A critical look at the parity doublet model

Functional Methods in Strongly Correlated Systems (FUNSCS2023)

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what is the parity-doublet model?

The parity-doublet model

Motivation: evidence for the existence of massive baryon parity doublet above chiral symmetry restoration transition

THE MODEL (DeTar, Kunihiro, 1989):

Consider two massless Dirac fermions of opposite parity

$$(\psi_a, \psi_b) \longrightarrow (\psi_a^L, \psi_a^R; \psi_b^L, \psi_b^R) \qquad \gamma_5 \psi_a^R = \psi_a^R, \;\; \gamma_5 \psi_a^L = -\psi_a^L$$

Under $SU(2)_R \times SU(2)_L$

$$\psi_a^R \to \mathrm{e}^{i\frac{\boldsymbol{\alpha}_R \cdot \boldsymbol{\tau}}{2}} \psi_a^R, \quad \psi_a^L \to \mathrm{e}^{i\frac{\boldsymbol{\alpha}_L \cdot \boldsymbol{\tau}}{2}} \psi_a^L$$

Chiral transformation

$$\psi_a^R \to \mathrm{e}^{i\frac{\boldsymbol{\alpha} \cdot \boldsymbol{\tau}}{2}} \psi_a^R, \quad \psi_a^L \to \mathrm{e}^{-i\frac{\boldsymbol{\alpha} \cdot \boldsymbol{\tau}}{2}} \psi_a^L \qquad \psi_a \to \mathrm{e}^{i\frac{\boldsymbol{\alpha} \cdot \boldsymbol{\tau}}{2}\gamma_5} \psi_a$$

"Mirror assignment" for the doublet

$$\psi_b \to \mathrm{e}^{-i\frac{\boldsymbol{\alpha} \cdot \boldsymbol{\tau}}{2}\gamma_5} \psi_b$$

Chirally invariant mass term

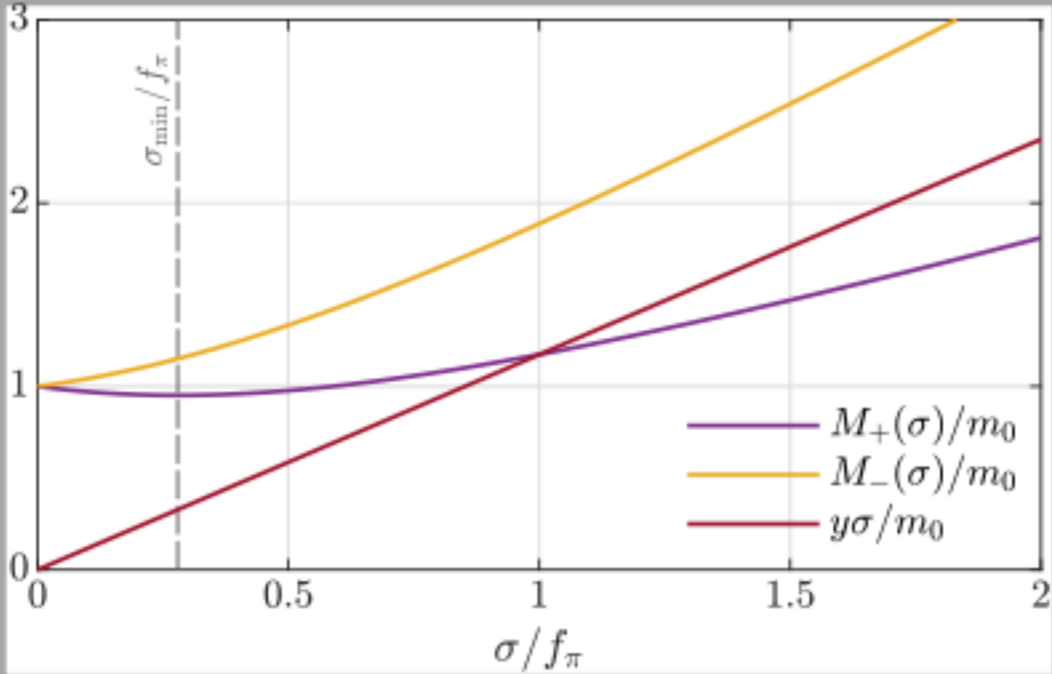
$$m_0(\bar{\psi}_a \gamma_5 \psi_b - \bar{\psi}_b \gamma_5 \psi_a)$$

Fermionic Lagrangian

$$\mathcal{L}_{\mathrm{F}} = (\bar{\psi}_a \bar{\psi}_b) \begin{pmatrix} \gamma_\mu(i\partial_\mu - g_v\omega_\mu) - y_a\,(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}) & m_0\gamma_5 \\ -m_0\gamma_5 & \gamma_\mu(i\partial_\mu - g_v\omega_\mu) - y_b\,(\sigma - i\gamma_5\vec{\pi}\cdot\vec{\tau}) \end{pmatrix} \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix}$$

Mass spectrum

$$M_\pm = \frac{1}{2}\left[\pm\sigma\,(y_a - y_b) + \sqrt{\sigma^2(y_a + y_b)^2 + 4m_0^2}\right]$$



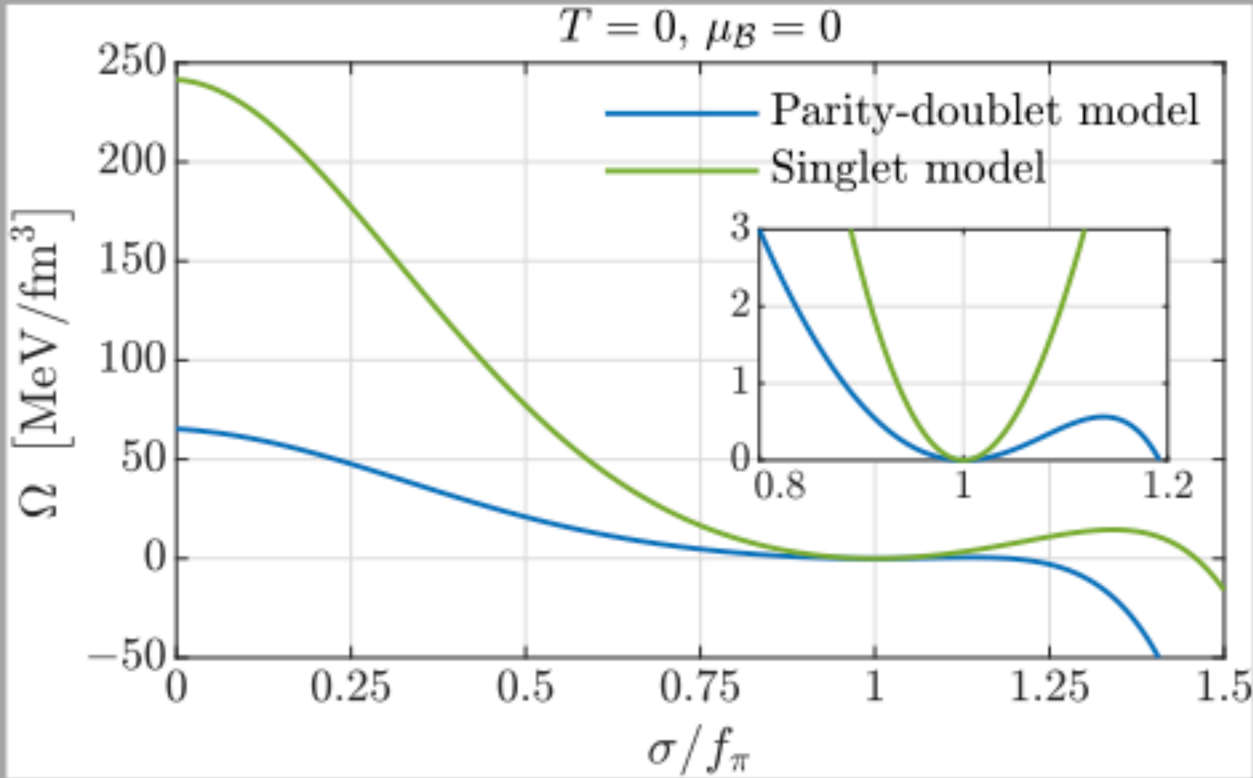
"Singlet" model

$$m_0 \to 0,\; y_a \to M_N/f_\pi$$

Mesonic Lagrangian

$$\mathcal{L}_{\mathrm{B}} = -V\left(\varphi^2\right) + h\left(\sigma - f_\pi\right) + \frac{1}{2} m_v^2 \omega^2 \qquad V\left(\varphi^2\right) = \sum_{n=1}^{4} \frac{\alpha_n}{2^n n!} \left(\varphi^2 - f_\pi^2\right)^n$$

$$\varphi = (\sigma, \vec{\pi})$$



NB. The potential near $\sigma = 0$ is sensitive to details of the potential near $\sigma = f_\pi$

→ correlates chiral transition to nuclear matter properties

Nuclear matter

Gap equation

Energy density

$$\mathcal{E}(n;\sigma) = \mathcal{E}_{\rm qp}(n;\sigma) + \frac{1}{2}G_v n^2 + U(\sigma)$$

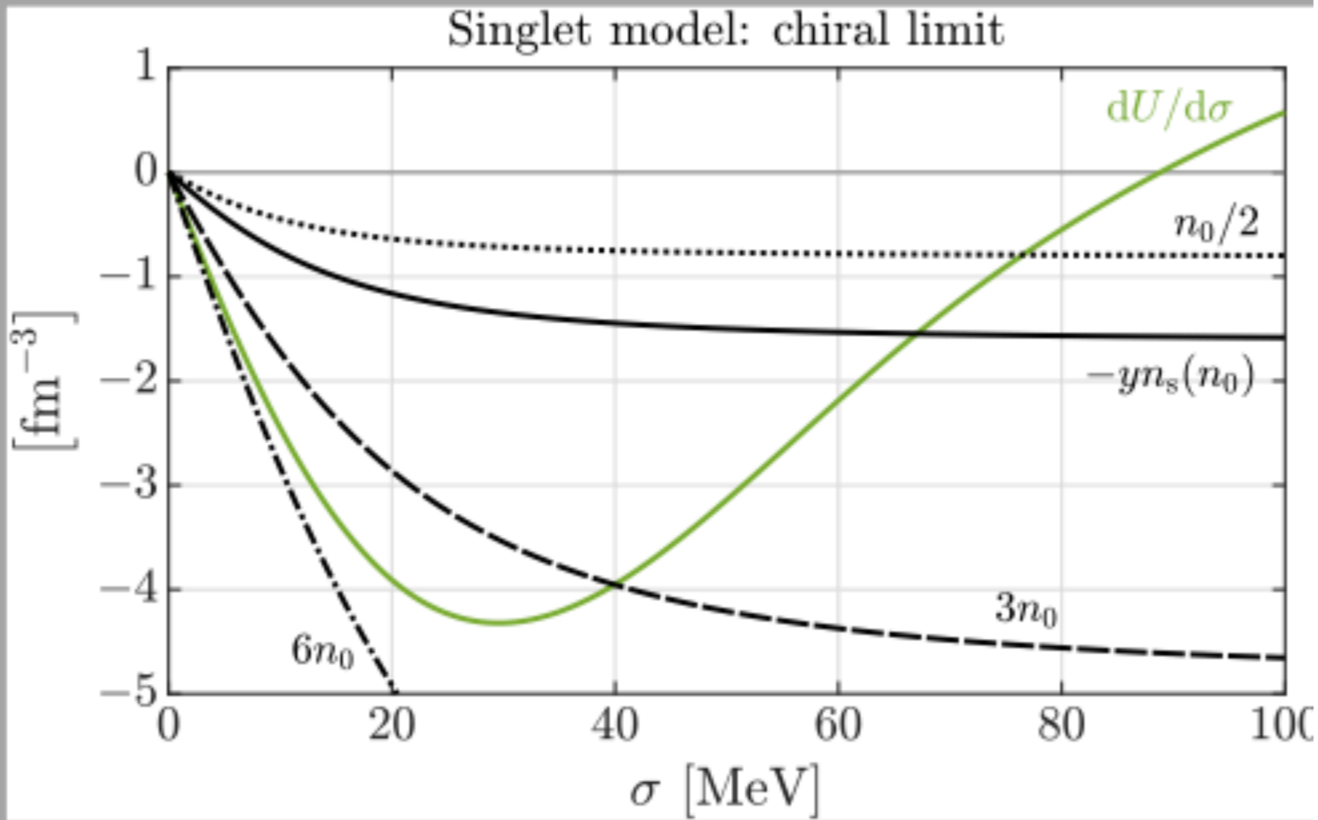
$$\mathcal{E}_{\rm qp}(n;\sigma) = 4\int_{|p|<p_F} \frac{d^3p}{(2\pi)^3}\sqrt{p^2 + M(\sigma)^2}$$

Scalar density

$$n_{\rm s} = 4M\int_{|\boldsymbol{p}|\le p_F} \frac{1}{\sqrt{\boldsymbol{p}^2 + M^2}}$$

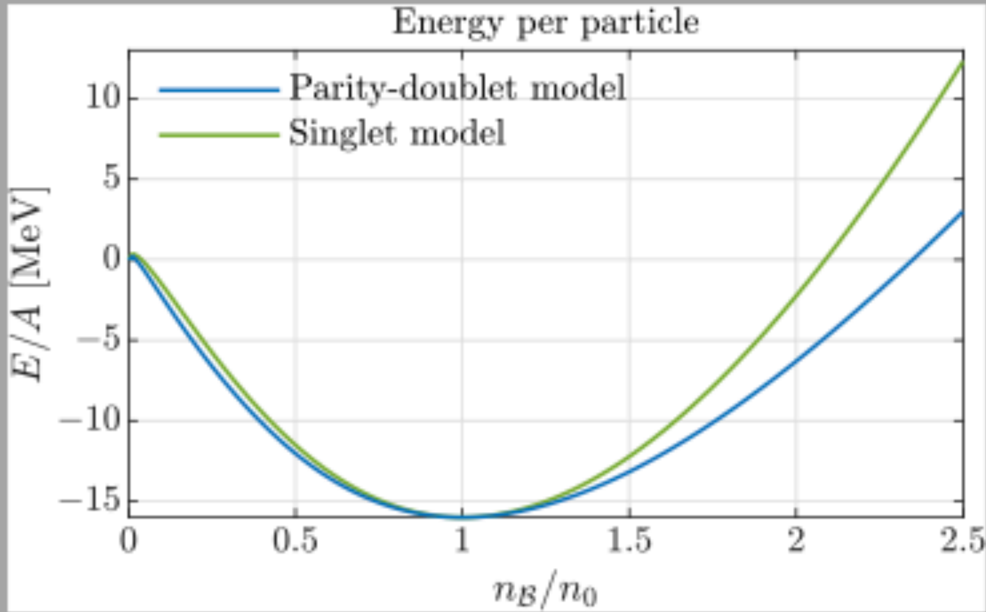
Gap equation

$$\frac{dU}{d\sigma} = -yn_s$$



Saturation curve

$$\frac{\mathcal{E}(n)}{n} = M_N + \frac{E}{A}$$



Note on surface energy

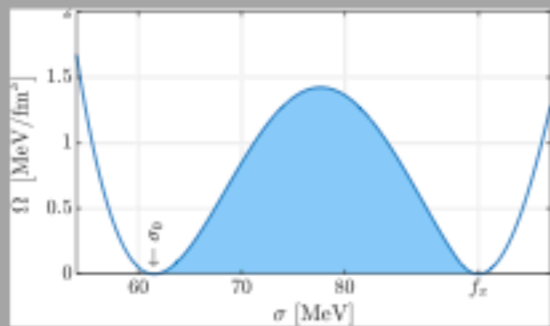
$$\Sigma[n] = \int_{-\infty}^{+\infty} \mathrm{d}z \left(\mathcal{E}(n) + \frac{C}{2n} \left(\frac{\mathrm{d}n}{\mathrm{d}z} \right)^2 - \mu_0 n \right)$$

variational → calculation

$$\Sigma = \int_0^{n_0} \mathrm{d}n \sqrt{2C \left(\frac{\mathcal{E}(n)}{n} - \mu_0 \right)}.$$

NB. This is NOT

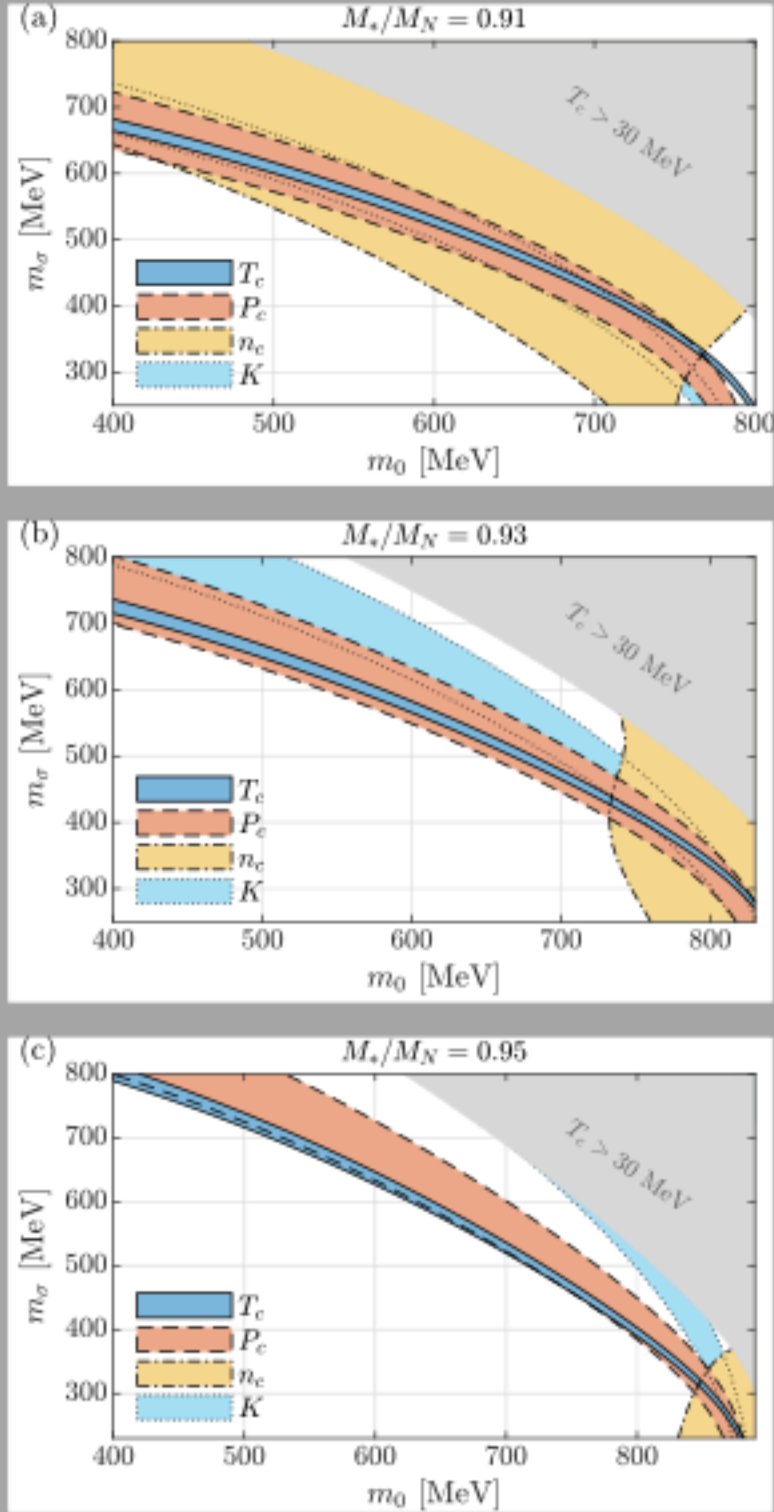
$$\Sigma = \int_{\sigma_0}^{f_\pi} \mathrm{d}\sigma \sqrt{2\Omega(\mu_0; \sigma)}$$



Remarks on parameters

Parameter	Numerical value
Chiral-invariant mass m_0 [MeV]	800
Isoscalar mass m_σ [MeV]	340
Landau effective mass M_*	$0.93 \times M_N$
Yukawa coupling y_a	6.9
Yukawa coupling y_b	13.0
In-medium condensate σ_0 [MeV]	65.9
Taylor coefficient α_3 [MeV^{-2}]	4.4×10^{-1}
Taylor coefficient α_4 [MeV^{-4}]	-7.8×10^{-5}
Vector coupling G_v [fm^2]	1.58
Compression modulus K [MeV]	242.8
Surface tension Σ [$\mathrm{MeV\,fm}^{-2}$]	1.28

Parameter	Numerical value
Isoscalar mass m_σ [MeV]	640
Landau effective mass M_*	$0.8 \times M_N$
Yukawa coupling y_a	10.1
In-medium condensate σ_0 [MeV]	69.7
Taylor coefficient α_3 [MeV^{-2}]	2.2×10^{-1}
Taylor coefficient α_4 [MeV^{-4}]	-4.3×10^{-5}
Vector coupling G_v [fm^2]	5.44
Compression modulus K [MeV]	299.2
Surface tension Σ [$\mathrm{MeV\,fm}^{-2}$]	1.43



The nucleon sigma term

The nucleon sigma term

$$\sigma_N = \bar{m} \left[\langle N | \bar{q}q | N \rangle - \langle \bar{q}q \rangle_0 \right] ;$$

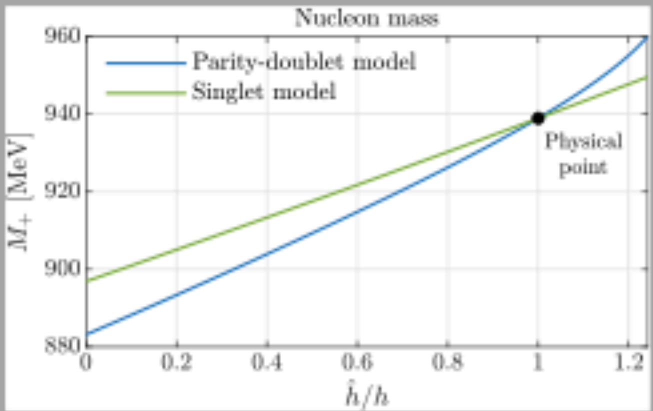
Value uncertain:

Empirical information from scattering amplitudes 60 MeV

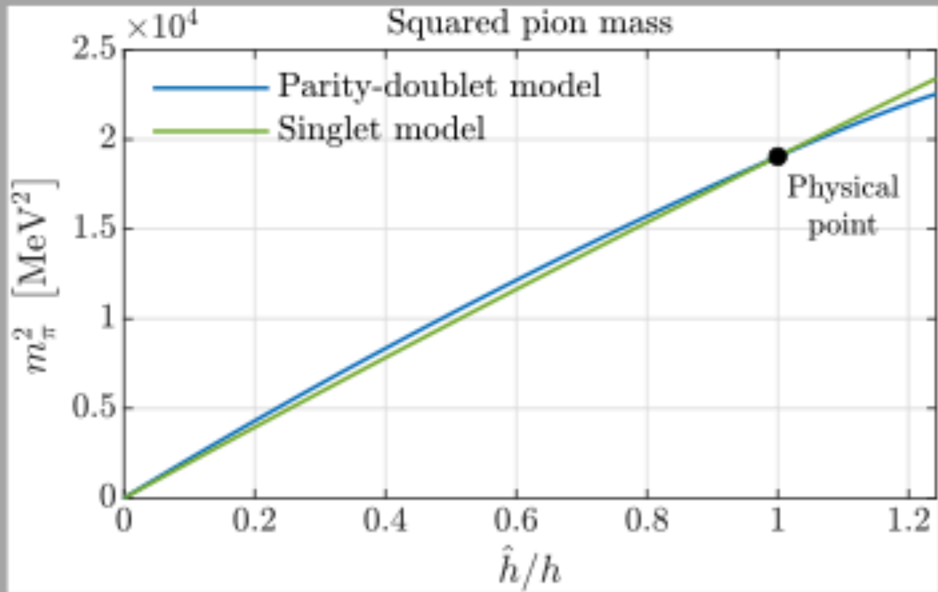
"Traditional" value (chiral PT) 45 MeV

Lattice calculations 40 MeV

Variational argument: $\sigma_N = \bar{m} \frac{\mathrm{d}M_N}{\mathrm{d}\bar{m}}$ $\quad M_N = \langle N | \mathcal{H} | N \rangle - \langle 0 | \mathcal{H} | 0 \rangle$



Non linearities between the chiral limit and the physical point



Estimate	Parity-doublet model	Singlet model
(0) [MeV]	55.8	42.1
(1) [MeV]	50.2	40.6
(2) [MeV]	43.1	38.8
(3) [MeV]	68.6	43.7

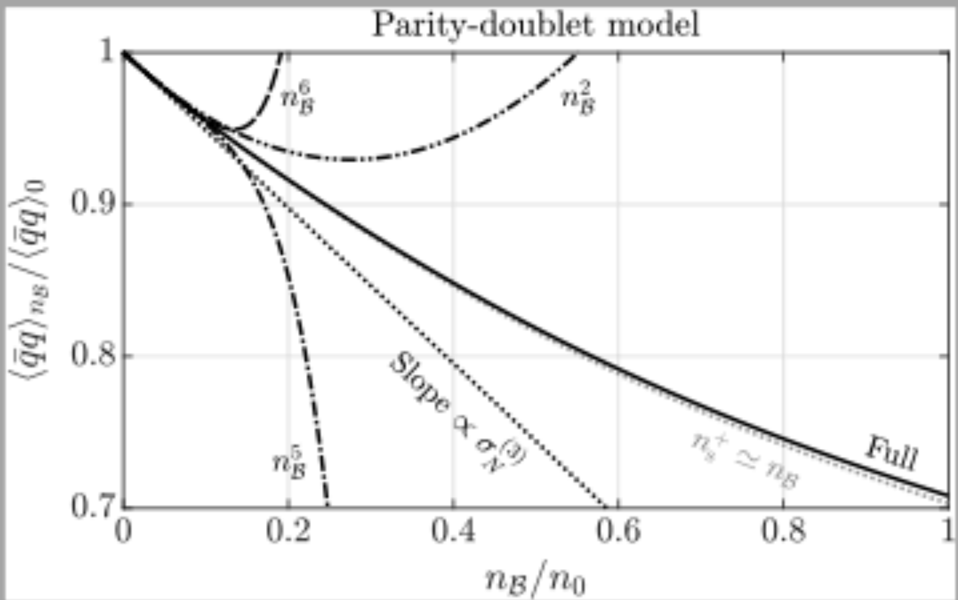
Density dependent corrections

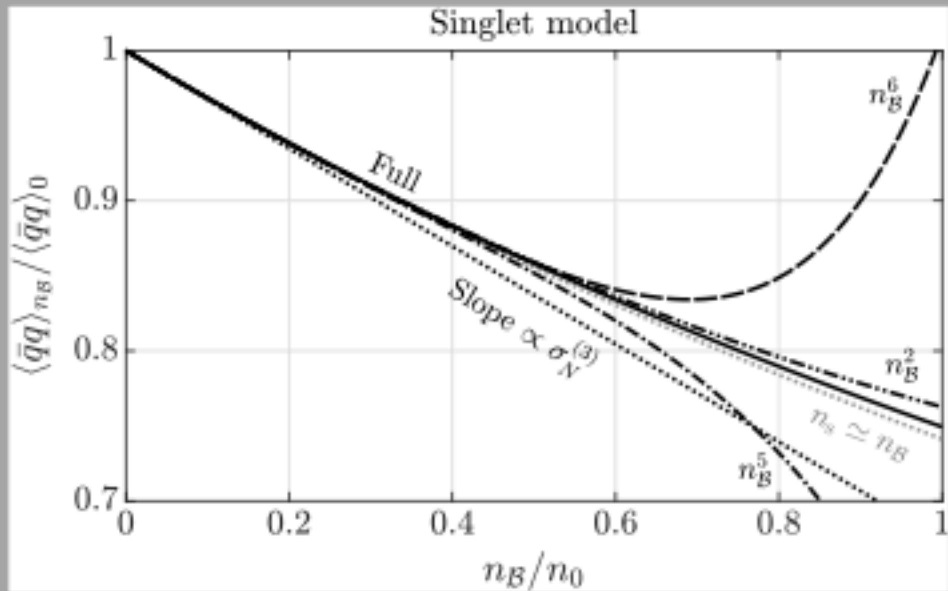
$$\bar{m}\,\langle\bar{q}q\rangle_{n_{\mathcal{B}}} = \bar{m}\,\langle\bar{q}q\rangle_0 + \bar{m}\frac{\mathrm{d}}{\mathrm{d}\bar{m}}\left[\mathcal{E}(n_{\mathcal{B}}) - \mathcal{E}(0)\right]$$

Low density approximation

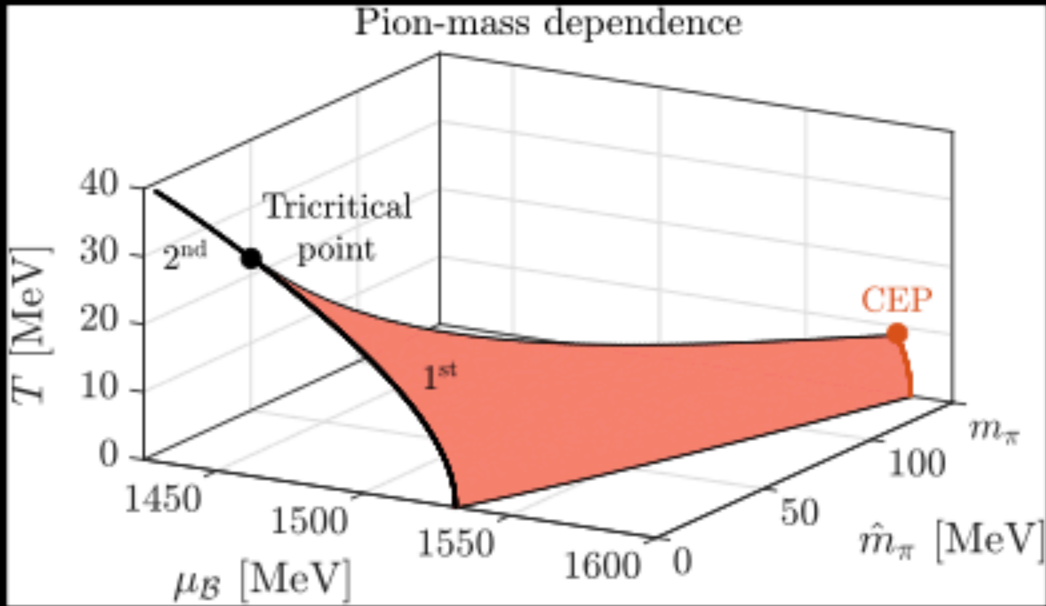
$$\frac{\langle\sigma\rangle_{n_{\mathcal{B}}}}{\langle\sigma\rangle_0} = \frac{\langle\bar{q}q\rangle_{n_{\mathcal{B}}}}{\langle\bar{q}q\rangle_0} \simeq 1 - \frac{\sigma_N^{(3)} n_{\mathrm{s}}}{f_\pi^2 m_\pi^2} \quad \longrightarrow \quad \frac{\langle\bar{q}q\rangle_{n_{\mathcal{B}}}}{\langle\bar{q}q\rangle_0} \simeq 1 - \frac{\bar{\sigma}_N(n_{\mathcal{B}})\,n_{\mathcal{B}}}{f_\pi^2 m_\pi^2}$$

Solve gap equation with ansatz $\sigma(n_{\mathcal{B}}) = \sigma^{(0)} + \sigma^{(1)} n_{\mathcal{B}} + \frac{1}{2}\sigma^{(2)} n_{\mathcal{B}}^2 + \cdots$





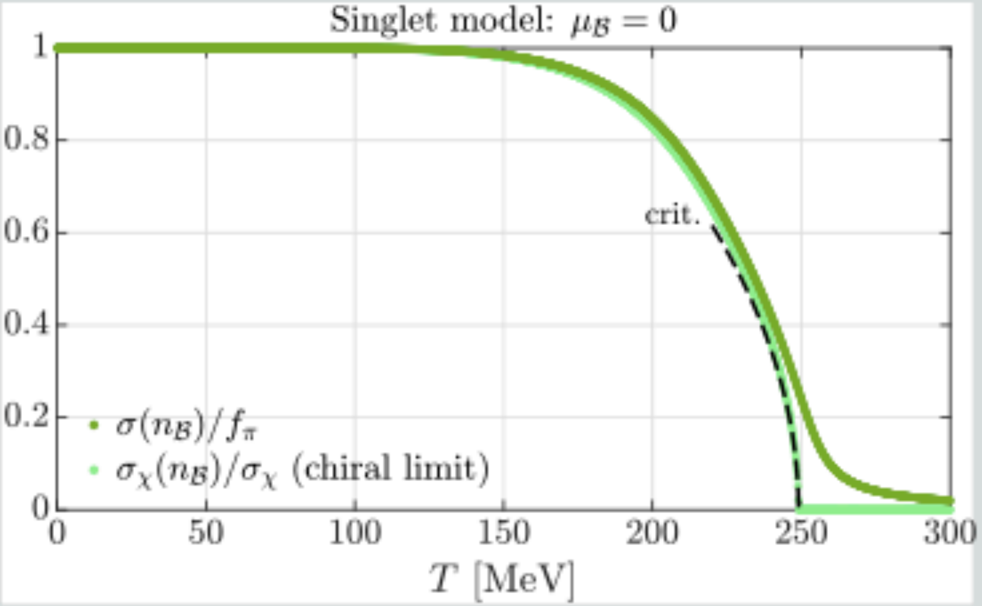
The chiral transition

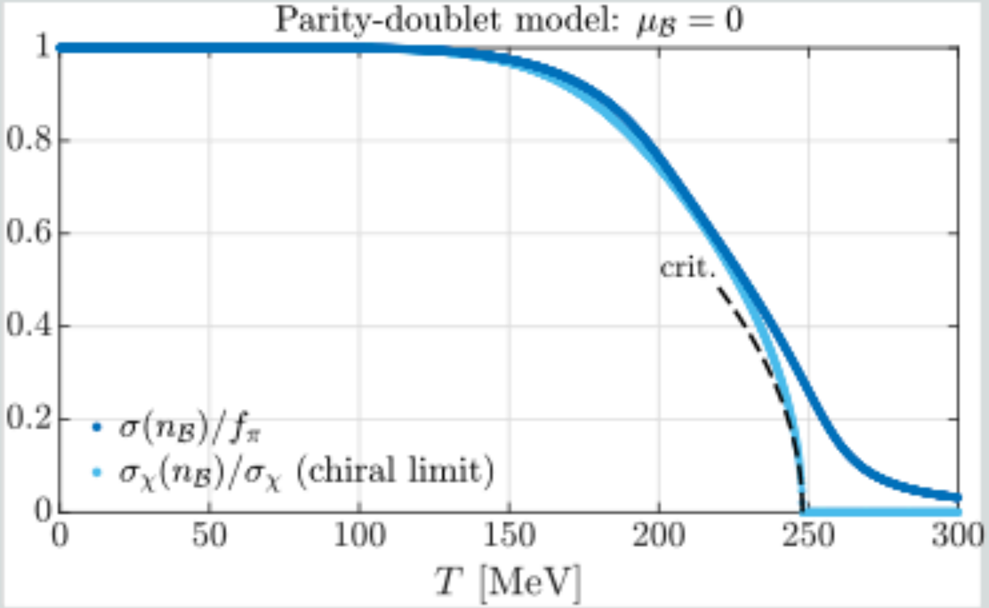


The chiral transition at $\mu_B = 0$

Second order transition in chiral limit
Continuous crossover for finite pion mass

$$U(\sigma) \simeq U_0 - \frac{r}{2}\sigma^2 + \frac{u}{4}\sigma^4 - l\sigma^4 \ln \frac{\sigma}{f_\pi}$$



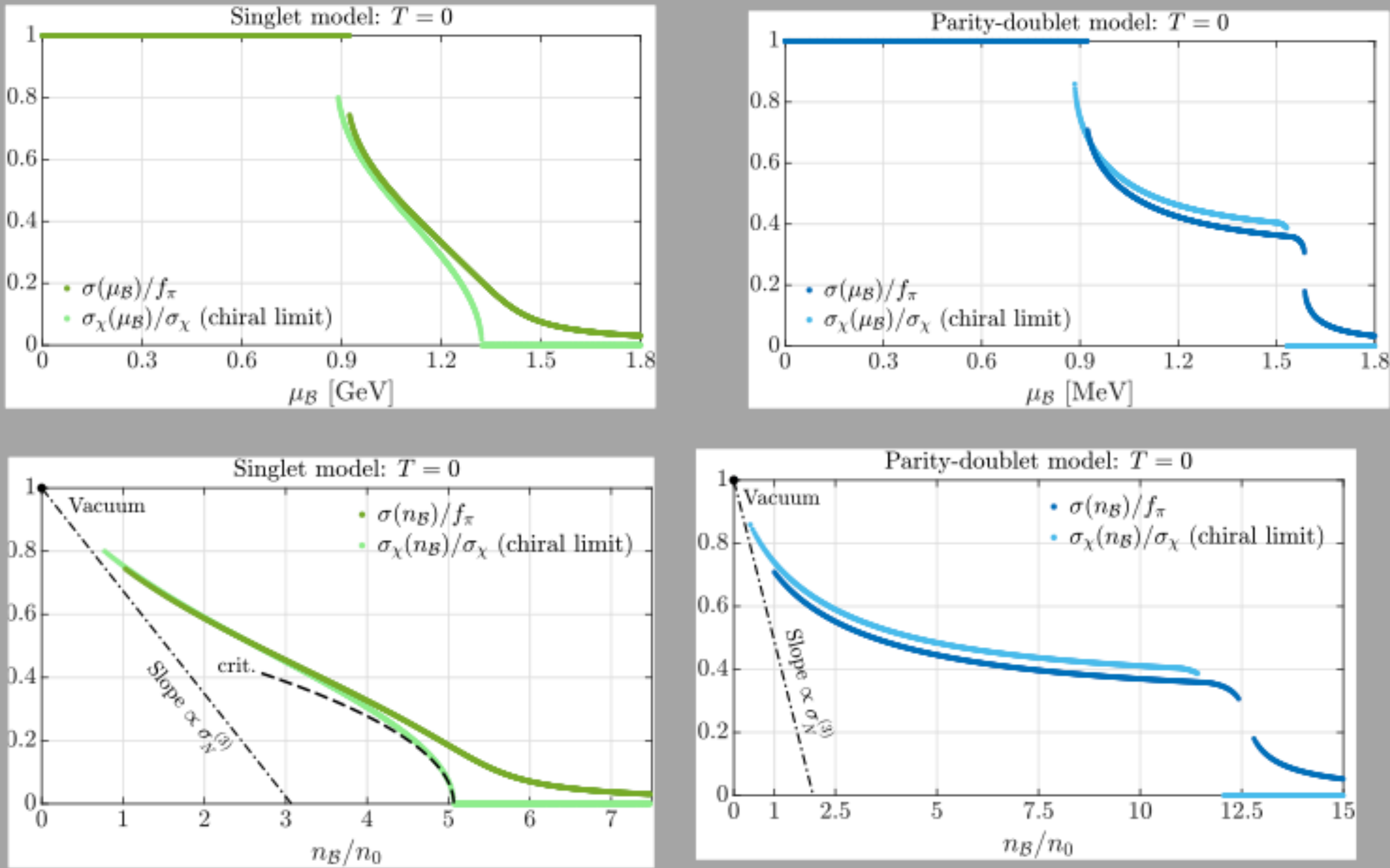


$$T_c = \sqrt{\frac{3r}{y^2}} \approx 249 \text{ MeV}$$

$$\sigma \simeq \left(\frac{4y^2 r}{3\tilde{u}^2}\right)^{1/4} \sqrt{T_c - T}$$

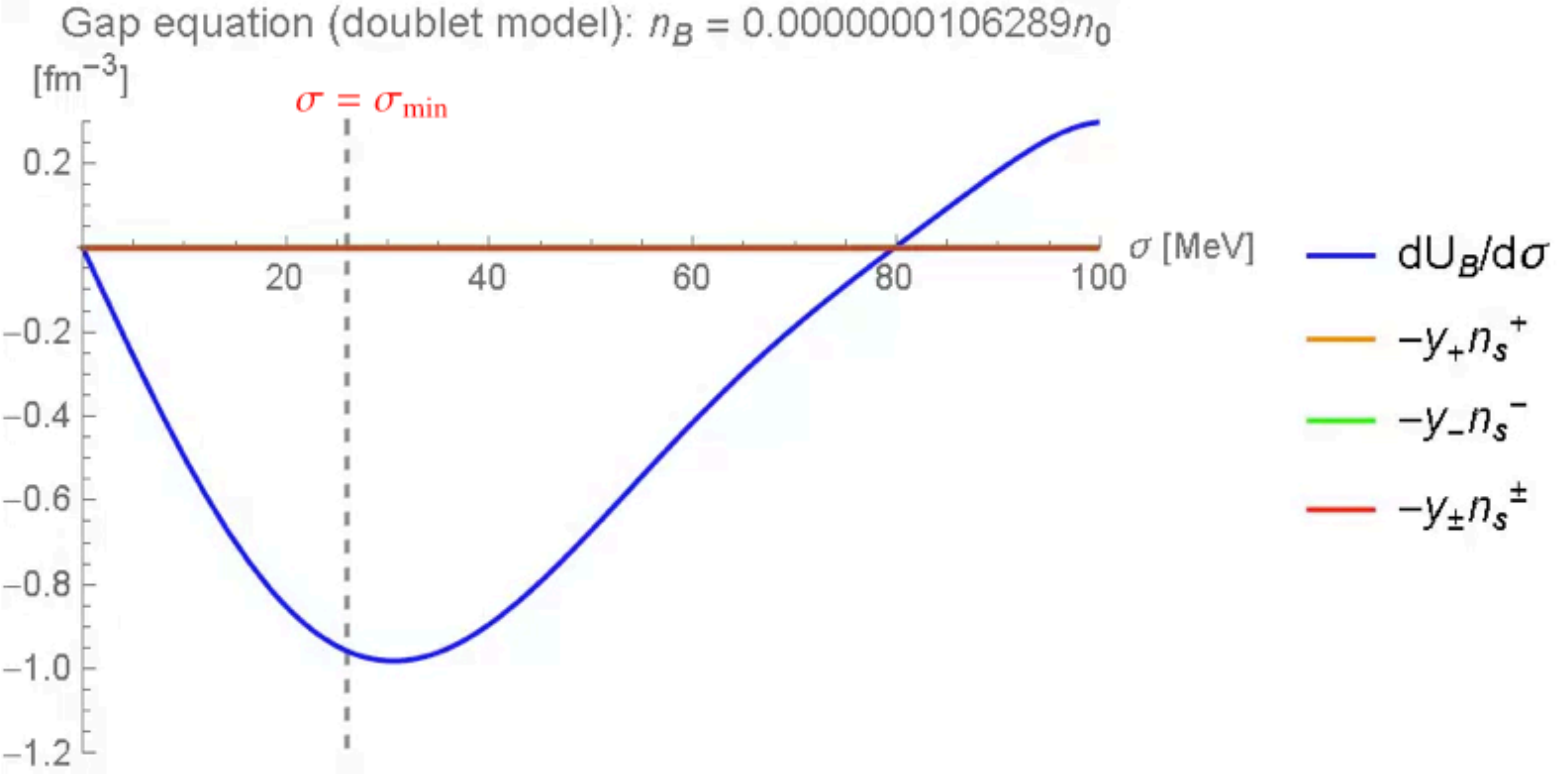
The chiral transition at T=0

($\mu_B \neq 0$)



The gap equation (doublet model)

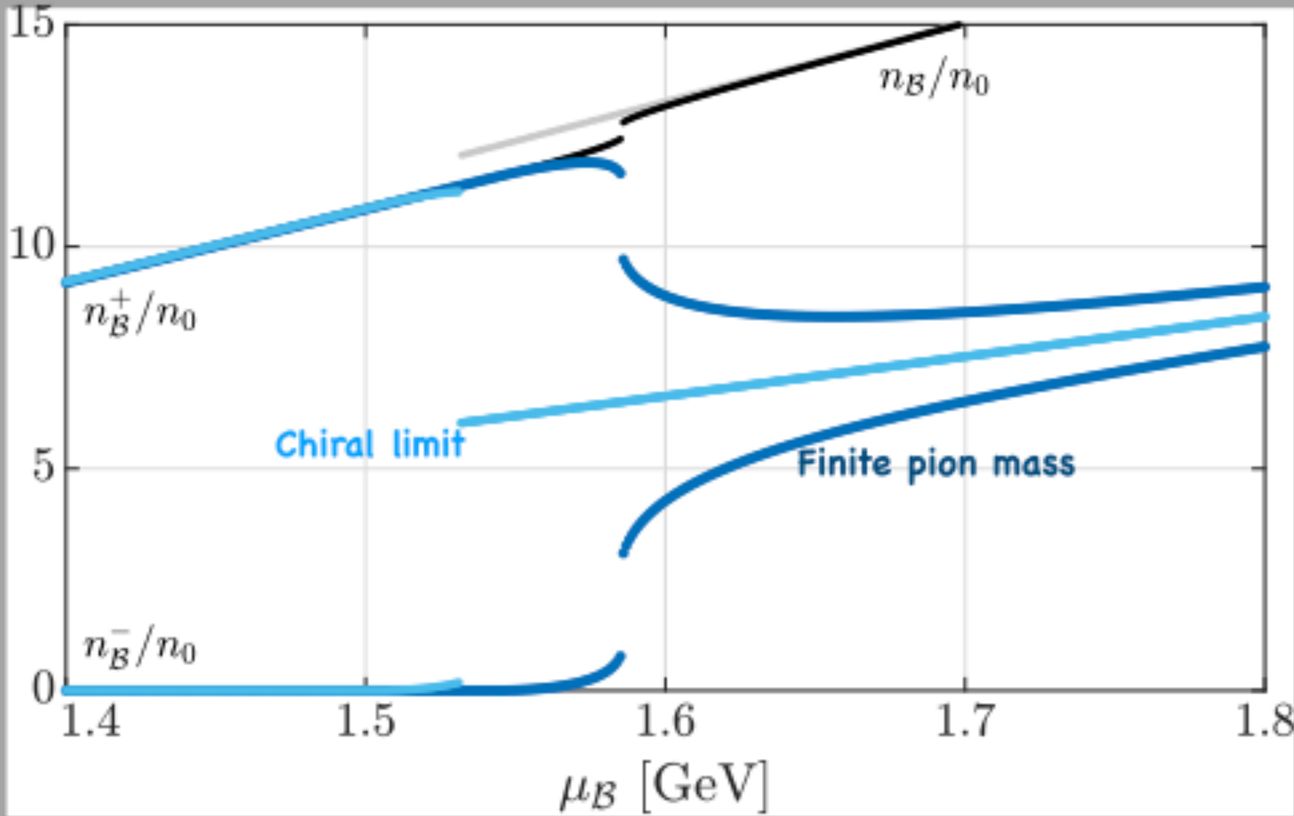
$$\left.\frac{\partial \mathcal{E}}{\partial \sigma}\right|_{n_{\mathcal{B}}^{+}, n_{\mathcal{B}}^{-}} = 0 = y_{+} n_{\mathrm{s}}^{+} + y_{-} n_{\mathrm{s}}^{-} + \frac{\mathrm{d}U}{\mathrm{d}\sigma}$$



Important features of the transition

- Rapid evolution of the populations of B+ and B- baryons
- Slow decrease of the scalar field with increasing density (role of m0)
- The minimum of the B+ spectrum $\sigma_{\min}$

Evolution of the populations

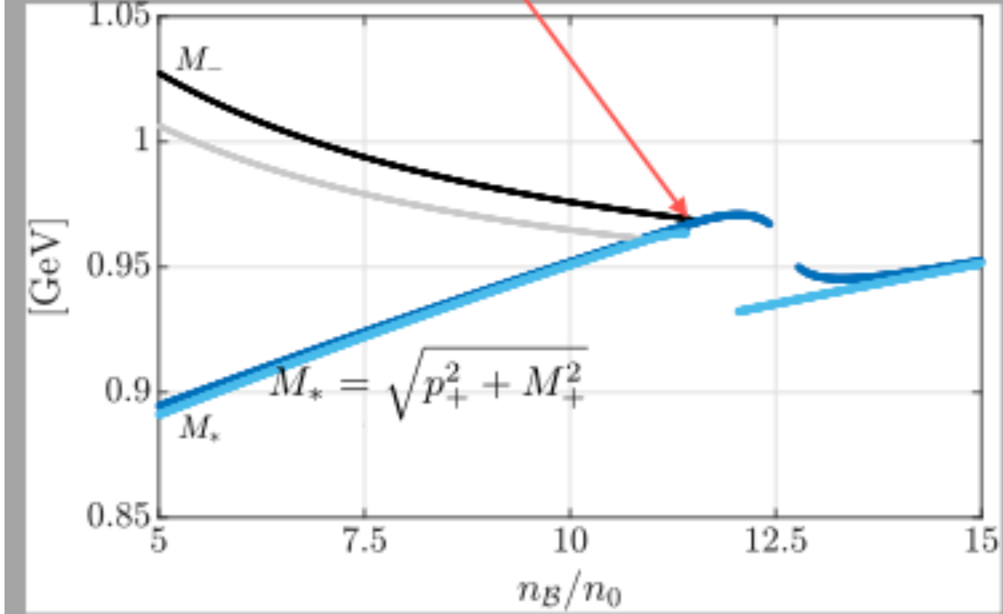


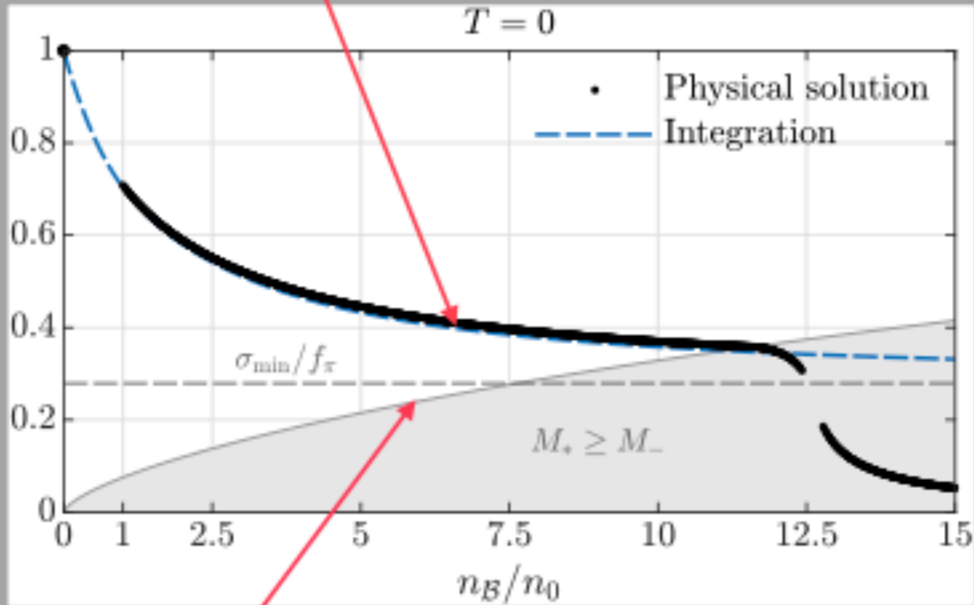
The B- threshold

gap equation as a differential equation:

$$M_- = \sqrt{p_F^2 + M_+^2}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}n_\mathcal{B}} \simeq -y_+ \left(\frac{\mathrm{d}^2 U}{\mathrm{d}\sigma^2} + n_\mathcal{B}\frac{\mathrm{d}^2 M^+}{\mathrm{d}\sigma^2}\right)^{-1}$$



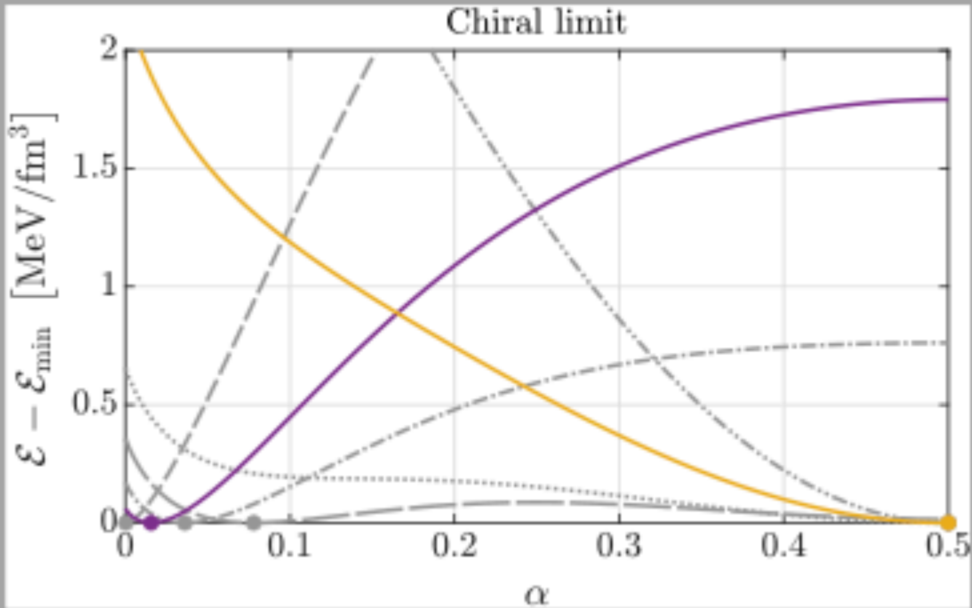


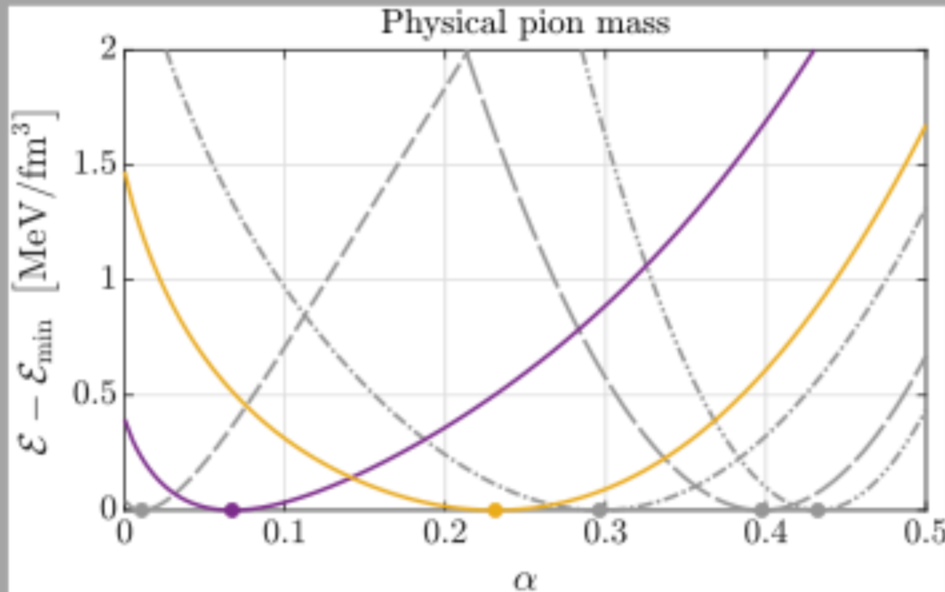
$$p_F^2 = \sigma(y_b - y_a)\sqrt{\sigma^2(y_a + y_b)^2 + 4m_0^2}.$$

The "symmetry energy"

$$\mathcal{E}(n_\mathcal{B}^+, n_\mathcal{B}^-; \sigma) = \mathcal{E}_{\rm qp}^+(n_\mathcal{B}^+; \sigma) + \mathcal{E}_{\rm qp}^-(n_\mathcal{B}^-; \sigma) + \frac{1}{2} G_v n_\mathcal{B}^2 + U(\sigma)$$

$$\alpha = \frac{n_\mathcal{B}^-}{n_\mathcal{B}} \qquad\qquad n_\mathcal{B} = n_\mathcal{B}^+ + n_\mathcal{B}^-$$



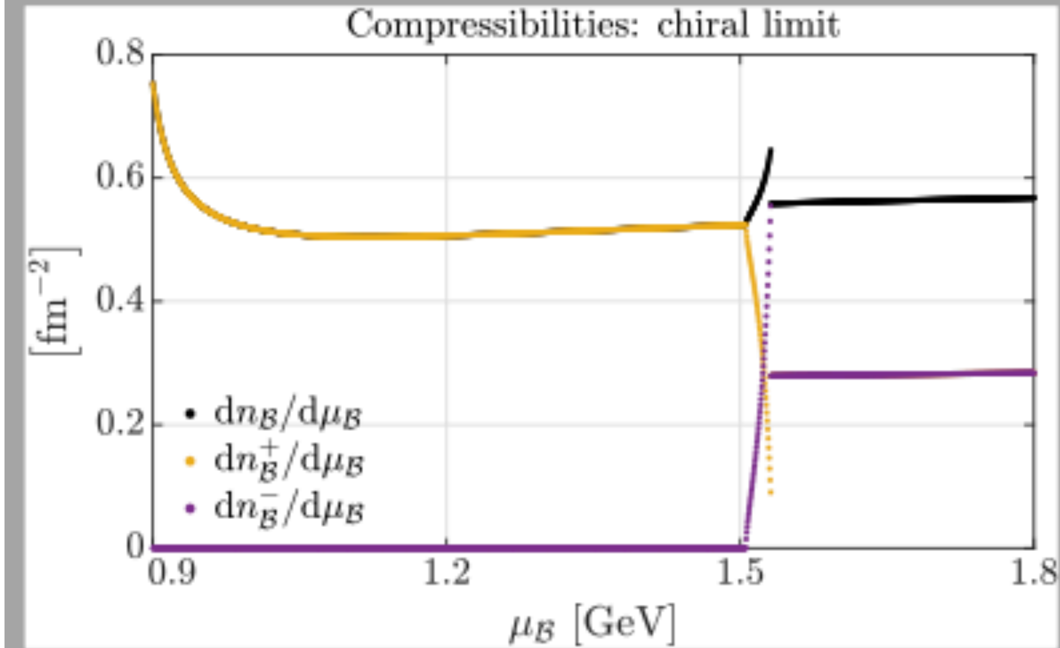


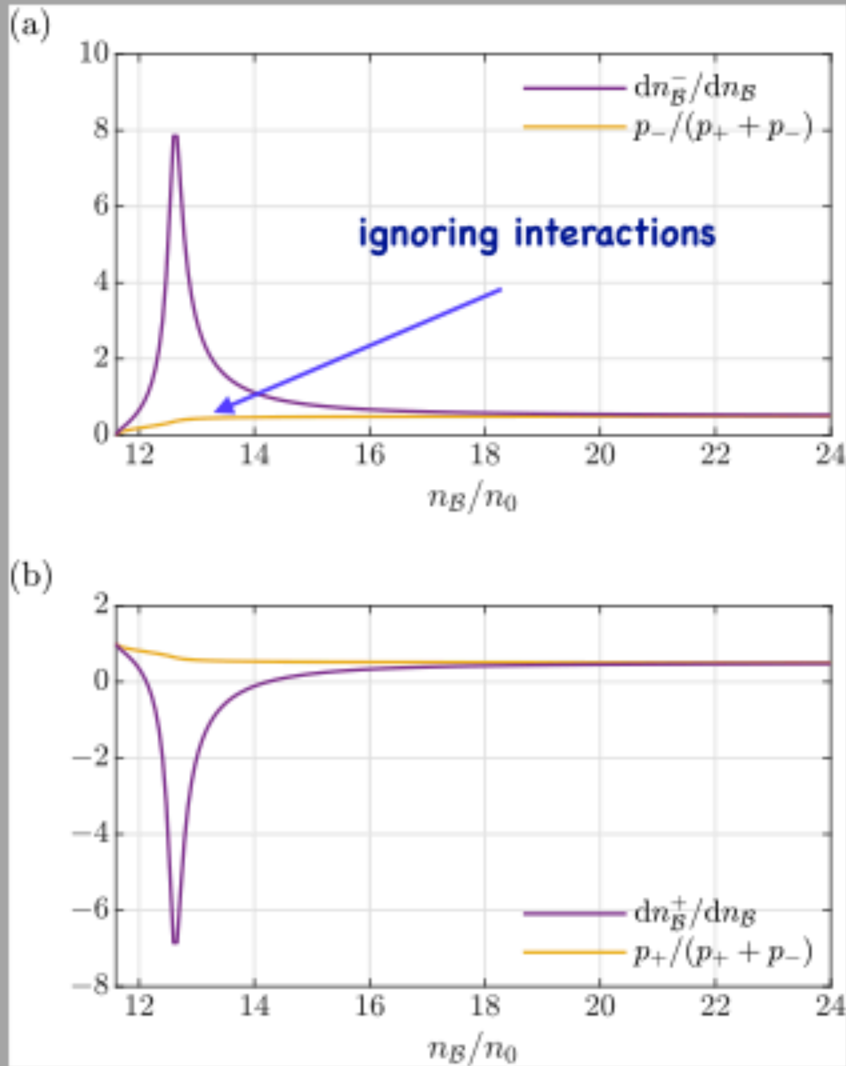
$$\delta^2\mathcal{E} \simeq \frac{1}{2}\left[\frac{1}{N_0^+} + \frac{1}{N_0^-} - \frac{(y_+M_+ - y_-M_-)^2}{m_\sigma^2 M_*^2}\right](\delta n_\mathcal{B}^-)^2$$

"Compressibilities"

$$\frac{\mathrm{d}n_{\mathcal{B}}^{+}}{\mathrm{d}n_{\mathcal{B}}} = \frac{\frac{1}{N_0^-} + \frac{y_- M_-}{m_\sigma^2 M_*^2}\left(y_+ M_+ - y_- M_-\right)}{\frac{1}{N_0^+} + \frac{1}{N_0^-} - \frac{1}{m_\sigma^2 M_*^2}\left(y_+ M_+ - y_- M_-\right)^2}$$

$$N_0^{\pm} = \frac{2 p_\pm M_*}{\pi^2}$$





Solving flow equations as an alternative to the gap equation

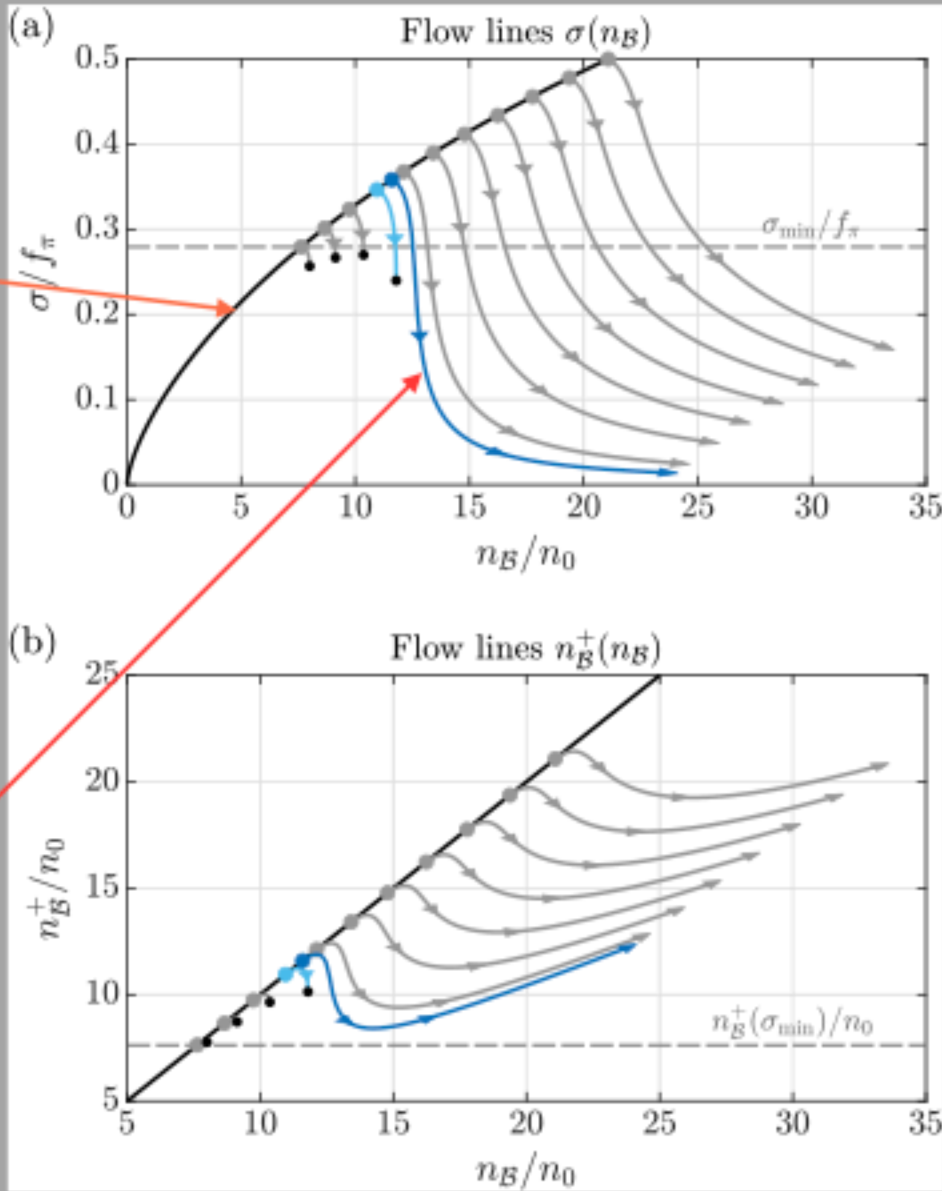
Before B- threshold

$$\frac{\mathrm{d}\sigma}{\mathrm{d}n_{\mathcal{B}}} = -\frac{y_+ M_+}{m_\sigma^2 M_*}$$

After B- threshold

$$\frac{\mathrm{d}\sigma}{\mathrm{d}n_{\mathcal{B}}} = -\frac{1}{m_\sigma^2 M_*}\left(y_+ M_+ \frac{\mathrm{d}n_{\mathcal{B}}^+}{\mathrm{d}n_{\mathcal{B}}} + y_- M_- \frac{\mathrm{d}n_{\mathcal{B}}^-}{\mathrm{d}n_{\mathcal{B}}}\right).$$

$$\frac{\mathrm{d}n_{\mathcal{B}}^+}{\mathrm{d}n_{\mathcal{B}}} = \frac{\frac{1}{N_0^-} + \frac{y_- M_-}{m_\sigma^2 M_*^2}\left(y_+ M_+ - y_- M_-\right)}{\frac{1}{N_0^+} + \frac{1}{N_0^-} - \frac{1}{m_\sigma^2 M_*^2}\left(y_+ M_+ - y_- M_-\right)^2}$$



Summary

- The parity doublet model, and its phase structure offers an interesting playground for many detailed calculations.
- Much can be learned by comparing singlet and doublet models. Also comparing physical pion mass with chiral limit.
- The dynamics of the chiral transition at finite density and zero temperature is very interesting.

BUT a model ... is a model

- Uncertainties in the phenomenological parametrisation of the effective potential for meson degrees of freedom.
- Correlation between the liquid gas transition and the chiral transition. Is that "physical"?
- Non linear effects are playing an important role (ambiguities in the value of the sigma term, long delays for the chiral transition, etc)
- Clearly input from QCD would be much needed