

Color superconductivity beyond Mean Field Approximation

Ugo Mire

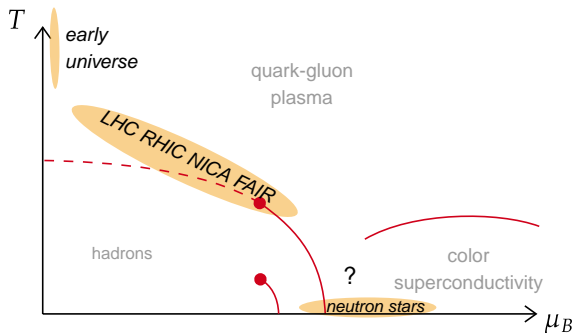
In collaboration with: Bernd-Jochen Schaefer

Hirschegg, September 12, 2023

1. Color Superconductivity and 2SC Phase
2. Functional Renormalization Group
3. Phase Structure and Astrophysical Applications

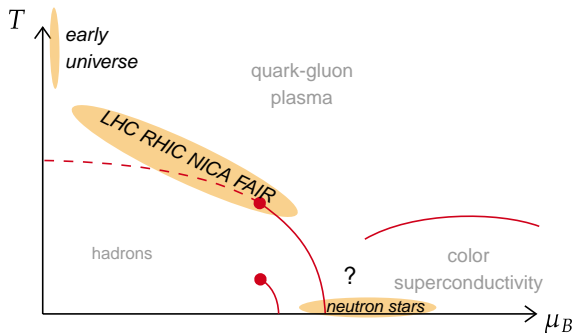
Color Superconductivity and 2SC Phase

Why study color superconductivity?



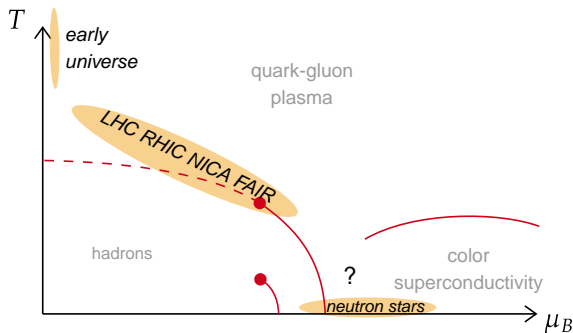
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- **Natural in QCD**: attractive gluon exchange \rightarrow Cooper pairing and superconductivity at high densities.

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- **Natural in QCD**: attractive gluon exchange \rightarrow **Cooper pairing** and superconductivity at high densities.

\uparrow
which pairing channel?

2SC Phase

2SC phase: condensates of the shape

$$\Delta_A = \langle q^T C \gamma_5 \tau_2 \lambda_A q \rangle, \quad A = 2, 5, 7$$

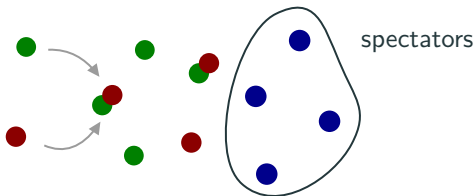
↖ some matrix structure in flavor,
color and Dirac space

Main characteristics:

- Chiral symmetry is not broken.
- Color symmetry is "broken" (Higgs mechanism), but not fully:

$$SU_c(3) \rightarrow SU_c(2).$$

- Only two color participate in superconductivity.



Functional Renormalization Group

Why use the FRG?

Four-quark interactions in QCD

- Low densities: **scalar-pseudoscalar** ($\sigma, \vec{\pi}$) channel dominates

$$(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau_i\psi)^2$$

→ quantitative description of chiral crossover using FRG.

[Fu, Pawłowski & Rennecke; 1909.02991]

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- High densities: **diquark** channel dominates

[Fu, Pawłowski & Rennecke; 1909.02991]

[Braun, Leonhardt & Pospiech; 1909.06298]

$$(q^T C\gamma_5\tau_2\lambda_A q)(\bar{q}\gamma_5\tau_2\lambda_A C\bar{q}^T)$$

→ a natural step is to include the diquark channel.

Quark-meson-diquark Model

Quark-meson-diquark model action:

$$\begin{aligned} S_{\text{QMD}} = \int_x \left\{ \bar{q} (\not{\partial} - \mu\gamma_0 + g_\phi (\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\tau})) q \right. \\ + \frac{g_\Delta}{2} (\Delta_A q^T C \gamma_5 \tau_2 \lambda_A q - \Delta_A^* \bar{q} \gamma_5 \tau_2 \lambda_A C \bar{q}^T) \\ + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 \\ + ((\partial_\nu + \delta_{\nu 0} 2\mu) \Delta_A^*) (\partial_\nu - \delta_{\nu 0} 2\mu) \Delta_A \\ \left. + U(\sigma^2 + \vec{\pi}^2, \Delta_A \Delta_A^*) - c\sigma \right\} \end{aligned}$$

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meson and diquark interactions,
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quark-diquark
interaction

diquark kinetic
term, couples to μ

meson and diquark interactions,
arbitrary potential constrained by
symmetries

explicit chiral symmetry
breaking term

Truncation and Regulator Choice

Some FRG details:

- **Local potential approximation** (LPA): only scale dependent quantity is the potential $U_k(\sigma, \Delta)$.
 - σ : chiral condensate.
 - Δ : diquark condensate.
 - $U_k \rightarrow \infty$ number of couplings.

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$$R_k = (k^2 - \vec{p}^2)\theta(k^2 - \vec{p}^2) .$$

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- **Initial conditions:**

$$U_\Lambda = a_1\sigma^2 + a_2\sigma^4 + b_1\Delta^2 + b_2\Delta^4$$

Flow Equation

$$\partial_t U_k(\sigma, \Delta) = - \text{[Diagram 1]} - \text{[Diagram 2]} + \frac{1}{2} \text{[Diagram 3]} + \frac{1}{2} \text{[Diagram 4]} + \frac{1}{2} \text{[Diagram 5]}$$

The equation is represented by five diagrams, each featuring a circle with a cross (⊗) at the top. Diagram 1 is a solid circle with two counter-propagating arrows and is labeled q_r, q_g . Diagram 2 is a solid circle with two counter-propagating arrows and is labeled q_b . Diagram 3 is a dashed circle and is labeled Δ_2, σ . Diagram 4 is a dashed circle and is labeled π . Diagram 5 is a dashed circle and is labeled Δ_5, Δ_7 .

Flow Equation

depends on $E_\Delta = \sqrt{(\epsilon_k \pm \mu)^2 + g_\Delta^2 \Delta^2}$

with $\epsilon_k = \sqrt{k^2 + g_\phi^2 \sigma^2}$

↓

$$\begin{aligned} \partial_t U_k(\sigma, \Delta) = & - \text{[solid circle with } \otimes \text{ at top, arrows clockwise, labeled } q_r, q_g \text{]} - \text{[solid circle with } \otimes \text{ at top, arrows clockwise, labeled } q_b \text{]} + \frac{1}{2} \text{[dashed circle with } \otimes \text{ at top, labeled } \Delta_2, \sigma \text{]} \\ & + \frac{1}{2} \text{[dashed circle with } \otimes \text{ at top, labeled } \pi \text{]} + \frac{1}{2} \text{[dashed circle with } \otimes \text{ at top, labeled } \Delta_5, \Delta_7 \text{]} \end{aligned}$$

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coupling between one diquark
and the sigma meson

$$\partial_t U_k(\sigma, \Delta) = - \begin{array}{c} \downarrow \\ \text{diagram 1} \end{array} - \begin{array}{c} \text{diagram 2} \end{array} + \frac{1}{2} \begin{array}{c} \downarrow \\ \text{diagram 3} \end{array} \\
 + \frac{1}{2} \begin{array}{c} \text{diagram 4} \end{array} + \frac{1}{2} \begin{array}{c} \text{diagram 5} \end{array}$$

The diagrams are:

- Diagram 1: A solid circle with a vertex at the top (circle with an 'X'). Two arrows on the circle indicate a clockwise loop. Labels q_r, q_g are at the bottom right.
- Diagram 2: A solid circle with a vertex at the top (circle with an 'X'). Two arrows on the circle indicate a clockwise loop. Label q_b is at the bottom right.
- Diagram 3: A dashed circle with a vertex at the top (circle with an 'X'). Label Δ_2, σ is at the bottom right.
- Diagram 4: A dashed circle with a vertex at the top (circle with an 'X'). Label π is at the bottom right.
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- Diagram 4: A dashed circle with a vertex \otimes at the top. An arrow points up to it from below.
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 problem with this part

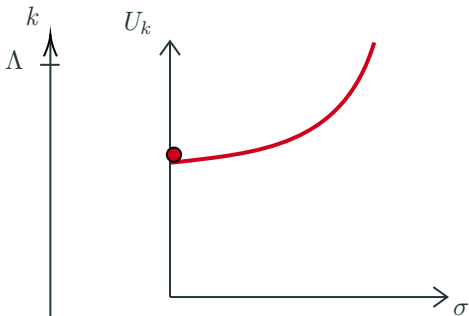
First Trouble: Bosonic Diquark Loops

Pion loops

$$\partial_t U_k = \frac{1}{\epsilon_\pi} \coth \frac{\epsilon_\pi}{2T} + \dots$$

with

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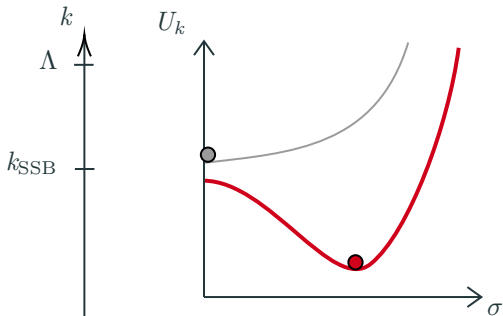
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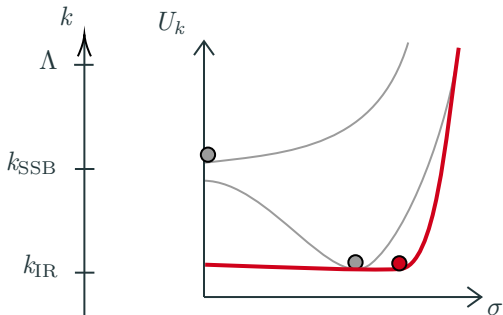
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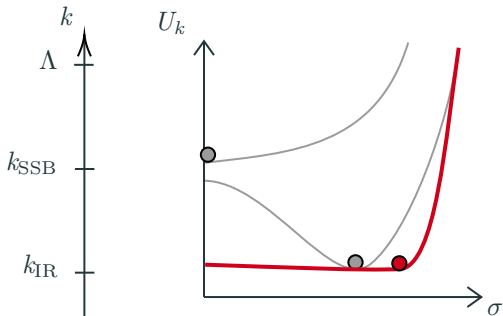
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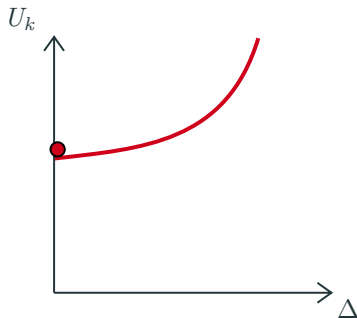


Diquark loops

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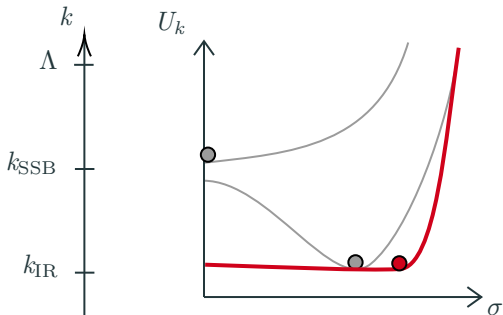
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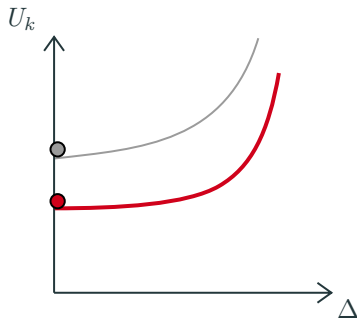


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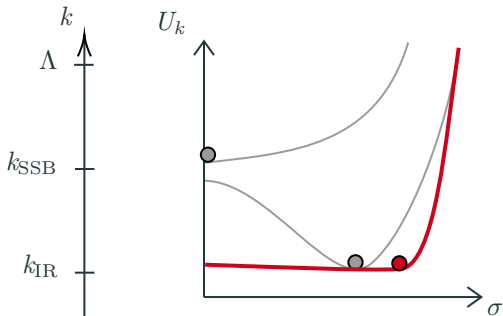
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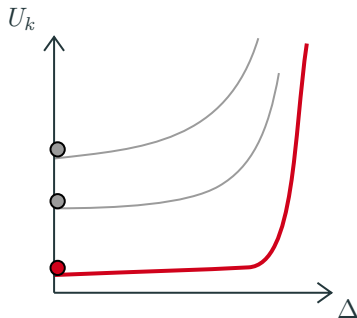


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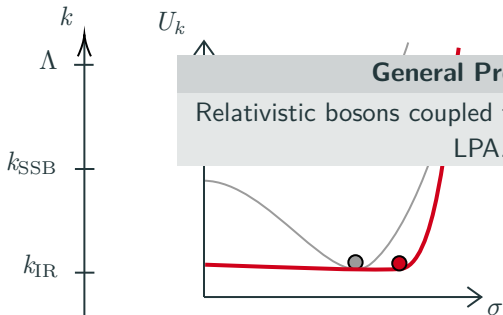
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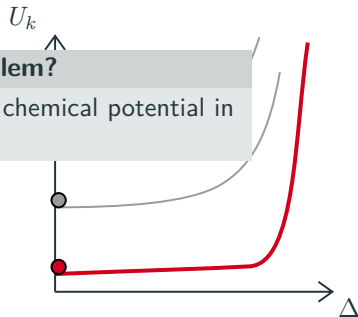


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General Problem?

Relativistic bosons coupled to chemical potential in LPA.

Second Trouble: Gapped Quark Loop

Gapped quark loop:



$$\propto \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k - \mu)^2 + g_\Delta^2 \Delta^2}} \xrightarrow{\partial_{\Delta^2}} \propto \frac{\epsilon_k - \mu}{((\epsilon_k - \mu)^2 + g_\Delta^2 \Delta^2)^{3/2}}$$

diverges at the Fermi-surface



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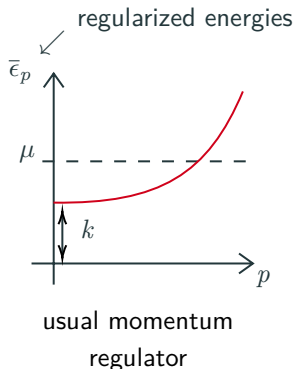
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Possible solution: **flow around the Fermi-surface**

[Braun, Dörfeld, Schallmo, Töpfel; 2008.05978]



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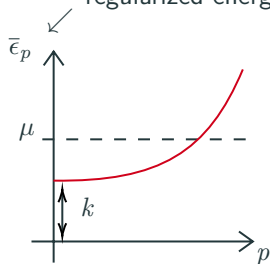
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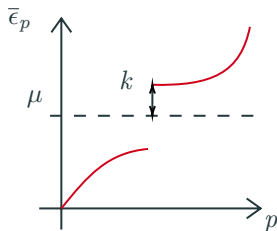
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regularized energies



usual momentum
regulator



Fermi-surface
regulator

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Gapped quark loop:



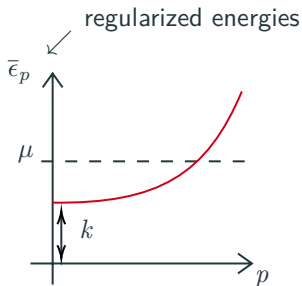
$$\propto \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k - \mu)^2 + g_\Delta^2 \Delta^2}} \xrightarrow{\partial_{\Delta^2}} \propto \frac{\epsilon_k - \mu}{((\epsilon_k - \mu)^2 + g_\Delta^2 \Delta^2)^{3/2}}$$

diverges at the Fermi-surface

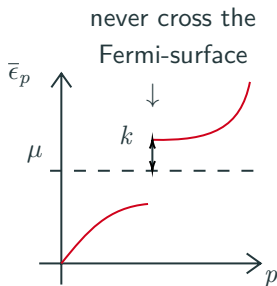


Possible solution: **flow around the Fermi-surface**

[Braun, Dörnfeld, Schallmo, Töpfel; 2008.05978]



usual momentum
regulator



Fermi-surface
regulator

Phase Structure and Astrophysical Applications

Quark-meson-diquark Model without Diquark Loops

Diquarks only at the mean field level: none of the previously mentioned problems are present.

$$\partial_t U_k = - \text{[diagram 1]} - \text{[diagram 2]} + \frac{1}{2} \text{[diagram 3]} + \frac{1}{2} \text{[diagram 4]}$$

The equation shows four diagrams representing terms in the derivative of the potential $\partial_t U_k$. Each diagram consists of a circle with a vertex at the top marked with a circled cross. The first two diagrams are solid lines with arrows pointing clockwise, labeled q_r, q_g and q_b respectively. The last two diagrams are dashed lines with arrows pointing clockwise, labeled σ and π respectively.

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The diagrams are:

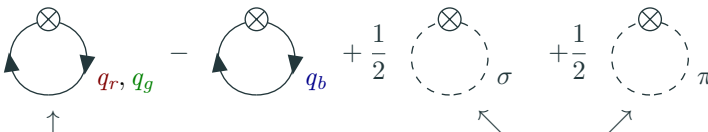
- Diagram 1: A solid circle loop with a vertex at the top (circle with an 'X'). Two arrows on the bottom half of the loop point clockwise. Labels q_r, q_g are at the bottom right.
- Diagram 2: A solid circle loop with a vertex at the top (circle with an 'X'). Two arrows on the bottom half of the loop point clockwise. Label q_b is at the bottom right.
- Diagram 3: A dashed circle loop with a vertex at the top (circle with an 'X').
- Diagram 4: A dashed circle loop with a vertex at the top (circle with an 'X').

diquarks still present here

$$E_{\Delta}^{\pm} = \sqrt{(\epsilon_k \pm \mu)^2 + g_{\Delta}^2 \Delta^2}$$

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The diagrammatic equation shows four terms. The first two terms are solid circles with arrows, representing diquark loops with quark flavors q_r, q_g and q_b . The last two terms are dashed circles with arrows, representing meson loops for σ and π . Each loop has a crossed circle at the top.

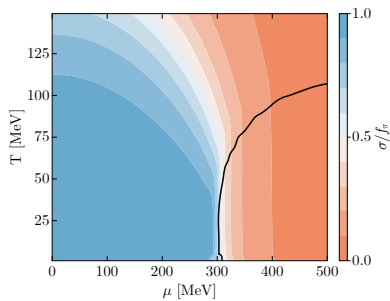
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- Look at phase structure.
- First astrophysical applications.

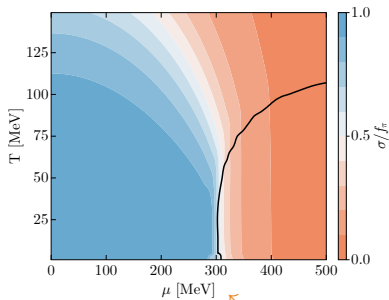
Phase Diagram



Phase diagram.

- Expected phase structure.

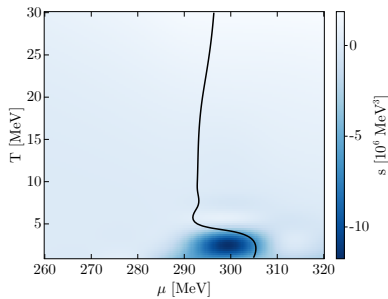
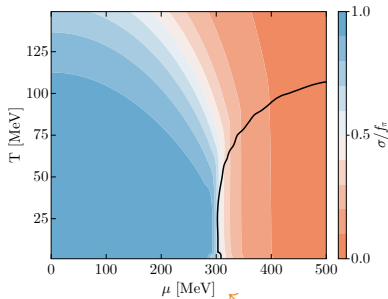
Phase Diagram



Phase diagram **What's going on here?**

- Expected phase structure.

Phase Diagram

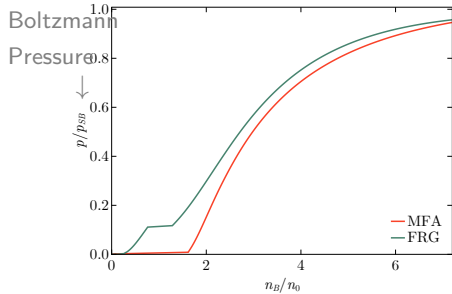


Phase diagram **What's going on here?** Entropy density s .

- Expected phase structure.
- Litim regulator in LPA: expect negative entropy **but** diquarks reduce the size of the region.

Equation of State

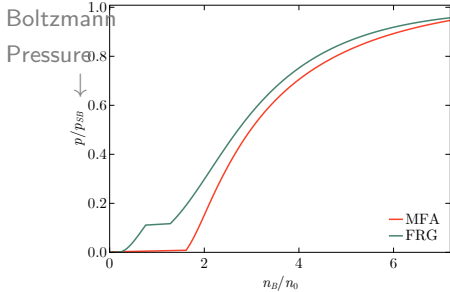
Stefan-



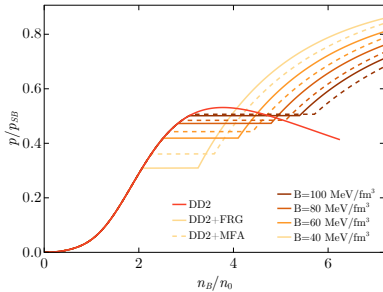
Equation of state $p(n)$ at $T = 1$

Equation of State

Stefan-



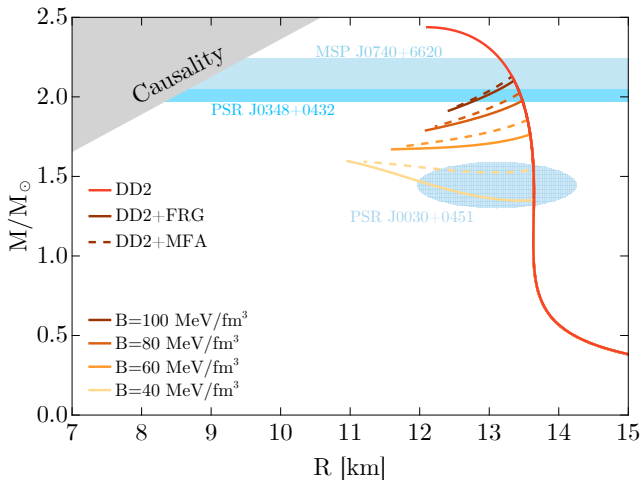
Equation of state $p(n)$ at $T = 1$



Maxwell construct with **hadronic eos DD2**

- Maxwell construct for astrophysical applications.
- No crossing with hadronic equation of state (DD2) → introduce **bag constant B** .

Mass-radius Relationship



- Superconducting core \rightarrow mostly unstable with current diquark parameters.

More realistic EoS: impose **neutrality conditions**

Charge neutrality (and β -equilibrium)

$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0 \quad (\text{and } u \leftrightarrow d + e^+ + \nu_e)$$

Need more down quark than up quark \rightarrow introduce different chemical potential: μ_{up} and μ_{down} .

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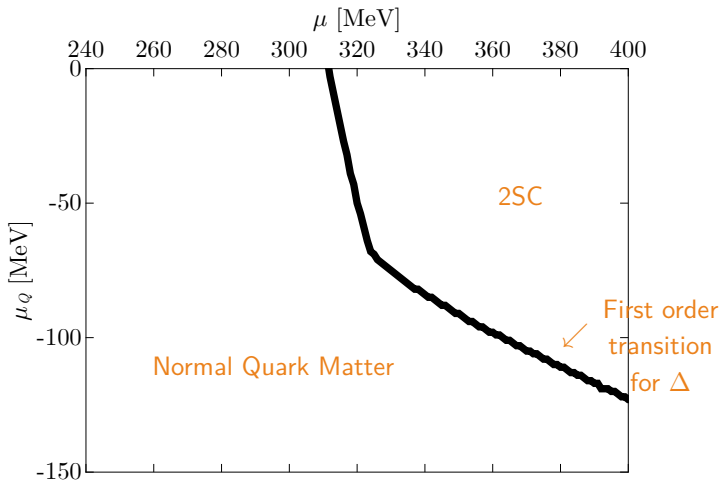
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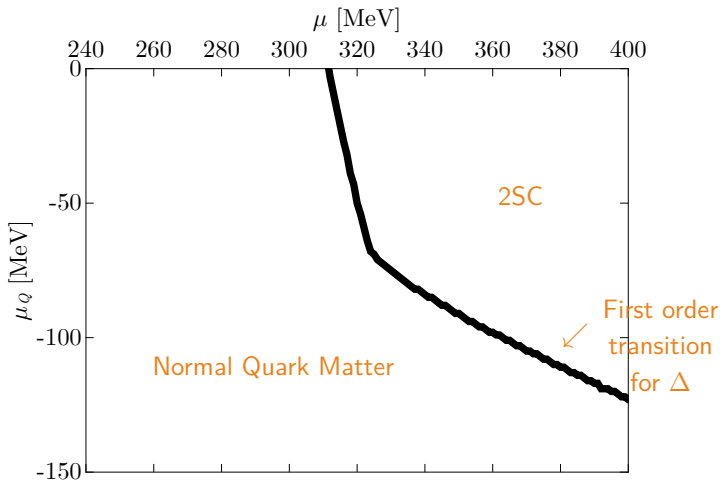
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Next: preliminary results in meanfield at $T = 1$ MeV.

Phase structure in the (μ, μ_Q) plane



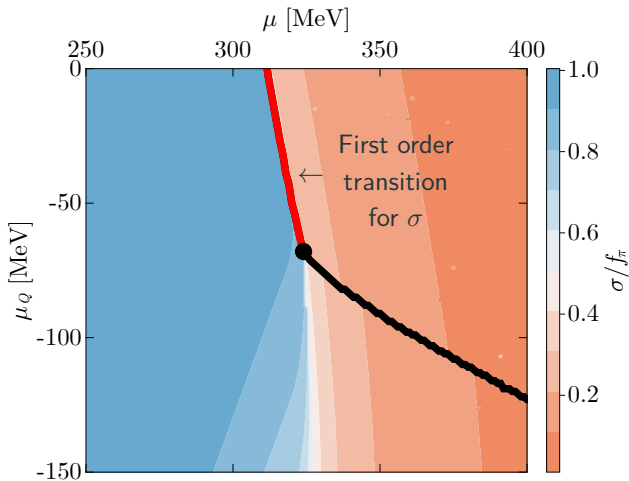
Phase structure in the (μ, μ_Q) plane



Question

What happens to the chiral condensate?

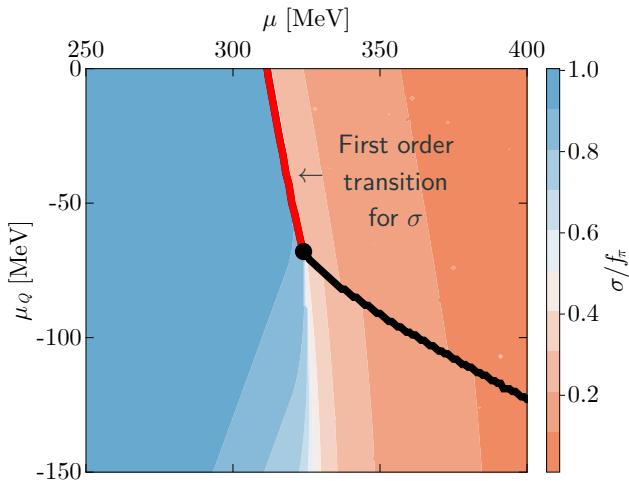
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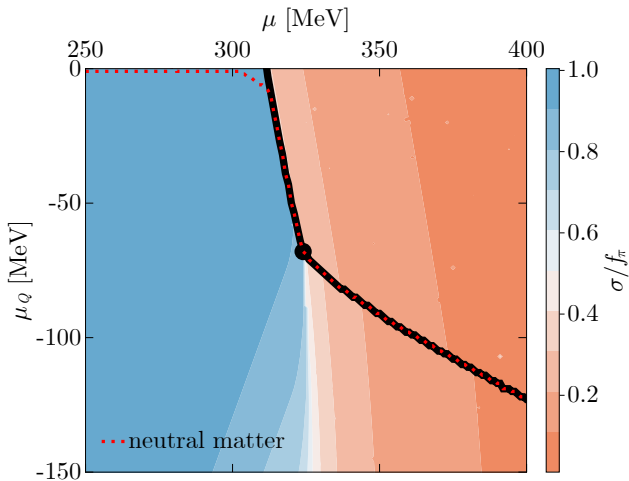
Phase structure in the (μ, μ_Q) plane



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Where is neutral matter in this phase diagram?

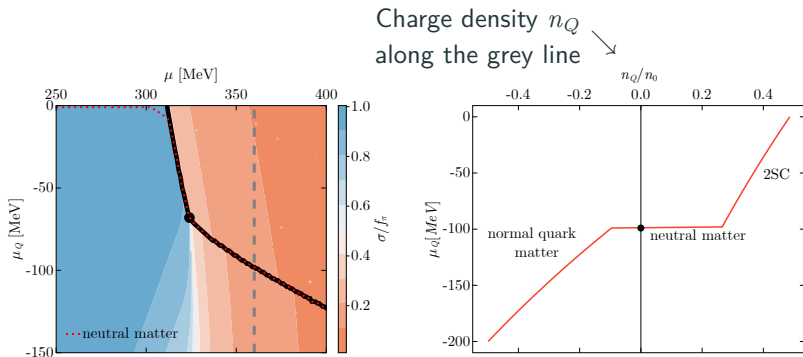
Phase structure in the (μ, μ_Q) plane



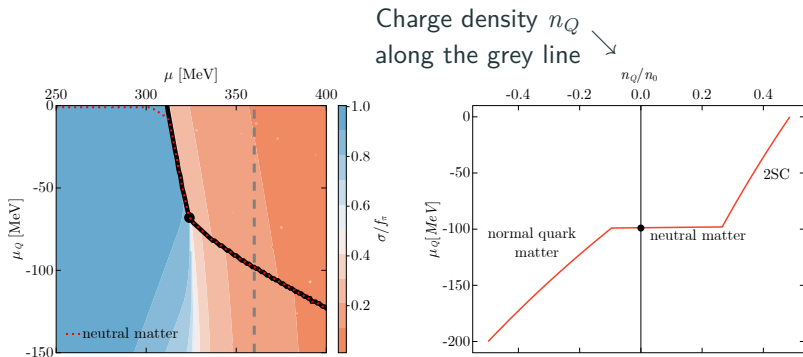
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Neutral Matter and First Order Transition



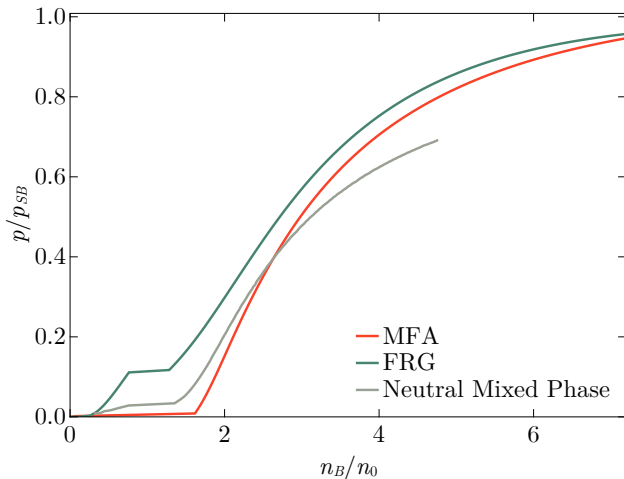
Neutral Matter and First Order Transition



- Density of neutral matter: inside a **first order region**.
- Suggest the presence of a **mixed phase**: normal quark matter + 2SC phase.

Mixed Equation of State

Neutral equation of state: right amount of normal quark matter and 2SC phase to reach neutrality.



Summary and Outlook

- Quark-meson-diquark model: model **chiral transition** and **2SC color superconducting phase**.
- FRG resolution faces two problems:
 - **Diquarks couple to μ** : cannot flow from symmetry restored phase to symmetry broken phase.
 - **Divergence at the Fermi-surface**: possible resolution with Fermi-surface regulator.
- Negative entropy after the chiral transition: better with diquarks at mean-field level.
- Neutrality condition suggest the presence of a **mixed phase**: normal quark matter and 2SC phase.

Backup Slides

Full Flow Equation

$$\begin{aligned}
 \partial_t U_k = \frac{k^5}{12\pi^2} & \left\{ \frac{3}{\epsilon_\pi} \coth \frac{\epsilon_\pi}{2T} + \frac{2}{\epsilon_{\Delta,0}} \left[\coth \frac{\epsilon_{\Delta,0} - 2\mu}{2T} + \coth \frac{\epsilon_{\Delta,0} + 2\mu}{2T} \right] \right. \\
 & + \sum_{i=1}^3 \frac{\alpha_2 z_i^4 - \alpha_1 z_i^2 + \alpha_0}{(z_i^2 - z_{i+1}^2)(z_i^2 - z_{i+2}^2)} \frac{1}{z_i} \coth \frac{z_i}{2T} \left. \right\} \\
 & - \frac{k^5}{3\pi^2} \left\{ \frac{2}{\epsilon_k} \left[\frac{E_k^+}{E_\Delta^+} \tanh \frac{E_\Delta^+}{2T} + \frac{E_k^-}{E_\Delta^-} \tanh \frac{E_\Delta^-}{2T} \right] \right. \\
 & \left. + \frac{1}{\epsilon_k} \left[\tanh \frac{\epsilon_k^+}{2T} + \tanh \frac{\epsilon_k^-}{2T} \right] \right\}
 \end{aligned}$$

with

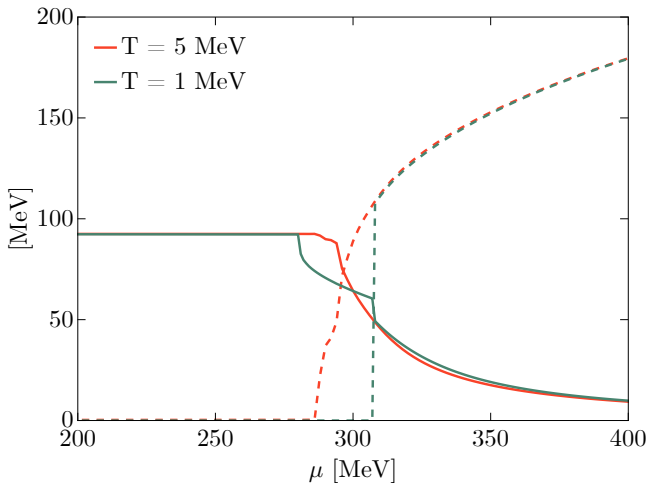
$$\epsilon_k^\pm = \sqrt{k^2 + g_\phi^2 \rho^2} \pm \mu = \epsilon_k \pm \mu$$

$$E_\pi = \sqrt{k^2 + 2U_{k,\rho}}$$

$$E_\Delta^\pm = \sqrt{(\epsilon_k \pm \mu)^2 + g_\Delta^2 d^2}$$

$$\epsilon_{\Delta,0} = \sqrt{k^2 + 2U_{k,d}}$$

Chiral and diquark condensates



Mixed Phase

