Deconfined Quantum (Pseudo-)Criticality from Non-Standard FRG Flow for 3D Wess–Zumino–Witten Theory

Functional Methods in Strongly Correlated Systems — Hirschegg, 12th September 2023

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DFG Deutsche Forschungsgemeinschaft

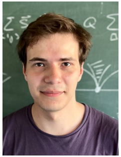
VILLUM FONDEN

SDU

Acknowledgement

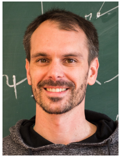
…Based on: ongoing work with

Astrid Eichhorn
(Odense)



Bilal Hawashin
(Bochum)

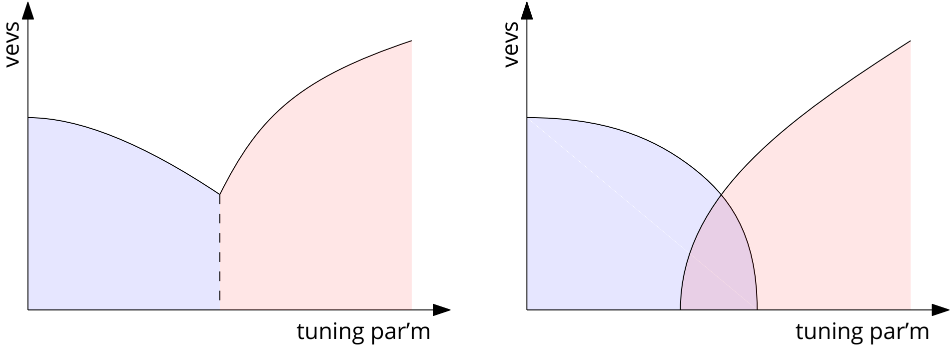
Lukas Janssen
(Dresden)



Michael Scherer
(Bochum)

Motivation: Critical phenomena beyond Ginzburg–Landau

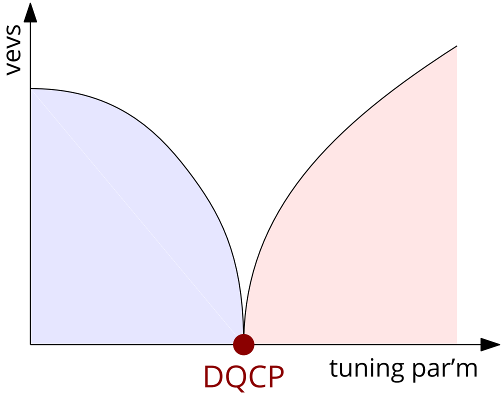
- Consider system with two global symmetries G_1, G_2
- Transitions between $\langle\phi_1\rangle \neq 0$ (SSB of G_1) and $\langle\phi_2\rangle \neq 0$ (SSB of G_2)?
  $\Rightarrow$ within Ginzburg–Landau paradigm only two allowed scenarios:



- Direct $\Rightarrow$ 1st order; 2nd order $\Rightarrow$ indirect (via *coexistence* phase)
- Direct 2nd order: forbidden

Motivation: Critical phenomena beyond Ginzburg–Landau

- Possibility beyond Ginzburg-Landau: *Deconfined quantum critical point* (DQCP)
  Senthil/Balents/Sachdev/Vishwanath/Fisher '04; …



- At DQCP: Enlarged symmetry $G \supset G_1, G_2$

Motivation: Critical phenomena beyond Ginzburg–Landau

- Possibility beyond Ginzburg-Landau: *Deconfined quantum critical point* (DQCP)
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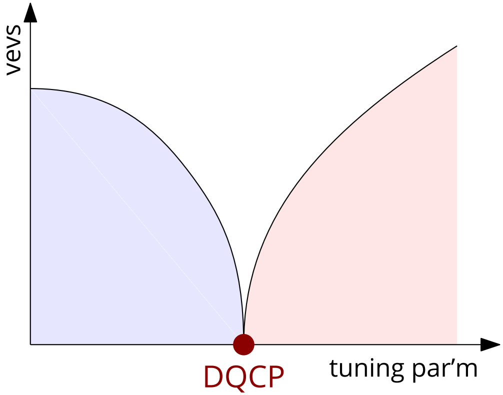
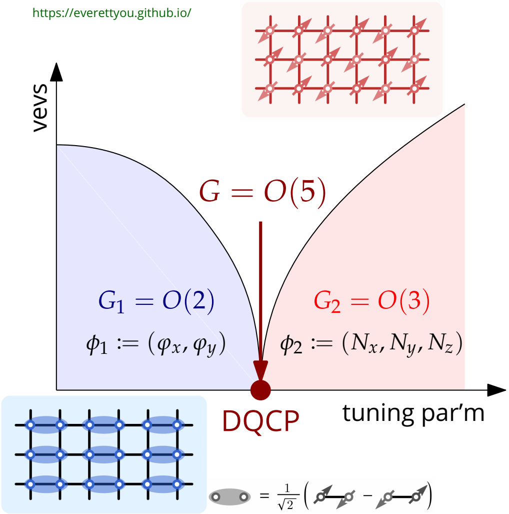


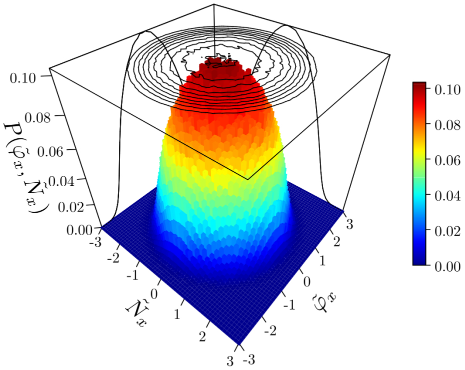
- At DQCP: Enlarged symmetry $G \supset G_1, G_2$
- Crucial ingredient to stabilise G: WZW term coupling the two OPs

Example: Transition between Néel and VBS order

Levin/Senthil ’04; Vishwanath/Balents/Senthil ’04; . . .



- (Computer) experiments: Signs of emergent O(5) symmetry . . . Nahum *et al.* ’15a



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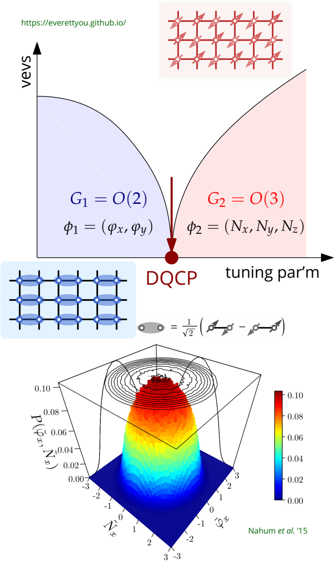
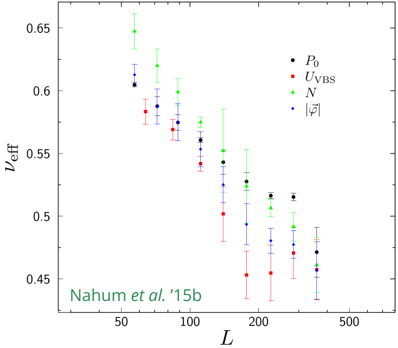


However . . .

- drift of critical exponents, unitarity bound violations
  Kuklov *et al.* '08; Jiang *et al.* '08; Chen *et al.* '13; d'Emidio *et al.* '21; . . .



Towards an explanation

Ma/Wang '20; Nahum '20

- EFT = $(d+2)$D WZW theory with target space S^{d+1} ($\simeq \mathrm{O}(d+2)\,/\,\mathrm{O}(d+1)$)

$$
\begin{aligned}
S &= S_{\mathrm{NL}\sigma\mathrm{M}} + 2\pi i k \Gamma_{\mathrm{WZW}} \\
S_{\mathrm{NL}\sigma\mathrm{M}} &= \frac{1}{4\pi g} \int d^d x (\partial_i \Phi_a)^2 \\
\Gamma_{\mathrm{WZW}} &= \frac{1}{(d+1)!\,\mathrm{Area}(S^{d+1})} \int_0^1 du \int d^d x\, \epsilon^{i_1 \dots i_{d+1}} \epsilon^{a a_1 \dots a_{d+1}} \Phi_a \Phi_{a_1, i_1} \dots \Phi_{a_{d+1}, i_{d+1}} \\
(d\mu(\Phi)) &= (d\Phi)\, \delta(|\Phi|^2 - 1)
\end{aligned}
$$

$$(\Phi_a) := (\phi_1, \phi_2) = (\varphi_x, \varphi_y, N_1, \dots N_d)$$

Towards an explanation

Ma/Wang '20; Nahum '20

$$S = S_{\mathrm{NL}\sigma\mathrm{M}} + 2\pi i k \Gamma_{\mathrm{WZW}}$$

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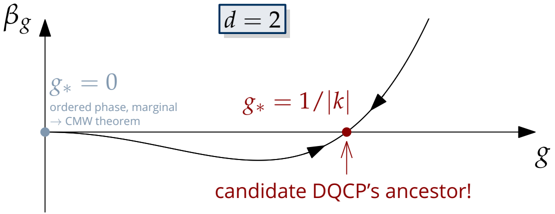
- $d = 2$: Solvable, fixed points known
  Polyakov/Wiegmann '83, Witten '84,
  Knizhnik/Zamolodchikov '84, Affleck/Haldane '87, …



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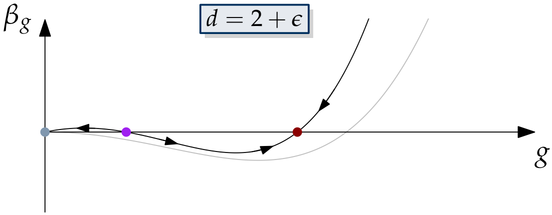
- $d = 2 + \epsilon, \epsilon \ll 1$
  Assume $\beta_g(d)$ analytic 'Cardy–Hamber trick'



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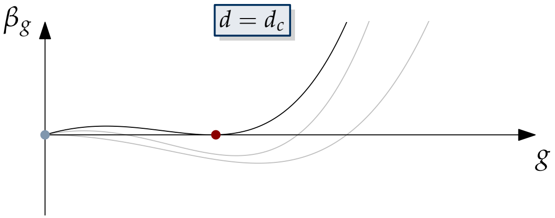

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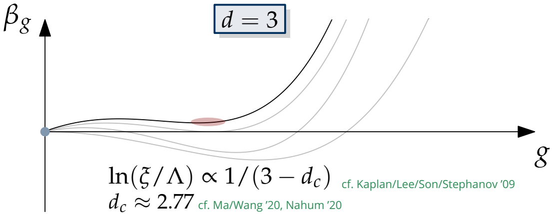
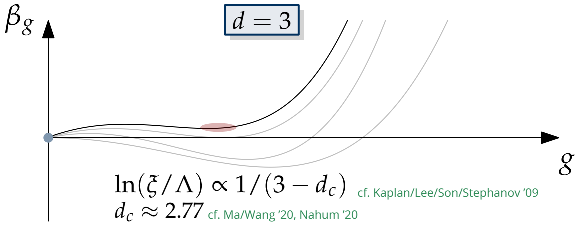
- $d = 2 + \epsilon, \epsilon \ll 1$
  Assume $\beta_g(d)$ analytic 'Cardy–Hamber trick'
- $d \to d_c$: Interacting stable and unstable FPs collide
- $d = 3$: No real FP, but large correlation length $\ln \xi \propto 1/(3 - d_c)$
  cf. Kaplan/Lee/Son/Stephanov '09



$\ln(\xi/\Lambda) \propto 1/(3 - d_c)$ cf. Kaplan/Lee/Son/Stephanov '09
$d_c \approx 2.77$ cf. Ma/Wang '20, Nahum '20

Towards an explanation

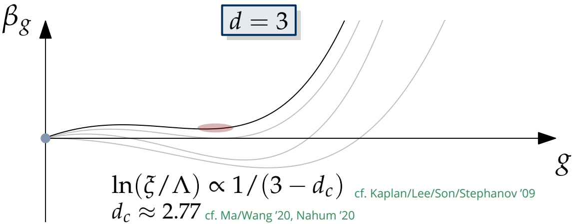
Ma/Wang '20; Nahum '20



- Qualitatively correct picture, but ...

Towards an explanation

Ma/Wang '20; Nahum '20



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- Caveat: Analytic continuation of WZW term across dim's mathematically dubious

$$\Gamma_{\text{WZW}} = \frac{1}{(d+1)!\,\text{Area}(S^{d+1})} \int_0^1 du \int d^d x\, \epsilon^{i_1 \ldots i_{d+1}} \epsilon^{a a_1 \ldots a_{d+1}} \Phi_a \Phi_{a_1, i_1} \ldots \Phi_{a_{d+1}, i_{d+1}}$$

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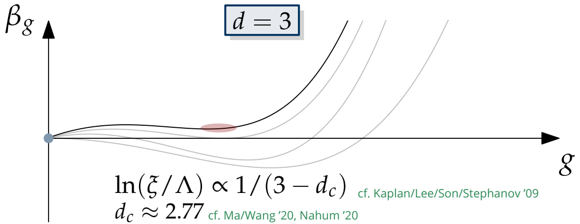

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- Here: Computation at fixed $d = 3$ using FRG; WZW level k instead of dim. d

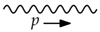
  k is just as RG-invariant as d; lattice input (cf. anomaly matching): physical case $\leftrightarrow k = 1$

Administrative details I: Parametrisation of S^5 and truncation

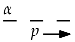
- 'Polyspherical' coordinates to eliminate $\delta(|\Phi|^2 - 1)$ from $(d\mu(\Phi))$ cf. Cardy '96; Ma/Wang '20, …

$$(\Phi_a) = \left(\tau_\alpha, \sqrt{1-\vec{\tau}^2}\cos\vartheta, \sqrt{1-\vec{\tau}^2}\sin\vartheta\right)$$

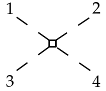
- Truncate classical action @ $\mathcal{O}(\text{fields}^4)$, relevant propagators & vertices:



$$= \frac{2\pi g}{p^2}$$

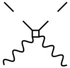


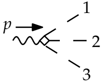
$$= \frac{2\pi g\,\delta_{\alpha\beta}}{p^2}$$



$$= \frac{1}{2\pi g}\left[\delta_{\alpha_1\alpha_2}\delta_{\alpha_3\alpha_4}(p_1+p_2)^2 + \text{perm.}\right]$$

analogous:





$$= \frac{ik}{3\pi}\epsilon^{ijk}\epsilon^{\alpha_1\alpha_2\alpha_3}(-ip_i)\left(l_{2j}l_{3k} \pm \text{perm.}\right)$$

- Minimal truncation ($\kappa \equiv$ RG scale):

$$\Gamma_\kappa[\vartheta,\tau_\alpha] = S[\vartheta,\tau_\alpha]_{g\to g_\kappa}$$

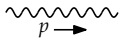
Administrative details II: Regularisation

- 'Natural' ansatz preserving $\mathrm{O}(5)$ symmetry:

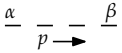
$$\Delta S_\kappa[\vartheta,\tau_\alpha] = \frac{1}{4\pi g_\kappa}\int_x \left(\sqrt{r(-\partial^2/\kappa^2)}\,\partial_i\Phi_a(\vartheta,\tau_\alpha)\right)^2$$

- Leads to:

(i) Regularised propagators and vertices + regulator-induced vertex



$$= \frac{2\pi g_\kappa}{P_\kappa(p^2)}$$



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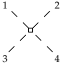
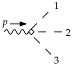


$$= \frac{1}{2\pi g_\kappa}\left[\delta_{\alpha_1\alpha_2}\delta_{\alpha_3\alpha_4}P_\kappa((p_1+p_2)^2) + \text{perm.}\right]$$

analogously,



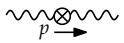
$$= \frac{ik}{3\pi}\epsilon^{ijk}\epsilon^{\alpha_1\alpha_2\alpha_3}(-ip_i)\,(l_{2j}l_{3k} \pm \text{perm.})$$

$$P_\kappa(p^2) := p^2[1 + r(p^2/\kappa^2)]$$

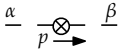
(ii) Regulator insertions

N.B.: Non-standard quartic insertion; analogous one for ϑ^4 exists but not needed here

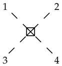
...



$$= \frac{-1}{2\pi g_\kappa}\left(\frac{\beta_g}{g_\kappa}p^2 r(p^2/\kappa^2) + \frac{2p^4}{\kappa^2}r'(p^2/\kappa^2)\right)$$



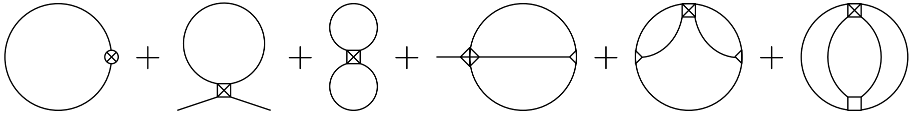
$$= \frac{-\delta_{\alpha\beta}}{2\pi g_\kappa}\left(\frac{\beta_g}{g_\kappa}p^2 r(p^2/\kappa^2) + \frac{2p^4}{\kappa^2}r'(p^2/\kappa^2)\right)$$



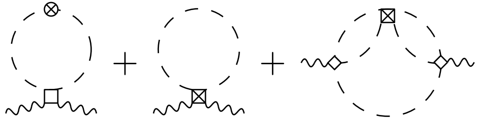
$$= \frac{-1}{2\pi g_\kappa}\left[\delta_{\alpha_1\alpha_2}\delta_{\alpha_3\alpha_4}\left(\frac{\beta_g}{g_\kappa}p_{12}^2 r(p_{12}^2/\kappa^2) + \frac{2p_{12}^4}{\kappa^2}r'(p_{12}^2/\kappa^2)\right) + \text{perm.}\right]$$

Administrative details III: Non-standard flow

Pawlowski *Annals Phys.* '07

$\kappa \partial_\kappa \Gamma_\kappa =$ 

- β_g fixed by normalisation of ϑ's propagator

$-\frac{\beta_g}{4\pi g_\kappa^2} =$ 

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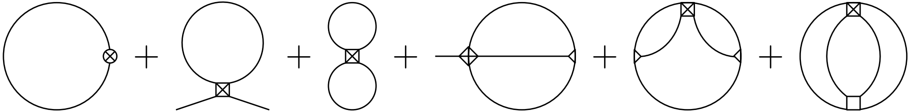

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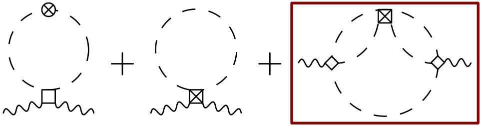

- Remark: Non-vanishing contribution of WZW term!

Results I: Beta function

$$\beta_g = g + \frac{1}{1 - \left(3I_{02} + I_{02}^{\text{nst}} + I_{01}^{\text{nst}}\right) g + 16\pi^3 I_{05} k^2 g^5} \left[- \left(3I_{12} + I_{12}^{\text{nst}} + I_{11}^{\text{nst}}\right) g^2 + 16\pi^3 I_{15} k^2 g^6 \right]$$

- $I_{\cdots}^{\cdots}$ depend on shape function r
- r-independently*, we have:

$$0 < 3I_{02}(r) + I_{02}^{\text{nst}}(r) + I_{01}^{\text{nst}}(r) =: C_{T,0}(r)$$
$$0 < 3I_{12}(r) + I_{12}^{\text{nst}}(r) + I_{11}^{\text{nst}}(r) =: C_{T,1}(r)$$
$$0 < I_{05}(r) =: C_{M,0}(r)$$
$$0 < I_{15}(r) =: C_{M,1}(r)$$

* to the best of our knowledge

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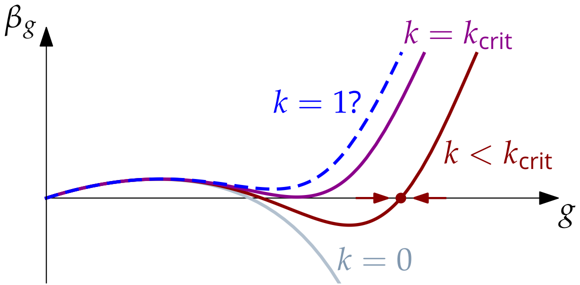

- k_{crit} is r-dependent $k_{\text{crit}}(r) = \frac{16}{25\sqrt{5}}\sqrt{\frac{\left(C_{T,1}(r)+C_{T,0}(r)\right)^5}{C_{M,1}(r)+C_{M,0}(r)}}$

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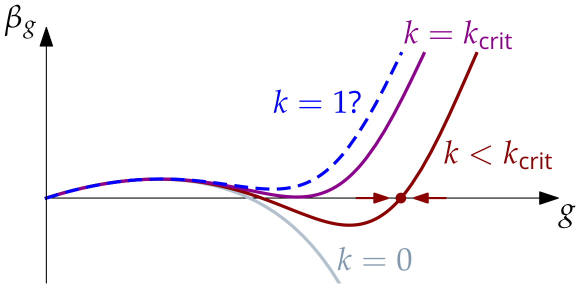
$$\beta_g = g + \frac{1}{1 - C_{T,0}(r)g + C_{M,0}(r)k^2 g^5}\left[-C_{T,1}(r)g^2 + C_{M,1}(r)k^2 g^6\right]$$



- k_{crit} is r-dependent $k_{\text{crit}}(r) = \frac{16}{25\sqrt{5}}\sqrt{\frac{\left(C_{T,1}(r)+C_{T,0}(r)\right)^5}{C_{M,1}(r)+C_{M,0}(r)}}$

  …consider family $r(x;A) = \frac{e^A}{x}e^{-Ax^2}$

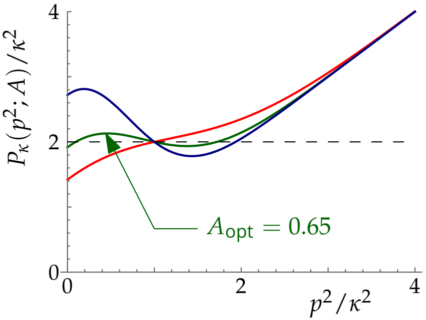
Results II: Estimate for k_{crit}

- Consider family

$$r(x;A) = \frac{e^A}{x} e^{-Ax^2}$$

- Optimal choice Litim '01

$$A_{\text{opt}} = \arg\max_{A>0} \inf_{p^2 \geqslant 0} P_\kappa(p^2; A)$$



Best estimate

$$k_{\text{crit}} \approx 0.7$$

Main source of numerical uncertainty: Monte-Carlo integration of $I_{15}(r)$ and $I_{05}(r)$

Conclusion

Summary

- Non-perturbative computation of critical WZW level at fixed $d = 3$
  …avoids analytic continuation of WZW term in spacetime dimension
- Result in rough agreement with numerics
- Systematic improvement of result possible within framework (e.g., truncation of Γ_κ by canonical dimension)

Outlook/ToDo

- Different families of regulators
- PMS vs gap criterion
- Extension to other coset spaces, e.g.:
  $\mathrm{O}(N)/[\mathrm{O}(N-n)\times\mathrm{O}(n)]$ *Grassmann* or $\mathrm{O}(N)/\mathrm{O}(4)$ *Stiefel*
  …Contains S^4 as special case [$(N,n) = (4,2)$ resp. $N = 5$]
  …Tracking of FPs as function of N, n at fixed $k = 1$ alternative perspective on pseudocriticality
- Higher-order regulators known pathway to gauge-invariant regulators (Yang-Mills, gravity, …) *in theory* Alexandre/Polonyi '01; Shapiro/Lavrov '13, '14; …

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  …possible also *in practice*?

Thank you!