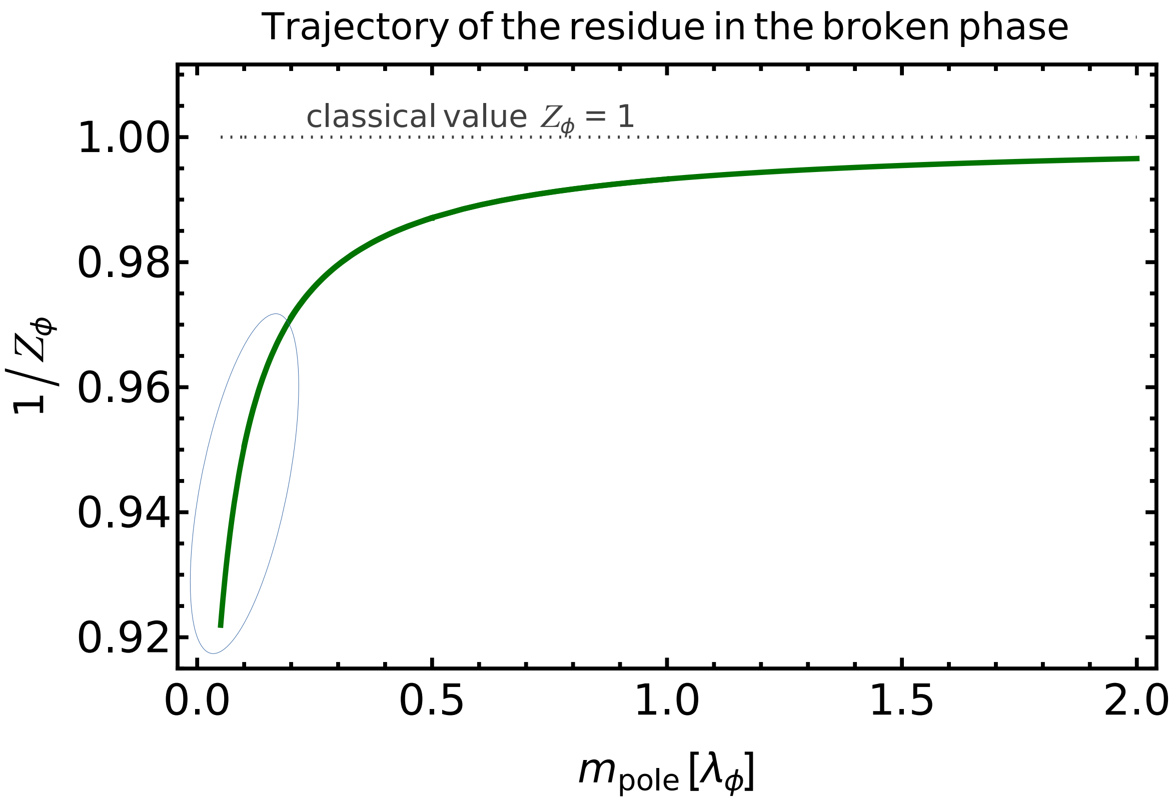


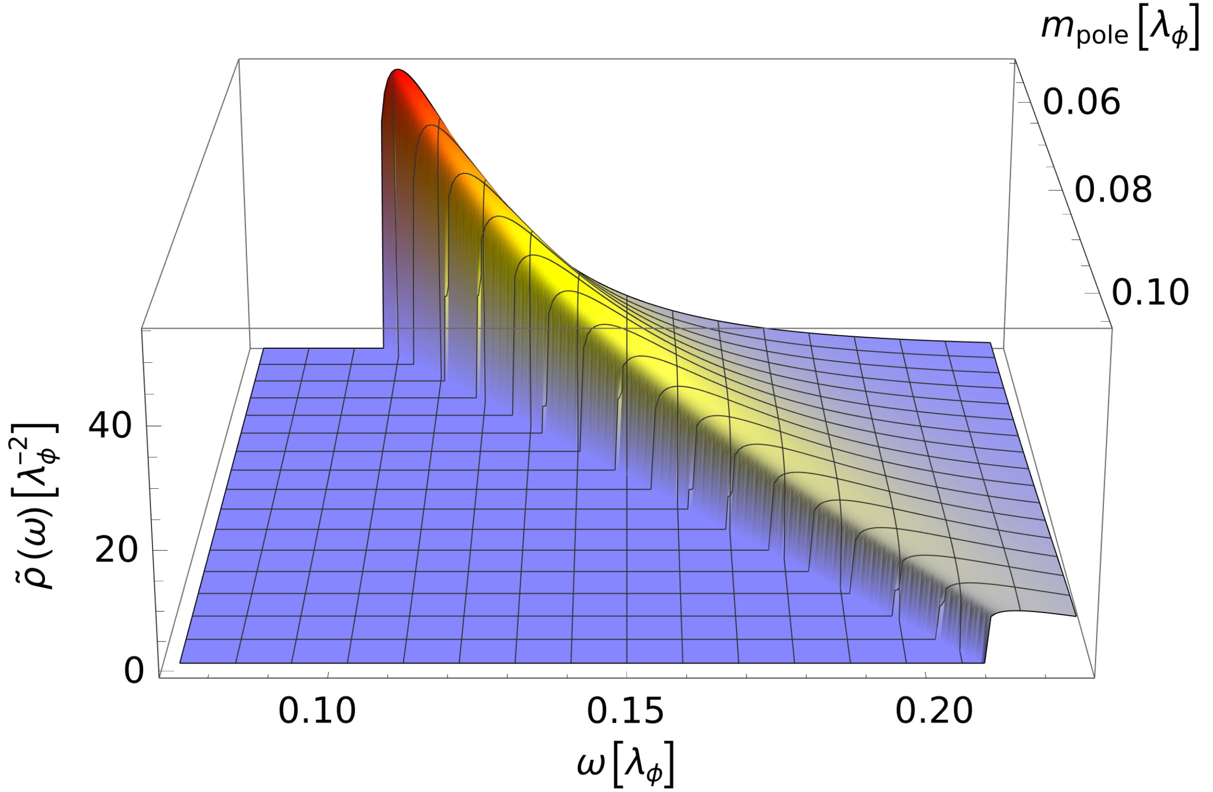


J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – arXiv:2303.16719

Results in the broken phase

$$\rho(\lambda) = \frac{2\pi}{Z_\phi}\delta(m_\phi^2 - \lambda^2) + \tilde{\rho}(\lambda)$$





Work to do and open questions – the scaling limit

What happens in the scaling limit?

$$\partial_t \phi_0 = -\frac{\partial_t V^{(1)}_{\text{eff}}(\phi_0)}{V^{(2)}_{\text{eff}}(\phi_0)} = -\frac{\partial_t V^{(1)}_{\text{eff}}(\phi_0)}{m^2_{\text{curv}}} \quad \Rightarrow \quad \phi_0 = \phi_{0,\Lambda}\,\sqrt{Z_\phi}\left(\frac{k}{\Lambda}\right)\,\exp\left\{\int_\Lambda^k \frac{dk'}{k'}\mathcal{D}(k')\right\}$$

How to extract critical exponent?

$$\bar{\phi}_0 \propto \tau^\beta\,, \qquad \beta = \frac{1}{2}\nu\,(1+\eta_\phi) \approx 0.3264$$

But what is the tuning parameter?

$$\xi \propto \tau^{-\nu} \qquad \xi \propto k^{-1}$$

$$\tau \propto k^{\frac{1}{\nu}}$$

$$\Rightarrow \quad \bar{\phi}_0 \propto k^{\frac{\beta}{\nu}}\,, \qquad \frac{\beta}{\nu} = \frac{1}{2}(1+\eta_\phi)$$

J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – arXiv:2303.16719

Wrap-up

- Spectral functional equations are powerful tool to calculate self-consistent spectral functions

- The spectral, functional Callan-Symanzik flow interpolates between correlation functions in the limit of high masses and their massless limit.

- Flowing (on-shell) renomalisation controls trajectory in theory space and eliminates related fine-tuning problems

- TODO: extend framework to finite temperature and chemical potential

- FRG specific: include a field-dependent effective potential

- Next goal: self-consistent quark spectral functions at finite T and μ

  → Diffusion coefficients and electric conductivity

Work to do and open questions – the scaling limit

How does this RG-prozedure relates to standart RG and scaling analysis?

→ Dynamical mapping between “usual” tuning parameter and on-shell mass

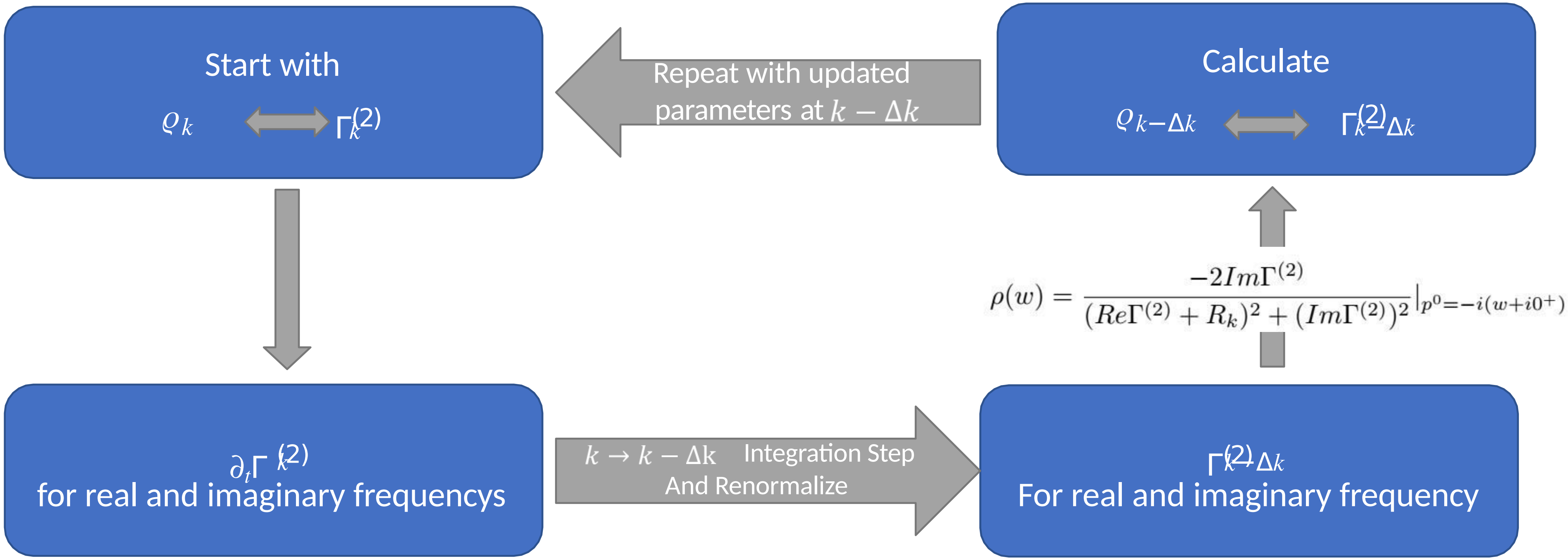
Information on ν in the countertermflow → work in progress

Is this “onshell/comoving” frame a suitable way to think about phase transition?

Back up

Application to a real scalar field in 3 dimension

local feedback



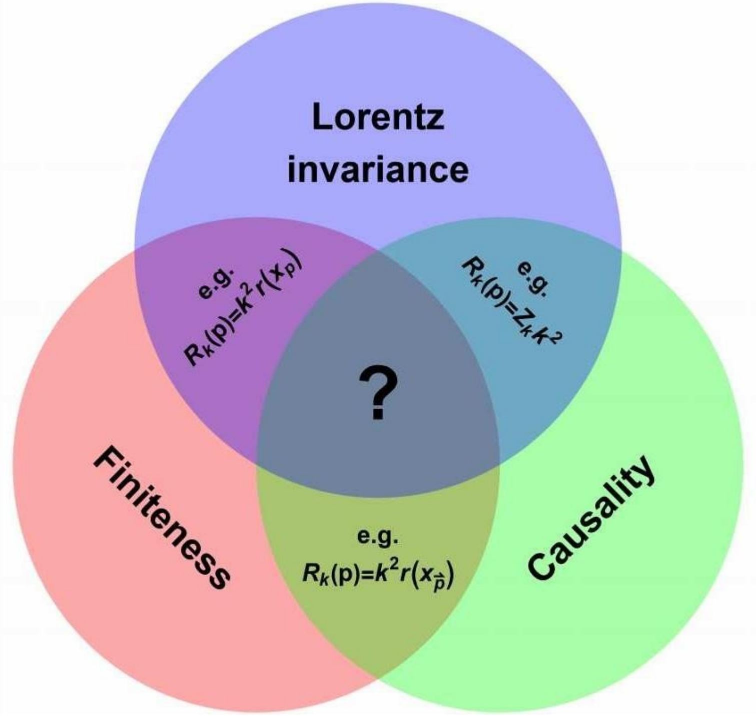
J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation

Spectral fRG and the Callan-Symanzik cut-off

arXiv:2206.10232

$$S[\phi] \to S[\phi] + \frac{1}{2}\int_q \phi(q) R_k(q^2)\phi(-q)$$

$$G_{(}p) = \frac{1}{\Gamma_k^2(p^2) + R_k(p^2)}$$

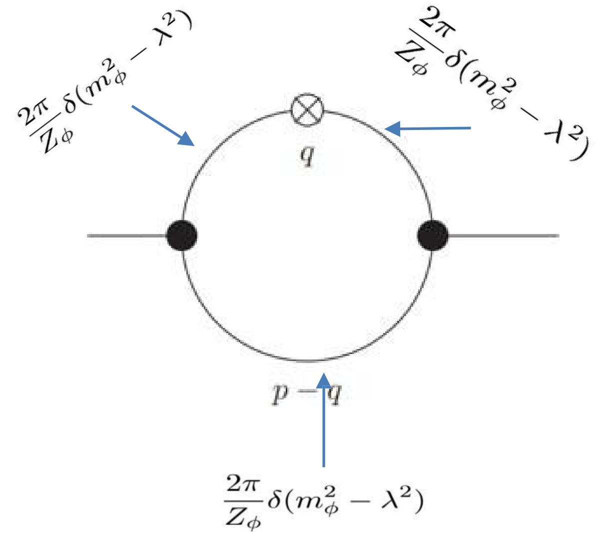


- Have to choose 2 out of 3 properties:
  - UV-regularisation
  - Lorentz invariance
  - Causal propagator at finite k

Backup

$$\rho(\lambda) = \frac{2\pi}{Z_\phi}\delta(m_\phi^2 - \lambda^2) + \tilde{\rho}(\lambda)$$

$$I_{\rm pol}(\lambda_1, \lambda_2, \lambda_3, p^2)$$
$$= \int \frac{d^3q}{(2\pi)^3} \left(\prod_{i=1}^{2} \frac{1}{(q^2+\lambda_i^2)}\right) \frac{1}{((q+p)^2+\lambda_3^2}$$



Pole-contributions:

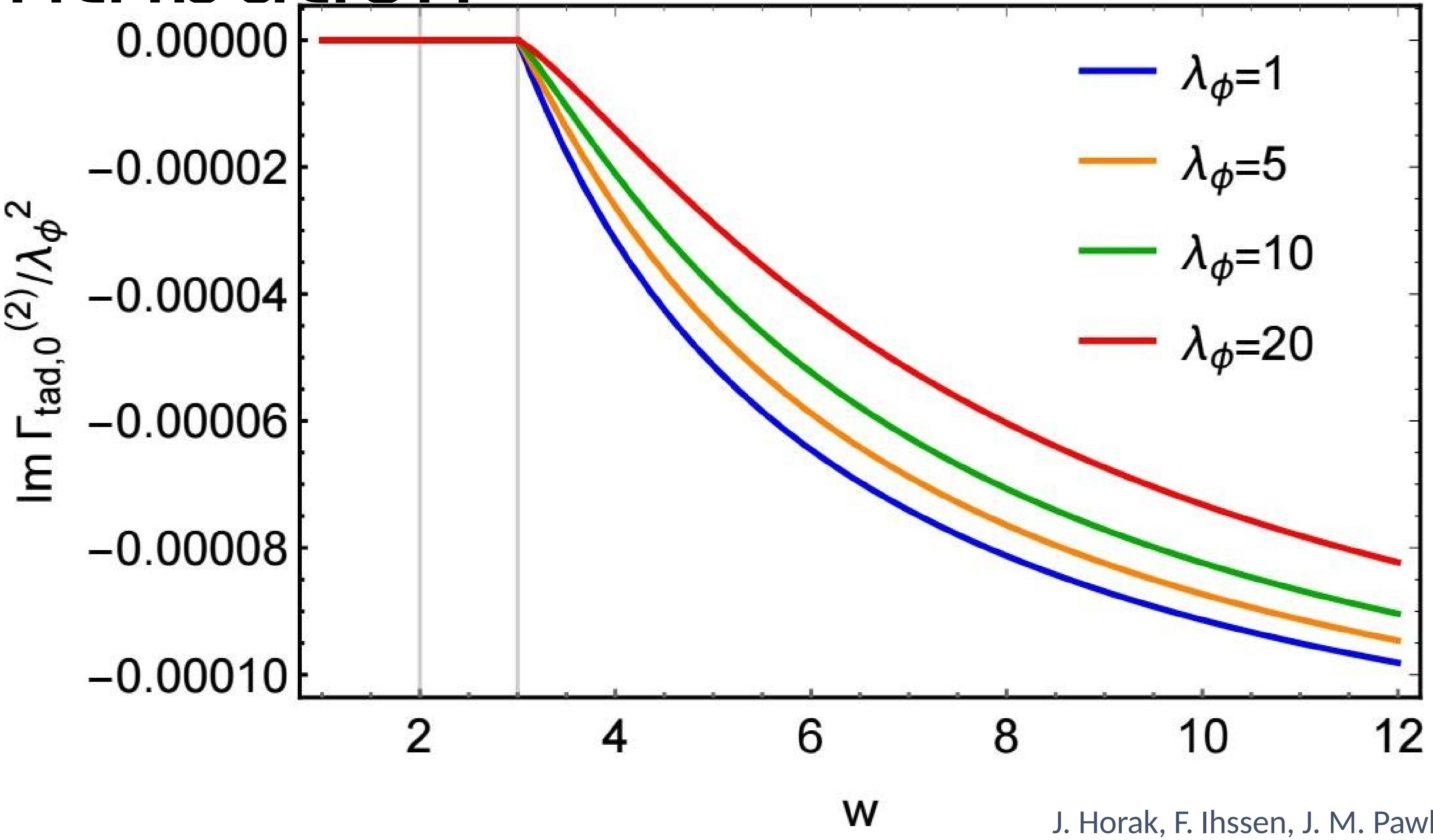
$\lambda_1^2 = \lambda_2^2 = \lambda_3^2 = k^2$

$$\frac{1}{(\lambda^2+q^2)^2} = \frac{-1}{2\lambda}\partial_\lambda \frac{1}{(\lambda^2+q^2)}$$

$$\partial_k \Gamma^{(2)}_{k,1^{st}-order}(p^2)$$
$$= -\frac{1}{2}\frac{(\Gamma_k^{(3)})^2}{Z_k^3}\frac{1}{4\pi p}\partial_k I_{pol}^{DSE}(m_k, m_k, p^2)$$

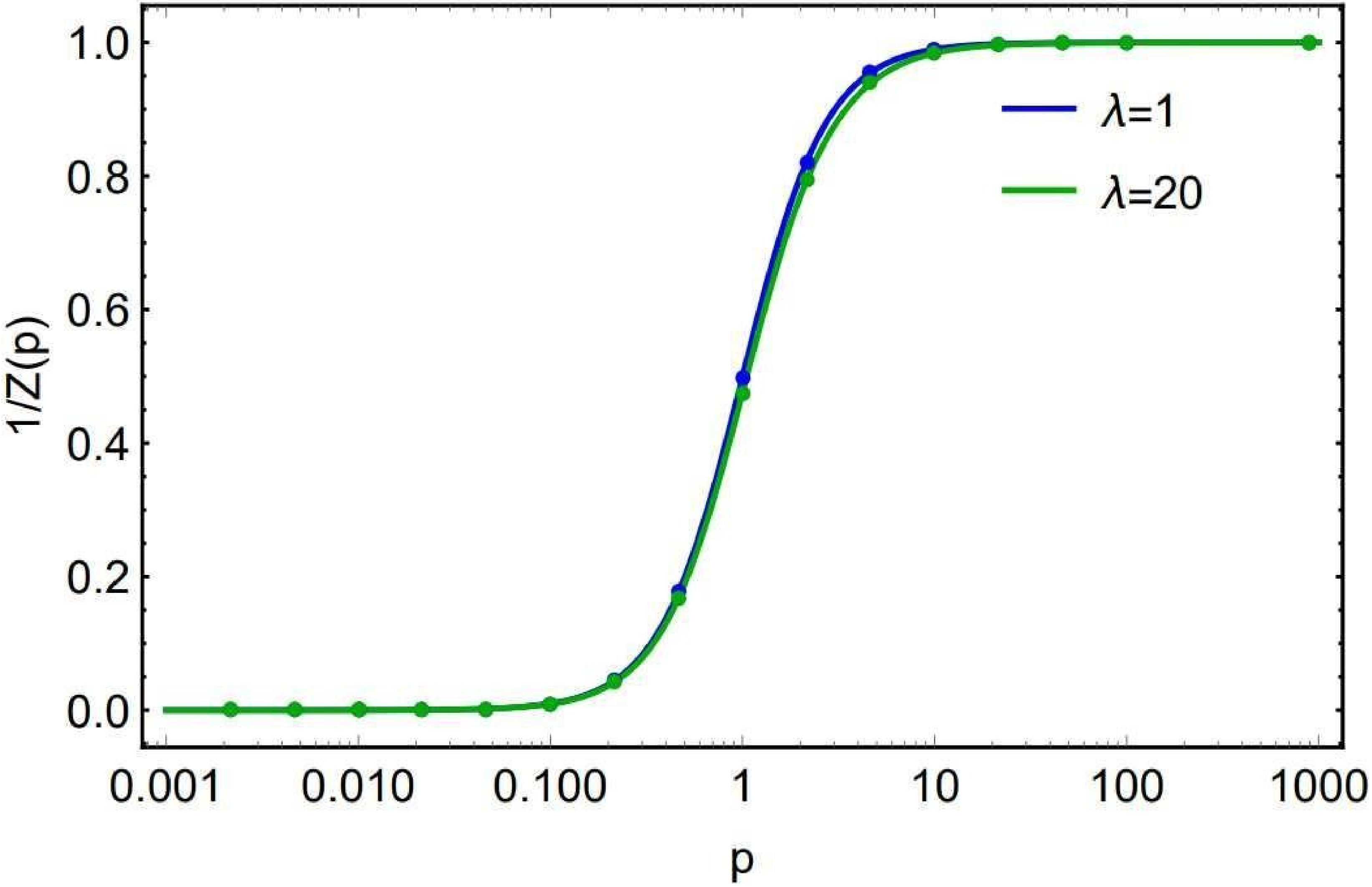
$$\int_k^{k_0} \partial_k \Gamma_k^{(2)}(p^2) = -\frac{1}{2}\left[\mathcal{F}(k) I_{\rm pol}^{DSE}(m_k, m_k, p^2)\right]_k^{k_0}$$
$$+ \frac{1}{8\pi p}\int_k^{k_0} dk\, \partial_k \mathcal{F}(k)\, \mathrm{Arctan}\left[\frac{p}{2m_k}\right]$$

Results: 1-cut (dynamical) Tadpole contribution



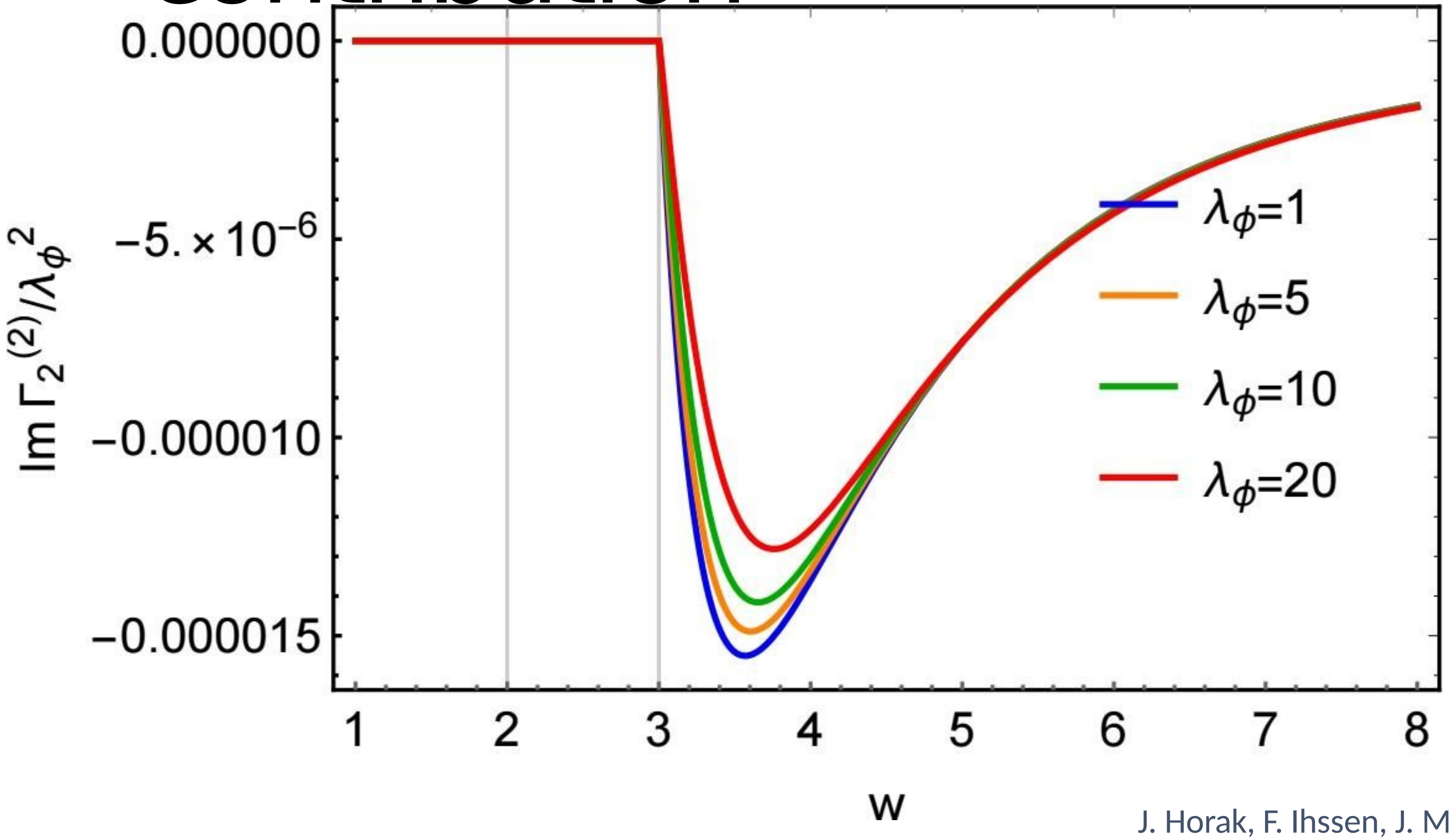
J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation

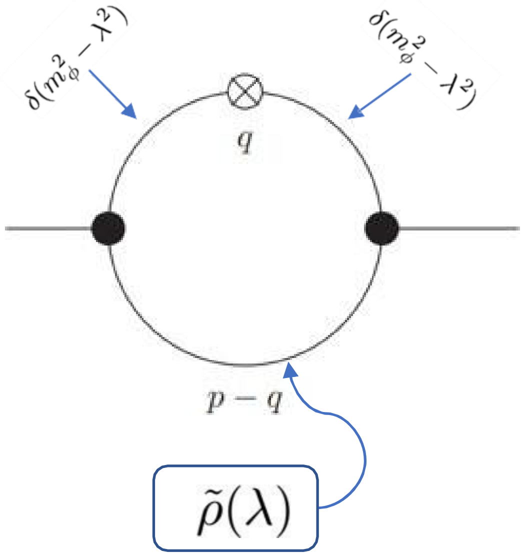
Results: Propagator dressing on the euclidean Axis



J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation

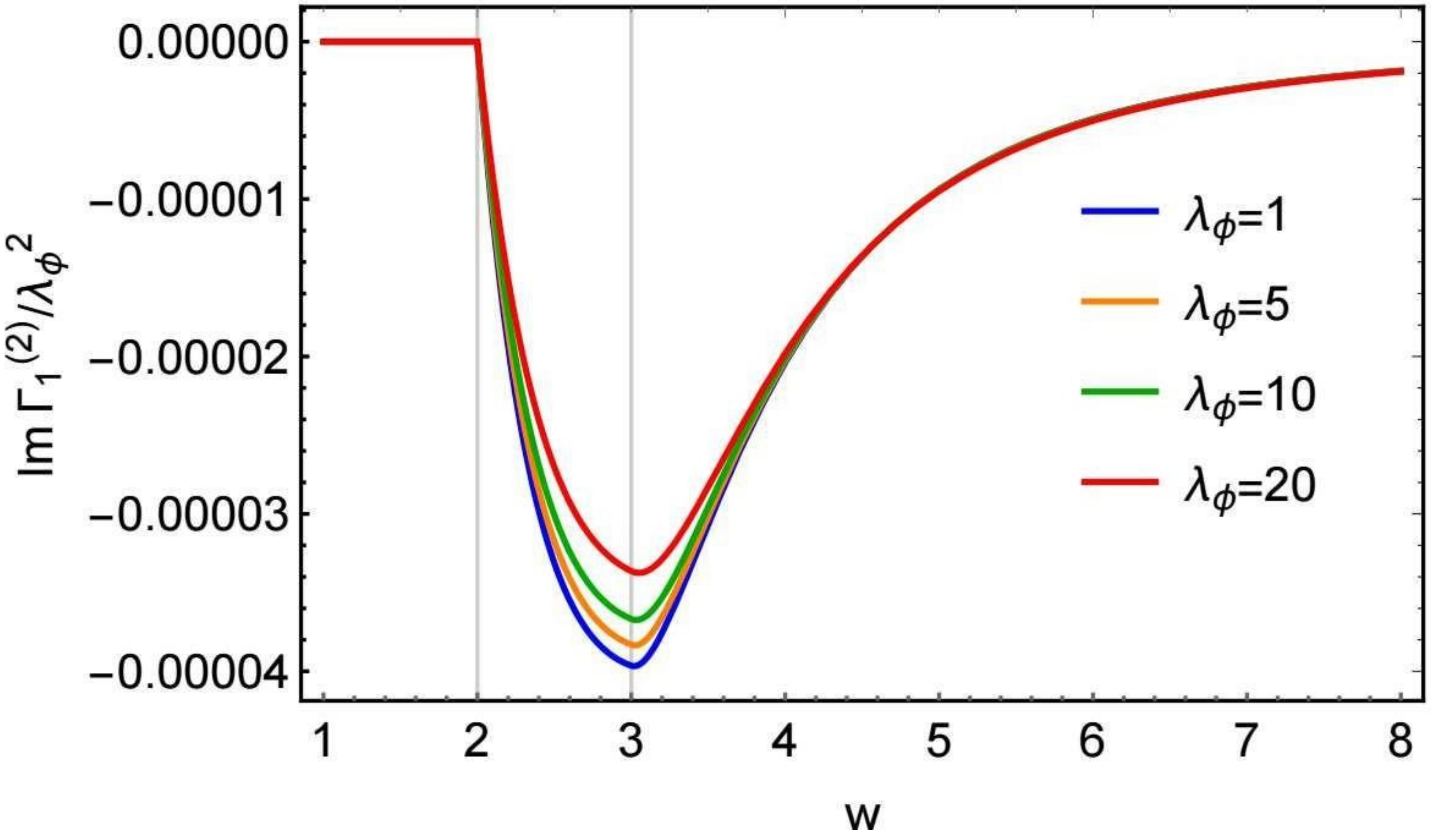
Results: 1-cut contribution





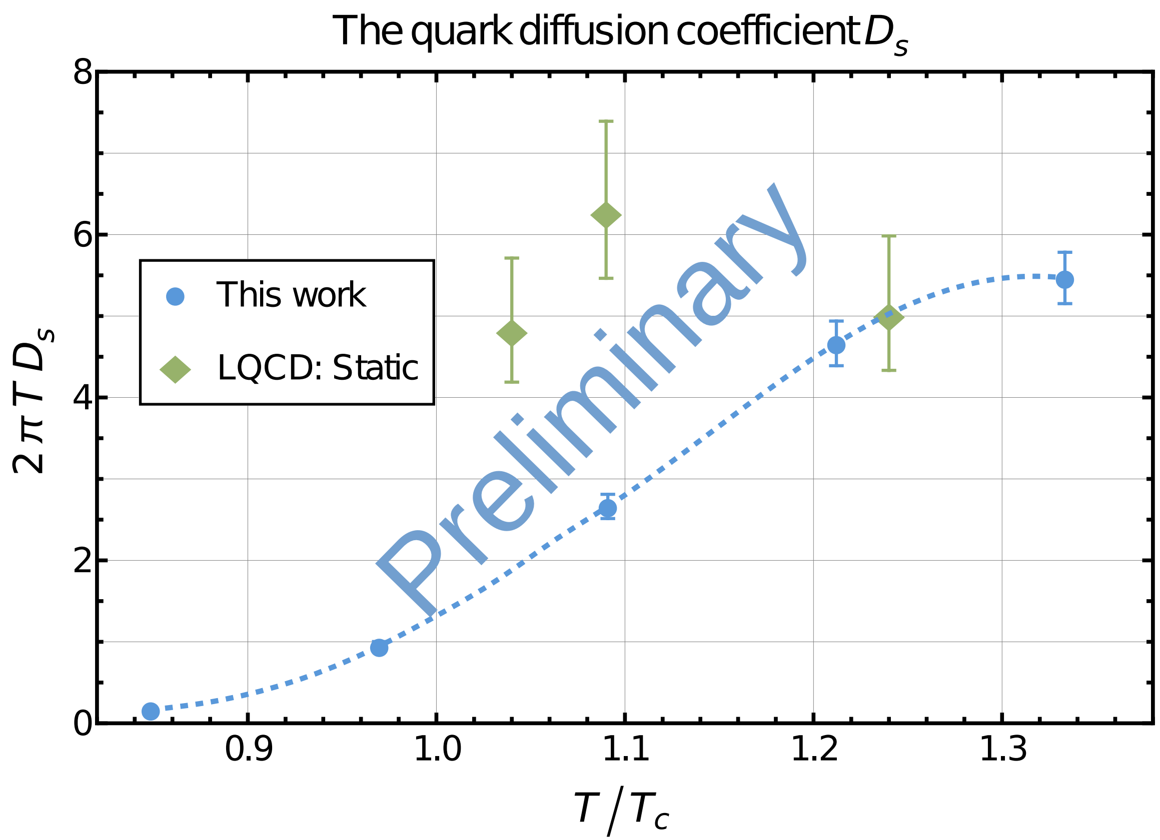
J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation

Results: 1-Cut contribution (1)



J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation

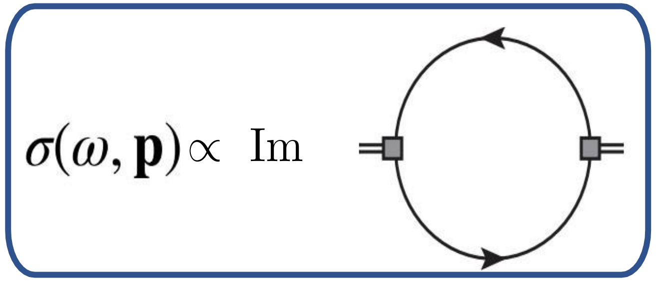
(Not so heavy) quark diffusion



Results from master thesis of Marcel Horstmann

Lattice data: Banerjee et al., Phys. Rev. D 85, 014510

Diffusion channel spectral function



- Quark propagator spectral function from improved massive HTL – computation
- Massive HTL:
  N. Haque, Phys. Rev. D 98, 014013
- Non-perturbative input:
  - Full, thermal pole mass and strong coupling from DSE data (light quarks):
    - F. Gao, J. M. Pawlowski,
      Phys. Rev. D 105, 094020
- Quark number susceptibility from lattice data:
  Borsányi et al., JHEP01(2012)138