Line of fixed points in Gross-Neveu theories

Charlie Cresswell-Hogg

based on 2207.10115, 2212.06815 & ongoing work with Daniel F. Litim

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Functional methods for strongly correlated systems 2023, Hirschegg

UNIVERSITY OF SUSSEX

Gross-Neveu

N Dirac fermions in *d* Euclidean dimensions

$$S = \int_x \left\{ \bar{\psi}_a \not{\partial} \psi_a + \tfrac{1}{2} G (\bar{\psi}_a \psi_a)^2 \right\}$$

Gross-Neveu

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GN universality classes

Gross-Neveu

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interacting 3d CFTs

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interacting 3d CFTs

asymptotically…

- ***free*** *in d = 2*
- ***safe*** *in d = 3*

Gross, Neveu ’74
Gawędzki, Kupiainen ’85
Rosenstein, Warr, Park ‘88
de Calan, Faria Da Veiga, Magnen, Seneor ’91

Gross-Neveu

N Dirac fermions in *d* Euclidean dimensions

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$$\psi_a \to \gamma^5 \psi_a \quad \bar{\psi}_a \to -\bar{\psi}_a \gamma^5 \quad \bar{\psi}_a \psi_a \to -\bar{\psi}_a \psi_a$$

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$$S = \int_x \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + \frac{1}{2} G (\bar{\psi}_a \psi_a)^2 + \frac{1}{3!} H (\bar{\psi}_a \psi_a)^3 \right\}$$

$$\psi_a \to \gamma^5 \psi_a \quad \bar{\psi}_a \to -\bar{\psi}_a \gamma^5 \quad \bar{\psi}_a \psi_a \to -\bar{\psi}_a \psi_a$$

CCH, Litim, 2207.10115

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(No) bosonisation

$$\bar{\psi}_a \partial\!\!\!/ \psi_a + \frac{1}{2} G (\bar{\psi}_a \psi_a)^2$$

bosonisation ⟷

$$\bar{\psi}_a \partial\!\!\!/ \psi_a + \sigma \, \bar{\psi}_a \psi_a - \frac{1}{2G} \sigma^2$$

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$$\frac{1}{2} (\partial \phi)^2 + \bar{\psi}_a \partial\!\!\!/ \psi_a + Y \phi \, \bar{\psi}_a \psi_a + U(\phi)$$

Gross-Neveu-Yukawa

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RG flow ↑

$$\frac{1}{2} (\partial \phi)^2 + \bar{\psi}_a \partial\!\!\!/ \psi_a + Y \phi\, \bar{\psi}_a \psi_a + U(\phi)$$

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This talk: fermionic FRG

Jakovác, Patkós, ’13; Jakovác, Patkós, Pósfay ’14

$$\frac{1}{2} (\partial \phi)^2 + \bar{\psi}_a \partial\!\!\!/ \psi_a + Y \phi \, \bar{\psi}_a \psi_a + U(\phi)$$

Gross-Neveu-Yukawa

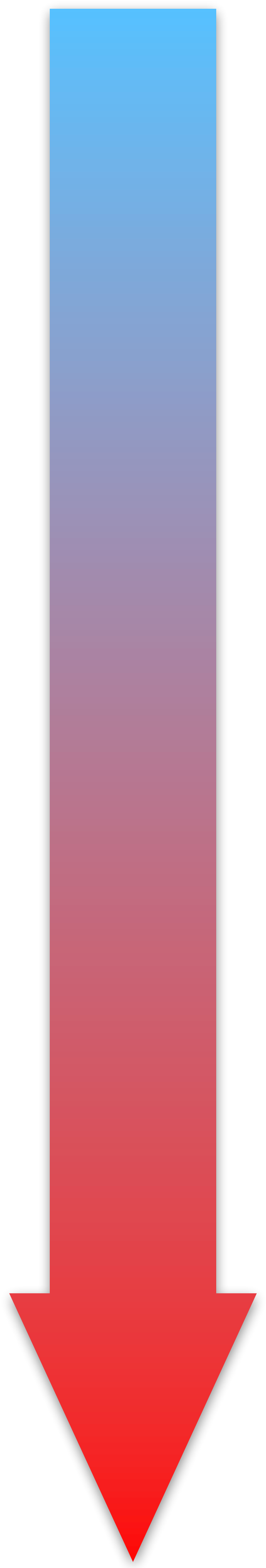
Map complete phase structure without chiral symmetry

Do this without introducing bosonic auxiliary fields

Verify correspondence with Yukawa models

Large N and the fermionic RG

UV



$$k = \Lambda : S_\Lambda$$

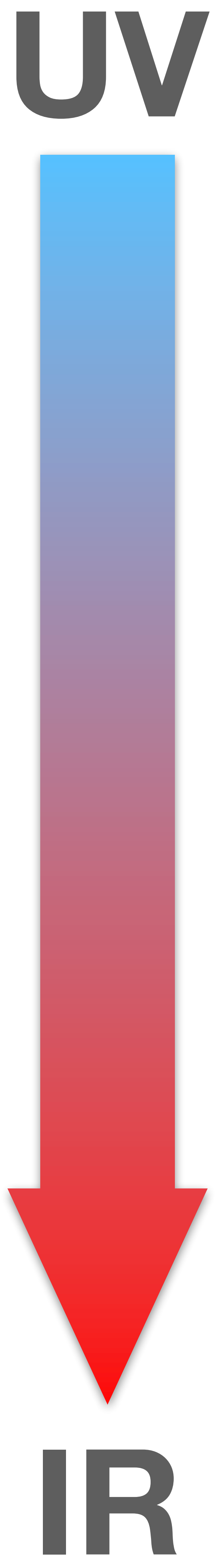
$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left\{ \left[\Gamma_k^{(2)} + R_k \right]^{-1} \cdot \partial_t R_k \right\}$$

Wetterich ’93; Morris ’94; Ellwanger ‘94

IR

$$k = 0 : \Gamma_{1\mathrm{PI}}$$

Large N and the fermionic RG



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Wetterich '93; Morris '94; Ellwanger '94

solve for effective action

$$k = 0 : \Gamma_{1\mathrm{PI}}$$

Large N and the fermionic RG

Exact solutions of form:

$$\Gamma_k[\psi,\bar{\psi}] = \int_x \bar{\psi}_a \partial\!\!\!/ \psi_a + F_k[\bar{\psi}_a \psi_a]$$

D'Attanasio, Morris '97; CCH, Litim, *in prep.*

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✓	✗
$f(\bar\psi_a\psi_a)$	$f(\bar\psi_a\psi_a)\,\bar\psi_b\partial\!\!\!/\psi_b$
$\partial_\mu(\bar\psi_a\psi_a)\partial^\mu(\bar\psi_b\psi_b)$	$(\bar\psi_a\partial\!\!\!/\psi_a)^2$

Large N and the fermionic RG

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All contributions to effective potential contained in:

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Large N and the fermionic RG

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fermionic local potential approximation

Jakovác, Patkós, '13; Jakovác, Patkós, Pósfay '14

Large N and the fermionic RG

$$\Gamma_k[\psi, \bar{\psi}] = \int_x \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + V_k(\bar{\psi}_a \psi_a) \right\}$$

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Exact flow for dimensionless potential:

$$\partial_t v = -dv + (d-1)\, z\partial_z v - \frac{1}{1+(\partial_z v)^2}$$

$$v(z,t) = k^{-d} V_k(k^{d-1} z)$$

Large N and the fermionic RG

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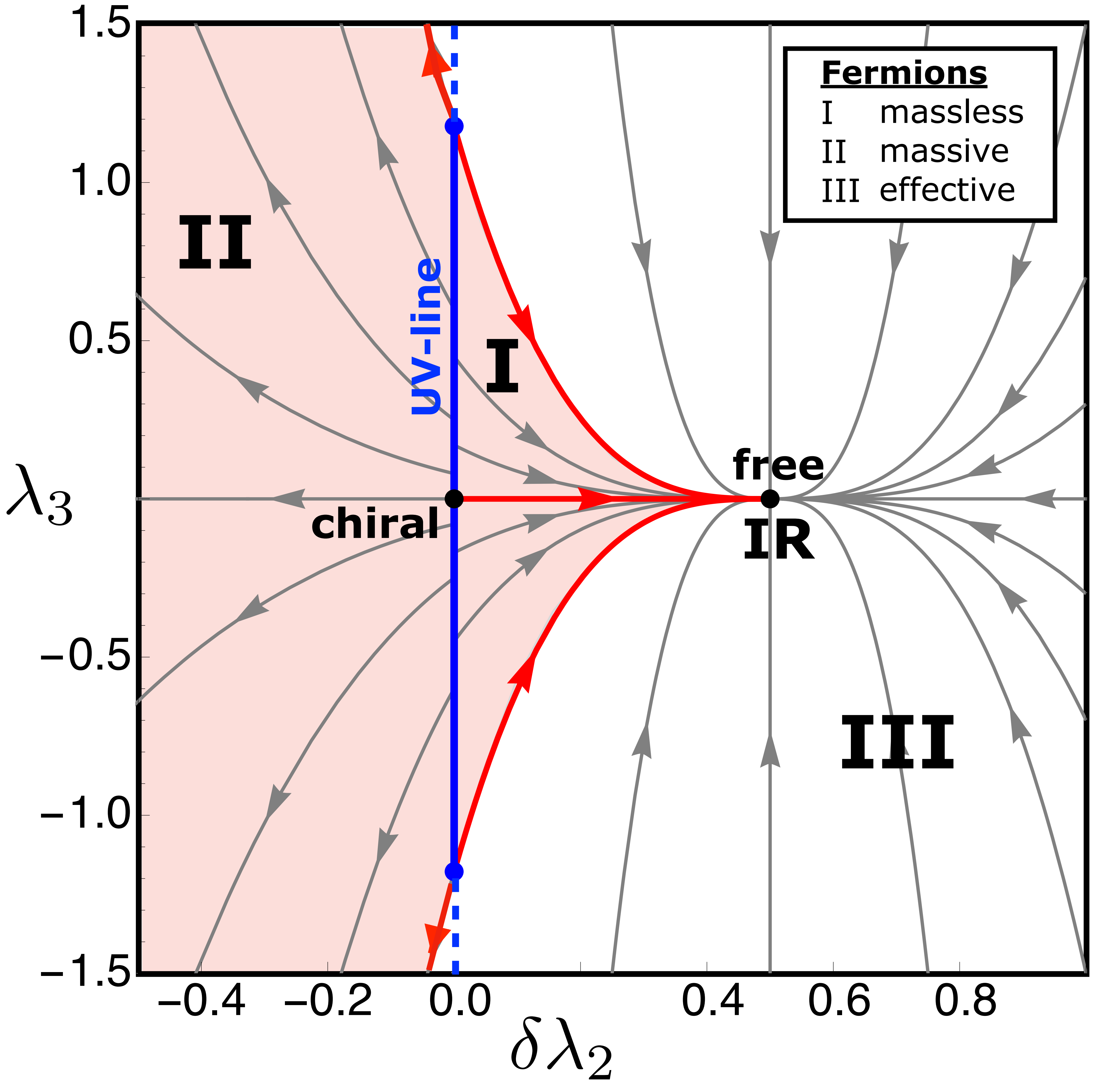
***d* = 3 from now on**

$$v(z,t) = k^{-d} V_k(k^{d-1} z)$$

Phase diagram

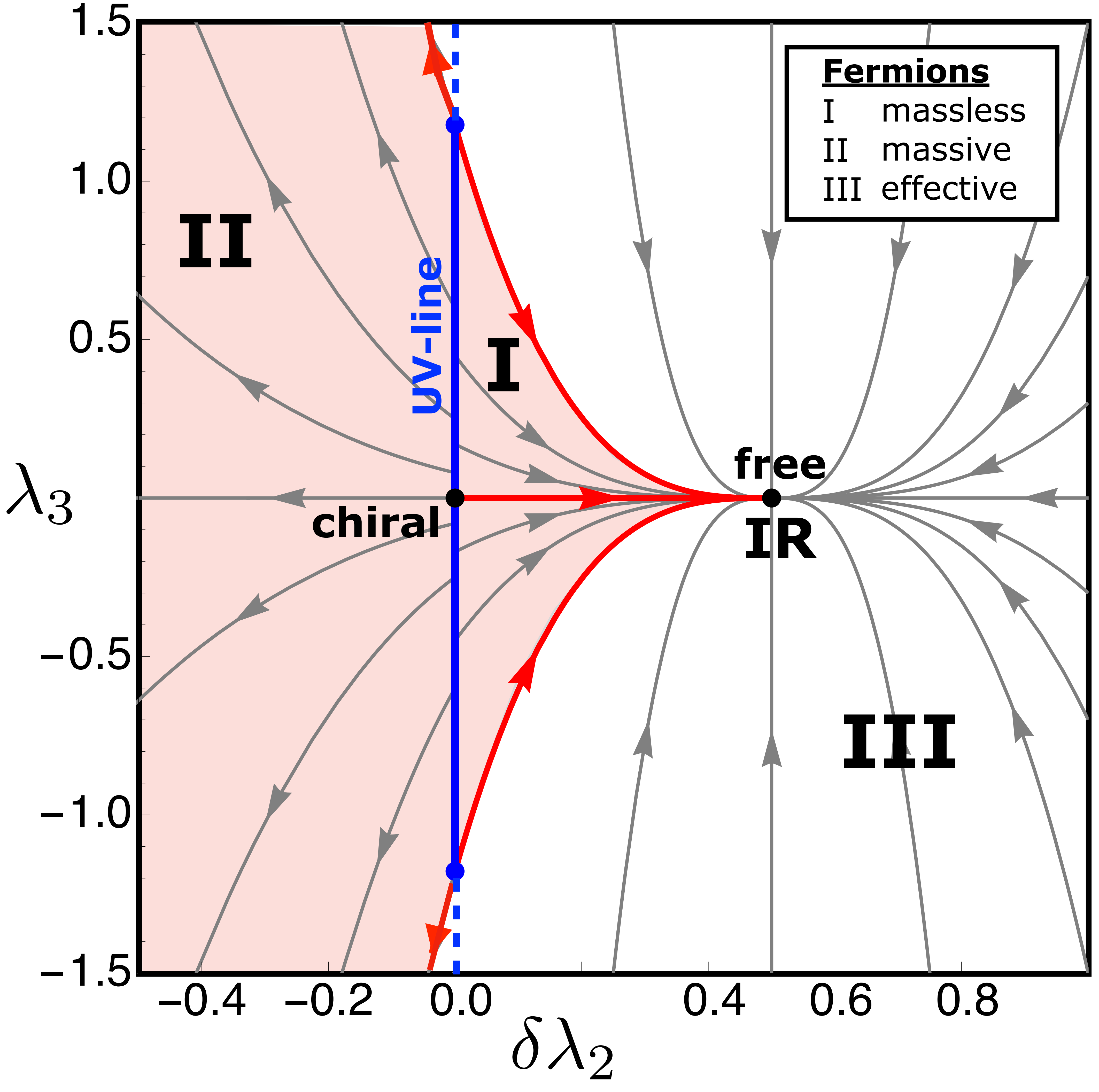
$$v(z,t) = \sum_{n=0}^{\infty} \frac{1}{n!}\lambda_n(t) z^n$$



Phase diagram

$$v(z,t) = \sum_{n=0}^{\infty} \frac{1}{n!}\lambda_n(t) z^n$$

$$\partial_t \lambda_1 = \lambda_1 \left(-1 + \frac{2\lambda_2}{(1+\lambda_1^2)^2} \right)$$



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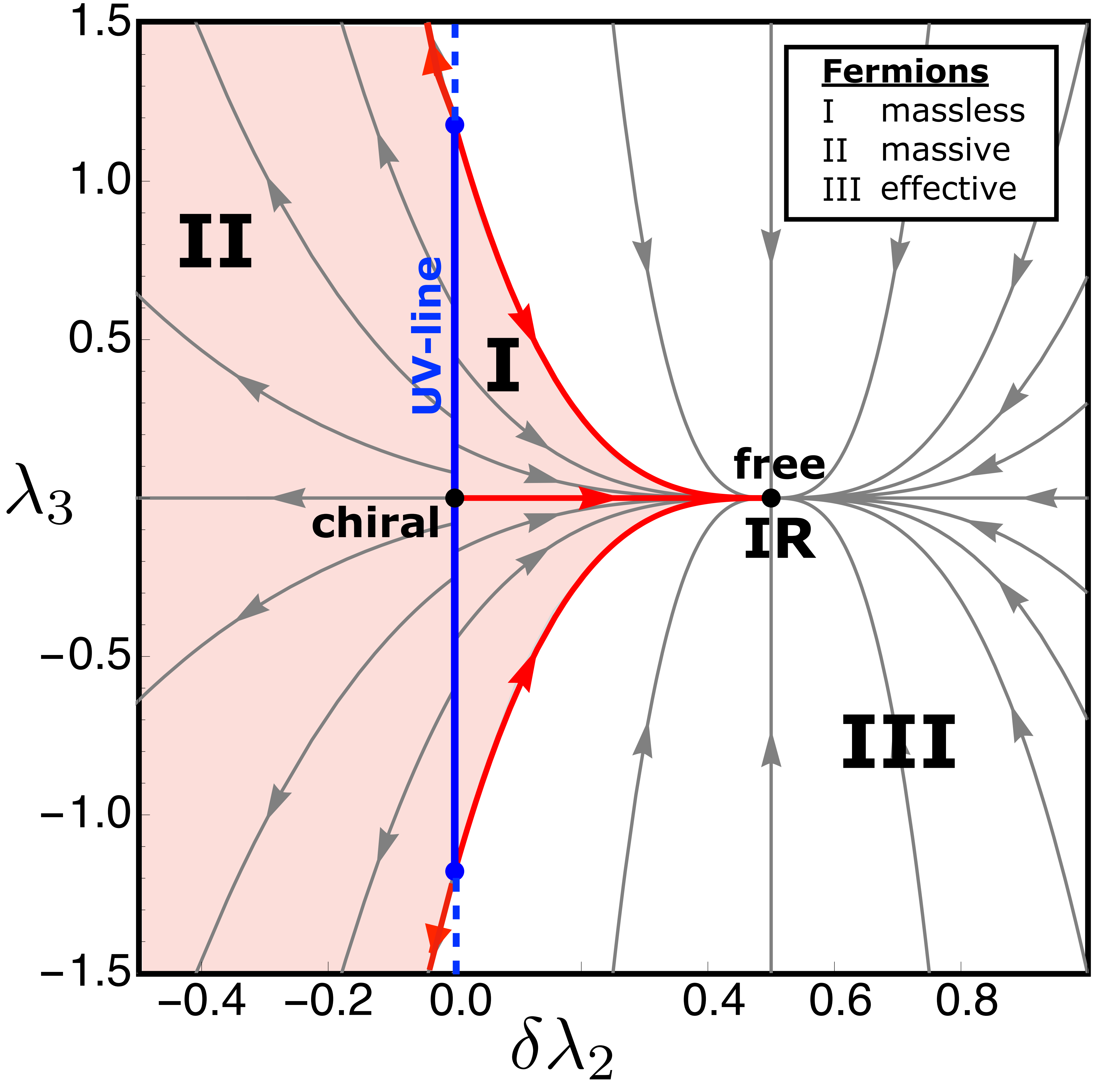
$$\lambda_1 = 0 \implies \partial_t \lambda_1 = 0$$

Massless fermions without chiral symmetry



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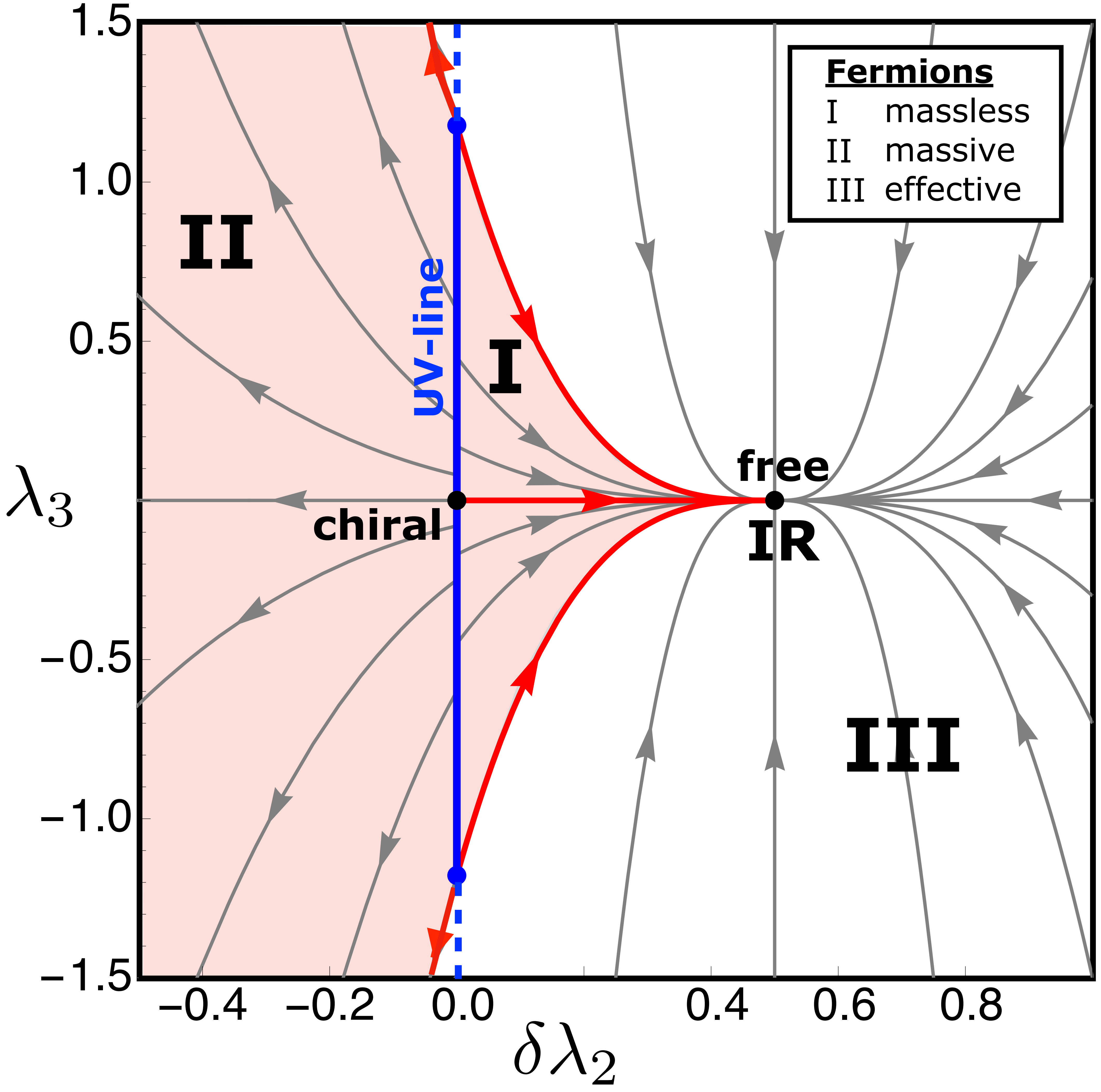
$$v(z,t) = \sum_{n=0}^{\infty} \frac{1}{n!}\lambda_n(t) z^n$$

$$\partial_t \lambda_2 = \lambda_2 \left(1 + 2\lambda_2\right)$$
$$\partial_t \lambda_3 = \lambda_3 \left(1 + 2\lambda_2\right)$$



Phase diagram

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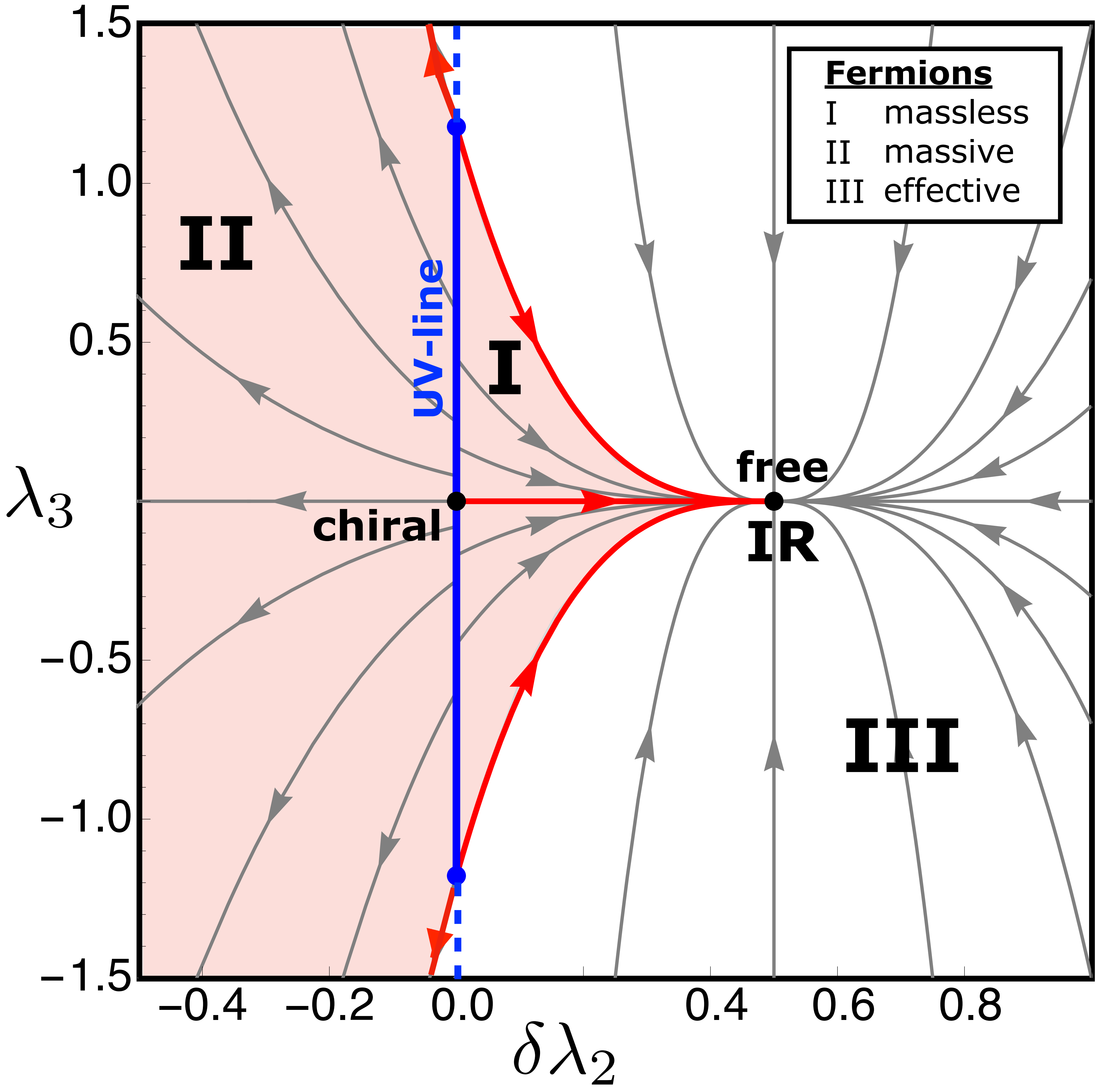
$$\partial_t \lambda_2 = \lambda_2 \left(1 + 2\lambda_2\right)$$
$$\partial_t \lambda_3 = \lambda_3 \left(1 + 2\lambda_2\right)$$

$$\lambda_2^* = 0$$
$$\lambda_3^* = 0$$



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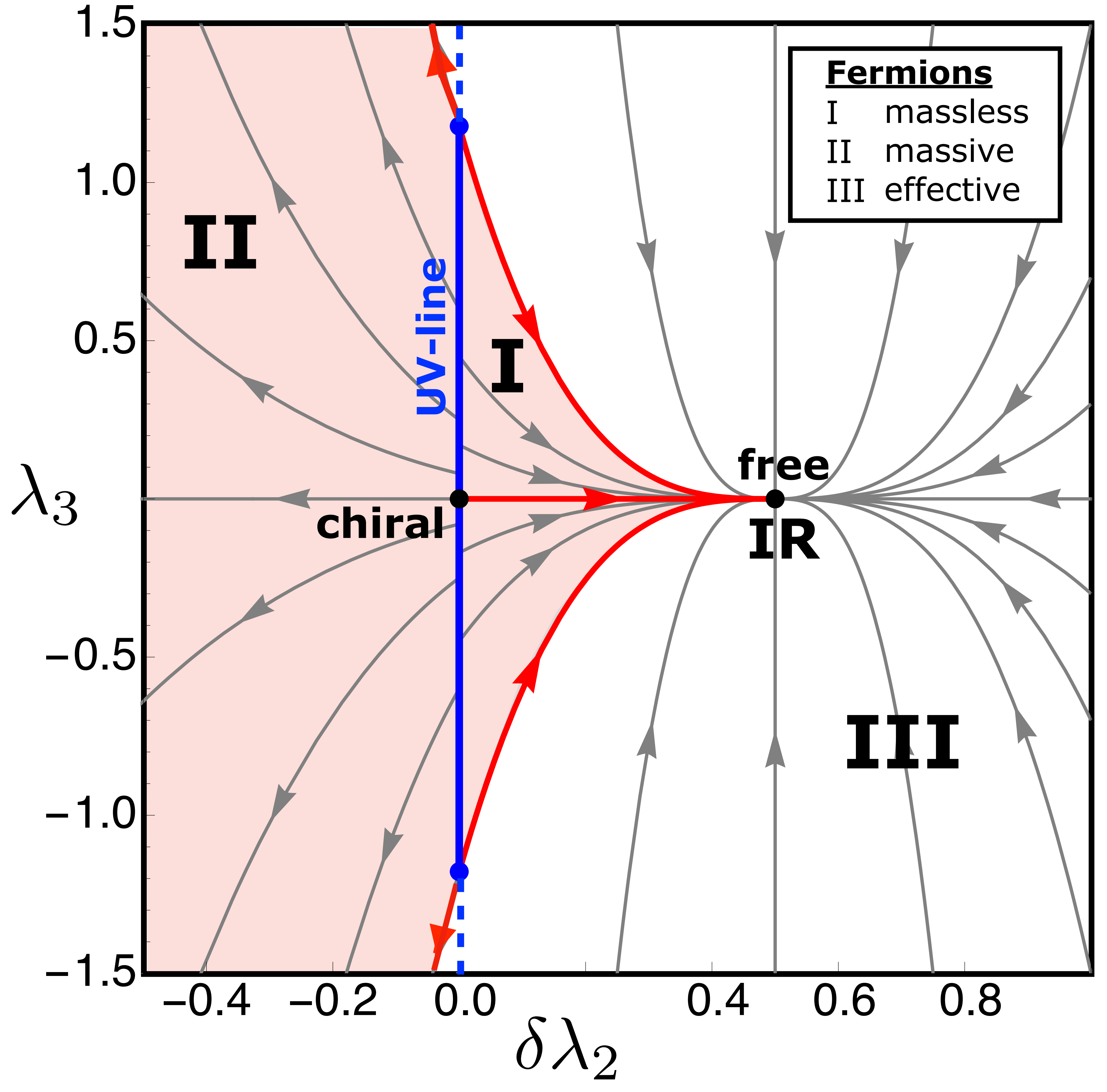
$$\lambda_2^* = -\frac{1}{2}$$
$$\lambda_3^* = \text{free parameter}$$



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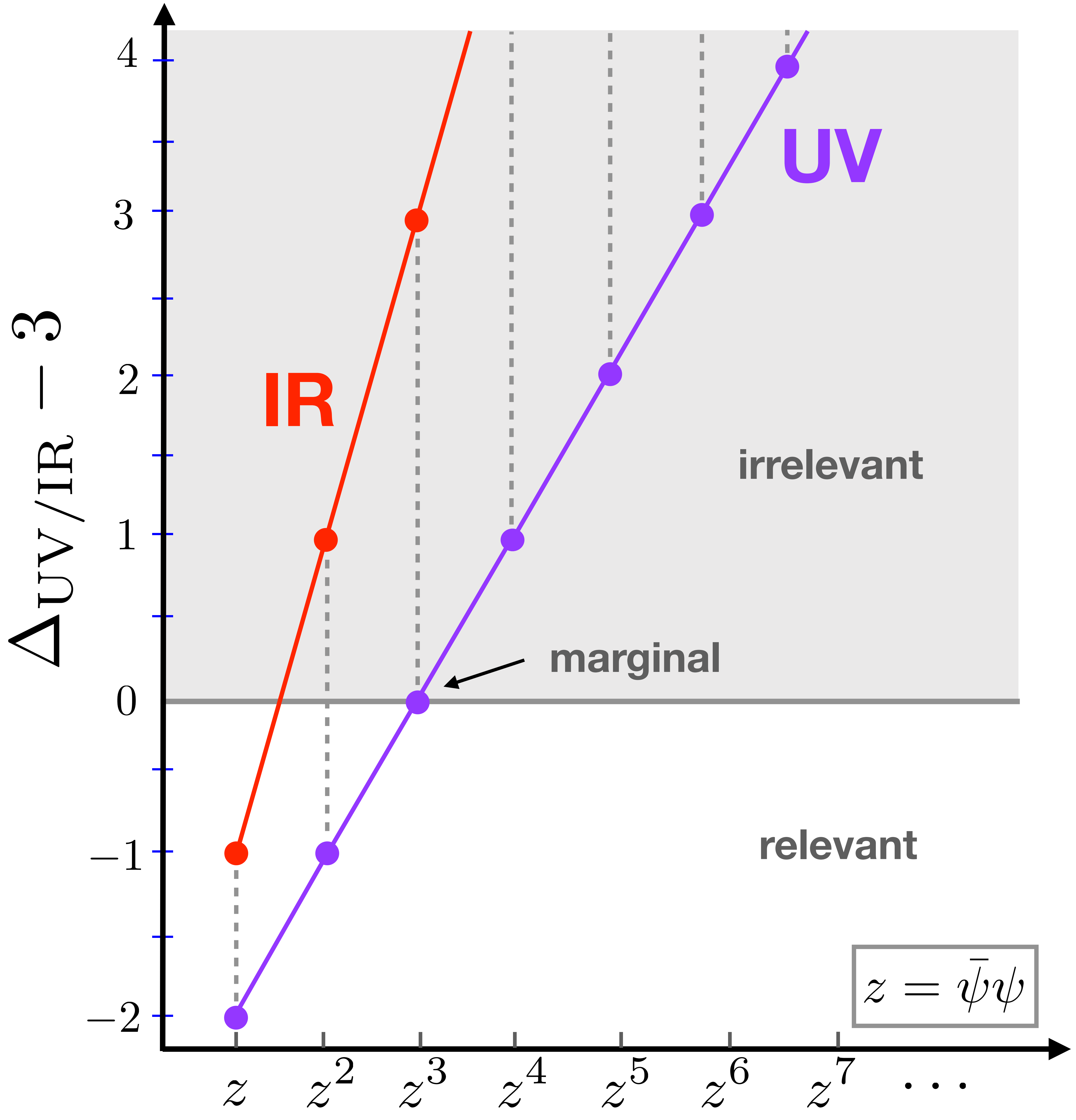

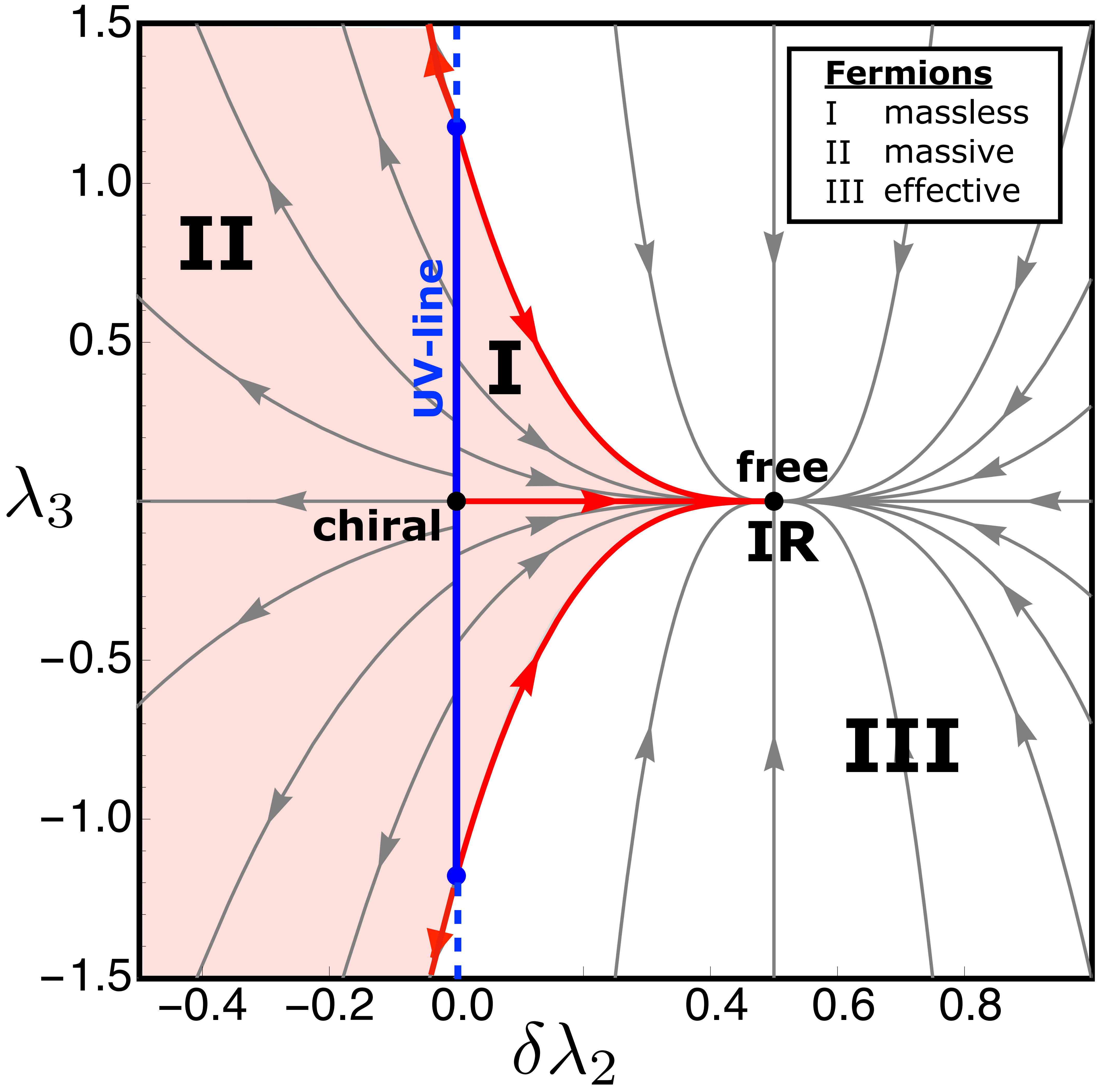

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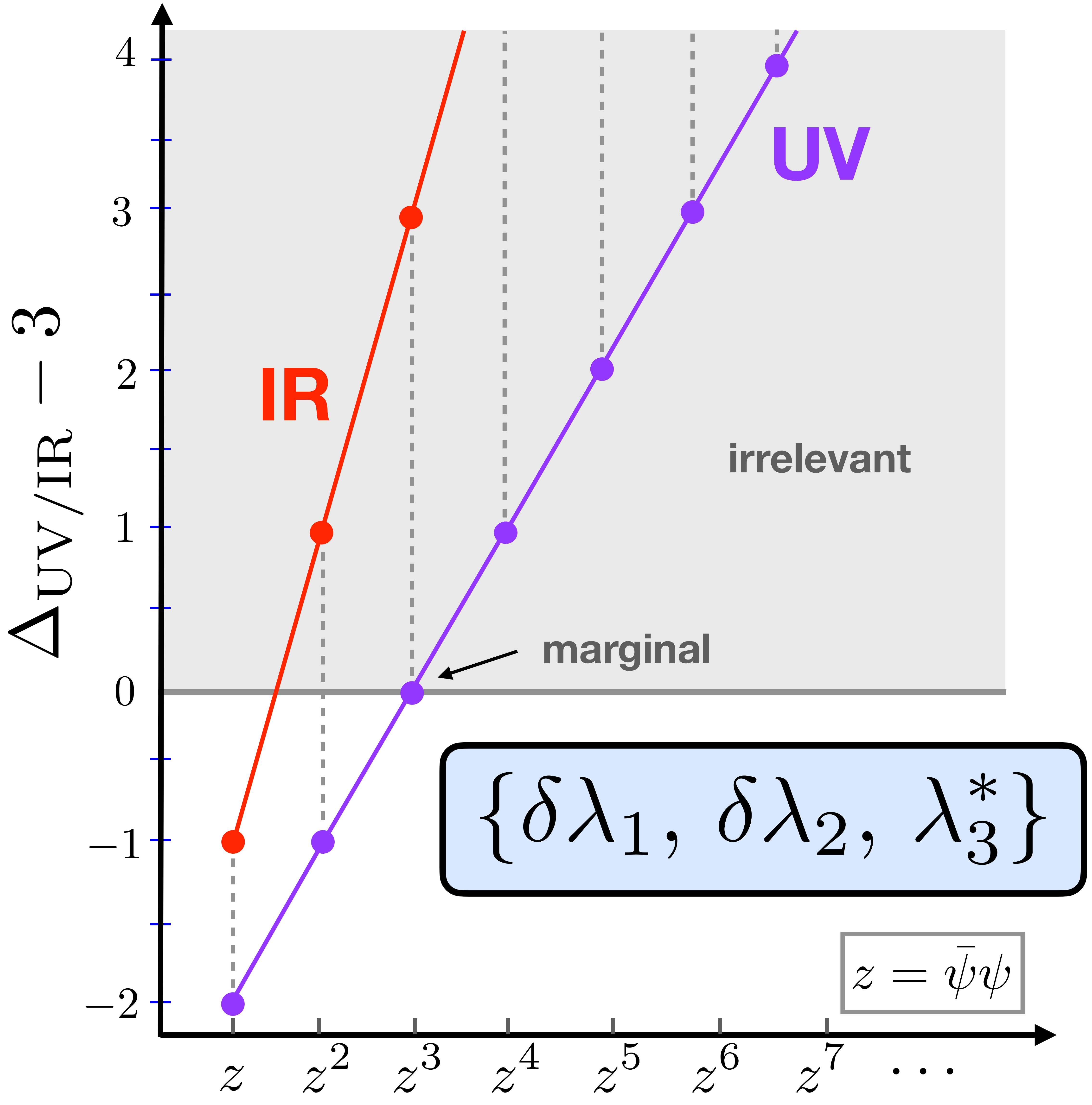
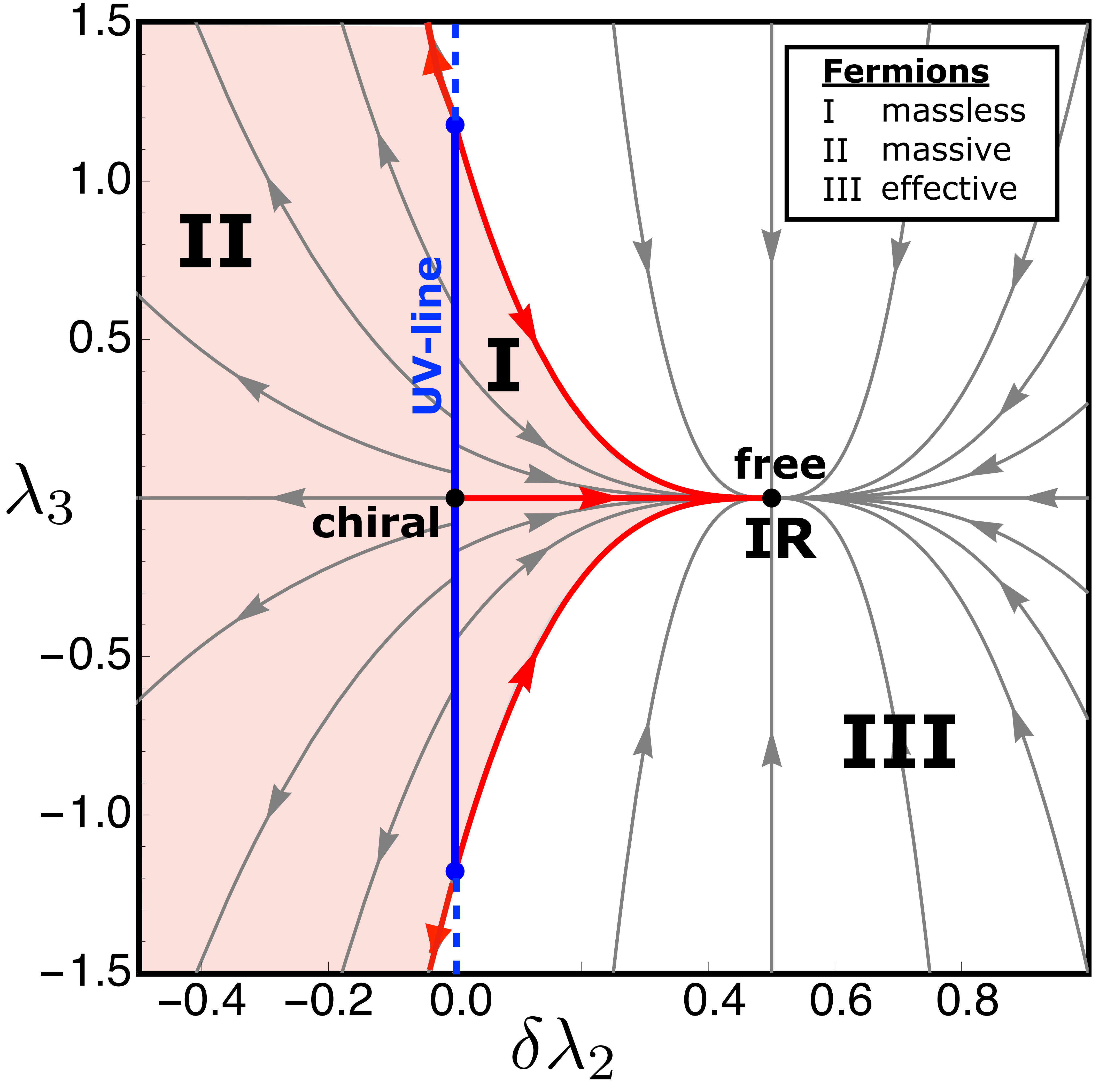


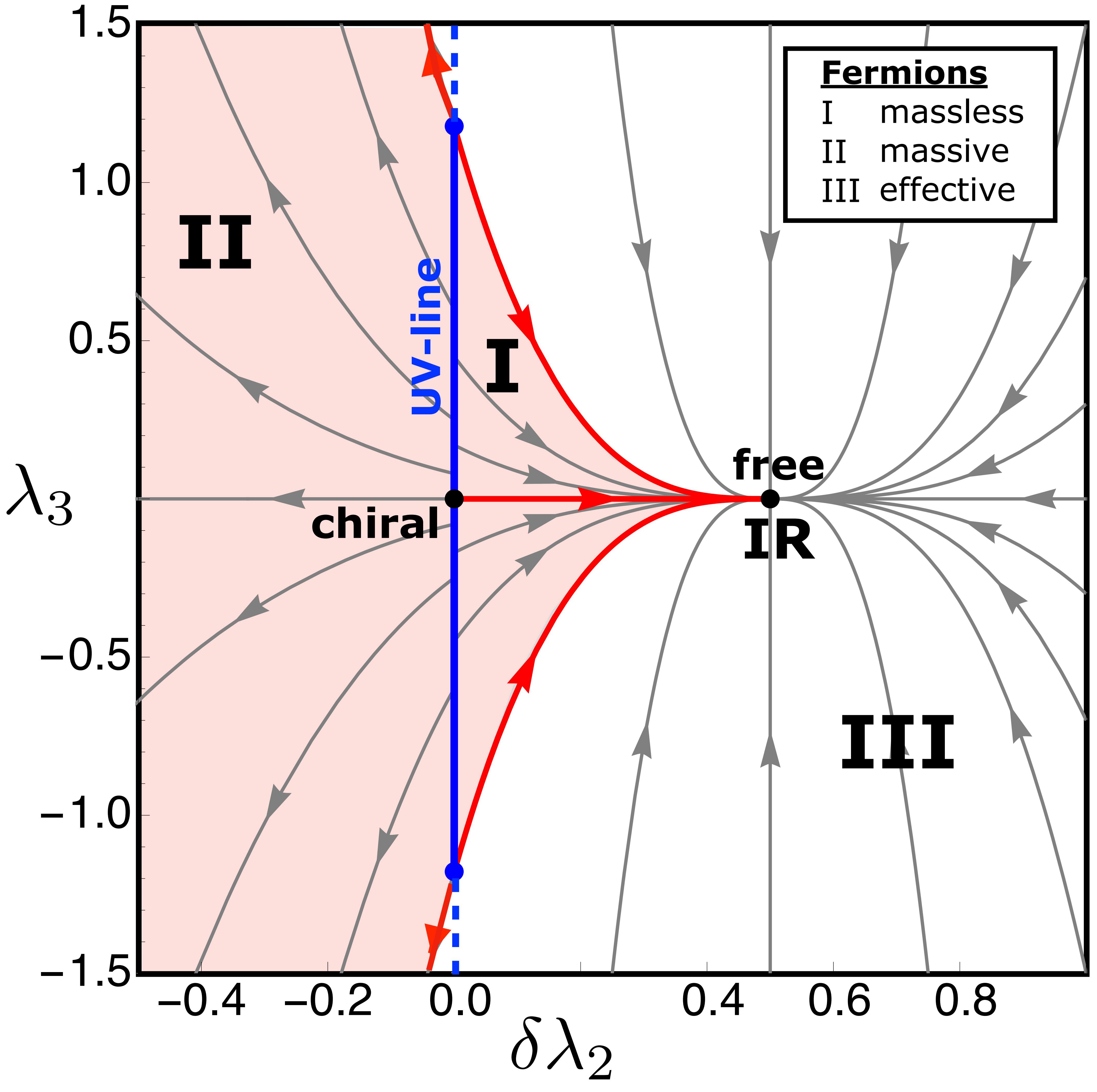


Phase diagram

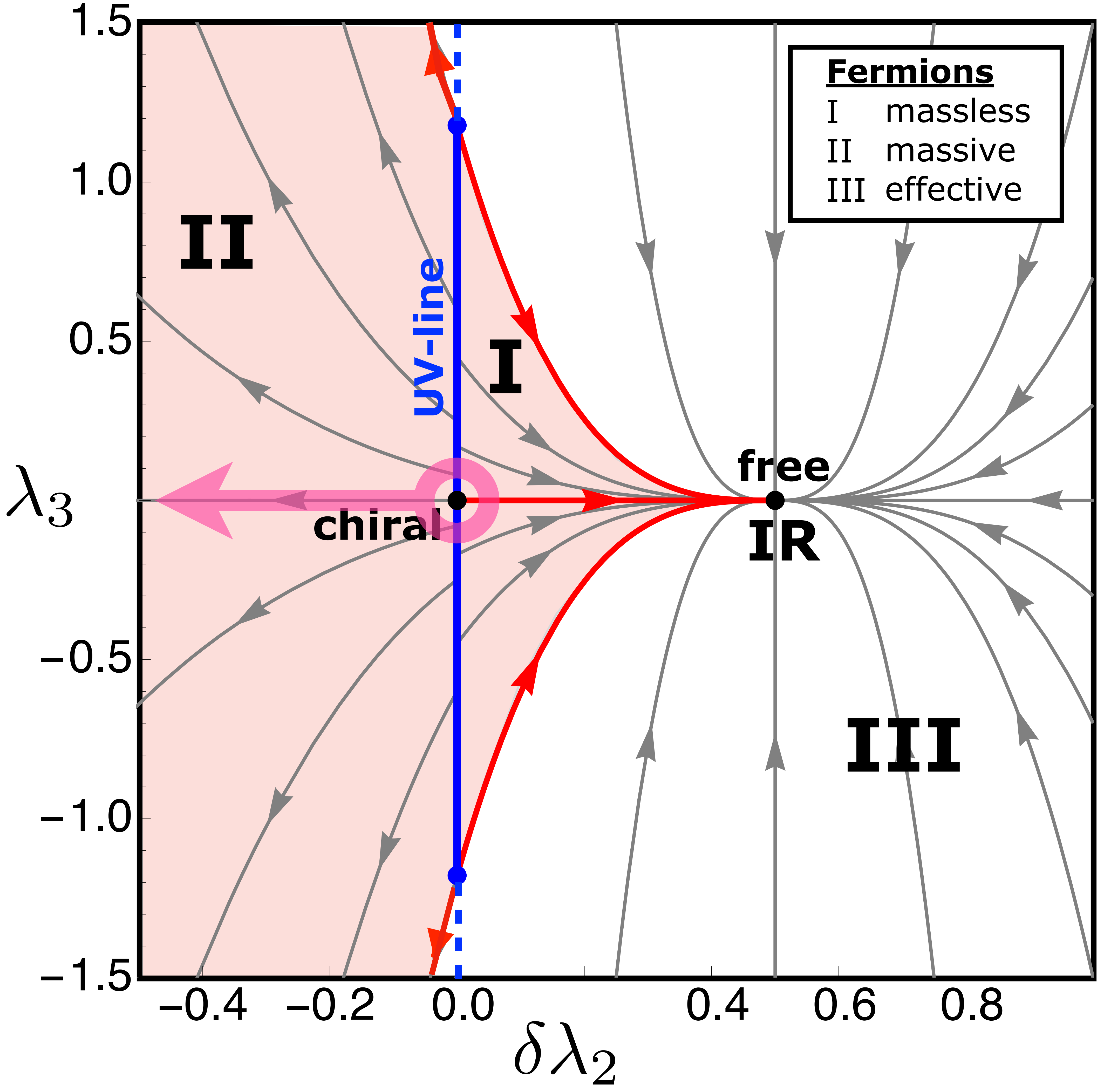
Region I: massless theories

Region II: massive theories

Region III: effective theories



Generation of mass



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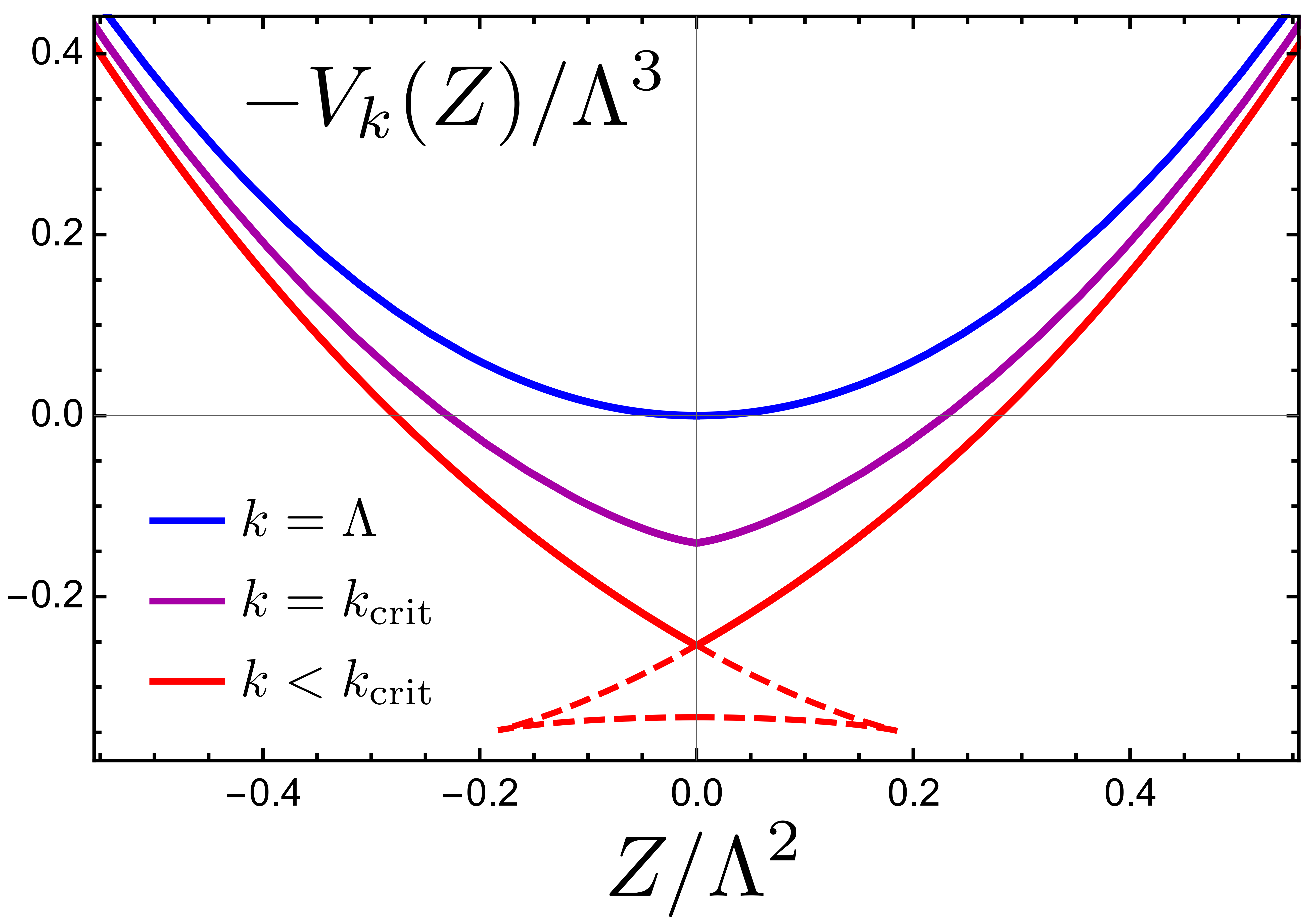

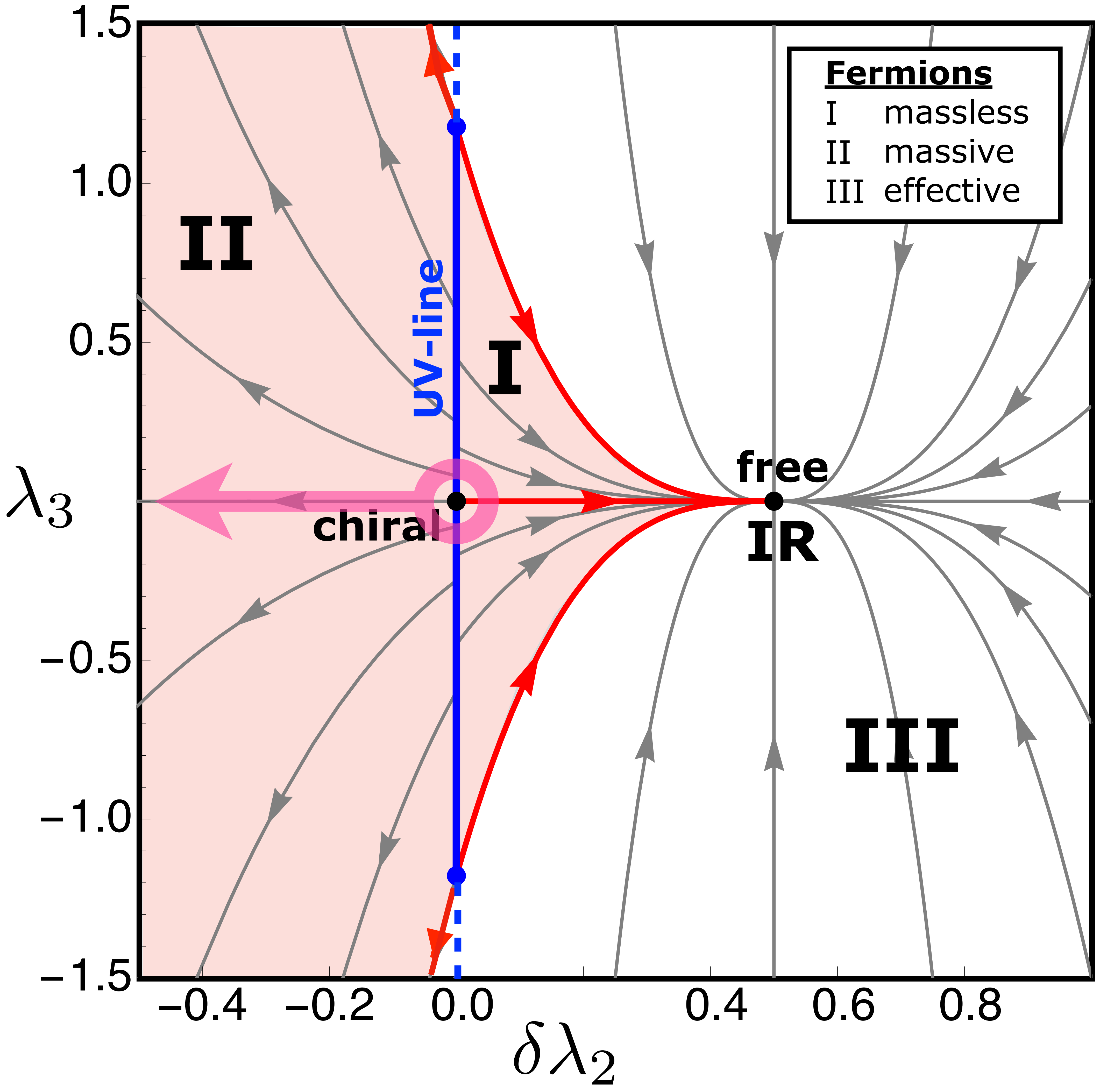

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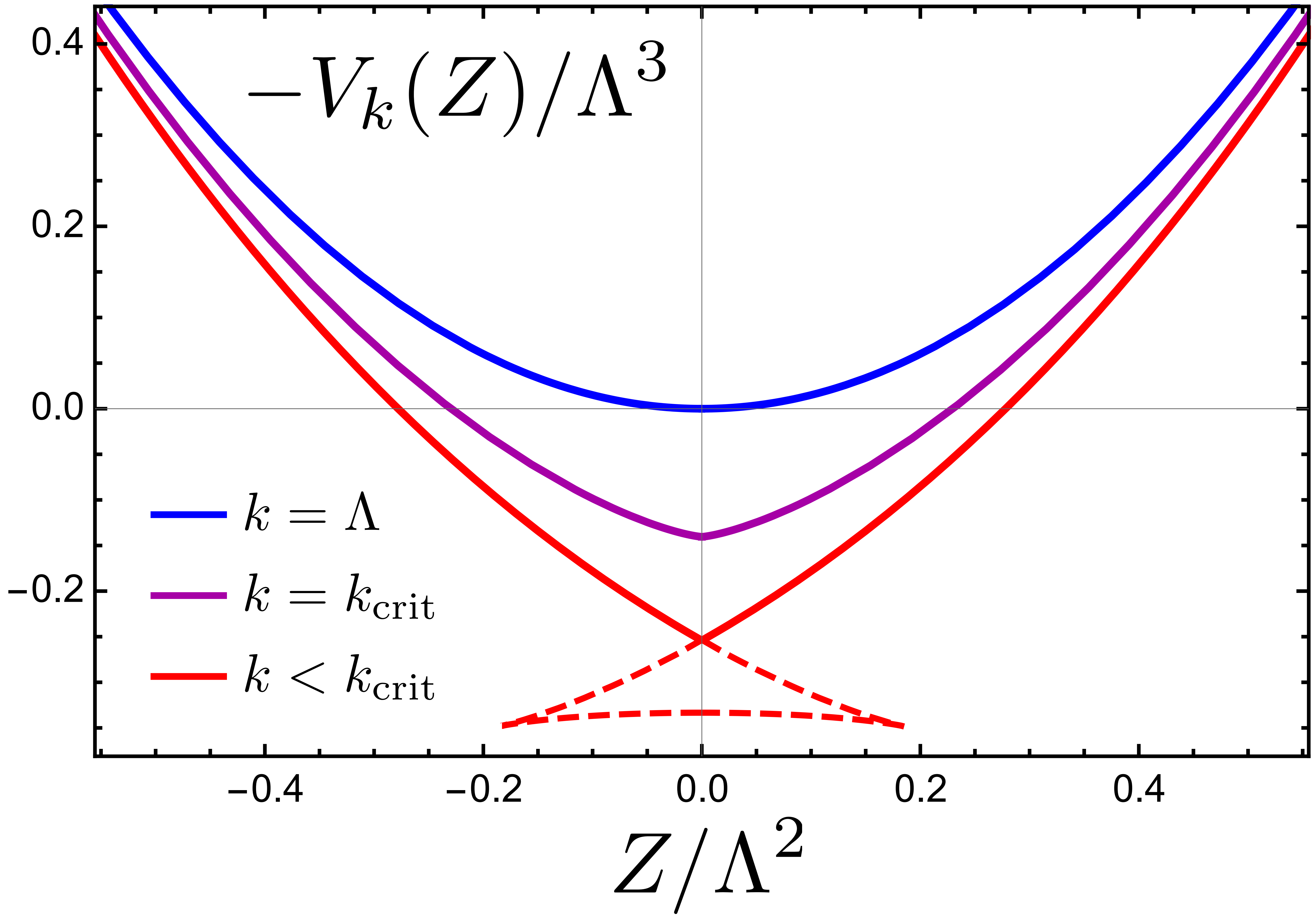
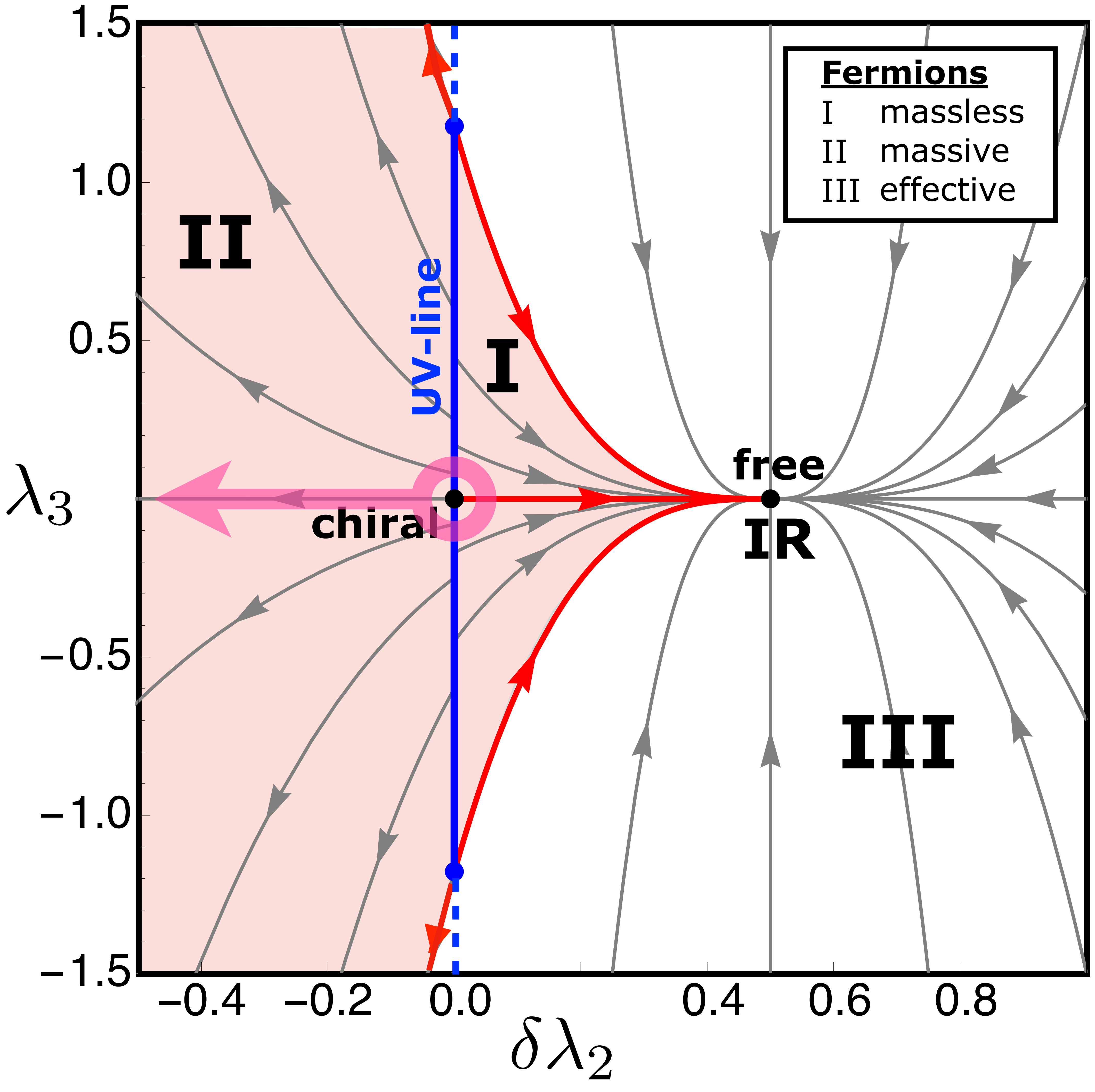
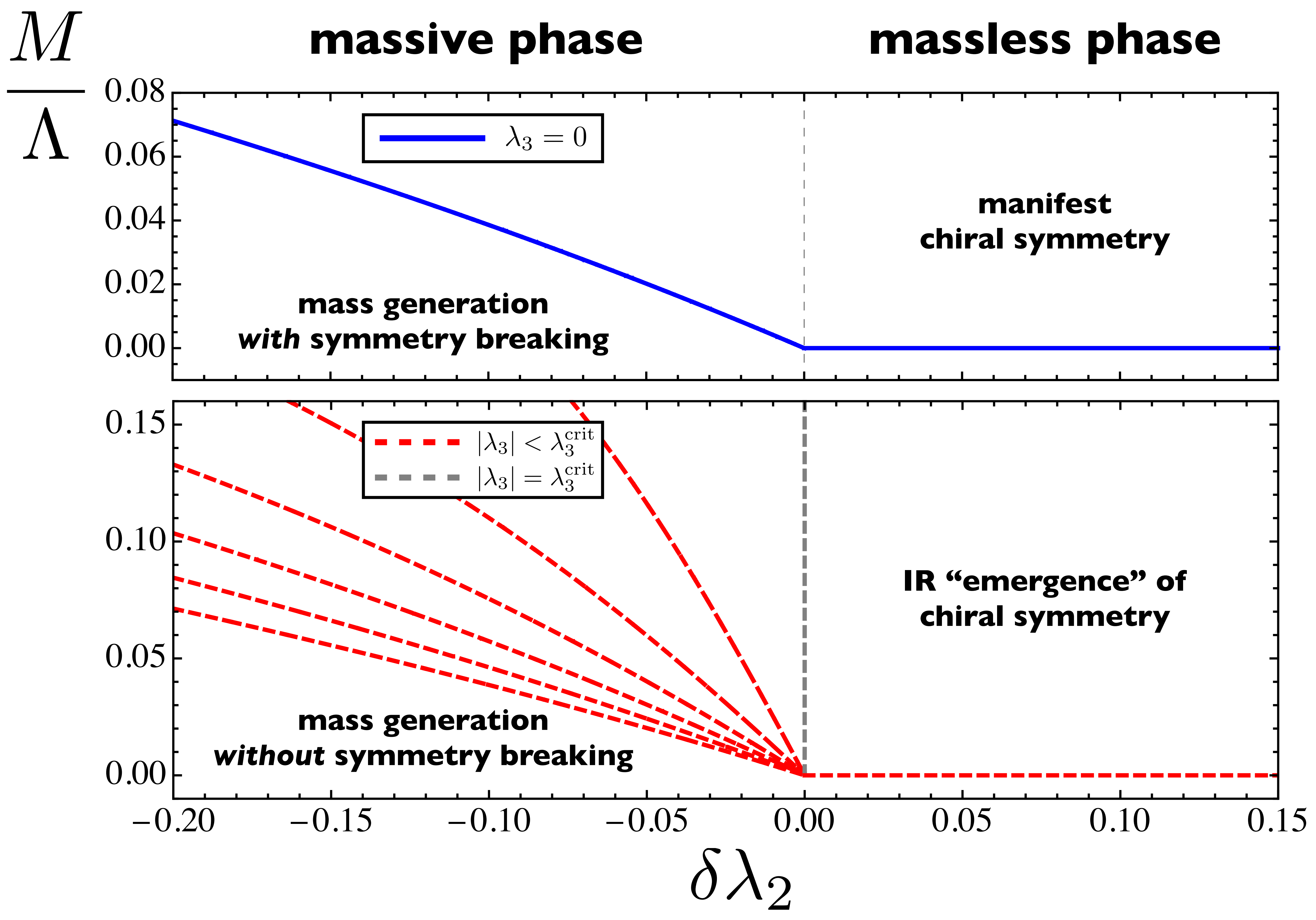


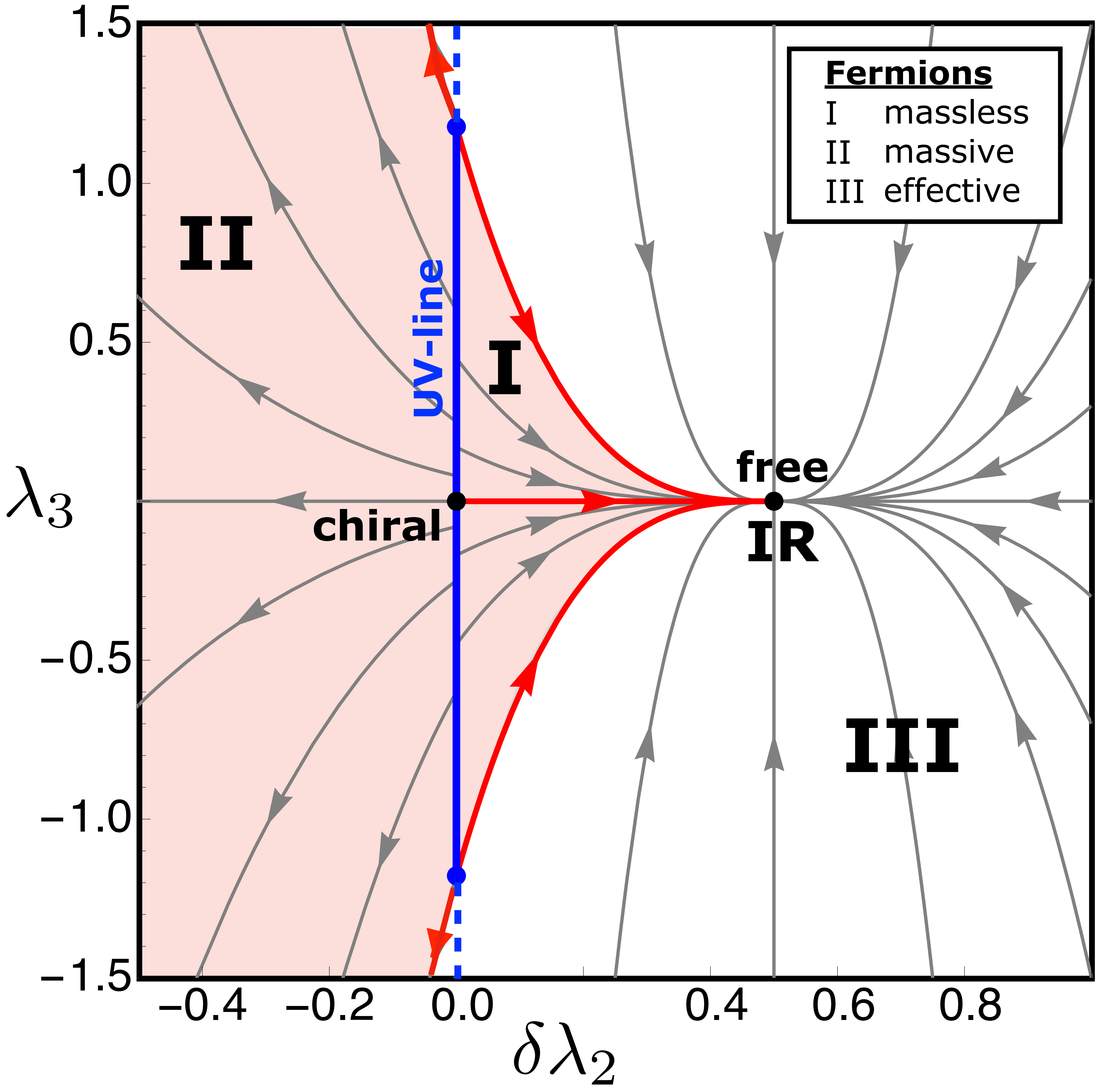


Weak solution: mass generated via cusp

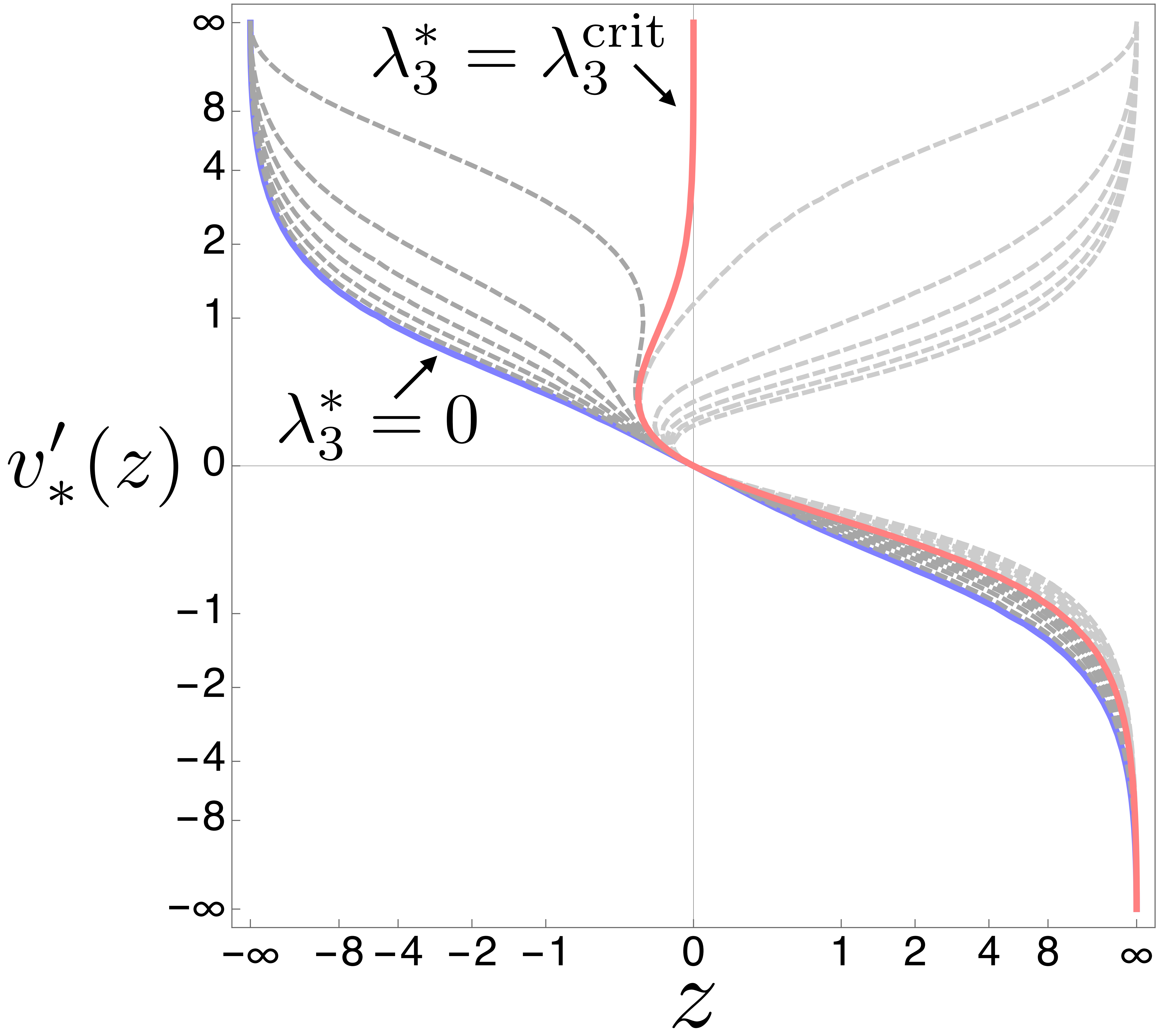
4d 4F: Aoki, Sato, 1304.5276; **O(N):** Grossi, Wink, 1903.09503 + **QCD & more**

Generation of mass





Spontaneous breaking of scale invariance



CCH, Litim, 2212.06815

cf. O(N) model:

Bander, Bardeen, Moshe, PRL ’83

David, Kessler, Neuberger, PRL ’85

Litim, Marchais, Mati, 1702.05749

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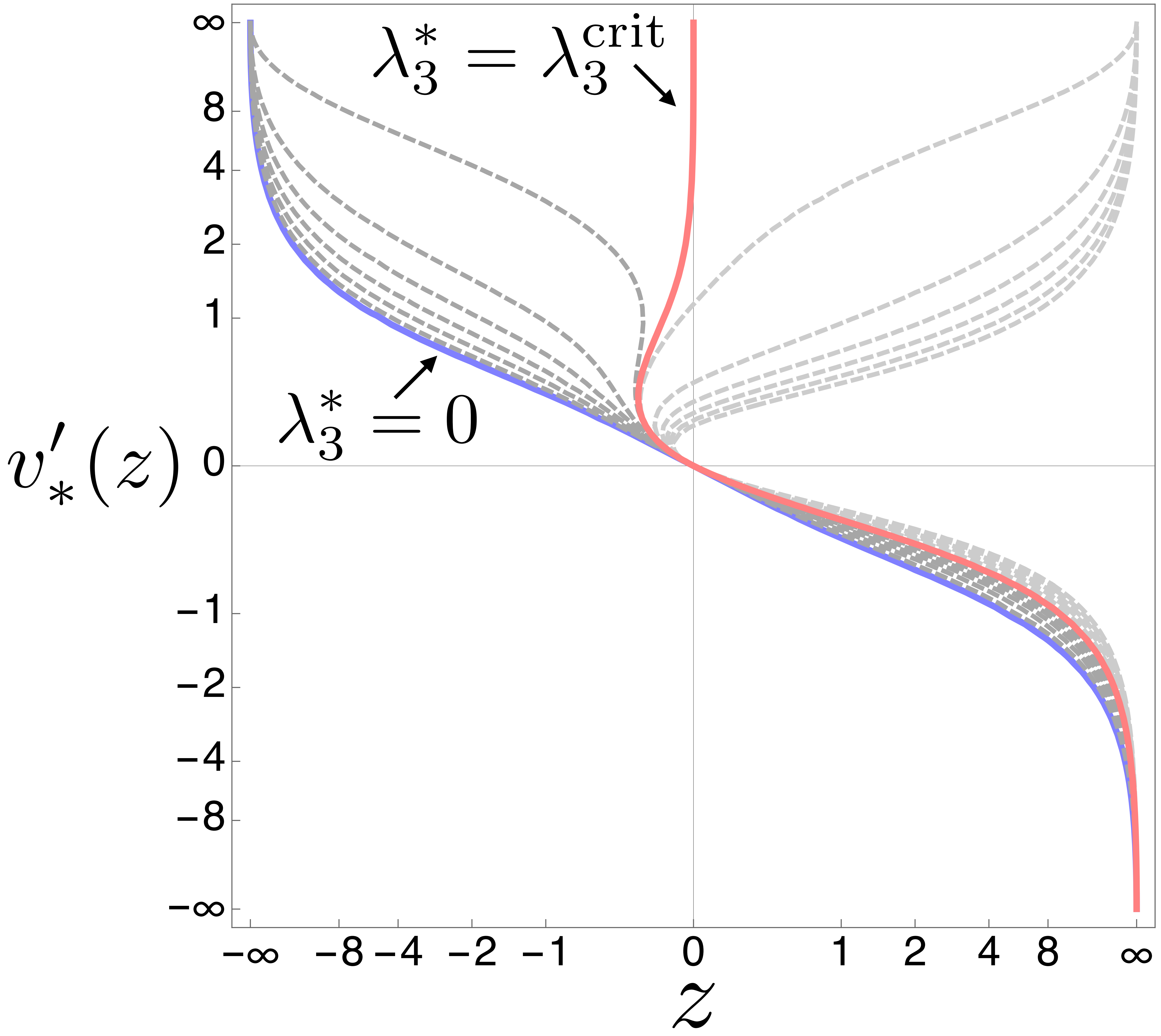

Physical fermion mass

$$M = \lim_{k\to 0} k\, v_*'(0)$$

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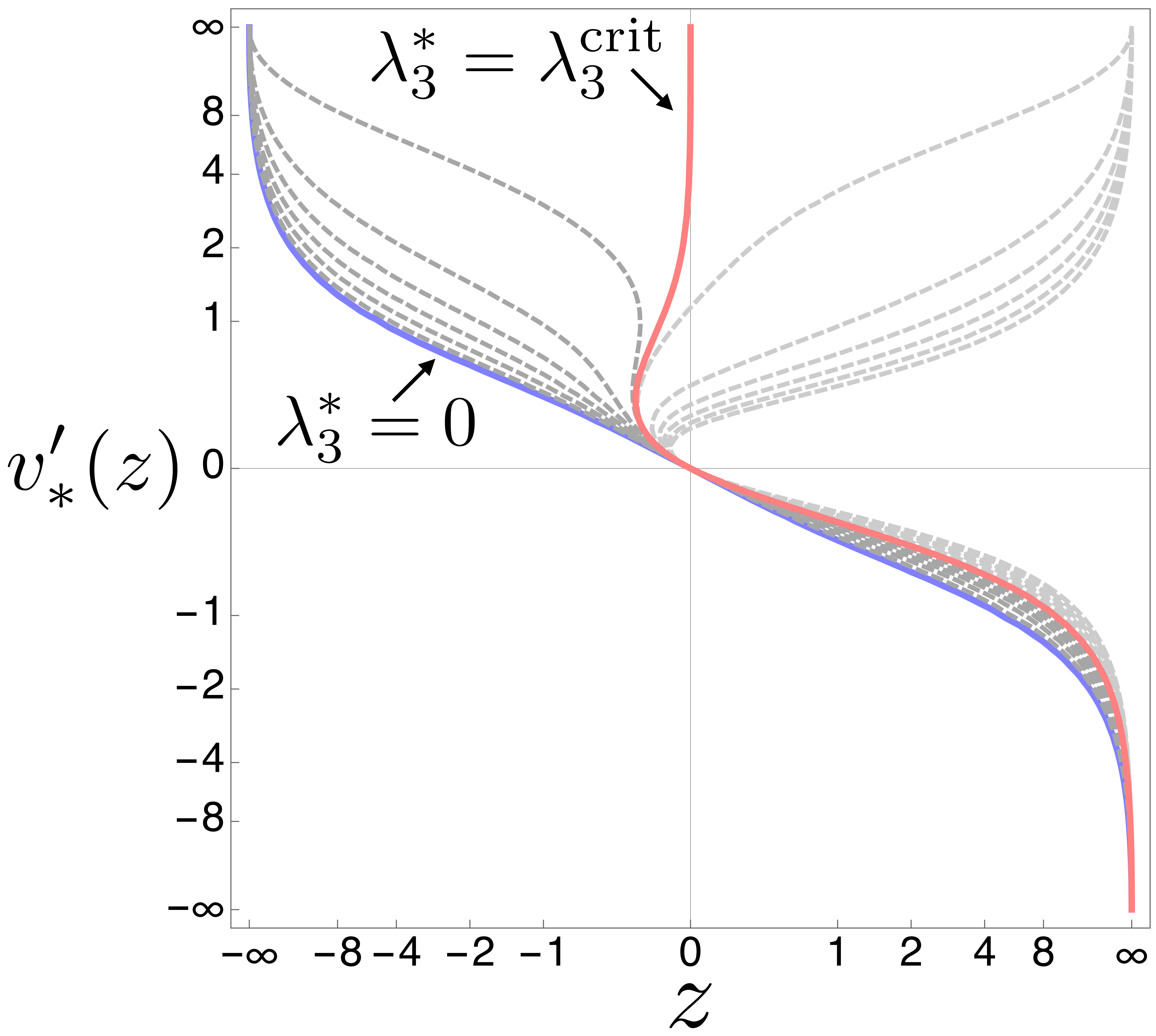

Physical fermion mass

$$M = \lim_{k\to 0} k\, v_*'(0)$$

satisfies gap equation

$$\left(\lambda_3^* - \lambda_3^{\rm crit}\right) M = 0$$

CCH, Litim, 2212.06815
cf. O(N) model:
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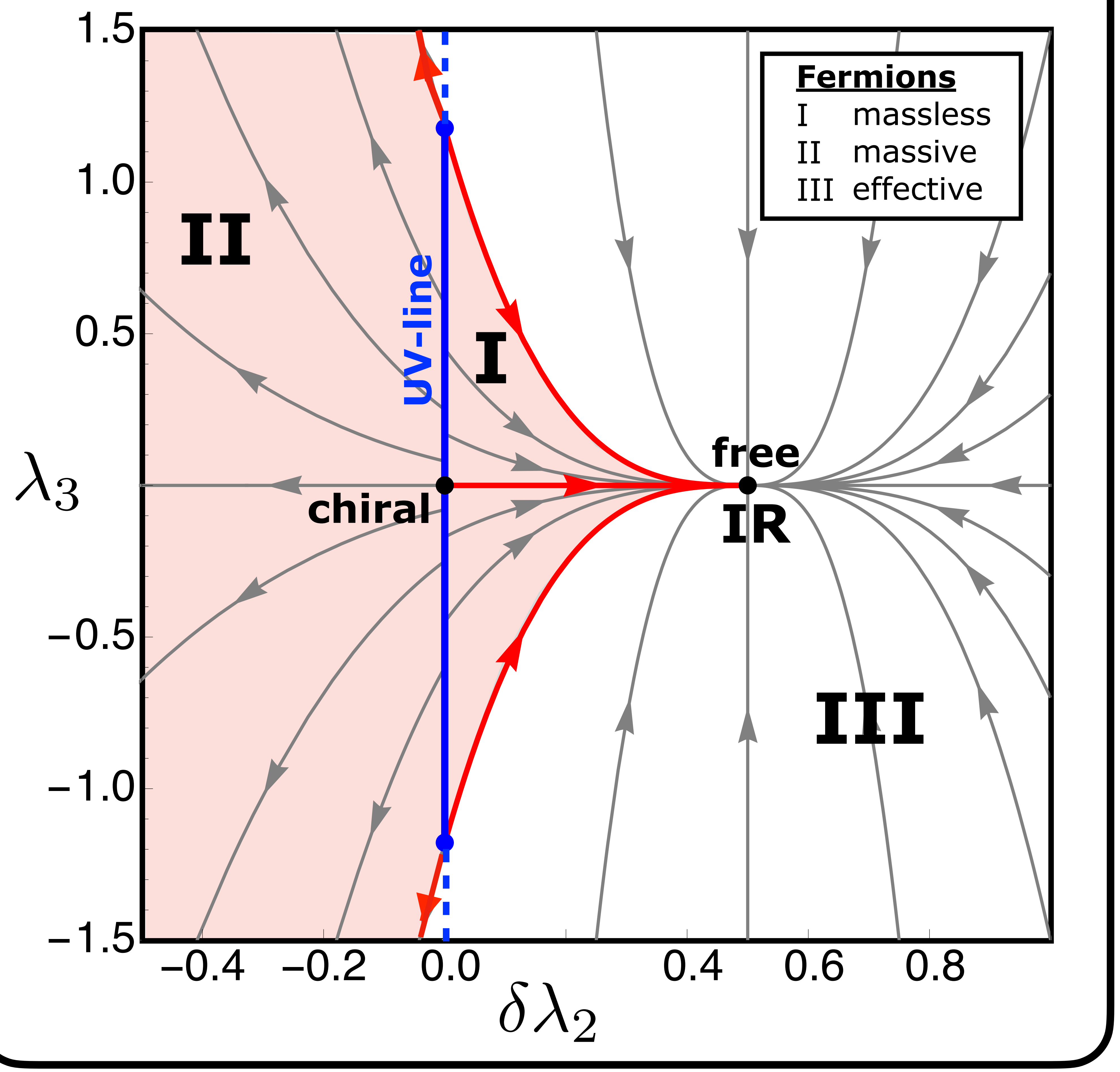
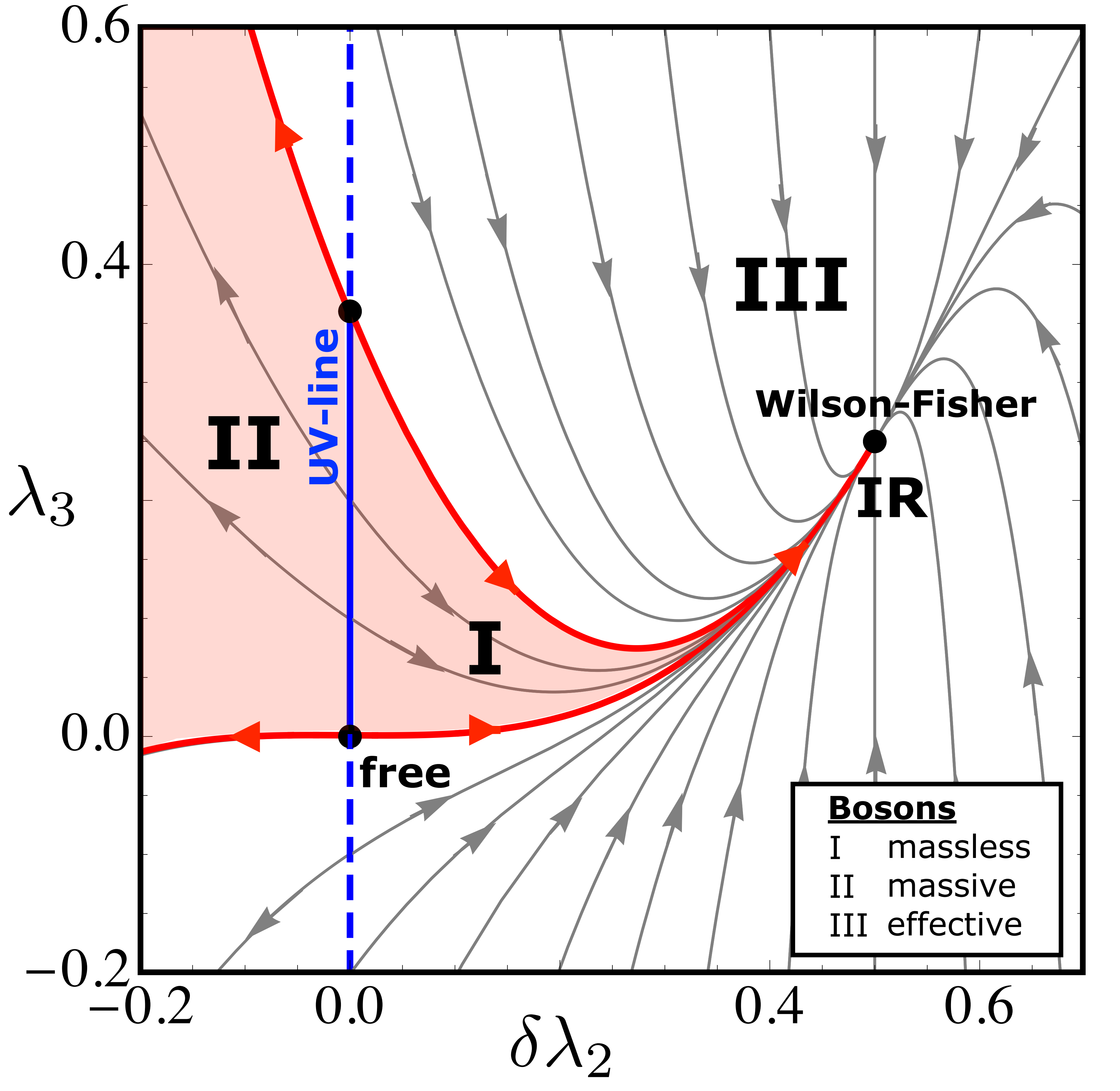
Fermions & bosons

$$(\lambda_2^* + \delta\lambda_2)\,(\bar\psi\psi)^2 + \lambda_3(\bar\psi\psi)^3$$



$$\delta\lambda_2(\phi^2)^2 + \lambda_3(\phi^2)^3$$



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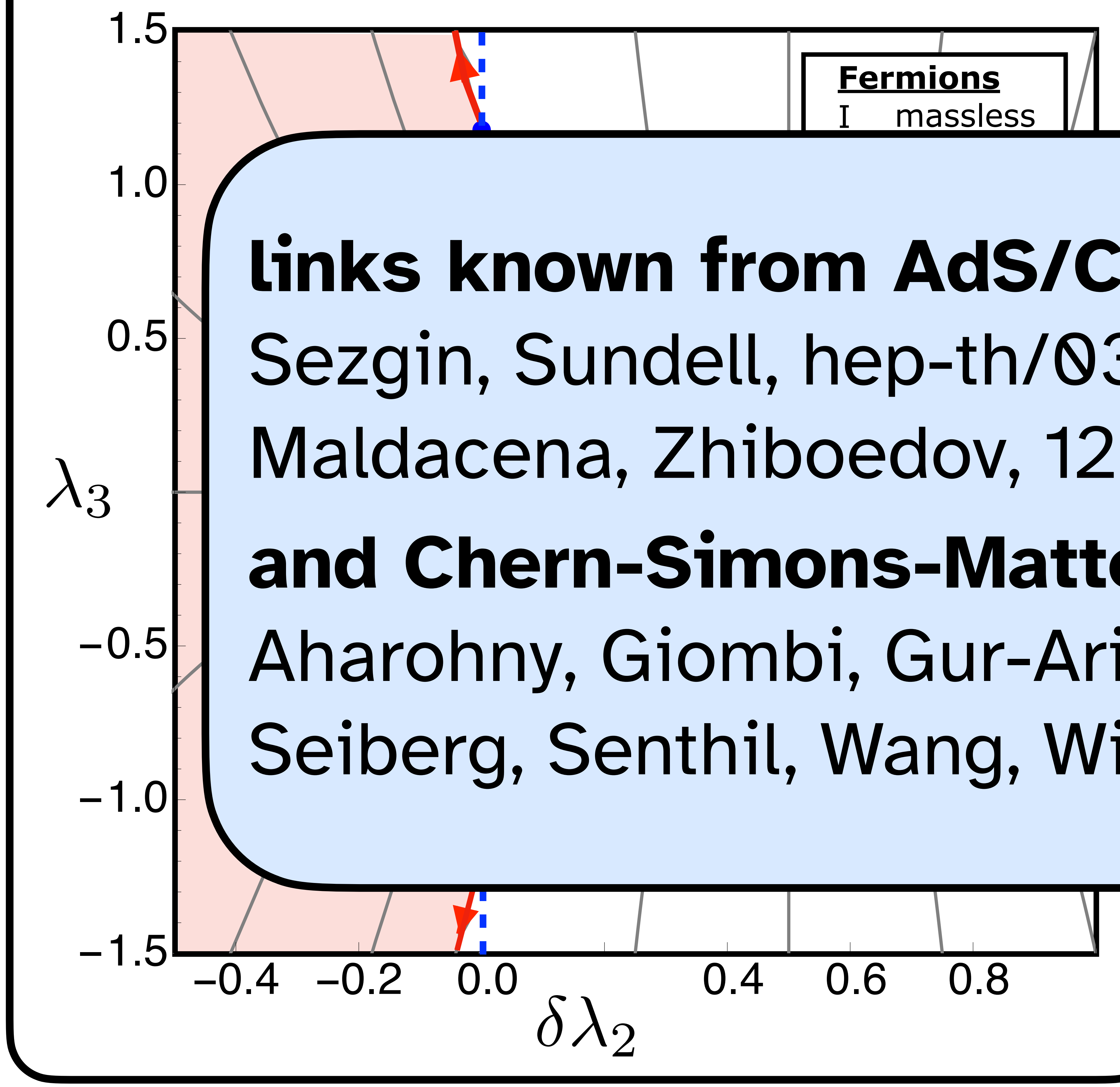
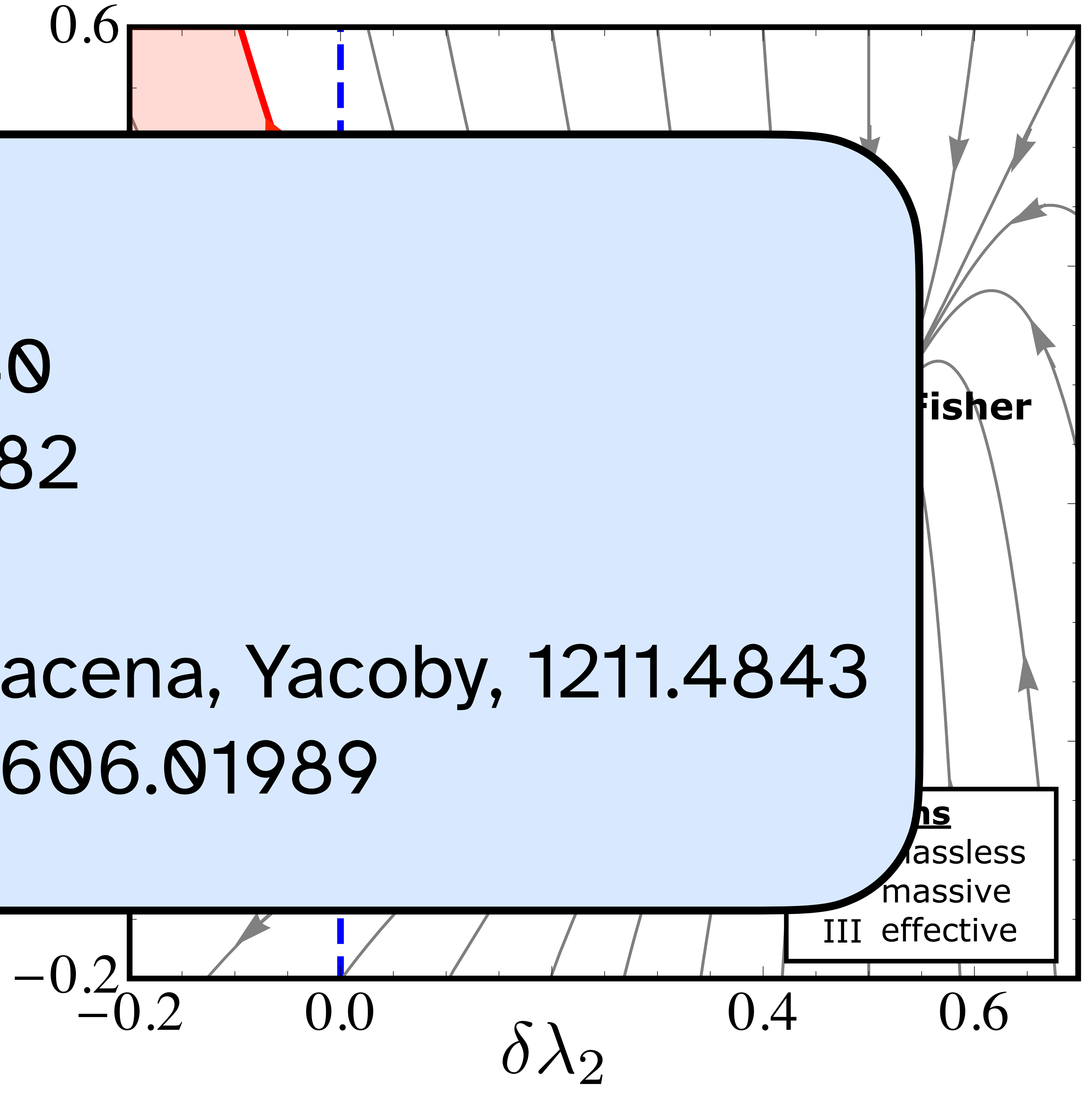


$\delta\lambda_2(\phi^2)^2 + \lambda_3(\phi^2)^3$



links known from AdS/CFT:

Sezgin, Sundell, hep-th/0305040

Maldacena, Zhiboedov, 1204.3882

and Chern-Simons-Matter:

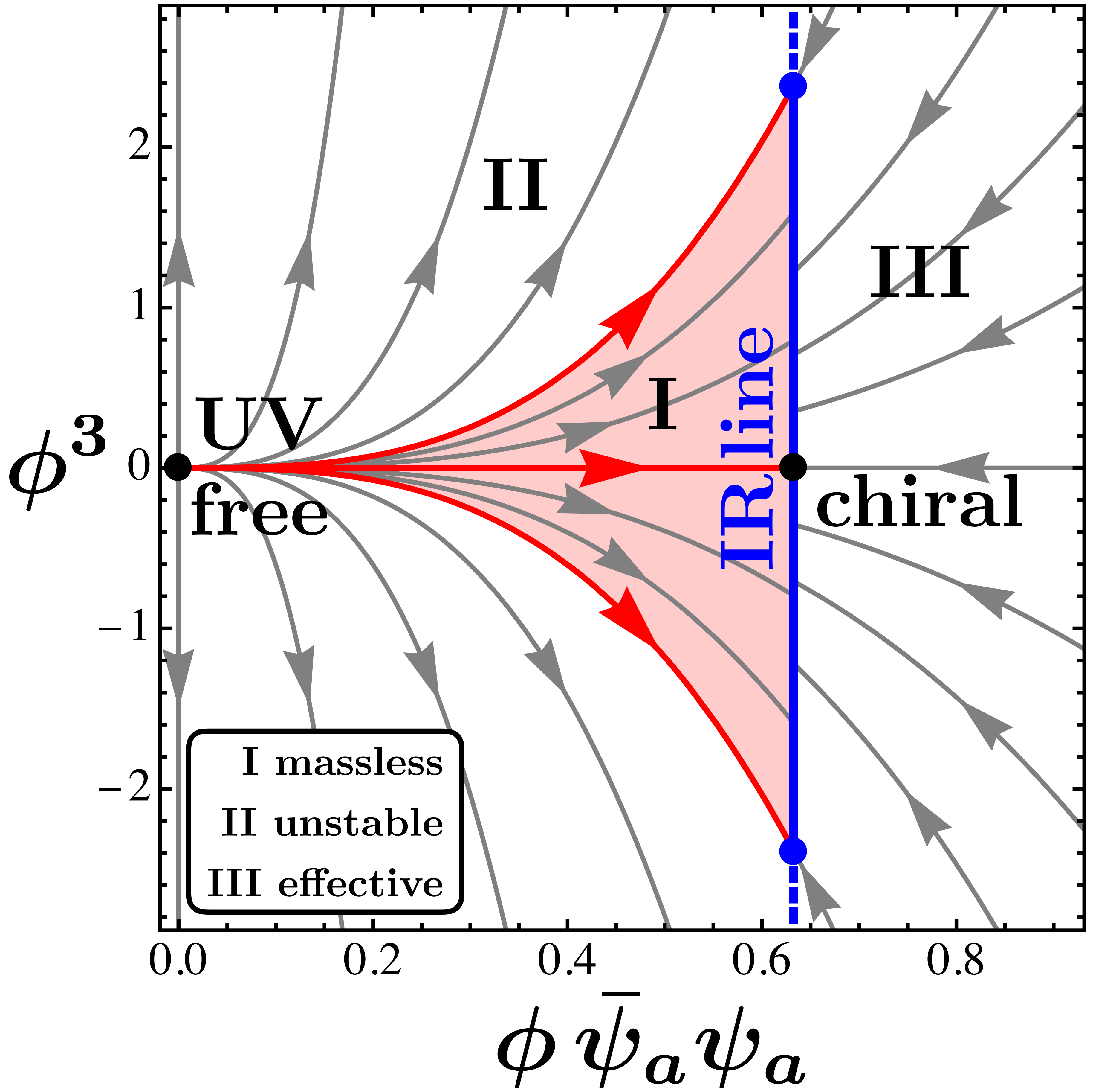
Aharohny, Giombi, Gur-Ari, Maldacena, Yacoby, 1211.4843

Seiberg, Senthil, Wang, Witten, 1606.01989

Gross-Neveu-Yukawa

$$S = \int_x \left\{ \tfrac{1}{2}(\partial\phi)^2 + \bar{\psi}_a \partial\!\!\!/ \psi_a + Y\phi\,\bar{\psi}_a\psi_a + U(\phi) \right\}$$

CCH, Litim, *in prep.*

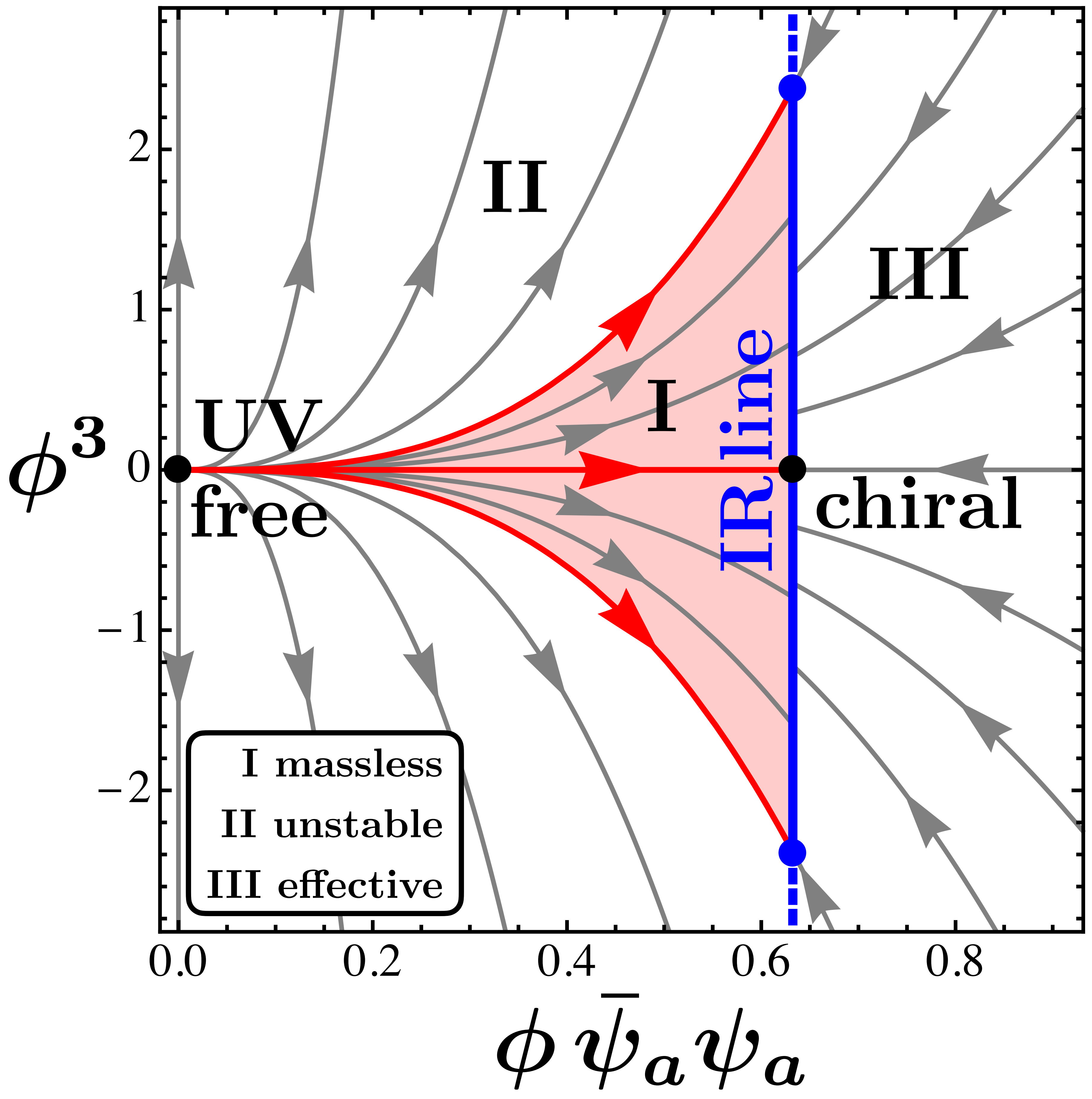


Gross-Neveu-Yukawa

$$S = \int_x \left\{ \tfrac{1}{2}(\partial\phi)^2 + \bar{\psi}_a \not{\partial} \psi_a + Y\phi\,\bar{\psi}_a\psi_a + U(\phi) \right\}$$

Asymptotically free: flows to GN CFT in IR

CCH, Litim, *in prep.*



Gross-Neveu-Yukawa

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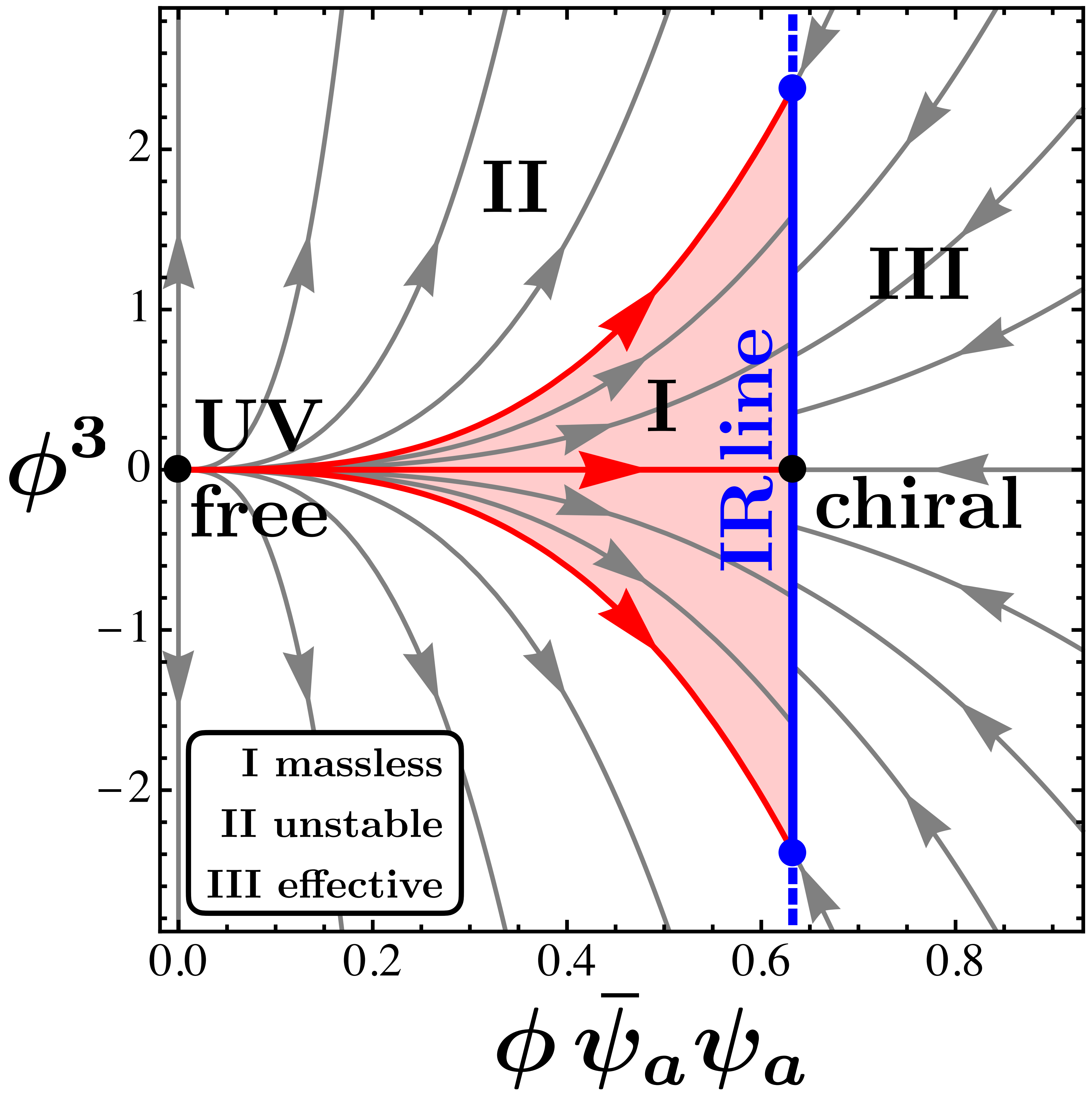
Asymptotically free: flows to GN CFT in IR

ϕ^3 exactly marginal

CCH, Litim, *in prep.*



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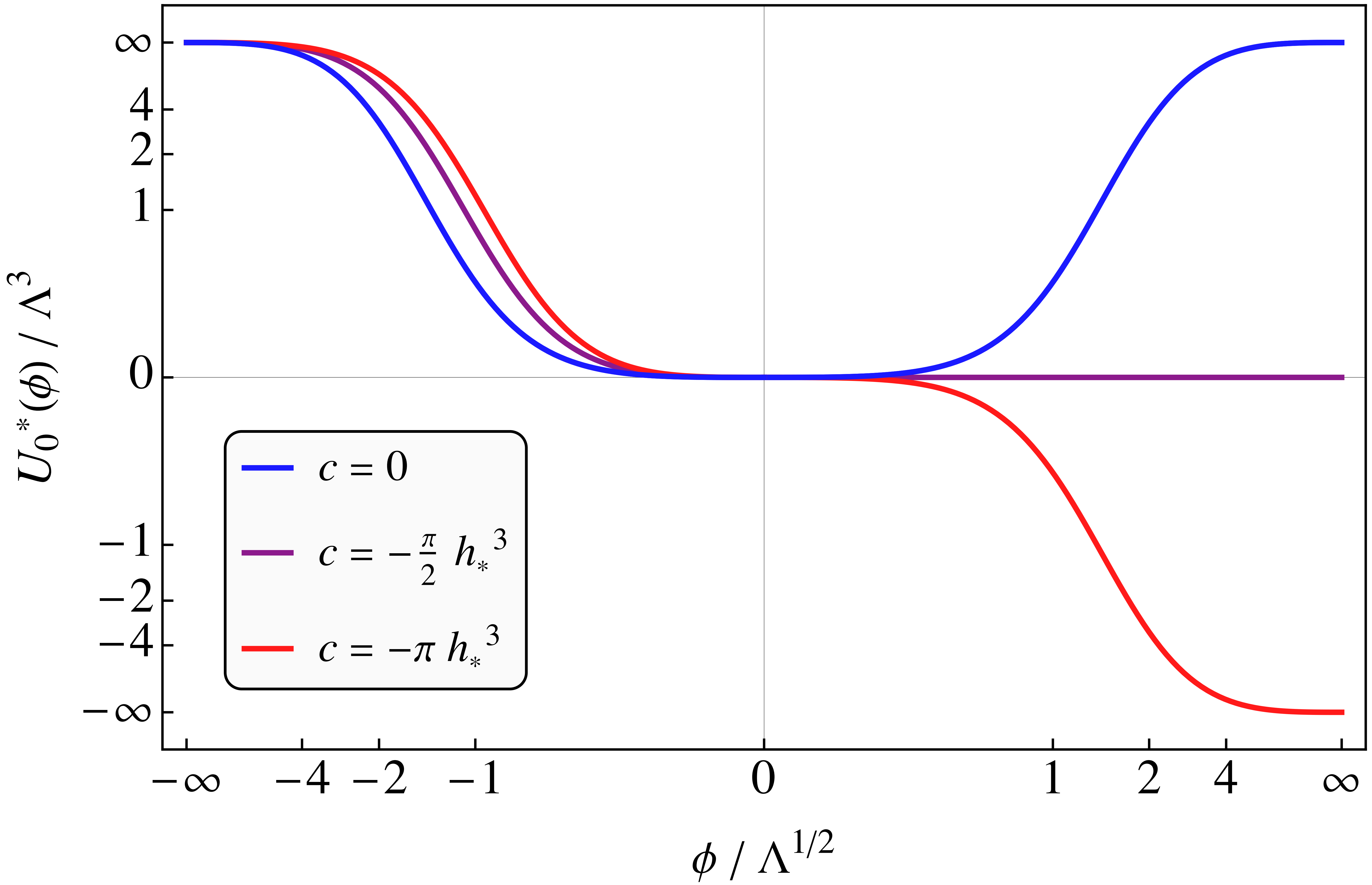

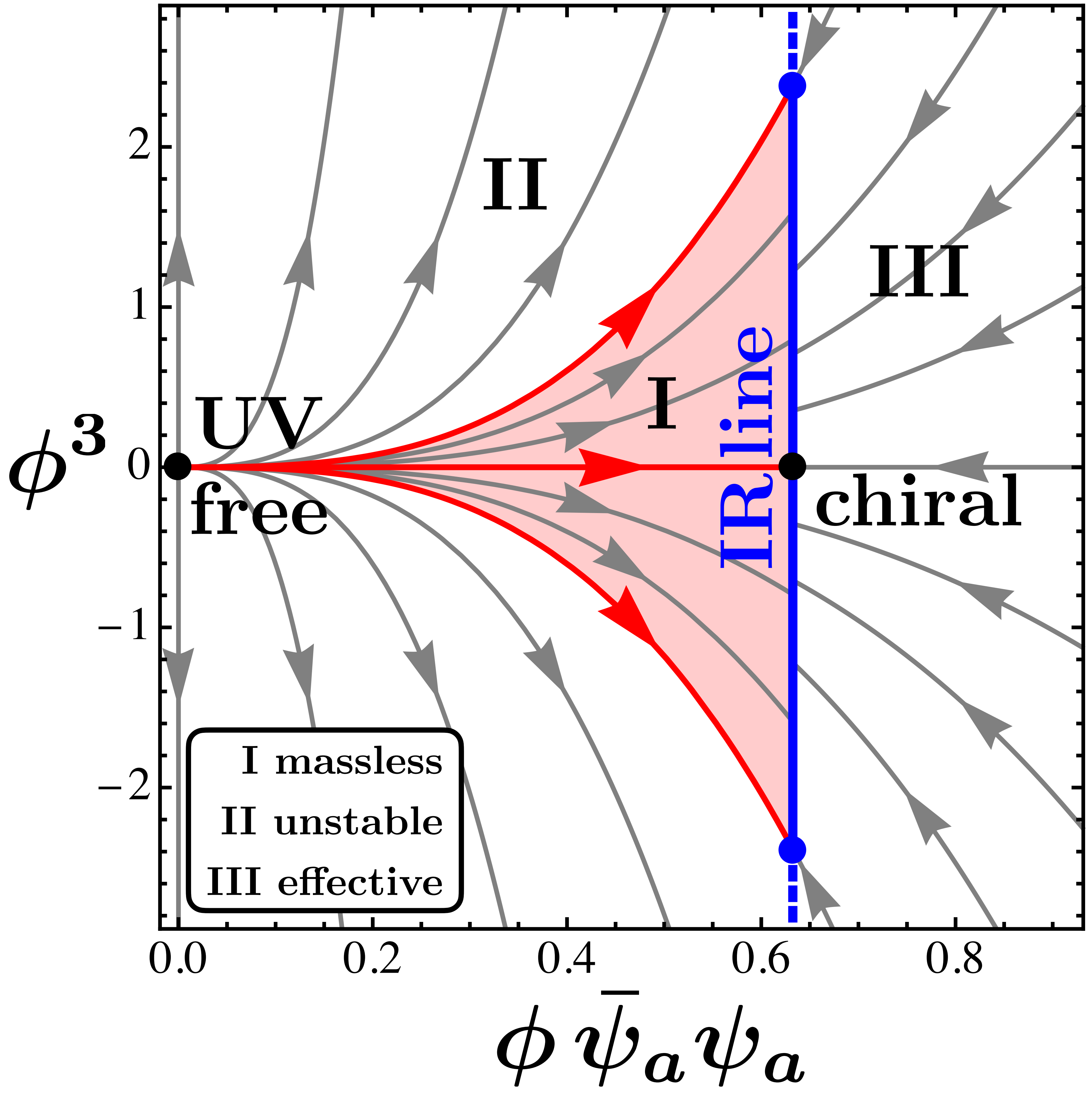

Gross-Neveu-Yukawa

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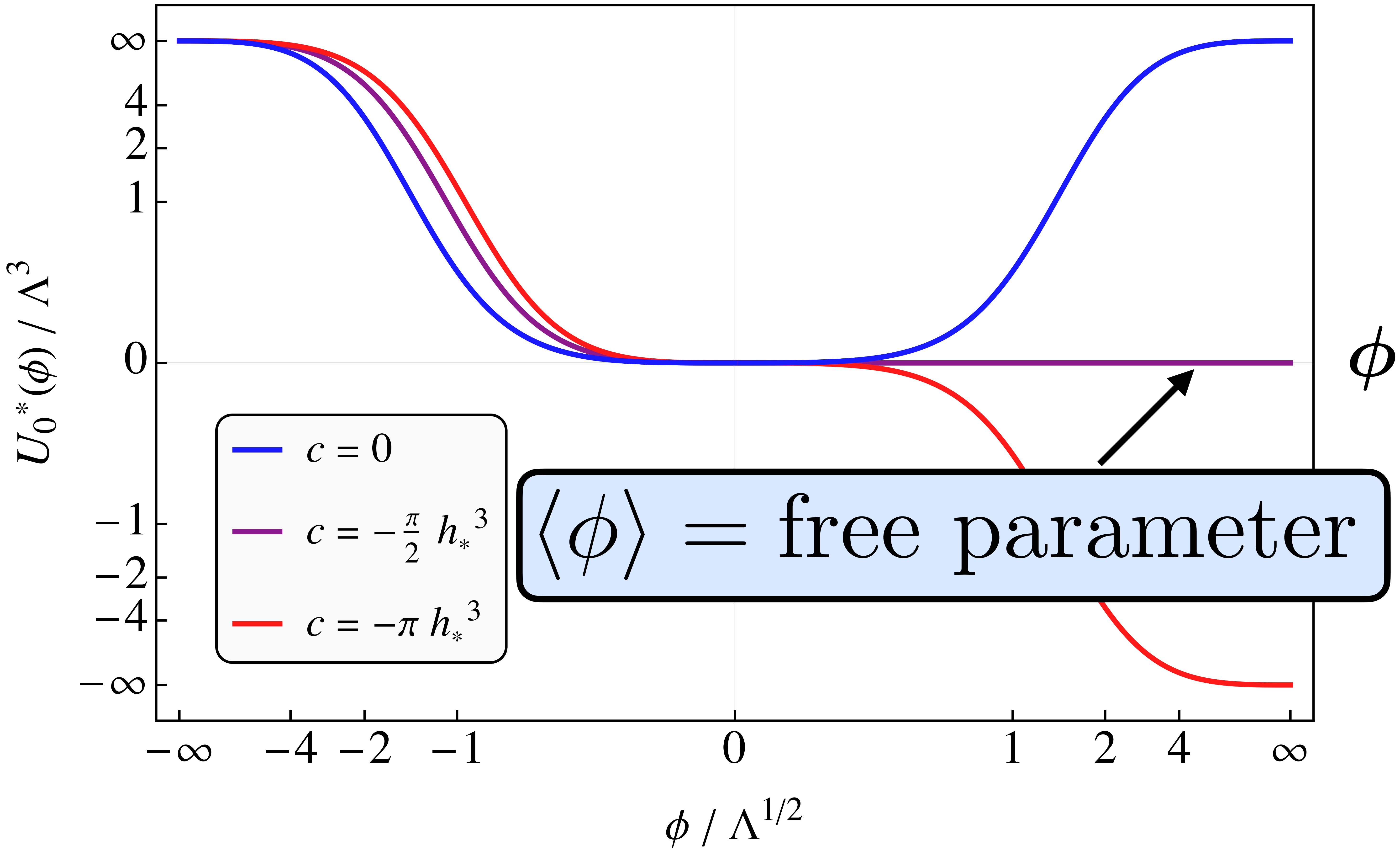
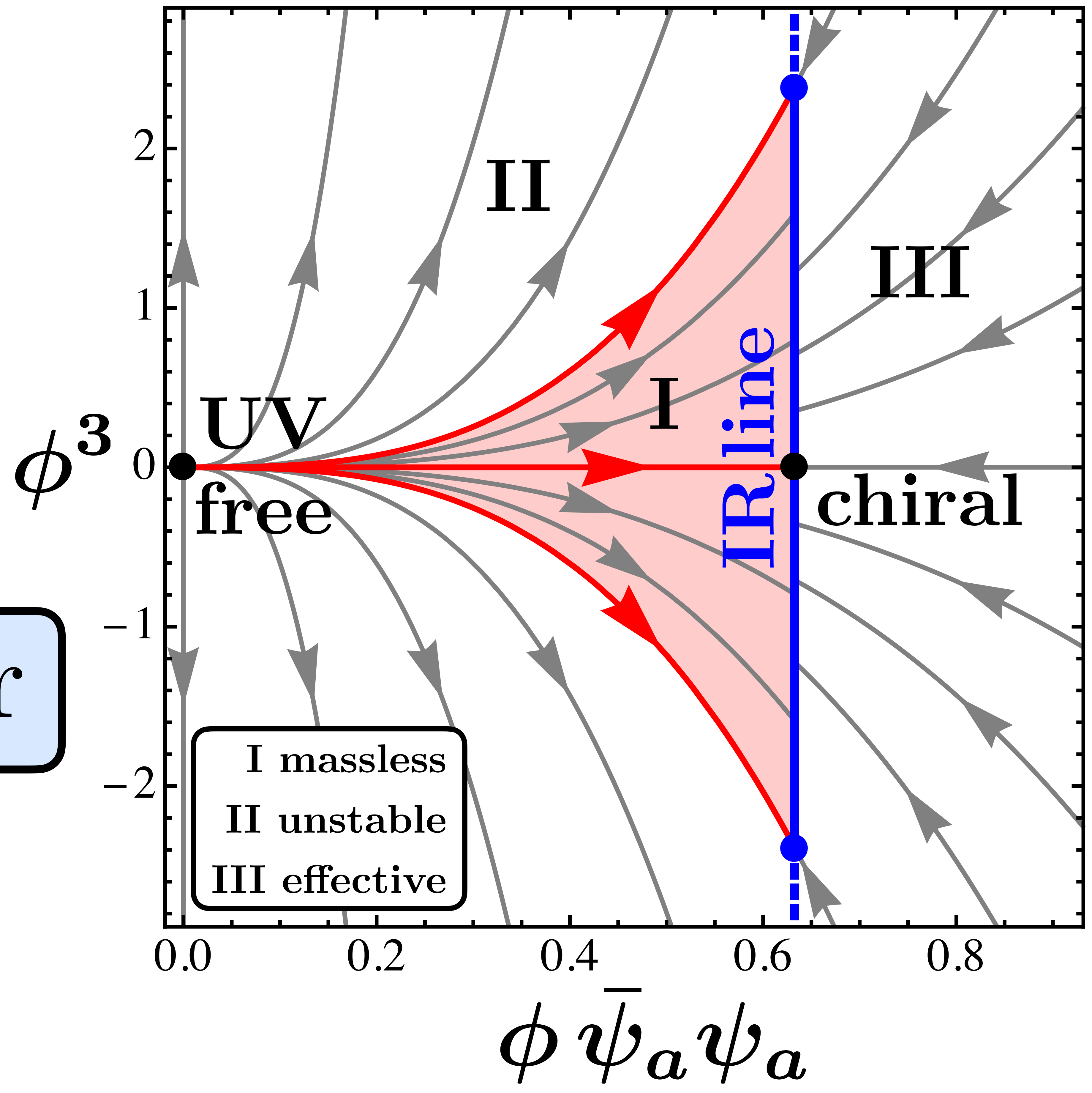
CCH, Litim, *in prep.*





Summary

- Fermionic RG allows exact solutions for effective potential at large N
- Explicit breaking of chiral symmetry opens up line of 6F fixed points
- Some theories exactly massless despite explicitly broken symmetry
- Evidence for duality with O(N) models, including fermionic BMB phenomenon
- Counterpart fixed points in Yukawa models with exactly marginal cubic term