Quantum critical scaling of long-range systems and beyond

- Constructing effective theories of (universal) phenomena-

"Long-range interacting quantum systems", N. Defenu, et al. Rev. Mod. Phys. 95, 035002 (2023)

Why not this

[Submitted on 21 May 2020]

The fate of $O(N)$ multi-critical universal behaviour

Nicolò Defenu, Alessandro Codello

The multi-critical fixed points of $O(N)$ symmetric models cease to exist in the $N \to \infty$ limit, but the mechanism regulating their annihilation still presents several enigmatic aspects. Here, we explore the evolution of high-order multi-critical points in the (d, N) plane and uncover a complex mosaics for their asymptotic behaviour at large N. This picture is confirmed by various RG approaches and constitutes a fundamental step towards the full comprehension of critical behaviour in $O(N)$ field theories.

Comments: 7 pages, 2 figures
Subjects: **Statistical Mechanics (cond-mat.stat-mech)**; High Energy Physics - Theory (hep-th)
Cite as: arXiv:2005.10827 **[cond-mat.stat-mech]**
(or arXiv:2005.10827v1 **[cond-mat.stat-mech]** for this version)
https://doi.org/10.48550/arXiv.2005.10827

Submission history

From: Nicolo Defenu Dr. [view email]
[v1] Thu, 21 May 2020 17:59:52 UTC (229 KB)

Long-range quantum Ising model

$$H = -\sum_{l<j} J_{ij}\sigma_l^x\sigma_j^x - h\sum_j \sigma_j^z,$$

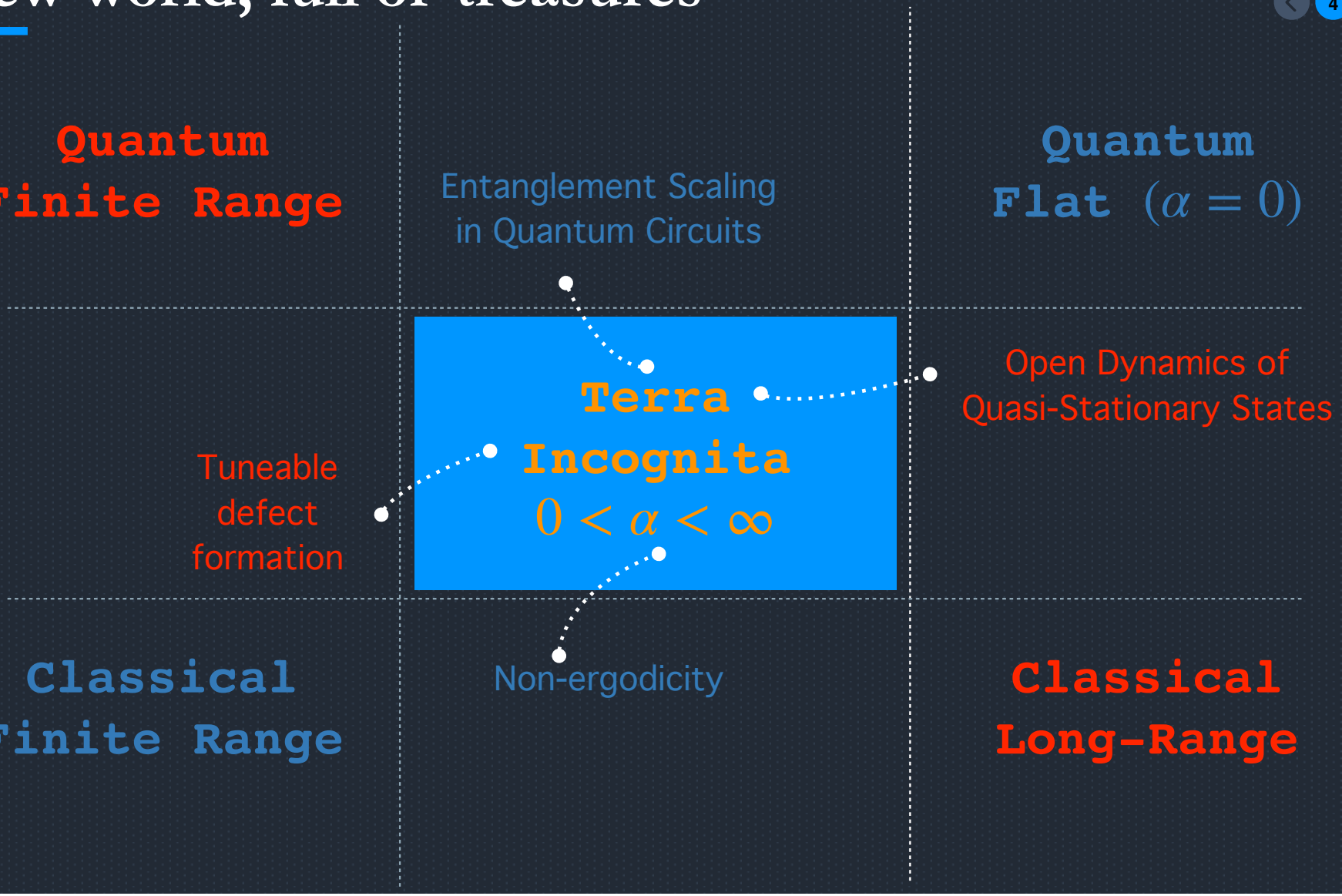
Long-range interactions: $J_{ij} = \frac{1}{N_\alpha}\frac{1}{|i-j|^\alpha}$ $\qquad N_\alpha^{-1} \approx \begin{cases} (1-\alpha)2^{(1-\alpha)}N^{\alpha-1} & \text{if } \alpha < 1 \\ 1/\log(N) & \text{if } \alpha = 1 \\ 1/\zeta(\alpha) & \text{if } \alpha > 1. \end{cases}$

$$J \gg h \;\Rightarrow\; |0\rangle \equiv \Pi_i |\uparrow\rangle_i \text{ or } \Pi_i |\downarrow\rangle_i \text{ and } \lim_{|i-j|\to\infty} \langle \sigma_i^z \sigma_j^z \rangle \propto N_0^2$$

$$J \ll h \;\Rightarrow\; |0\rangle \equiv \Pi_i |\rightarrow\rangle_i \text{ with } |\rightarrow\rangle \equiv |\uparrow\rangle + |\downarrow\rangle \text{ and } \lim_{|i-j|\to\infty} \langle \sigma_i^z \sigma_j^z \rangle \propto e^{-\frac{|i-j|}{\xi}}$$

Quantum Critical Point at $h_c > 0$

A new world, full of treasures



Spin-wave approximation

$$H = -\sum_{l<j} J_{ij}\sigma_l^x\sigma_j^x - h\sum_j \sigma_j^z,$$

Holstein-Primakoff transformation

$$\sigma_\ell^x \simeq \frac{a_\ell + a_\ell^\dagger}{2} \qquad \sigma_\ell^y \simeq \frac{a_\ell - a_\ell^\dagger}{2i} \qquad \sigma_\ell^z = \frac{1}{2} - a_\ell^\dagger a_\ell$$

$$H = -\sum_{l<j} J_{ij}(a_i^\dagger a_j + h.c.) + h\sum_j a_j^\dagger a_j,$$

Long-range interactions

$$J_k = \sum_{\ell=1}^{N} \frac{\cos(k\ell)}{\ell^{\alpha}} \approx \mathrm{Li}_{\alpha}\left(e^{ik}\right) + \mathrm{Li}_{\alpha}\left(e^{-ik}\right)$$

$$\lim_{k\to 0} V_k = h_c + c\,k^{\sigma} + O(k^2), \quad \sigma = \alpha - 1$$

Continuous limit: $\lim_{k\to 0} \omega_k = ck^{\sigma} - \mu$

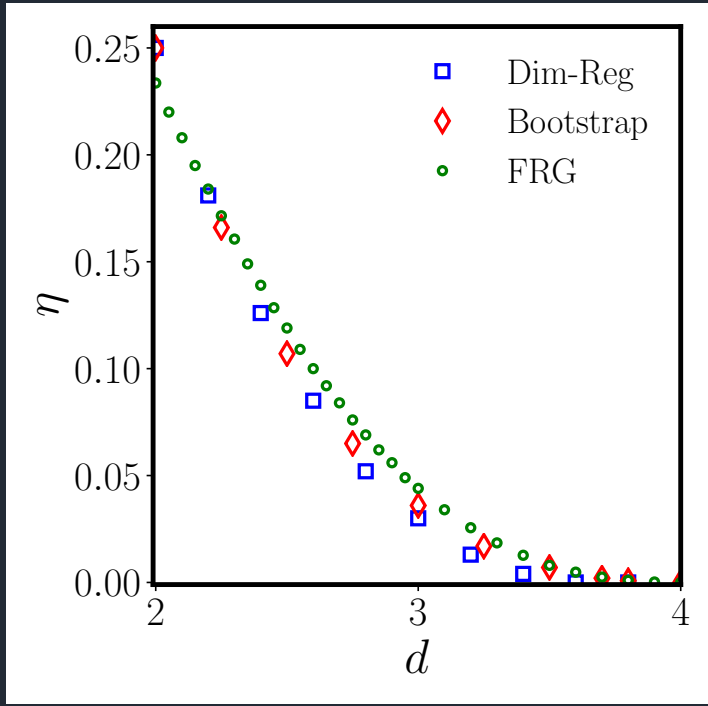
Bose gas with non-analytic dispersion relation

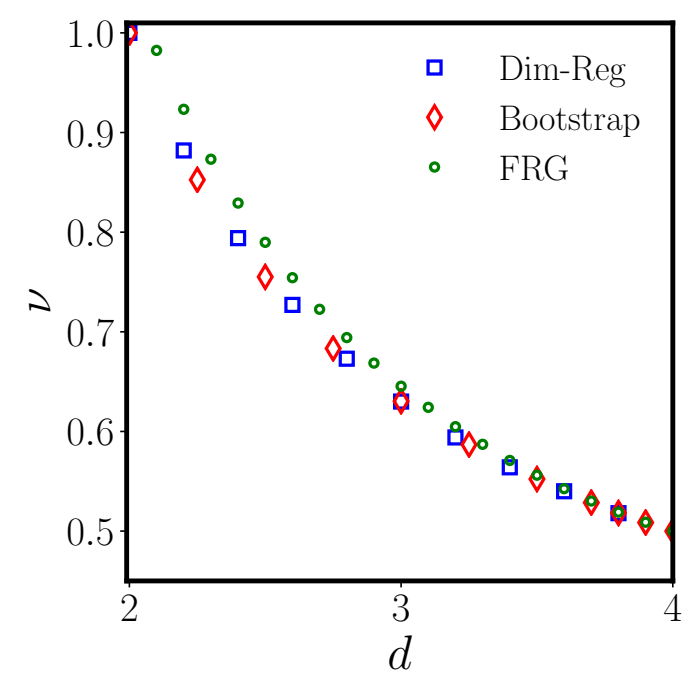
φ^4 Theories in fractional dimension

$$H = \int d^d x \left\{ \partial_\mu \psi \partial_\mu \psi + m\psi^2 + g\psi^4 \right\}$$

$$\xi \propto \varepsilon^{-\nu}$$

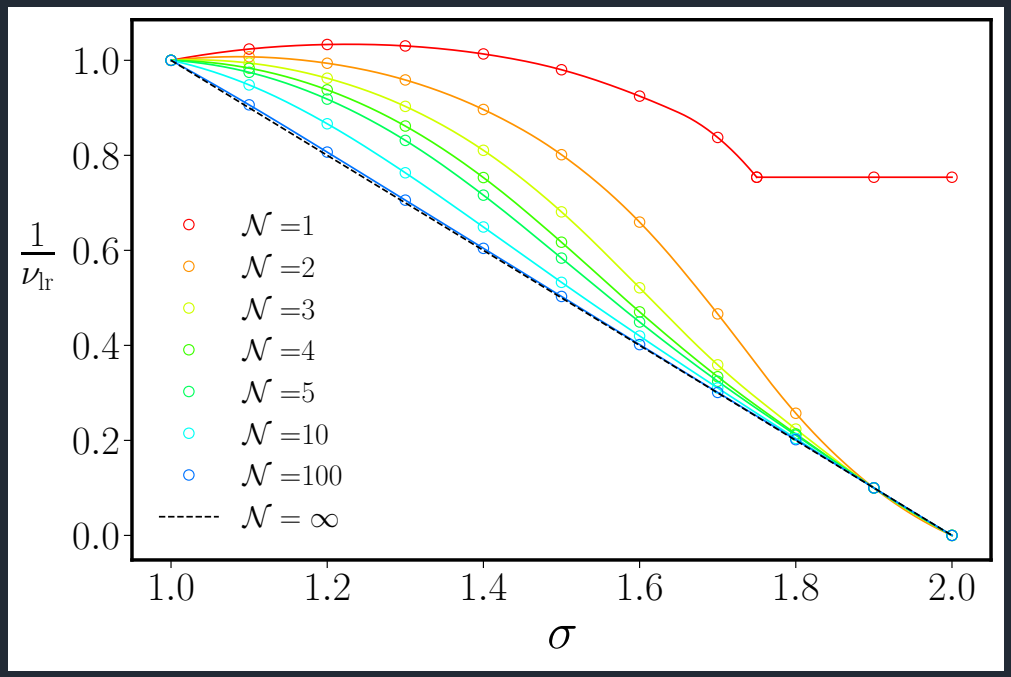
$$\langle \psi(0)\psi(r) \rangle \propto r^{d-2+\eta}$$

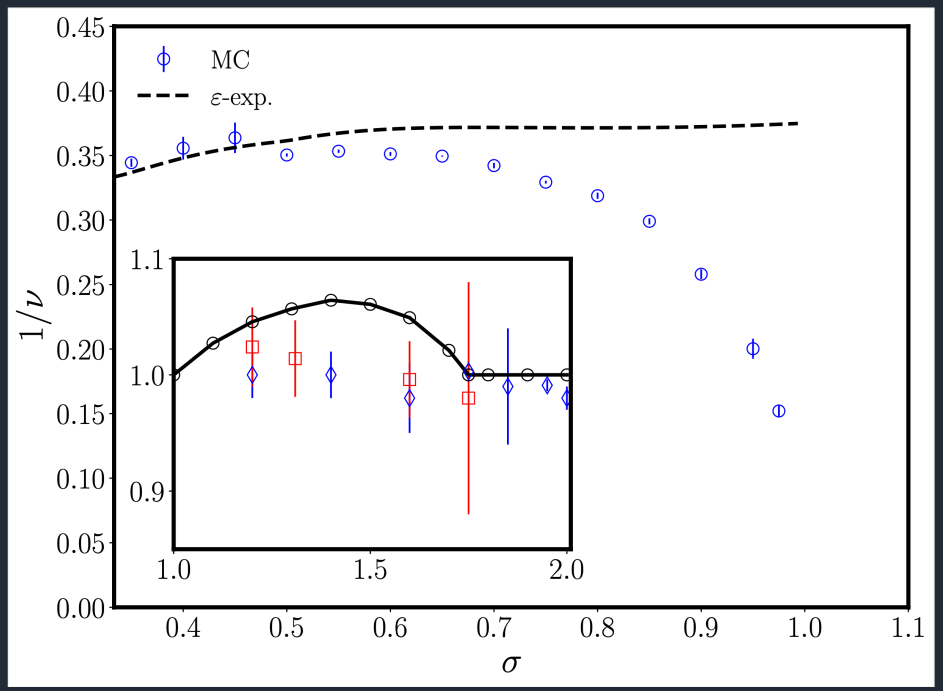




1) Y. Holovatch, *Theor. Math. Phys.* **96**, 1099–1109 (1993).
2) S. *El-Showk et al., J. Stat. Phys.* ***157,*** *869–914 (2014).*
3) A. Codello, ND, G. D'Odorico, Phys. Rev. D ***91****, 105003 (2015)*

What is the meaning of non-integer dimensions?





- ND, A Trombettoni, A Codello, Phys. Rev. E 92, 052113 (2015)
- ND, et al., J. High Energ. Phys. 2015, 141 (2015).
- ND, A Trombettoni, S Ruffo, Phys. Rev. B 94, 224411 (2016)
- ND, A Trombettoni, S Ruffo, Phys. Rev. B 96, 104432 (2017).

New tools are needed

Theoretical understanding is limited as the scaling of couplings becomes super-extensive

All to all couplings severely reduce numerical performance

The Spectral Dimension

Vibrational spectrum

At low frequencies the vibrational spectrum of coupled oscillators on a network obeys:

$$\rho(\omega) \propto \omega^{d_s - 1}$$

Random Walk Return Rates

A random walker hopping on networks will return at its starting point with probability:

$$P_0(t) \propto t^{-d_s/2}$$

Mermin-Wagner Theorem

Spontaneous symmetry breaking of continuous symmetries is forbidden for

$$d_s \leq 2$$

Does it control universal properties?

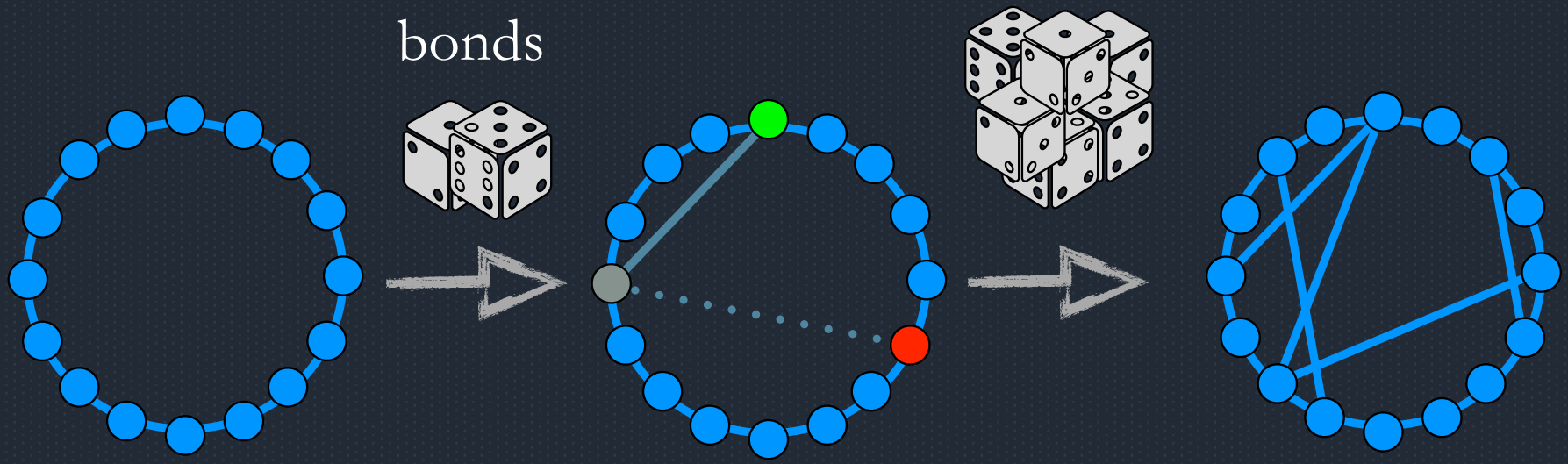
What about discrete symmetries?

1) R. Burioni, D. Cassi, *Phys. Rev. Lett.* ***76****, 1091 (1996).*
2) R. Burioni, D. Cassi, A. Vezzani, *Phys. Rev.* ***E 60****, 1500 (1999).*

My solution

Additional random bonds

~N new bonds



Regular lattice in d-dimension

$$p \sim \frac{1}{r_{ij}^{\alpha}}$$

Self-averaging graph structure

A. P. Millán, G. Gori, F. Battiston, T. Enss, N. Defenu, *Phys. Rev. Res.* 3, 023015 (2021).

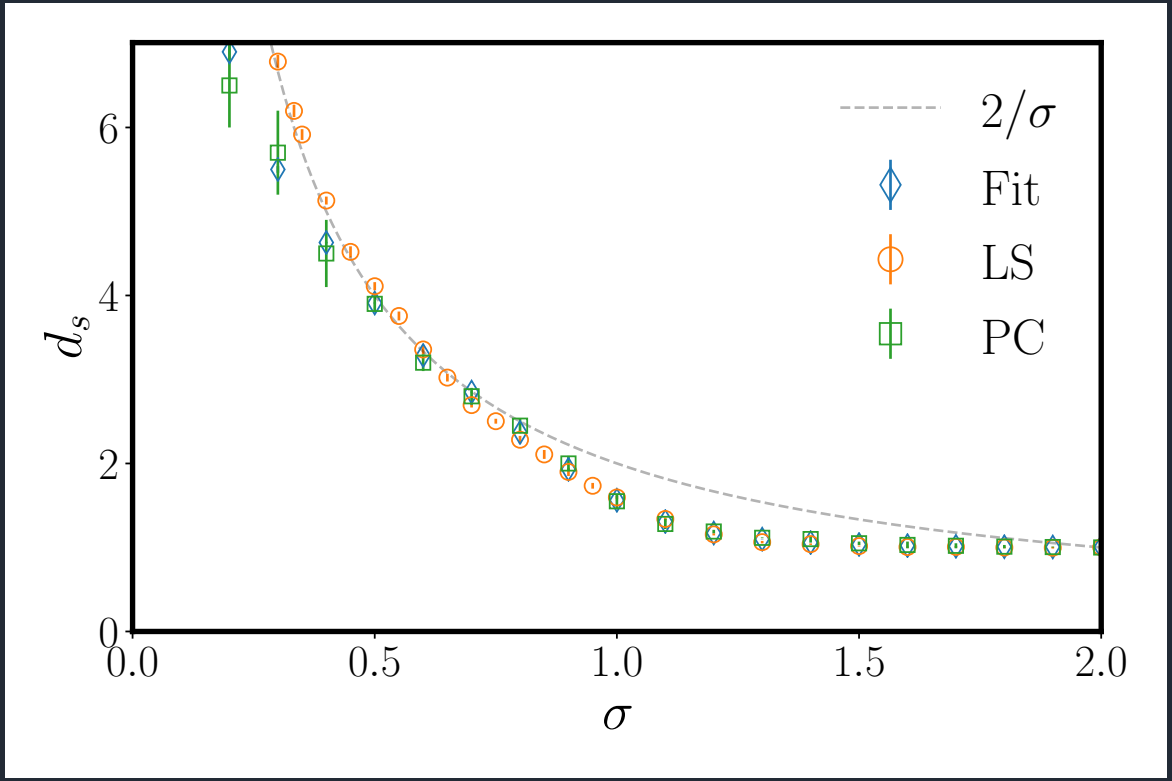
Complex networks with tuneable dimension

Collapse of the return probabilities

The average of the random walker return rates for several network realisations with different number of sites are collapsed to obtain an estimation of the spectral dimension.

Scaling of the smallest eigenvalues

The power law scaling of the first eigenvalue of the laplacian operator as a function of the network size is studied for large network sizes.



Research funded by the STRUCTURES collaboration under exploratory project (EP) 2.3: “Universality on Network Structures from Quantum Dynamics to Big Data”. PI: ND

Anomalous Dimension?

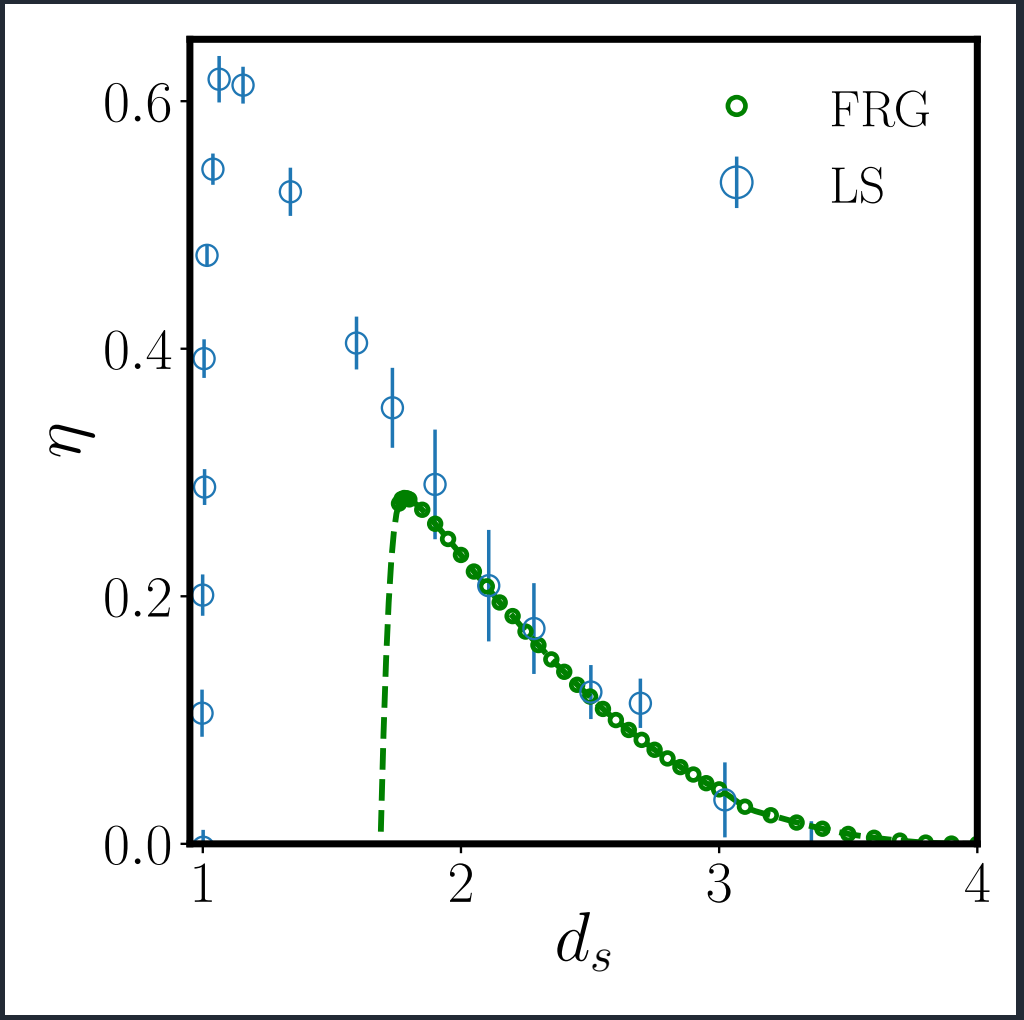
Anomalous Dimension of Ising model

Calculated by the local potential approximation of Functional RG.

Correction to the spectral dimension of QLR-Net

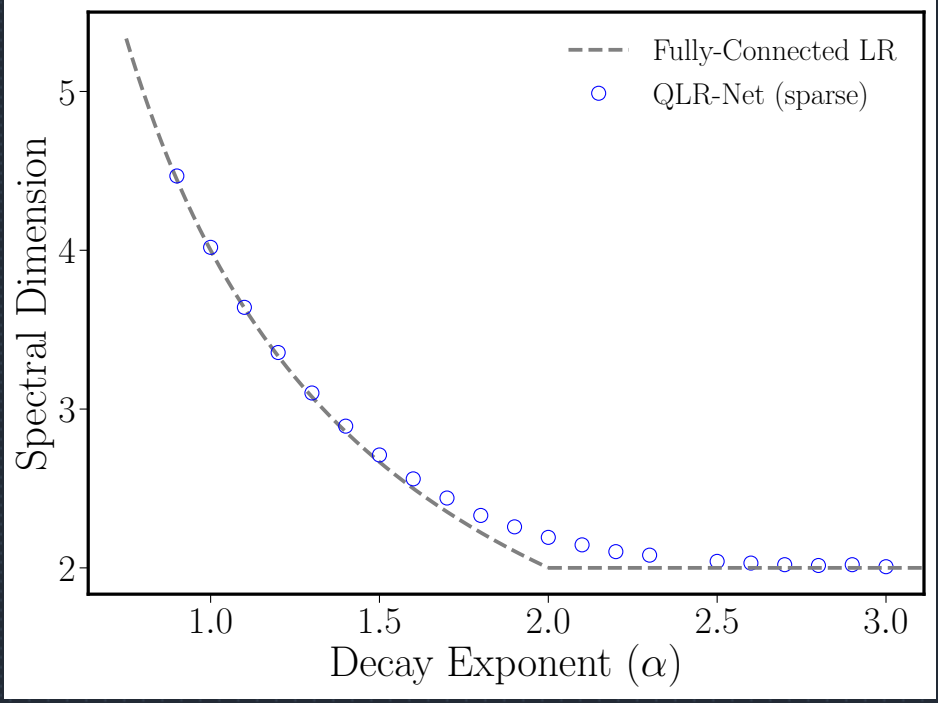
The green points have been obtained by the formula:

$$d_s = (2 - \eta_{\text{graph}})/\sigma$$

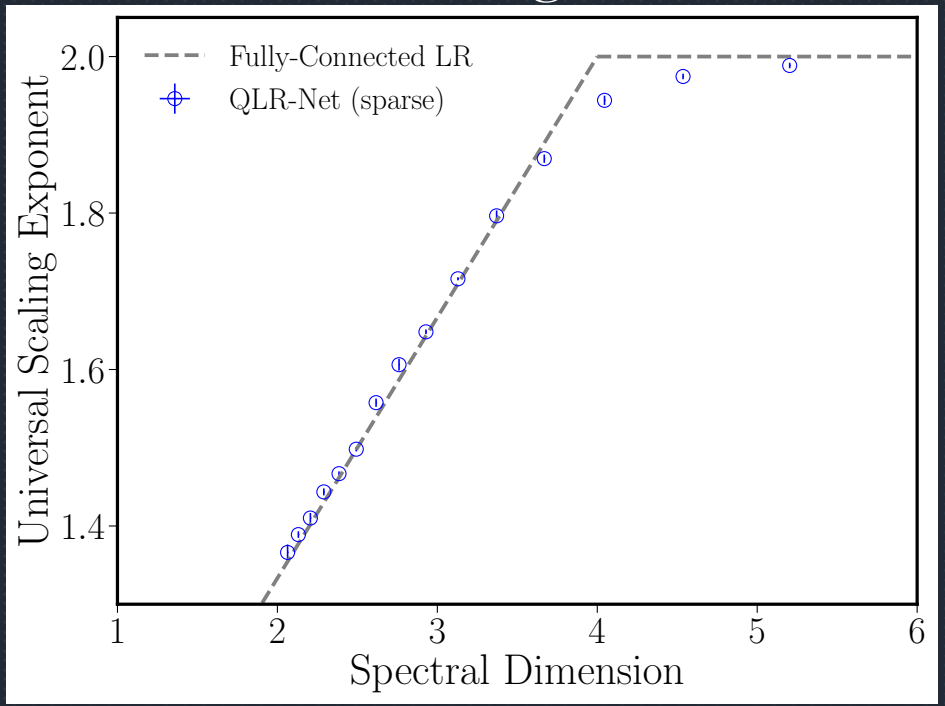


The universality conjecture

Match the low-energy spectra

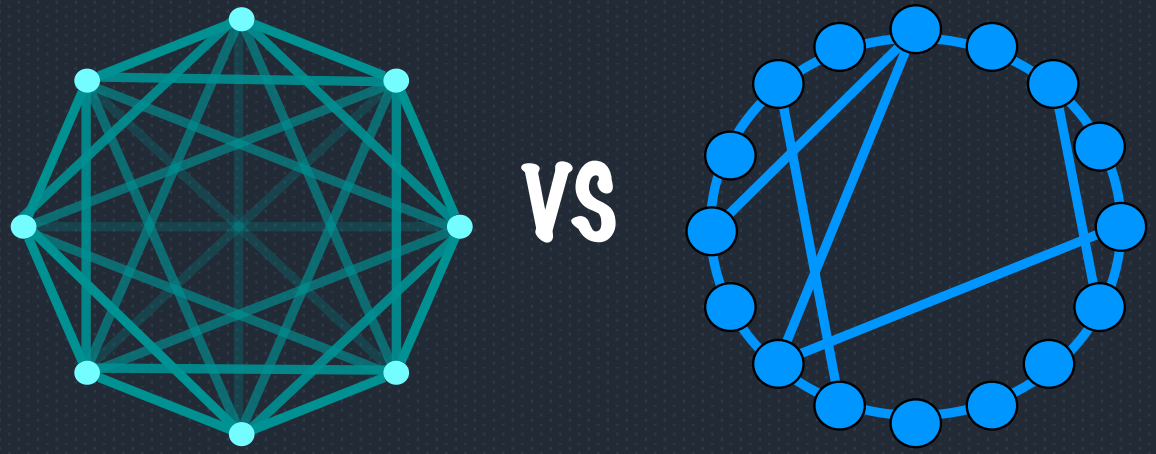


Reproduce the universal scaling



G. Bighin, T. Enss, N. Defenu, arXiv:2211.13302 (2022).

My goal



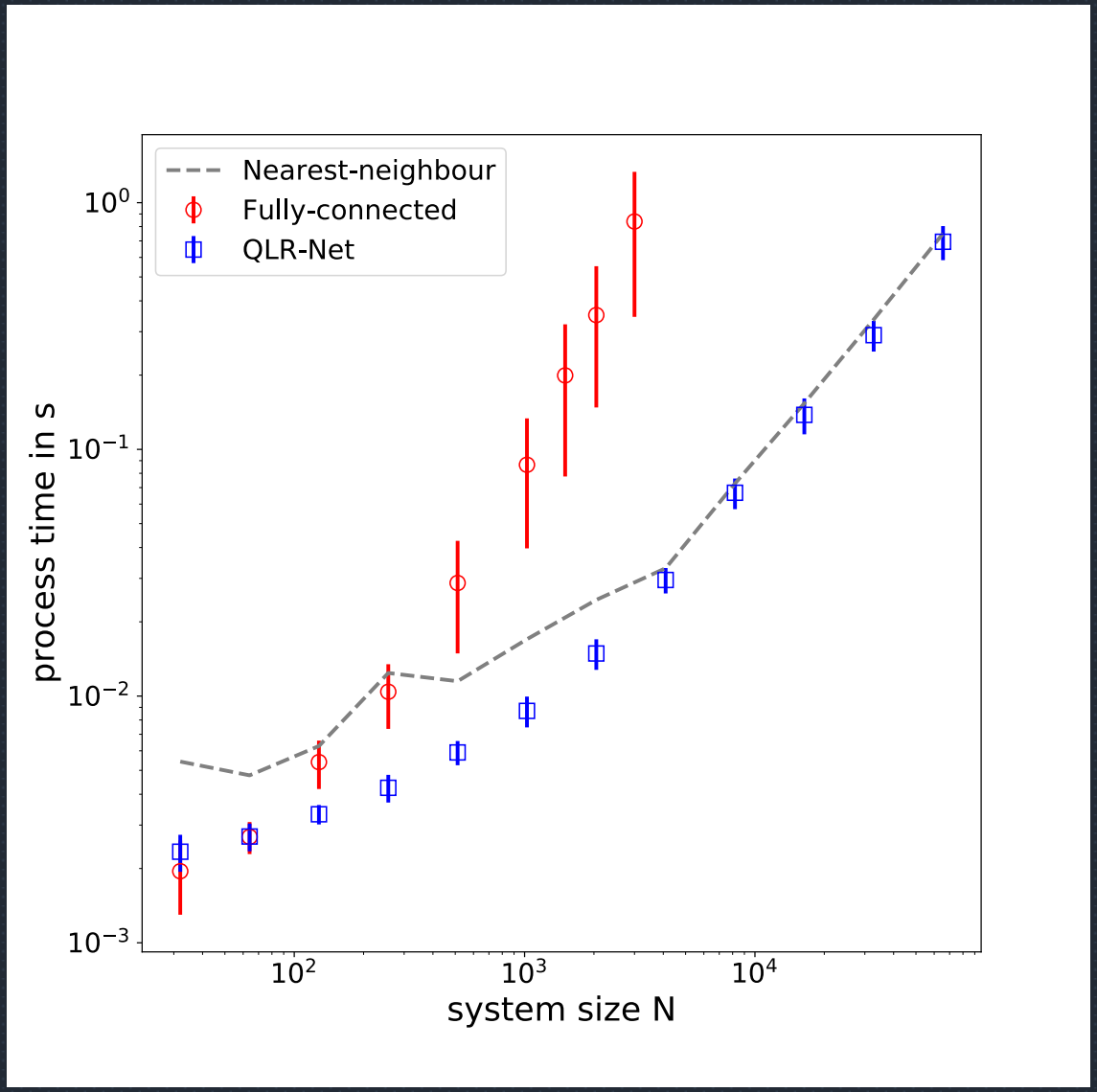
Fully connected

Sparse

Advantages:

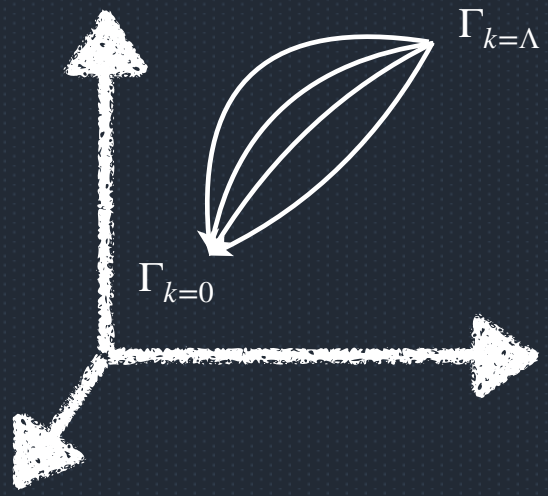
- Fast entanglement spreading inherited by long-range couplings
- Amenable numerics thanks to sparse coupling matrices
- Universal tool to describe long-range interacting systems

Speed up


Nearest-neighbour
Fully-connected
QLR-Net
process time in s
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{2}
10^{3}
10^{4}
system size N

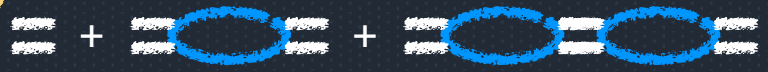
Methodology

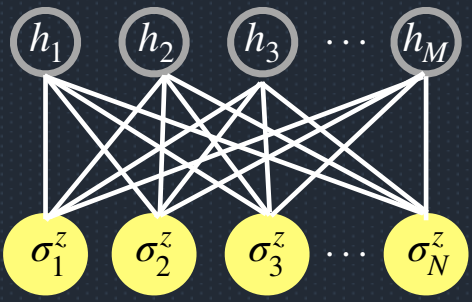
(Out-of-equilibrium)
RG study at $\alpha > d$



Synergy of theoretical tools

$1/N$-expansion at $\alpha < d$



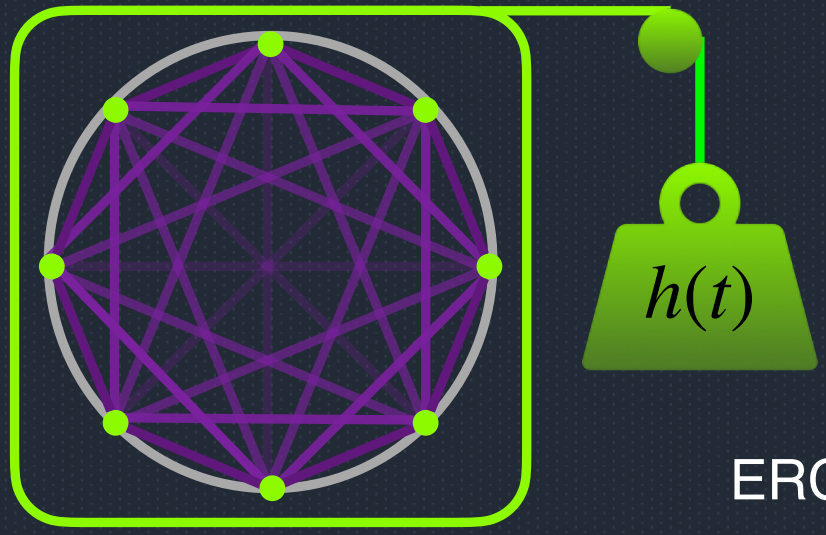


(Variational) Quantum MC
simulations at all α

Entering a new era of quantum information

Manipulate fundamental physics laws via complex geometric structures

Tuneable Spectrum + Strong Interactions



Novel fundamental physics phenomena

Novel proposals for Quantum Technologies

ERC-Stg QLR-Net (2022)

Opinionism

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CORRESPONDENCE | 28 March 2023

Funding bias: nurture European researchers' independence

Sandeep Chowdhary, Nicolò Defenu, Federico Musciotto & Federico Battiston

We are concerned over indicators that research funded by the European Union relies disproportionately on US collaborations. Such a bias risks compromising the independence of European researchers.

ERG 2024

SRS
SwissMAP Research Station in Les Diablerets

12th International Conference on the Exact Renormalization Group 2024 (ERG2024)

22–27 Sept 2024
Maison des Congrès
Europe/Zurich timezone

Overview
Participant List

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The ERG conferences, held every second year, are intended to bring together researchers from different branches of theoretical physics who apply modern functional renormalization group (fRG) methods to describe and understand a variety of physical phenomena. For example, applications include quantum many-body systems, statistical mechanics, particle physics, nuclear physics and quantum theories of gravity. The conference is intended to foster collaboration and scientific exchange both within the growing fRG community and among the various communities working on non-perturbative methods, renormalizationgroup (RG) approaches, and modern developments in field-theoretical methods. To this end, besides providing ample space for a detailed account of the recent developments of the fRG method and its applications, the event will also host plenary review talks on different research directions of topical interest for the contemporary theoretical-physics community.

Starts 22 Sept 2024, 19:00
Ends 27 Sept 2024, 14:00
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Andrea Trombettoni

Giacomo Gori

Federico Battiston

Starting Grant: Quantum Long-Range Networks

Project Funding [200021_207537]

Thank you