Color superconductivity beyond Mean Field Approximation

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JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

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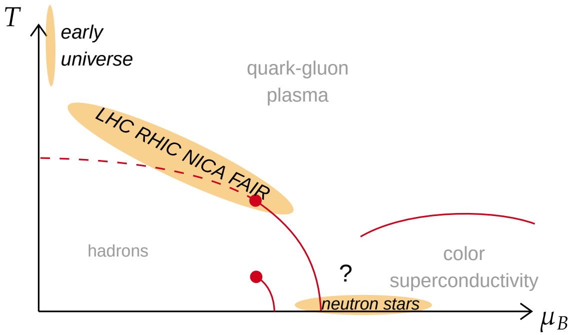
Outline

1. Color Superconductivity and 2SC Phase
2. Funcitonal Renormalization Group
3. Phase Structure and Astrophysical Applications

Color Superconductivity and 2SC Phase

Why study color superconductivity?



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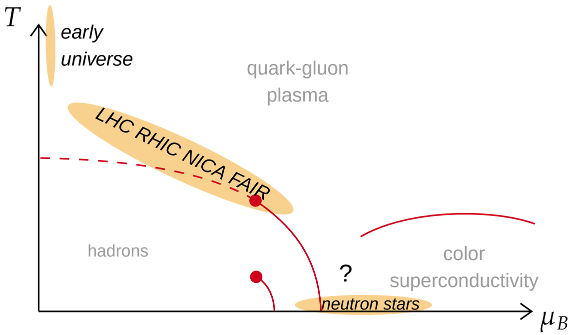

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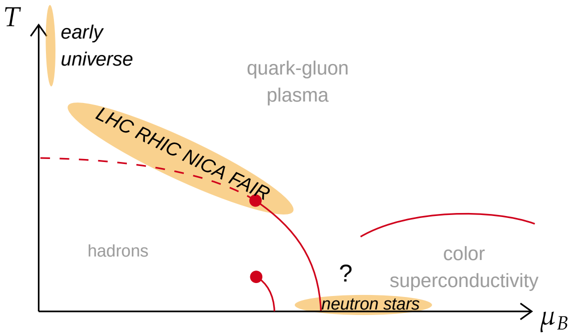

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↑

which pairing channel?

2SC Phase

2SC phase: condensates of the shape

$$\Delta_A = \langle q^T C \gamma_5 \tau_2 \lambda_A q \rangle \,, \quad A = 2, 5, 7$$

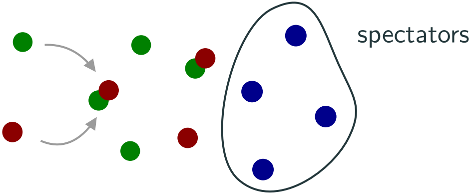
↖ some matrix structure in flavor, color and Dirac space

Main characteristics:

- Chiral symmetry is not broken.
- Color symmetry is "broken" (Higgs mechanism), but not fully:

$$SU_c(3) \to SU_c(2) \,.$$

- Only two color participate in superconductivity.



Funcitonal Renormalization Group

Why use the FRG?

Four-quark interactions in QCD

- Low densities: scalar-pseudoscalar (σ, $\vec{\pi}$) channel dominates

$$(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau_i\psi)^2$$

$\rightarrow$ quantitative description of chiral crossover using FRG.

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- High densities: diquark channel dominates

[Braun, Leonhardt & Pospiech; 1909.06298]

$$(q^T C\gamma_5\tau_2\lambda_A q)(\bar{q}\gamma_5\tau_2\lambda_A C\bar{q}^T)$$

$\rightarrow$ a natural step is to include the diquark channel.

Quark-meson-diquark Model

Quark-meson-diquark model action:

$$
\begin{aligned}
S_{\mathrm{QMD}} = \int_x \Big\{ & \bar{q}\left(\not{\partial} - \mu\gamma_0 + g_\phi\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right) q \\
& + \frac{g_\Delta}{2}\left(\Delta_A\, q^T C\gamma_5\tau_2\lambda_A\, q - \Delta_A^* \bar{q}\gamma_5\tau_2\lambda_A\, C\bar{q}^T\right) \\
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↖ explicit chiral symmetry breaking term

Truncation and Regulator Choice

Some FRG details:

- Local potential approximation (LPA): only scale dependent quantity is the potential $U_k(\sigma, \Delta)$.
  - σ: chiral condensate.
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$$R_k = (k^2 - \vec{p}^2)\theta(k^2 - \vec{p}^2) \; .$$

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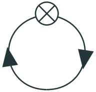
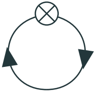
- Initial conditions:

$$U_\Lambda = a_1\sigma^2 + a_2\sigma^4 + b_1\Delta^2 + b_2\Delta^4$$

Flow Equation

$\partial_t U_k(\sigma, \Delta) = -$  q_r, q_g $-$  q_b $+ \frac{1}{2}$  Δ_2, σ

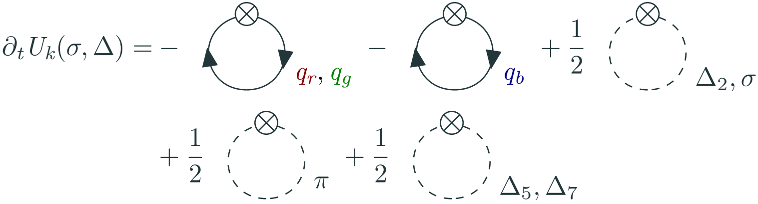
$+ \frac{1}{2}$  π $+ \frac{1}{2}$  Δ_5, Δ_7

Flow Equation

depends on $E_\Delta = \sqrt{(\epsilon_k \pm \mu)^2 + g_\Delta^2 \Delta^2}$

with $\epsilon_k = \sqrt{k^2 + g_\phi^2 \sigma^2}$

$\downarrow$



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coupling between one diquark and the sigma meson

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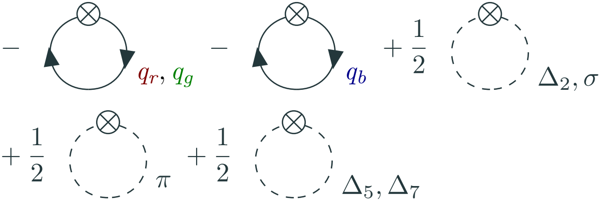

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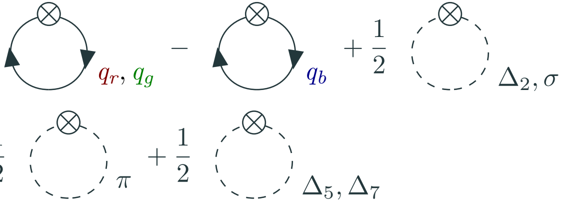

↗
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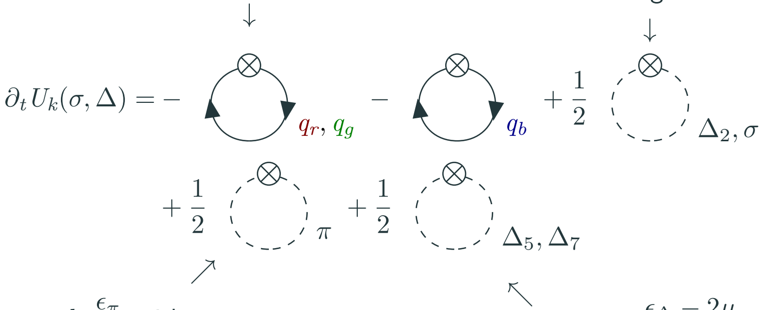

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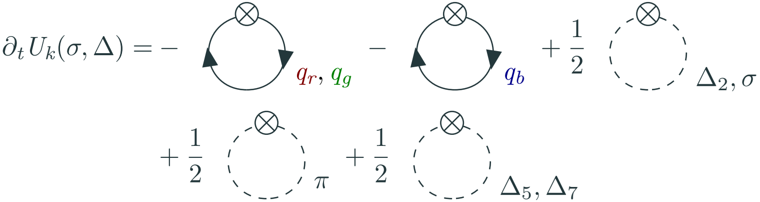

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problem with this part

First Trouble: Bosonic Diquark Loops

Pion loops

$$\partial_t U_k = \frac{1}{\epsilon_\pi} \coth \frac{\epsilon_\pi}{2T} + \dots$$

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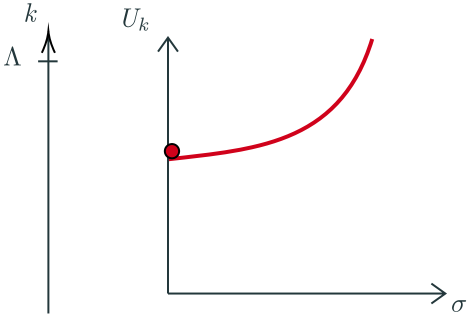

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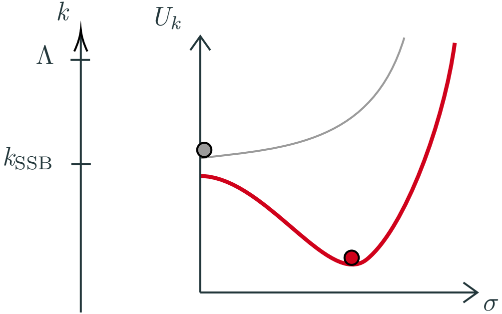

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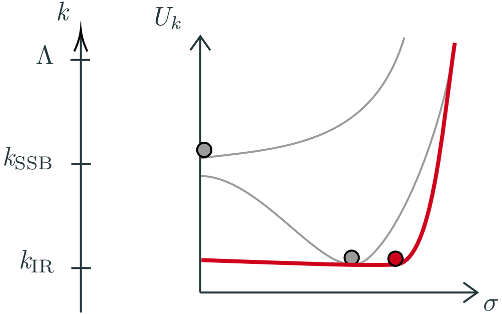

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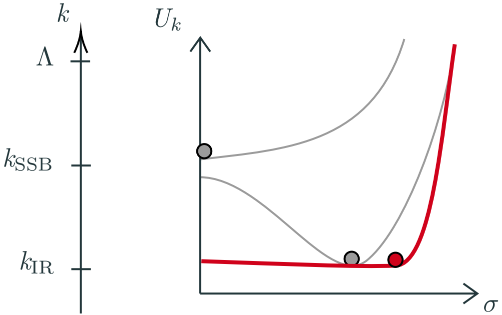

Diquark loops

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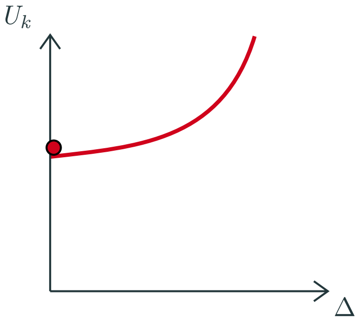

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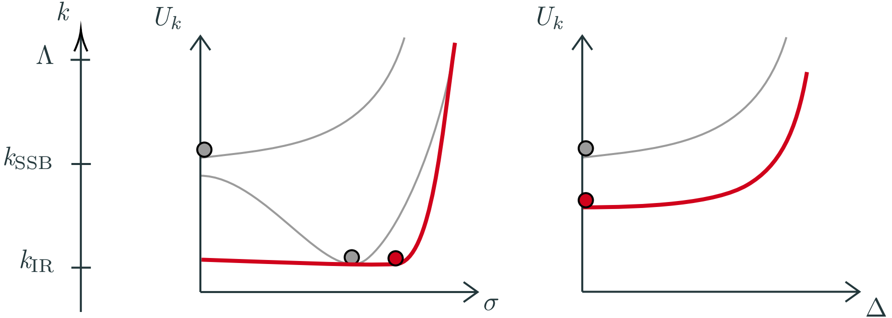

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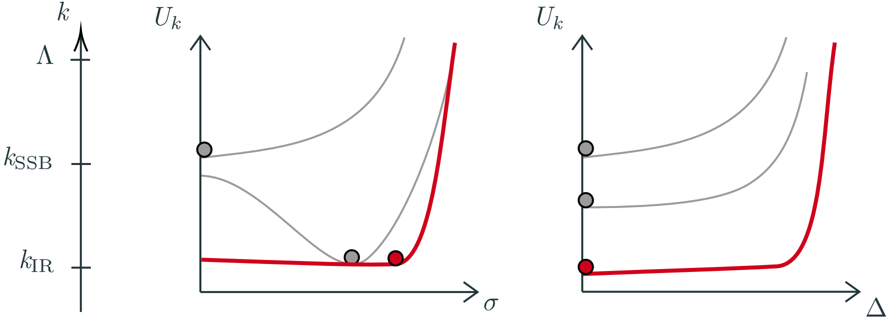

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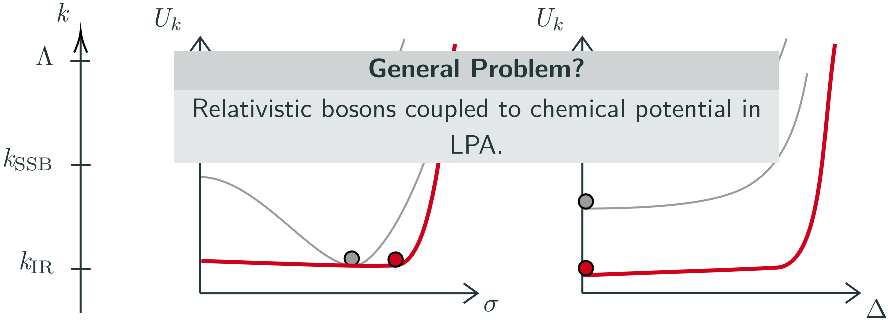


General Problem?

Relativistic bosons coupled to chemical potential in LPA.

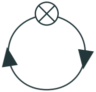
Second Trouble: Gapped Quark Loop

Gapped quark loop:

diverges at the Fermi-surface

↓

$-$



$$q_r,\, q_g \quad \propto \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k - \mu)^2 + g_\Delta^2 \Delta^2}} \xrightarrow{\partial_{\Delta^2}} \propto \frac{\epsilon_k - \mu}{\left((\epsilon_k - \mu)^2 + g_\Delta^2 \Delta^2\right)^{3/2}}$$

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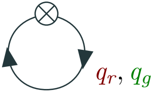
$-$ 

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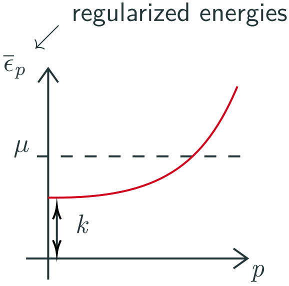
Possible solution: flow around the Fermi-surface [Braun, Dörnfeld, Schallmo, Töpfel; 2008.05978]



usual momentum regulator

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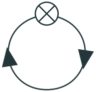

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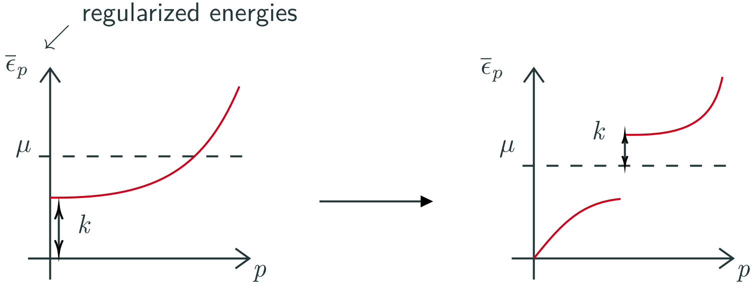

usual momentum regulator

Fermi-surface regulator

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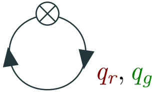
$-$ 

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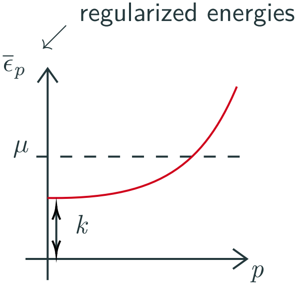

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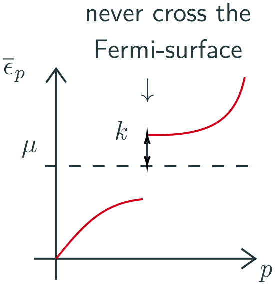
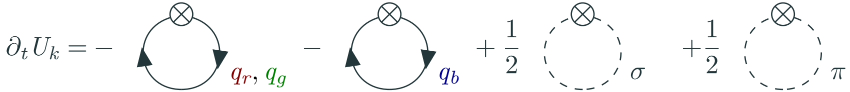


Fermi-surface regulator

Phase Structure and Astrophysical Applications

Quark-meson-diquark Model without Diquark Loops

Diquarks only at the mean field level: none of the previously mentioned problems are present.



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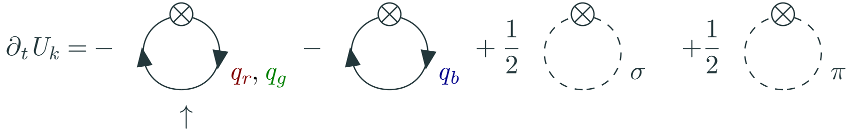

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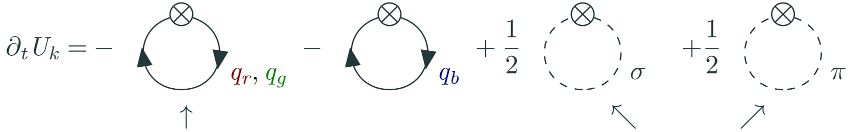

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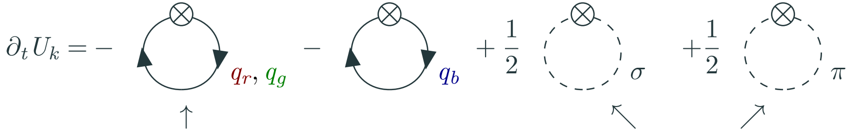

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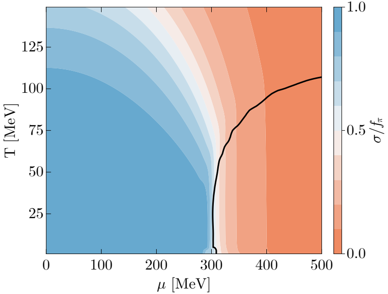
still include fluctuations from pion and sigma mesons

- Look at phase structure.
- First astrophysical applications.

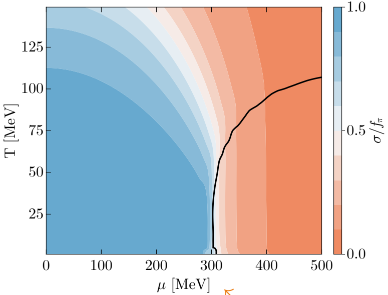
Phase Diagram



Phase diagram.

- Expected phase structure.

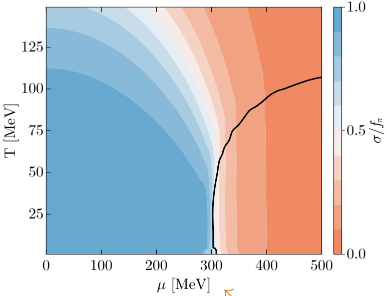
Phase Diagram



Phase diagram. What's going on here?

- Expected phase structure.

Phase Diagram



Phase diagram.

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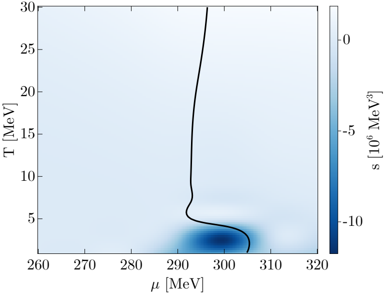
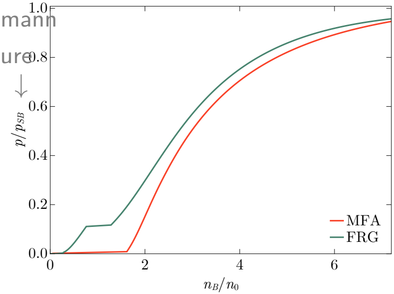


Entropy density s.

- Expected phase structure.
- Litim regulator in LPA: expect negative entory **but** diquarks reduce the size of the region.

Equation of State

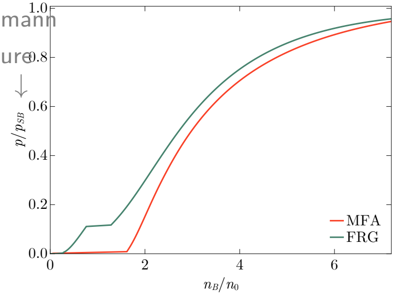
Stefan-
Boltzmann
Pressure
↓



Equation of state $p(n)$ at $T = 1$

Equation of State

Stefan-Boltzmann Pressure ↓



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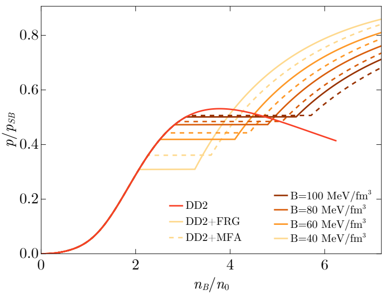
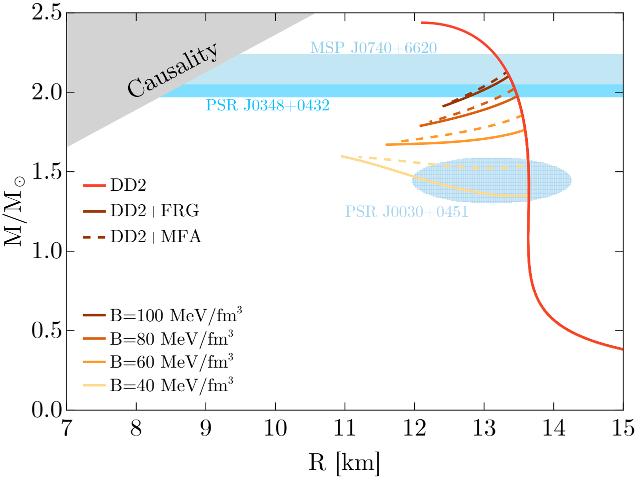


Maxwell construct with hadronic eos DD2

- Maxwell construct for astrophysical applications.
- No crossing with hadronic equation of state (DD2) $\rightarrow$ introduce bag constant B.

Mass-radius Relationship



- Superconducting core → mostly unstable with current diquark parameters.

Charge Neutral Matter

More realistic EoS: impose neutrality conditions

Charge neutrality (and β-equilibrium)

$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0 \quad (\text{and} \quad u \leftrightarrow d + e^+ + \nu_e)$$

Need more down quark than up quark $\rightarrow$ introduce different chemical potential: μ_{up} and μ_{down}.

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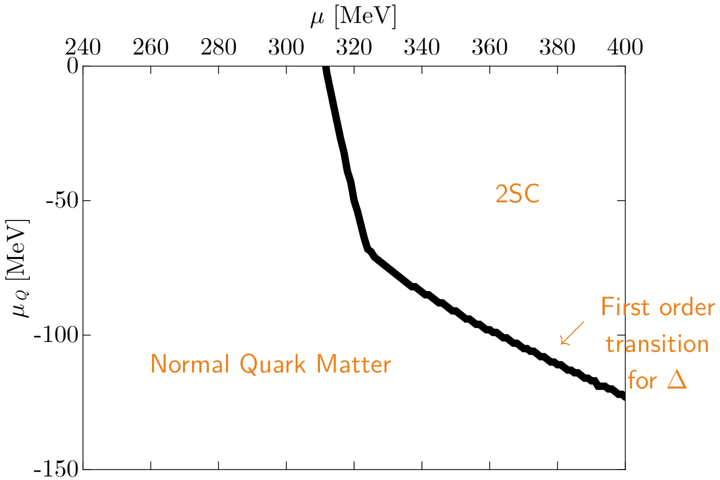
Next: preliminary results in meanfield at $T = 1$ MeV.

Phase structure in the (μ, μ_Q) plane



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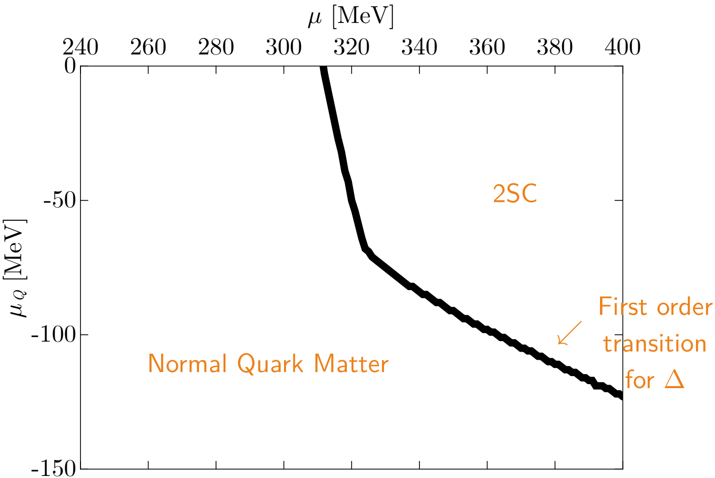
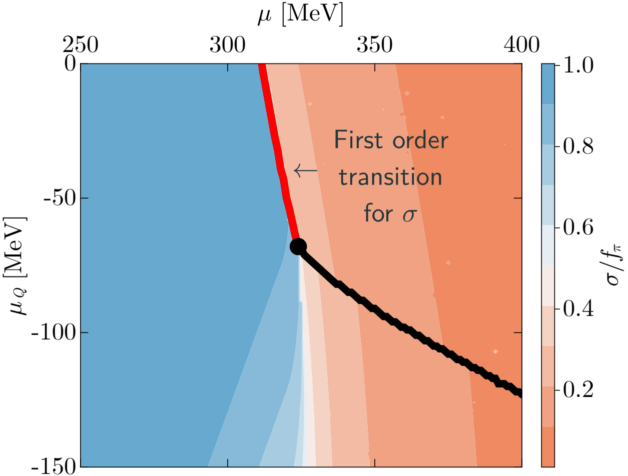


Question

What happen to the chiral condensate?

Phase structure in the (μ, μ_Q) plane



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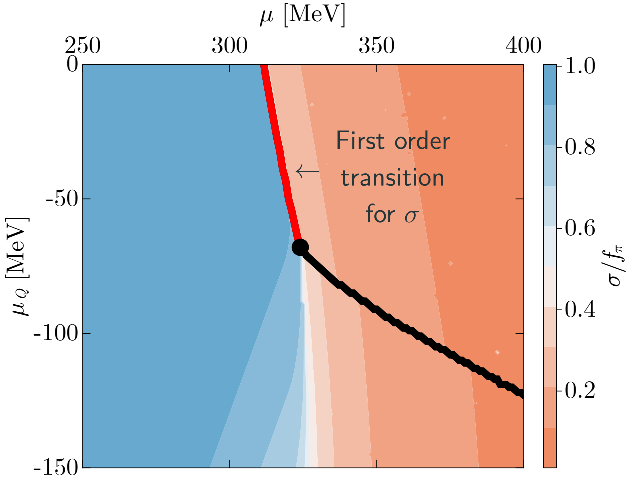

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Where is neutral matter in this phase diagram?

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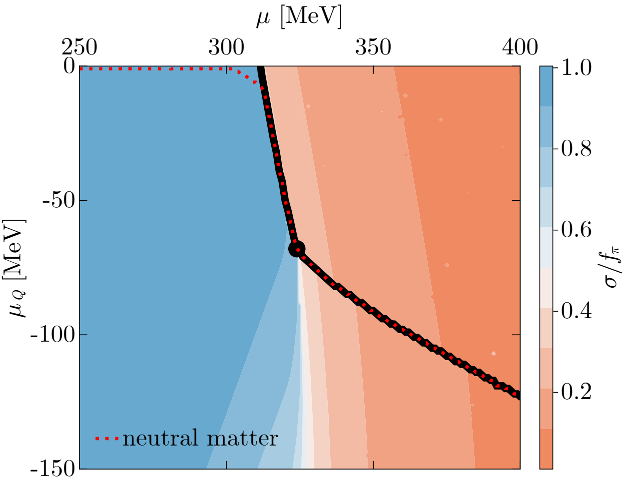

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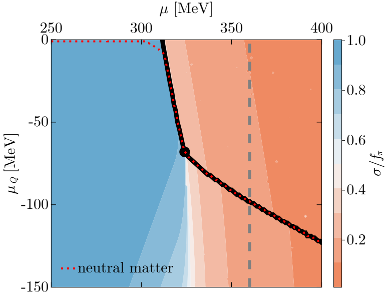
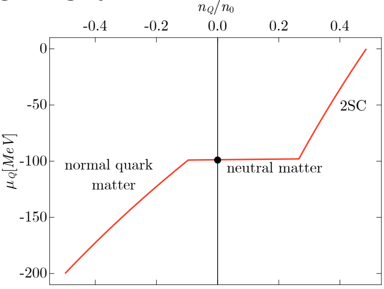
Neutral Matter and First Order Transition

Charge density n_Q
along the grey line





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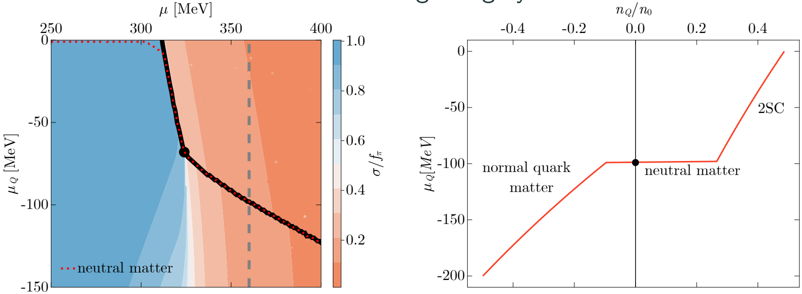
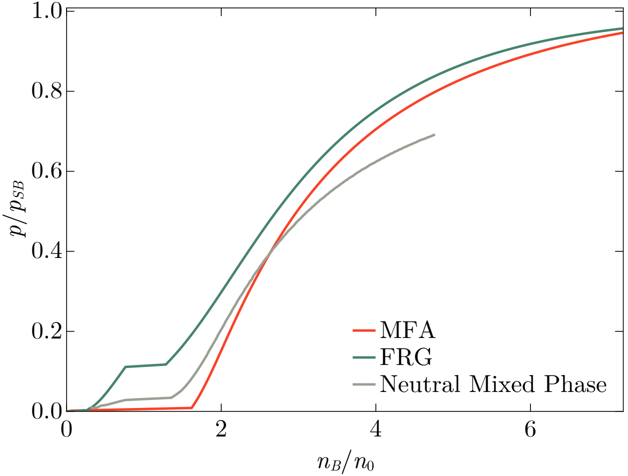


- Density of neutral matter: inside a first order region.
- Suggest the presence of a mixed phase: normal quark matter + 2SC phase.

Mixed Equation of State

Neutral equation of state: right amount of normal quark matter and 2SC phase to reach neutrality.



Summary and Outlook

- Quark-meson-diquark model: model chiral transition and 2SC color superconducting phase.
- FRG resolution faces two problems:
  - Diquarks couple to μ: cannot flow from symmetry restored phase to symmetry broken phase.
  - Divergence at the Fermi-surface: possible resolution with Fermi-surface regulator.
- Negative entropy after the chiral transition: better with diquarks at mean-field level.
- Neutrality condition suggest the presence of a mixed phase: normal quark matter and 2SC phase.

Backup Slides

Full Flow Equation

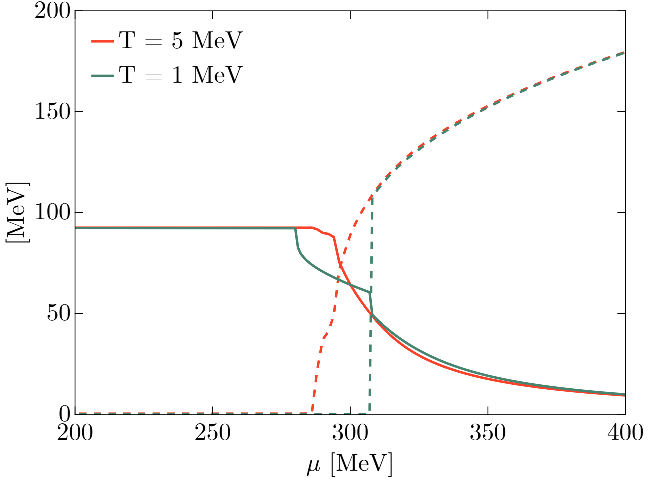
$$
\begin{aligned}
\partial_t U_k = \frac{k^5}{12\pi^2} \Bigg\{ & \frac{3}{\epsilon_\pi} \coth \frac{\epsilon_\pi}{2T} + \frac{2}{\epsilon_{\Delta,0}} \left[\coth \frac{\epsilon_{\Delta,0} - 2\mu}{2T} + \coth \frac{\epsilon_{\Delta,0} + 2\mu}{2T} \right] \\
& + \sum_{i=1}^{3} \frac{\alpha_2 z_i^4 - \alpha_1 z_i^2 + \alpha_0}{(z_i^2 - z_{i+1}^2)(z_i^2 - z_{i+2}^2)} \frac{1}{z_i} \coth \frac{z_i}{2T} \Bigg\} \\
& - \frac{k^5}{3\pi^2} \Bigg\{ \frac{2}{\epsilon_k} \left[\frac{E_k^+}{E_\Delta^+} \tanh \frac{E_\Delta^+}{2T} + \frac{E_k^-}{E_\Delta^-} \tanh \frac{E_\Delta^-}{2T} \right] \\
& + \frac{1}{\epsilon_k} \left[\tanh \frac{\epsilon_k^+}{2T} + \tanh \frac{\epsilon_k^-}{2T} \right] \Bigg\}
\end{aligned}
$$

with

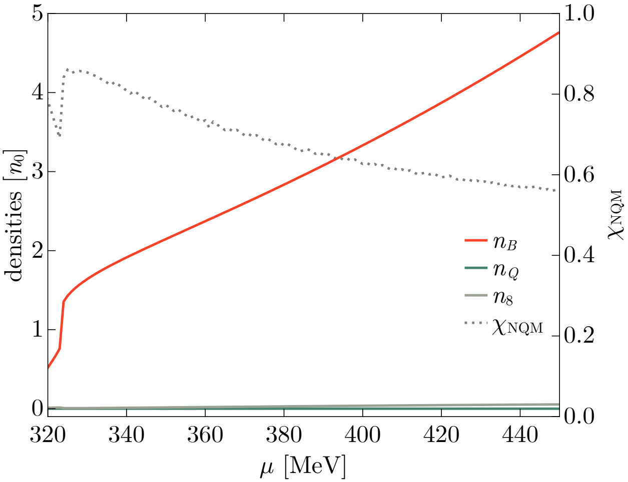
$$
\epsilon_k^\pm = \sqrt{k^2 + g_\phi^2 \rho^2} \pm \mu = \epsilon_k \pm \mu \qquad E_\pi = \sqrt{k^2 + 2\,U_{k,\rho}}
$$

$$
E_\Delta^\pm = \sqrt{(\epsilon_k \pm \mu)^2 + g_\Delta^2\, d^2} \qquad \epsilon_{\Delta,0} = \sqrt{k^2 + 2\,U_{k,d}}
$$

Chiral and diquark condensates



Mixed Phase



densities $[n_0]$
χ_{NQM}
μ [MeV]
n_B
n_Q
n_8
χ_{NQM}
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0
320
340
360
380
400
420
440