

# Quark matter in Neutron Stars

**Bernd-Jochen Schaefer**



September 12<sup>th</sup>, 2023



**EMMI Workshop**

**Functional Methods in Strongly Correlated  
Systems (FUNSCS2023)**

Hirschegg, Kleinwalsertal, Austria, September 10 - September 15, 2023

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# agenda

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- **Hybrid and quark star matter based on a nonperturbative equation of state**

[Konstantin Otto \(Giessen U.\)](#), [Micaela Oertel \(LUTH, Meudon\)](#), [Bernd-Jochen Schaefer \(Giessen U.\)](#)

Published in: *Phys.Rev.D* 101 (2020) 10, 103021 • e-Print: [1910.11929](#) [hep-ph]

- **Nonperturbative quark matter equations of state with vector interactions**

[Konstantin Otto \(Giessen U.\)](#), [Micaela Oertel \(LUTH, Meudon\)](#), [Bernd-Jochen Schaefer \(Giessen U.\)](#)

Published in: *Eur.Phys.J.ST* 229 (2020) 22-23, 3629-3649 • e-Print: [2007.07394](#) [hep-ph]

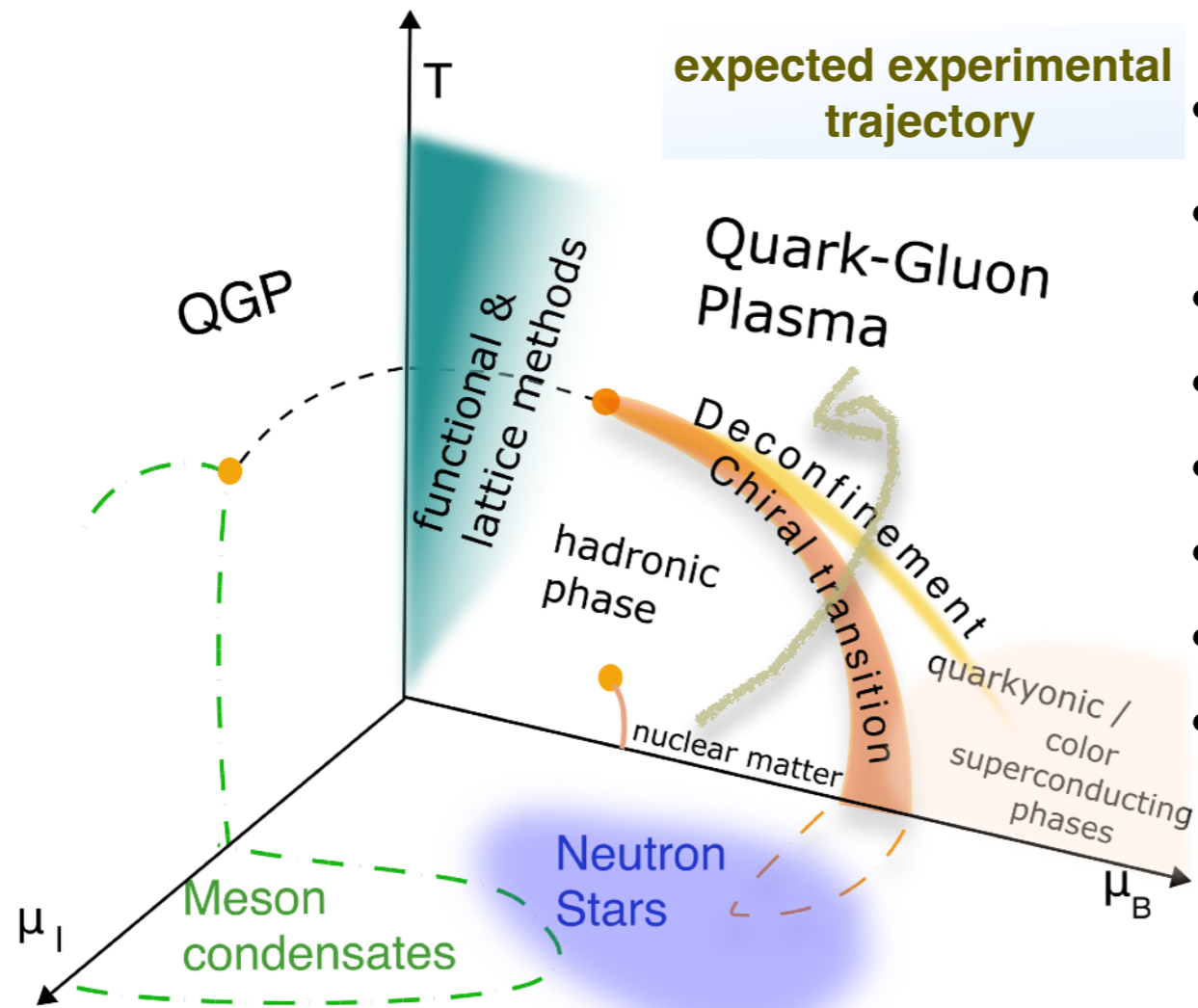
- **Regulator scheme dependence of the chiral phase transition at high densities**

[Konstantin Otto \(Giessen U.\)](#), [Christopher Busch \(Giessen U.\)](#), [Bernd-Jochen Schaefer \(Giessen U.\)](#)

Published in: *Phys.Rev.D* 106 (2022) 9, 094018 • e-Print: [2206.13067](#) [hep-ph]

# conjectured QCD phase structure

Open issues

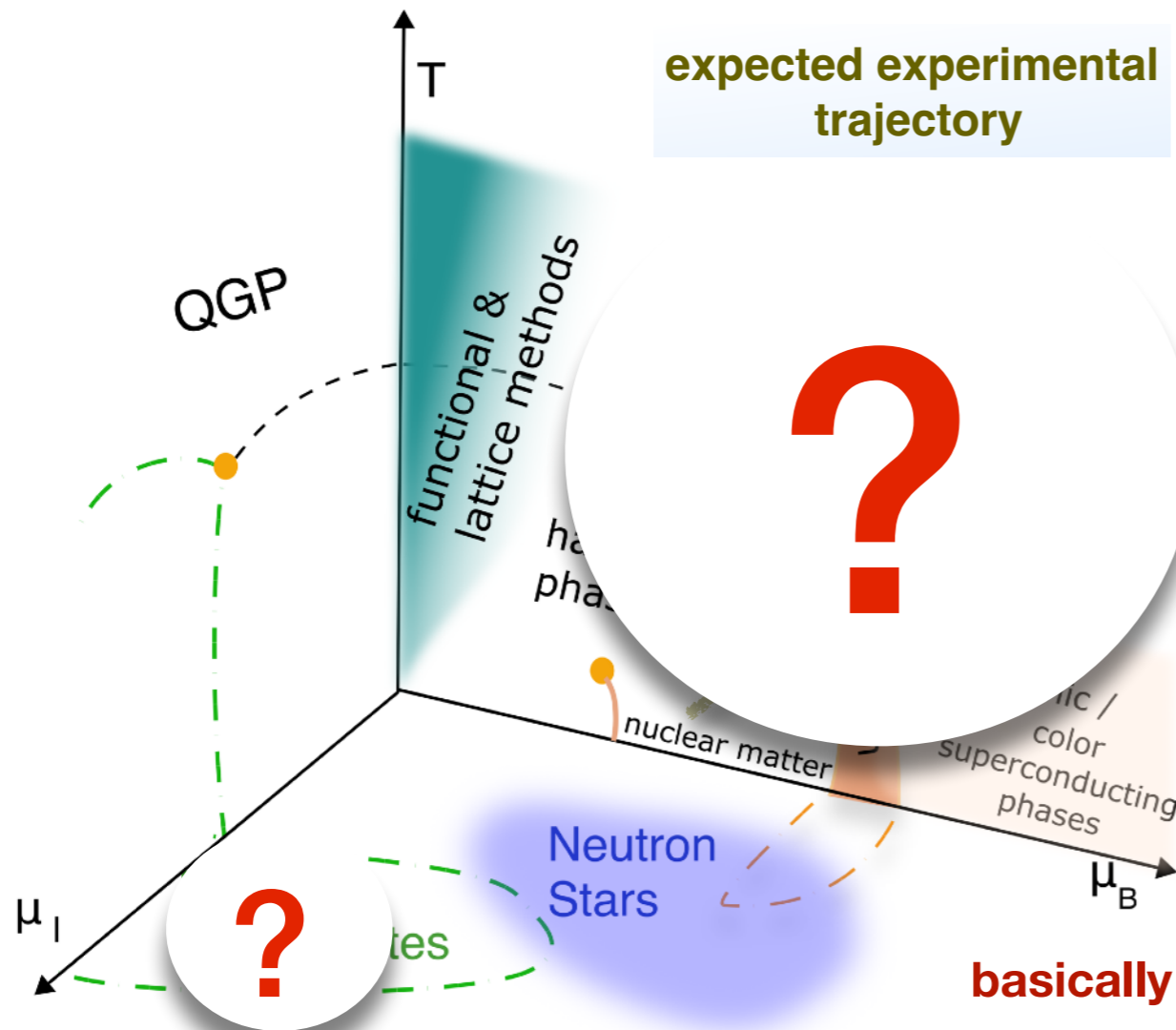


- **Critical endpoint (CEP)?** chiral  $\leftrightarrow$  deconfinement?
- **Chiral-Spin symmetry / Quarkyonic phase/s?**
- inhomogeneous phase/s?
- axial anomaly restoration?
- finite volume effects?
- role of fluctuations?
- experimental signatures?
- ....

usual assumptions: equilibrium, homogeneous phases, infinite volume, ....

# conjectured QCD phase structure

Open issues



expected experimental trajectory

- **Critical endpoint (CEP)?** chiral  $\leftrightarrow$  deconfinement?
- **Chiral-Spin symmetry / Quarkyonic phase/s?**
- **inhomogeneous phase/s?**
- **axial anomaly restoration?**
- **finite volume effects?**
- **role of fluctuations?**
- **experimental signatures?**
- ....

basically only corners known from first principle QCD

**alternative to HIC** to probe cold dense QCD matter  $\rightarrow$  **massive neutron star** (e.g. PSR J0348+0432 )

cold dense QCD matter: **only effective low-energy realisation** of QCD: e.g. (P)QM models

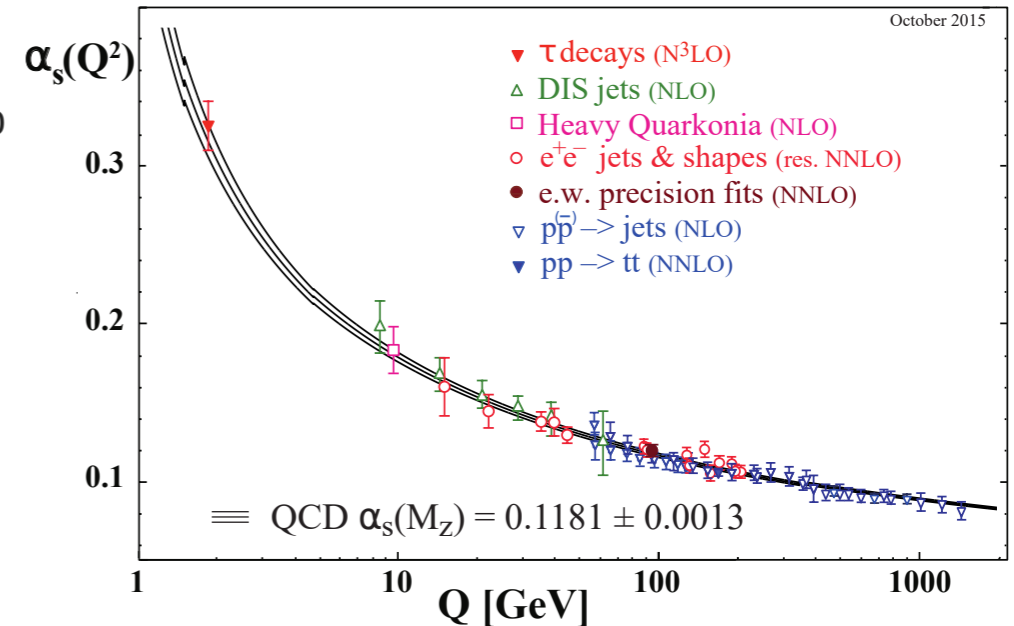
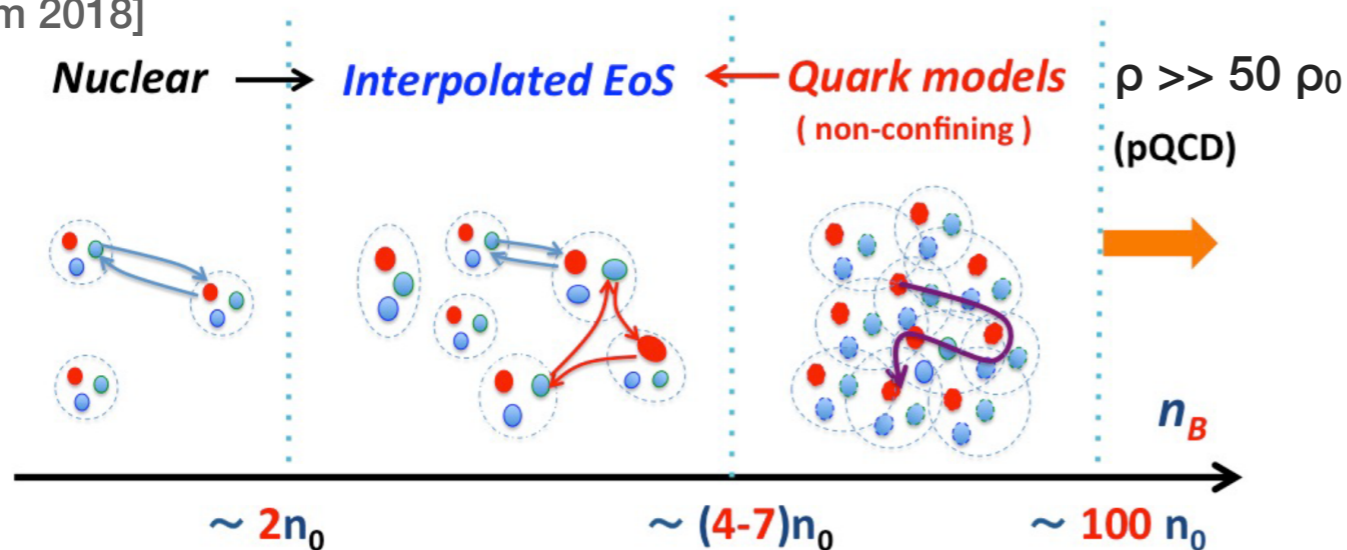
ultimate goal: microscopic description of EoS guided by QCD first principle



# EoS for dense matter

Kojo et al.: three window model of dense matter

[Baym 2018]



Nuclear phase:  
1-2 meson/quark  
exchanges

interpolated EoS  
many meson/quark  
exchanges

Quark phase:  
quarks no longer  
specific to baryons

mostly mean-field investigations  
like NJL-type or phenomenological  
models

EoS from  
**nuclear physics**  
 $\rho < 2\rho_0$   $\chi$ EFT

system gradually changes  
from hadronic to quark matter  
- **diquarks, colored quarks virtually ...**  
- role of strangeness / hyperons

**→ upgrade with FRG methods**

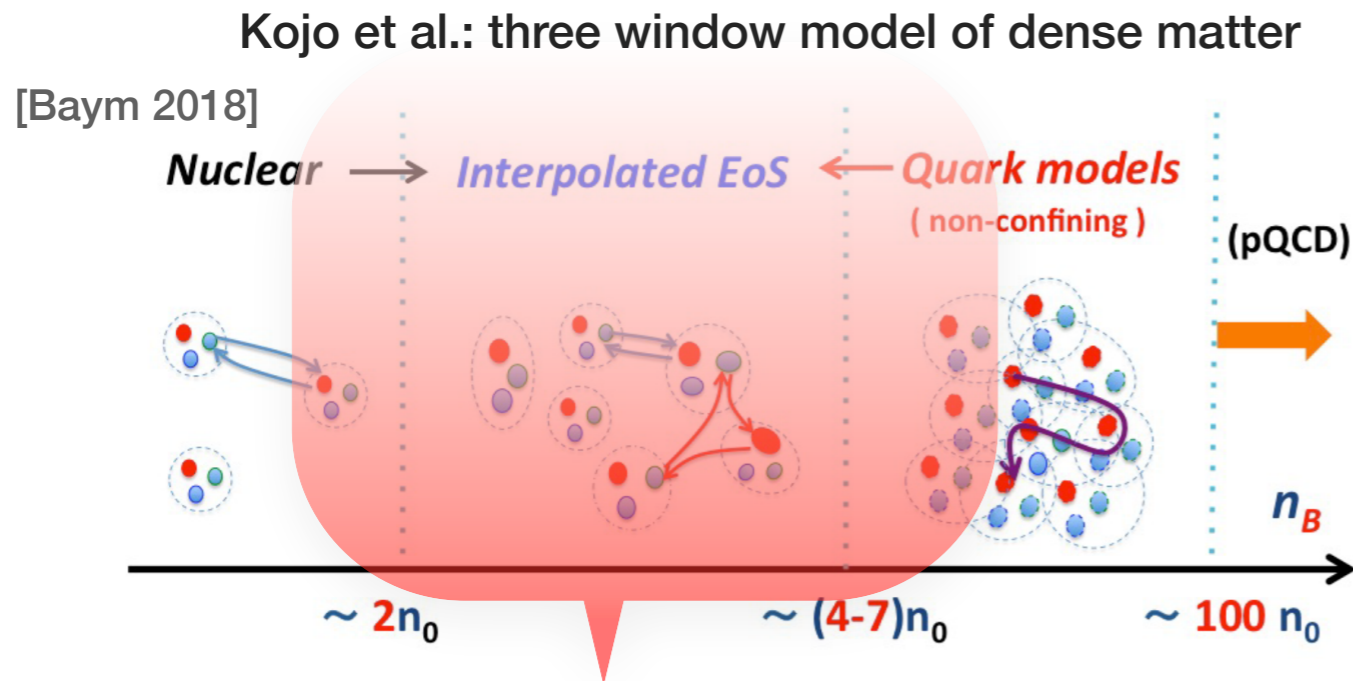
$2\rho_0 < \rho < 7\rho_0$  Neutron stars

[Hebeler, Lattimer, Pethick, Schwenk et al. 2010]

[Schaffner-Bielich et al. 2008]

[Blaschke, Fischer, Oertel et al. 2018]

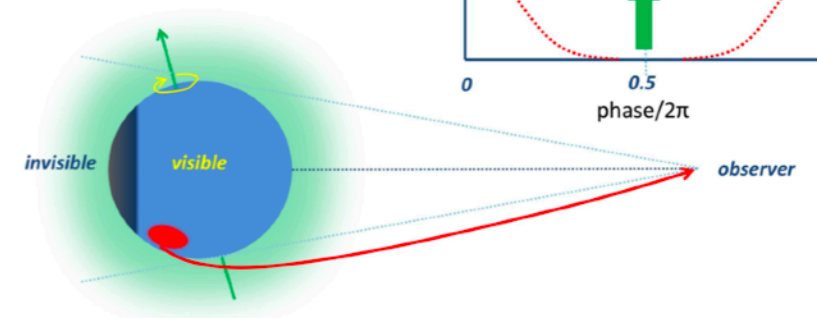
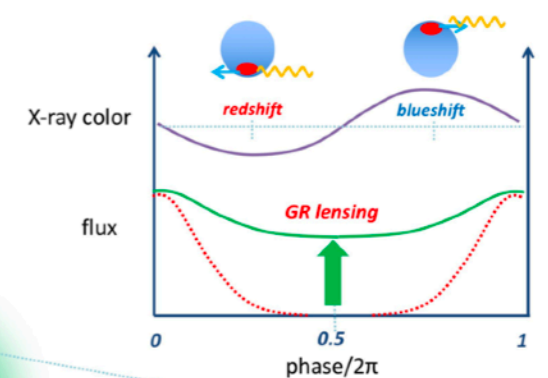
# experimental facts



nearest ~ 400 ly

$R \sim 10 - 13 \text{ km}$

$M > 2 M_{\text{sol}}$

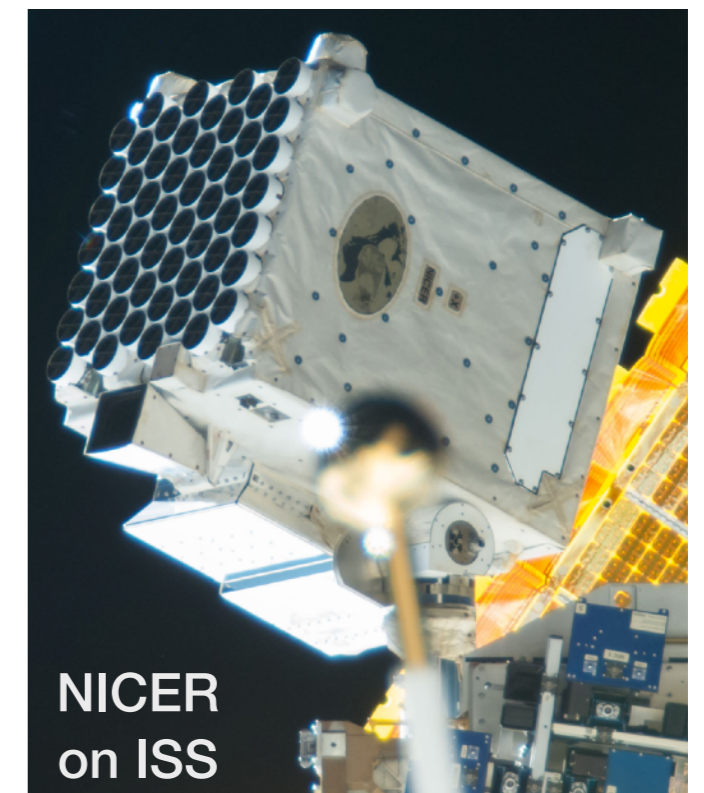


future: ~ 2035

running „accelerator“

- $M_{\text{max}} = 2.05 M_{\odot}$  &  $R \sim 12.8 \text{ km}$  ;  $\rho_{\text{central}} \sim 5\rho_0$
- $M = 1.4 M_{\odot}$  &  $R = 12,8 \text{ km}$  ;  $\Delta R = 0$
- $M = 2.4 M_{\odot}$  &  $R = 12,8 \text{ km}$  ; ~ as  $1.4 M_{\odot}$  stars
- PSR J0740+6620 & J0030+0451  
**both  $M (> M_{\odot})$  &  $R$  measured**  
 GW170817 constraints tidal deformability  $\rightarrow R$
- **detection phase transition possible:**  
 increase post-merger dominant oscillation frequency  
 only a few events expected (even w/ 3rd generation detectors)

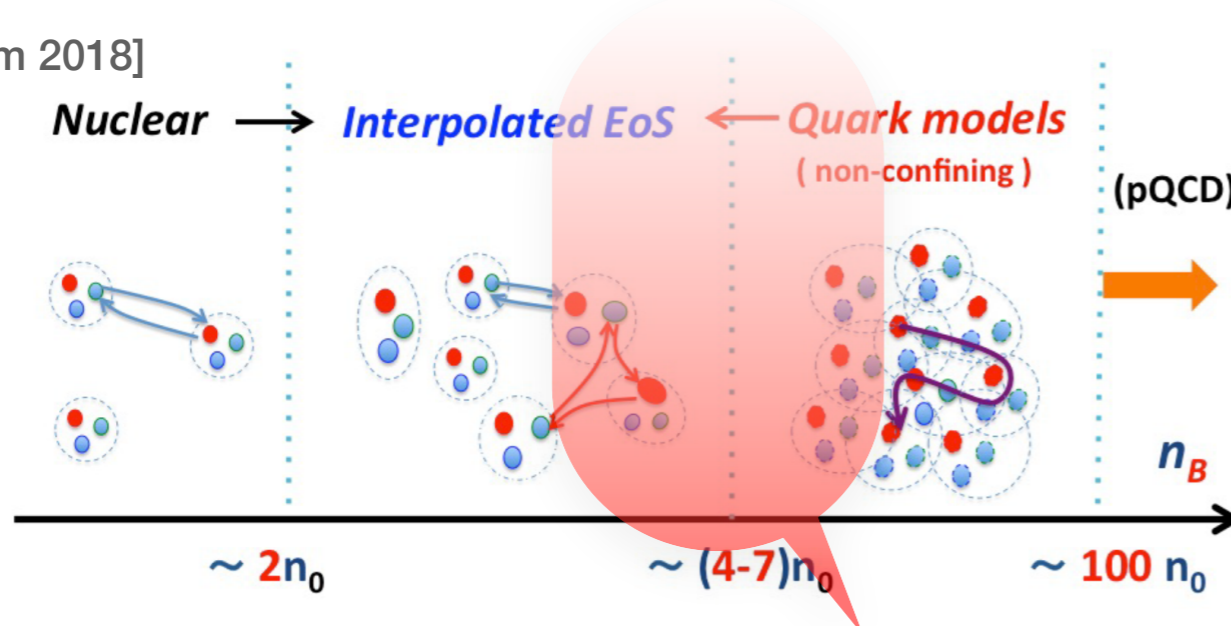
- new 3<sup>rd</sup> generation detectors:
- (American) Cosmic Explorer
- (European) Einstein Telescope



NICER on ISS

# transition from hadronic to quark matter

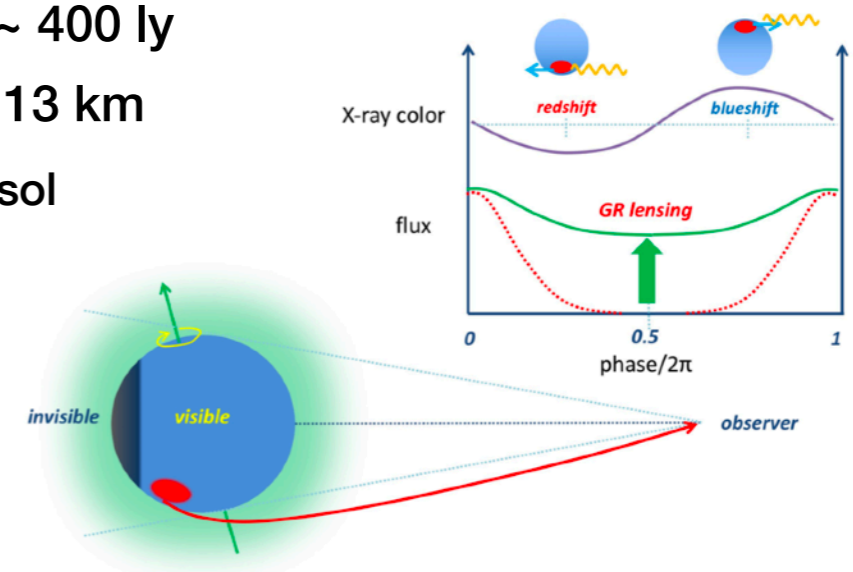
[Baym 2018]



nearest  $\sim 400$  ly

$R \sim 10 - 13$  km

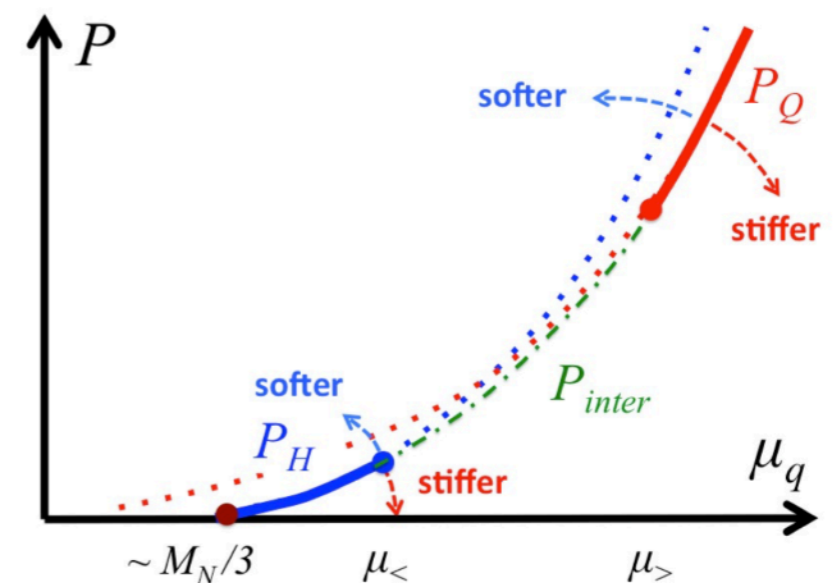
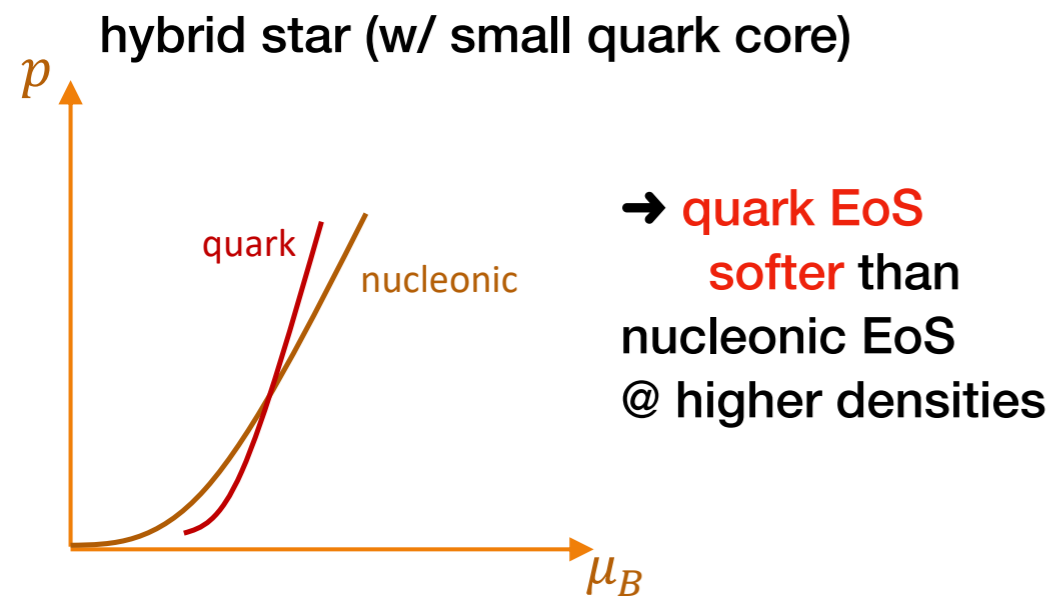
$M > 2 M_{\text{sol}}$



several possibilities (if transition): Maxwell construction or continuous interpolation

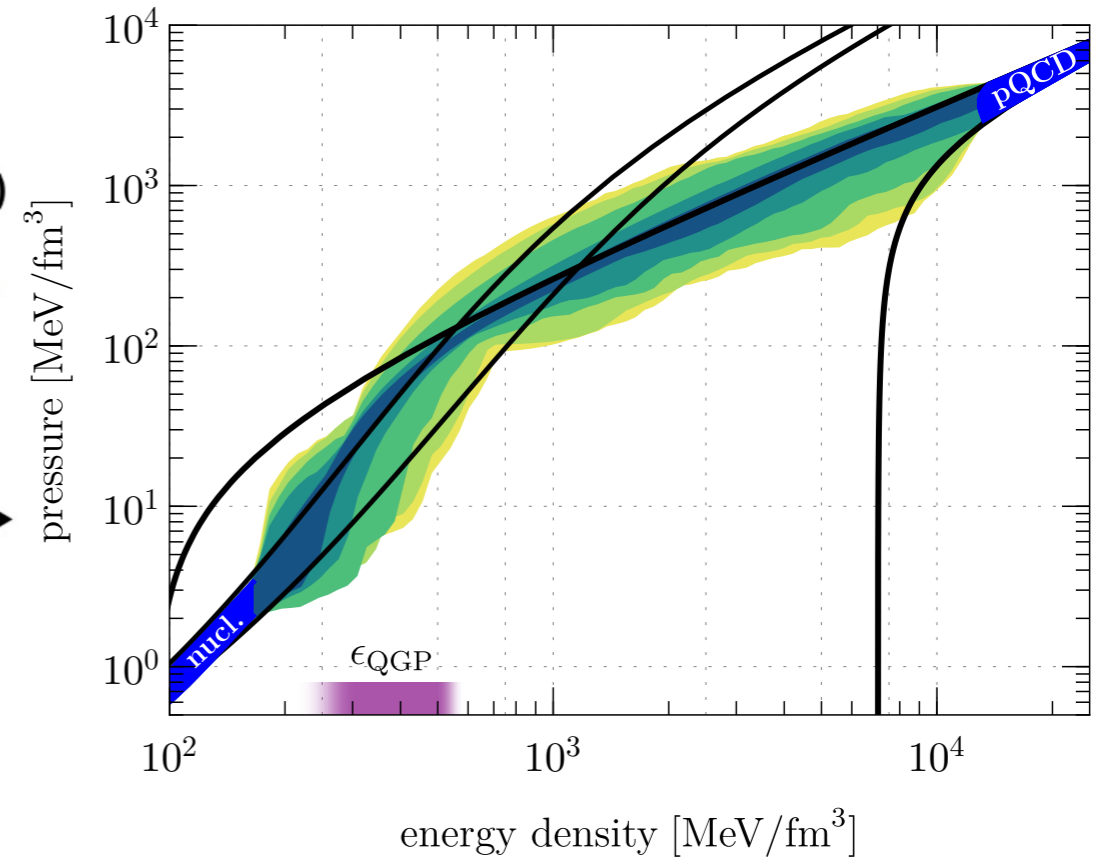
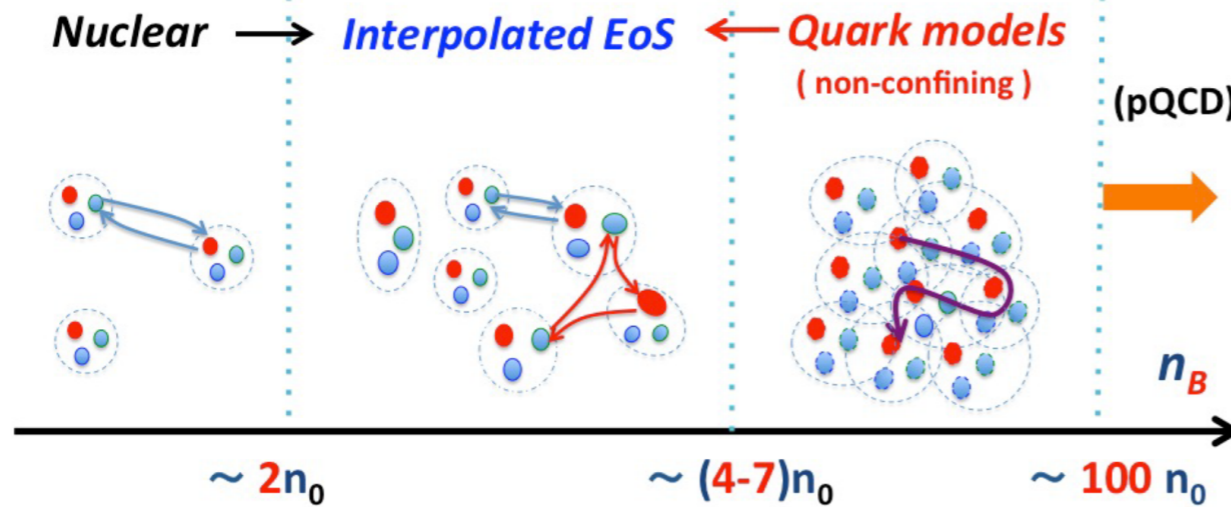
first-order transition

quark-hadron continuity



# conflicting constraints on EoS

[Baym 2018]

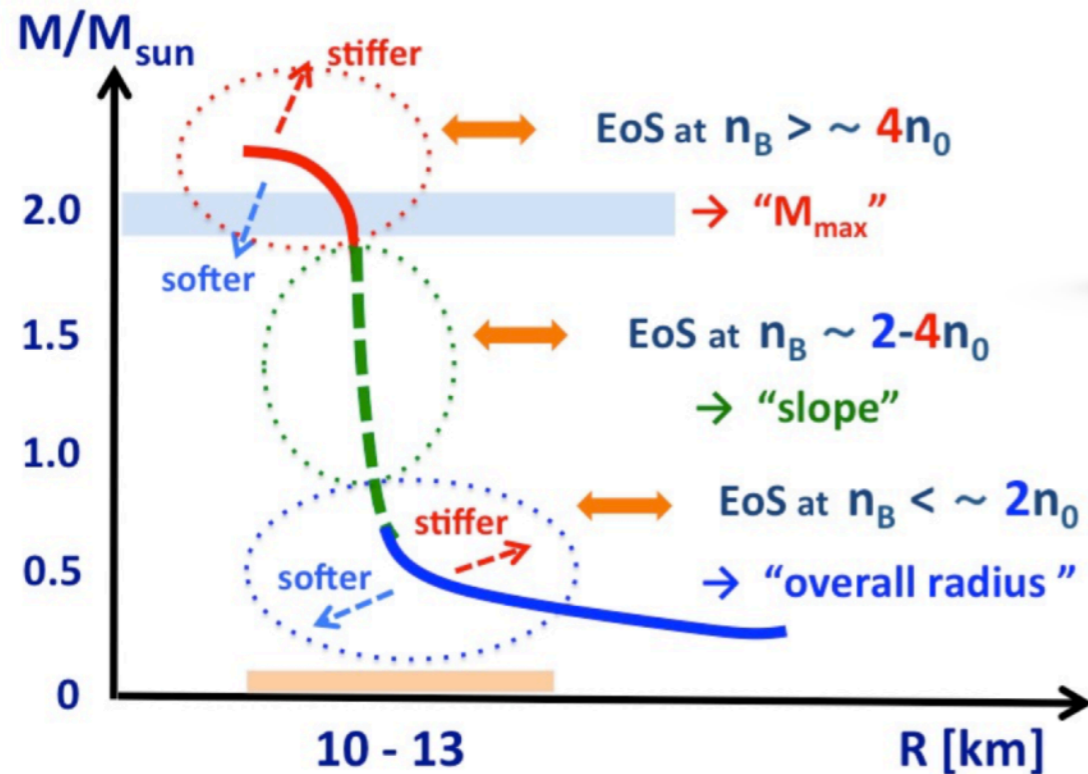
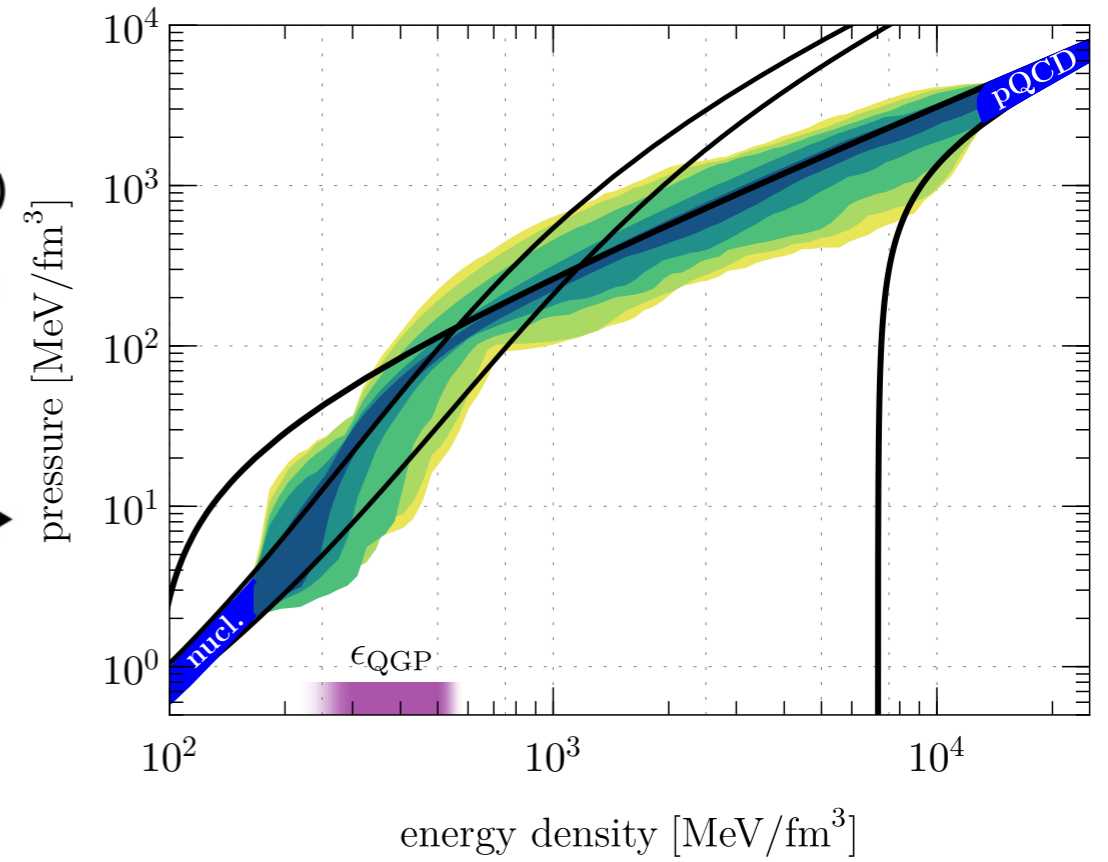
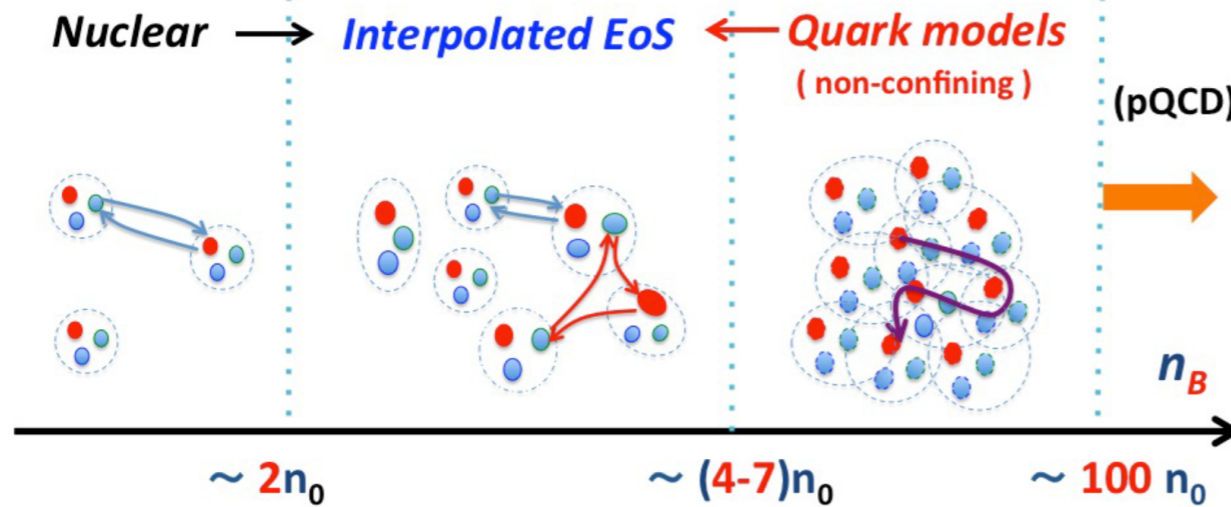


**EoS  $\leftrightarrow$  TOV equation  $\leftrightarrow$  M-R relation (observables)**



# conflicting constraints on EoS

[Baym 2018]



**EoS ↔ TOV equation ↔ M-R relation (observables)**

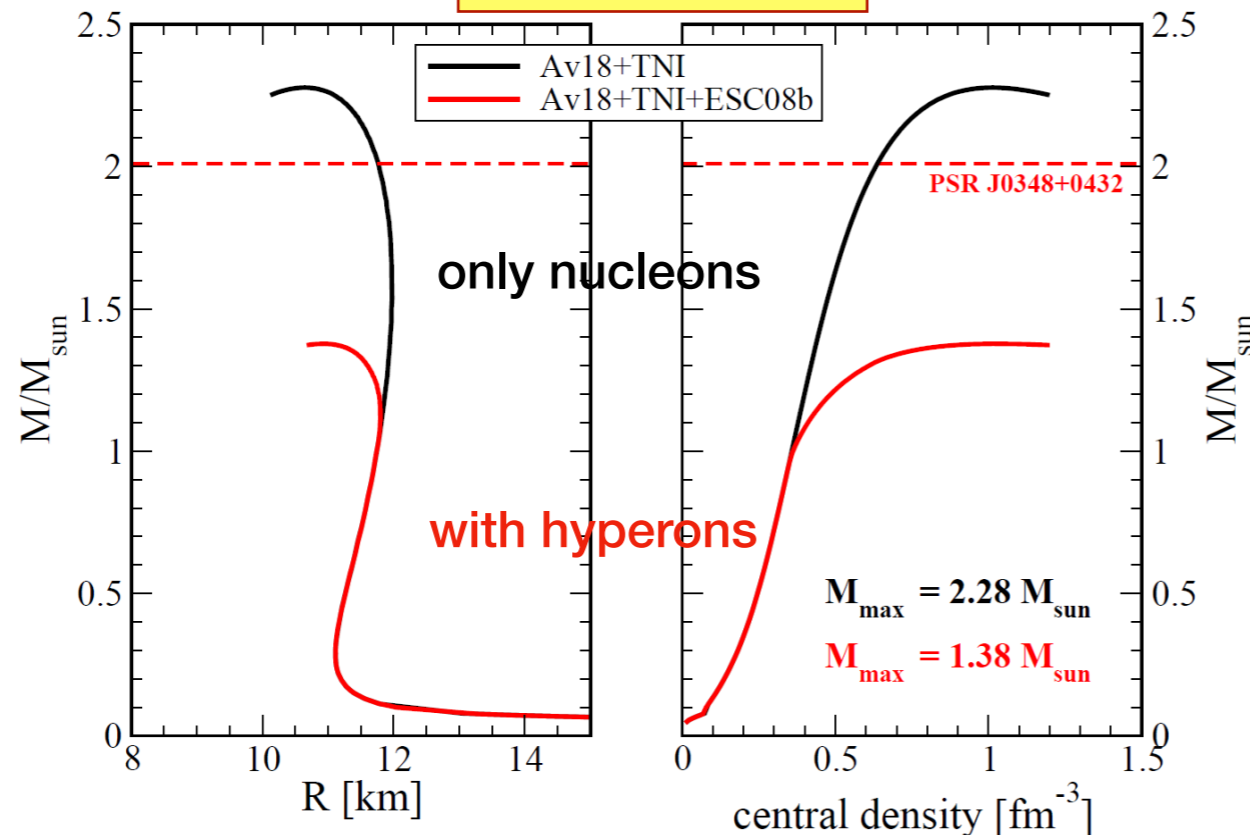
three constraints on the EoS:

1. stiff enough (@high density) →  $2M_{\odot}$
2. soft enough (@low density) → Radius
3. speed of sound  $< 1$

# unsolved puzzles / open issues

[Bombaci 2016]

## hyperons puzzle



### Further constraints:

causality

charge neutrality:  $n_p = n_e + n_\mu$

$\beta$ -equilibrium:  $\mu_n = \mu_p + \mu_e$

simplification:

→ electrons and muons as  
**free Fermi gas in EoS**

### General problems (physical theory input required):

#### → hyperon puzzle

onset of strangeness in hadronic phase or quark phase

→ soften EoS

[Djapo, BJS, Wambach 2010]

#### → masquerade problem

many EoS look similar → similar M-R relation

increasing #dof **soften** EoS,

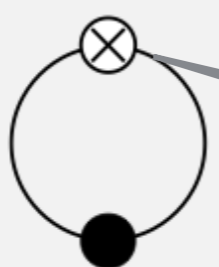
repulsive interactions **stiffen** EoS

[Alvarez-Castillo, Blaschke 2014]

# Functional Renormalization Group

## Wetterich Equation (average effective action)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \partial_t R_k \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right)$$

$$k \partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$


[Wetterich 1993]

$$t = \ln(k/\Lambda)$$

$$\Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi}$$

$R_k$  regulators

## Ansatz effective action Quark-Meson truncation in LPA (LO derivative expansion)

$$\Gamma_k = \int d^4x \bar{q} [i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)] q + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 + V_k(\phi^2)$$

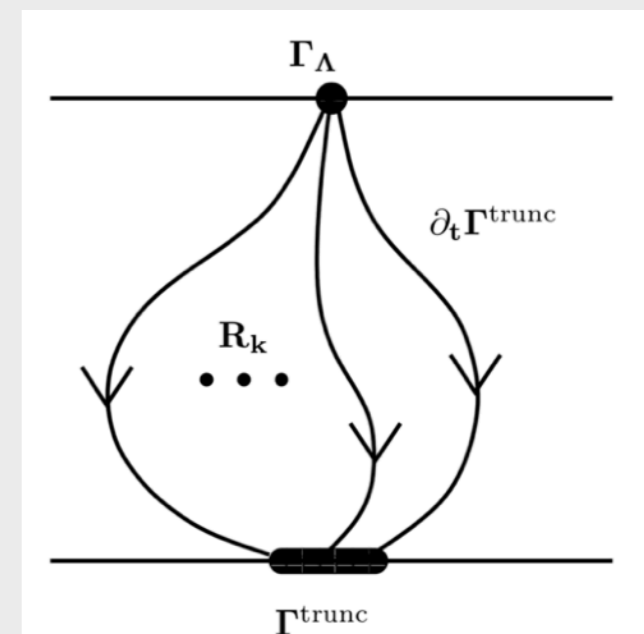
$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

arbitrary potential

## shape function conditions:

$$R_k(p^2) = p^2 r(p^2/k^2)$$

- $\lim_{p^2/k^2 \rightarrow \infty} R_k(p^2) = 0$
- $\lim_{p^2/k^2 \rightarrow 0} R_k(p^2) > 0 (= k^2)$
- $\lim_{k \rightarrow \infty} R_k(p^2) \rightarrow \infty$



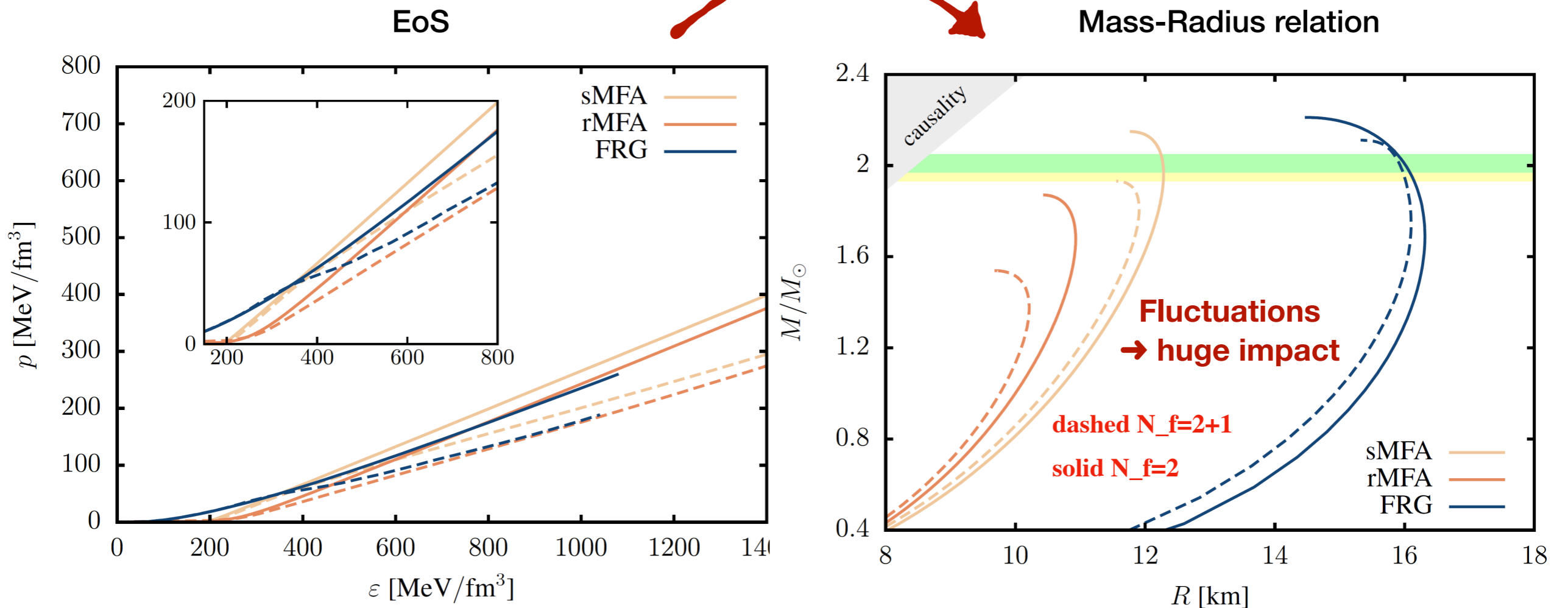
# Impact of fluctuations on EoS

[Otto, Oertel, BJS 2020]

Impose  $\beta$ -equilibrium and charge neutrality conditions

$$\begin{aligned}\mu_u &= \mu_q - \frac{2}{3}\mu_e \\ \mu_d &= \mu_q + \frac{1}{3}\mu_e \\ \mu_s &= \mu_q + \frac{1}{3}\mu_e\end{aligned}$$

Tolman-Oppenheimer-Volkoff equations



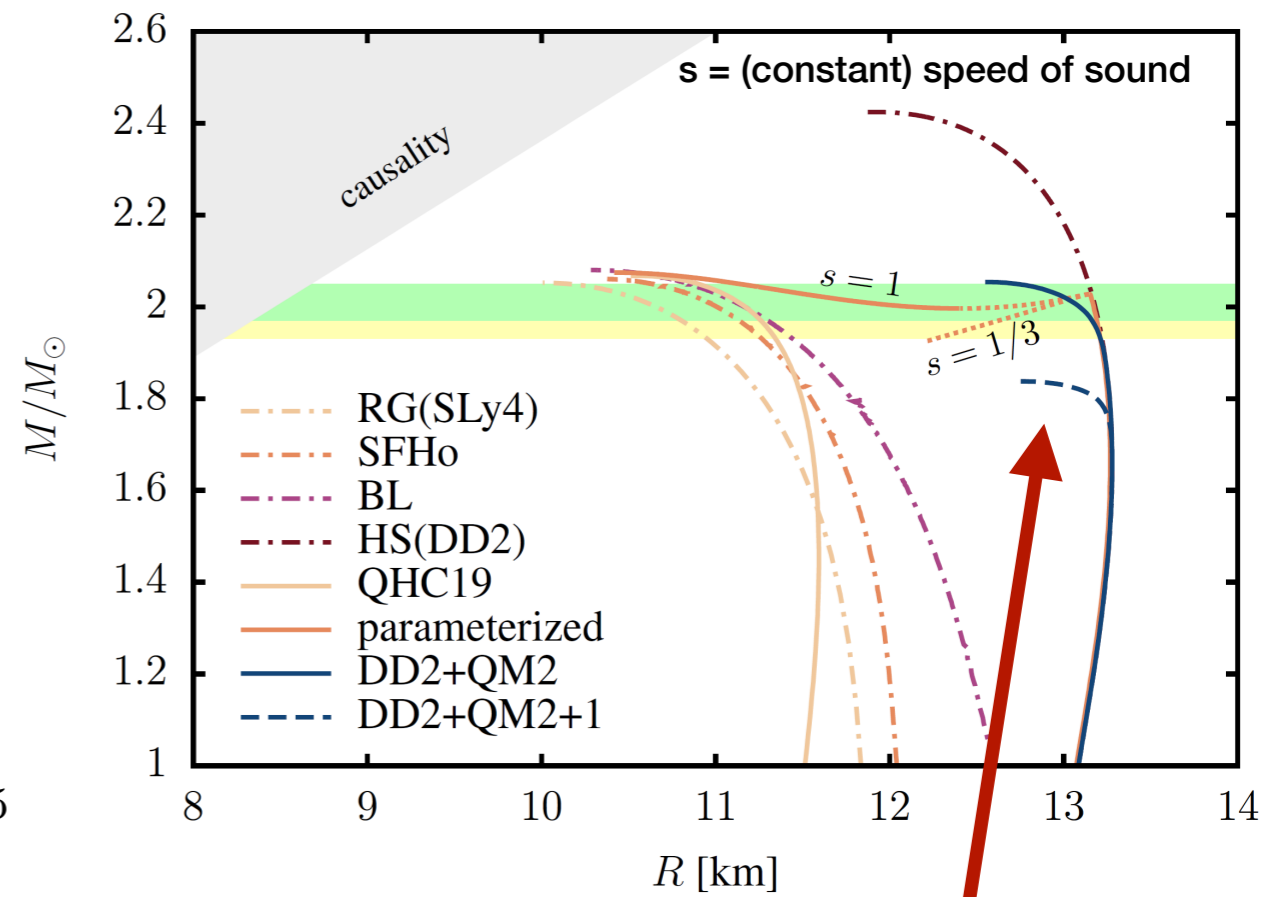
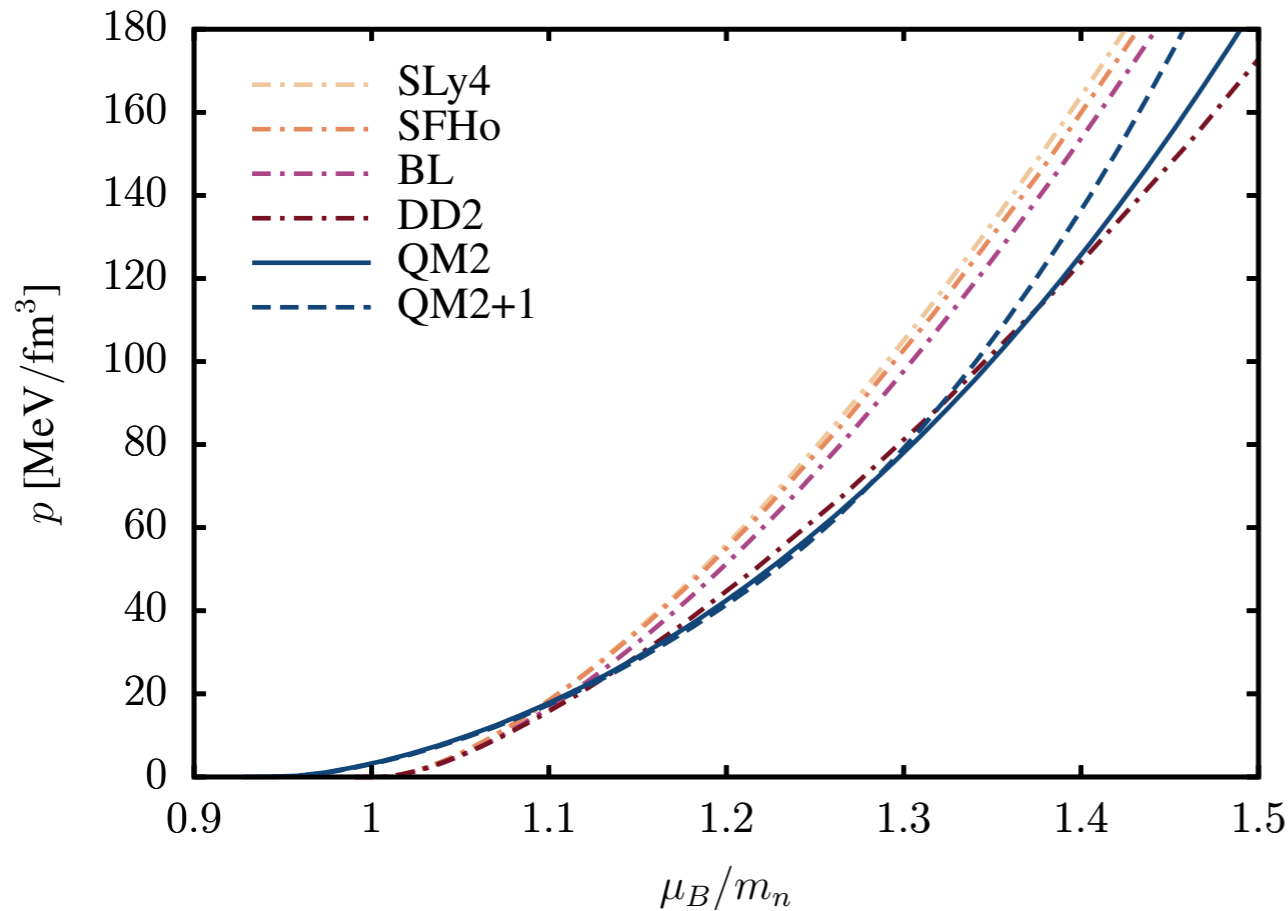


# Hybrid star construction possible? - yes

[Otto, Oertel, BJS 2020]

combine nuclear EoS (DD2) with FRG QM truncation

→ continuous nuclear-hybrid branch



**2 M<sub>⊙</sub> limit violated for N<sub>f</sub>= 2+1**

**can a repulsive vector interaction remedy this behavior?**

# vector mesons & the FRG EoS

[Otto, Oertel, BJS 2020]

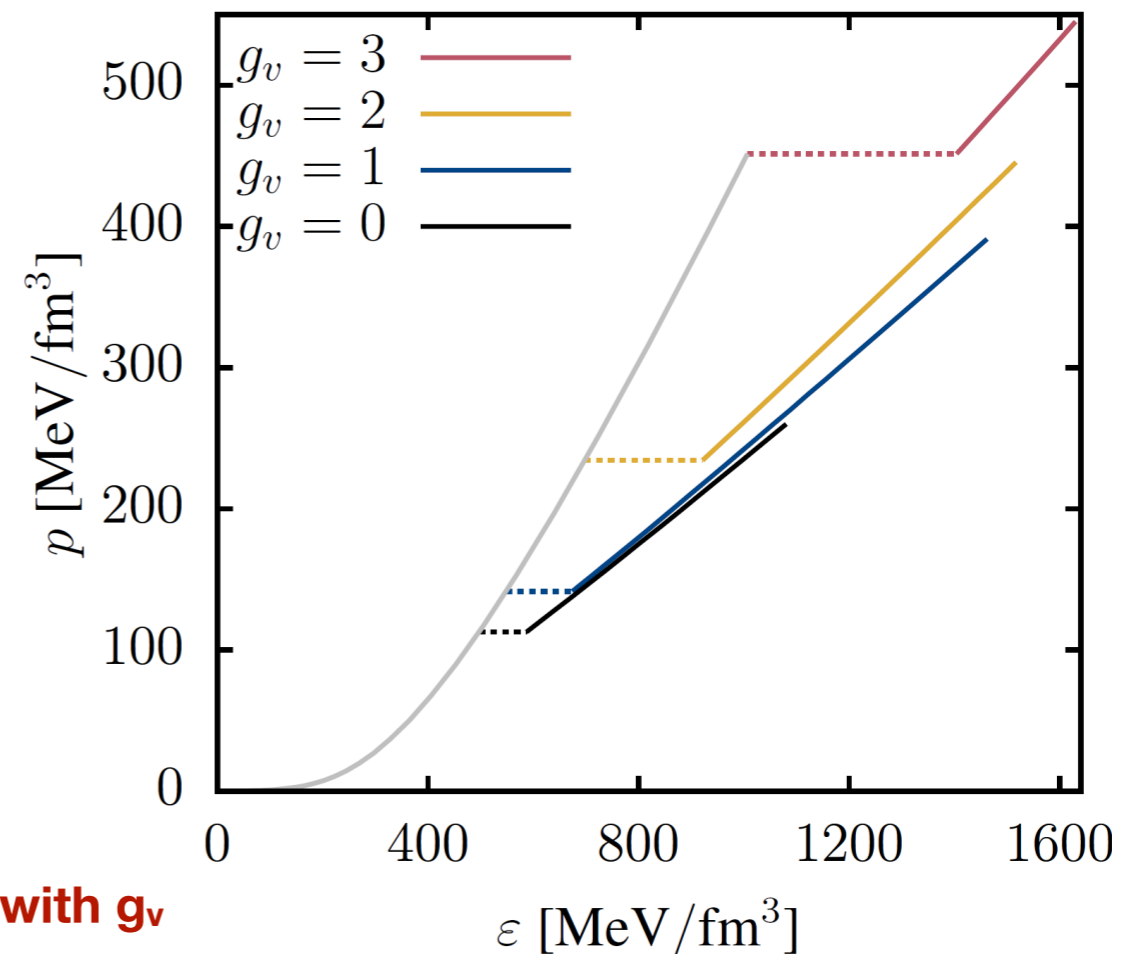
[Rennecke 2015]  
[Pereira, Stiele, Costa 2020]

- Yukawa type interaction of temporal component and mean-field potential

$$\Gamma_{\text{vec}} = \int_x \left[ \frac{g_v}{2} \bar{q} \gamma_0 \text{diag}_f(\omega, \omega, \sqrt{2}\phi) q - \frac{1}{2} (m_\omega^2 \omega^2 + m_\phi^2 \phi^2) \right]$$

- effectively shifts the chemical potentials:

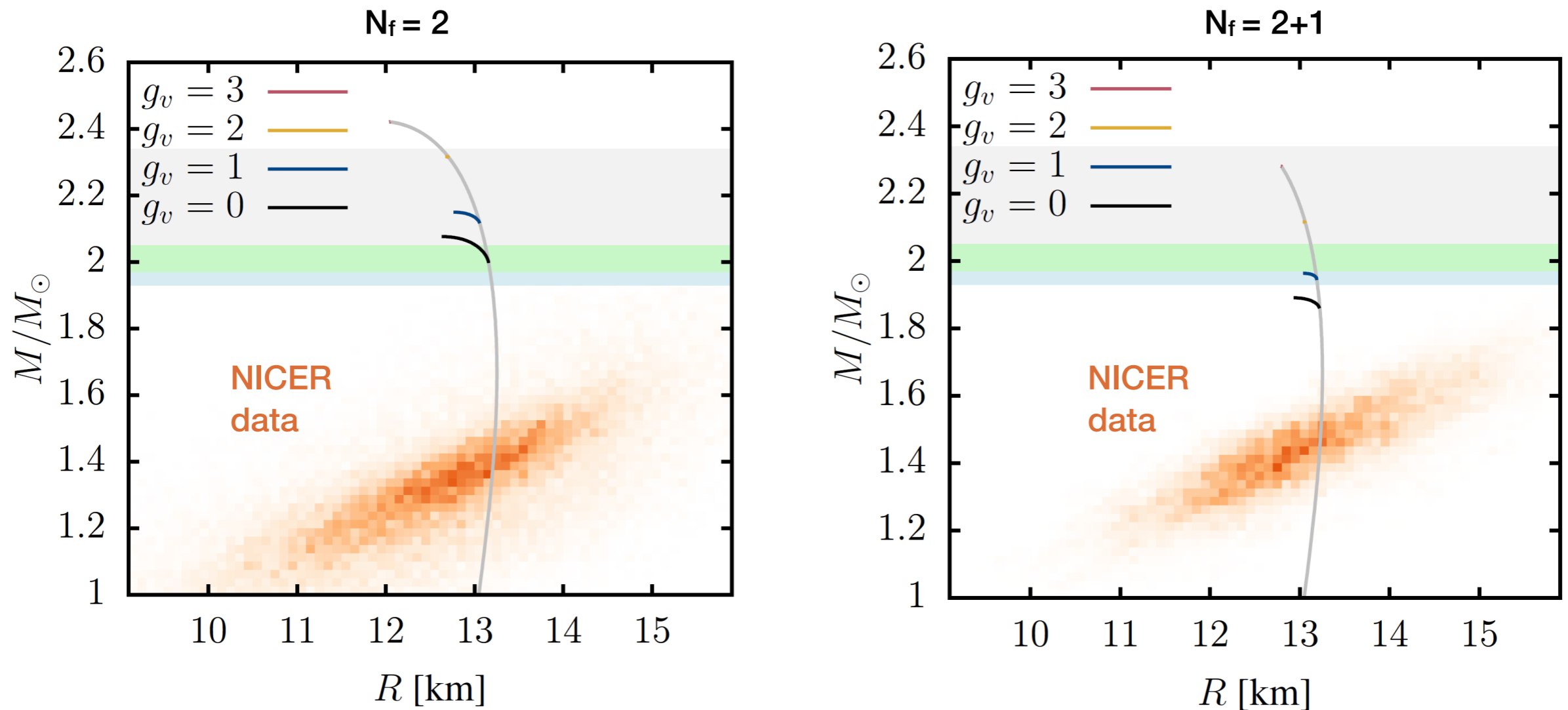
$$\begin{aligned} \tilde{\mu}_u &= \mu_q - \frac{2}{3}\mu_e - \frac{g_v}{2}\omega \\ \tilde{\mu}_d &= \mu_q + \frac{1}{3}\mu_e - \frac{g_v}{2}\omega \\ \tilde{\mu}_s &= \mu_q + \frac{1}{3}\mu_e - \frac{g_v}{\sqrt{2}}\phi \end{aligned}$$



→ energy gap and transition pressure increases with  $g_v$

# Mass-Radius relations

[Otto, Oertel, BJS 2020]



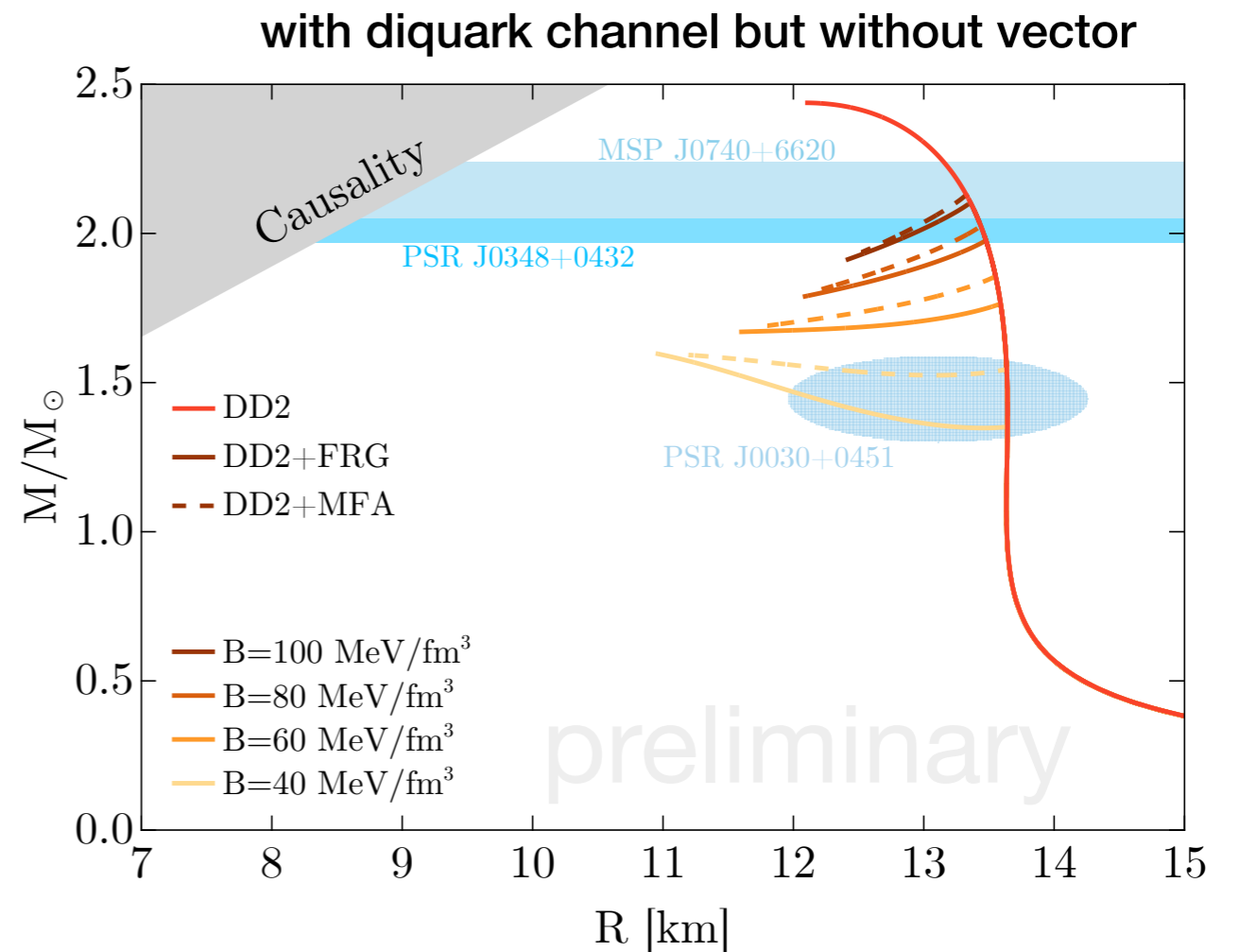
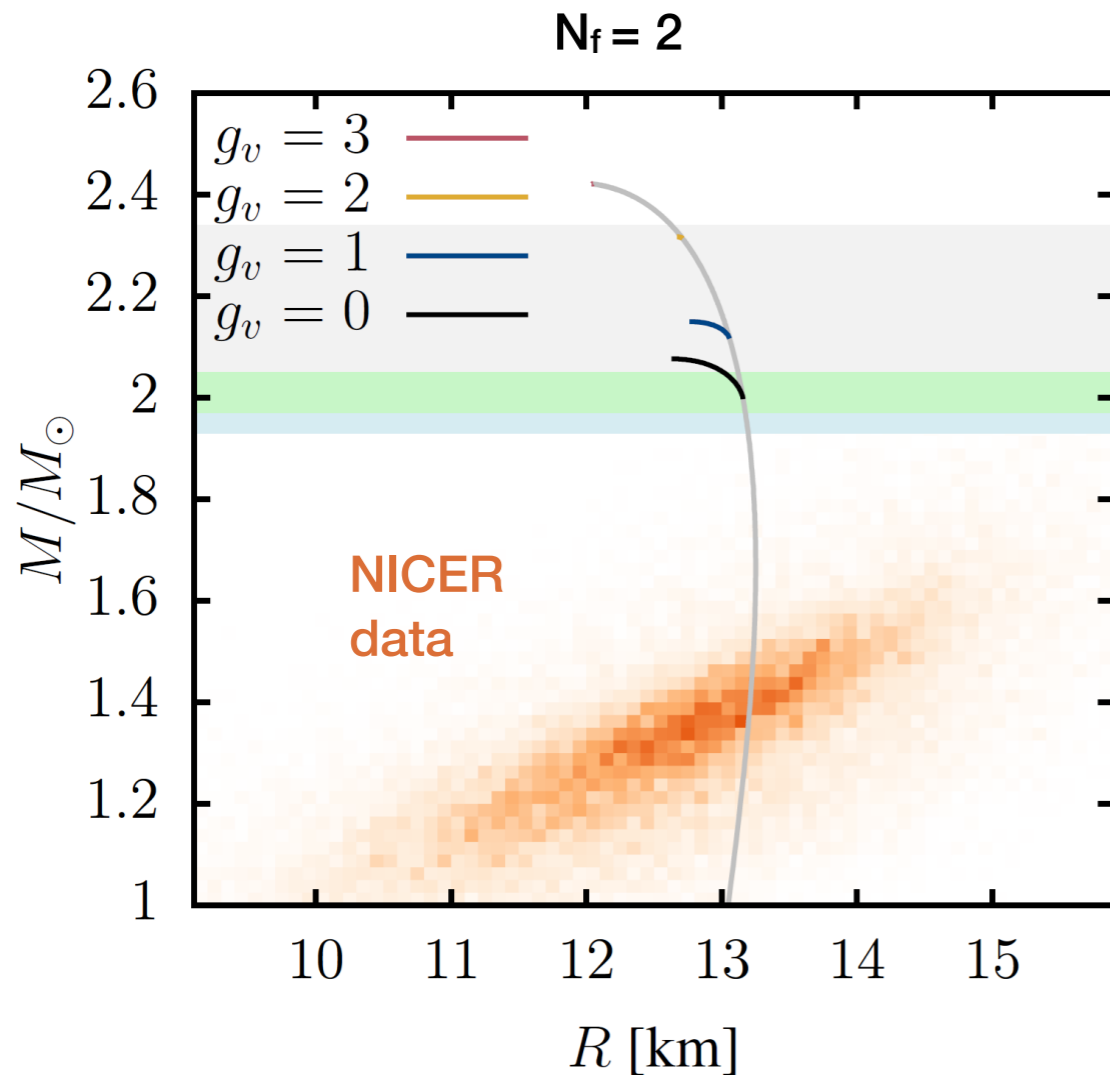
→ including strange quarks: **finite vector coupling is needed to achieve  $2M_{\odot}$  limit**

→ at the same time: **larger vector coupling lead to smaller quark cores!**

# Mass-Radius relations

[Otto, Oertel, BJS 2020]

[Mire, BJS ... next talk]



→ including strange quarks: **finite vector coupling is needed to achieve  $2M_\odot$  limit**

→ at the same time: **larger vector coupling lead to smaller quark cores!**



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so far so good ... BUT

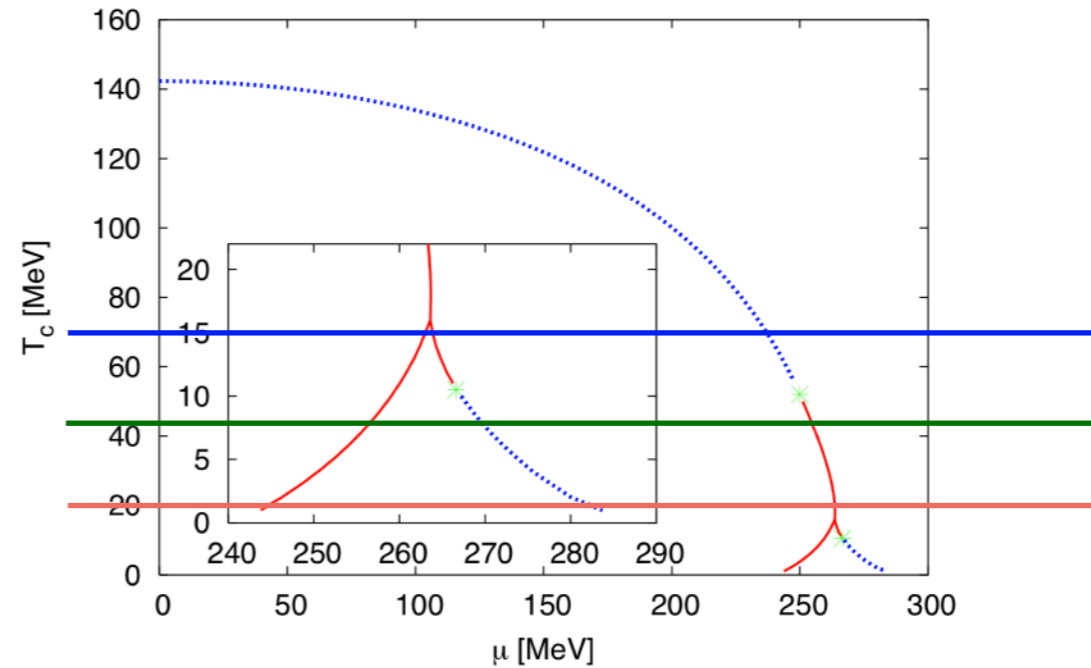
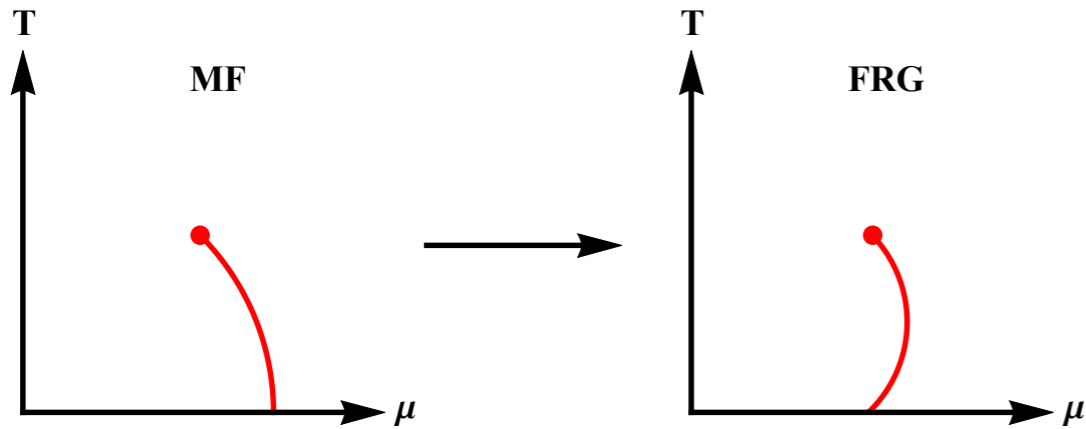
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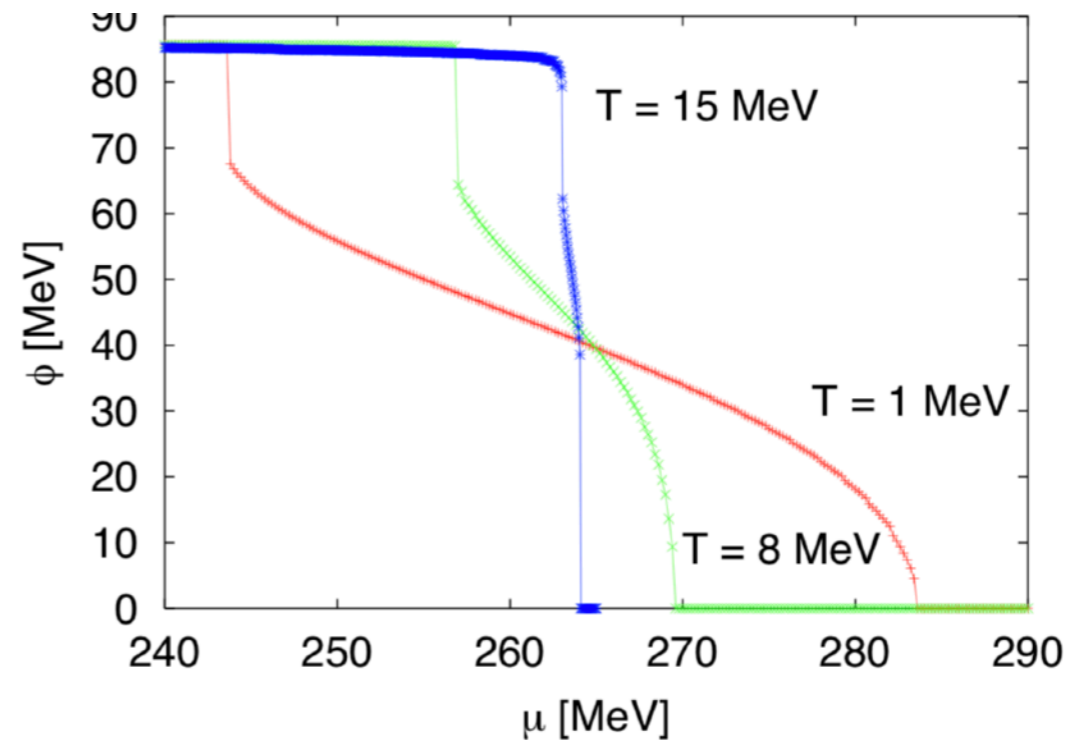
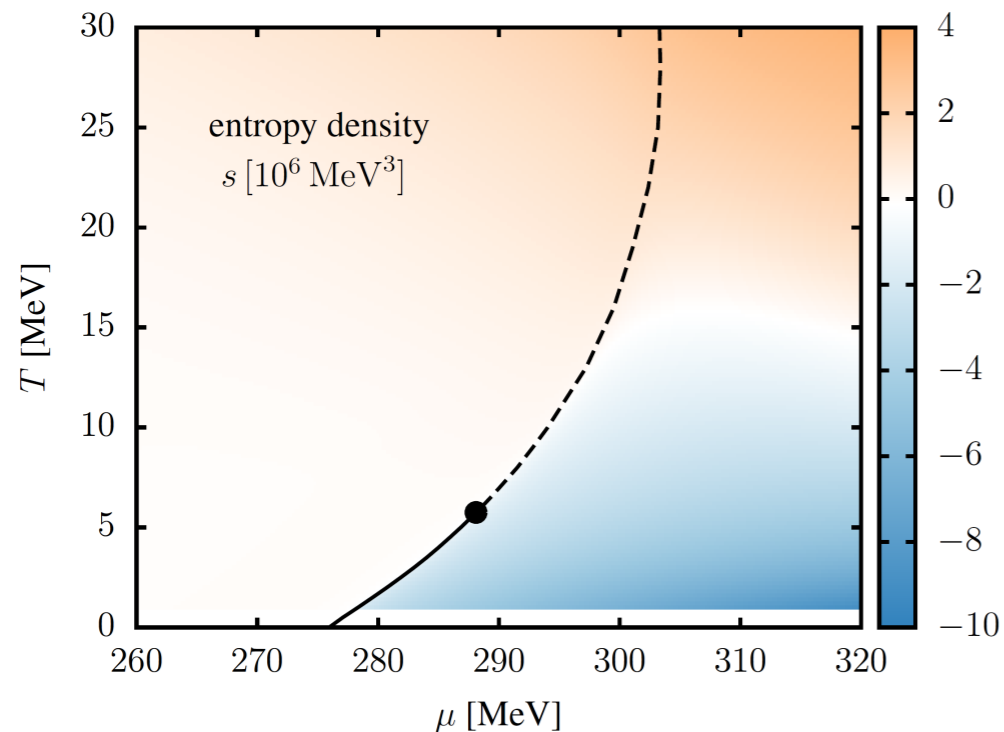
# back-bending / negative entropy density

[R-A Tripolt, BJS, L von Smekal, J Wambach 2018]

[BJS, Wambach 2005]

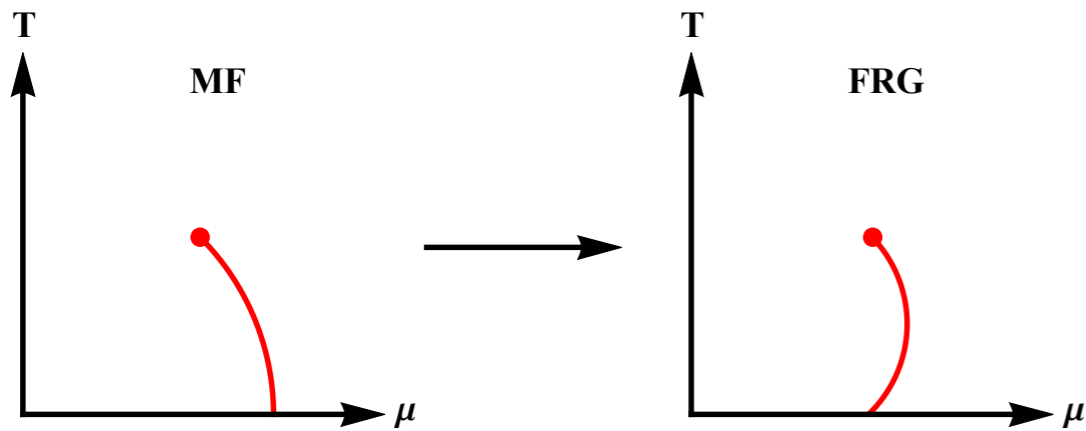


- Phase diagram quark-meson model
- Entropy density:  $s/T^3$

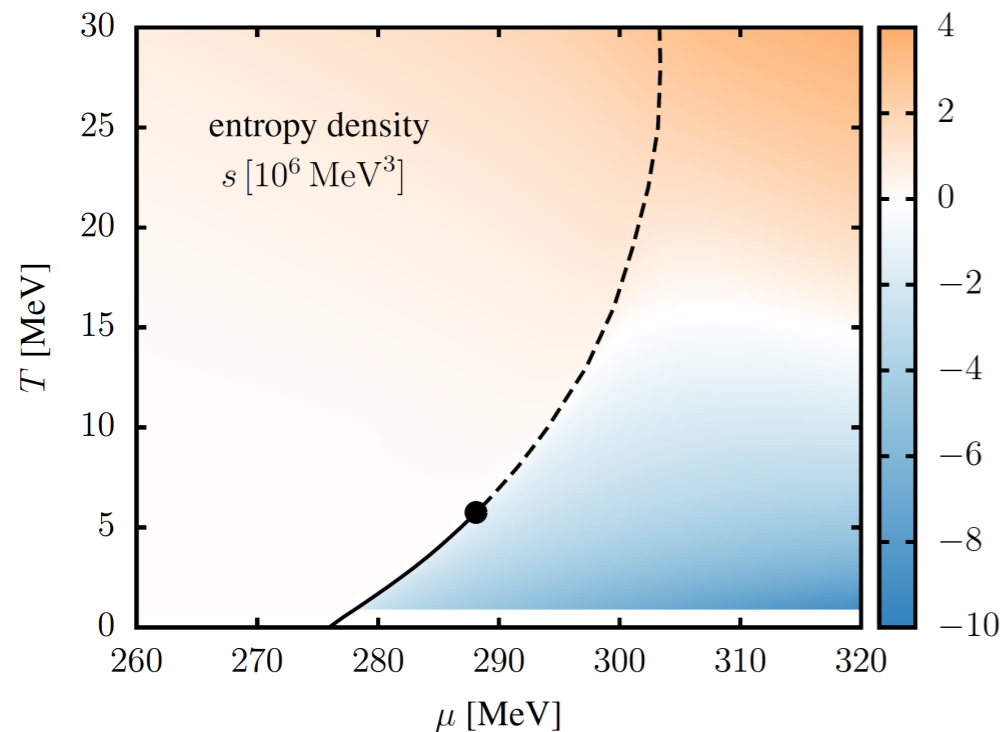


# back-bending / negative entropy density

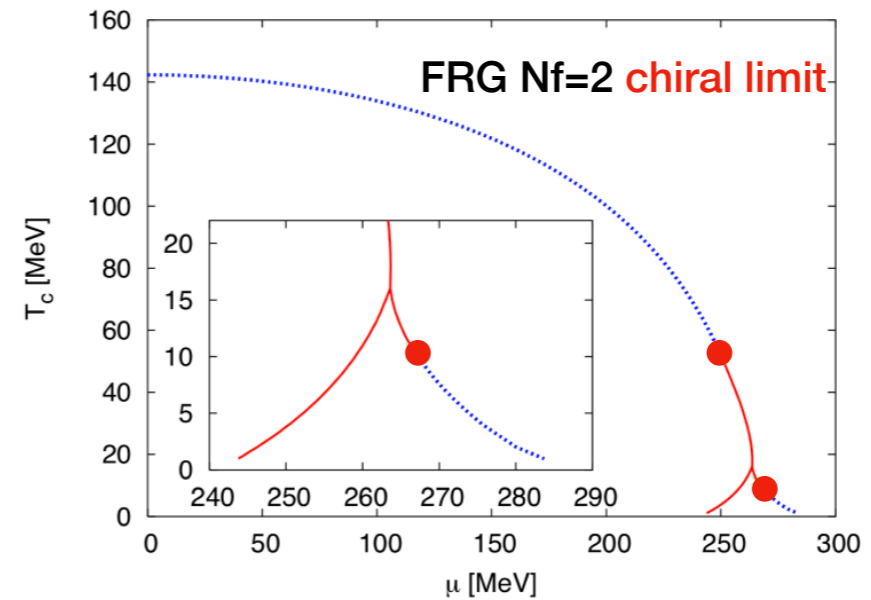
[R-A Tripolt, BJS, L von Smekal, J Wambach 2018]



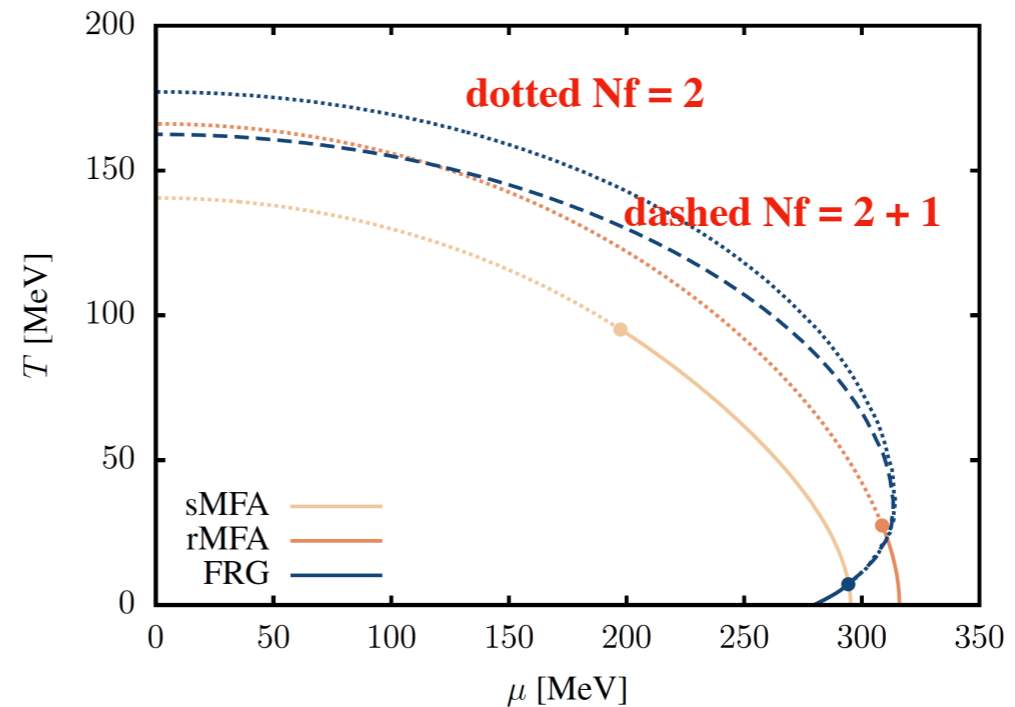
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[BJS, Wambach 2005]

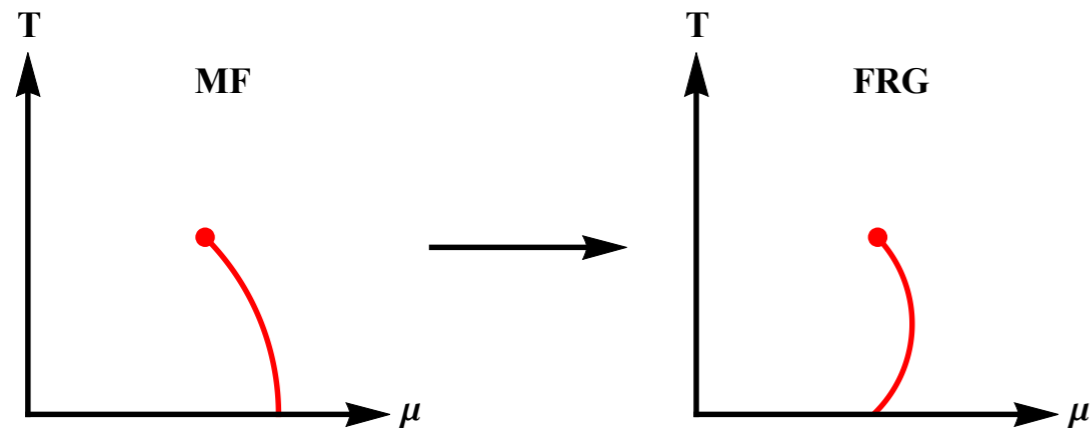


[Otto, Oertel, BJS 2020]



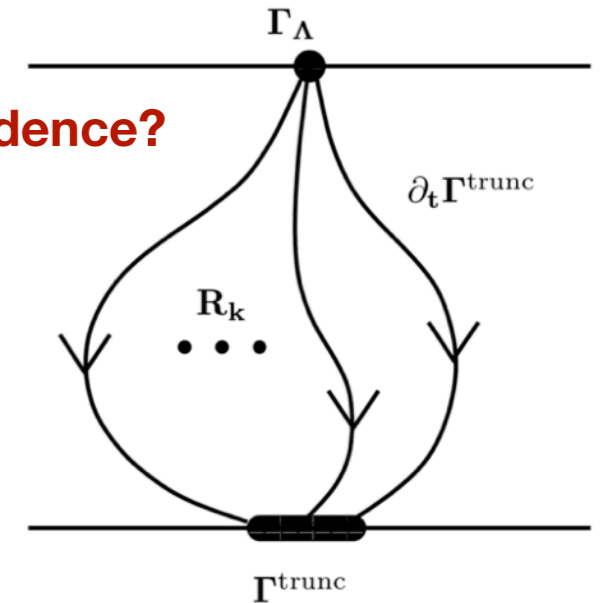
# back-bending / negative entropy density

[R-A Tripolt, BJS, L von Smekal, J Wambach 2018]



→ **Regulator scheme dependence?**

less pronounced when more channels are included  
e.g. pairing channel  
s. next talk by Ugo Mire



• fermionic regulator

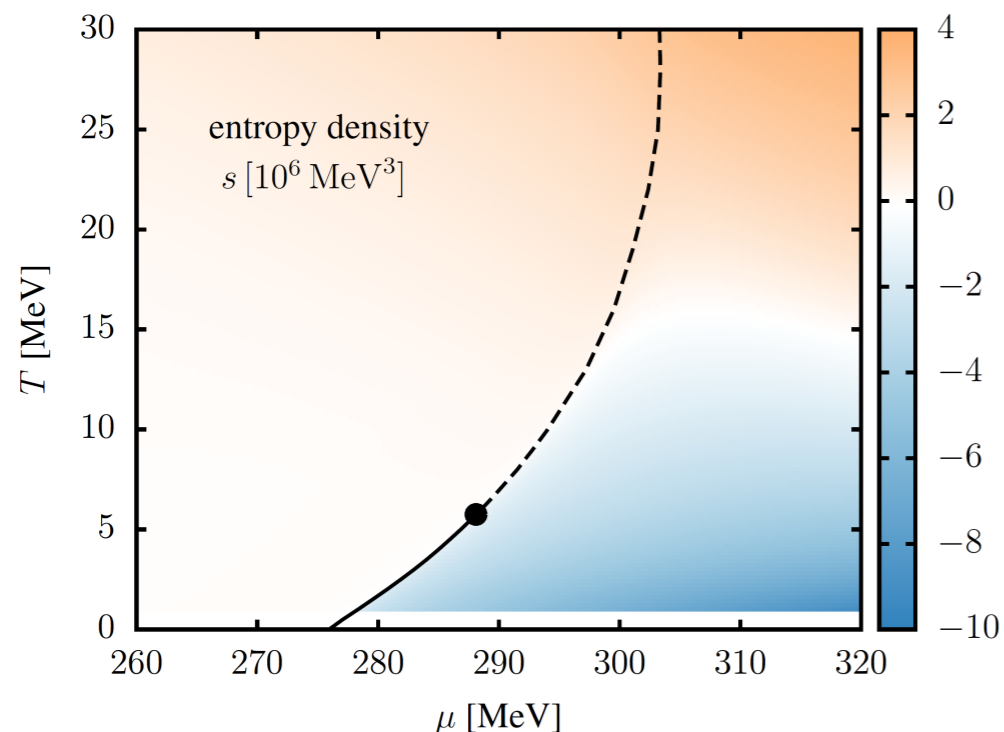
$$R_k^F(p, \mu) = R_k^F(\tilde{p}, 0) \quad \tilde{p} = \begin{pmatrix} p_0 + i\mu \\ \vec{p} \end{pmatrix}$$

shift required to preserve **Silver Blaze** property (T=0)  
(necessary but not sufficient)

• relative shift in the cutoff-scales  
between bosonic & fermionic regulator  
→ is needed beyond LPA

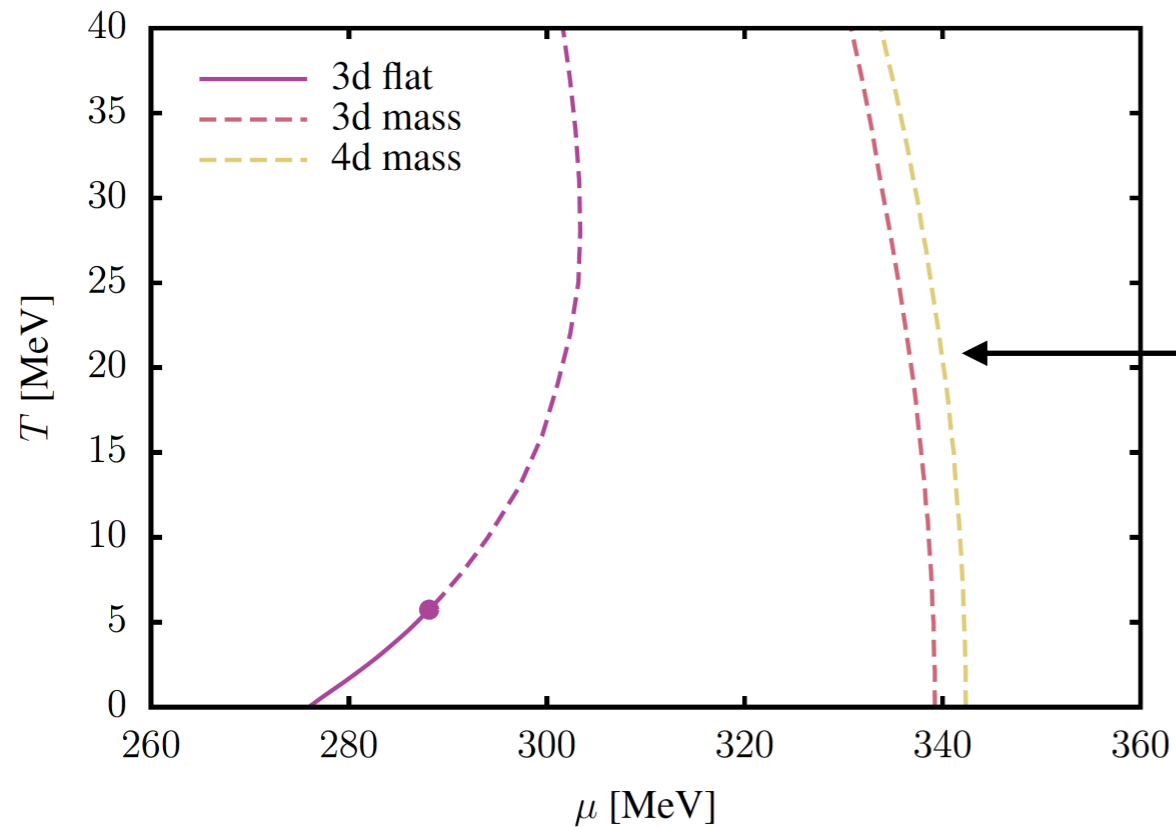
e.g. field-dependent Yukawa-coupling  
(multi quark-antiquark-meson scatterings)

- Phase diagram quark-meson model
- Entropy density:  $s/T^3$





# Chiral transition at low temperature



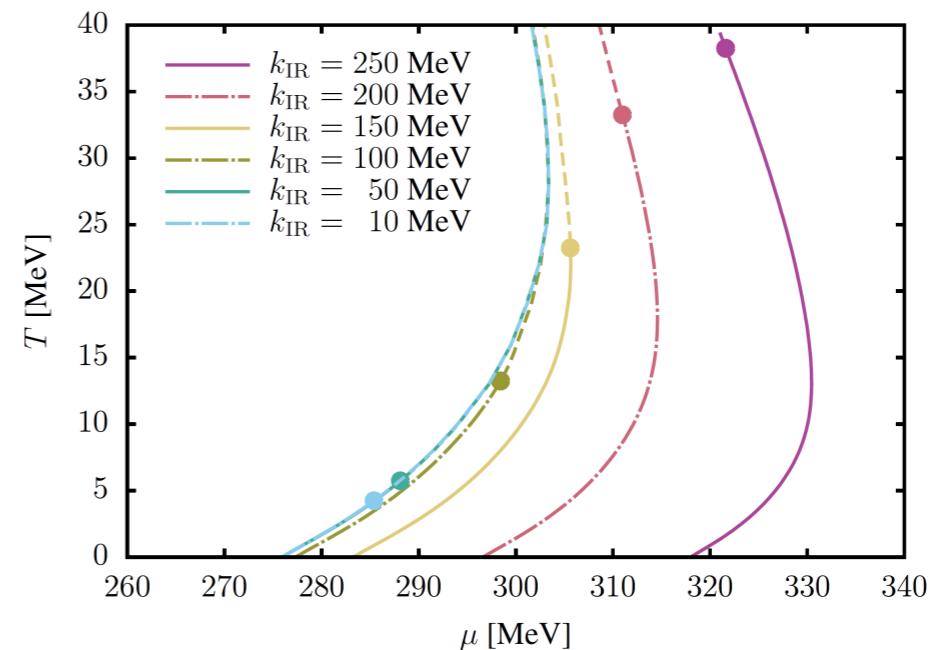
no back-bending with  
Callan-Symanzik mass-like regulator

small differences between 3d and 4d regulators

purely crossover  $\rightarrow$  pseudocritical  $\mu_c$   
larger than  $m_q$

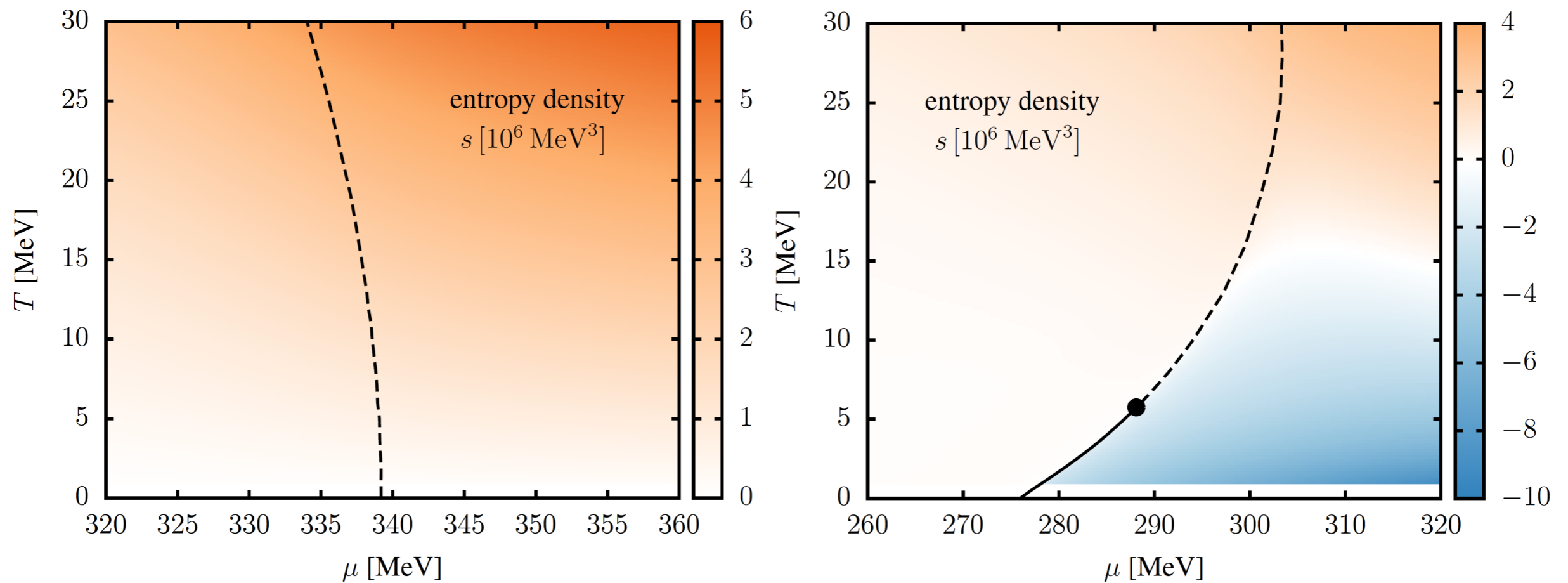
could finite IR cutoff play a role?  $\rightarrow$  No

transition line shifts and CEP moves down  
but back-bending over large  $k_{IR}$  range



# Chiral transition at low temperature

→ no negative entropy density anymore for Callan-Symanzik mass-like regulator



# EoS from QCD

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Braun et al. 2012++]

$\Lambda \sim O(10\text{GeV})$     quarks, gluons



▸ symmetric regime

quarks, gluons → mesons

$k_{\Phi}$



▸ symmetry-broken regime

quarks, diquarks, mesons etc

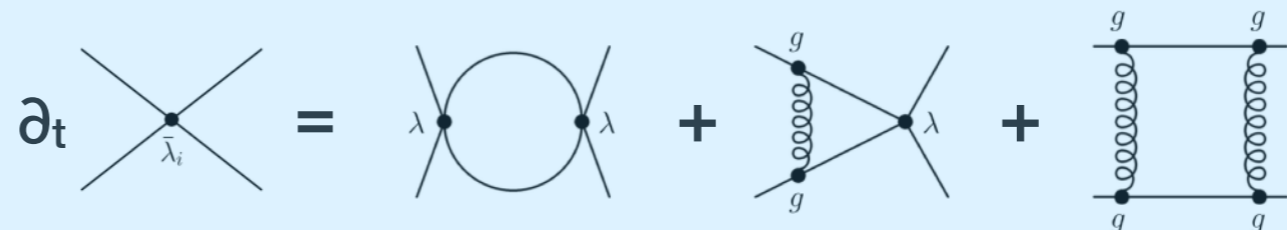
$k_{\text{IR}}$

$$S = \int d^4x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi} (i\not{\partial} + \bar{g}A + i\gamma_0\mu) \psi \right\}$$

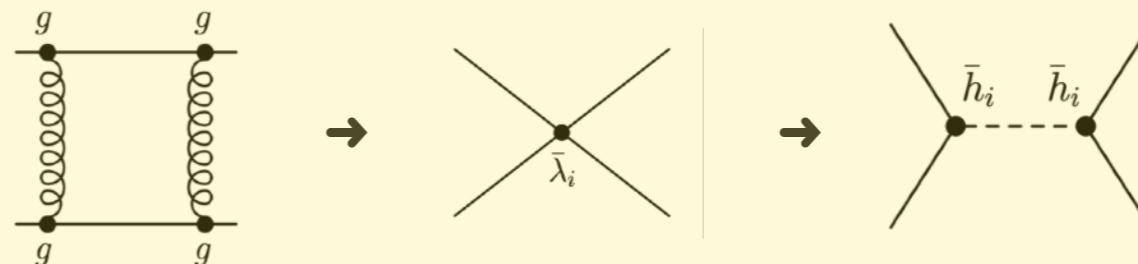
- quark-gluon vertex → many quark self-interaction channels

- dynamical hadronisation:

4-quark correlators → bound states / resonances



- general picture:



# EoS from QCD

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Braun et al. 2012++]

$\Lambda \sim O(10\text{GeV})$     quarks, gluons



▸ symmetric regime

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$k_\Phi$

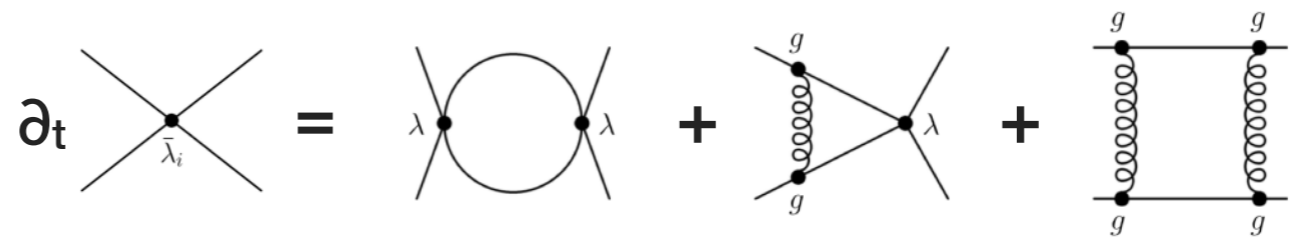


▸ symmetry-broken regime

quarks, diquarks, mesons etc

$k_{\text{IR}}$

$$S = \int d^4x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi} (i\not{\partial} + \bar{g}A + i\gamma_0\mu) \psi \right\}$$



- symmetry breaking → condensates

→ onset Landau-pole-type behavior  
 $\lambda \sim 1/m$

→ Ginzburg-Landau effective potential

Quark-meson-diquark truncation

# EoS from QCD

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Braun et al. 2012++]

$\Lambda \sim O(10\text{GeV})$  quarks, gluons

▸ symmetric regime

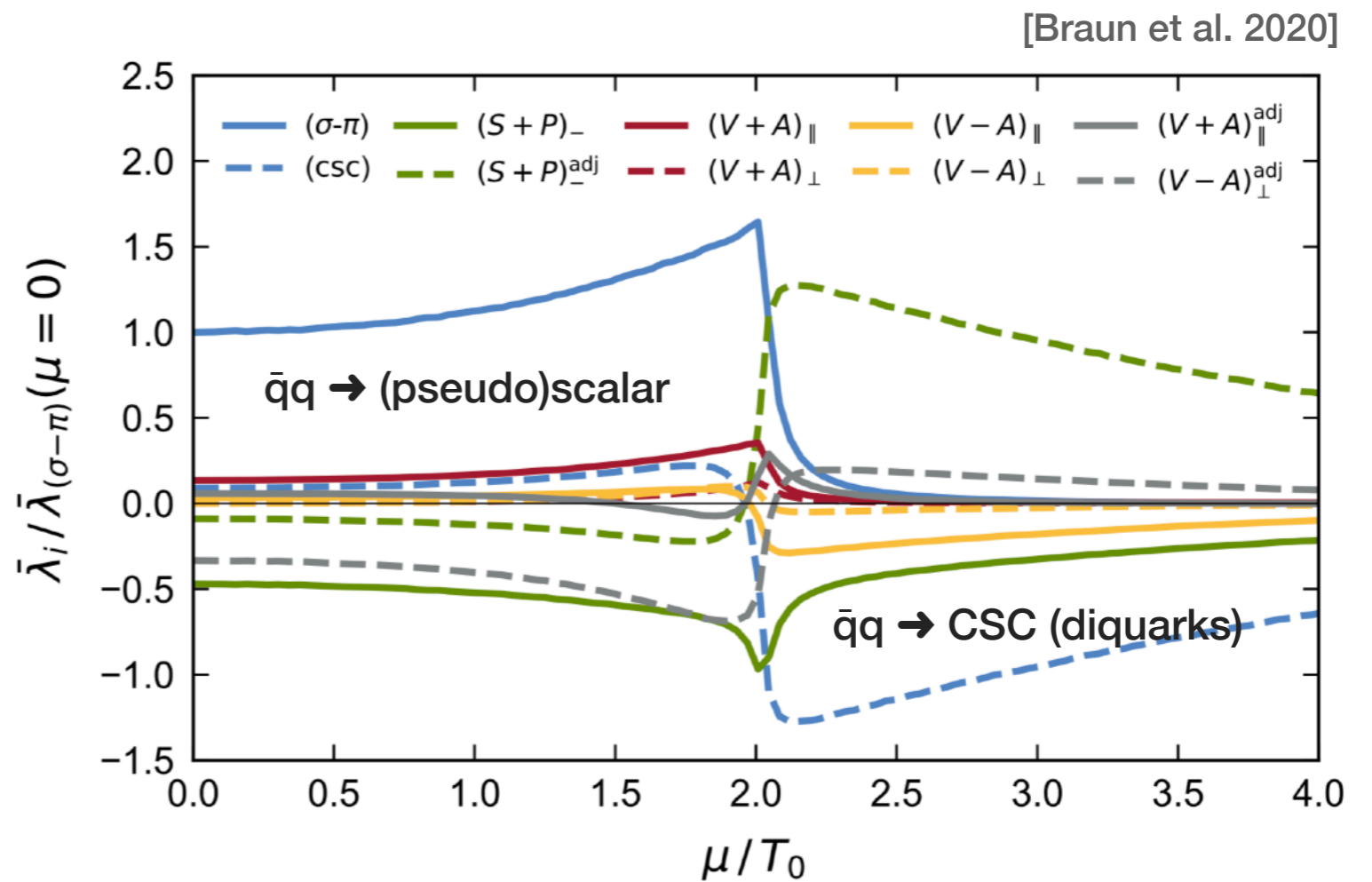
quarks, gluons  $\rightarrow$  mesons

$k_\Phi$

▸ symmetry-broken regime

quarks, diquarks, mesons etc

$k_{\text{IR}}$



# Summary

Three home messages:

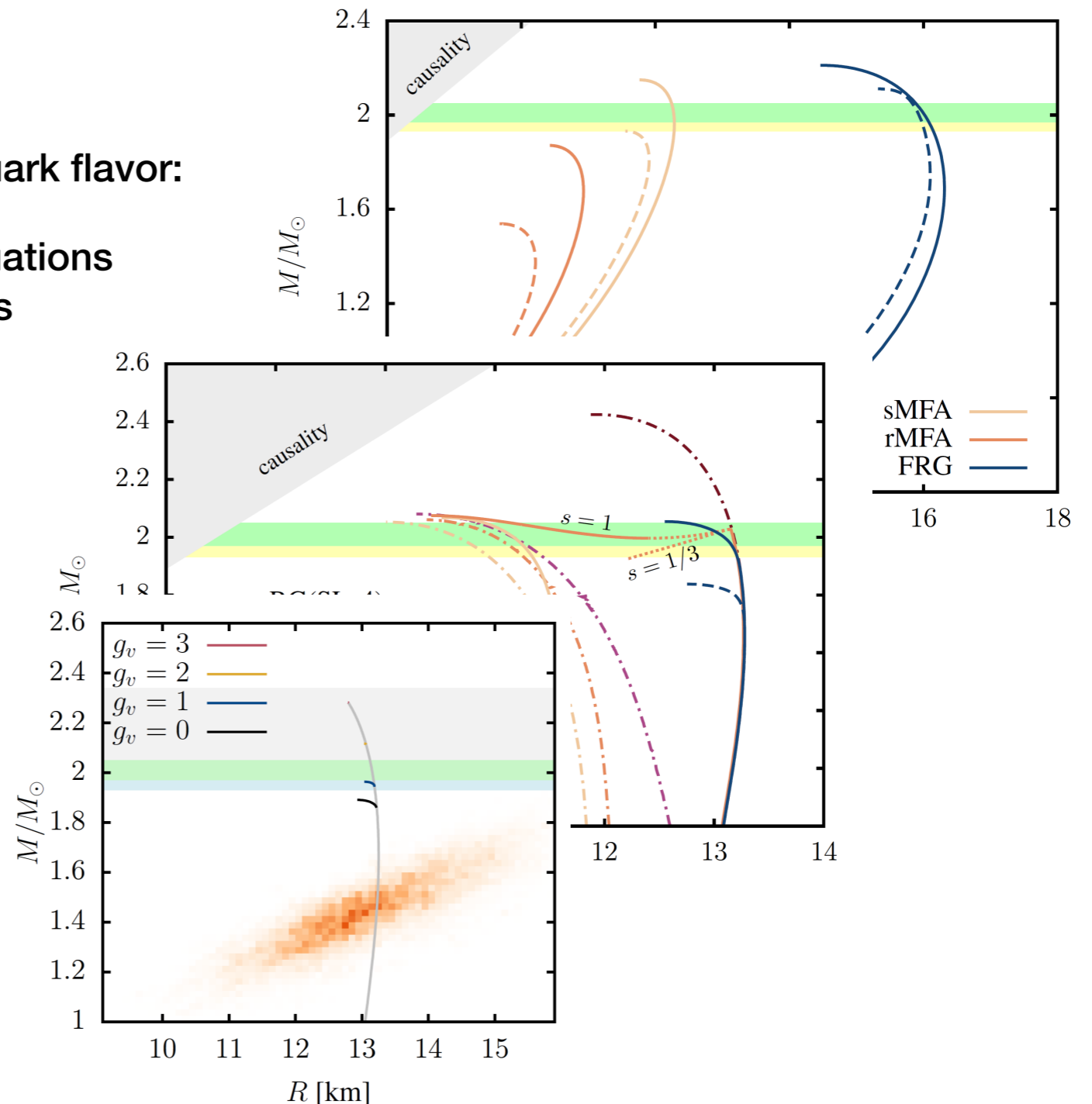
1. EoS with the FRG for two and three quark flavor:

→ significant impact of fluctuations on M-R relation for NSs

2. hybrid stars are possible

Non-zero vector coupling needed  
→ to reach  $2 M_{\odot}$  with strangeness

similar findings with pairing channels  
see next talk by Ugo Mire





# Summary

3. in LPA no back-bending / negative entropy density  
for  
CS mass-like regulators

CS type regulators closer to  
poles compared to flat regulator

→ (vacuum) flows numerically harder

