FunSCS 2023 workshop

Non-perturbative characteristics of (QCD) spectral functions at finite temperature

Peter Lowdon

(Goethe University Frankfurt)

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC-TR 211 Strong-interaction matter under extreme conditions

Talk outline

1. Non-perturbative QFT for $T > 0$
2. Causality constraints
3. Spectral properties from Euclidean data
4. Revisiting $T > 0$ perturbation theory
5. Summary & outlook

1. Non-perturbative QFT for $T > 0$

- QFTs can be defined using a core set of physically-motivated axioms

  $\longrightarrow$ Applies to simple QFTs, but generally a work in progress...

Axiom 1 (Hilbert space structure). *The states of the theory are rays in a Hilbert space $\mathcal{H}$ which possesses a continuous unitary representation $U(a, \alpha)$ of the Poincaré spinor group $\overline{\mathscr{P}_{+}^{\uparrow}}$.*

Axiom 2 (Spectral condition). *The spectrum of the energy-momentum operator P^{μ} is confined to the closed forward light cone $\overline{V}^{+} = \{p^{\mu} \mid p^{2} \geq 0,\, p^{0} \geq 0\}$, where $U(a, 1) = e^{iP^{\mu}a_{\mu}}$.*

Axiom 3 (Uniqueness of the vacuum). *There exists a unit state vector $|0\rangle$ (the vacuum state) which is a unique translationally invariant state in $\mathcal{H}$.*

Axiom 4 (Field operators). *The theory consists of fields $\varphi^{(\kappa)}(x)$ (of type (κ)) which have components $\varphi_{l}^{(\kappa)}(x)$ that are operator-valued tempered distributions in $\mathcal{H}$, and the vacuum state $|0\rangle$ is a cyclic vector for the fields.*

Axiom 5 (Relativistic covariance). *The fields $\varphi_{l}^{(\kappa)}(x)$ transform covariantly under the action of $\overline{\mathscr{P}_{+}^{\uparrow}}$:*

$$U(a, \alpha)\varphi_{i}^{(\kappa)}(x)U(a, \alpha)^{-1} = S_{ij}^{(\kappa)}(\alpha^{-1})\varphi_{j}^{(\kappa)}(\Lambda(\alpha)x + a)$$

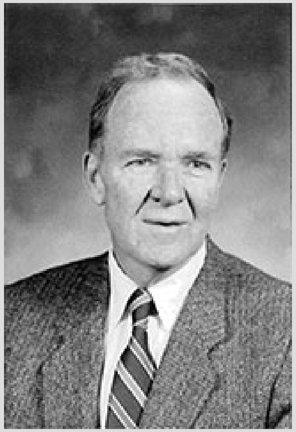
where $S(\alpha)$ is a finite dimensional matrix representation of the Lorentz spinor group $\overline{\mathscr{L}_{+}^{\uparrow}}$, and $\Lambda(\alpha)$ is the Lorentz transformation corresponding to $\alpha \in \overline{\mathscr{L}_{+}^{\uparrow}}$.

Axiom 6 (Local (anti-)commutativity). *If the support of the test functions f, g of the fields $\varphi_{l}^{(\kappa)}, \varphi_{m}^{(\kappa')}$ are space-like separated, then:*

$$[\varphi_{l}^{(\kappa)}(f), \varphi_{m}^{(\kappa')}(g)]_{\pm} = \varphi_{l}^{(\kappa)}(f)\varphi_{m}^{(\kappa')}(g) \pm \varphi_{m}^{(\kappa')}(g)\varphi_{l}^{(\kappa)}(f) = 0$$

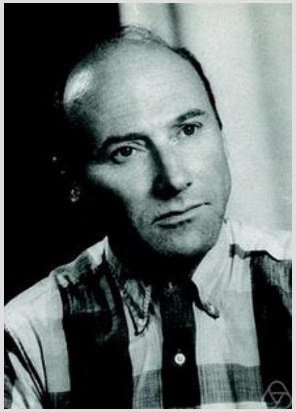
when applied to any state in $\mathcal{H}$, for any fields $\varphi_{l}^{(\kappa)}, \varphi_{m}^{(\kappa')}$.

- <u>Conclusion</u>: correlation functions $\langle 0|\varphi_{l_1}^{(\kappa_1)}(x_1)\cdots\varphi_{l_n}^{(\kappa_n)}(x_n)|0\rangle$ encode **all** of the dynamical information $\longrightarrow$ *what properties do these have?*



A. Wightman

[R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and all that* (1964).]

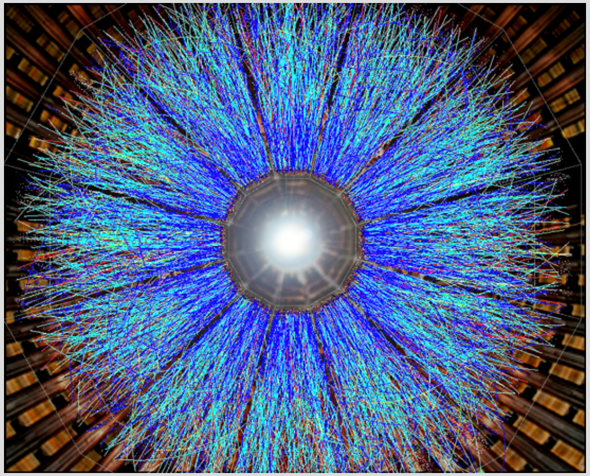


R. Haag

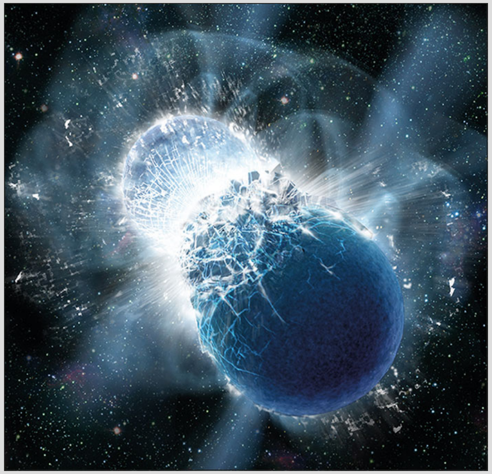
[R. Haag, *Local Quantum Physics*, Springer-Verlag (1996).]

1. Non-perturbative QFT for $T > 0$

- To describe physical phenomena in "extreme environments" one must understand of how QFT applies to systems that are hot, dense, or both



[Brookhaven National Lab]



[Skyworks Digital Inc.]

- Therefore need to figure out how the inclusion of temperature $T=1/\beta$ or density modifies the standard QFT assumptions, and what effect this has on the correlation functions.

$\rightarrow$ In this talk I will restrict to $T > 0$ and vanishing density

1. Non-perturbative QFT for $T > 0$

- **Idea**: Look for a generalisation of the standard axioms that is compatible with $T > 0$, and approaches the vacuum case for $T \to 0$

Axiom 1 (**Hilbert space structure**). *The states of the theory are rays in a Hilbert space $\mathcal{H}$ which possesses a continuous unitary representation $U(a, \alpha)$ of the Poincaré spinor group $\overline{\mathscr{P}_+^\uparrow}$.*

→ **H_β is defined for fixed $\beta = 1/T$**

Axiom 2 (**Spectral condition**). *The spectrum of the energy-momentum operator P^μ is confined to the closed forward light cone $\overline{V}^+ = \{p^\mu \mid p^2 \geq 0,\ p^0 \geq 0\}$, where $U(a, 1) = e^{iP^\mu a_\mu}$.*

→ **Replaced by the KMS condition**

$$\langle \Omega_\beta | \phi(x_1) \cdots \phi(x_k) \phi(x_{k+1}) \cdots \phi(x_n) | \Omega_\beta \rangle$$
$$= \langle \Omega_\beta | \phi(x_{k+1}) \cdots \phi(x_n) \phi(x_1 + i(\beta, \vec{0})) \cdots \phi(x_k + i(\beta, \vec{0})) | \Omega_\beta \rangle$$

Axiom 3 (**Uniqueness of the vacuum**). *There exists a unit state vector $|0\rangle$ (the vacuum state) which is a unique translationally invariant state in $\mathcal{H}$.*

→ **Instead, thermal background state $|\Omega_\beta\rangle$**

Axiom 4 (**Field operators**). *The theory consists of fields $\varphi^{(\kappa)}(x)$ (of type (κ)) which have components $\varphi_l^{(\kappa)}(x)$ that are operator-valued tempered distributions in $\mathcal{H}$, and the vacuum state $|0\rangle$ is a cyclic vector for the fields.*

✓ **Fields are still distributions**

Axiom 5 (**Relativistic covariance**). *The fields $\varphi_l^{(\kappa)}(x)$ transform covariantly under the action of $\overline{\mathscr{P}_+^\uparrow}$:*

$$U(a, \alpha) \varphi_i^{(\kappa)}(x) U(a, \alpha)^{-1} = S_{ij}^{(\kappa)}(\alpha^{-1}) \varphi_j^{(\kappa)}(\Lambda(\alpha)x + a)$$

where $S(\alpha)$ is a finite dimensional matrix representation of the Lorentz spinor group $\overline{\mathscr{L}_+^\uparrow}$, and $\Lambda(\alpha)$ is the Lorentz transformation corresponding to $\alpha \in \overline{\mathscr{L}_+^\uparrow}$.

→ **The fields no longer transform under general unitary Lorentz transformations**

Axiom 6 (**Local (anti-)commutativity**). *If the support of the test functions f, g of the fields $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$ are space-like separated, then:*

$$[\varphi_l^{(\kappa)}(f), \varphi_m^{(\kappa')}(g)]_\pm = \varphi_l^{(\kappa)}(f)\varphi_m^{(\kappa')}(g) \pm \varphi_m^{(\kappa')}(g)\varphi_l^{(\kappa)}(f) = 0$$

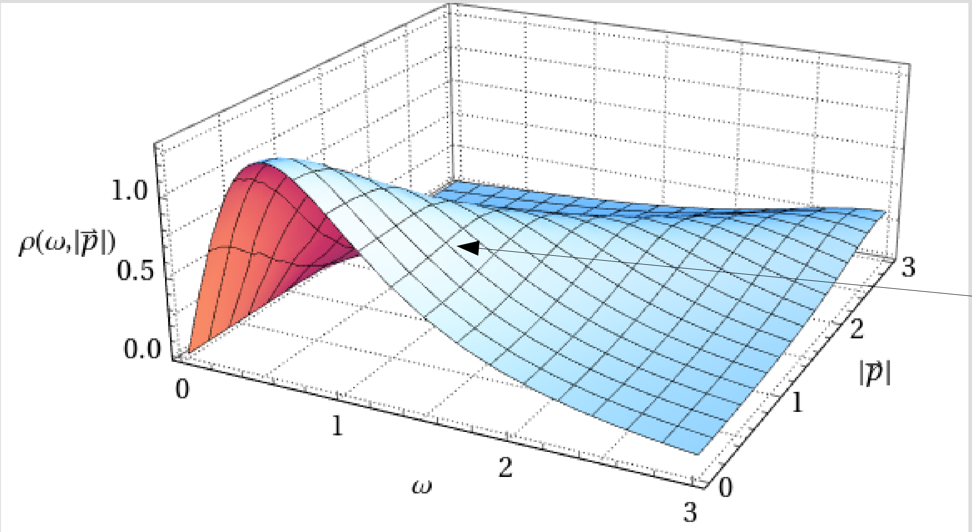
when applied to any state in $\mathcal{H}$, for any fields $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$.

✓ **Locality is unaffected by the properties of the background state. *This is important!***

1. Non-perturbative QFT for $T > 0$

- At finite T spectral functions $\rho(\omega,\boldsymbol{p})$ play a particularly important role

$$\rho(\omega,\vec{p}) = \int d^4x \; e^{i(\omega x_0 - \vec{p}\cdot\vec{x})} \langle\Omega_\beta| \left[\phi(x),\phi(0)\right] |\Omega_\beta\rangle$$



Peak locations and their dispersion are related to the dynamics of the medium and the underlying degrees of freedom of the theory

- Spectral functions also enter into the calculation of numerous important observables (transport coefficients, particle production rates, etc.)

Important question: *Can general spectral function characteristics be disentangled from model-dependent effects?*

2. Causality constraints

- Let's consider the general properties of the spectral function of a scalar field

Field locality ⇒

$[\Phi(x),\Phi(y)]=0$ for $(x-y)^2 < 0$

$$\rho(\omega,\vec{p}) = \int_0^\infty ds \int d^4u\, \epsilon(\omega-u_0)\, \delta\big((\omega-u_0)^2 - (\vec{p}-\vec{u})^2 - s\big)\, \Psi(u,s)$$

→ *"Jost-Lehmann-Dyson (JLD) representation"* [R. Jost, H. Lehmann Nuovo Cim. 5, 1957; F.J. Dyson, Phys. Rev. 110, 1958]: precursor to <u>all</u> causal spectral representations!

Imposing **Lorentz invariance** ⇒

$$\rho(\omega,\vec{p}) = 2\pi\epsilon(\omega)\int_0^\infty ds\, \delta\big(p^2 - s\big)\, \varrho(s)$$

e.g. $\rho(s)=\delta(s-m^2)$ for free theory

- <u>Note</u>: the splitting $\rho(\omega,\vec{p}) = \widetilde{\mathcal{W}}(\omega,\vec{p}) - \widetilde{\mathcal{W}}(-\omega,-\vec{p})$ does *not* uniquely relate the (p-space) two-point function to the spectral function $\rho(\omega,\boldsymbol{p})$

  But… if we impose the **spectral condition** ⇒ $\widetilde{\mathcal{W}}(\omega,\vec{p}) = \theta(\omega)\rho(\omega,\vec{p})$

- From this, all the standard vacuum QFT results follow, including the propagator *Källén-Lehmann representation*

$$D(p) = \int_0^\infty ds\, \frac{\varrho(s)}{p^2 - s + i\varepsilon}$$

2. Causality constraints

- But what about the situation when $T > 0$?

  - **Field locality** ✓ $\rightarrow$ the JLD representation is still valid
  - **Lorentz invariance** ✗ $\rightarrow$ but can retain rotational invariance
  - **Spectral condition** ✗ $\rightarrow$ replaced by the KMS condition, which implies the relation: $\widetilde{\mathcal{W}}(\omega,\vec{p}) = \dfrac{\rho(\omega,\vec{p})}{1-e^{-\beta\omega}}$

- Taking all of the $T > 0$ constraints into account one finds*

$$\rho(\omega,\vec{p}) = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\, \epsilon(\omega)\, \delta\big(\omega^2 - (\vec{p}-\vec{u})^2 - s\big)\, \widetilde{D}_\beta(\vec{u},s)$$

"Thermal spectral density"

- This is the $T > 0$ generalisation of the *Källén-Lehmann* representation

  $\rightarrow$ In *position space* the two-point function has the form:

$$\mathcal{W}(x) = \langle \Omega_\beta | \phi(x)\phi(0) | \Omega_\beta \rangle = \int_0^\infty ds\, \mathcal{W}_\beta^{(s)}(x) D_\beta(\vec{x},s)$$

Superposition of free correlators modulated by the factors $D_\beta(\boldsymbol{x},s)$

* See: *J. Bros and D Buchholz, Z. Phys. C* 55 (1992), *Ann. Inst. H.Poincare Phys.Theor.* 64 (1996)

2. Causality constraints

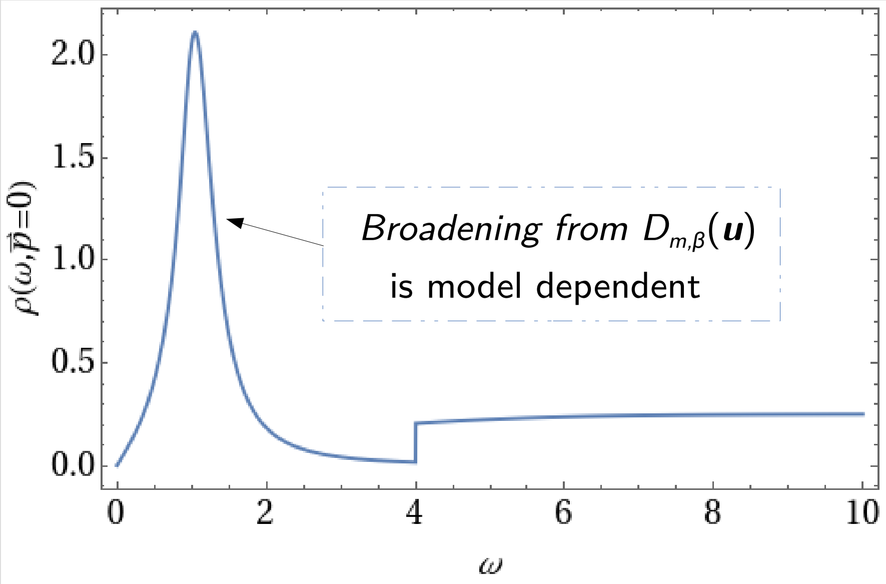
- <u>Proposition</u>: the medium contains "Thermoparticles": particle-like constituents which differ from collective quasi-particle excitations, and show up as ***discrete*** contributions [Bros, Buchholz, *NPB* 627 (2002)]

$$\widetilde{D}_{\beta}(\vec{u},s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s-m^2) + \widetilde{D}_{c,\beta}(\vec{u},s)$$

→ Thermoparticle components $\widetilde{D}_{\beta}(\boldsymbol{u})\delta(s\text{-}m^2)$ reduce to those of a vacuum particle state with mass m in the limit $T \to 0$

→ Non-trivial "Damping factor" $\widetilde{D}_{\beta}(\boldsymbol{u})$ results in thermally-broadened peaks in the spectral function, i.e. parametrises the effects of collisional broadening

→ Component $\widetilde{D}_{c,\beta}(\boldsymbol{u},s)$ contains all other types of excitations, including those that are *continuous* in s



- In many instances *Euclidean* data is used to calculate $T > 0$ observables, e.g. spectral functions $\rho_\Gamma(\omega,\boldsymbol{p})$ from $C_\Gamma(\tau,\vec{x}) = \langle O_\Gamma(\tau,\vec{x})\, O_\Gamma(0,\vec{0})\rangle_T$ where O_Γ is some particle-creating operator

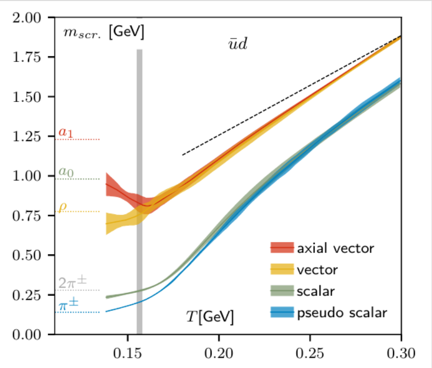
$$\widetilde{C}_\Gamma(\tau,\vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh\left[\left(\frac{\beta}{2} - |\tau|\right)\omega\right]}{\sinh\left(\frac{\beta}{2}\omega\right)}\, \rho_\Gamma(\omega,\vec{p})$$

$\rightarrow$ Determine $\rho_\Gamma(\omega,\boldsymbol{p})$ given $\widetilde{C}_\Gamma(\tau,\boldsymbol{p})$: *problem is ill-conditioned, need more information!*

- Another quantity of interest in lattice studies is the *spatial* correlator

$$C_\Gamma(x_3) = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} d\tau\, C_\Gamma(\tau,\vec{x}) = \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} e^{ip_3 x_3} \int_0^\infty \frac{d\omega}{\pi\omega}\, \rho_\Gamma(\omega, p_1 = p_2 = 0, p_3)$$

- Large-x_3 behaviour $C_\Gamma(x_3) \sim \exp(-m_{scr}|x_3|)$ used to extract "screening masses" $m_{scr}(T)$



[HotQCD collaboration, *PRD* 100 (2019)]

3. Spectral properties from Euclidean data

<u>Goal</u>: Use the additional constraints imposed by causality to better understand how spectral features manifest themselves in Euclidean data

- Causality implies a general connection between the spatial correlator and thermal spectral density [P.L., *PRD* 106 (2022); P.L., O. Philipsen, *JHEP* 10, 161 (2022)]

$$C(x_3) = \frac{1}{2}\int_0^\infty ds \int_{|x_3|}^\infty dR \; e^{-R\sqrt{s}} D_\beta(R,s)$$

Thermal spectral density in position space

→ Thermoparticle states give rise to $C(x_3)$ contributions that are particularly significant in the large-x_3 region

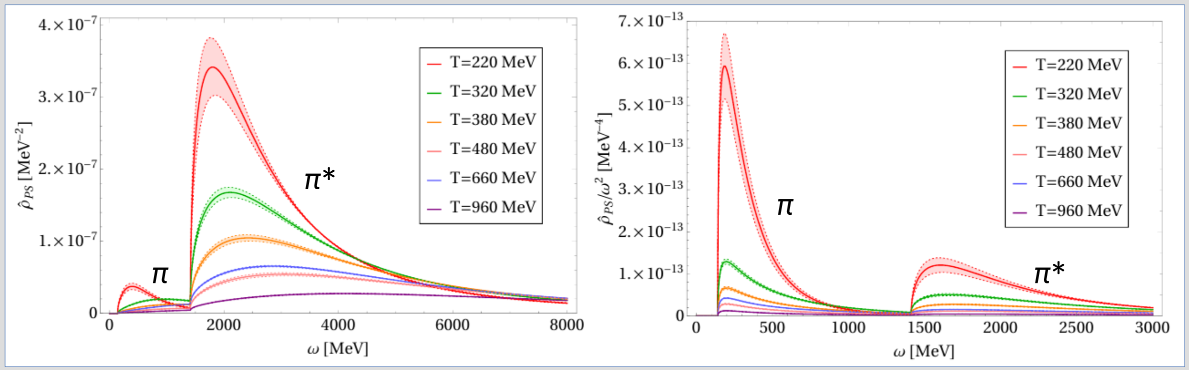
$$C(x_3) \approx \frac{1}{2}\sum_{i=1}^{n} \int_{|x_3|}^\infty dR \; e^{-m_i R} D_{m_i,\beta}(R)$$

- Once the damping factors of these states are know one can use the $T > 0$ spectral representation to compute their analytic contribution to $\rho(\omega,\boldsymbol{p})$

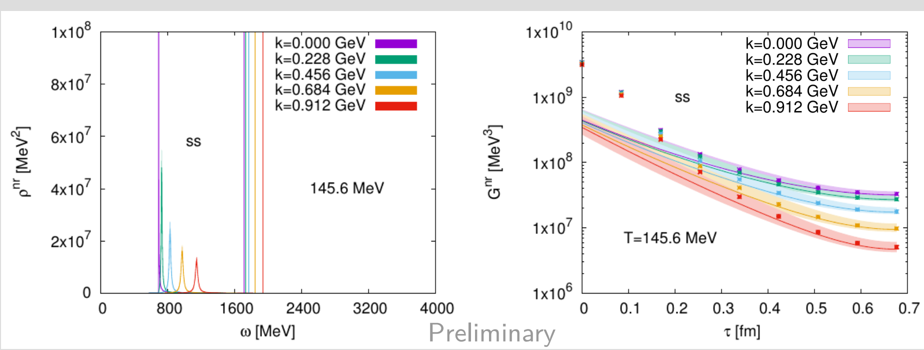
3. Spectral properties from Euclidean data

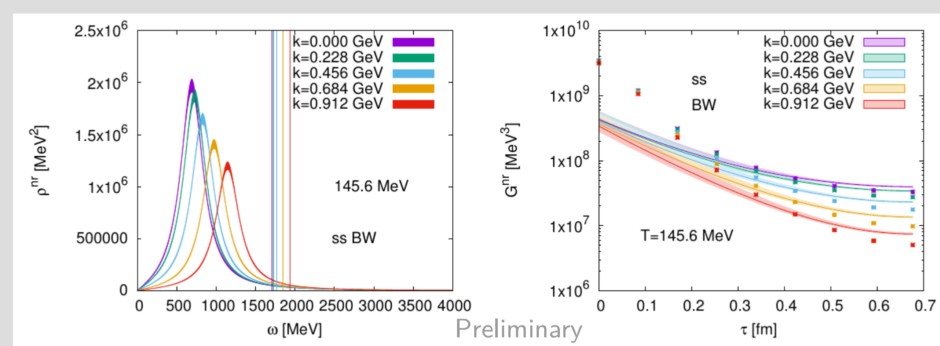
- Can now apply these relations to QCD lattice data → simple case is the spatial correlator $C_{\mathrm{PS}}(x_3)$ of pseudo-scalar meson operator $\boxed{\mathcal{O}^a_{\mathrm{PS}} = \overline{\psi}\gamma_5 \frac{\tau^a}{2}\psi}$

  → Pseudo-scalar mesons composed of two light quarks [P.L., O. Philipsen, 2022]



- Work is ongoing [D. Bala, O. Kaczmarek, P. Lowdon, O. Philipsen, and T. Ueding] to extend this to heavier quarks (light-strange and strange-strange)





→ This approach can discriminate between different ground state hypotheses

4. Revisiting $T > 0$ perturbation theory

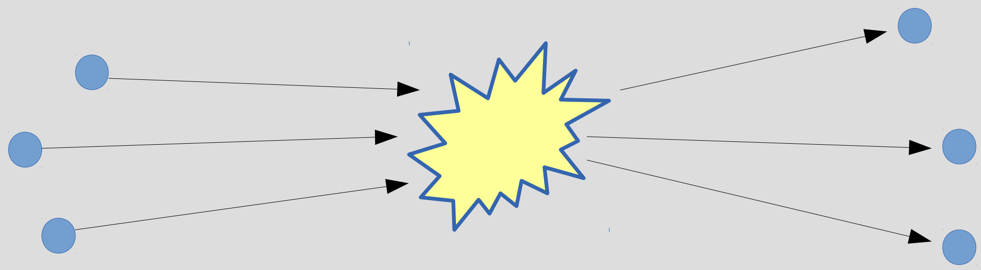
- It has long been understood that $T > 0$ perturbation theory has complications: non-analytic contributions, IR divergences, ...

- In fact, Weldon [*PRD* 65 (2002)] showed that the perturbative procedure in Φ^4 theory *fails* at 2-loop order because the self-energy $\Pi(k)$ has a branch point on the perturbative mass shell $k_0 = E(k)$

  → This is a generic feature of perturbative computations that use free thermal propagators, or in fact any propagators that have a real dispersion relation $p_0 = E(p)$

$$G_R(p) = -\frac{1}{(p_0 + i\epsilon)^2 - E(p)^2}$$

- *Why?* → Interactions with the thermal medium persist, even for large times, so need to take this into account in the definition of scattering states!



4. Revisiting $T > 0$ perturbation theory

- QFT reason: the KMS condition is incompatible with on-shell scattering states and non-zero interactions → "*Narnhofer-Requardt-Thirring Theorem*" [Commun. *Math. Phys.* 92, 247 (1983)]

- Idea: Start with propagators that are off shell [Landsman,1988; Weldon, 2002]

  → Then perform perturbative calculations with *these* propagators instead of free field one. Hypothesised that this could give rise to an IR-regularised perturbative expansion for $T > 0$ [Bros, Buchholz hep-th/9511022]

  → *But what form should these propagators take?*

- A few ideas [Landsman, 1988; Bros, Buchholz, 2002; Weldon 2002], but seems that thermoparticle propagators would be a natural candidate

$$\widetilde{G}_{\beta}^{(-)}(k_0,\vec{p}) = \frac{1}{4|\vec{p}|\kappa}\ln\left[\frac{-k_0^2+m^2+(|\vec{p}|+\kappa)^2}{-k_0^2+m^2+(|\vec{p}|-\kappa)^2}\right]$$

Φ^4 theory case (*Width parameter* $\kappa \sim \sqrt{|\lambda|}T$)

- Can then compare non-perturbative data (lattice, fRG, etc.) with the results from this perturbative expansion [PL, O. Philipsen, *in preparation*]

Summary & outlook

- Causality imposes **non-perturbative** constraints for $T > 0$ which have significant implications

  → Spectral properties of thermal correlation functions

  → Connection between real-time observables and Euclidean correlators

- So far, only real scalar fields Φ with $T > 0$ have been considered, but this approach *can* be extended

  → Other hadronic states (baryons, exotic states, ...)

  → Higher spin fields/states (fermions, vectors, ...)

  Work in progress!

  → Non-vanishing density, $|\mu|>0$

- Ultimately, these constraints and methods can help in gaining a better understanding of physically relevant theories, including QED and QCD