# Dissipative dynamic of scalar field

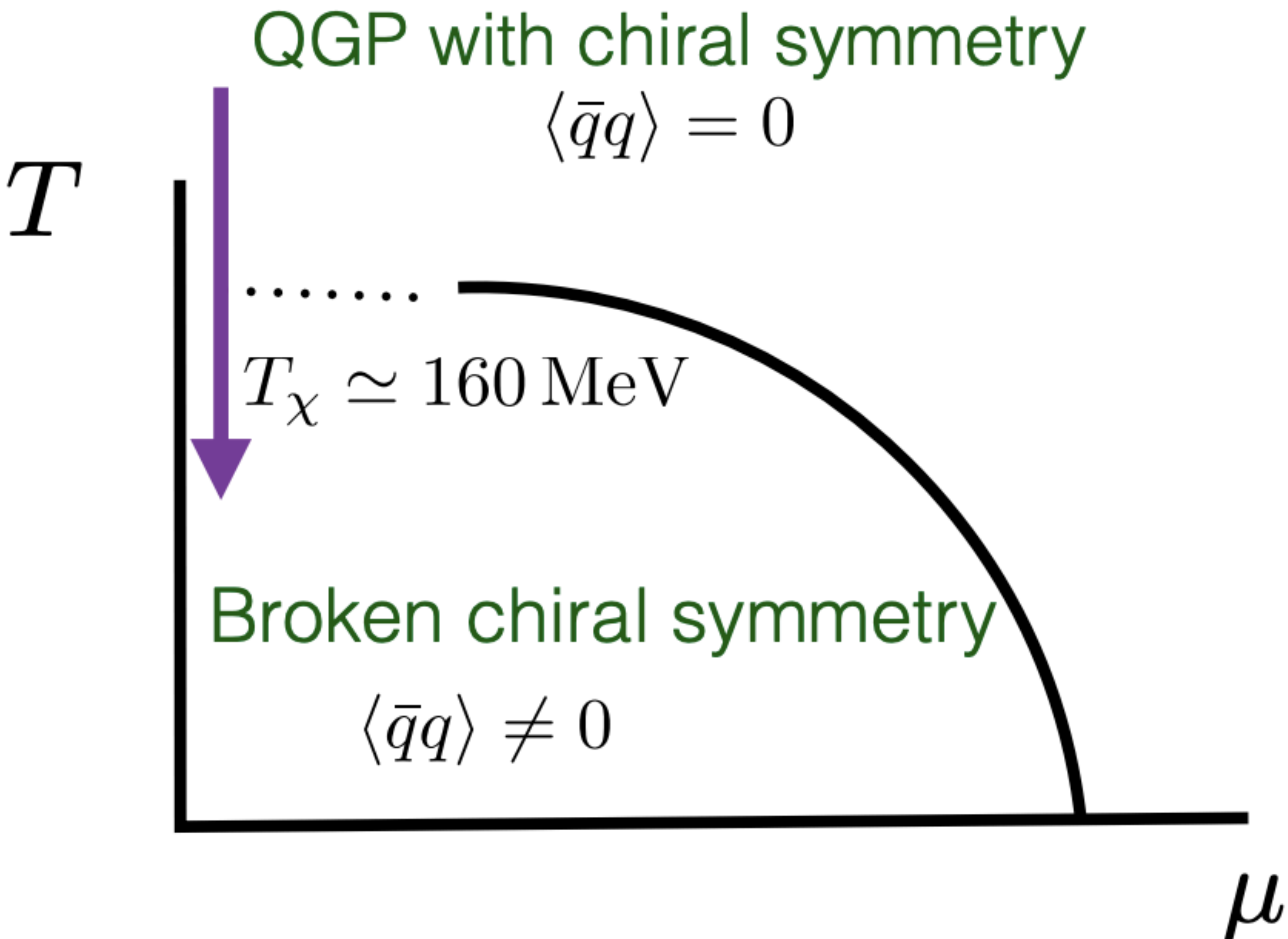
Eduardo Grossi
Florence University and INFN Florence

Istituto Nazionale di Fisica Nucleare
SEZIONE DI FIRENZE

L. Batini, E. Grossi and N. Wink, arXiv:2309.......

Hirschegg, 10/09/2023

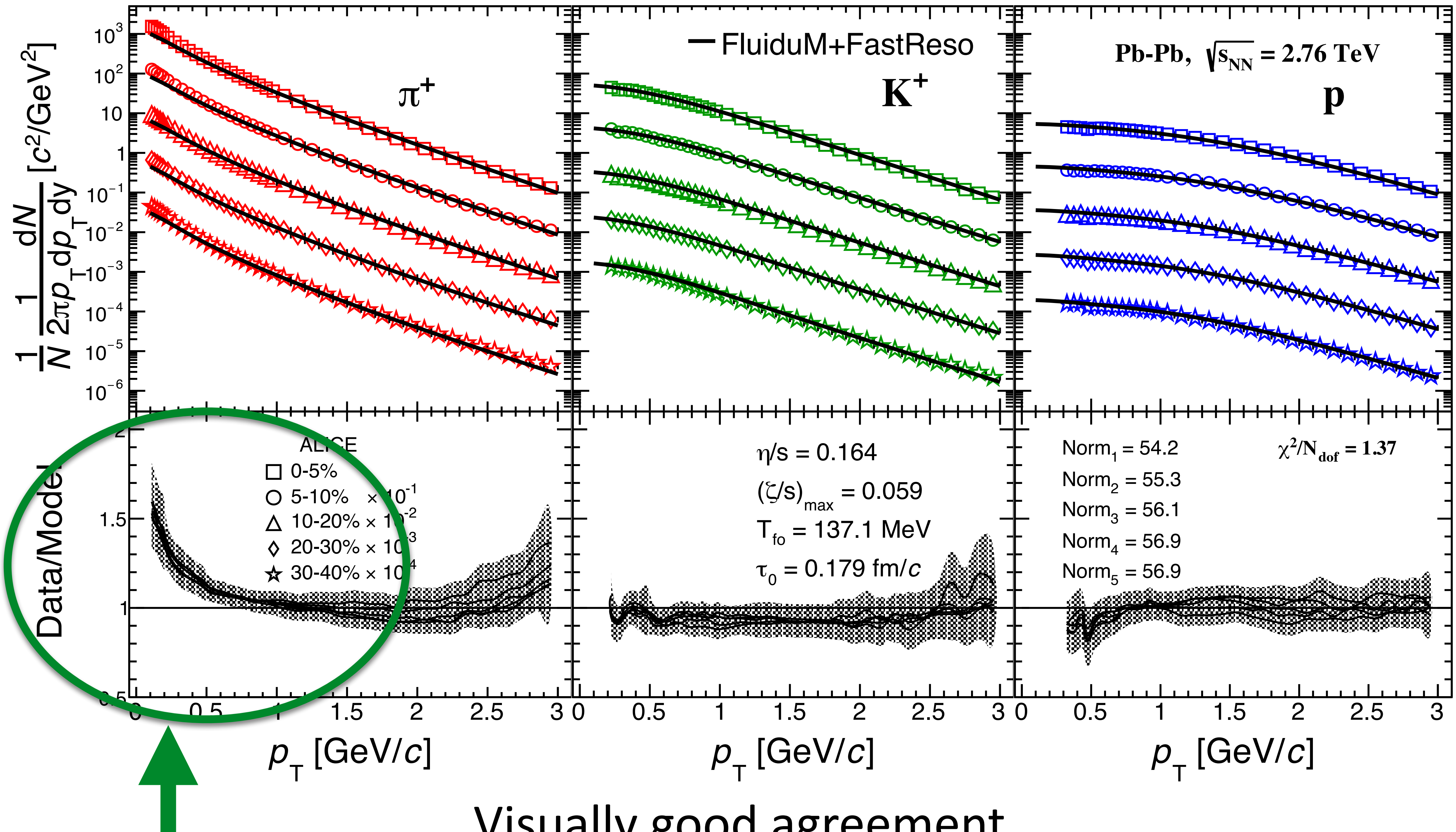
# Motivation 1




We are neglecting any hydro-dynamics of the chiral condensate !

# Maybe in the data ?

Fit the pt spectra of pions in the first five centralities




Visually good agreement,

but not amazing fit D. Devetak , et al JHEP (2019)

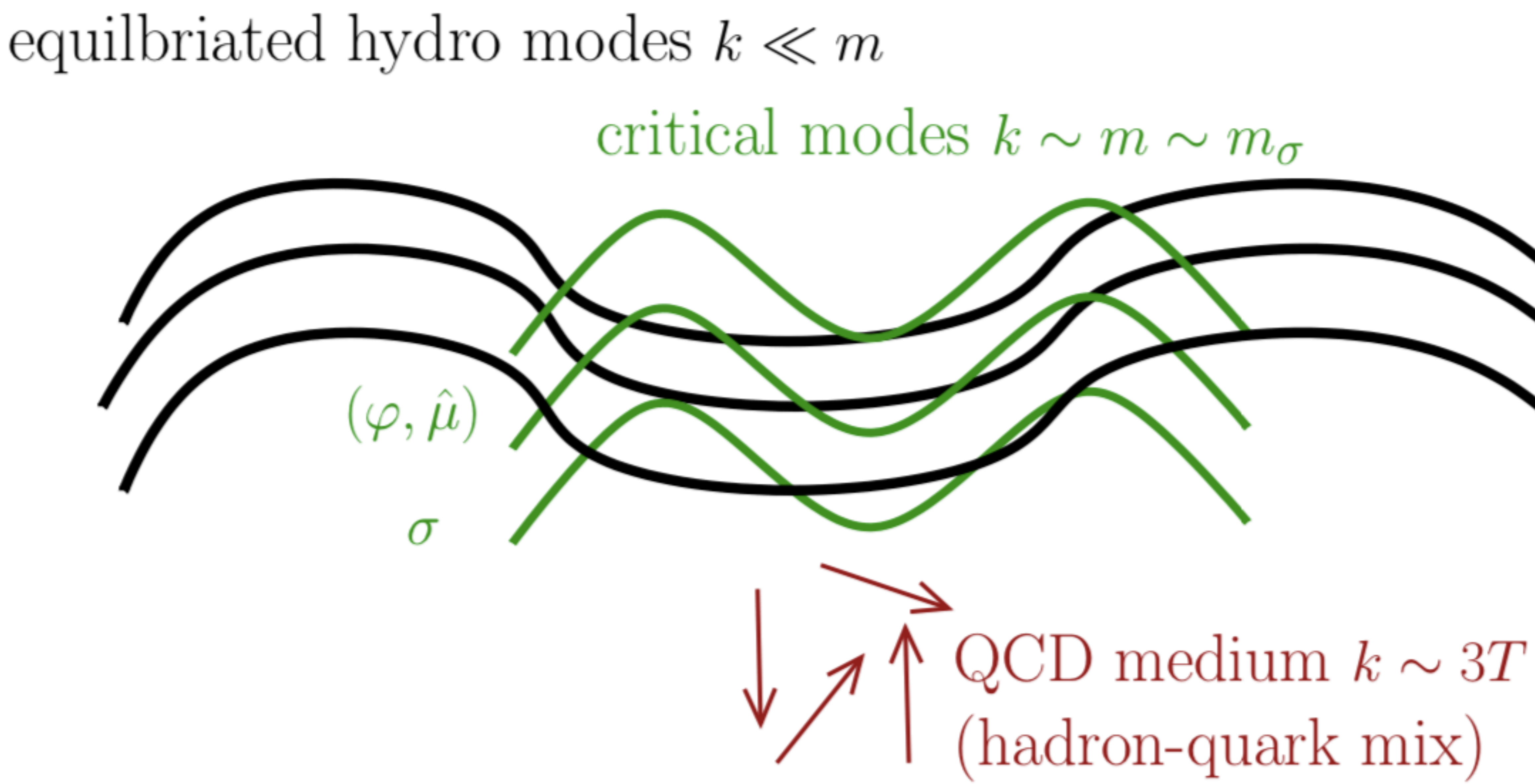
The main discrepancy is for pions at low pt

# Physical picture T~Tc

Working regime

Rajagopal Wilczek hep-ph/9210253
Son and Stephanov hep-ph/020422

$$k \ll m \sim m_\sigma \ll \pi T_C \sim \pi\Lambda_{QCD}$$




Soft pions+sigma mode on Hydro

# Equation of motion (Model G)

Rajagopal Wilczek (93)

Chiral condensate $\phi_a$ + Axial and Vector charge $n_{ab} = \chi_0 \mu_{ab}$

$$\partial_t \phi_a + g_0\, \mu_{ab} \phi_b = \Gamma_0 \nabla^2 \phi_a - \Gamma_0 (m_0^2 + \lambda \phi^2) \phi_a + \Gamma_0 H_a + \theta_a \,,$$

$$\partial_t n_{ab} + g_0\, \nabla \cdot (\nabla \phi_{[a} \phi_{b]}) + H_{[a} \phi_{b]} = D_0 \nabla^2 n_{ab} + \partial_i \Xi^i_{ab} \,.$$

Ideal part — Dissipative part — Gaussian Noise

- The ideal part is charge conservation and Josephson constraint
- Two dissipative coefficient $\Gamma_0$ and $D_0$ and noise

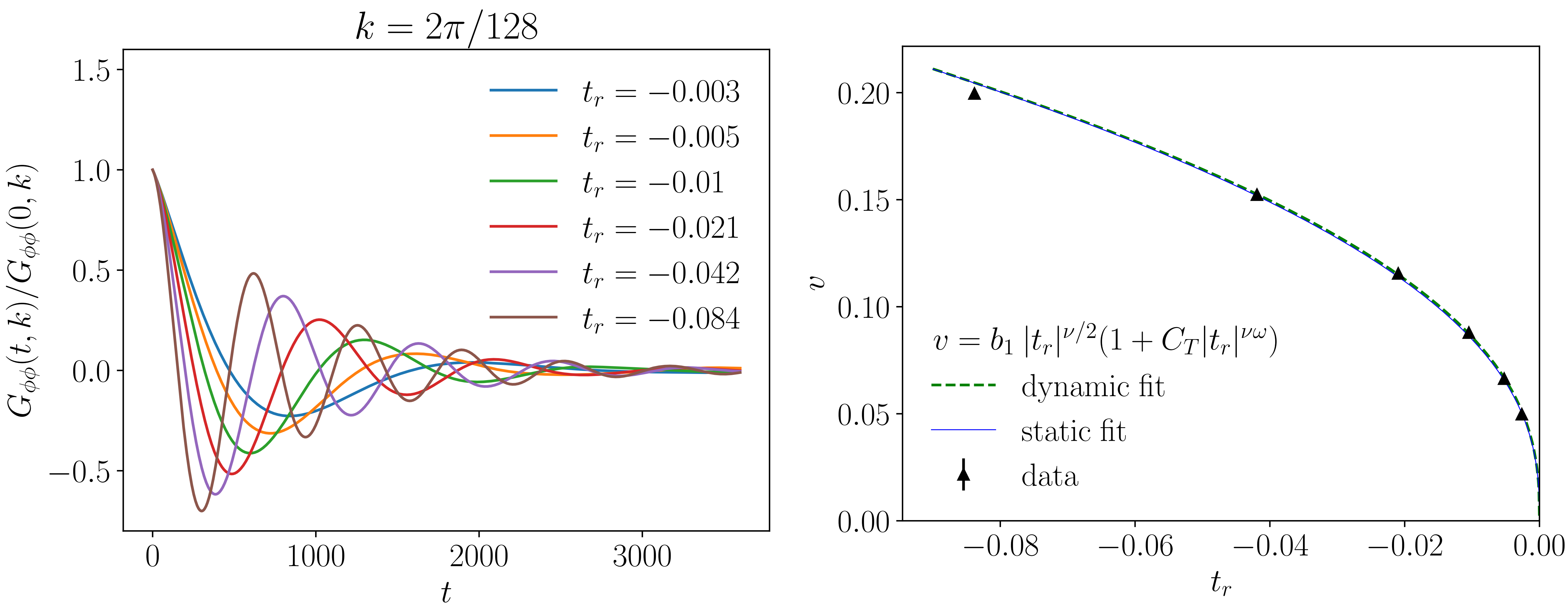
Diffusion at high temperature, pion propagation at low temperature as the vev develops

A. Florio, E.G., A. Soloviev, D, Teaney PRD (2022)

A. Florio, E.G., D, Teaney (2023)

# Snap shoot of what is going on

In the broken phase one has pion waves




$$\omega(k) = vk + v_1 k^2.$$

The dispersion relation of the of the waves actually is determine by the GOR relation

$$v^2 \propto \langle \phi^2 \rangle$$

# Langevin Dynamics

## Stochastic Process with a phase transition

$$\partial_t \phi(\mathbf{x}, t) = -\frac{1}{X} \frac{\delta \mathcal{H}[\phi]}{\delta \phi(t, \mathbf{x})} + \xi(t, \mathbf{x})$$

$$\langle \xi(t, \mathbf{x}) \rangle = 0$$

$$\big\langle \xi(t, \mathbf{x}) \xi(t', \mathbf{x}') \big\rangle = \frac{2T}{X} \delta^{(d)} \big(\mathbf{x} - \mathbf{x}'\big) \delta\big(t - t'\big)$$

## One can cast the stochastic process in a action

$$Z = \int \mathscr{D} \left[\phi_a, \phi_r\right] e^{iS\left[\phi_a, \phi_r\right]}$$

## Martin-Siggia-Rose action

$$\Gamma[\phi_a, \phi_r] = \int_{t,\mathbf{x}} \left[ \phi_a \frac{\delta \mathcal{H}}{\delta \phi_r} + X\big(\phi_a \partial_t \phi_r - i\phi_a^2\big) \right]$$

Hohenberg and B. I. Halperin, Rev. Mod. Phys. 49, 435 (1977),
P. Glorioso and H. Liu, arXiv:1805.09331
L. Canet, H. Chaté, B. Delamotte, arXiv:1106.4129,....

# Real time generating function

To undersand the renormalization is convienient introduce the Close time path integrals with a thermal density matrix

$$\mathcal{Z}[J_+, J_-] = \int \mathcal{D}\varphi_+ \mathcal{D}\varphi_- \rho[\varphi_+, \varphi_-] e^{iS[\varphi_+, J_+] - iS[\varphi_-, J_-]} ,$$

In thermal equilibrium we have the KMS symmetry

$$\mathcal{T}_\beta \varphi_\pm(t, \boldsymbol{x}) = \varphi_\pm(-t \pm i\beta/2, \boldsymbol{x})$$

The good basis is the r/a defined as

$$\varphi_r(t, \boldsymbol{x}) = \frac{1}{2}(\varphi_+(t, \boldsymbol{x}) + \varphi_-(t, \boldsymbol{x})) ,$$

$$\varphi_a(t, \boldsymbol{x}) = \varphi_+(t, \boldsymbol{x}) - \varphi_-(t, \boldsymbol{x})$$

In this basis there are two symmetry that one can exploit to Construct consitent truncation for high temperature

# Symmetry of the effective action

For high temperature one obtain a local disctrete symmetry

$$\mathcal{T}_\beta \varphi_r(x) = \Theta\varphi_r(x) = \varphi_r\left(t_R, \boldsymbol{x}\right) \ ,$$

$$\begin{aligned}\mathcal{T}_\beta \varphi_a(x) &\simeq \Theta\varphi_a(x) + i\Theta\partial_t\varphi_r(x) \\ &= \varphi_a\left(t_R, \boldsymbol{x}\right) - i\partial_{t_R}\varphi_r\left(t_R, \boldsymbol{x}\right) \ ,\end{aligned}$$

The time derivative of the calssical filed is related to the quantum one

No terms with no a field are allowd

$$\mathcal{Z}[J_+ = J, J_- = J] = 1$$

$$(-i)^{n+1}\frac{\delta^n \log \mathcal{Z}[J_r, J_a]}{\delta J_r \cdots \delta J_r} = G_{a\cdots a} = 0$$

# High temperature expansion

P. Glorioso and H. Liu, arXiv:1805.09331

L. Canet, H. Chaté, B. Delamotte, arXiv:1106.4129

The time derivate mix with a field then is natural to organize the action in monomials

$$\Gamma[\Phi] = \int_{t,\boldsymbol{x}} \mathcal{L} = \int_{t,\boldsymbol{x}} \left\{ E\phi_a + X\phi_a^2 + Y\phi_a^3 \right\},$$

Expand in power of time time derivative

$$E = E_0 + E_1 + E_2 \,,$$ Up to 2 derviative

$$X = X_0 + X_1 \,,$$ Up to 1 derviative

$$Y = Y_0 \,.$$ Only constant term

Impose the flactuation dissiaption theorem

$$\Gamma[\Phi] = \Gamma[\mathcal{T}_\beta \Phi] + \int_{t,\boldsymbol{x}} \partial_\mu V^\mu$$

# High temperature expansion

The action up to first order time derivative

$$\Gamma_1[\Phi] = \int_{t,\boldsymbol{x}} \left\{ \phi_a \frac{\delta \mathcal{H}}{\delta \phi_r} + X(\phi_a \partial_t \phi_r - \frac{i}{\beta}\phi_a^2) \right\}$$

The action up to second order  derivative

$$\Gamma_2[\Phi] = \int_{t,\boldsymbol{x}} \Big\{ - i\frac{2}{\beta} Z_t \partial_t^2 \phi_r \phi_a$$

$$- 2i\frac{1}{\beta^3} \partial_{\phi_r} Z_t \left( \phi_a^3 - \frac{3i\beta}{2} \left(\partial_t \phi_r\right) \phi_a^2 - \frac{1}{2}\beta^2 (\partial_t \phi_r)^2 \phi_a \right) \Big\} \,.$$

The coefficient can be field dependent

# Schwinger Keldysh

Truncation up to first order in the derivative with Field depndence potential and relaxation rate

$$\Gamma_k[\Phi] = \int_{t,\mathbf{x}} \phi_a \Big( X_k(\phi_r)(\partial_t \phi_r - i\phi_a) - \nabla^2 \phi_r + U_k^{(1)}(\phi_r) \Big),$$

Relaxation rate ($X_k$)

Effective potential derivative ($U_k^{(1)}$)

$$\Gamma_{k=\Lambda} = \mathcal{S} \longrightarrow \Gamma_{k=0} = \Gamma$$

$$U_\Lambda(\phi) \equiv -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

$$X_\Lambda(\phi) \equiv 1$$

With Spatial Litim regulator

Already studied in detail by many authors

Hohenberg and B. I. Halperin, Rev. Mod. Phys. 49, 435 (1977),
L. Canet, H. Chaté, B. Delamotte, arXiv:1106.4129

# Numerical implementation

- Litim regulator

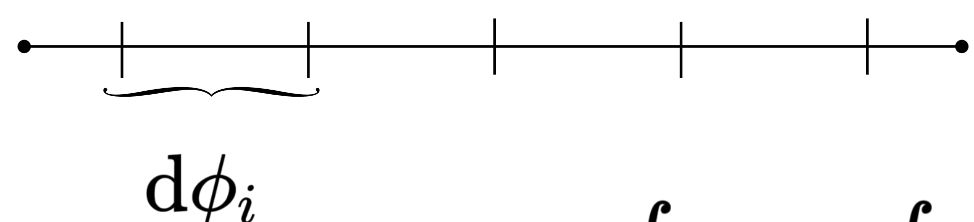
$$\partial_k U^{(1)}(\phi) = \frac{\partial}{\partial \phi}\left(\frac{\Omega_d}{(2\pi)^d}\frac{k^{d+1}}{d}\frac{1}{U^{(2)}+k^2}\right)$$

Transport equation

$$\longrightarrow \quad \partial_t u = \partial_\phi f$$

$$\partial_k X(\phi) = \frac{\Omega_d k^{d+1}}{2(2\pi)^d}\left[3(\partial_\phi G)^2 X + 4\partial_\phi(G^2)X^{(1)} + 2G^2 X^{(2)}\right] \qquad G = \frac{1}{k^2+U^{(2)}}$$

- Field discretization




- First-order upwind scheme

$$\partial_t u_i + \frac{f_{i+1}-f_i}{\mathrm{d}\phi_i} = 0$$

- Alternating pattern $\longrightarrow$

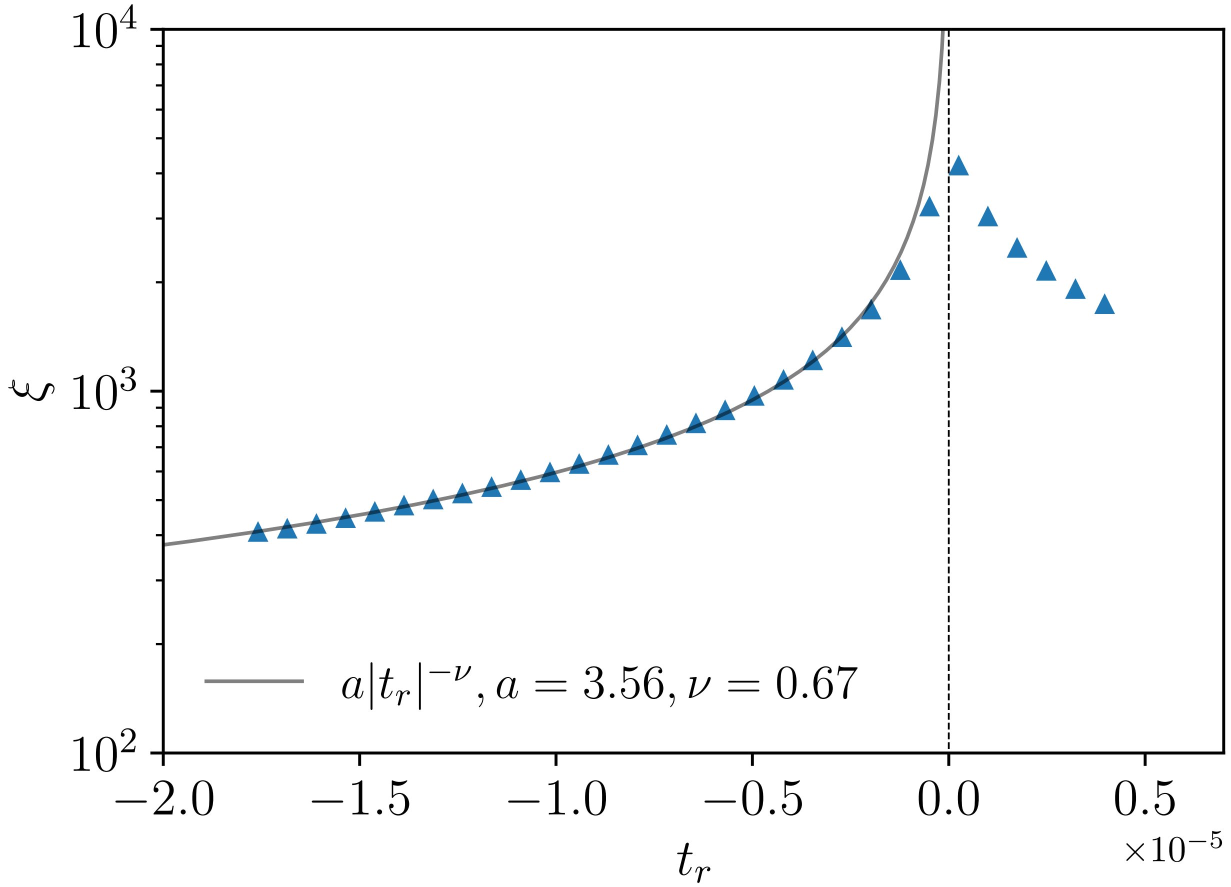
$$\partial_t u_i = \frac{f^R_{i+1} - f^L_i}{\mathrm{d}\phi_i}.$$

$$f_i^R = f\left(t, \frac{u_{i+1}-u_i}{\mathrm{d}\phi_i}\right), \quad f_i^L = f\left(t, \frac{u_i - u_{i-1}}{\mathrm{d}\phi_i}\right)$$

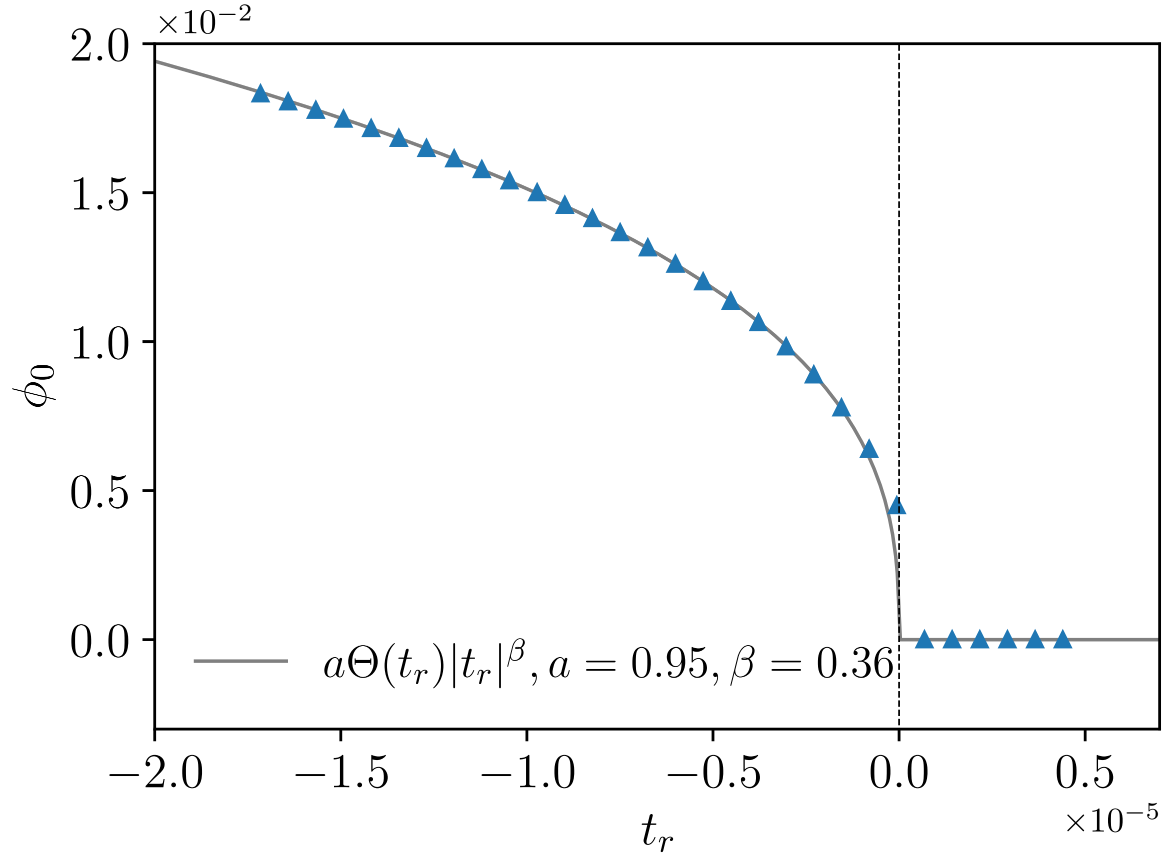
- Similar for the relaxation rate equation

# Static exponent

## Correlation lenght




## Field expectation value




$$\xi(t_r) = \xi_{\pm}|t_r|^{-\nu}$$

$$\phi_0(t_r) = a|t_r|^{\beta},$$

$$\xi(t_r) \equiv \sqrt{\frac{1}{U^{(2)}}}$$

$$U^{(1)} = 0$$

# Critical Slowing Down

Near the critical point

- Correlation length diverges
- Correlation time diverges $\xi \sim |T - T_c|^{-\nu}$

$$t_c \sim \xi^z \sim |T - T_c|^{-z\nu}$$

dynamical critical exponent

Correlation time

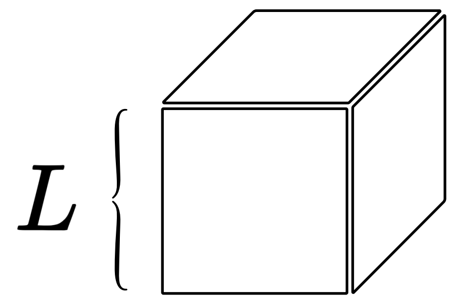
Correlation length

Critical temperatur

# Finite size Critical temperature

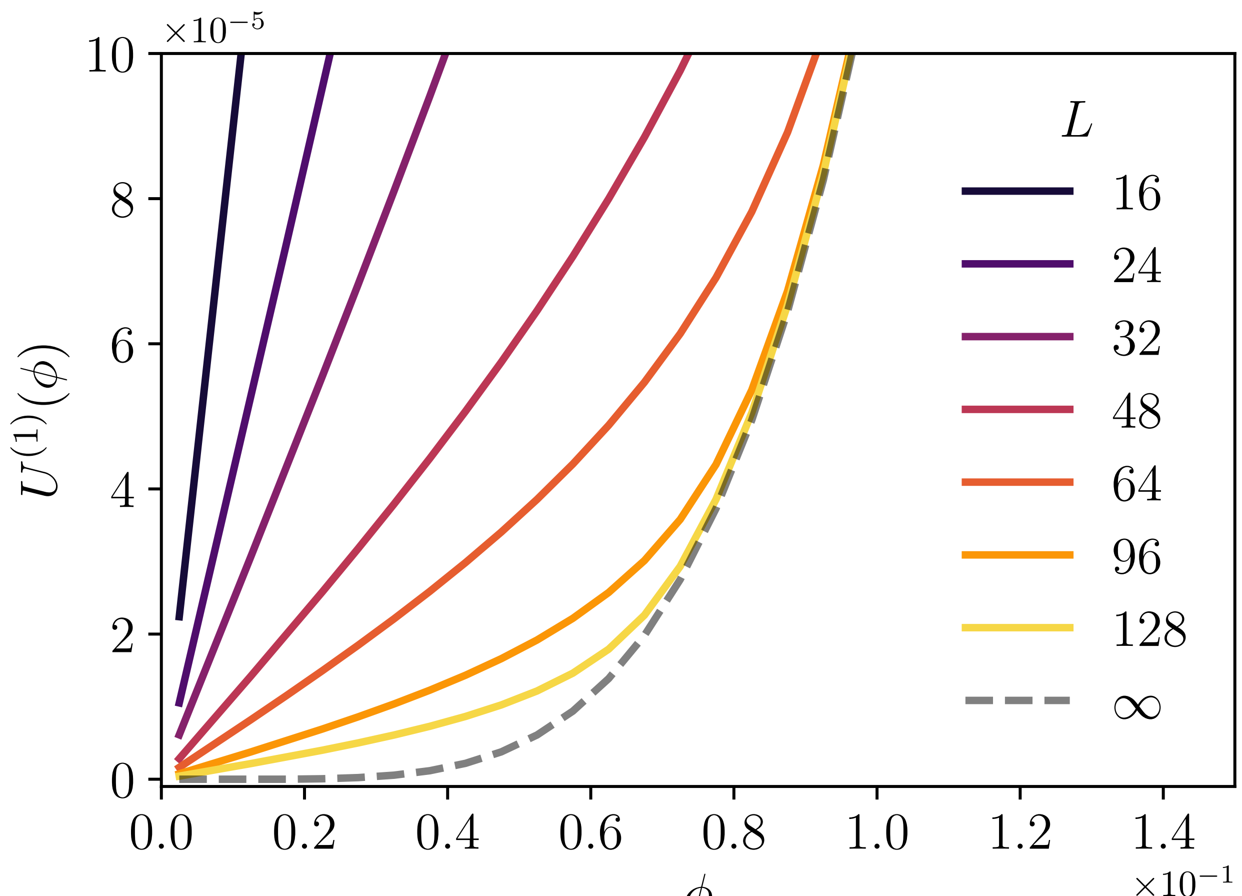
- Periodic boundary conditions
- Discretize the momenta in a box

$$\int_{-\infty}^{\infty} dp_i \to \frac{2\pi}{L} \sum_{n_i \in \mathbb{Z}}$$

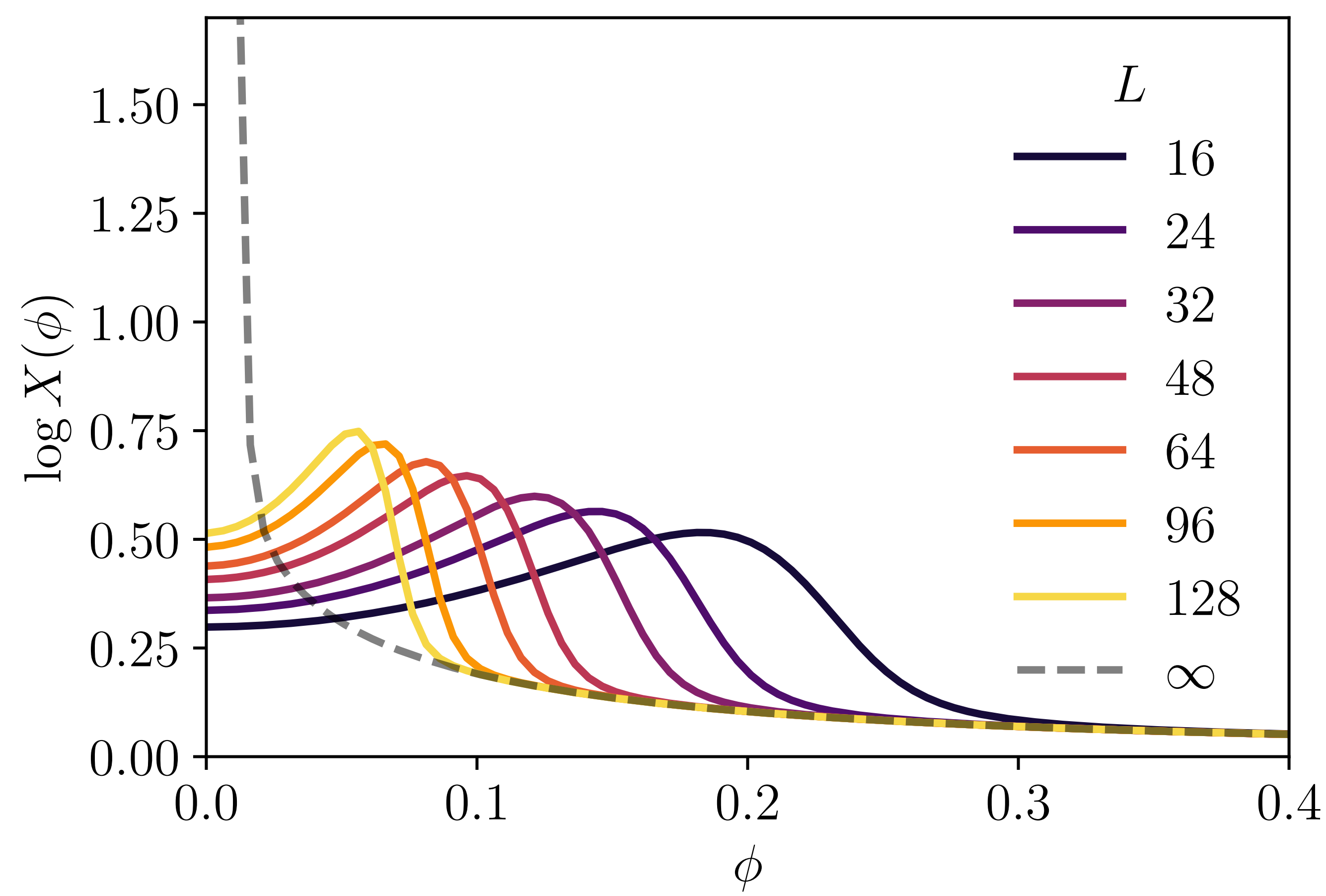



$$t_c \equiv \frac{X(\phi_c)}{U^{(2)}(\phi_c)}$$

Eff. Potential derivative




Relaxation rate



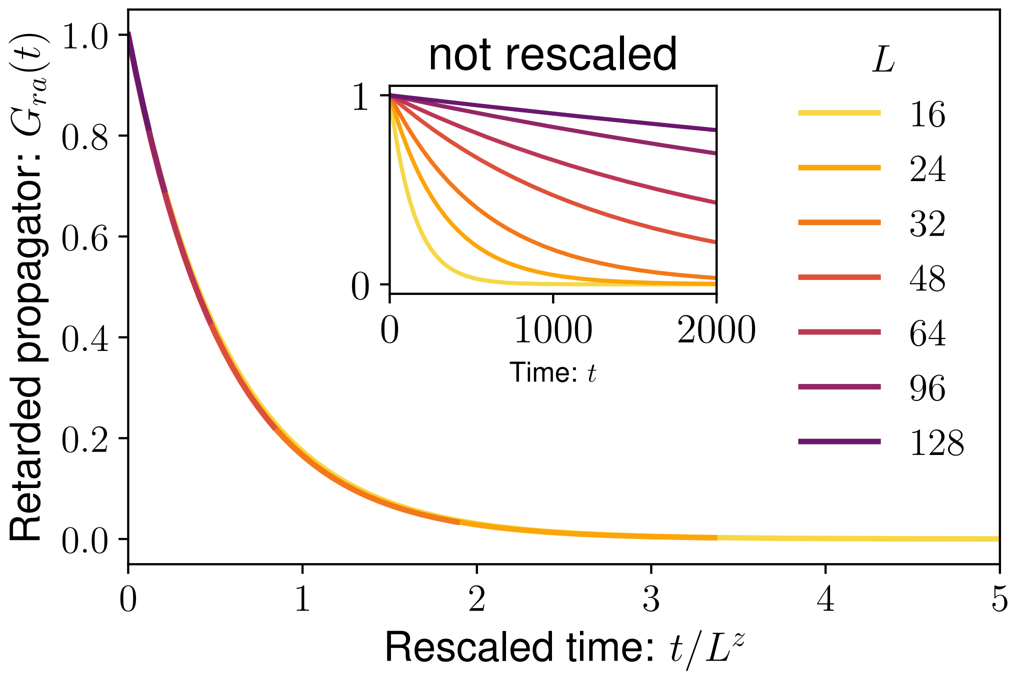
# Critical scaling

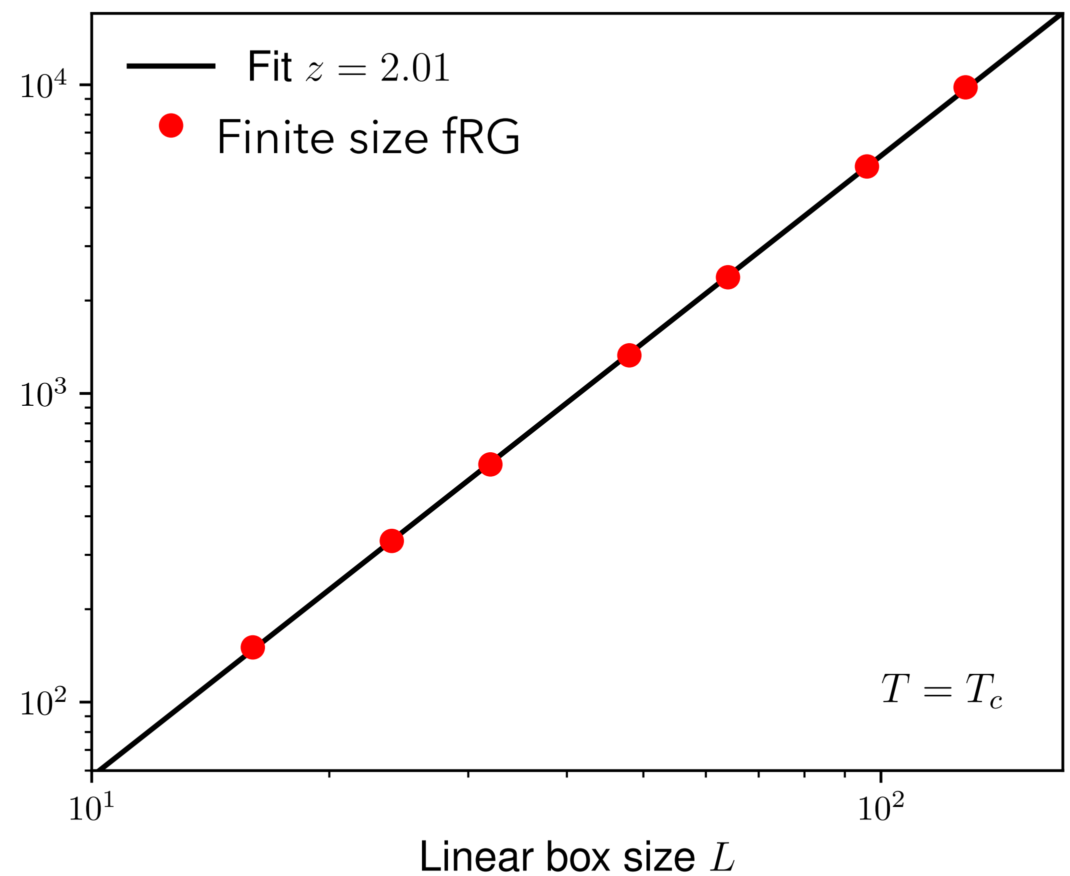
- Scaling form
- Retarded function

$$t_c(T,L) = L^z f_{t_c}\left(\Delta T L^{1/\nu}\right) \approx L^z f(0)$$

$$G_{ra}(t, p=0) = \exp\left(-\frac{U^{(2)}}{X}t\right)\Theta(t)$$

Relaxation time $t_c \equiv \dfrac{X(\phi_c)}{U^{(2)}(\phi_c)}$




$z \approx 2.05$ fRG LPA L. Canet and H. Chaté (2007) J. Phys. A: Math. Theor. 40 1937

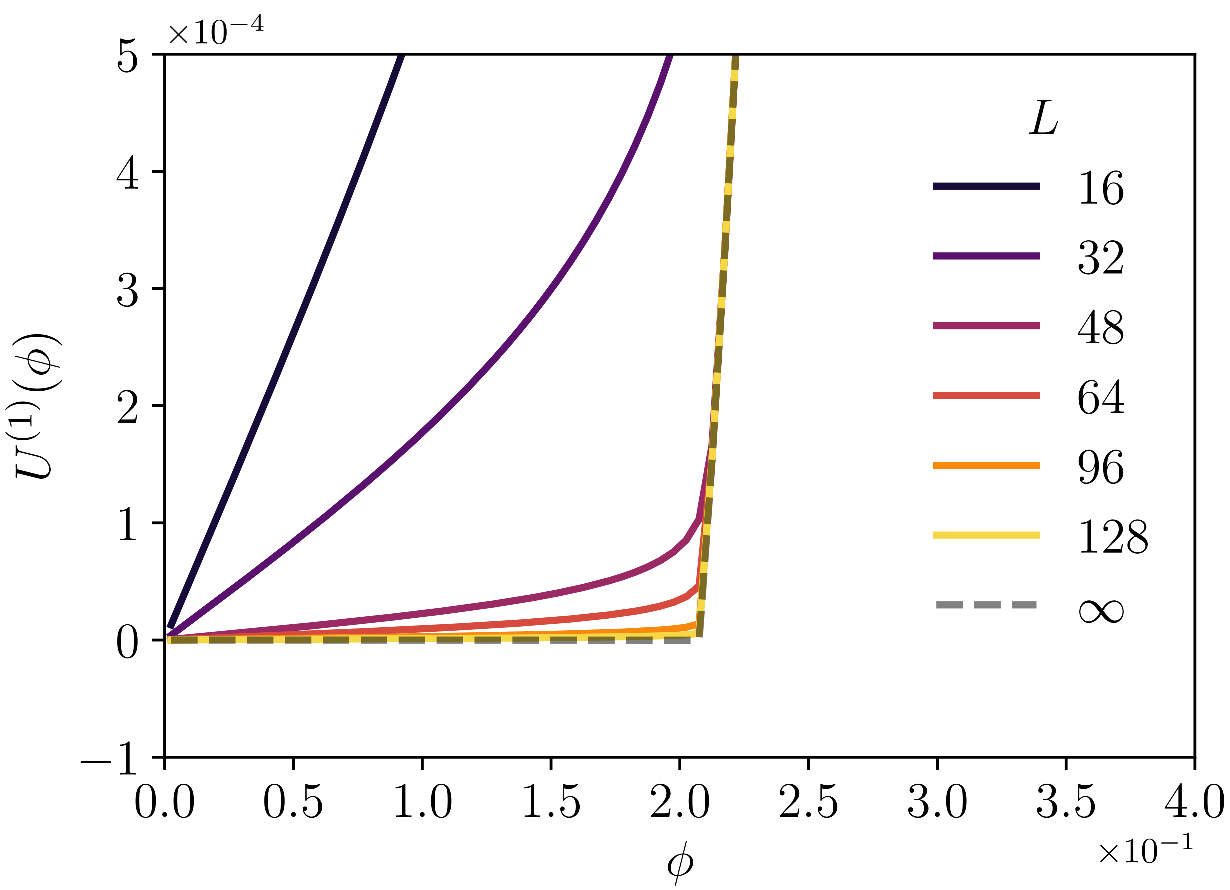
$z \approx 2.03$ MC N. Ito, et al. (2000) J. Phys. Soc. Japan 69 1931

Really close to mean field

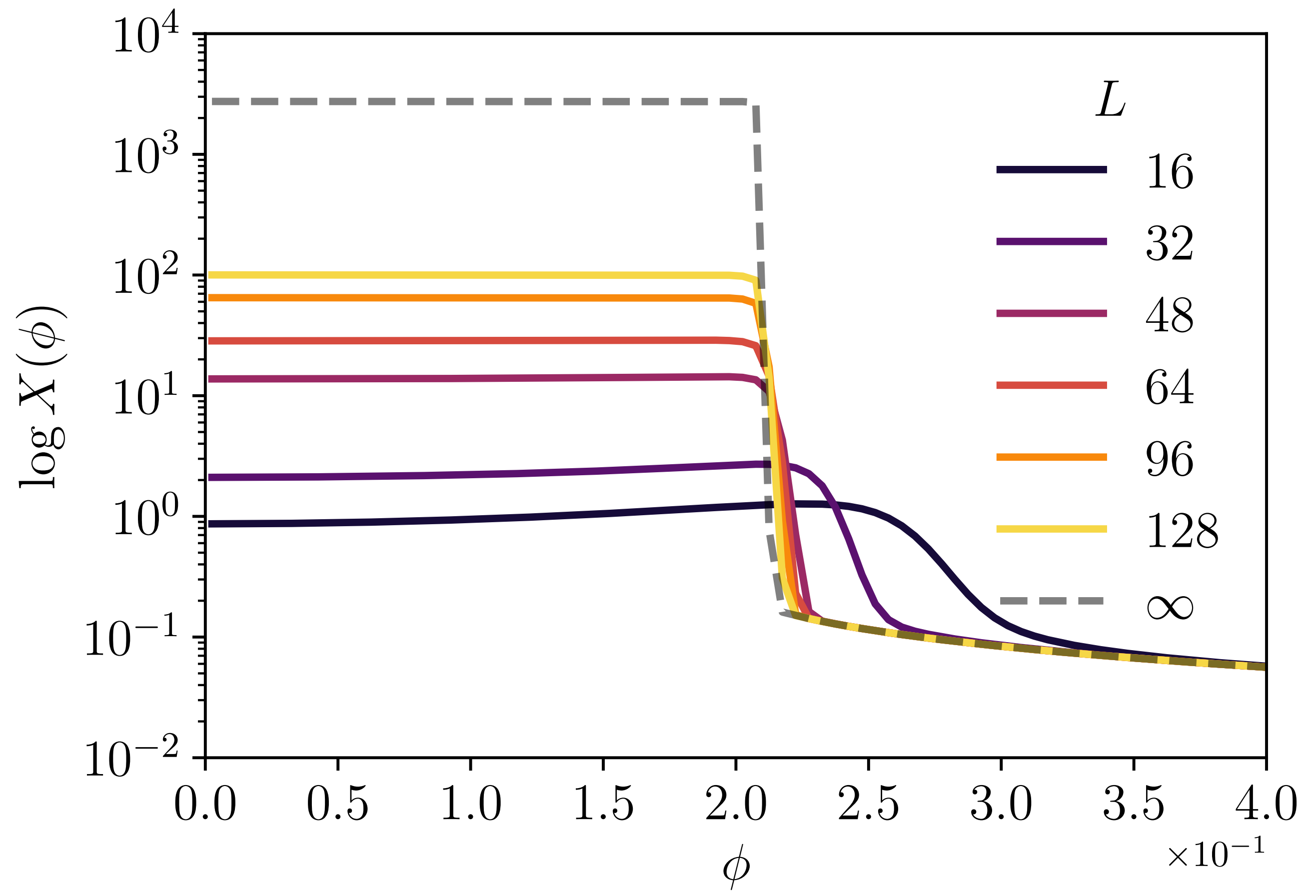
# Finite size in broken phase

In the broken phase things are getting weird

The potential get flat and




The relaxation rate get infinity




The flat region of the is non accessible from the field

$$t_c \equiv \frac{X(\phi_c)}{U^{(2)}(\phi_c)}$$

# Summary and Outlook

- We studied the the field dependence of the relaxation rate for Ising model
- The critical exponent are consitent with previous extraction
- In the broken phase the field dependece diverge in the flat region of the potential
- Can be extendend with a lot of pain to the O(4) model.
- About the wave function renormalization?
- How much it is for Model G in real QCD? Is small is big compare to the expantion rate of the fireball?

Thank you for your attention!