

Unfolding strongly correlated QCD

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Universität Heidelberg & ExtreMe Matter Institute

Hirschegg, September 11th 2023

GEFÖRDERT VOM



Bundesministerium
für Bildung
und Forschung



STRUCTURES
CLUSTER OF
EXCELLENCE



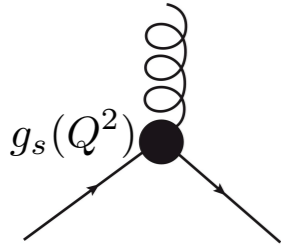
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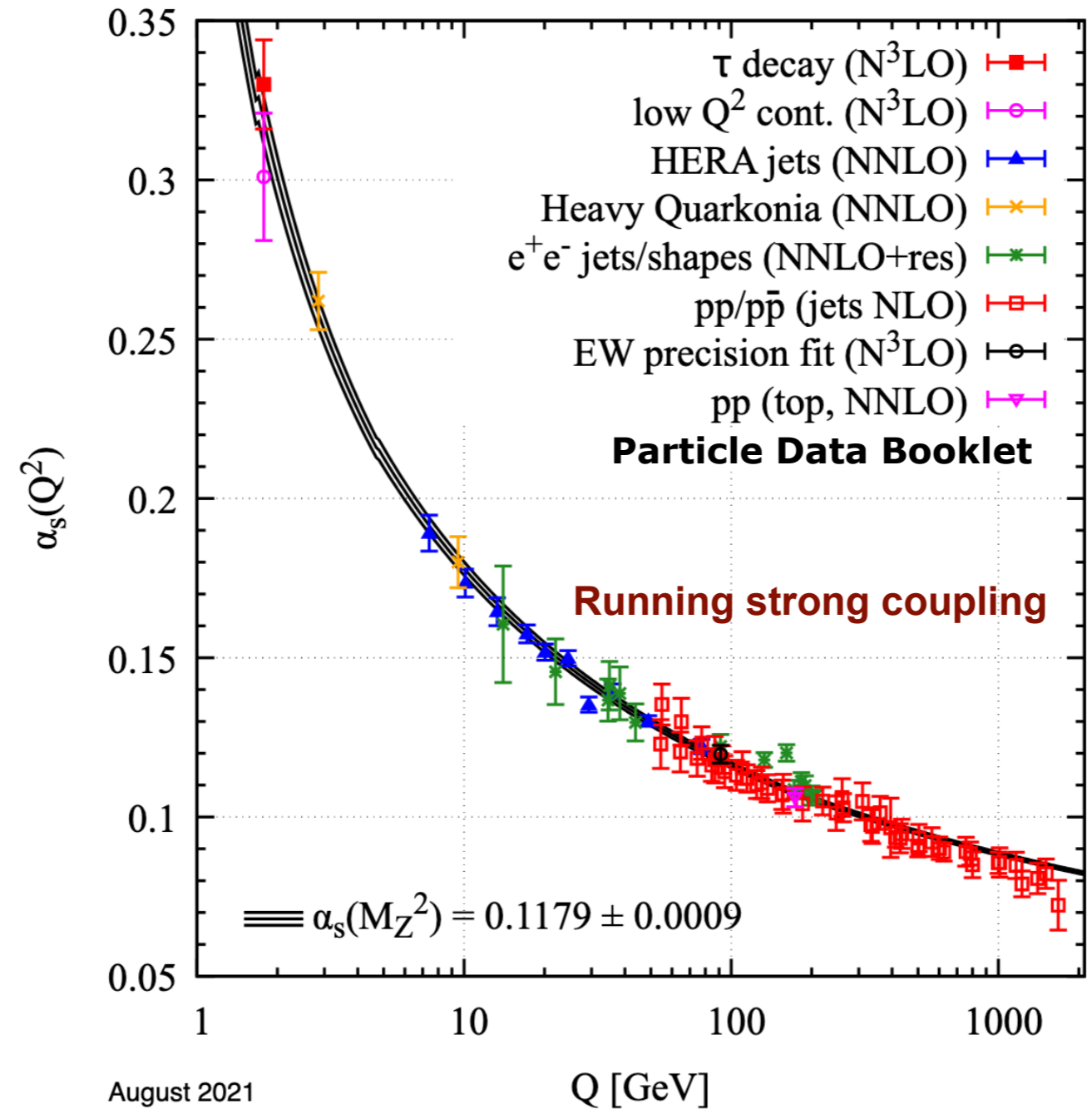
What's hot in QCD?

Low Temperatures, densities

Large Temperatures, densities



$$\alpha_s(Q^2) = \frac{g_s(Q^2)}{4\pi}$$

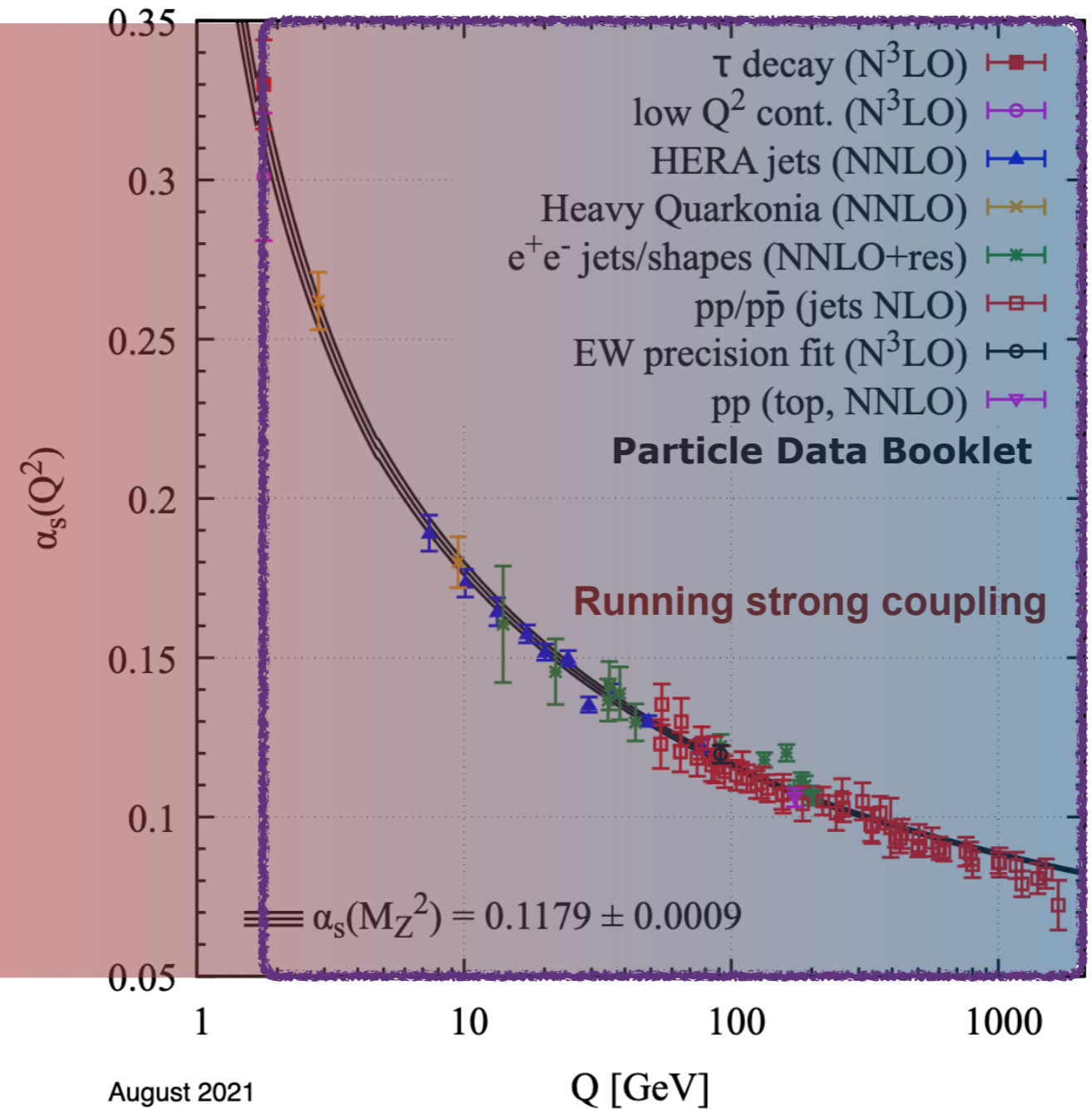


August 2021

What's hot in QCD?

Low Temperatures, densities

Large Temperatures, densities



QCD background
for
SM & BSM physics

crucial for 'unfolding'

deconvolution
reconstruction

⋮

August 2021

UV: weakly coupled quarks and gluons

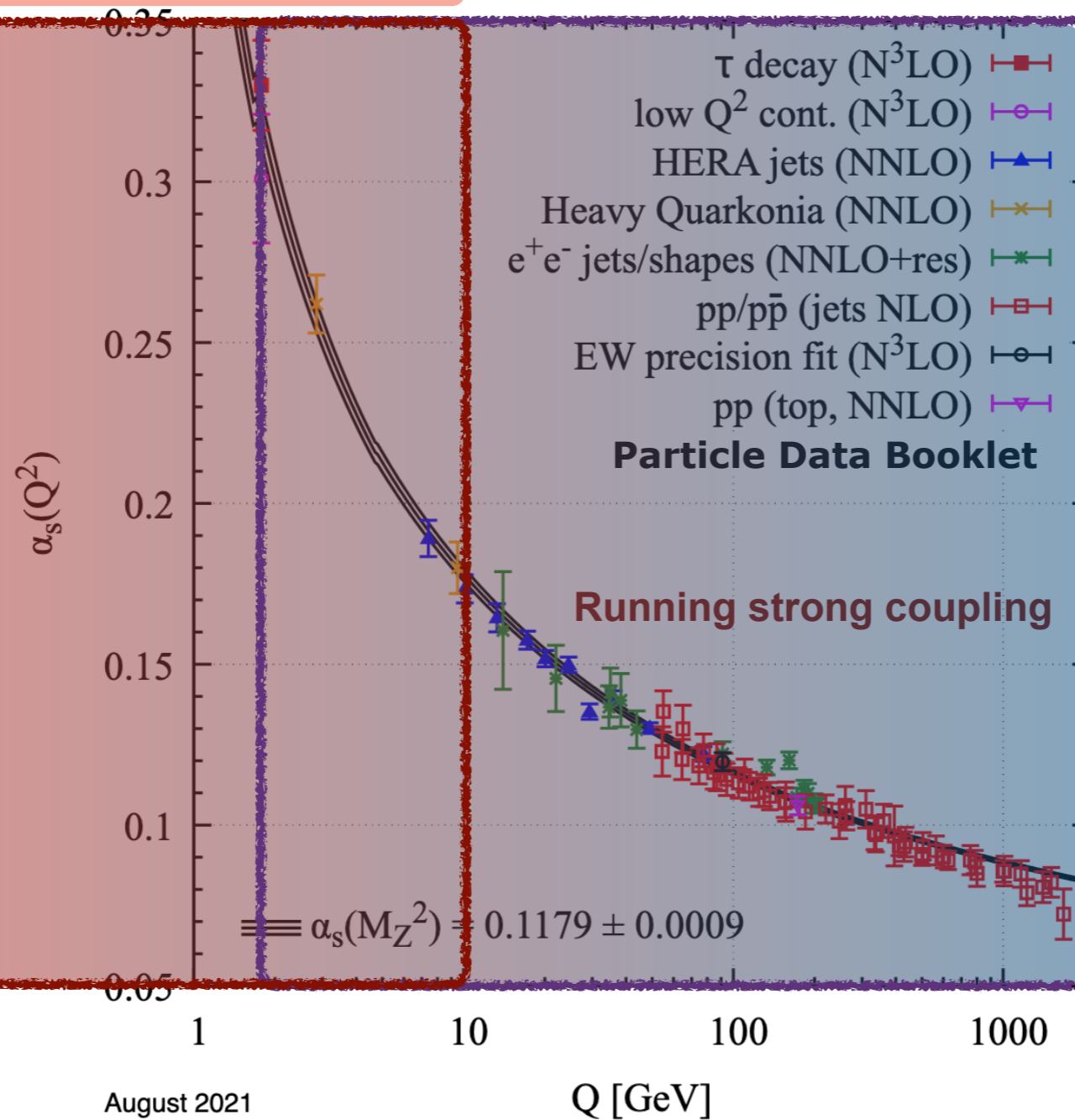
What's hot in QCD?

The cool stuff!

For the purpose of the talk/workshop!

IR: confinement & chiral symmetry breaking

Strongly correlated



QCD background
for
SM & BSM physics

crucial for 'unfolding'

deconvolution
reconstruction

⋮

August 2021

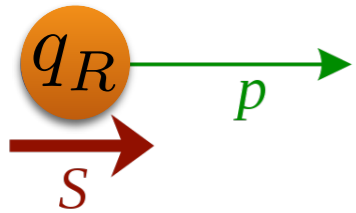
UV: weakly coupled quarks and gluons

Chiral symmetry breaking

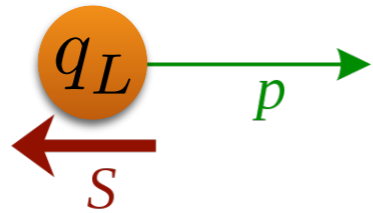
Chirality for massless particles

helicity

Right-handed:



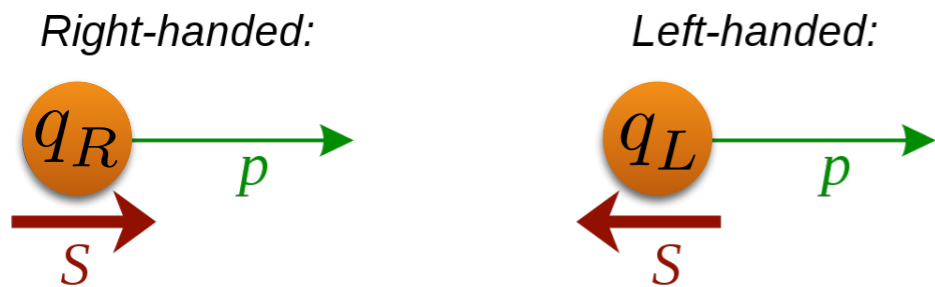
Left-handed:



Chiral symmetry breaking

Chirality for massless particles

helicity



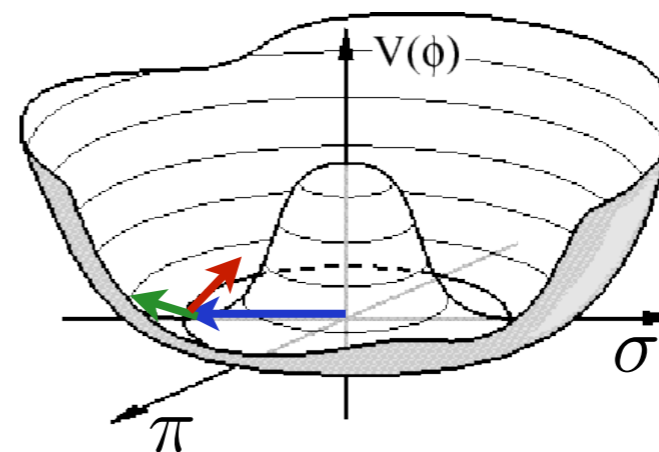
Order parameter

$$\sigma \propto \langle \bar{q}q \rangle = \langle q_L^\dagger q_R + q_R^\dagger q_L \rangle$$

chiral condensate

- Chiral symmetry $\sigma = 0$
- Symmetry broken $\sigma \neq 0$

Meson potential

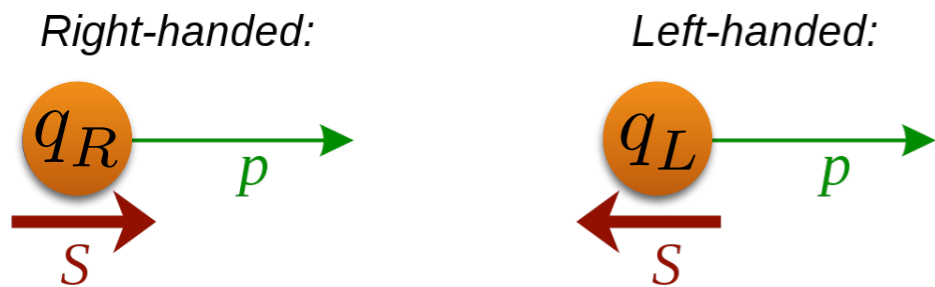


$$\vec{\pi} \simeq i \langle \bar{q} \vec{\sigma} \gamma_5 q \rangle$$

Chiral symmetry breaking

Chirality for massless particles

helicity



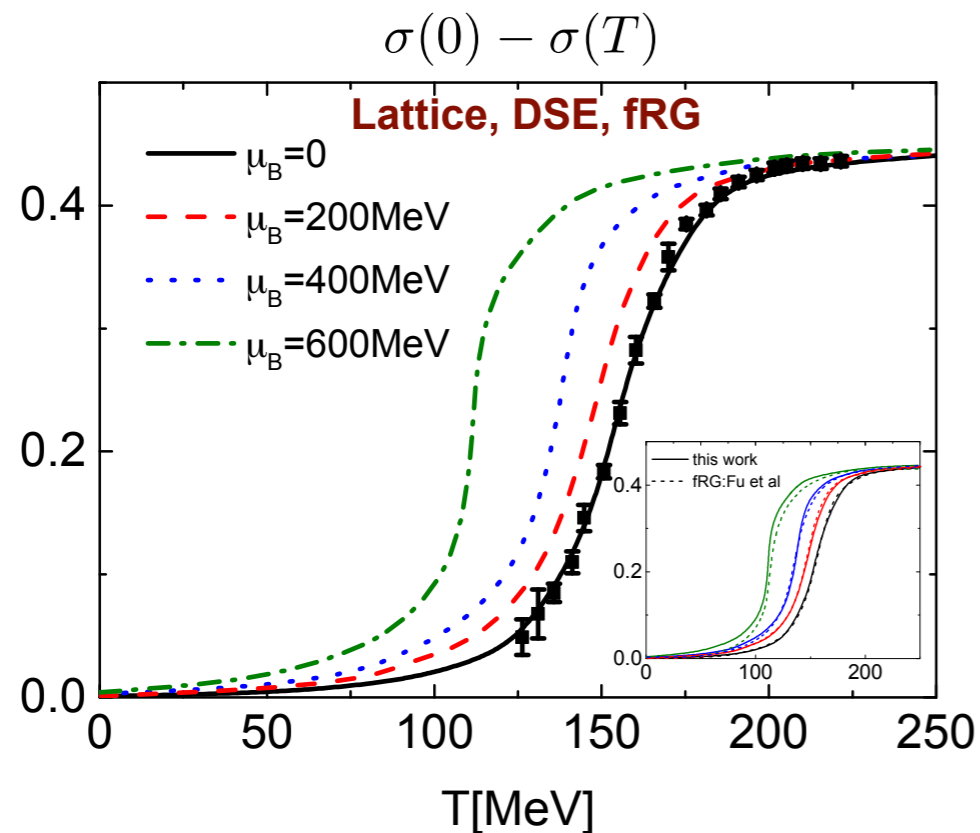
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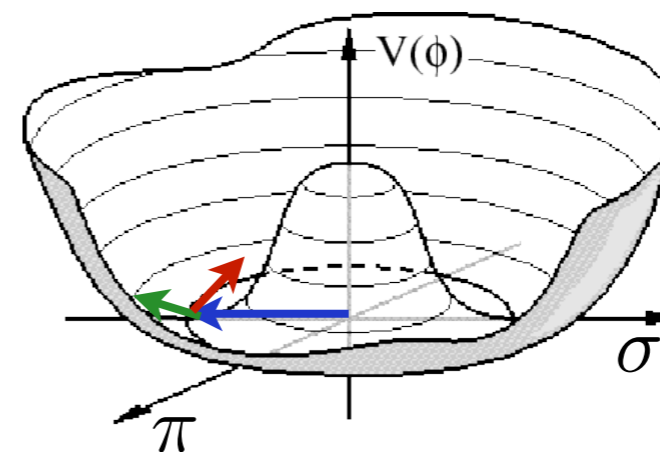
chiral condensate

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- Symmetry broken $\sigma \neq 0$

renormalised light condensate



Meson potential



$$\vec{\pi} \simeq i \langle \bar{q} \vec{\sigma} \gamma_5 q \rangle$$

Chiral symmetry breaking

chiral symmetry

$$\langle \bar{q}q \rangle = 0$$

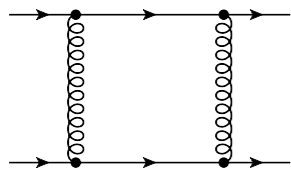
Mean field expansion

$$\int_x (\bar{q}q)^2 = \int_x \langle \bar{q}q \rangle \bar{q}q + \dots$$

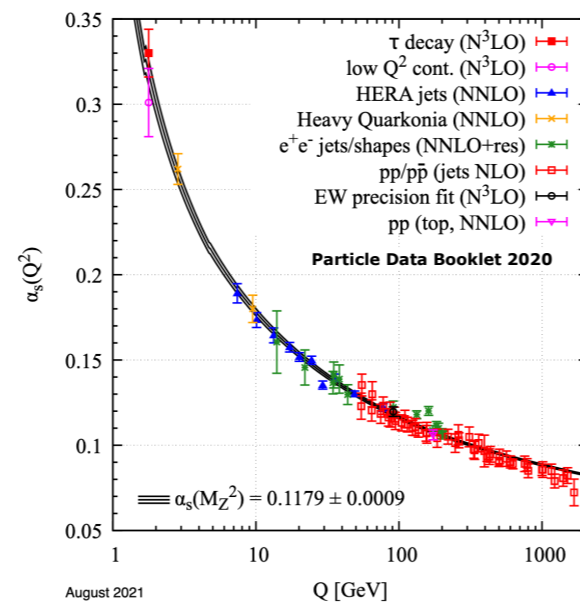
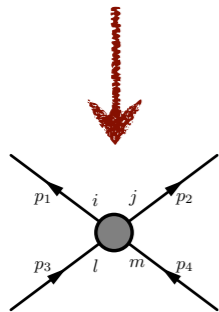
Chiral symmetry breaking

chiral symmetry

$$\langle \bar{q}q \rangle = 0$$

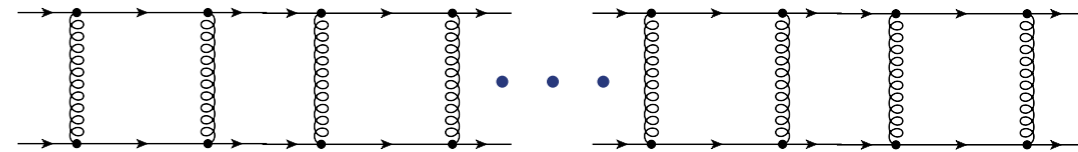


$$\propto \alpha_s^2$$



Mean field expansion

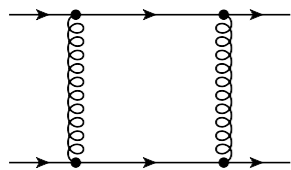
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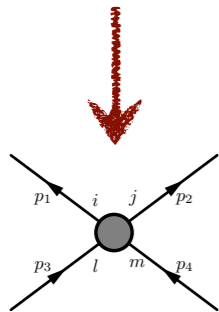
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chiral symmetry

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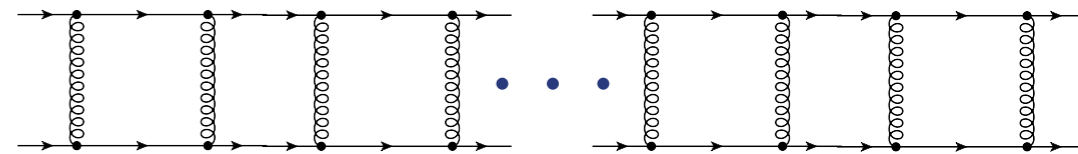


$$\langle \bar{q}q \rangle \neq 0$$

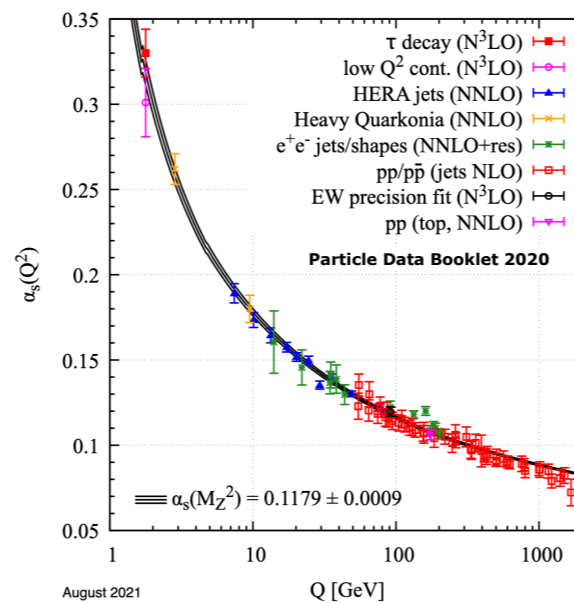
chiral symmetry broken

Mean field expansion

$$\int_x (\bar{q}q)^2 = \int_x \langle \bar{q}q \rangle \bar{q}q + \dots$$



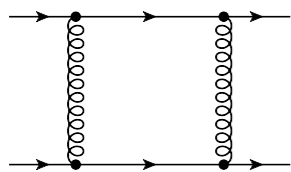
critical strong coupling α_s^{CR}



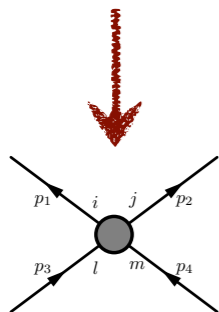
Chiral symmetry breaking

chiral symmetry

$$\langle \bar{q}q \rangle = 0$$



$$\propto \alpha_s^2$$

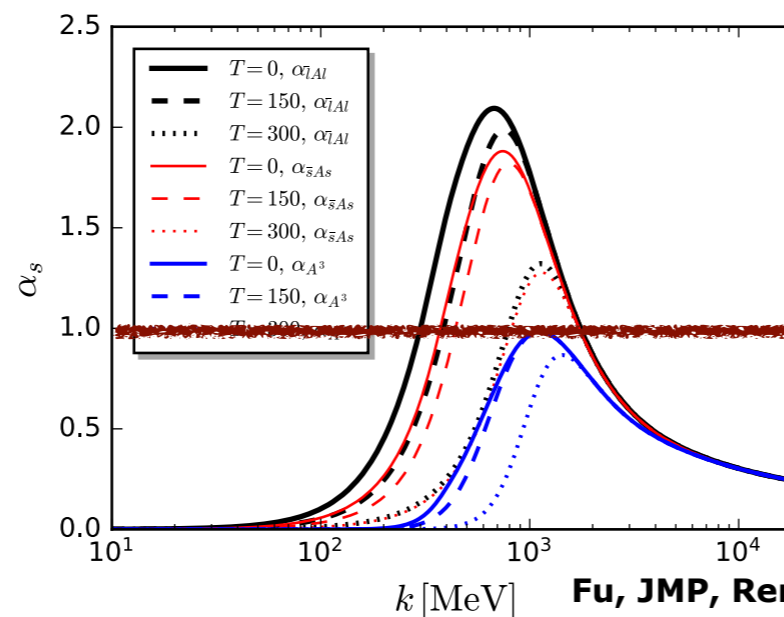
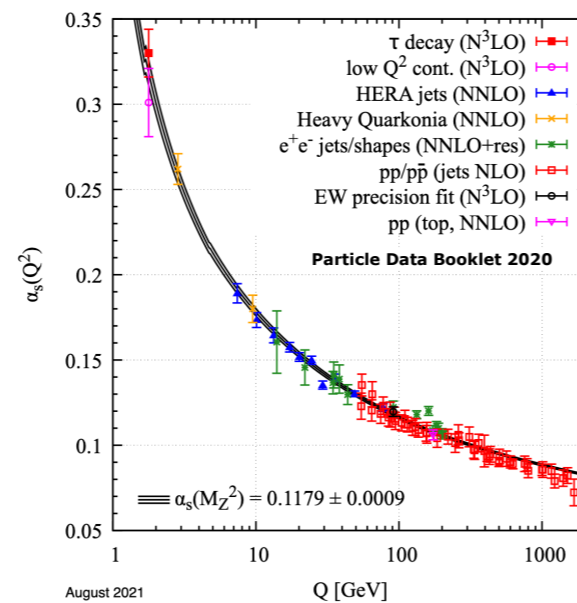
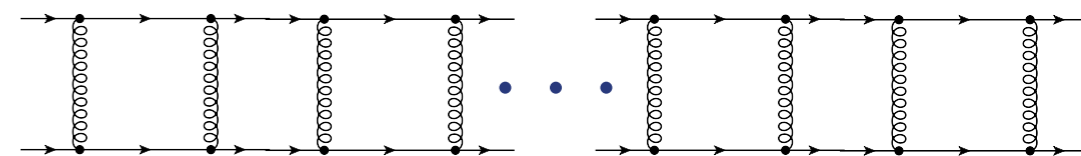


$$\langle \bar{q}q \rangle \neq 0$$

chiral symmetry broken

Mean field expansion

$$\int_x (\bar{q}q)^2 = \int_x \langle \bar{q}q \rangle \bar{q}q + \dots$$



critical strong coupling α_s^{CR}

Chiral symmetry breaking

physical masses

strong chiral symmetry breaking $\Delta m_{\chi SB} \approx 400 \text{ MeV}$



up



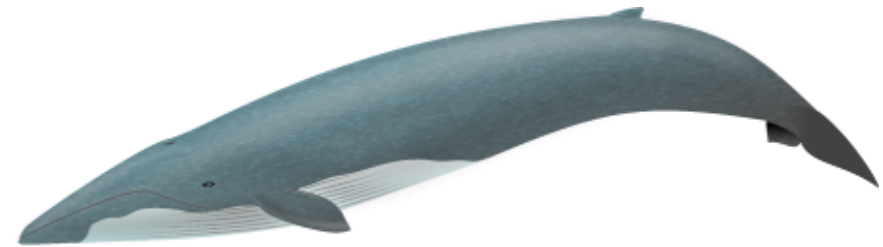
down



charm



top



strange



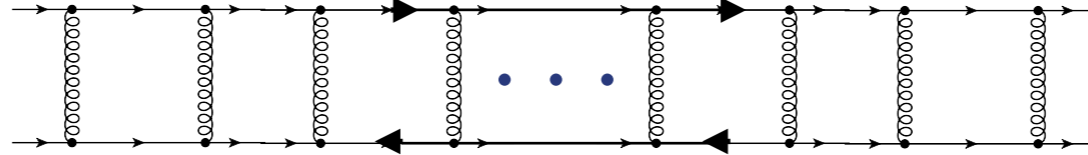
bottom



2 light flavours, one heavy flavour 2+1

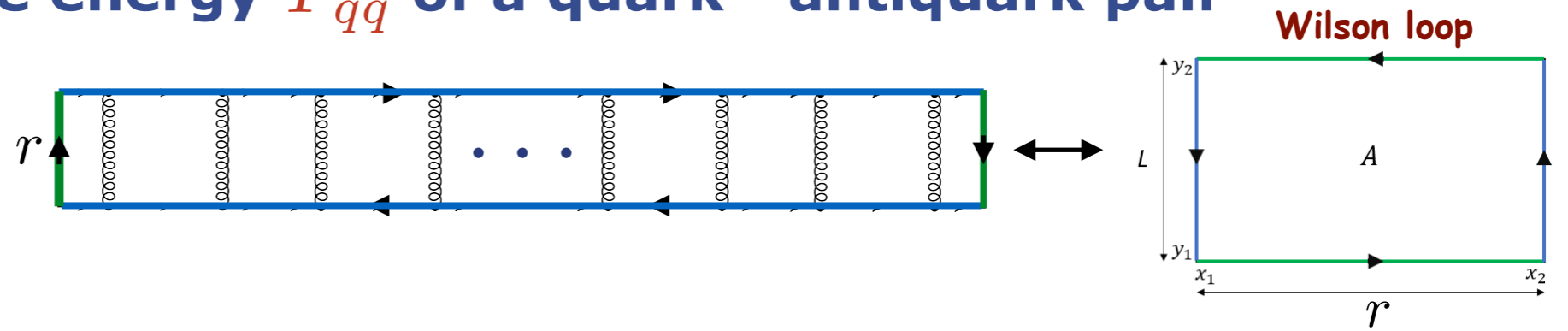
Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair



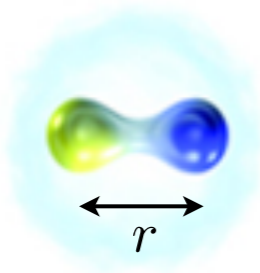
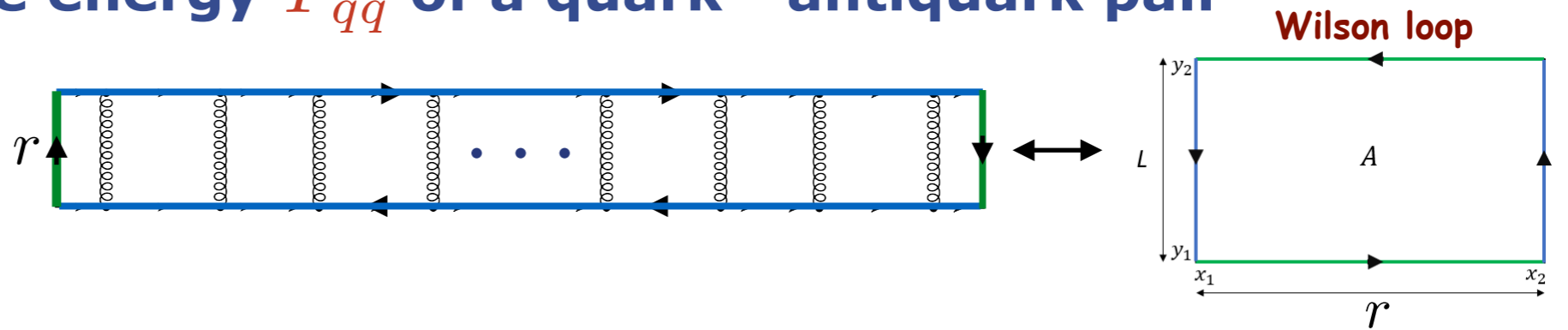
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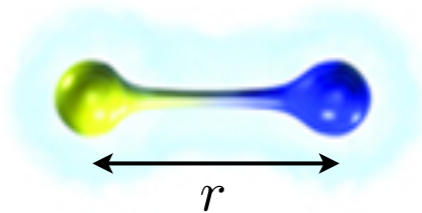


Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

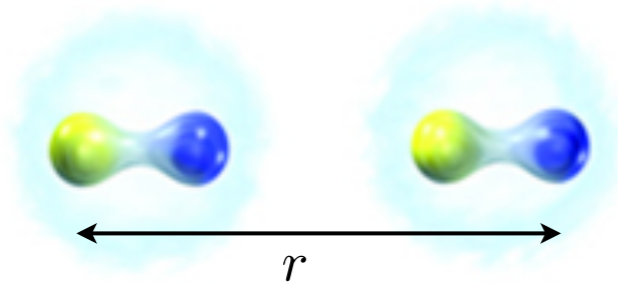
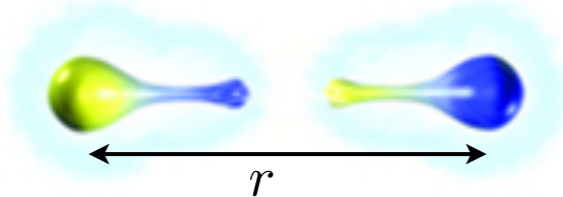


$$F_{q\bar{q}} \simeq -\frac{1}{r}$$



$$F_{q\bar{q}} \simeq \sigma r$$

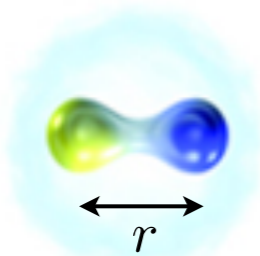
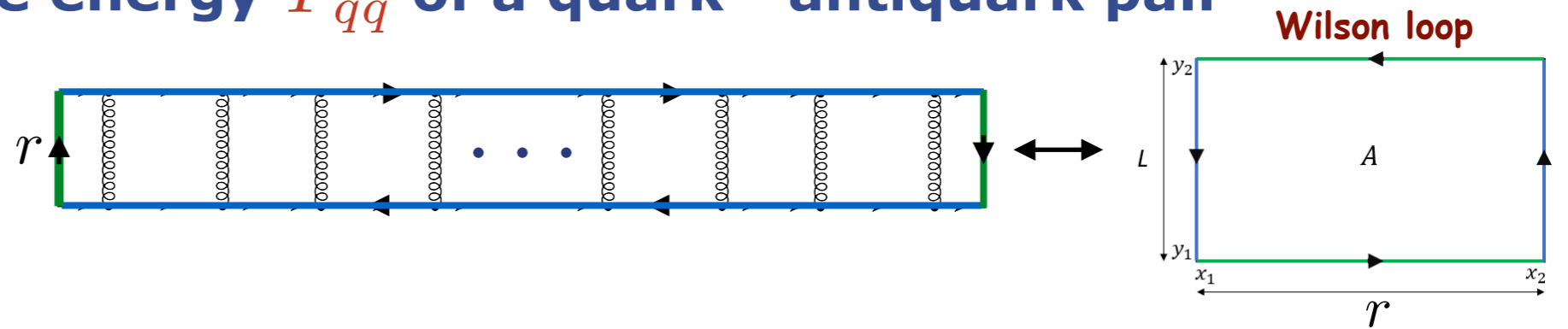
string breaking at $r \approx 1\text{fm}$



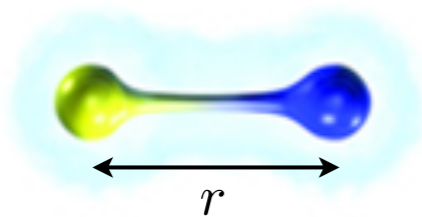
$$F_{q\bar{q}} \simeq \text{const.}$$

Confinement

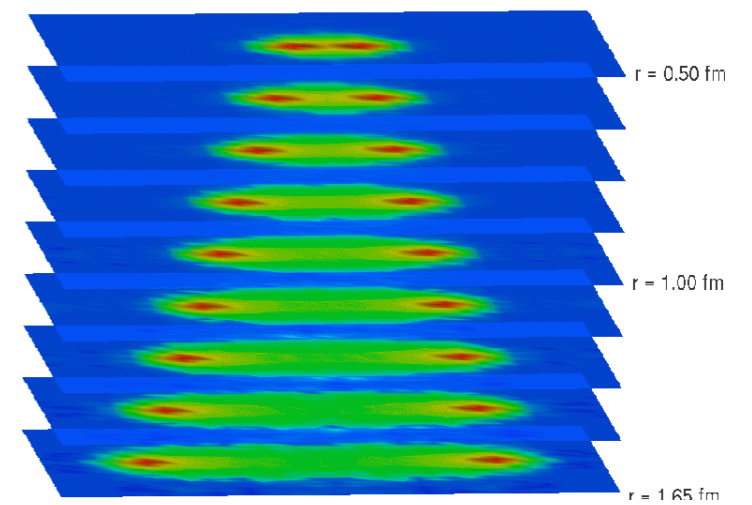
Free energy $F_{q\bar{q}}$ of a quark - antiquark pair



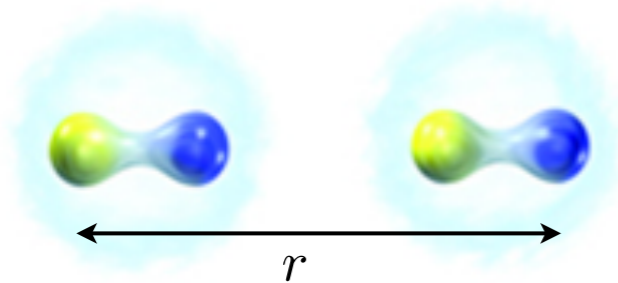
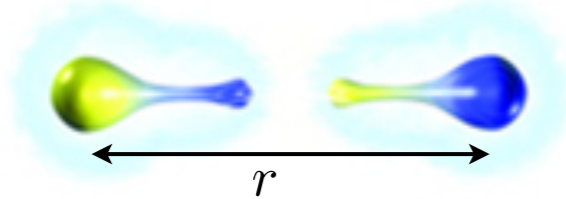
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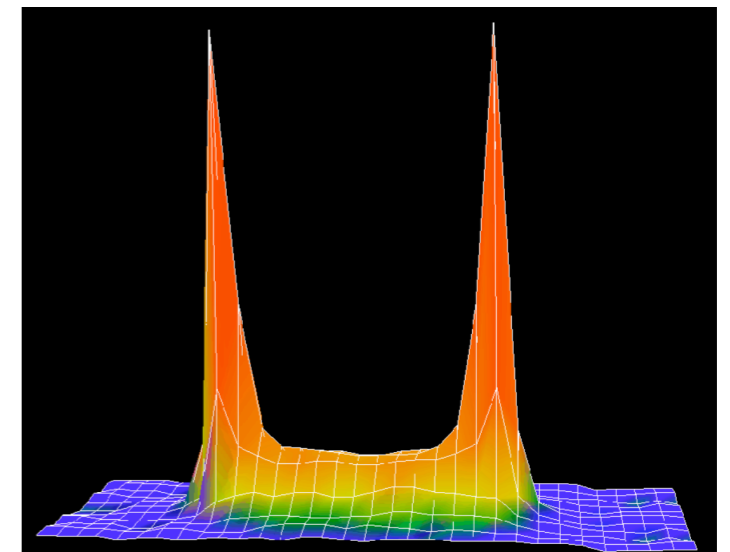
string breaking at $r \approx 1$ fm



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Energy density

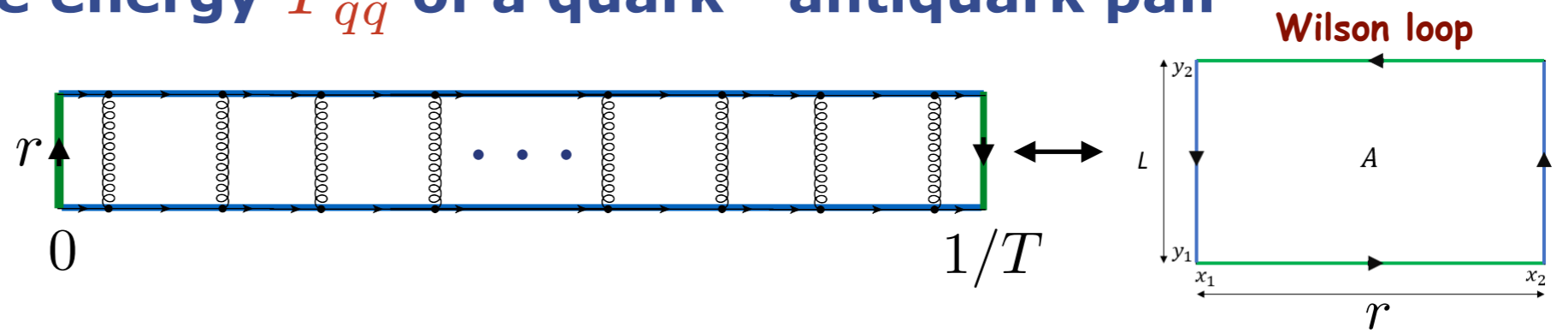
Bali et al. '94



Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

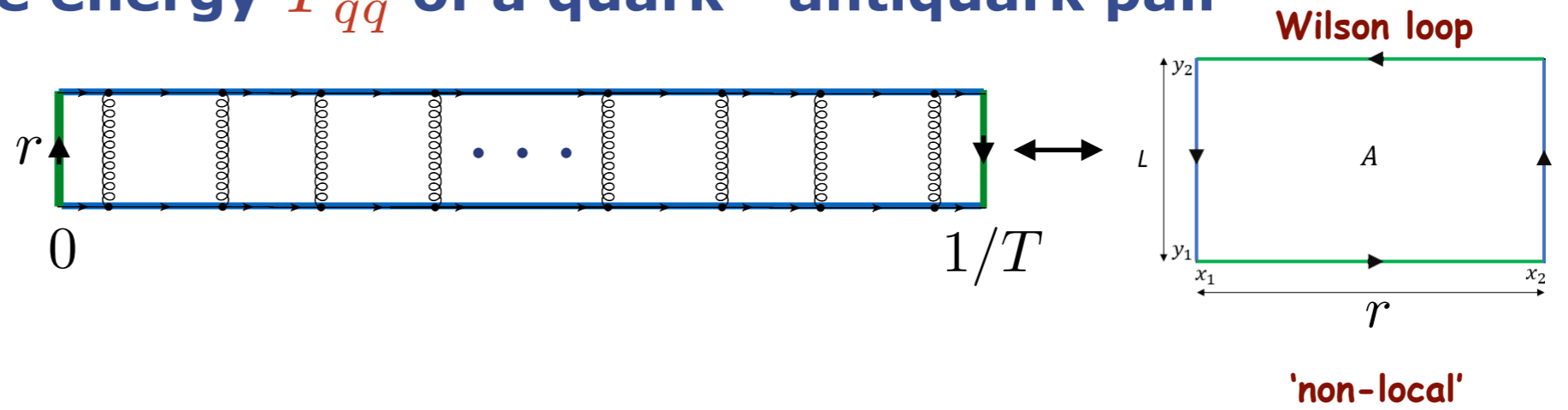
finite T



Confinement

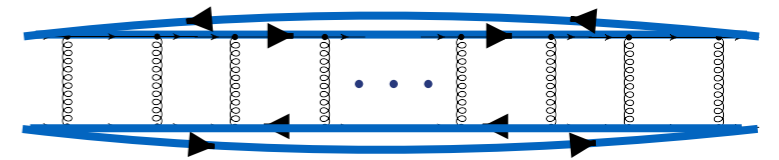
Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

finite T



Polyakov loop correlator

$$\langle q(0)\bar{q}(r) \rangle \sim \langle P(0)P^\dagger(r) \rangle \sim V(r, T)/T$$

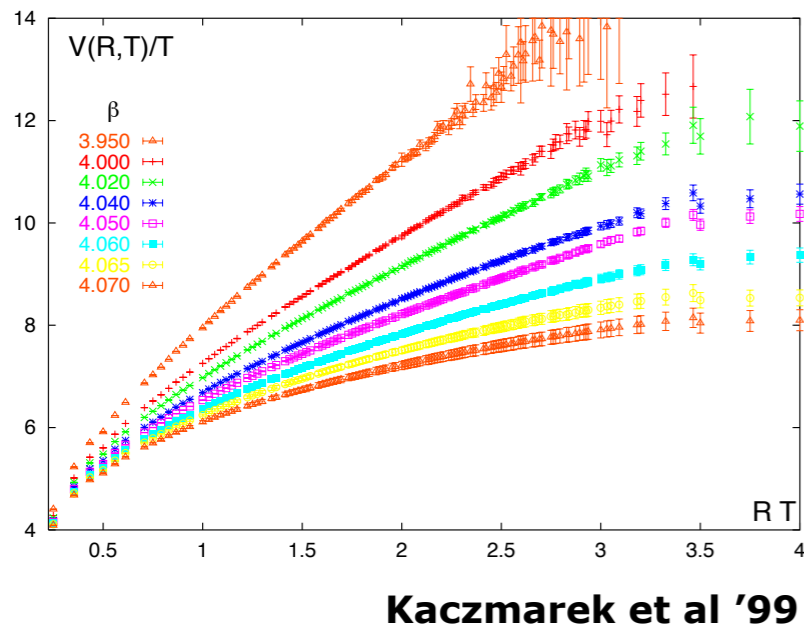
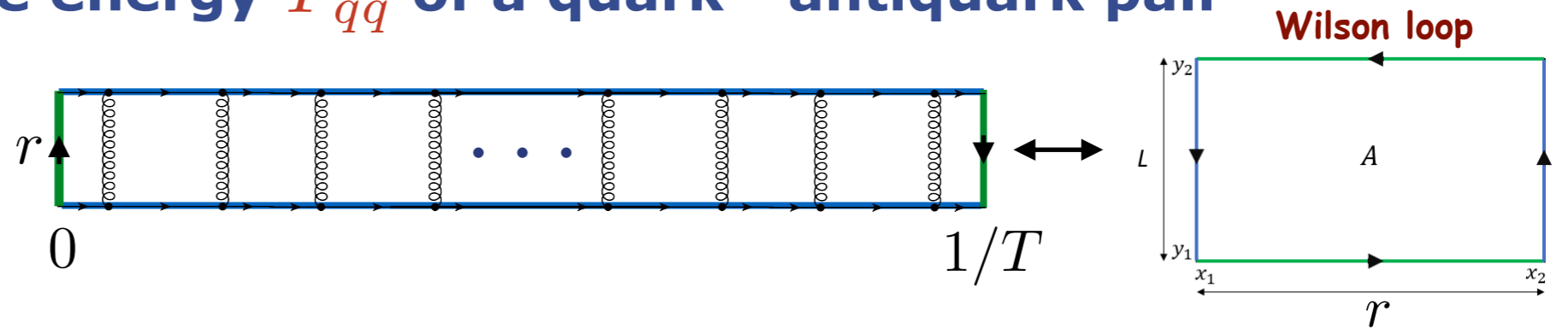


'local order parameter'

Confinement

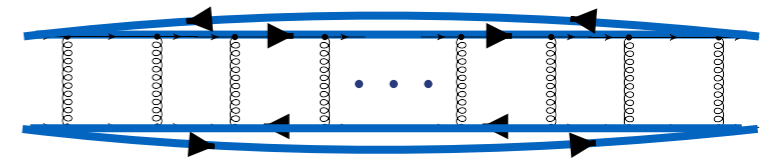
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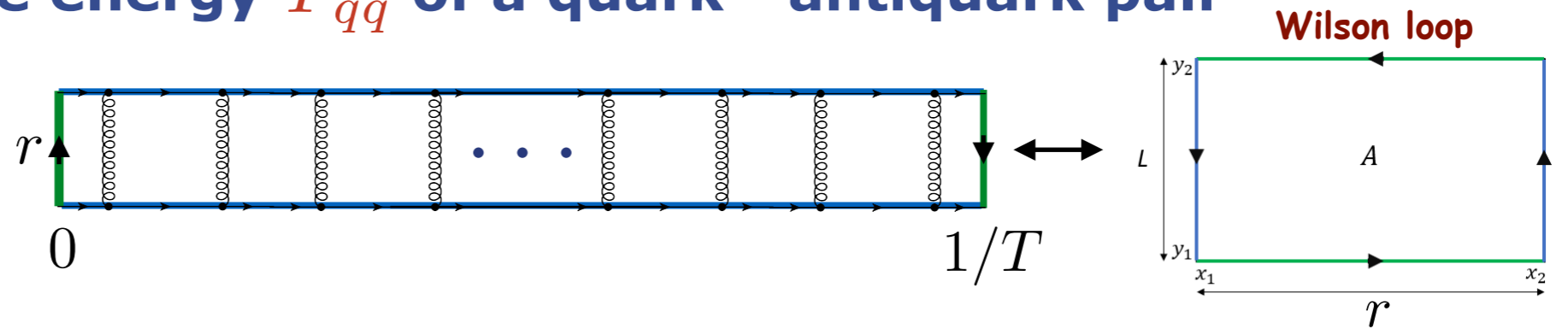


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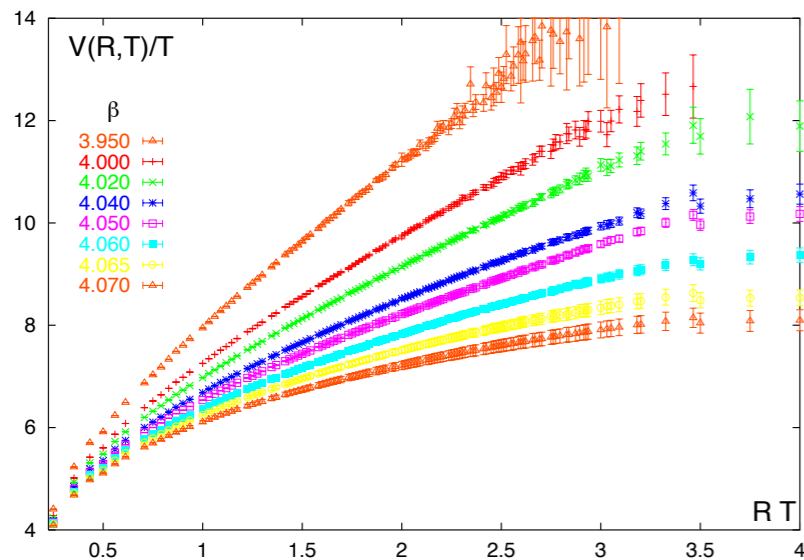
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finite T



Wilson loop

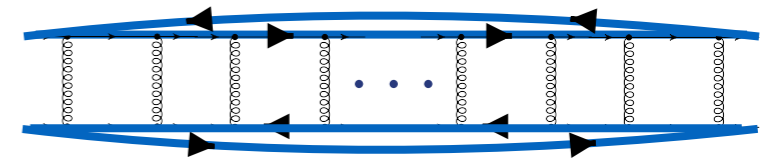
'non-local'



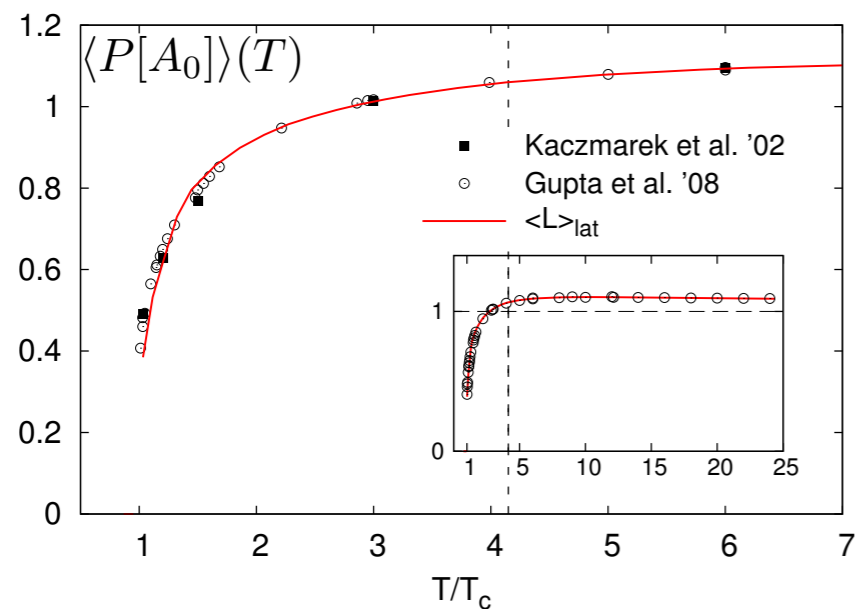
Kaczmarek et al '99

Polyakov loop correlator

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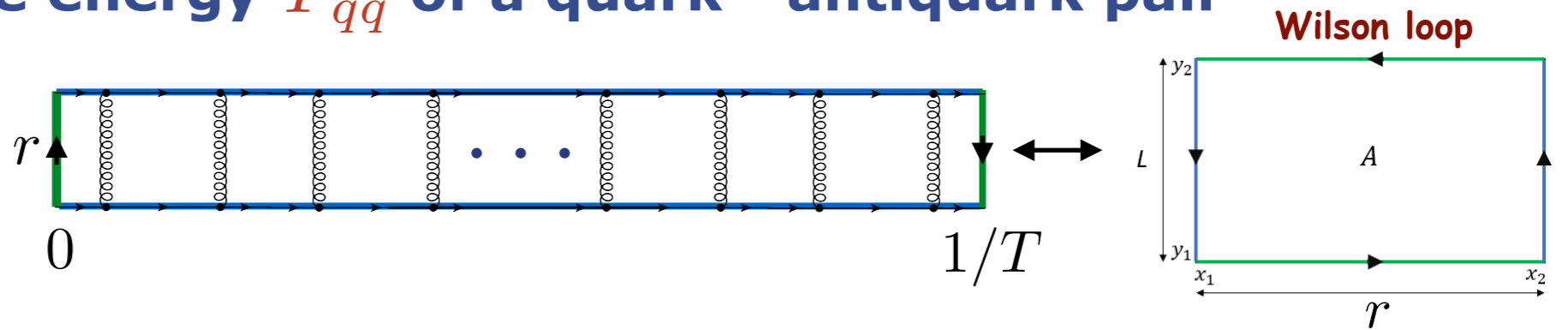


Herbst, Lücker, JMP, 2015

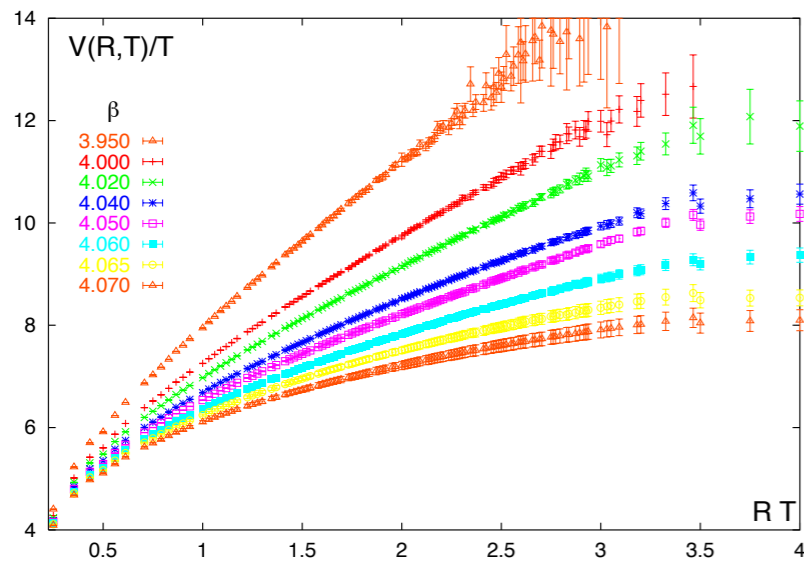
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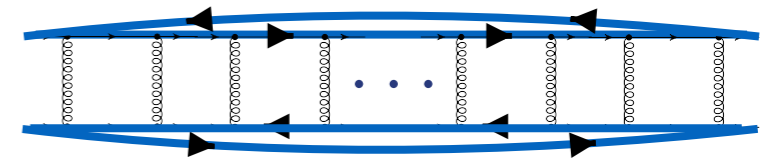
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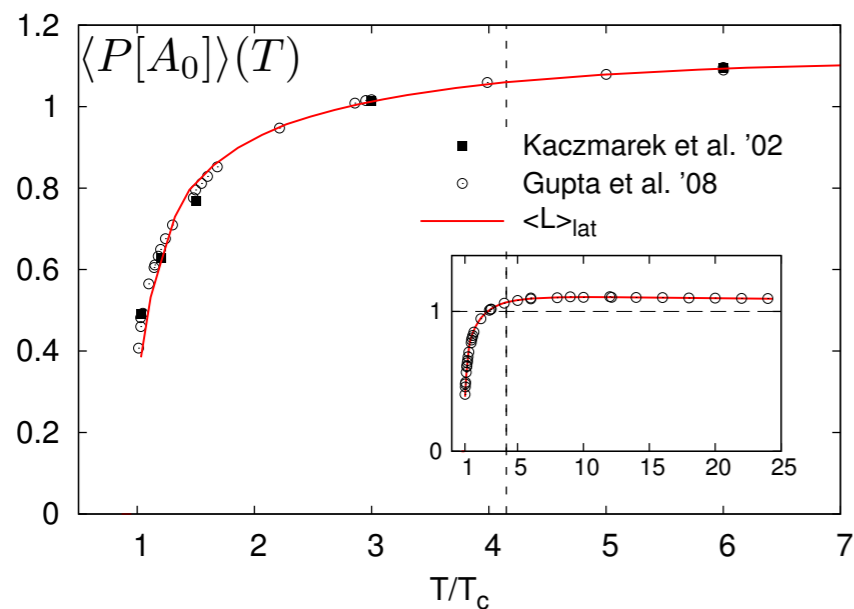


'local order parameter'

Center symmetry of the gauge group

SU(2): Z_2 Ising

SU(3): Z_3 Potts

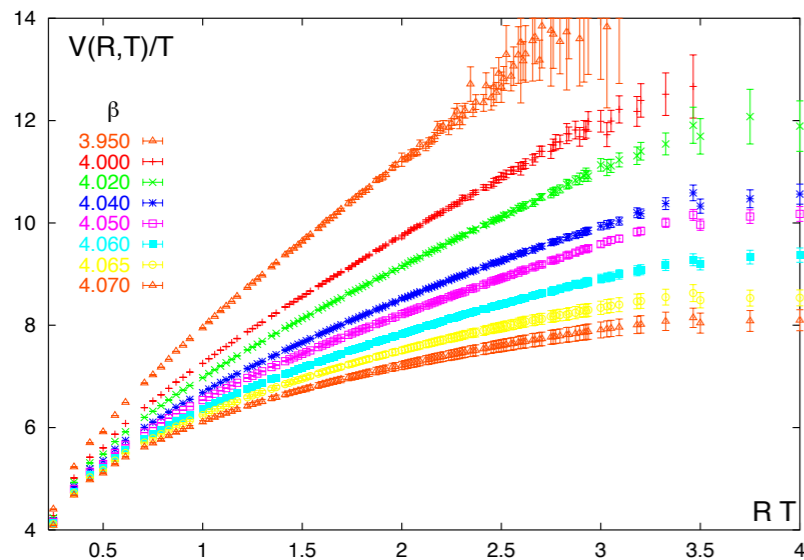
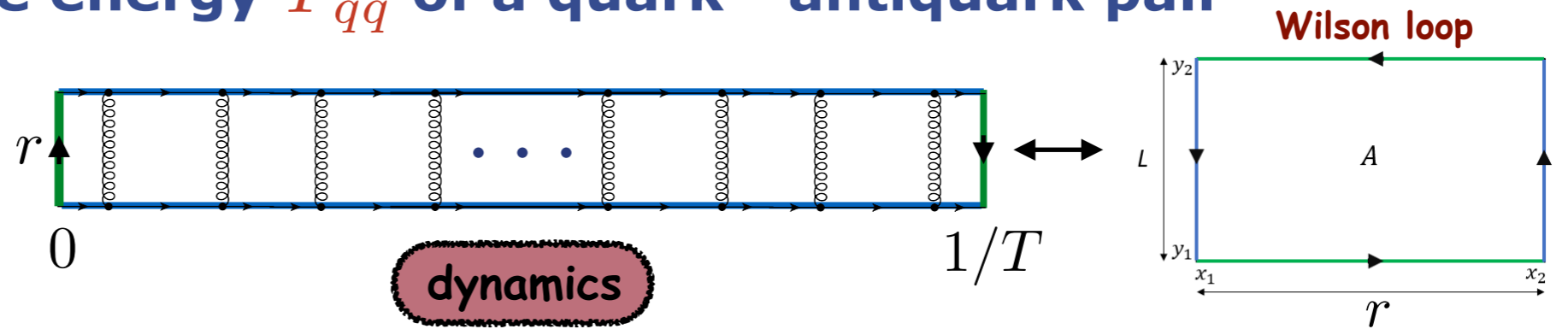


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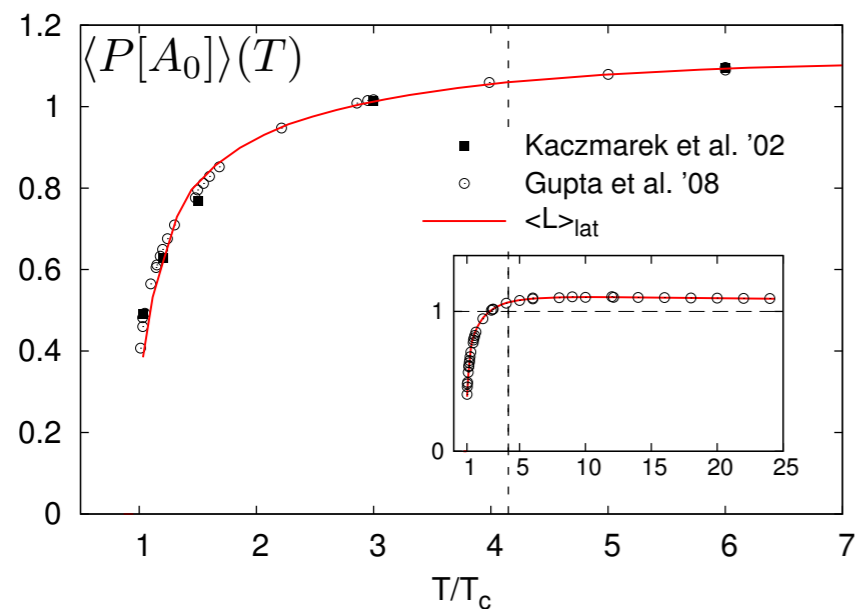
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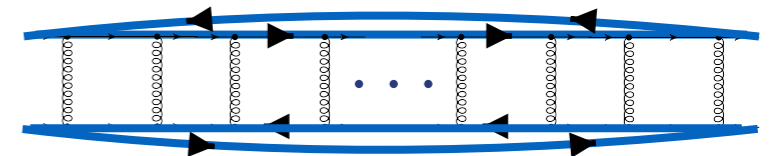
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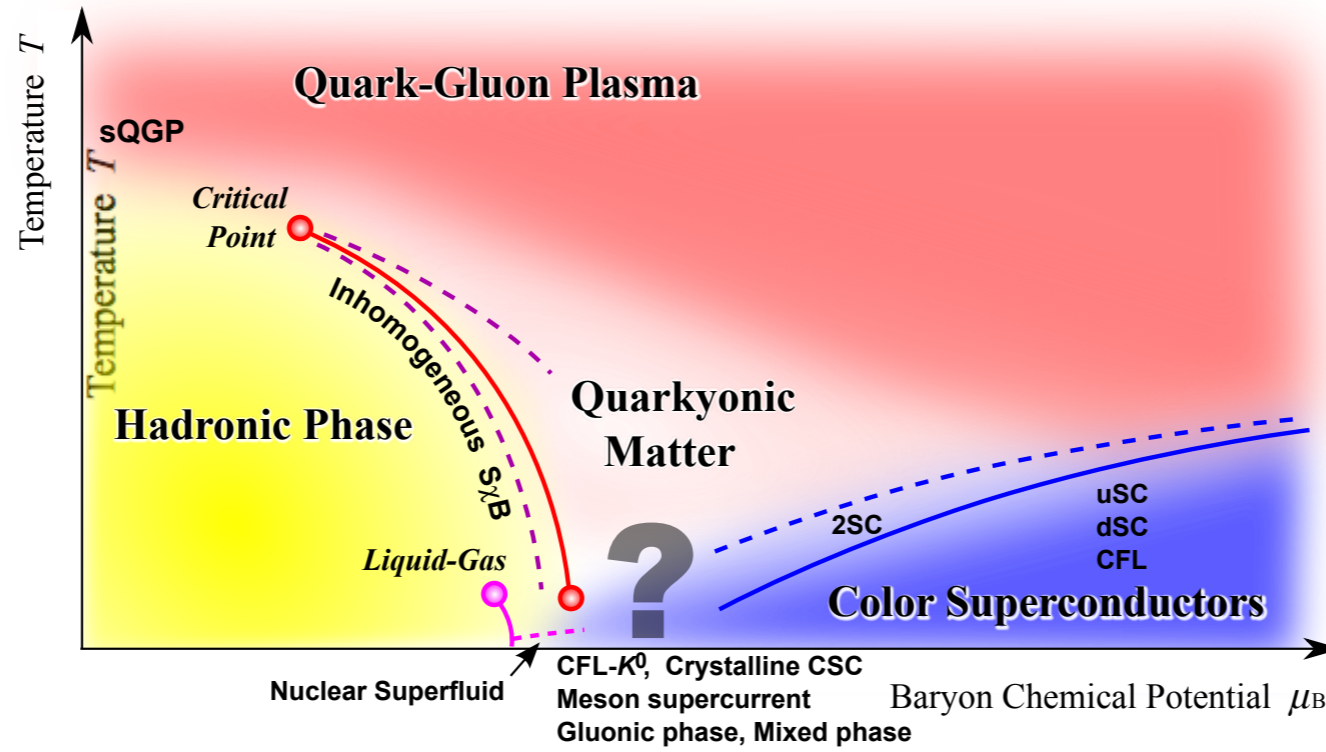
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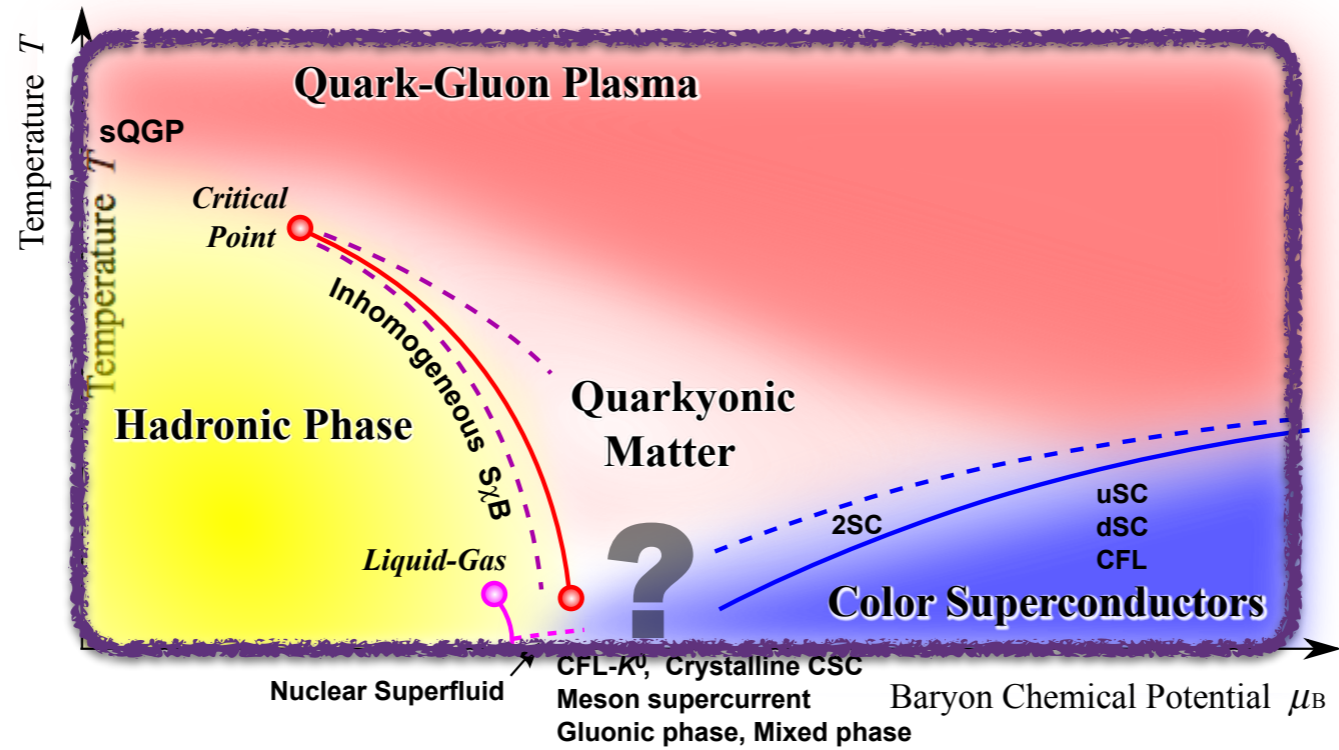
$$\text{SU}(3): Z_3 \quad \text{Potts}$$

It's the dynamics, stupid!

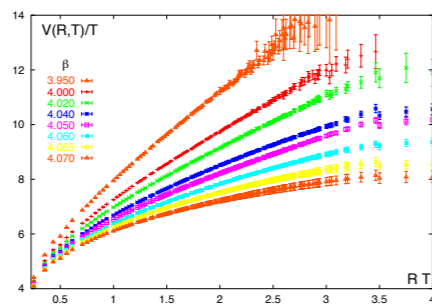
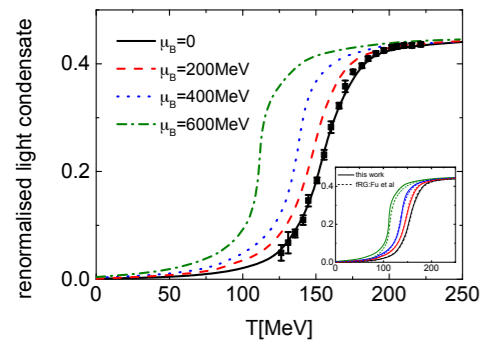
Strongly correlated QCD



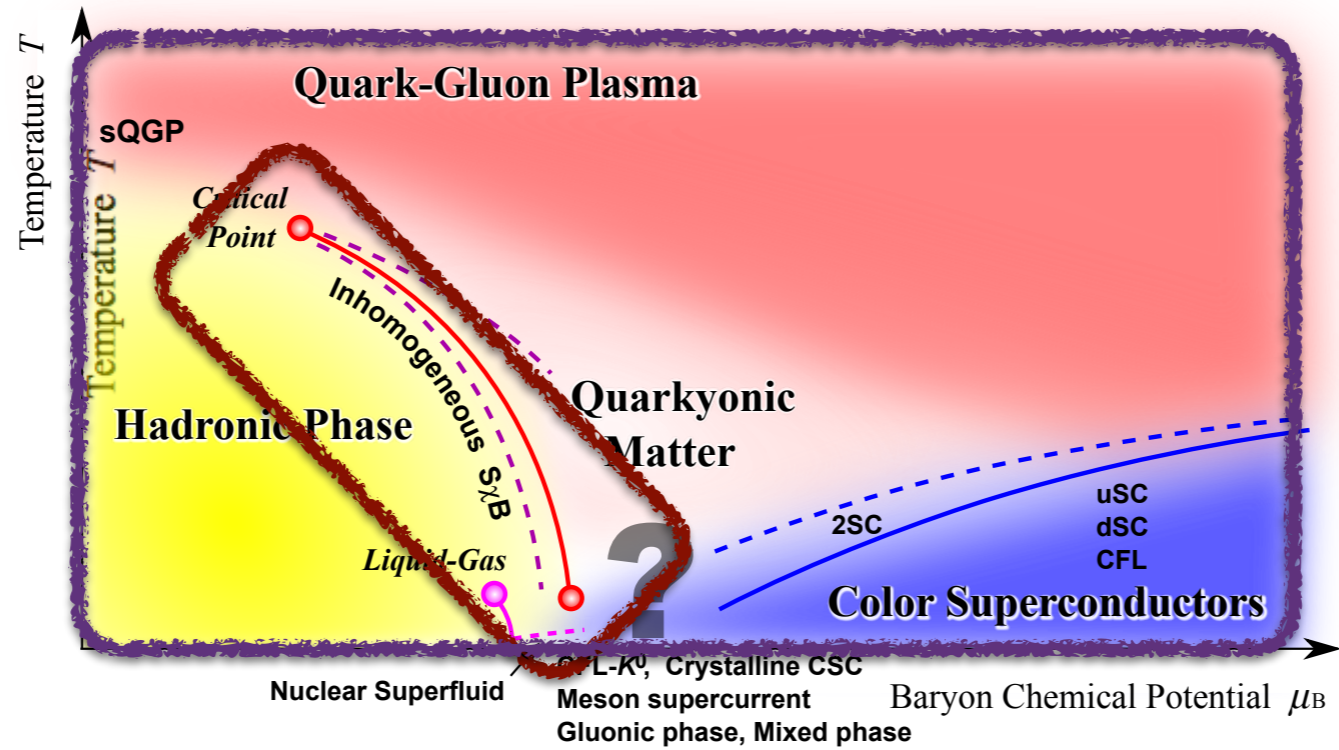
Strongly correlated QCD



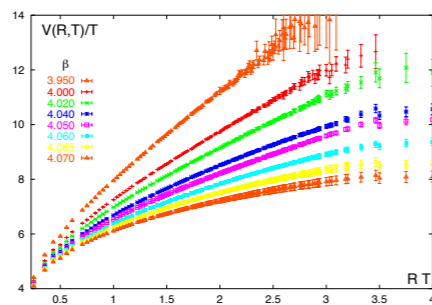
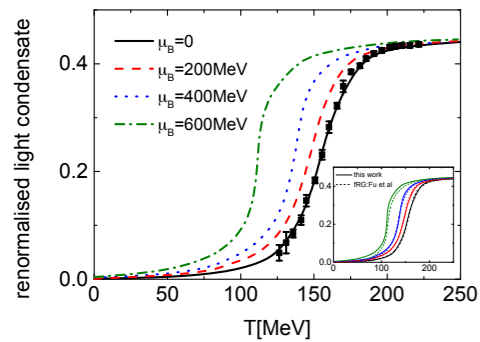
Chiral symmetry breaking & Confinement



Strongly correlated QCD



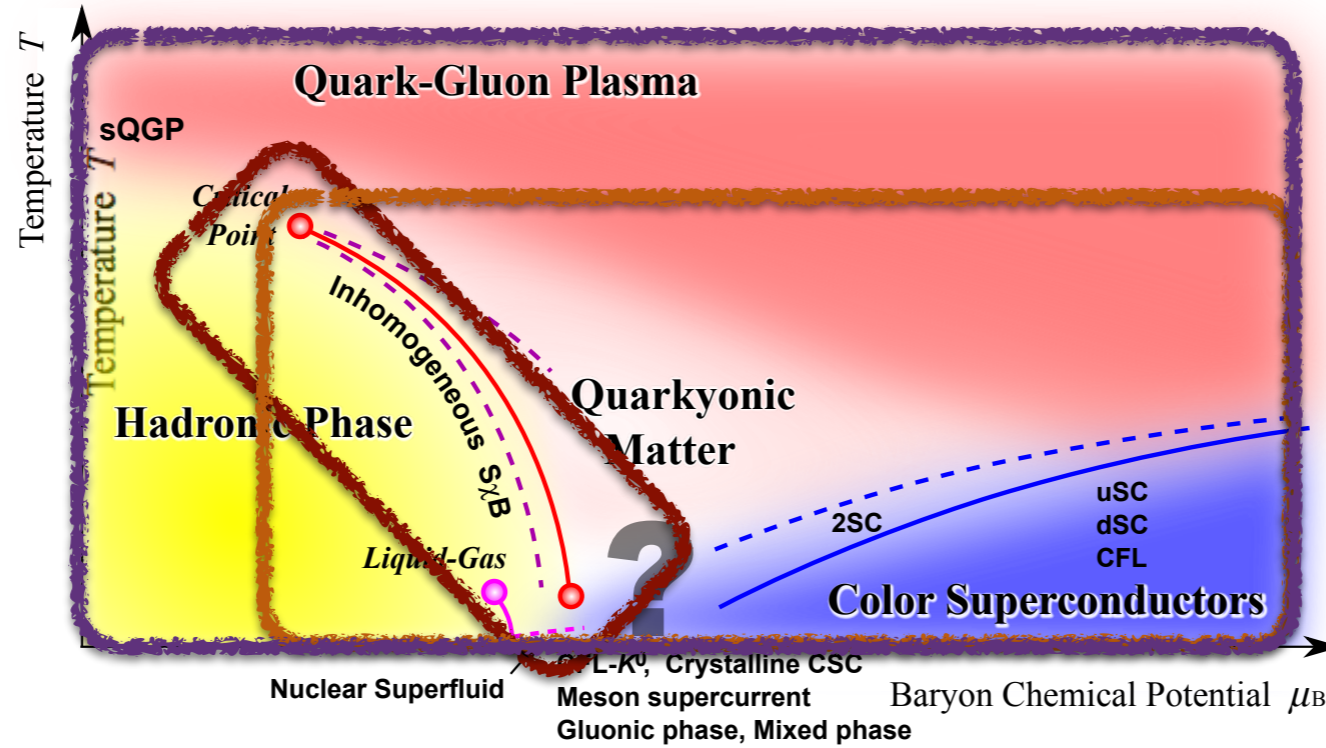
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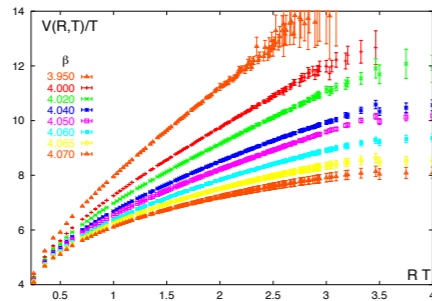
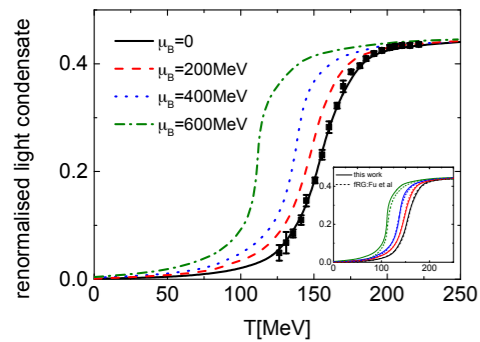
Moat regime & inhomogeneous phases



Strongly correlated QCD



Chiral symmetry breaking & Confinement



Different & competing orders

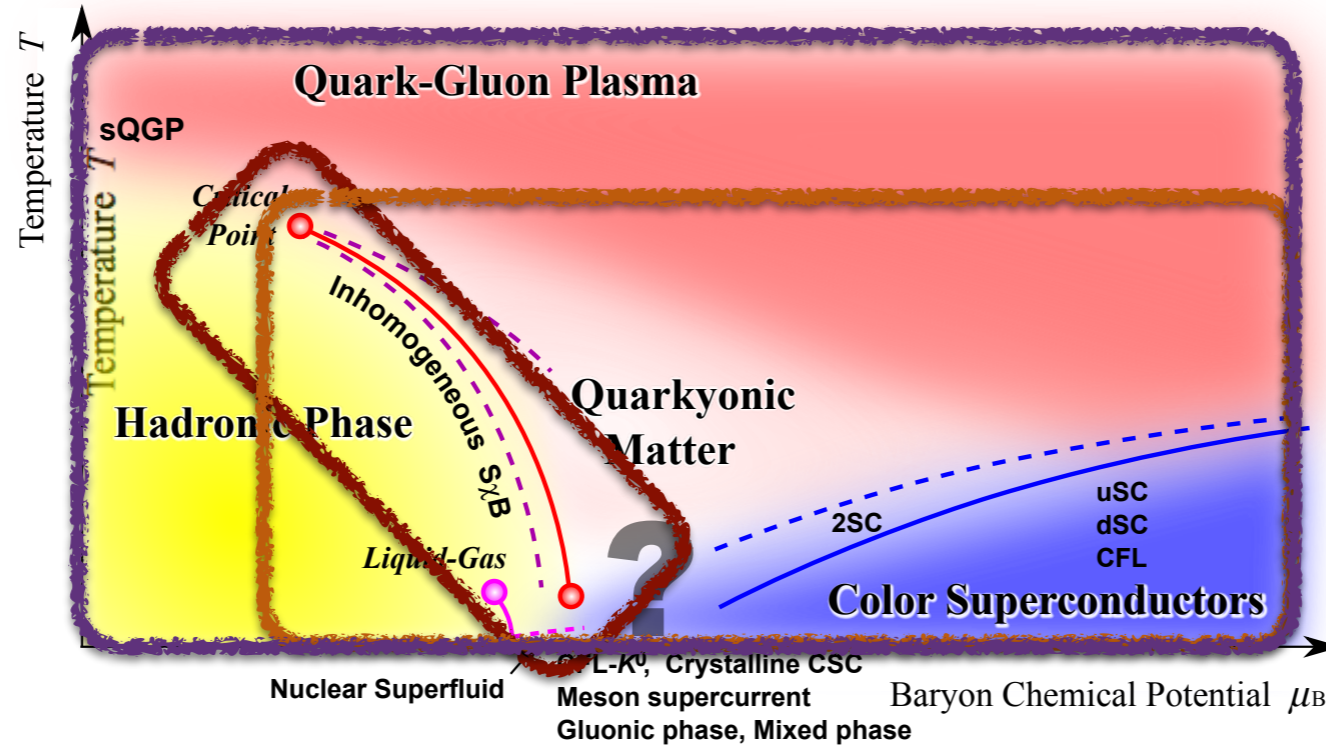
$$(\bar{q} T_i q)^2$$

$$(q \mathcal{E}_j q)^2$$

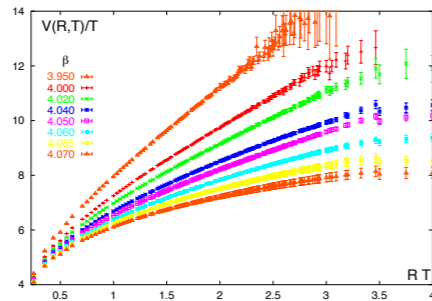
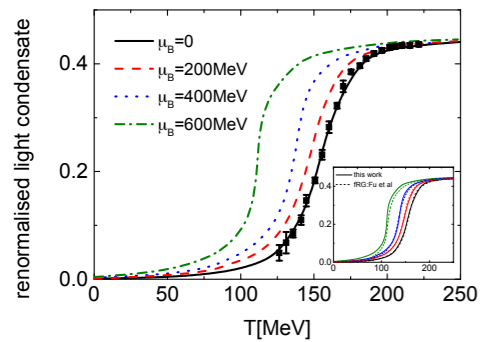
Moat regime & inhomogeneous phases



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Different & competing orders

$$(\bar{q} \mathcal{T}_i q)^2$$

$$(q \mathcal{E}_j q)^2$$

$$\frac{\mu_B}{T} \lesssim 6 \quad \vec{\pi}, \sigma$$

$$\frac{\mu_B}{T} \gtrsim 6 \quad d$$

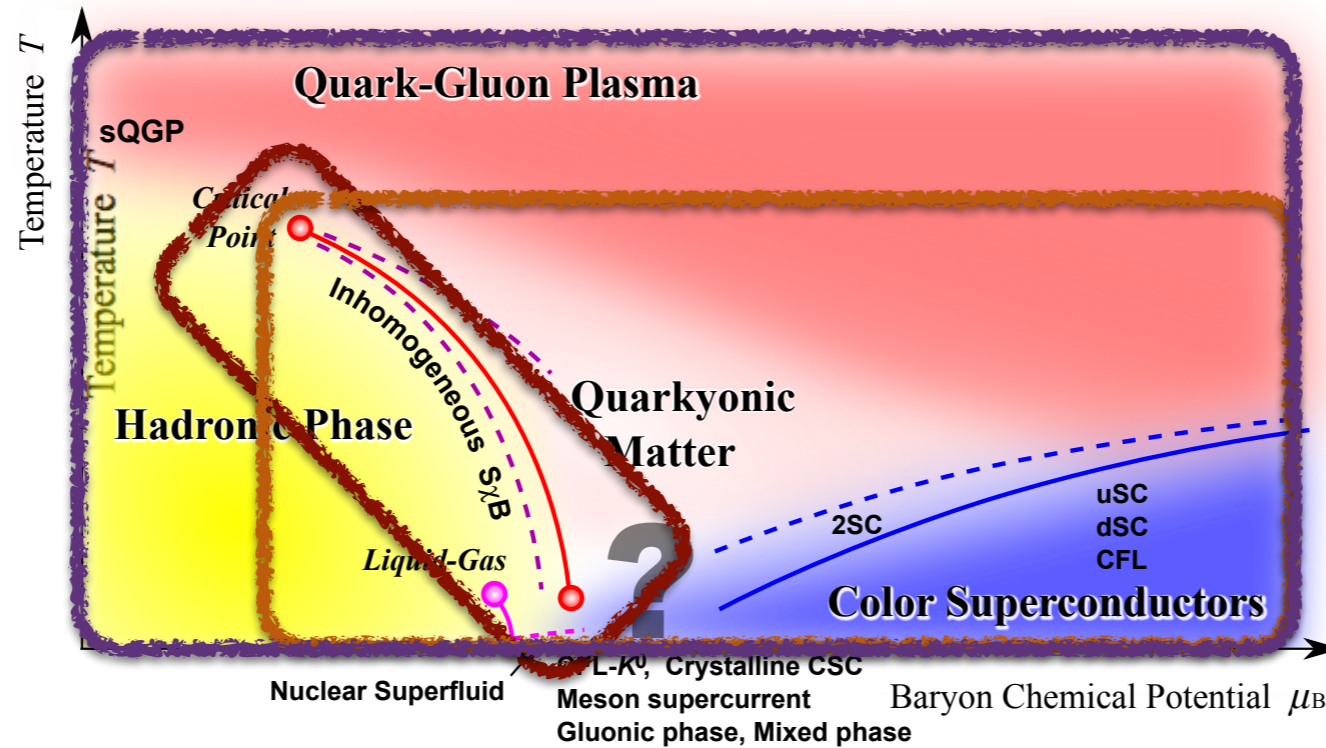
Moat regime & inhomogeneous phases



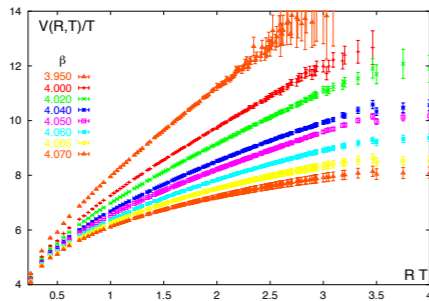
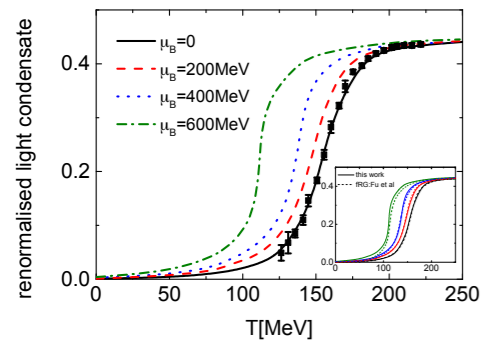
Pion, sigma mode

diquark

Strongly correlated QCD



Chiral symmetry breaking & Confinement



early Universe, HIC, nuclear Astrophysics

Different & competing orders

$$(\bar{q} \mathcal{T}_i q)^2$$

$$(q \mathcal{E}_j q)^2$$

$$\frac{\mu_B}{T} \lesssim 6 \quad \vec{\pi}, \sigma$$

$$\frac{\mu_B}{T} \gtrsim 6 \quad d$$

Moat regime & inhomogeneous phases



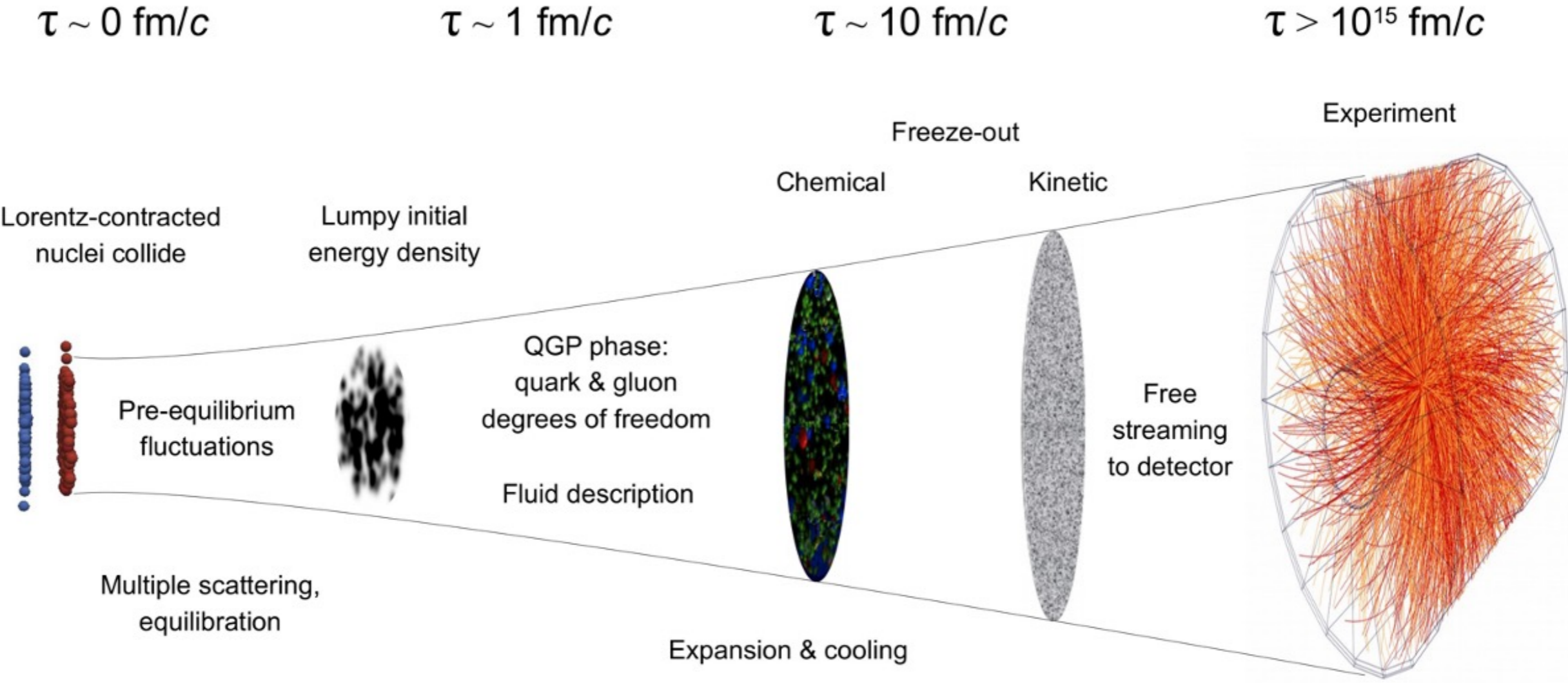
HIC, nuclear Astrophysics

Pion, sigma mode

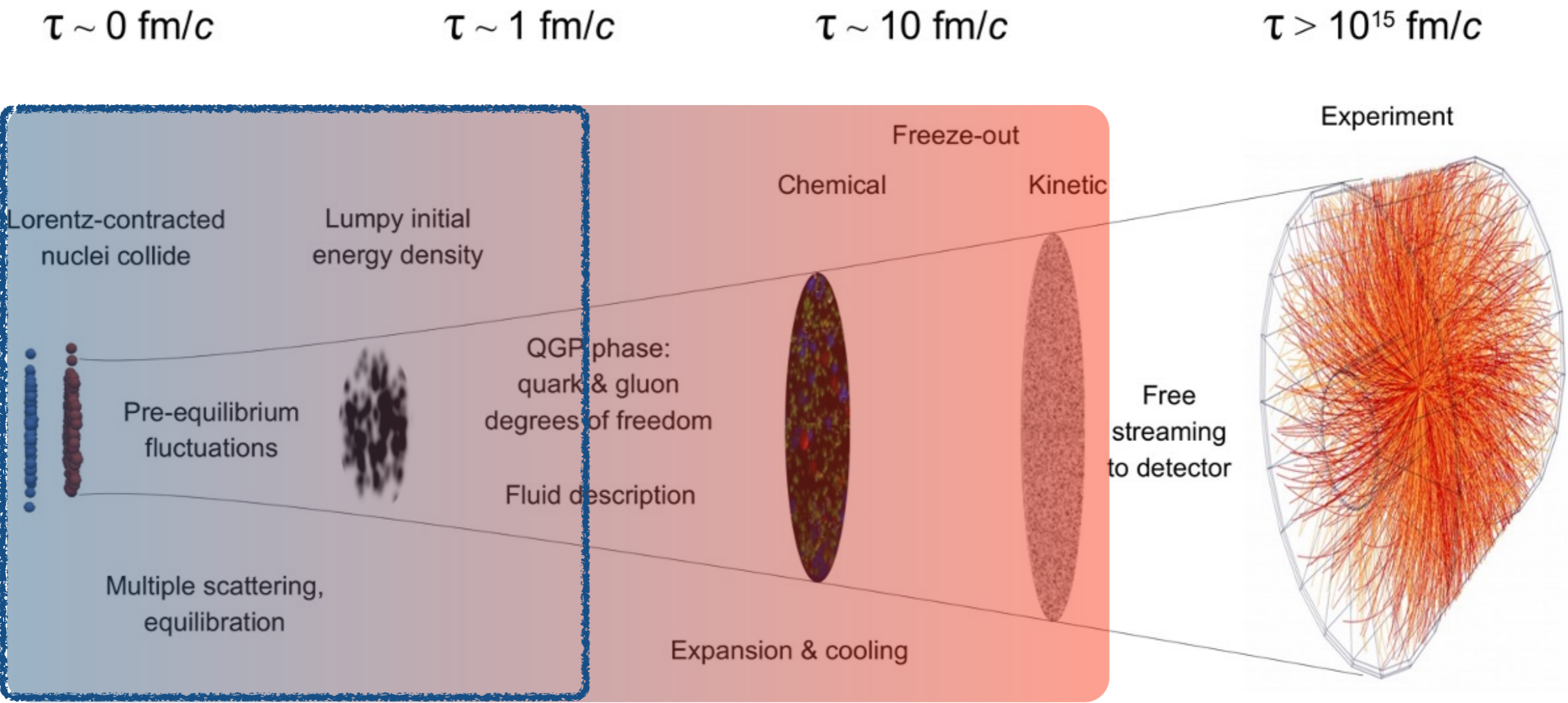
diquark

HIC, nuclear Astrophysics

Unfolding strongly correlated QCD with heavy ion collisions

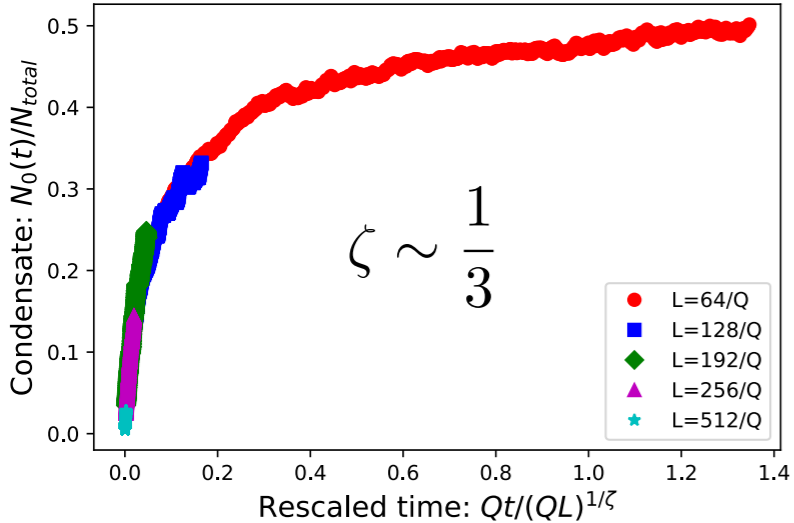


Unfolding strongly correlated QCD with heavy ion collisions



large occupancies

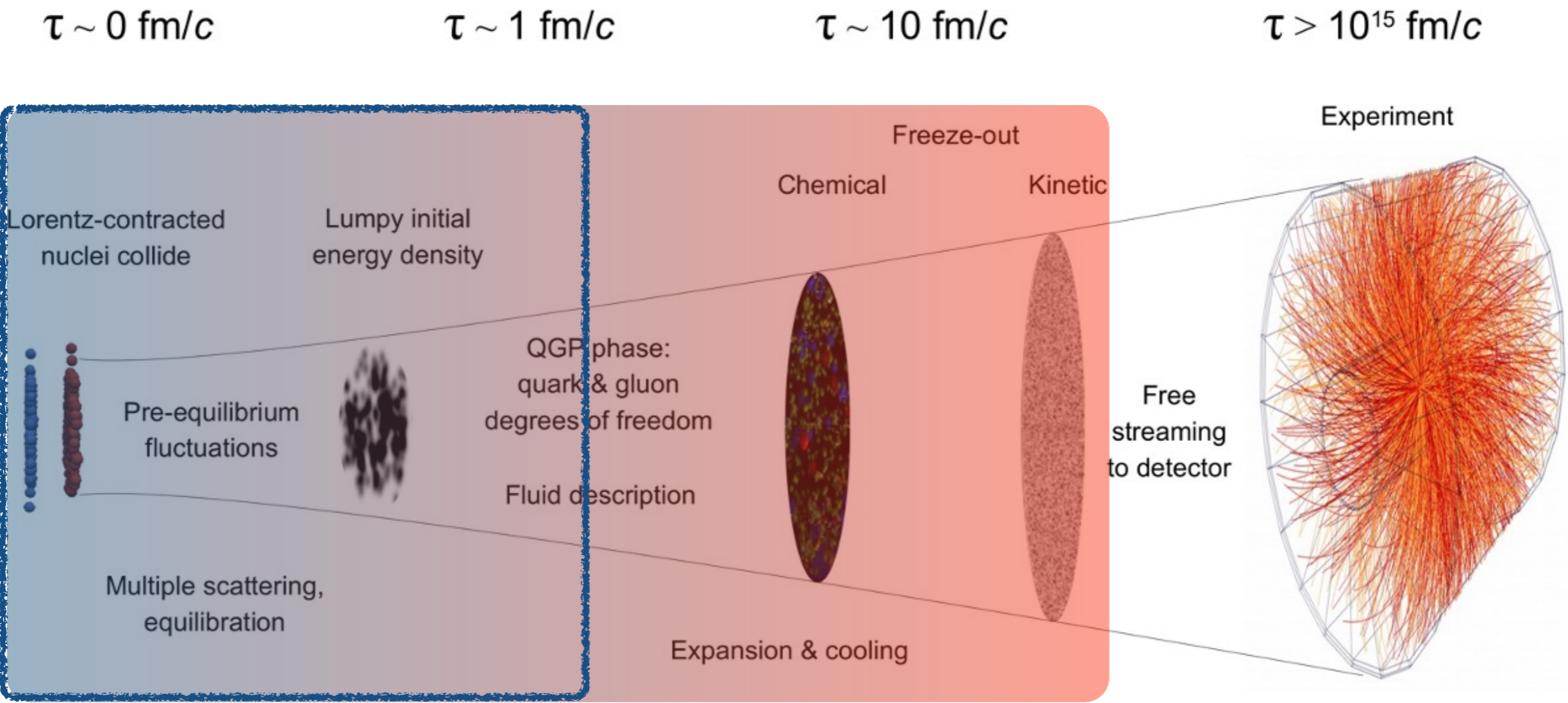
non-thermal fixed point



$$\sim \int_{\text{vol}} \langle P(0)P^\dagger(r) \rangle$$

Classical-statistical

Unfolding strongly correlated QCD with heavy ion collisions



large occupancies

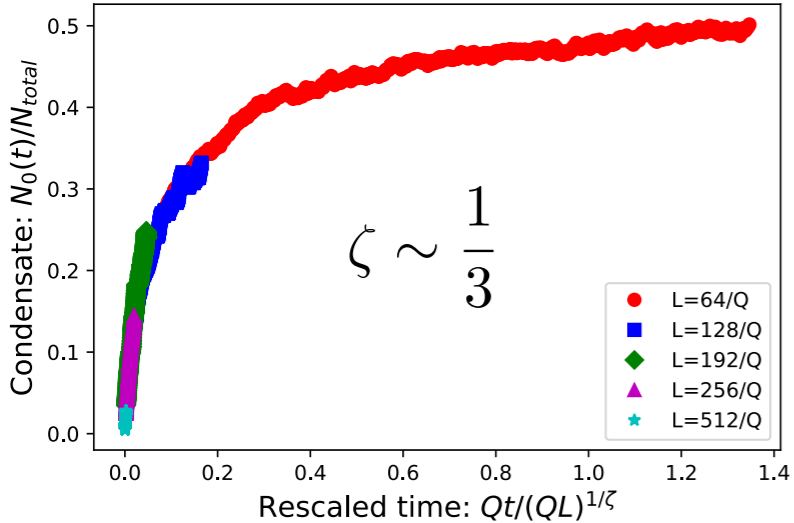
non-thermal fixed point

statistics + collective behaviour

&

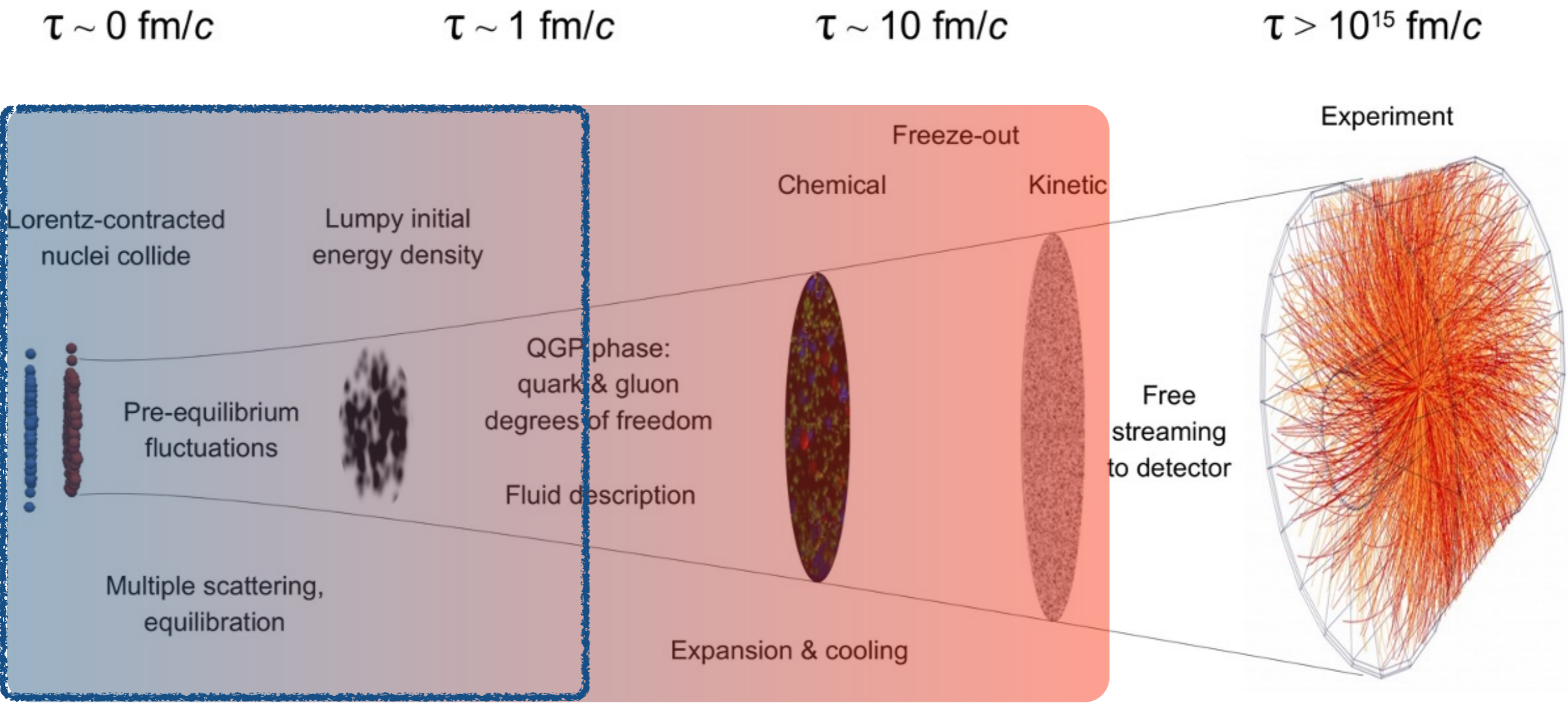
strongly correlated quantum dynamics

Classical-statistical



$$\sim \int_{\text{vol}} \langle P(0)P^\dagger(r) \rangle$$

Unfolding strongly correlated QCD with heavy ion collisions



large occupancies

statistics + collective behaviour

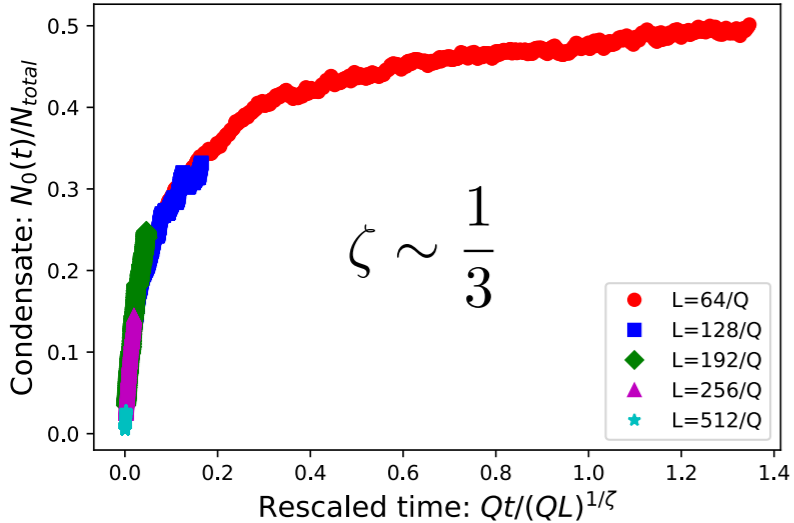
Background

non-thermal fixed point

&

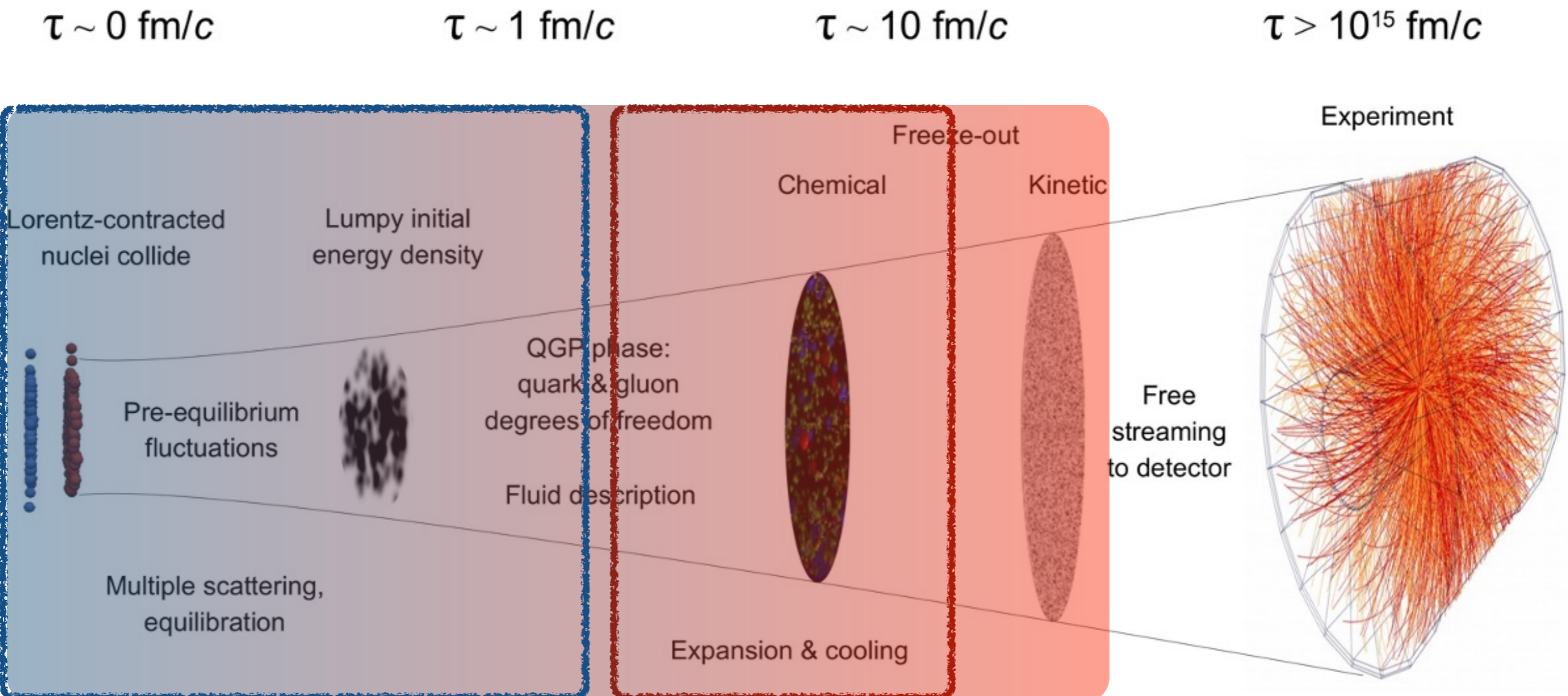
strongly correlated quantum dynamics

Classical-statistical



$$\sim \int_{\text{vol}} \langle P(0)P^\dagger(r) \rangle$$

Unfolding strongly correlated QCD with heavy ion collisions



large occupancies

non-thermal fixed point

statistics + collective behaviour

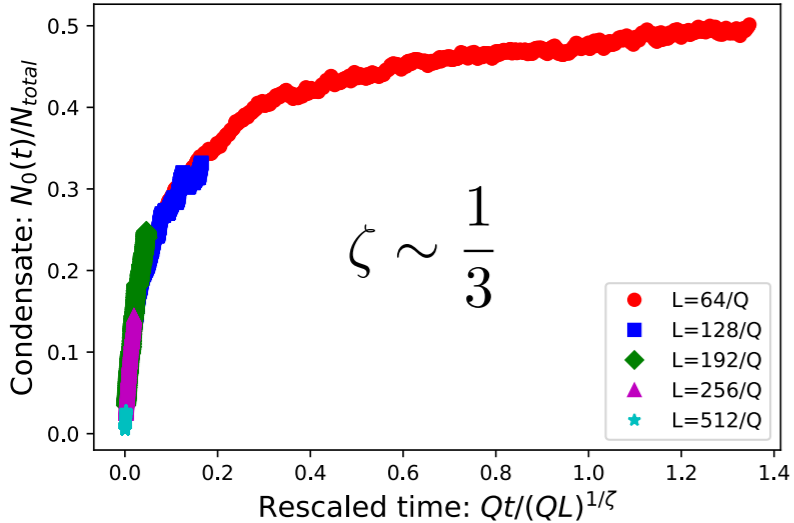
Background

&

strongly correlated quantum dynamics

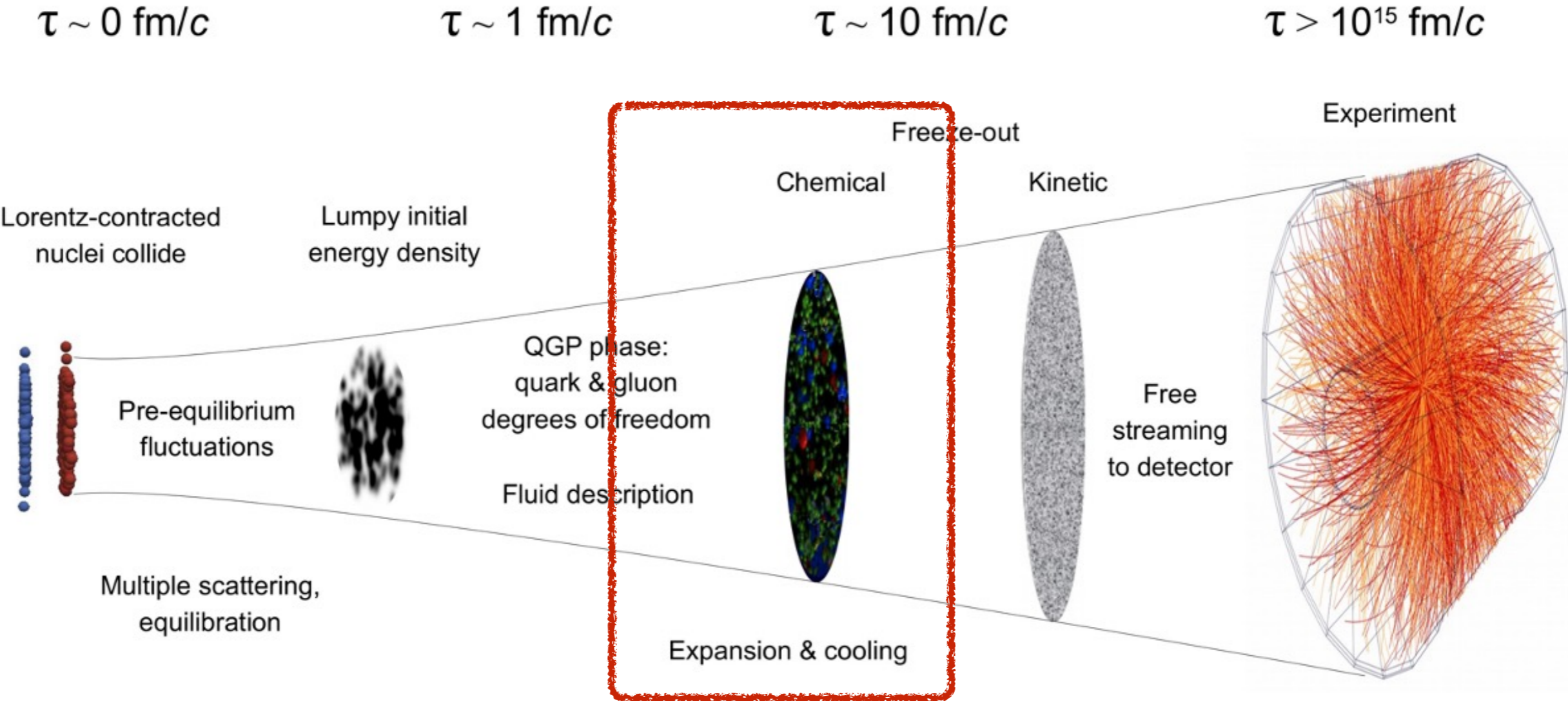
The cool stuff!

Classical-statistical



$$\sim \int_{\text{vol}} \langle P(0)P^\dagger(r) \rangle$$

Unfolding strongly correlated QCD with heavy ion collisions



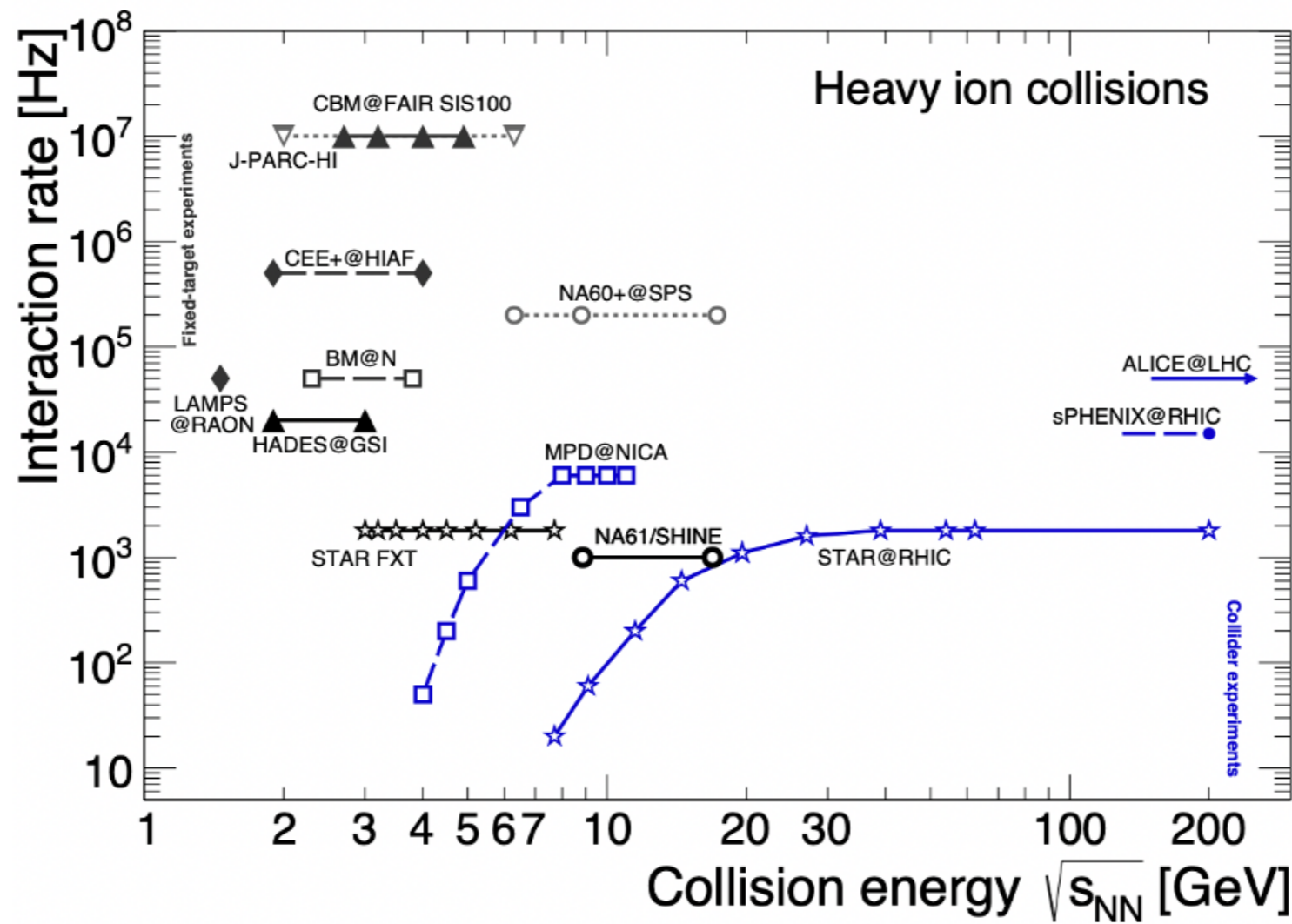
← How to unfold: inverse problem!

strongly correlated quantum dynamics

Experimental landscape

'The (experimental) future is bright'

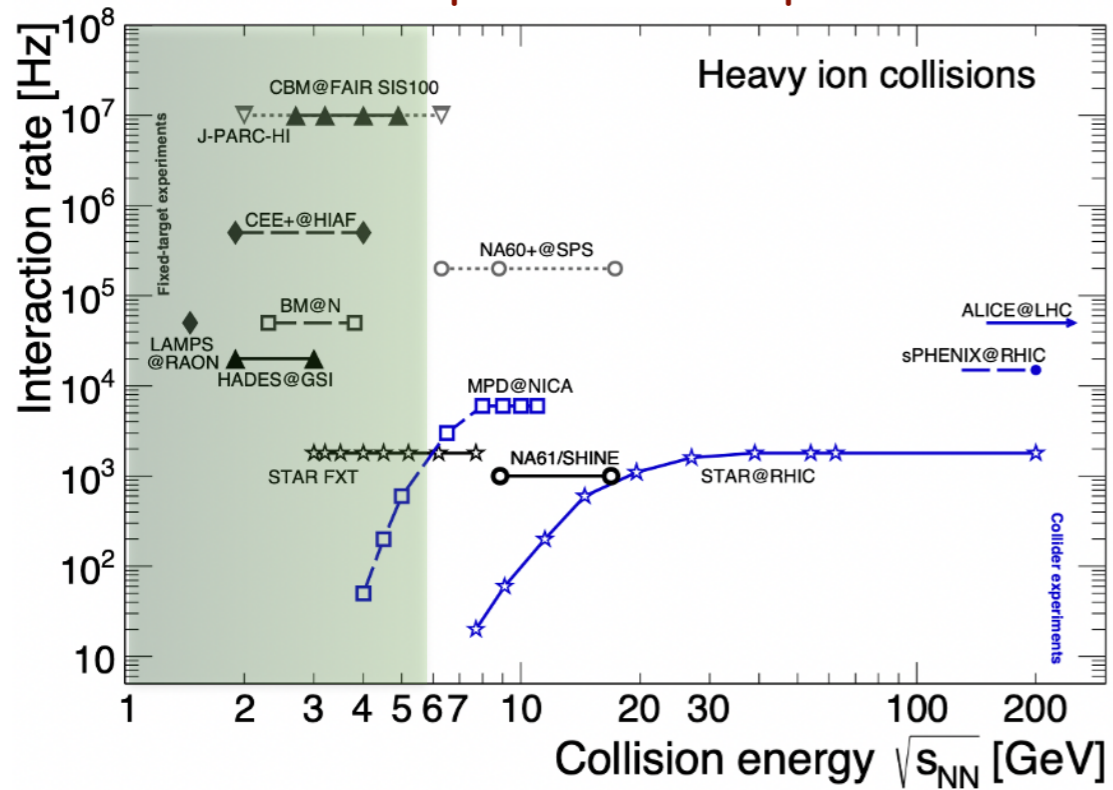
Tetyana Galatyuk, Erice 2021



Galatyuk, A982 (2019) update 2021; CBM, EPJA 53 3 (2017) 60

How bright does it get?

Experimental landscape

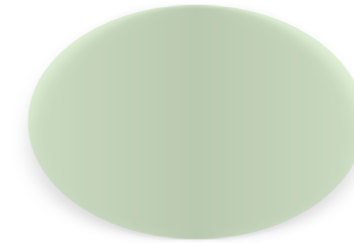


Galatyuk, A982 (2019) update 2021
CBM, EPJA 53 3 (2017) 60

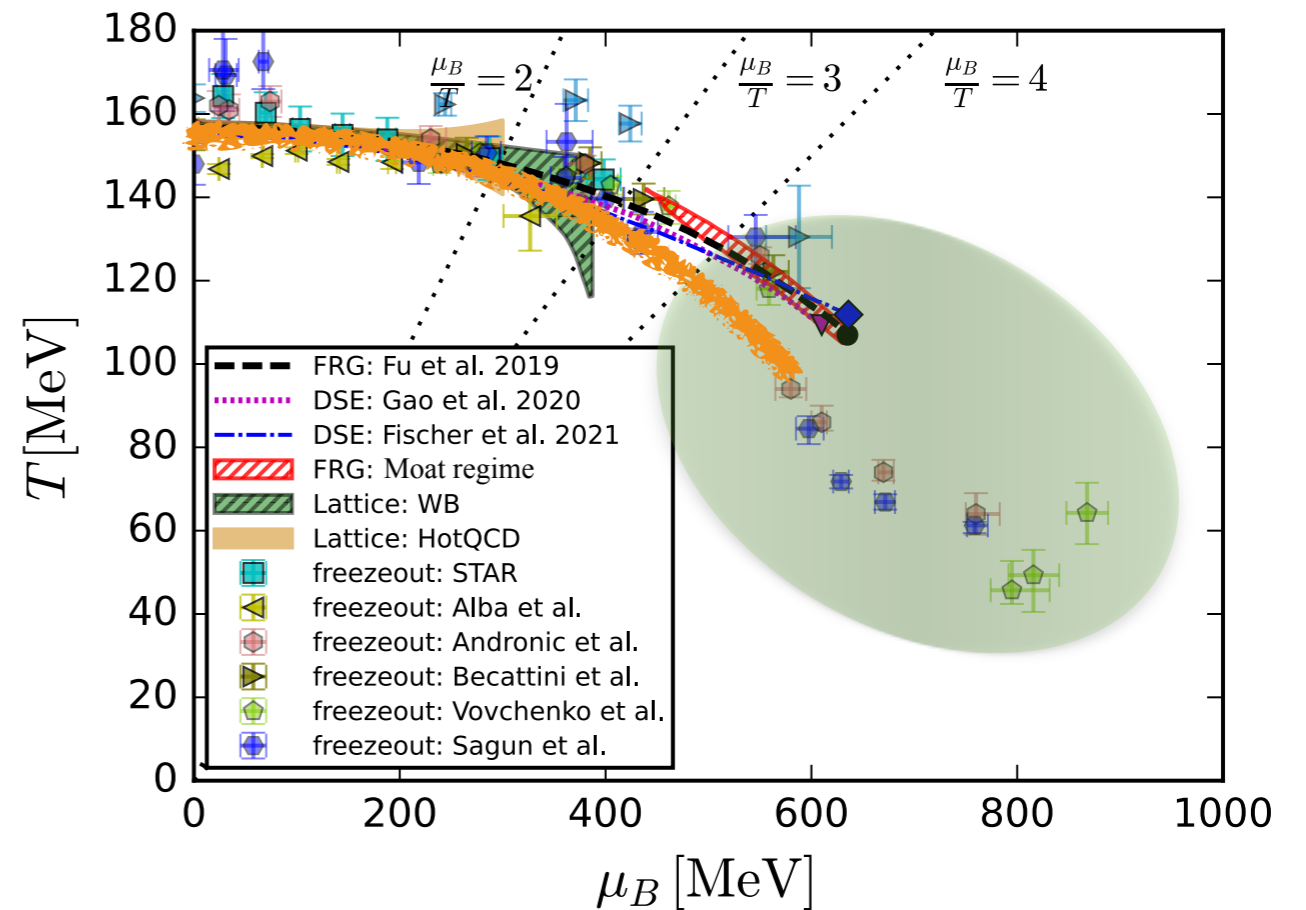
Freezeout curve

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

CEP or other NEW physics/phases



Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)



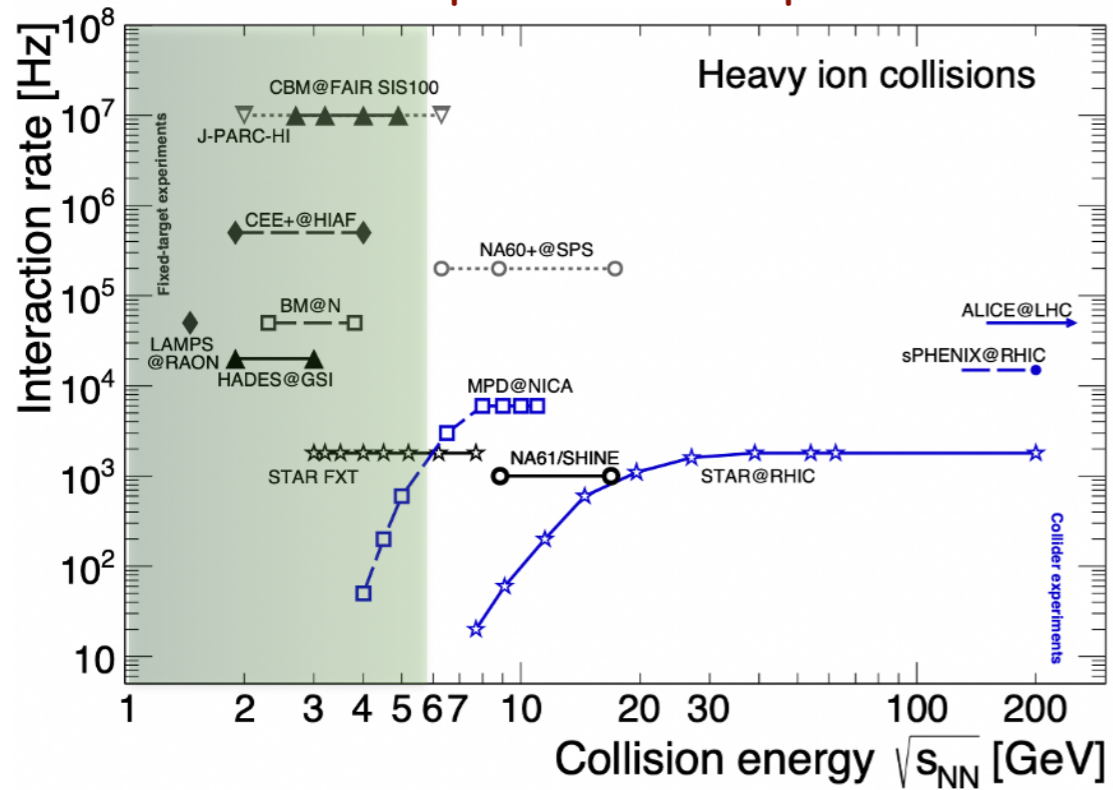
Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

How bright does it get?

Experimental landscape



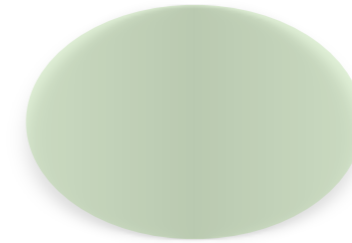
Galatyuk, A982 (2019) update 2021
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Freezeout curve

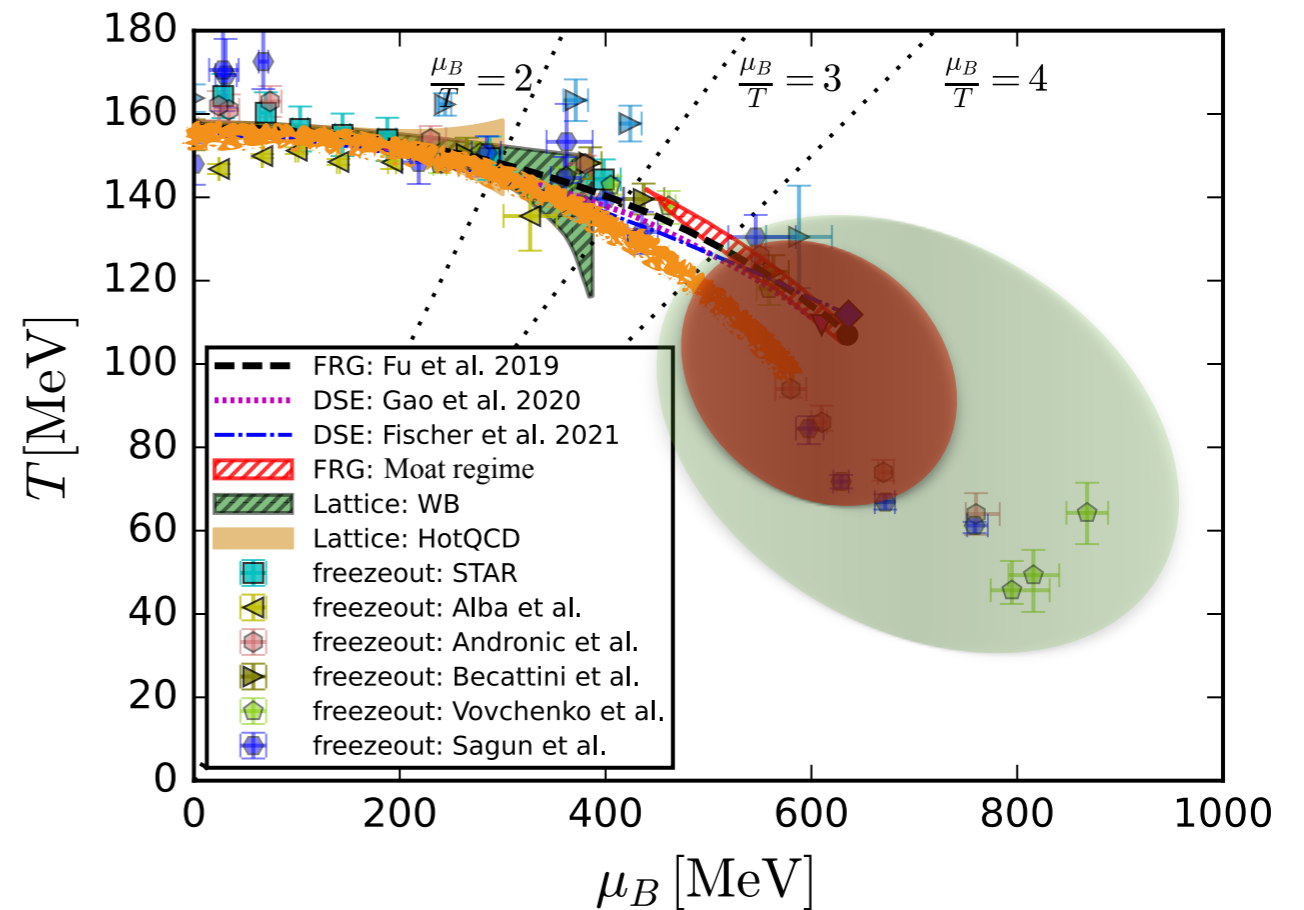
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9



CEP or other NEW physics/phases



Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)



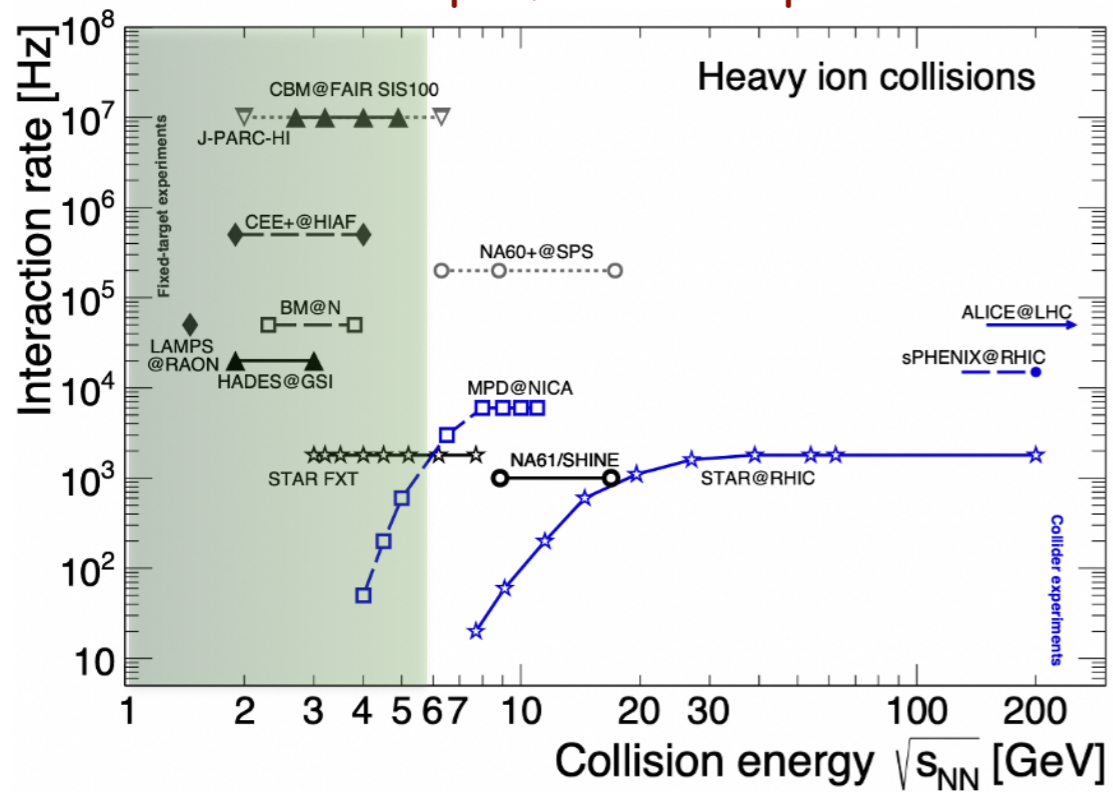
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Experimental landscape



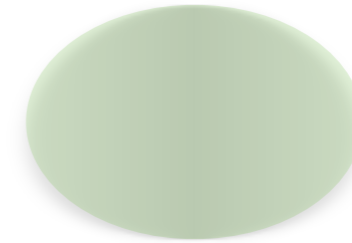
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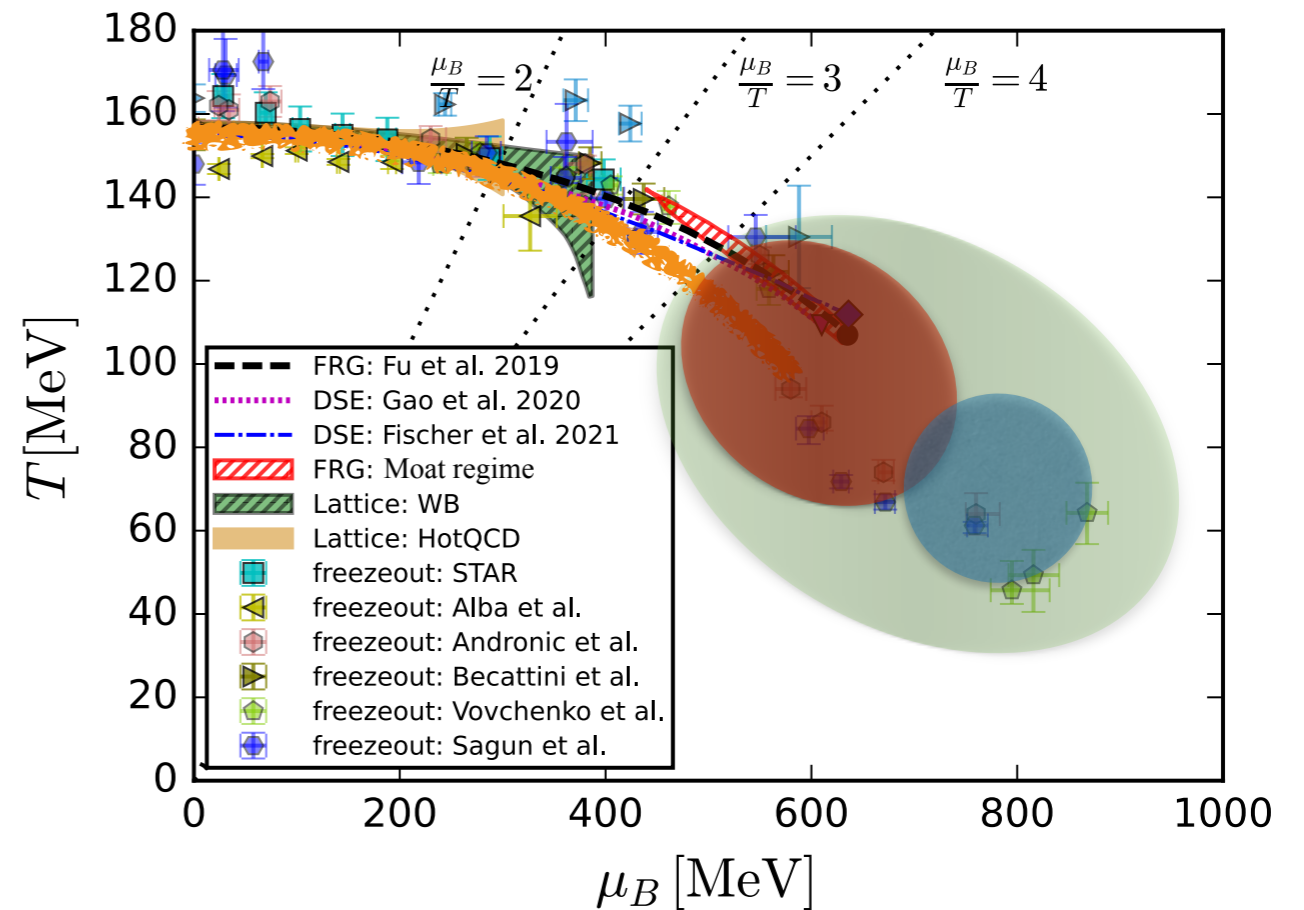
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9



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Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)



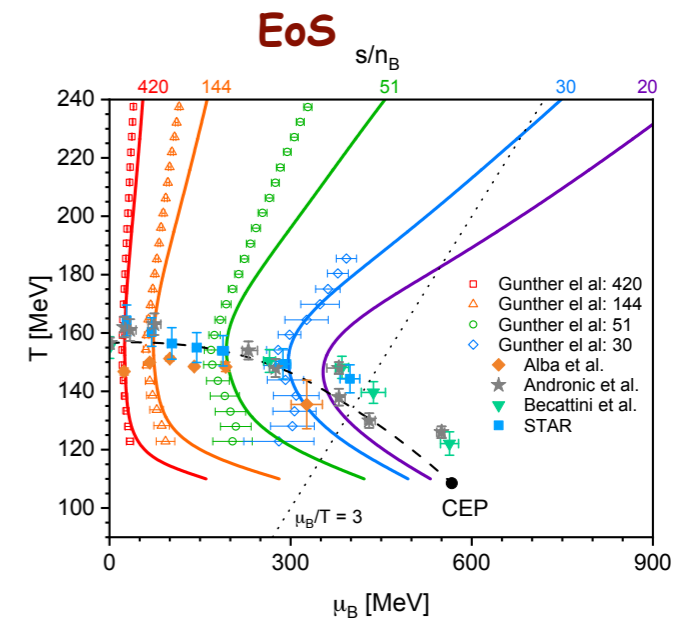
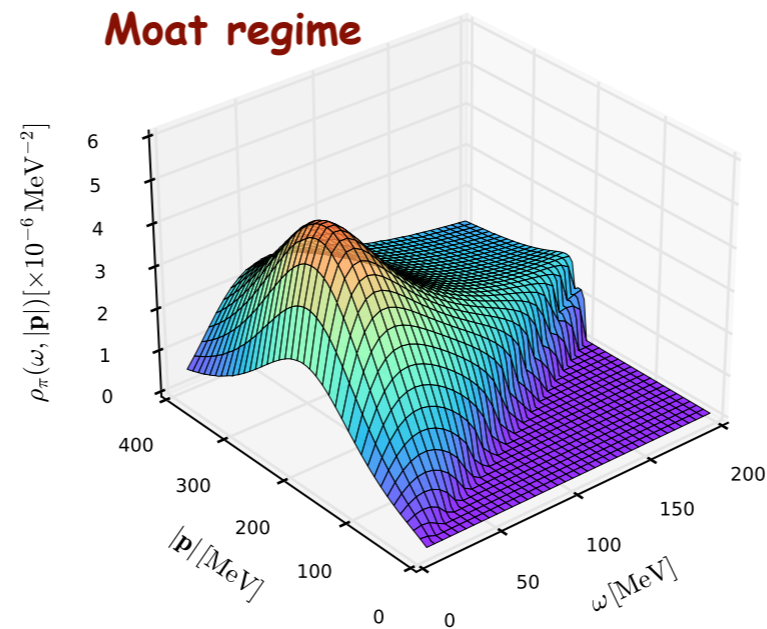
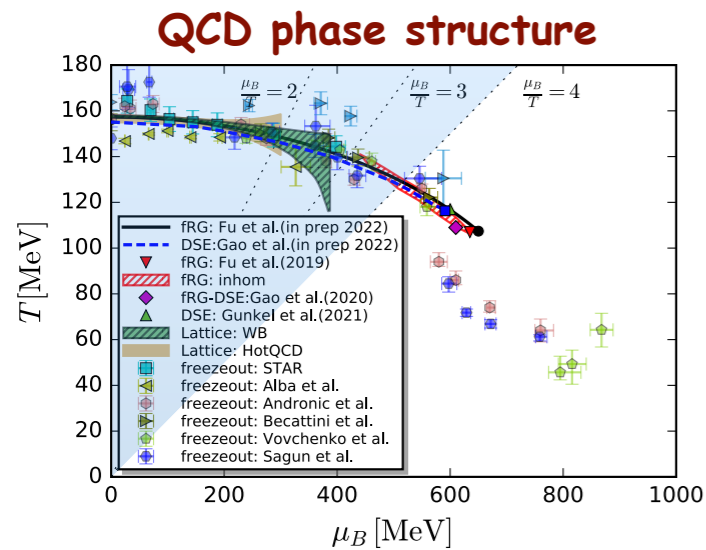
Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

Sneak preview on some results

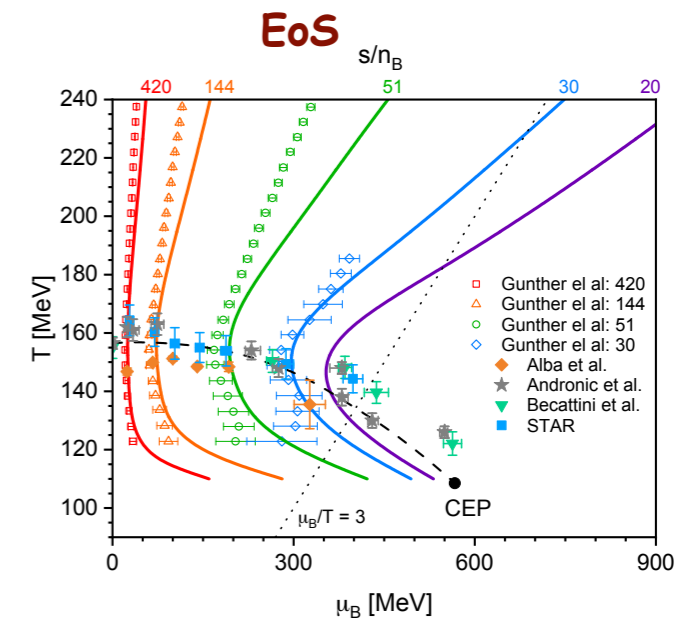
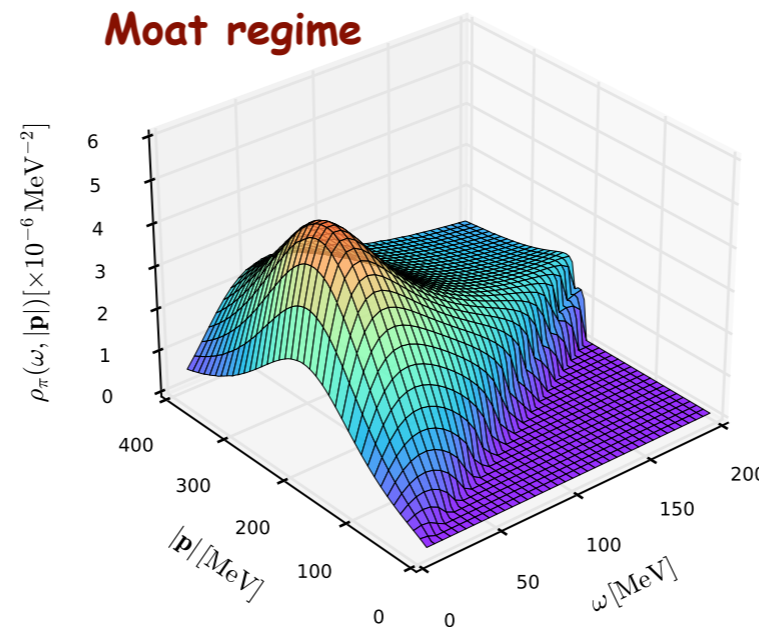
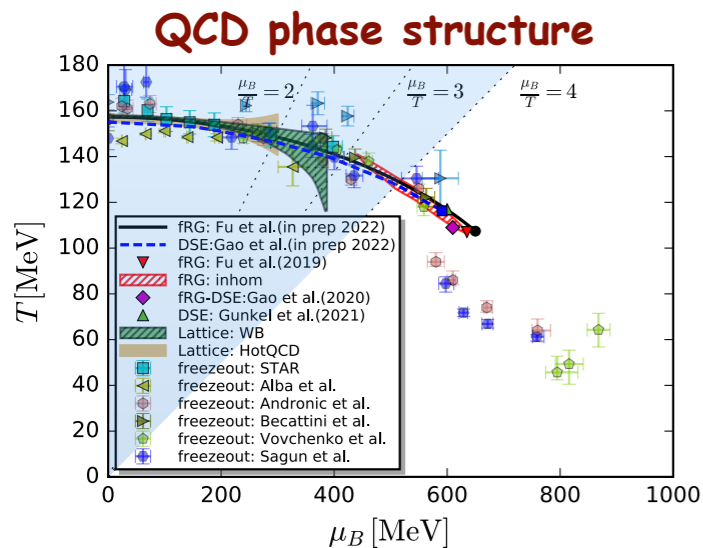
- QCD phase structure, 'CEP' estimate & phenomenological applications



- (Non-critical) chiral dynamics & fluctuations of observed charges

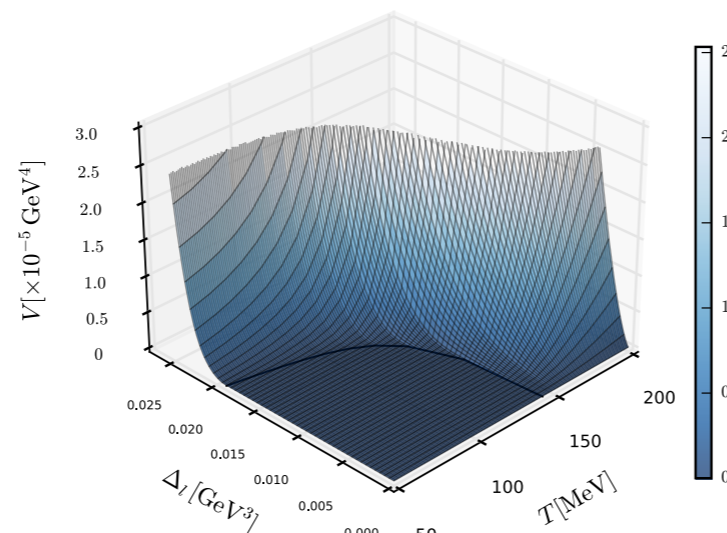
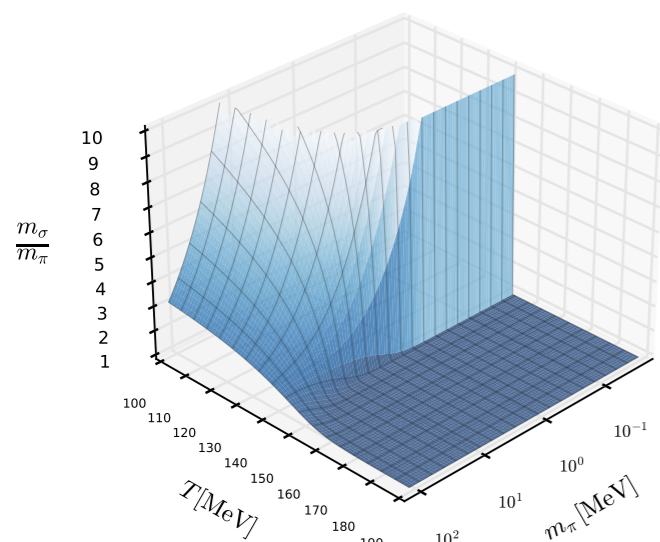
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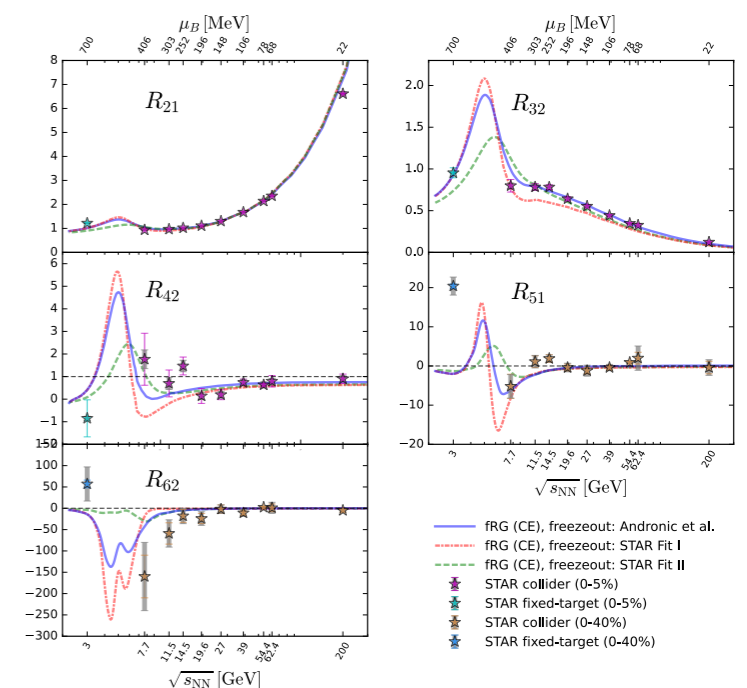


- (Non-critical) chiral dynamics & fluctuations of observed charges

Chiral dynamics



Fluctuations of conserved charges



Additional material & technical details

fQCD collaboration

**Braun, Chen, Fu, Gao, Geissel, Horak, Huang, Lu, Ihssen, JMP, Rennecke,
Sattler, Schallmo, Tan, Töpfel, Turnwald, Wen, Wessely, Wink, Yin**

Dalian, Beijing, Darmstadt, Heidelberg, Gießen

Outline

- QCD phase structure: Where do we stand?

- Chiral dynamics

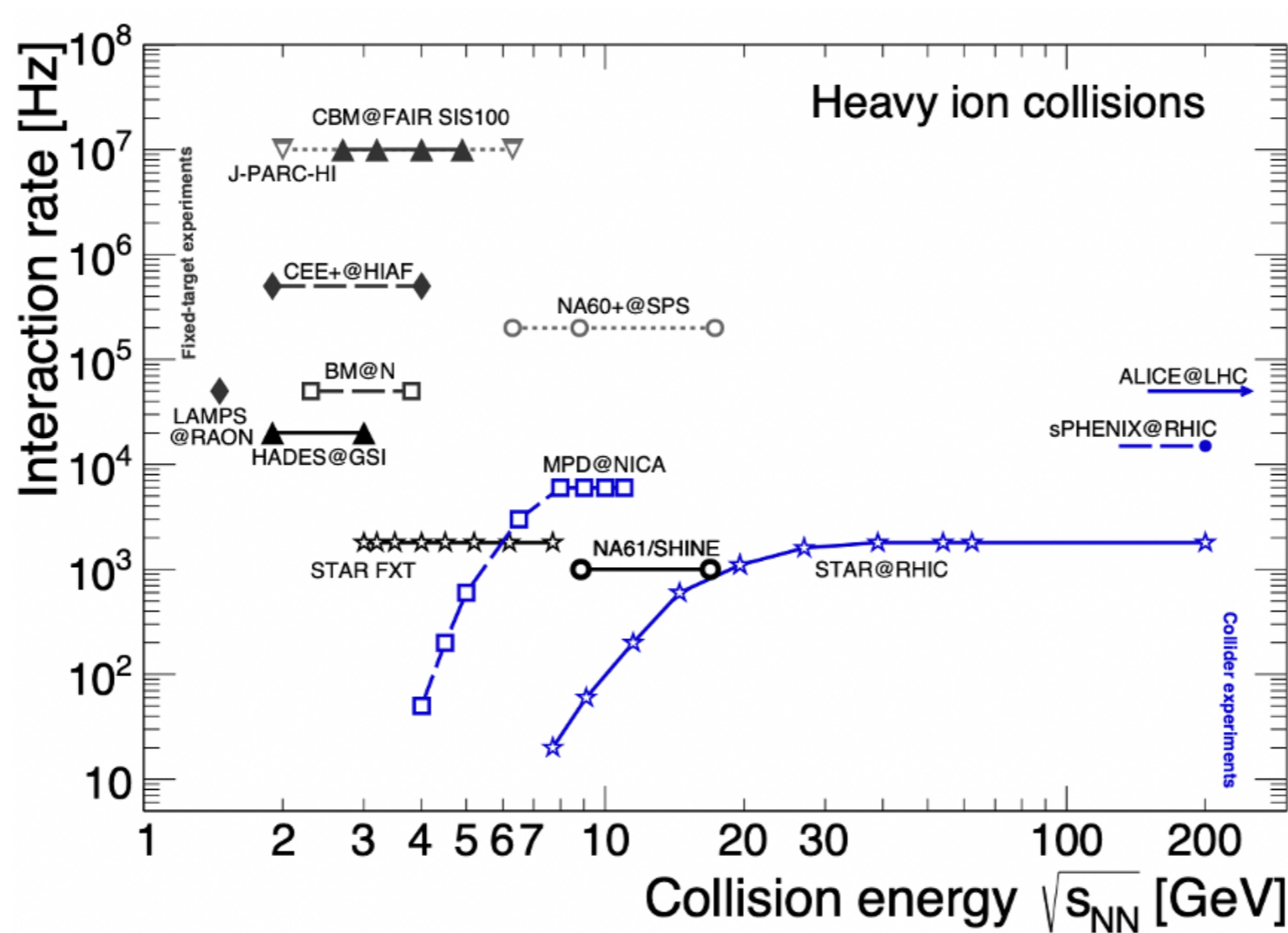
- Fluctuations of conserved charges

- Summary & outlook

Experimental landscape

'The (experimental) future is bright'

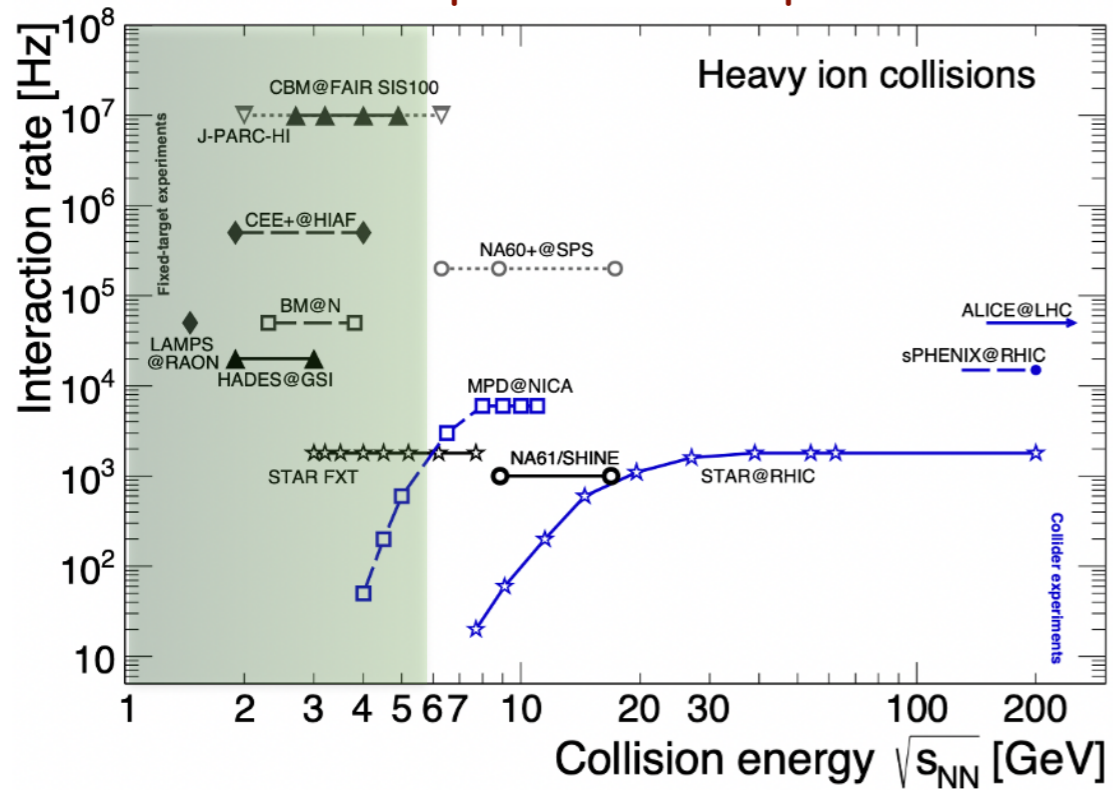
Tetyana Galatyuk, Erice 2021



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How bright does it get?

Experimental landscape



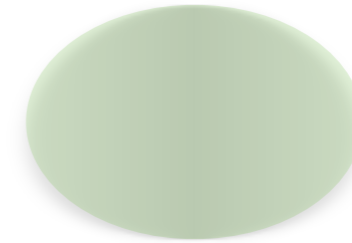
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Freezeout curve

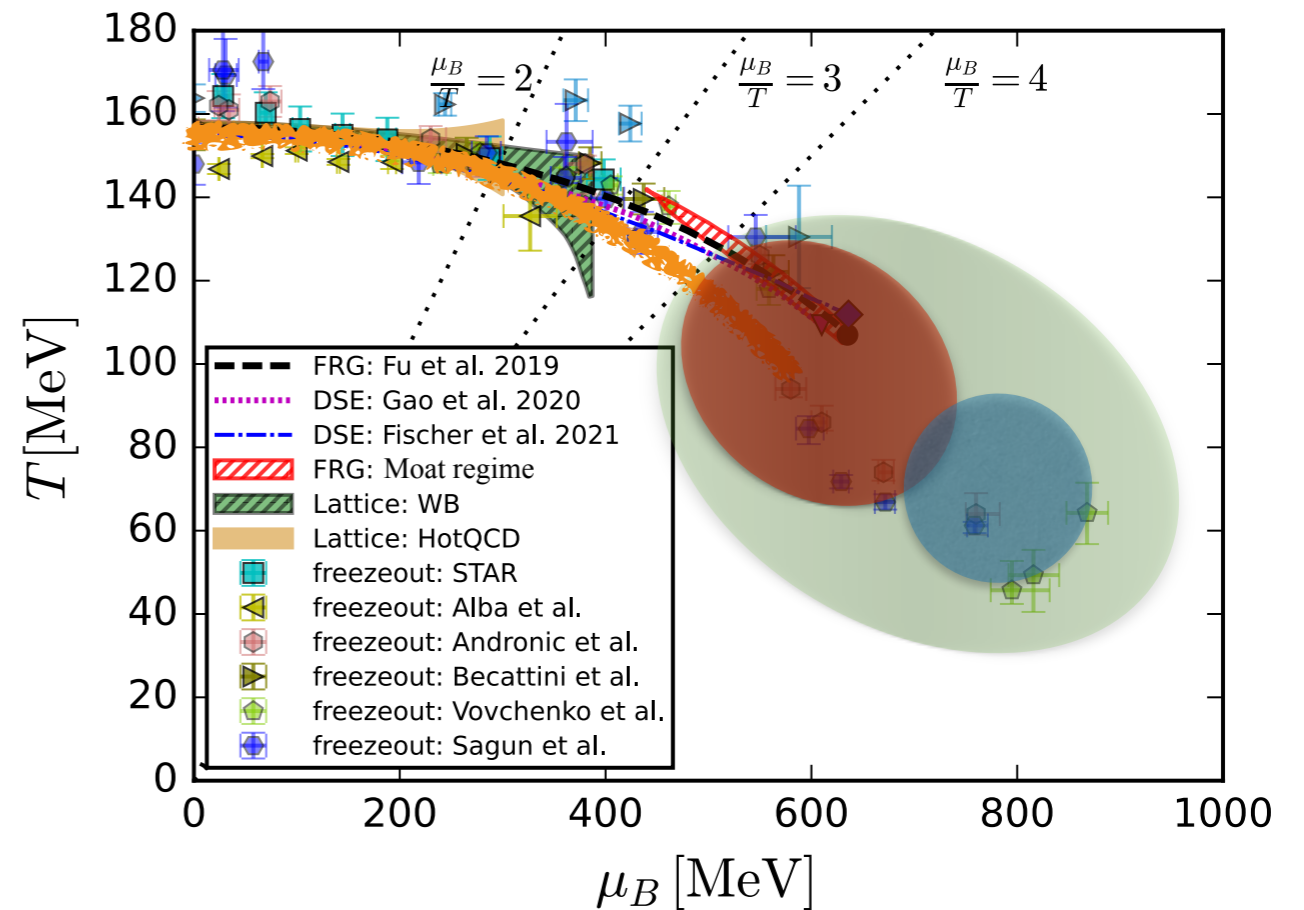
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9



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Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)



Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

QCD with Functional Approaches

ab initio & closed form

Diagrammatic functional relations for the free energy $\Gamma[\phi]$

functional RG:

free energy/
grand potential

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{glue quantum fluctuations} - \text{quark quantum fluctuations} + \frac{1}{2} \text{hadronic quantum fluctuations}$$

glue quantum fluctuations

quark quantum fluctuations

hadronic quantum fluctuations

RG-scale k : $t = \ln k$

functional DSE :

A_0 : background field

$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2} \text{glue loop} - \text{quark loop} - \frac{1}{6} \text{glue loop with vertices} + \text{quark loop with vertices}$$

fRG: Fu, Commun.Theor.Phys. 74 (2022) 097304

Dupuis et al, Phys.Rept. 910 (2021) 1

DSE: Fischer, PPNP 105 (2019) 1

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{glue quantum fluctuations} - \text{quark quantum fluctuations} + \frac{1}{2} \text{hadronic quantum fluctuations}$$

free energy/
grand potential

The diagram illustrates the functional RG equation for the free energy/grand potential. It shows four terms in a sequence, separated by minus and plus signs. The first term is a circle of orange wavy lines, labeled 'glue quantum fluctuations'. The second term is a dashed circle, labeled 'quark quantum fluctuations'. The third term is a solid circle, labeled 'quark quantum fluctuations'. The fourth term is a dashed circle, labeled 'hadronic quantum fluctuations'. Each diagram has a small circle with an 'X' on top. The equation is: $\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{glue quantum fluctuations} - \text{quark quantum fluctuations} + \frac{1}{2} \text{hadronic quantum fluctuations}$. Below the equation, the text 'free energy/grand potential' is written.

Correlation functions

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{glue quantum fluctuations} - \text{quark quantum fluctuations} \right) + \frac{1}{2} \text{hadronic quantum fluctuations}$$

free energy/
grand potential

Correlation functions

gluon propagator

$$\langle A_\mu A_\nu \rangle(p)$$

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{glue quantum fluctuations} - \text{quark quantum fluctuations} \right) + \frac{1}{2} \left(\text{hadronic quantum fluctuations} \right)$$

free energy/
grand potential

Correlation functions

gluon propagator

$$\langle A_\mu A_\nu \rangle(p)$$

Pure glue

$$\partial_t \text{---}^{-1} = \text{---} \text{---} + \text{---} \text{---}$$

$$\partial_t \text{---}^{-1} = \text{---} \text{---} - 2 \text{---} \text{---} - \frac{1}{2} \text{---} \text{---}$$

$$\partial_t \text{---} = - \text{---} - \text{---} + \text{perm.}$$

$$\partial_t \text{---} = - \text{---} + 2 \text{---} + \text{---} + \text{perm.}$$

$$\partial_t \text{---} = + \text{---} + \text{---} - 2 \text{---} - \text{---} + \text{perm.}$$

+ matter loops

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{glue quantum fluctuations} - \text{quark quantum fluctuations} \right) + \frac{1}{2} \text{hadronic quantum fluctuations}$$

free energy/
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$$\langle A_\mu A_\nu \rangle(p)$$

quark propagator

$$\langle q \bar{q} \rangle(p)$$

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left[\text{glue quantum fluctuations} - \text{quark quantum fluctuations} \right] + \frac{1}{2} \text{hadronic quantum fluctuations}$$

free energy/
grand potential

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$$\langle A_\mu A_\nu \rangle(p)$$

quark propagator

$$\langle q\bar{q} \rangle(p)$$

quark-gluon vertex

$$\langle q\bar{q} A_\mu \rangle(p_1, p_2)$$

Eight transverse tensor structures

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{glue quantum fluctuations} - \text{quark quantum fluctuations} \right) + \frac{1}{2} \text{hadronic quantum fluctuations}$$

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Eight transverse tensor structures

quark-anti-quark scatterings

$$\langle q\bar{q}q\bar{q} \rangle(p_1, p_2, p_3)$$

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{glue quantum fluctuations} - \text{quark quantum fluctuations} \right) + \frac{1}{2} \text{hadronic quantum fluctuations}$$

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Eight transverse tensor structures

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$$\langle q\bar{q}q\bar{q} \rangle(p_1, p_2, p_3)$$

$$\partial_t \text{---}^{-1} = \text{gluon self-energy diagrams}$$

$$\partial_t \text{---} = \text{quark-gluon vertex diagrams}$$

$$\partial_t \text{---} = \text{quark-anti-quark scattering diagrams}$$

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{glue quantum fluctuations} - \text{quark quantum fluctuations} \right) + \frac{1}{2} \text{hadronic quantum fluctuations}$$

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Eight transverse tensor structures

quark-anti-quark scatterings

$$\langle q\bar{q}q\bar{q} \rangle(p_1, p_2, p_3)$$

$$\partial_t \text{gluon}^{-1} = \text{gluon loop} + \text{quark loop} + \frac{1}{2} \text{ghost loop} + \text{ghost loop} + \text{quark loop} - \text{quark loop}$$

$$\partial_t \text{quark-gluon vertex} = - \text{gluon loop} - \text{quark loop} - \text{quark loop} - \text{quark loop} - \frac{1}{2} \text{quark loop} + 2 \text{quark loop} - \text{quark loop} + \text{perm.}$$

$$\partial_t \text{quark-anti-quark scattering} = 2 \text{quark loop} - \text{quark loop} - \text{quark loop} - \text{quark loop} - \text{quark loop} - \text{quark loop} - \text{quark loop} + \text{perm.}$$

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left[\text{glue quantum fluctuations} - \text{quark quantum fluctuations} \right] + \frac{1}{2} \left[\text{hadronic quantum fluctuations} \right]$$

free energy/
grand potential

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$$\langle q\bar{q}q\bar{q} \rangle(p_1, p_2, p_3)$$

$$\partial_t \text{---}^{-1} = \text{---} + \text{---} + \frac{1}{2} \text{---} + \text{---} + \text{---} - \text{---}$$

$$\partial_t \text{---} = - \text{---} - \text{---} - \text{---} - \text{---} - \frac{1}{2} \text{---} + 2 \text{---} - \text{---} + \text{perm.}$$

$$\partial_t \text{---} = 2 \text{---} - \text{---} - \text{---} - \text{---} - \text{---} - \text{---} - \text{---} + \text{perm.}$$

Emergent composites

$$\text{---} \xrightarrow{s} \text{---} = \text{---} - \text{---} + \text{---}$$

where

$$\left. \begin{array}{c} \vec{p}_1 \\ \vec{p}_2 \end{array} \right\} \text{---} \left. \begin{array}{c} \vec{p}_4 \\ \vec{p}_3 \end{array} \right\} (\phi) = 0$$

$(p_1 + p_3)^2 = 0$
 $(p_2 + p_4)^2 = 0$

Functional Methods for QCD

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left[\text{glue quantum fluctuations} - \text{quark quantum fluctuations} + \text{hadronic quantum fluctuations} \right]$$

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Eight transverse tensor structures

quark-anti-quark scatterings

$$\langle q\bar{q}q\bar{q} \rangle(p_1, p_2, p_3)$$

$$\partial_t \text{---}^{-1} = \text{diagrams}$$

$$\partial_t \text{---} = \text{diagrams} - \frac{1}{2} \text{diagram} + 2 \text{diagram} - \text{diagram} + \text{perm.}$$

$$\partial_t \text{---} = 2 \text{diagrams} - \text{diagram} - \text{diagram} - \text{diagram} - \text{diagram} - \text{diagram} - \text{diagram} + \text{perm.}$$

Emergent composites

$$\partial_t \text{---}^{-1} = -2 \text{diagram} + \text{diagram} + \frac{1}{2} \text{diagram}$$

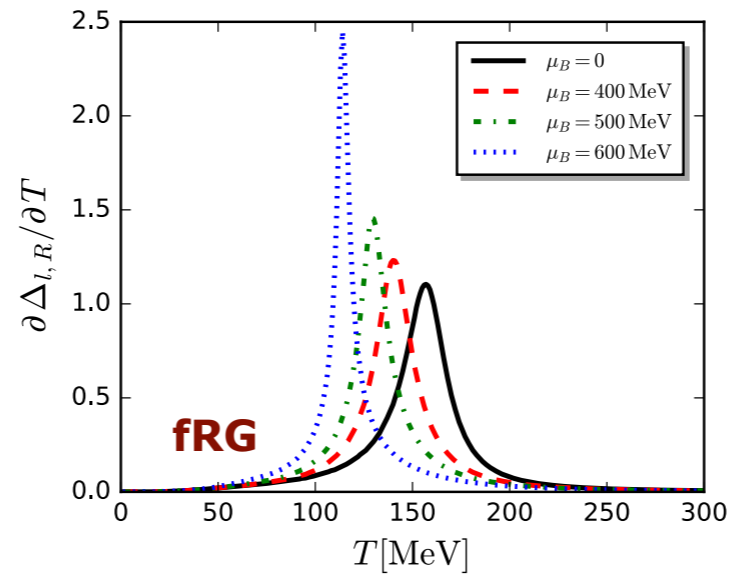
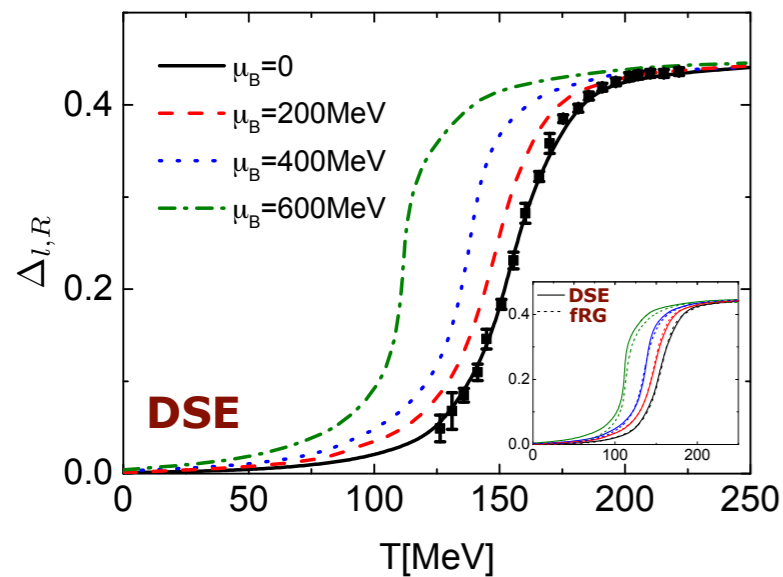
$$\partial_t \text{---} = \text{diagrams} + 2 \text{diagram} + \text{perm.}$$

Emergent composites for mesons, diquarks, baryons

Fukushima, JMP, Strodthoff, AP 446 (2022) 169106

Chiral condensates

renormalised condensate



$$\Delta_{l,R}(T, \mu_B) \simeq \Delta_l(T, \mu_B) - \Delta_l(0, 0)$$

$$\Delta_q(T, \mu_B) = \frac{T}{\mathcal{V}} m_q^0 \int_x \langle \bar{q}(x) q(x) \rangle$$

lattice: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010)

DSE: quark condensates

See also

Fischer, Luecker, PLB 718 (2013) 1036

Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

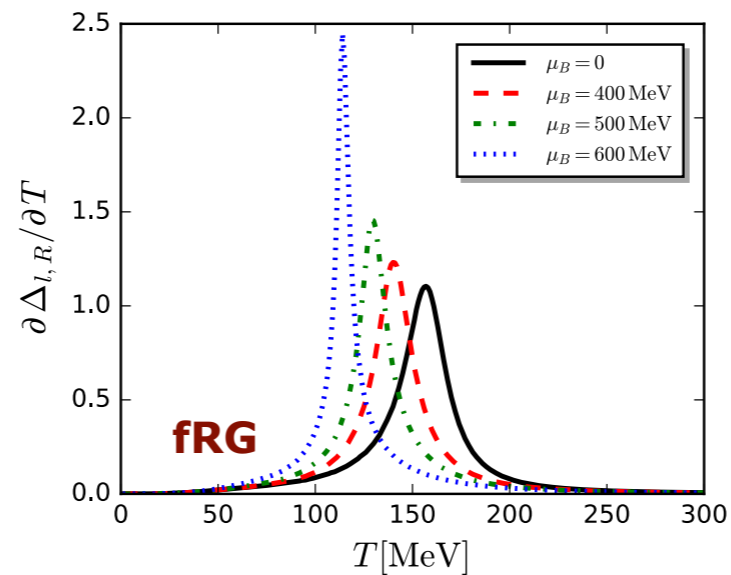
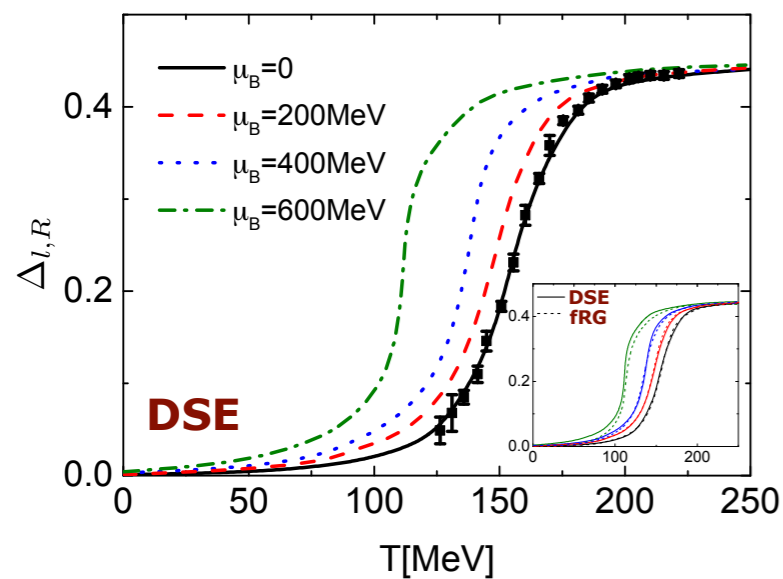
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

fRG: Fu, JMP, Rennecke, PRD 101 (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

Chiral condensates

renormalised condensate

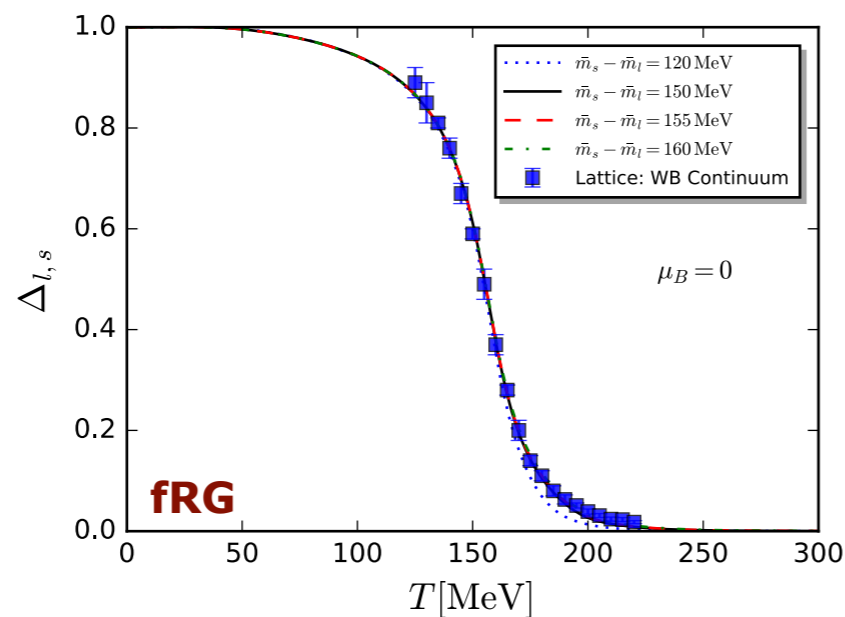


$$\Delta_{l,R}(T, \mu_B) \simeq \Delta_l(T, \mu_B) - \Delta_l(0, 0)$$

$$\Delta_q(T, \mu_B) = \frac{T}{\mathcal{V}} m_q^0 \int_x \langle \bar{q}(x) q(x) \rangle$$

lattice: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010)

reduced condensate

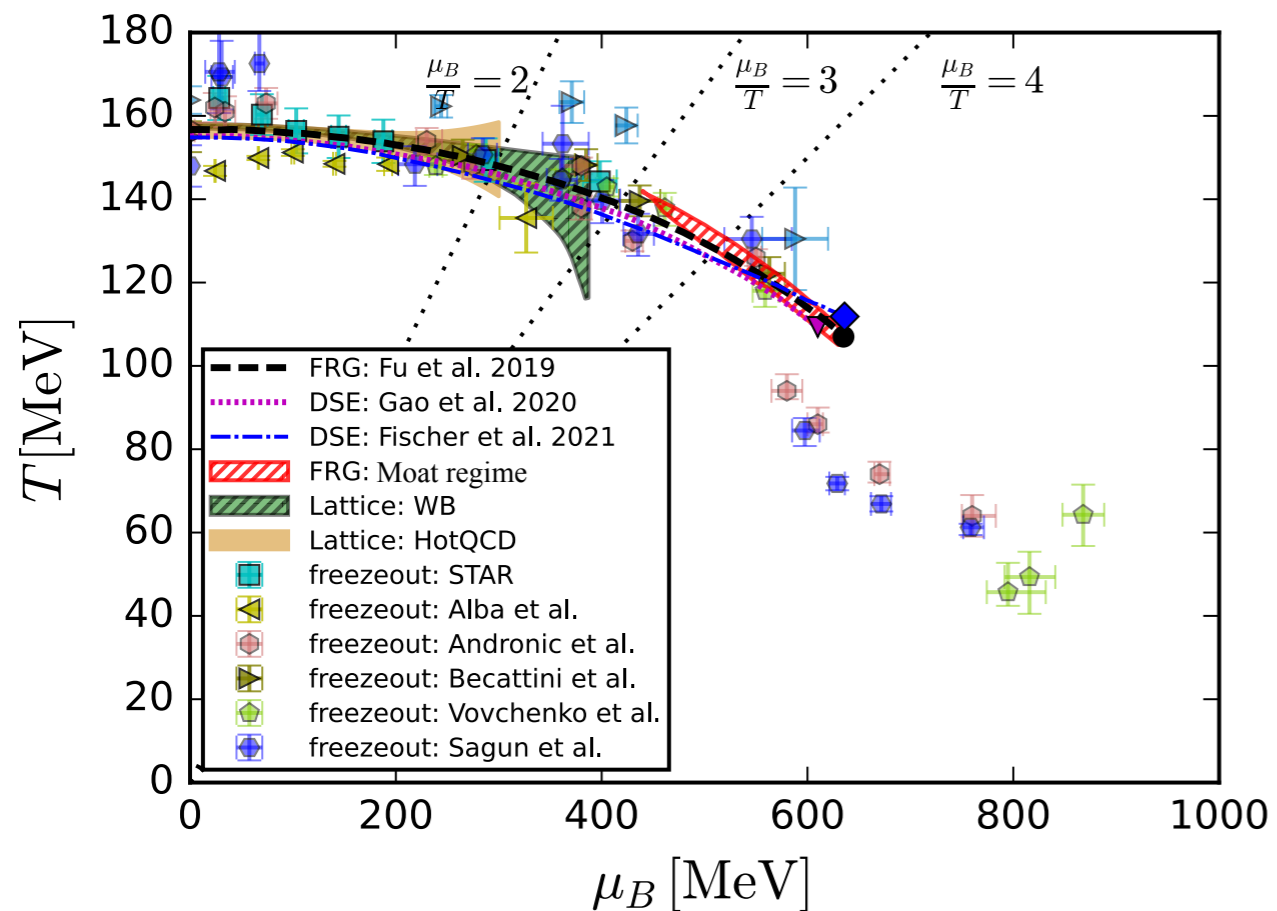


$$\Delta_{l,s}(T, \mu_B) = \frac{\Delta_l(T, \mu_B) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T, \mu_B)}{\Delta_l(0, 0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0, 0)}$$

fRG: Fu, JMP, Rennecke, PRD 101 (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

Curvature of the chiral transition line



fQCD

$$\kappa_{\text{FRG}} = 0.0142(2)$$

$$\kappa_{\text{DSE}} = 0.0147(5)$$

lattice

$$\kappa_{\text{WB}} = 0.0149(21)$$

WB, PLB 751 (2015) 559

$$\kappa_{\text{hotQCD}} = 0.015(4)$$

hotQCD, PLB 795 (2019) 15

Quantitative reliability: $\mu_B/T \lesssim 4$

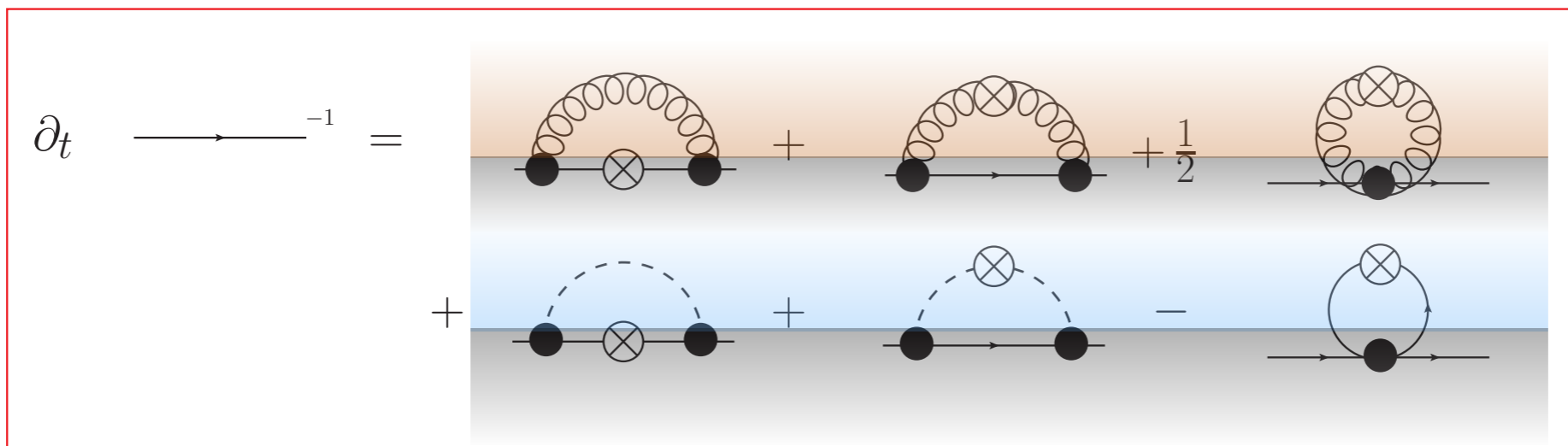
fRG: Fu, JMP, Rennecke, PRD 101 (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584
Gunkel, Fischer, PRD 104 (2021) 054022

Analyticity considerations at finite density

- Self-consistent truncations to functional relations define analytic functions in μ_B , eg:

$$\partial_t \langle q(x) \bar{q}(y) \rangle(\mu_B) = \text{Loop} \left[\langle q(x) \bar{q}(y) \rangle(\mu_B), \langle q(x) A_\mu(y) \bar{q}(z) \rangle(\mu_B), \dots; \mu_B \right]$$

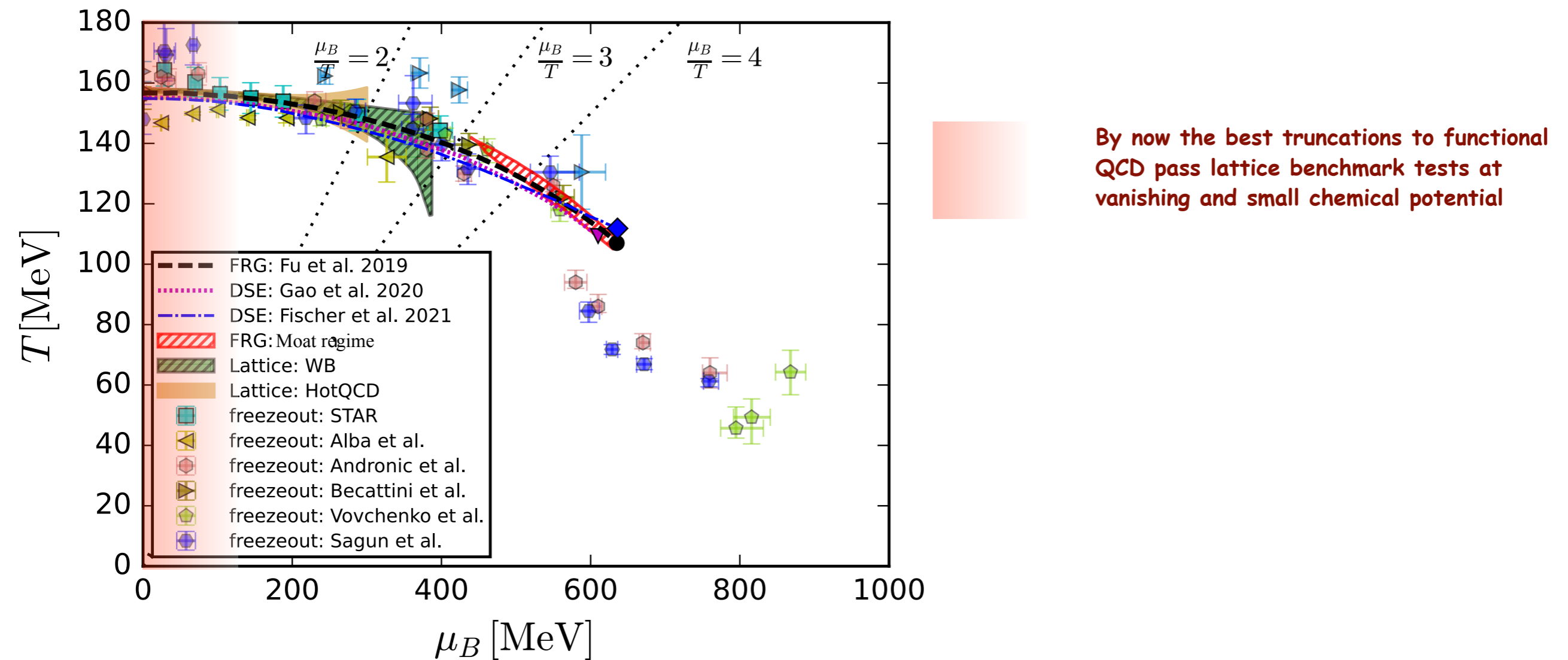


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- Consequences for functional QCD predictions at finite density

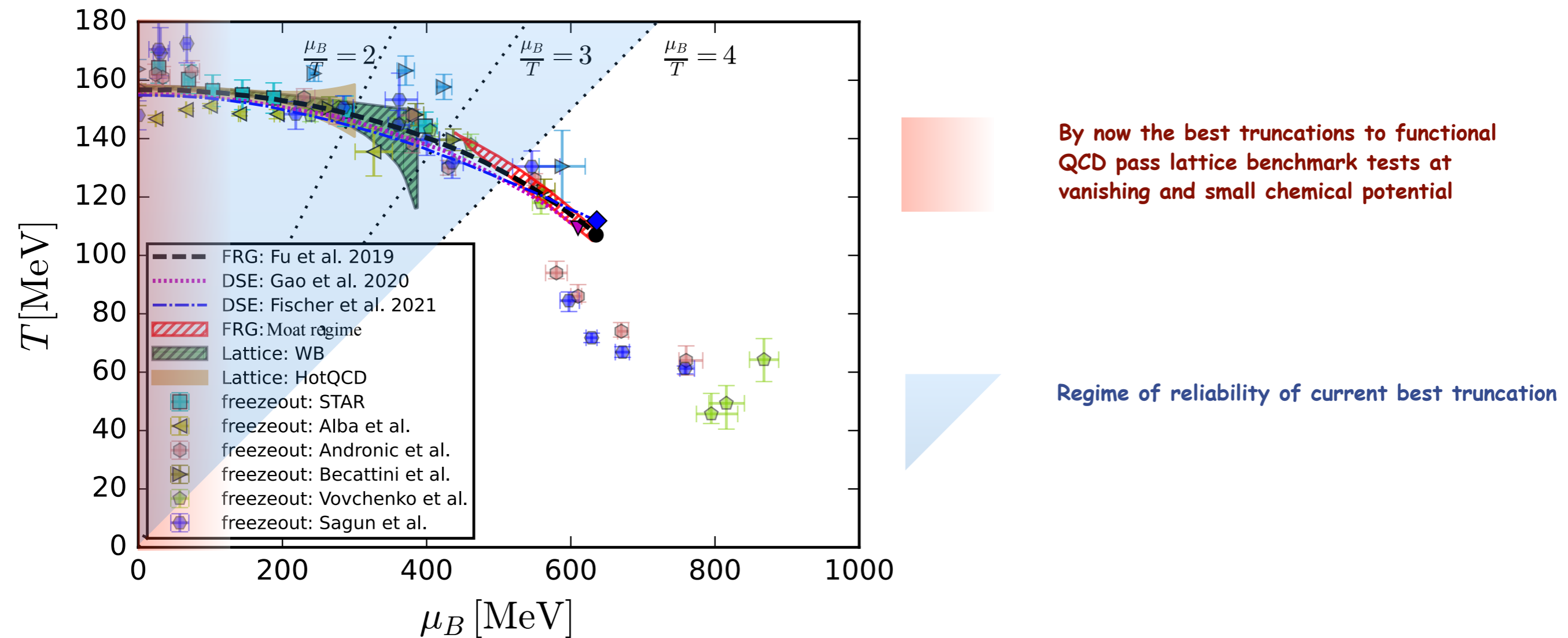


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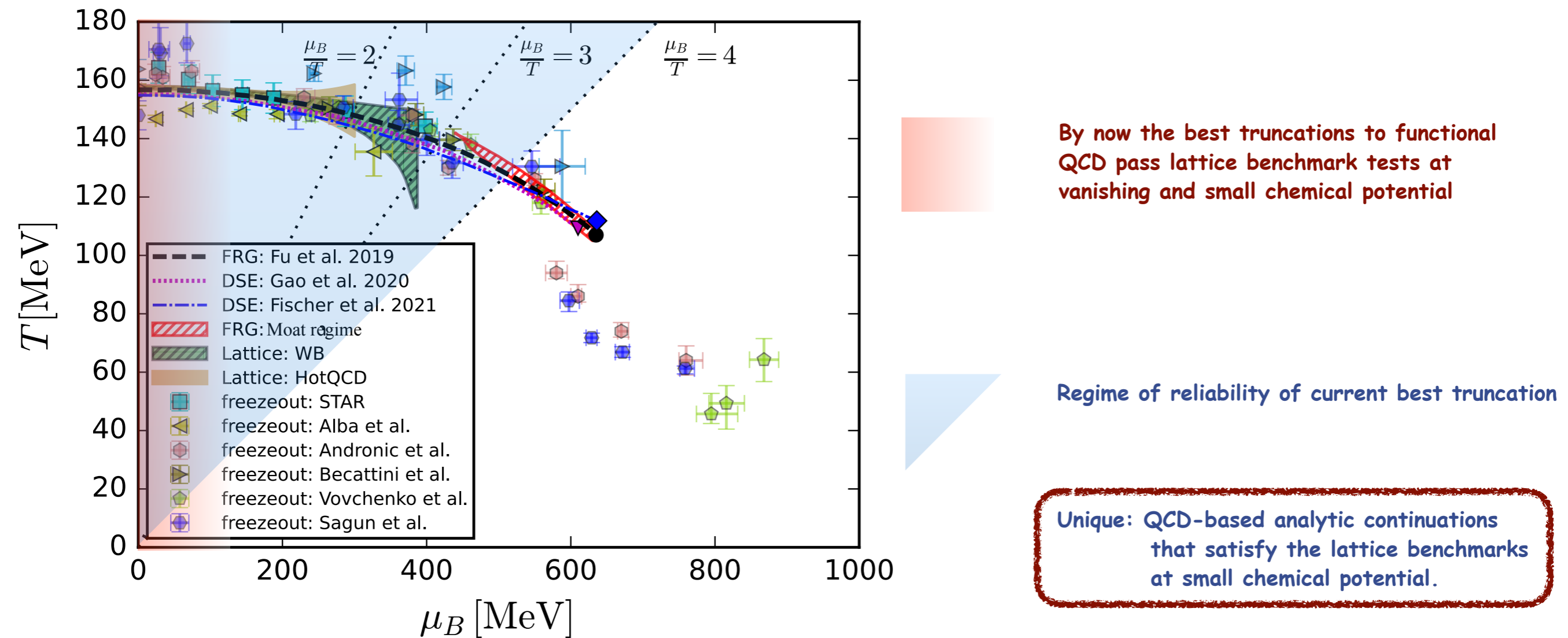


Analyticity considerations at finite density

- Self-consistent truncations to functional relations define analytic functions in μ_B , eg:

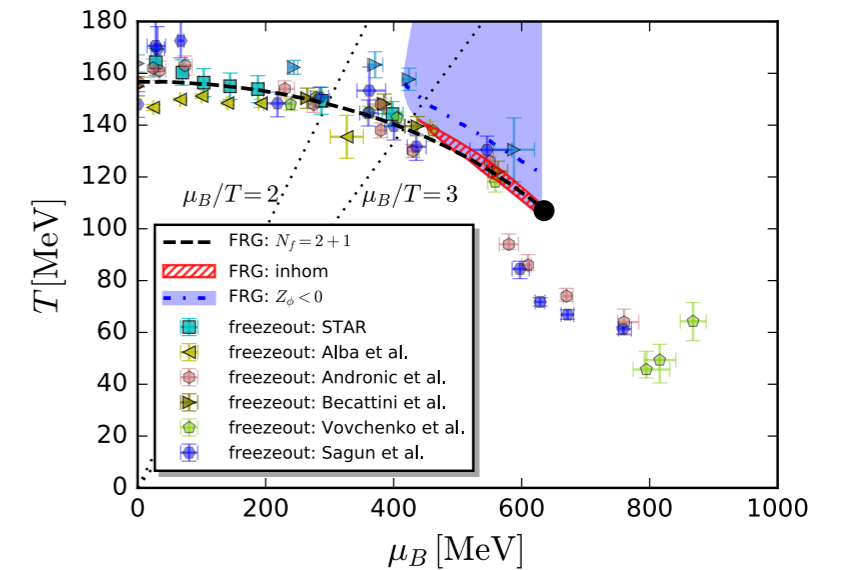
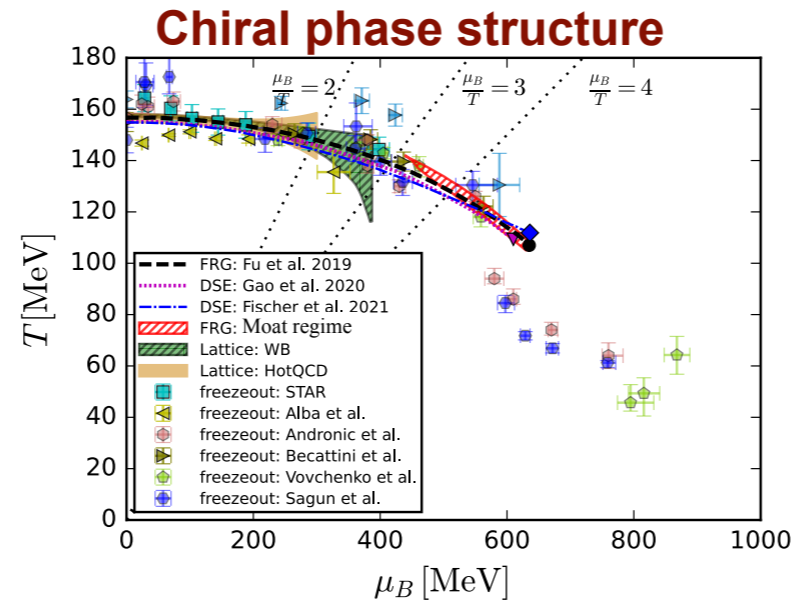
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- Consequences for functional QCD predictions at finite density



Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Sneak preview on the QCD moat



Fu, JMP, Rennecke, PRD 101 (2020) 054032

Chinese moat



Wikipedia

German moat



Wikimedia commons: © Günter Seggebäing, CC BY-SA 3.0

US moat



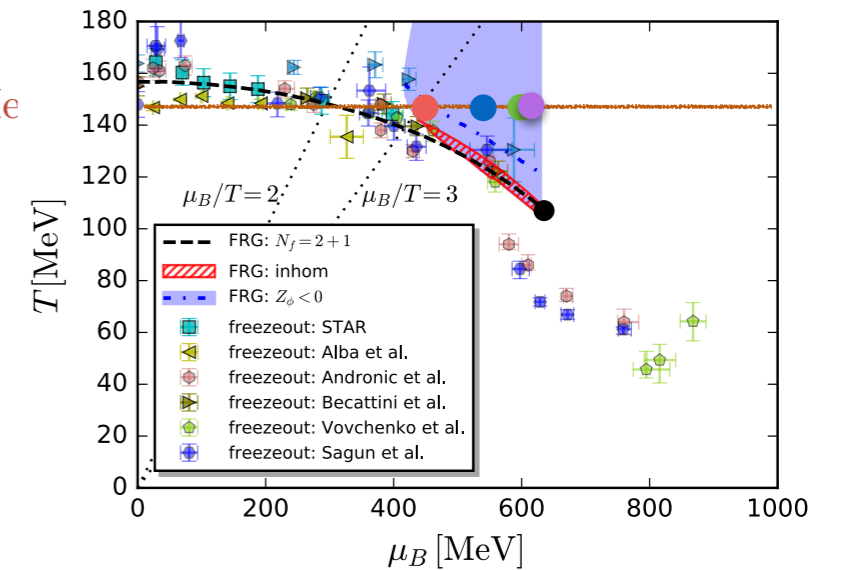
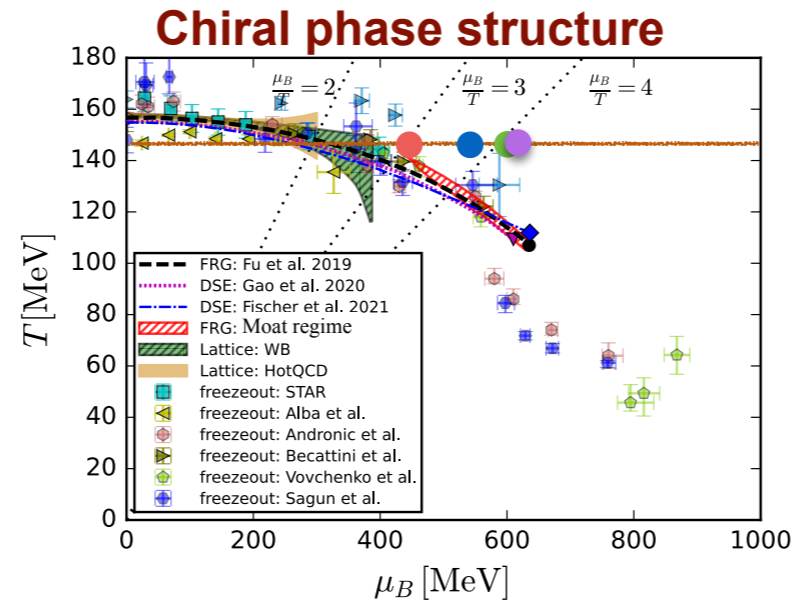
Wikipedia

Physics moat

Pisarski, Rennecke, PRL 127 (2021) 152302

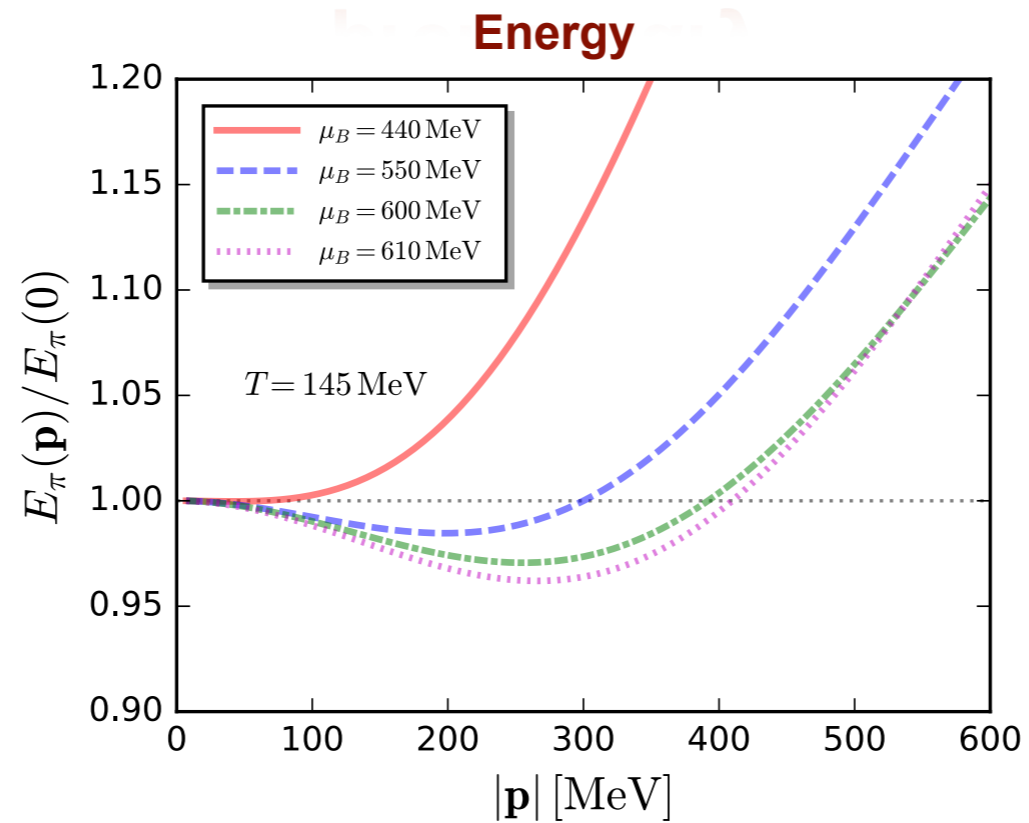
Rennecke, Pisarski, Rischke, PRD 107 (2023) 11, 11601

Sneak preview on the QCD moat



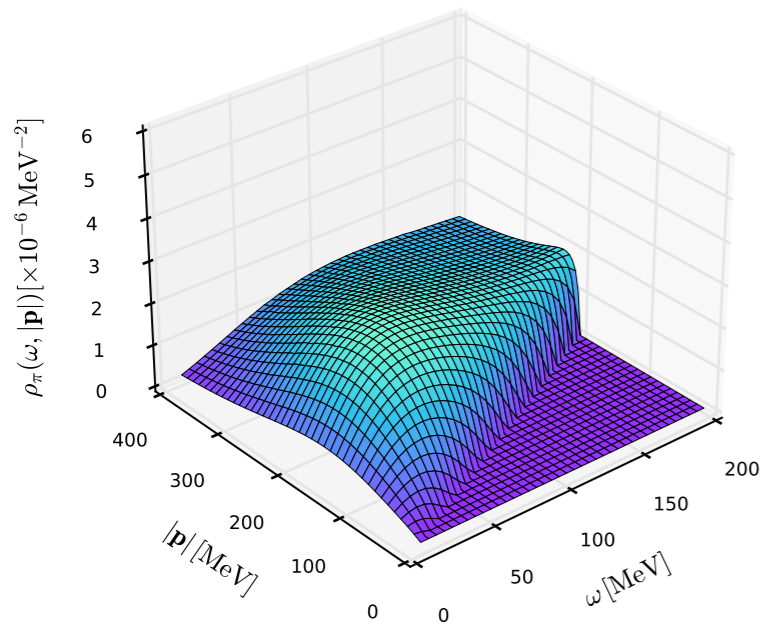
Fu, JMP, Rennecke, PRD 101 (2020) 054032

preliminary

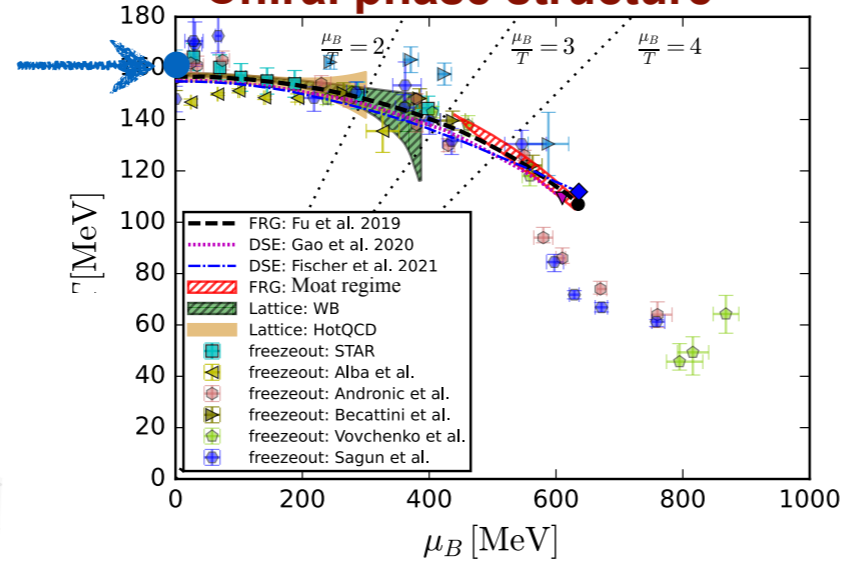


Sneak preview on the QCD moat

Pion spectral function



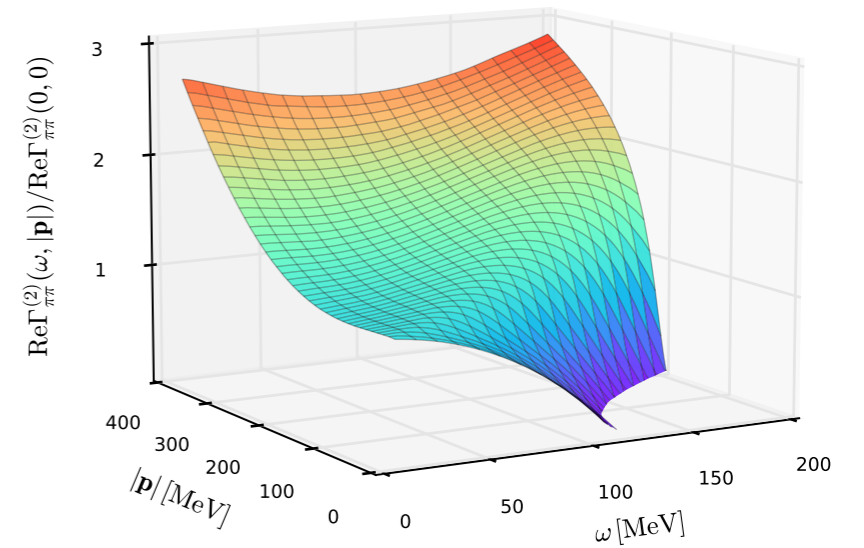
Chiral phase structure



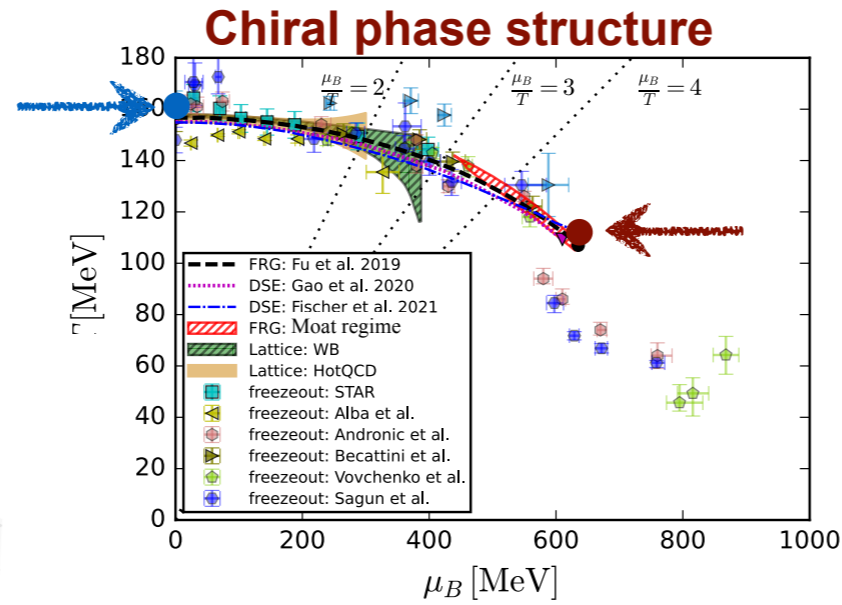
preliminary

$(T, \mu_B) = (160 \text{ MeV}, 0)$

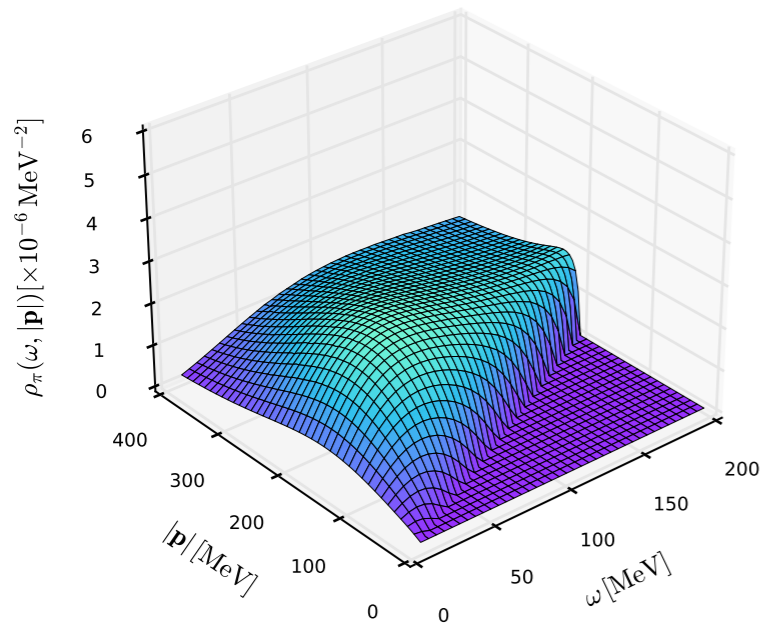
Pion correlation function



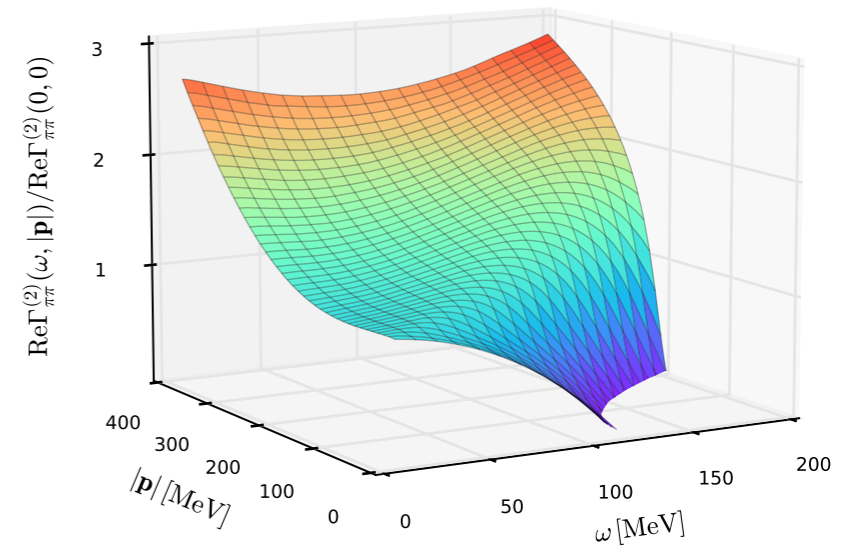
Sneak preview on the QCD moat



Pion spectral function



Pion correlation function

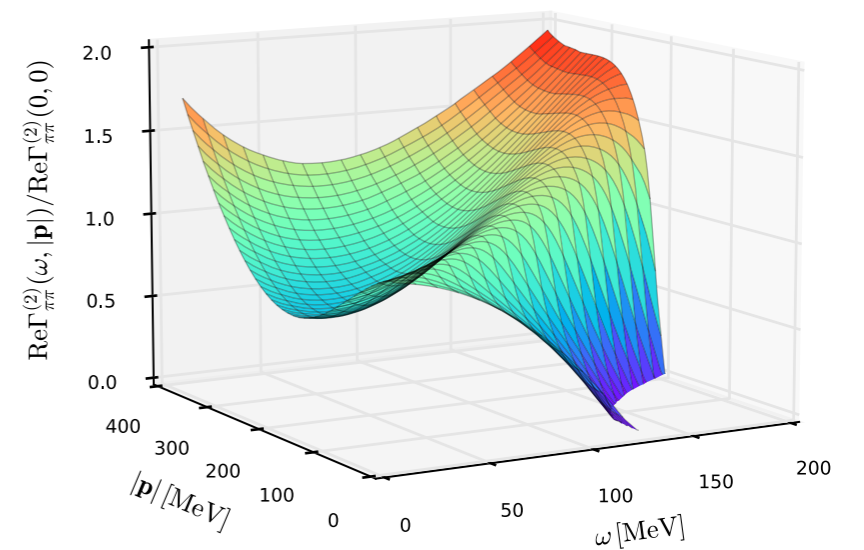
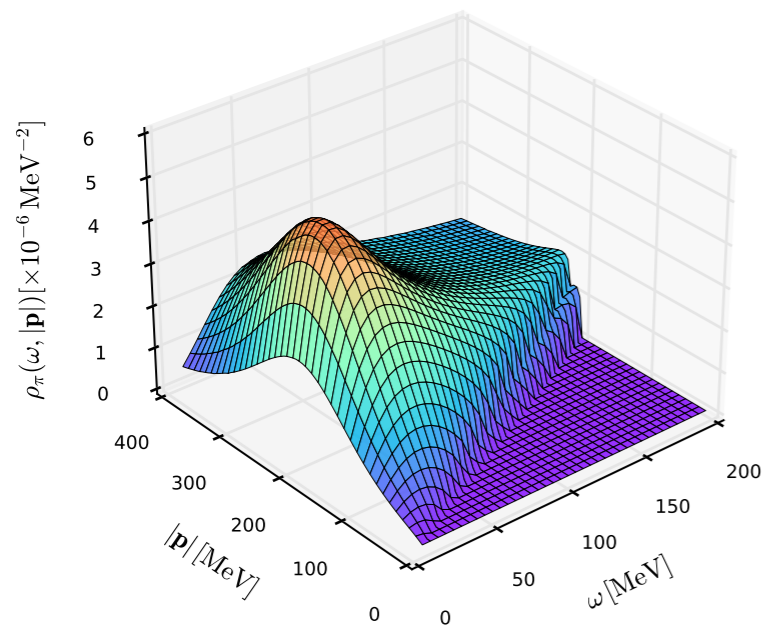


preliminary

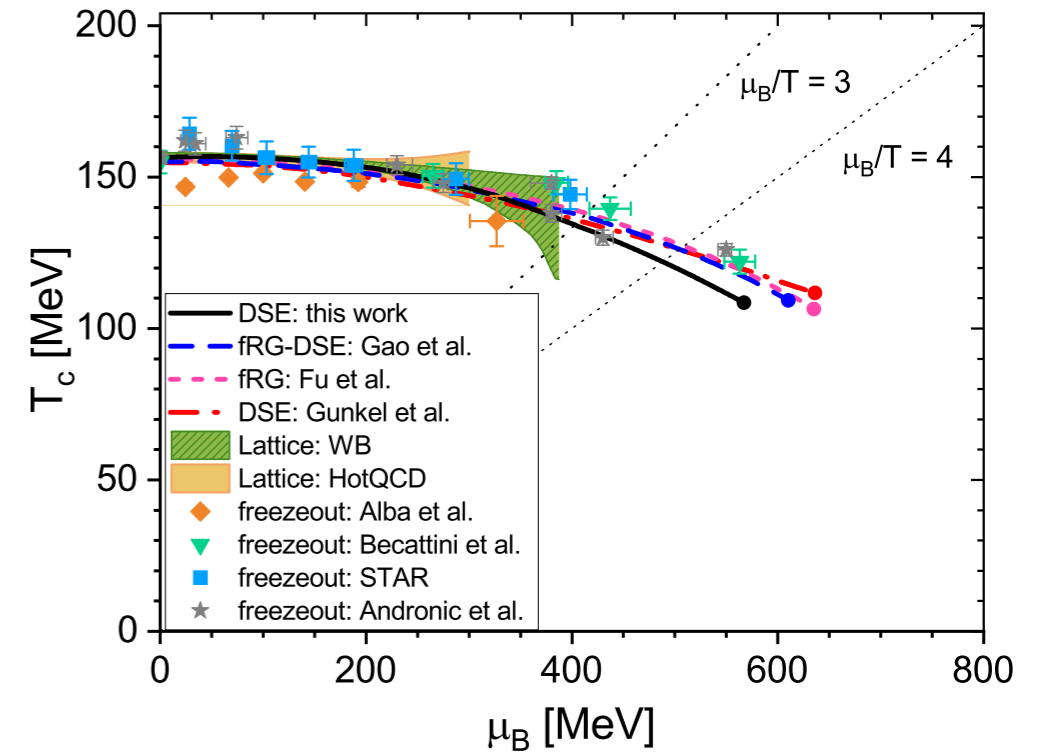
$(T, \mu_B) = (160 \text{ MeV}, 0)$

Moat

$(T, \mu_B) = (114 \text{ MeV}, 630 \text{ MeV})$

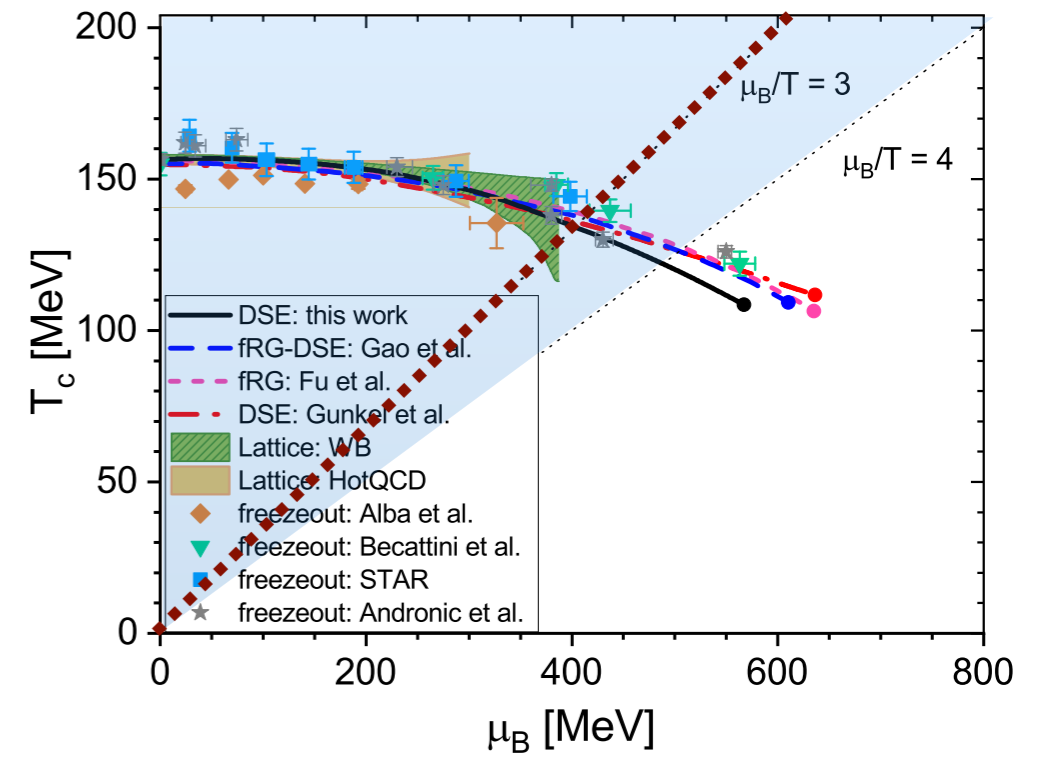


EoS with the minimal DSE



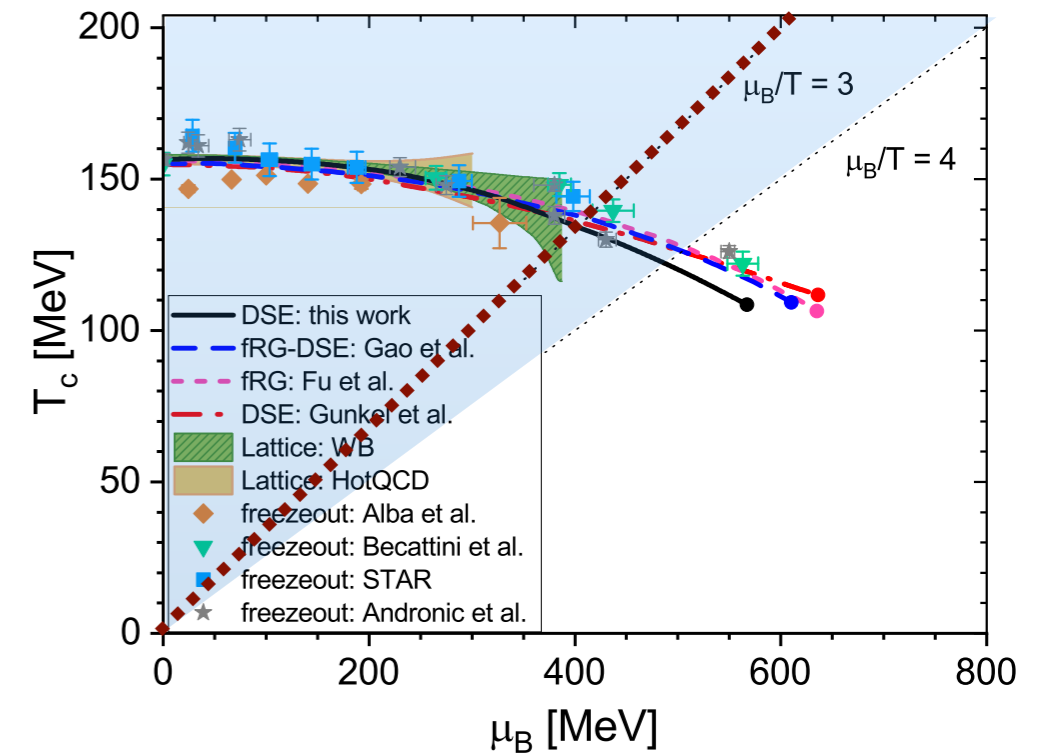
preliminary

EoS with the minimal DSE



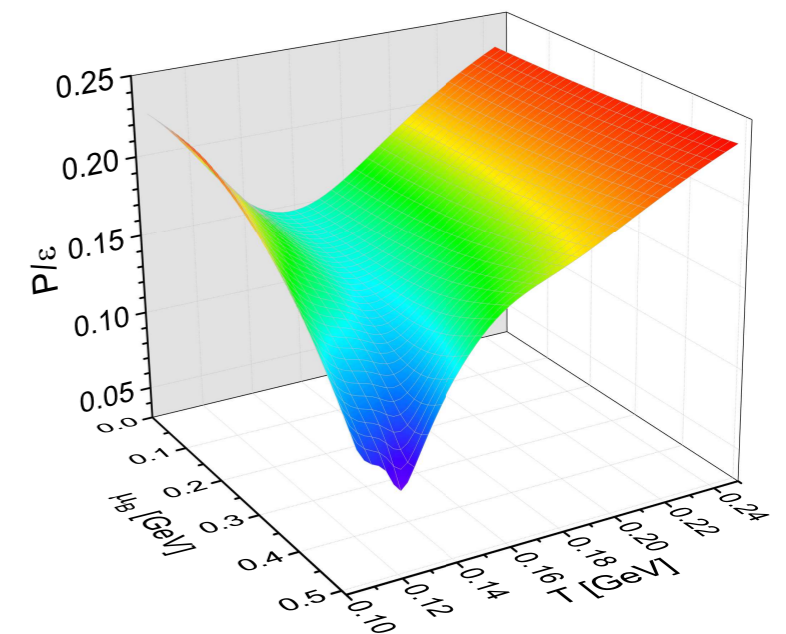
preliminary

EoS with the minimal DSE

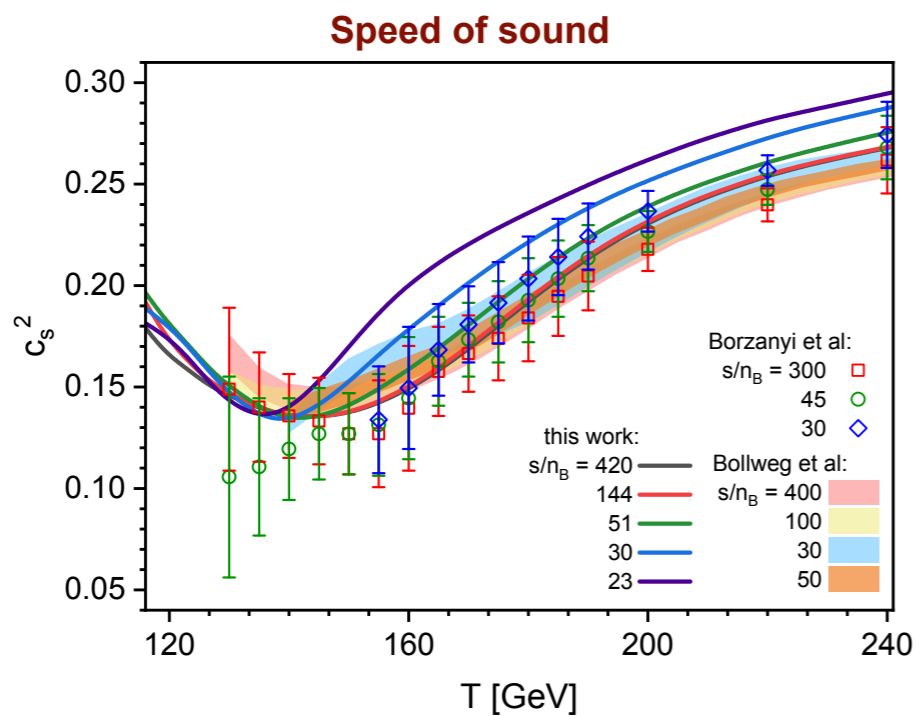
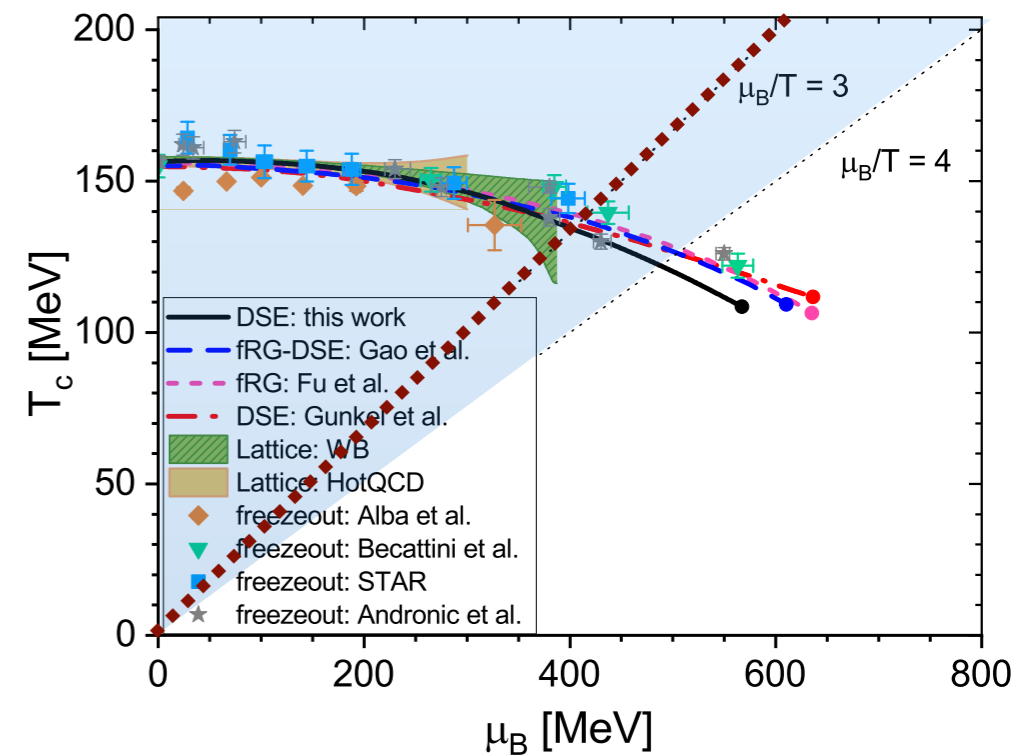
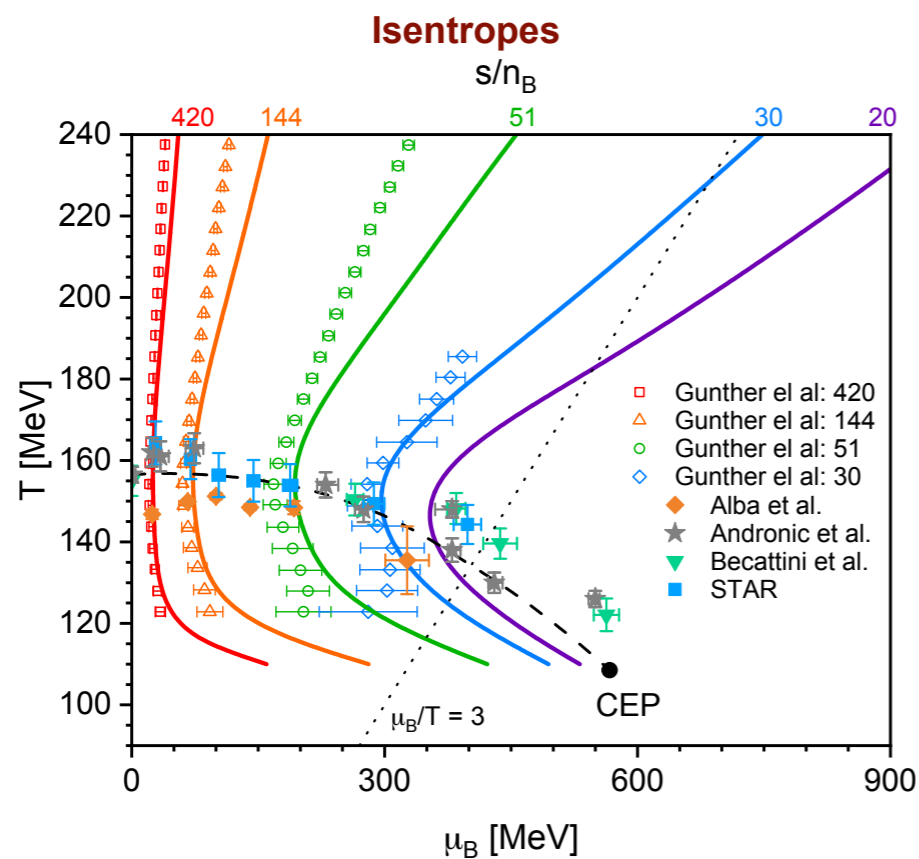


preliminary

Pressure over energy density

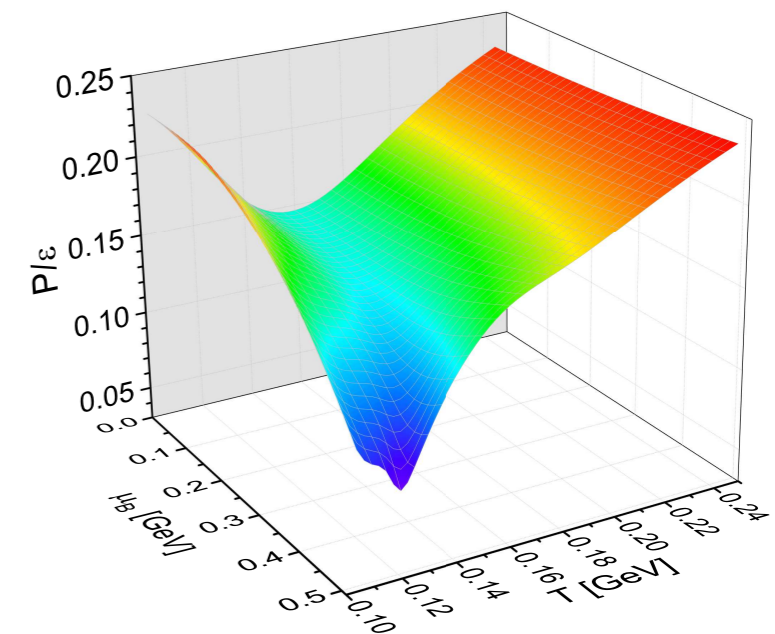


EoS with the minimal DSE



preliminary

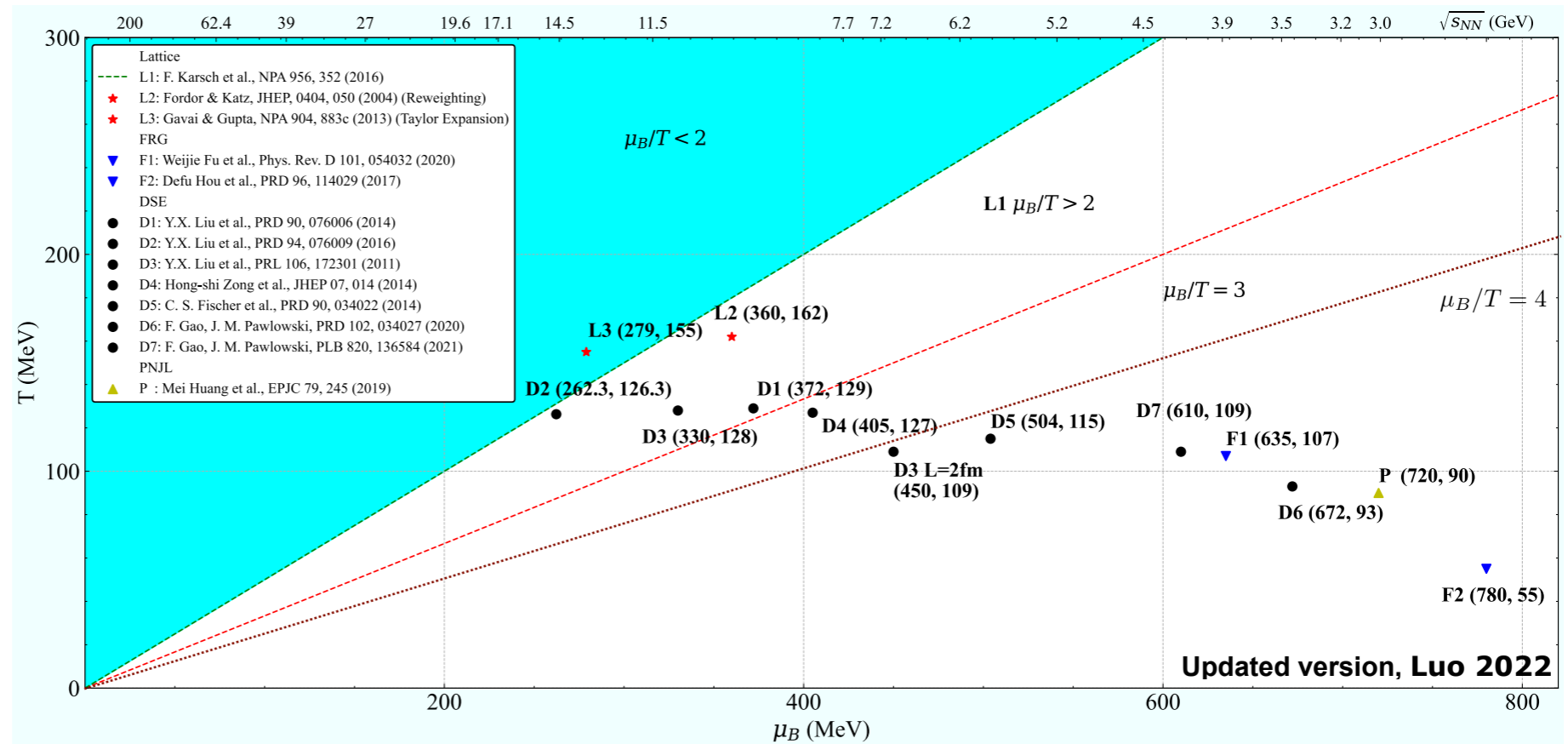
Pressure over energy density





Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

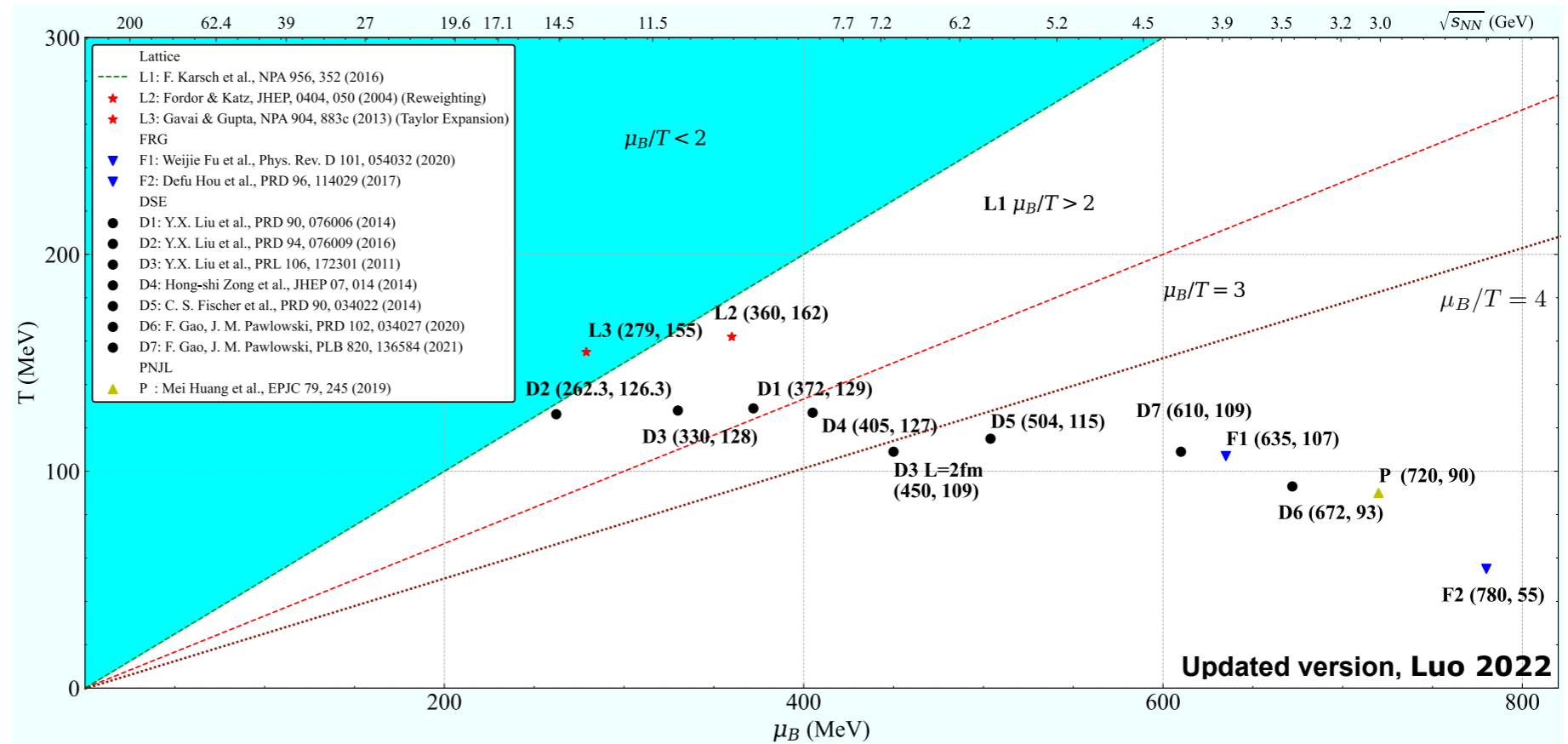
Lack of predictive power for CEP-predictions is no quality measure!

CEP is standing for 'regime with new physics'



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Common folklore since ~2004



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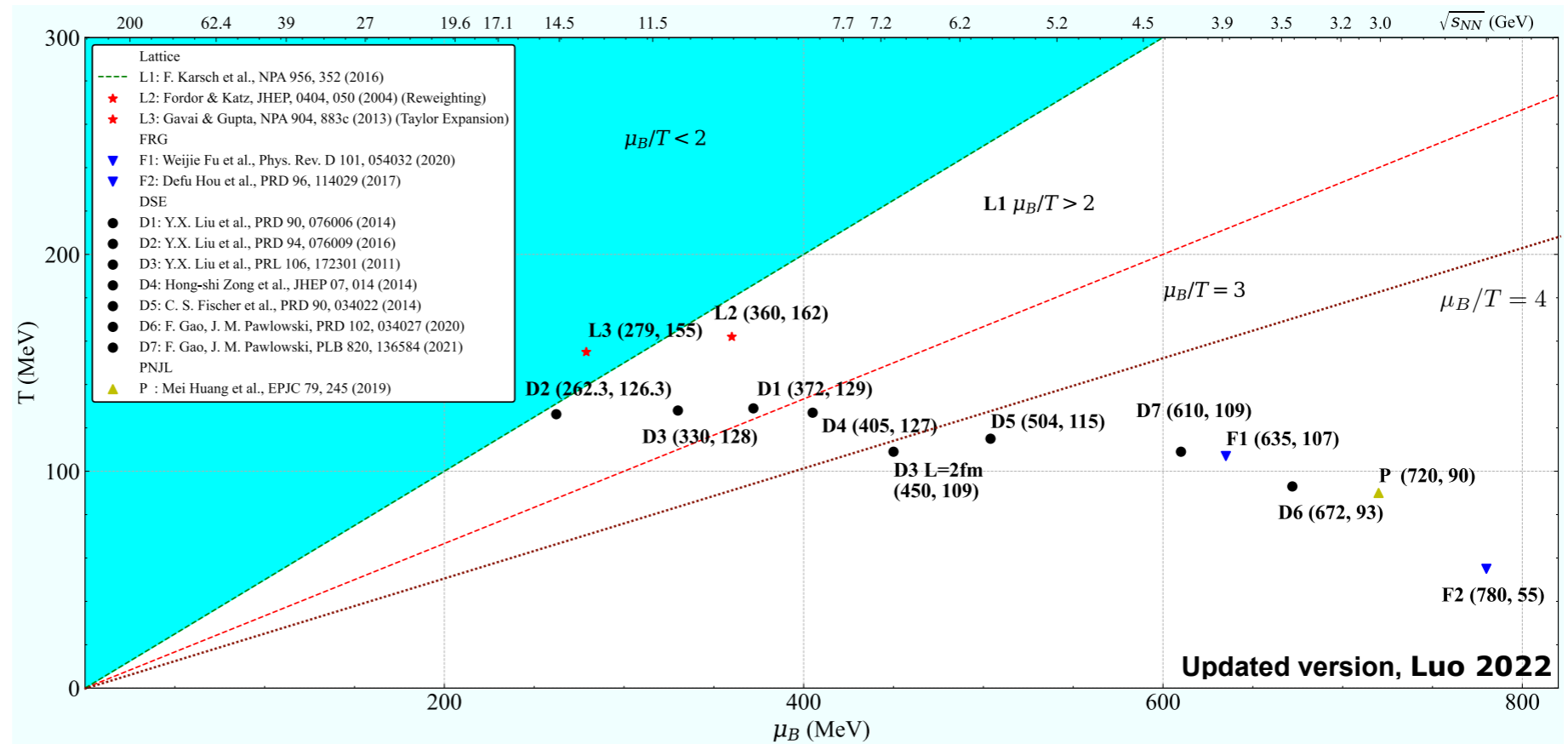
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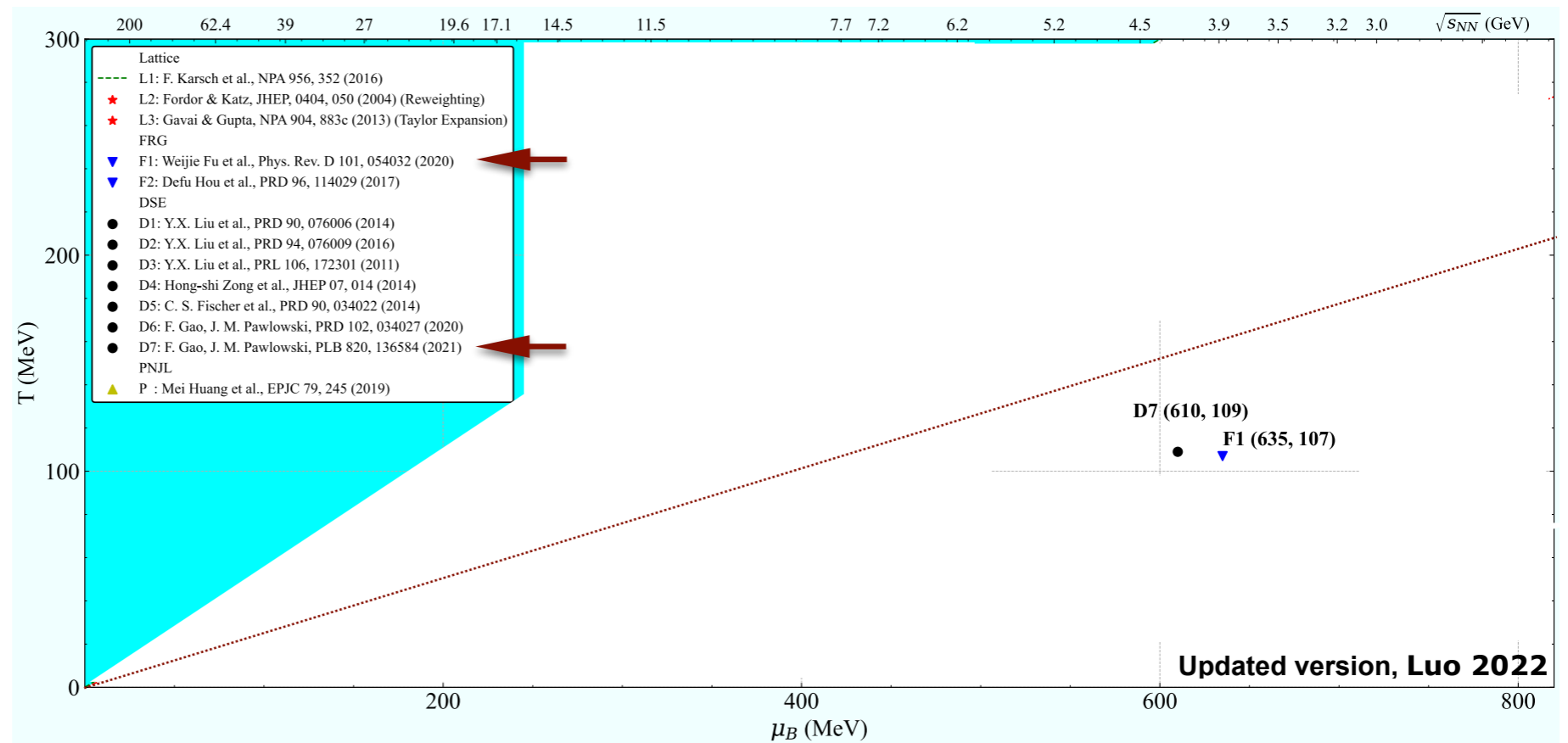
Remove CEP-predictions

- (i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)
- (ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at $\mu_B = 0$
- (iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)



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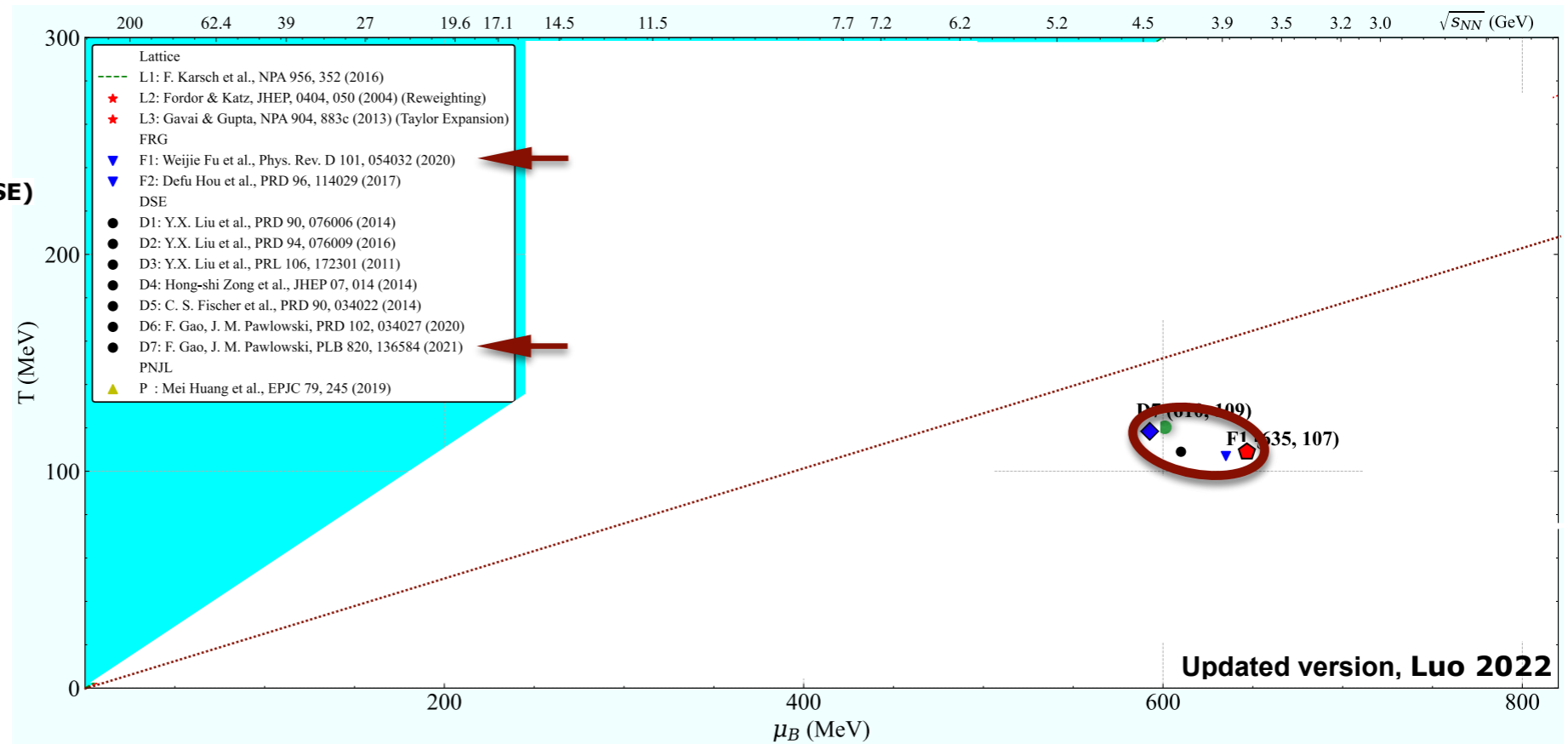
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Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2023)

- ◆ Gao, JMP, Schneider, in prep (DSE)
- ◆ Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)
- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)



Large uncertainties for the estimation of CP location.

Remove CEP-predictions

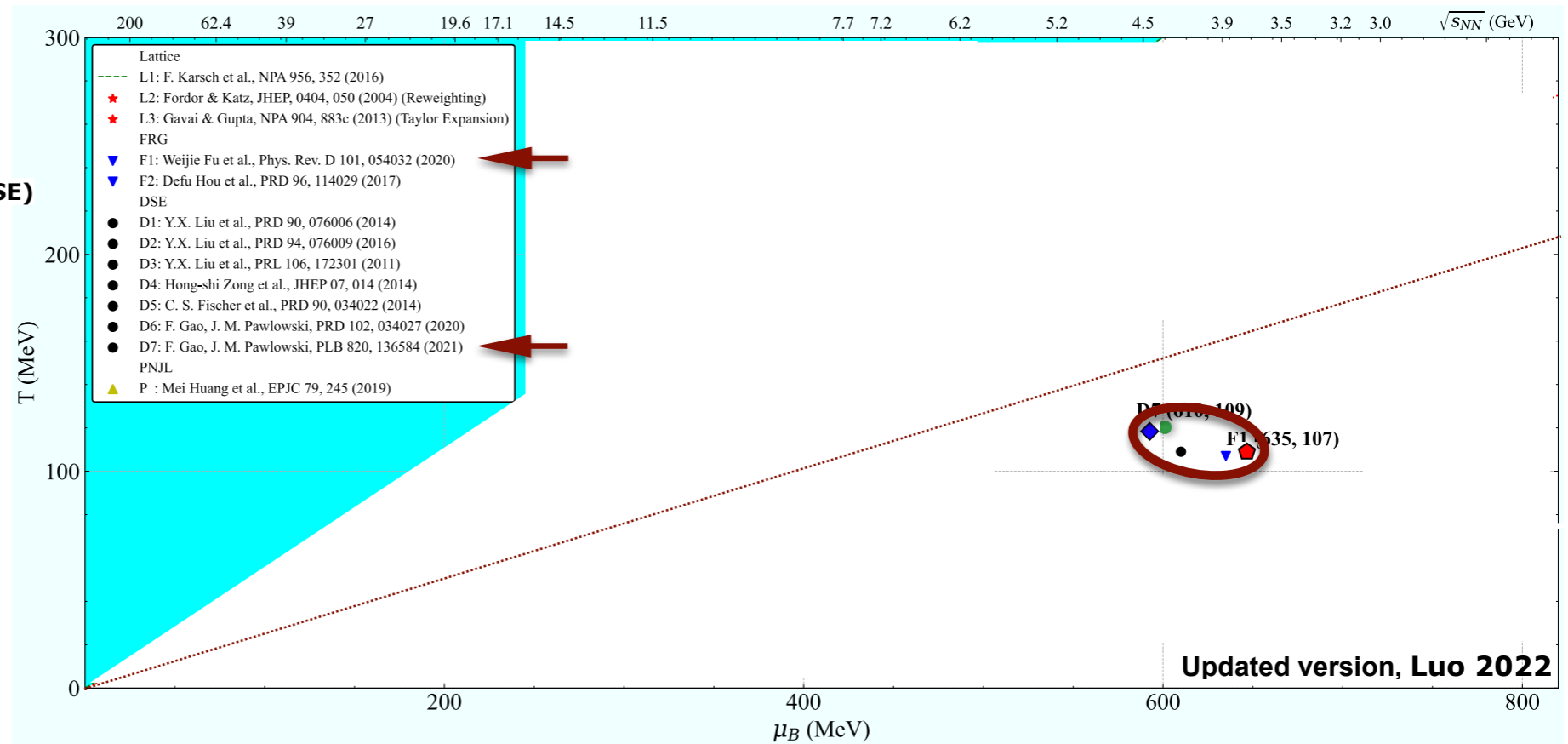
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Still uncertainties for the estimation of CP location.

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Outline

- QCD phase structure: Where do we stand?

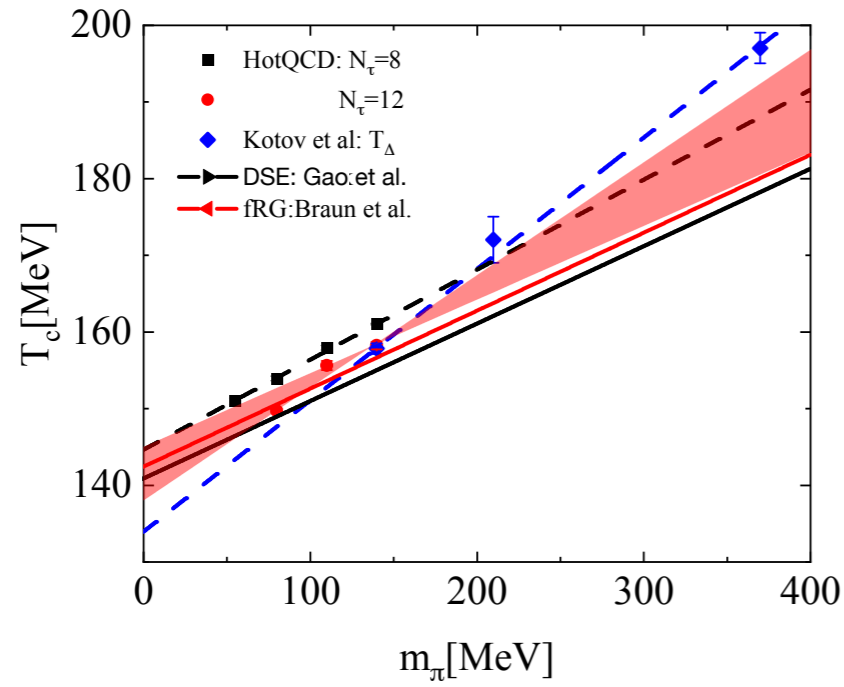
- Chiral dynamics

- Fluctuations of conserved charges

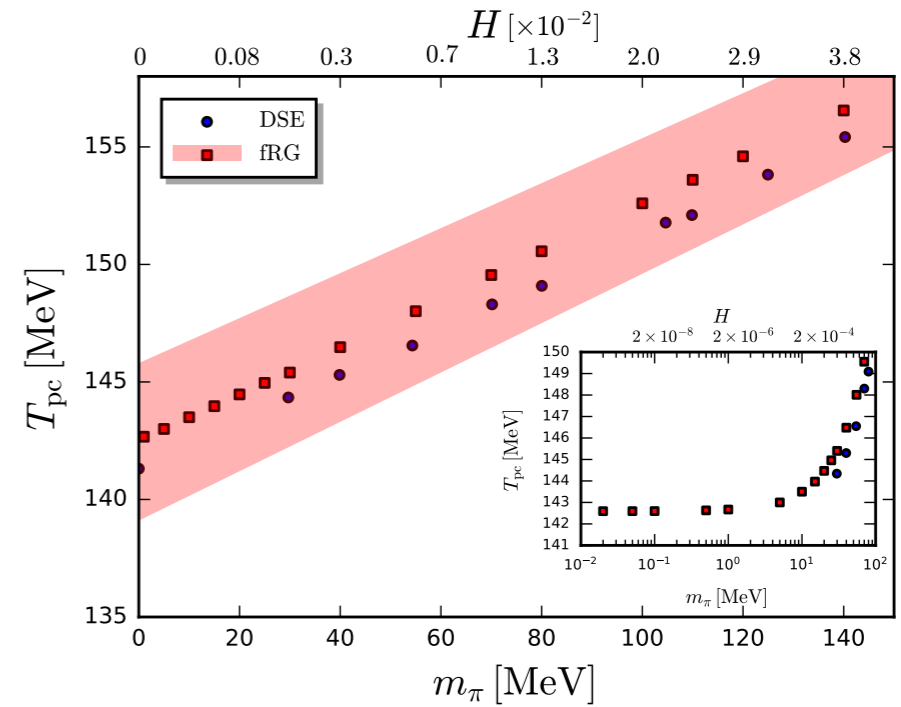
- Summary & outlook

To be (critical) or not (to be)

Chiral transition temperature

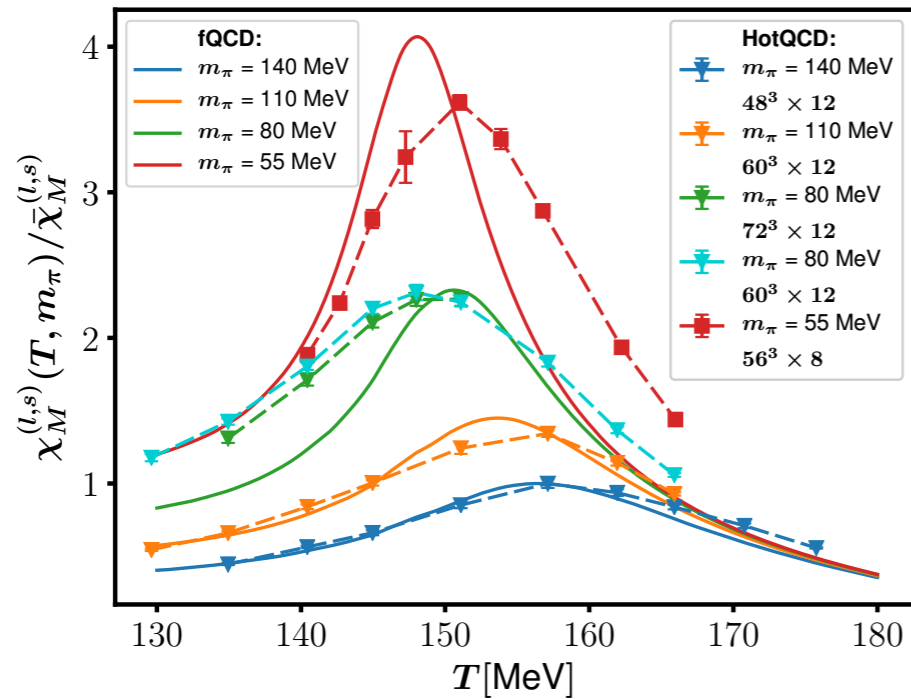


$$H = \frac{m_l}{m_s}$$



fQCD collaboration, in preparation

Magnetic susceptibility

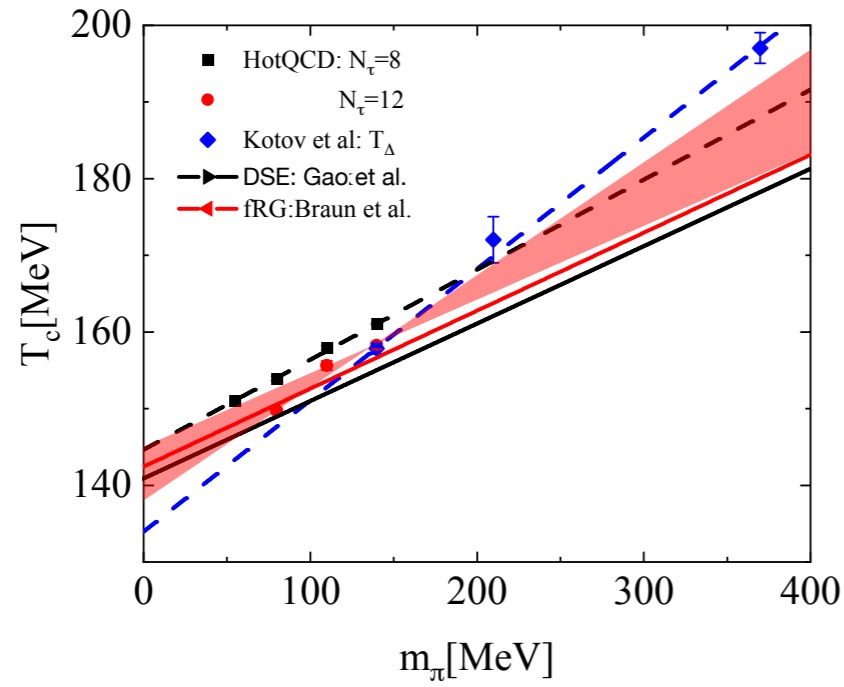


Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

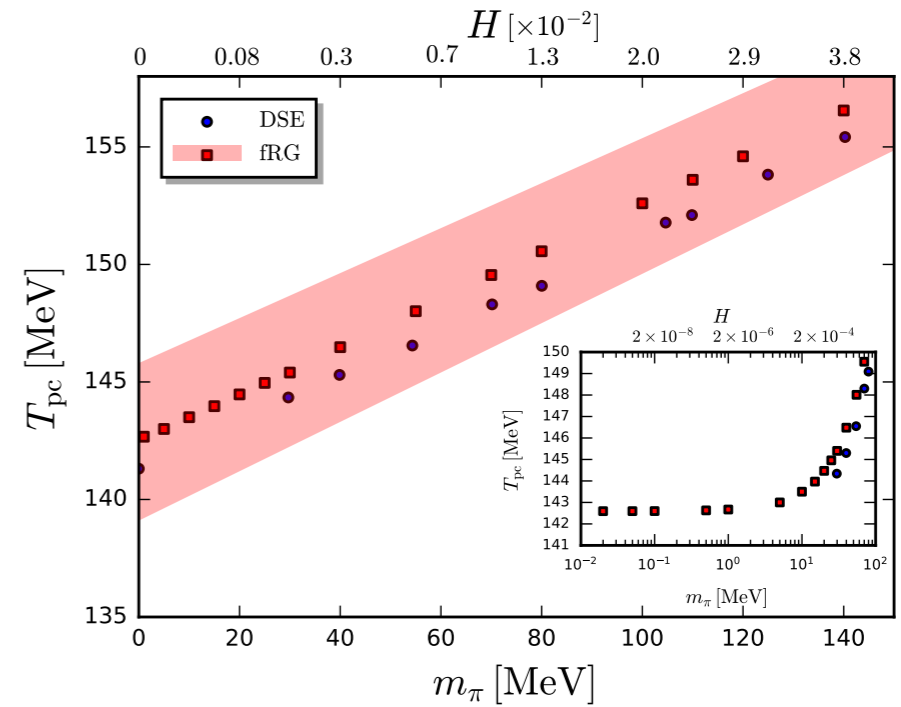
Gao, JMP, PRD 105 (2022) 094020

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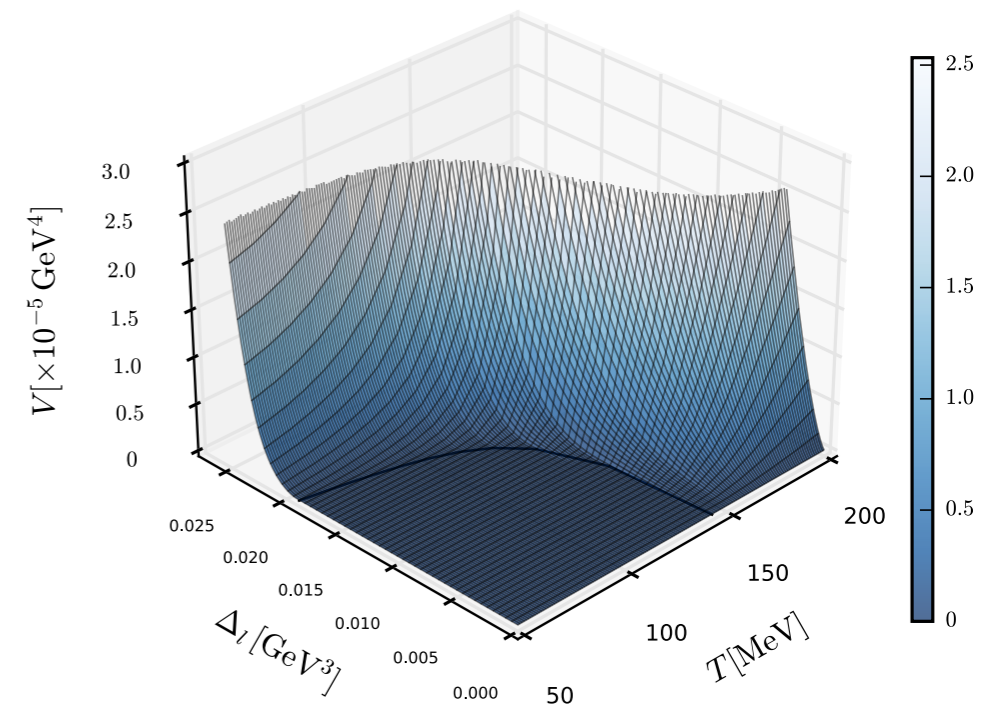
fQCD collaboration, in preparation

$$V_\chi^{(fRG)} \approx V_\chi^{(DSE)}$$

Order parameter potential & scaling

$$V_\chi \approx \Delta_l^n \iff \Delta_l(H) \propto H^{\frac{1}{n-1}}$$

(Critical) exponent: $\frac{1}{\delta} = \frac{1}{n-1}$

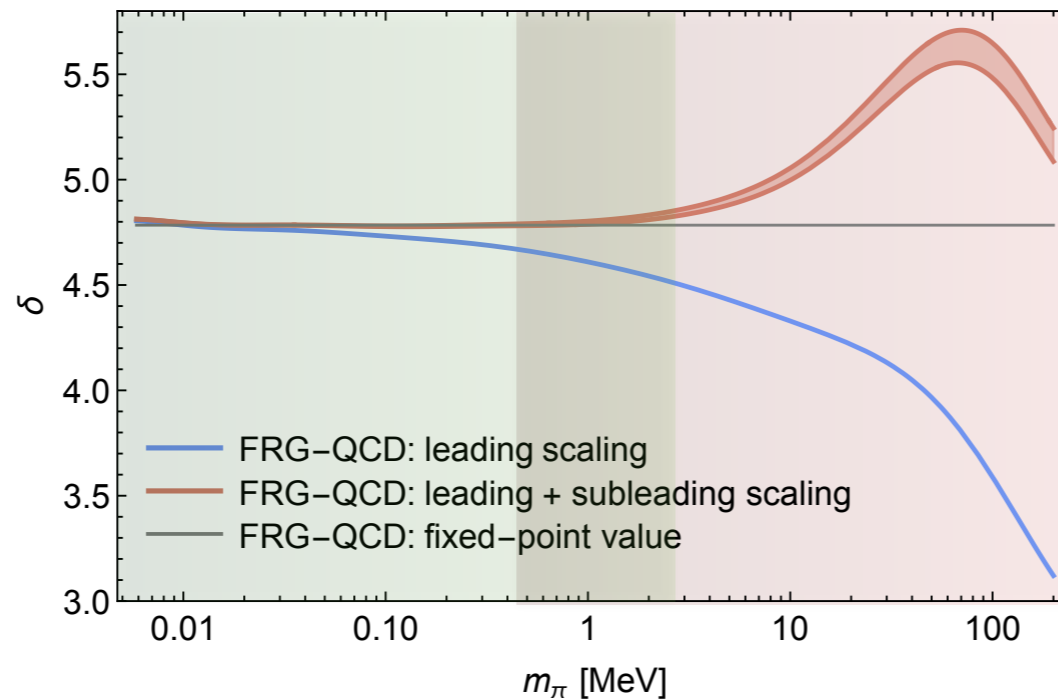


Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



'chiral scaling'

Trivial $\Delta_l^{1+\delta}$ scaling

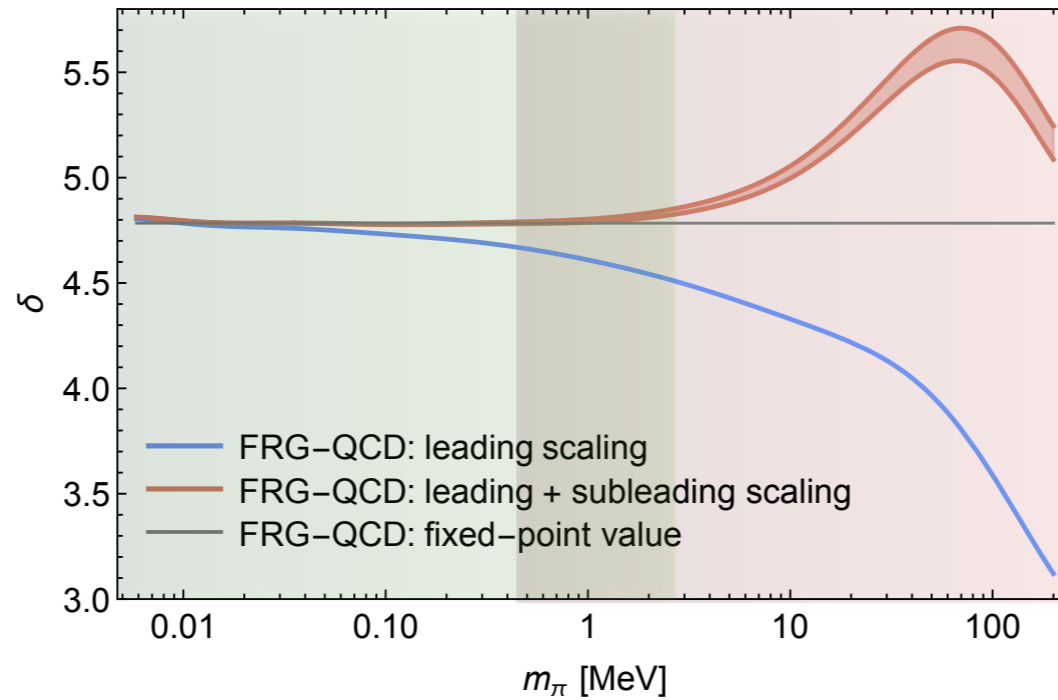
fQCD collaboration, in preparation

QM: Chen, Wen, WF, PRD 104 (2021) 054009

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta} [1 + a_m m_\pi^{2\theta_H} + \dots]$$

Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



Critical O(4) scaling

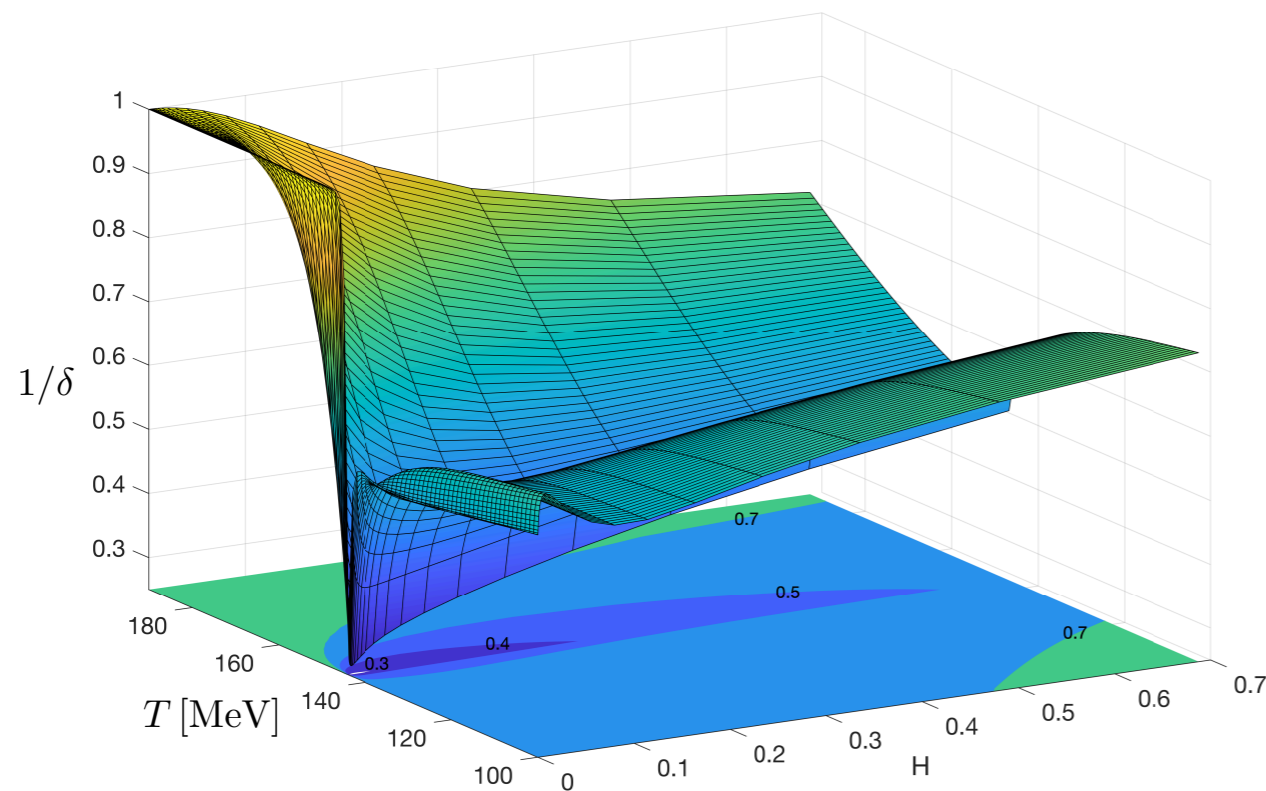
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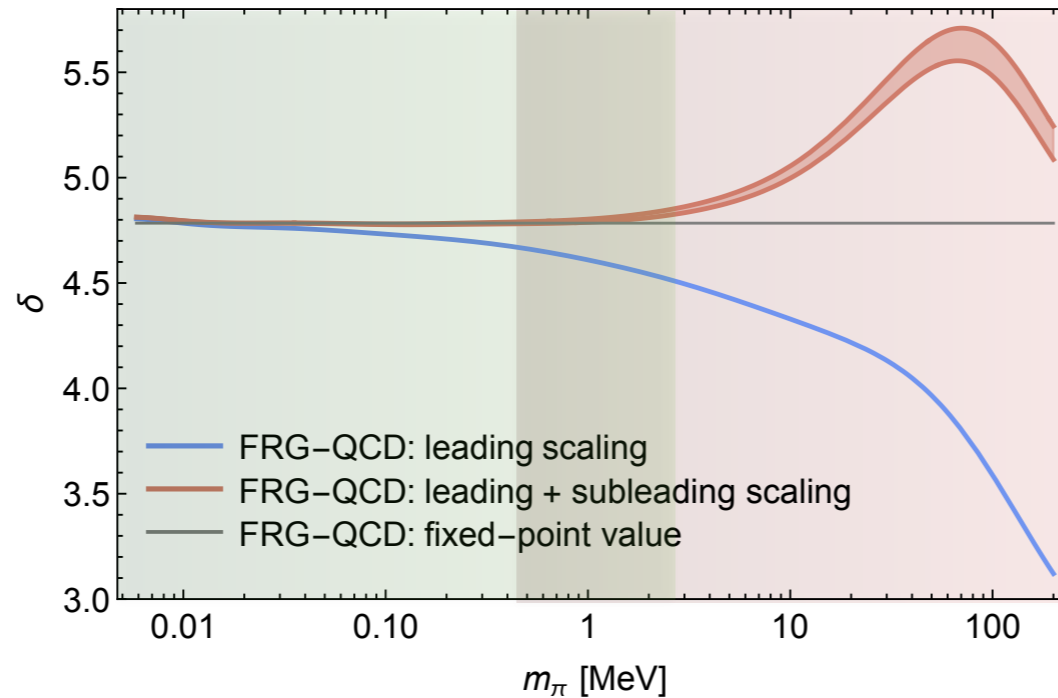
Small chiral scaling regime



Small critical regime around pot. CEP

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Scaling coefficient as function of the pion mass



Critical O(4) scaling

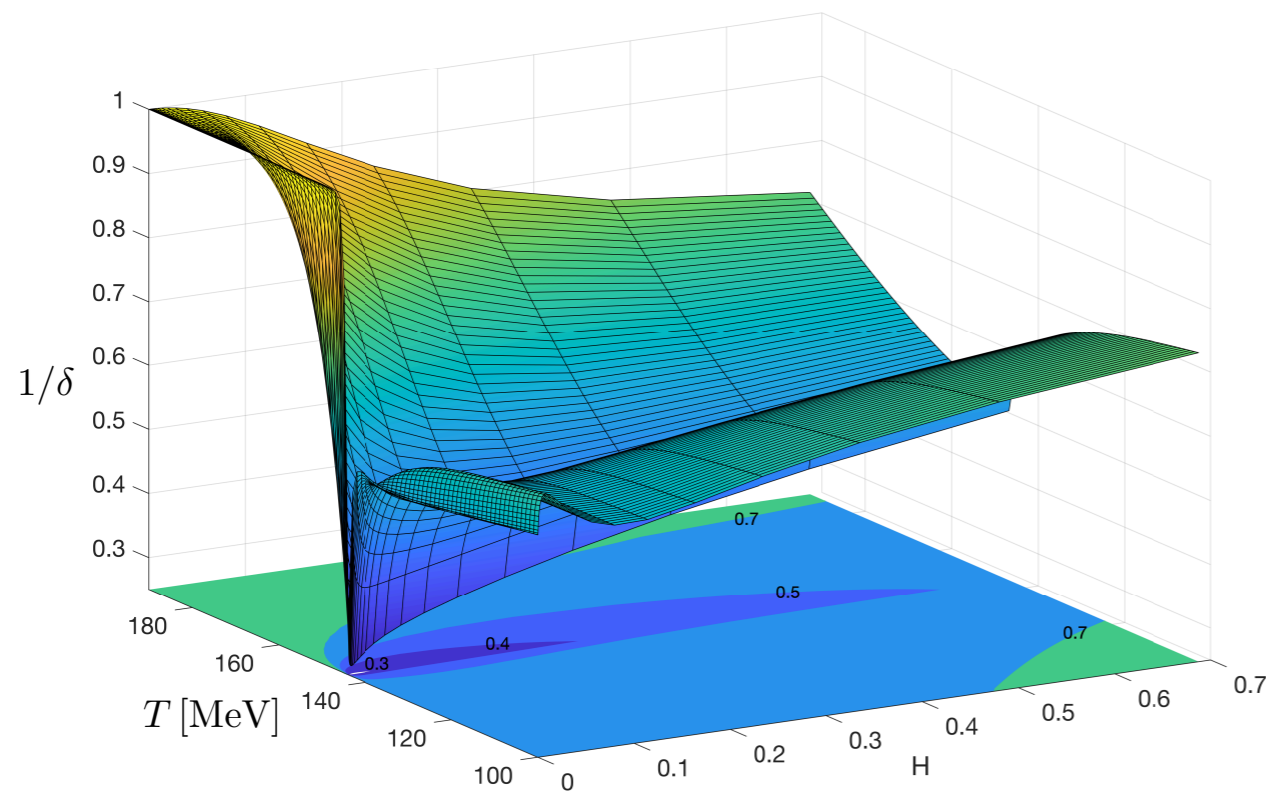
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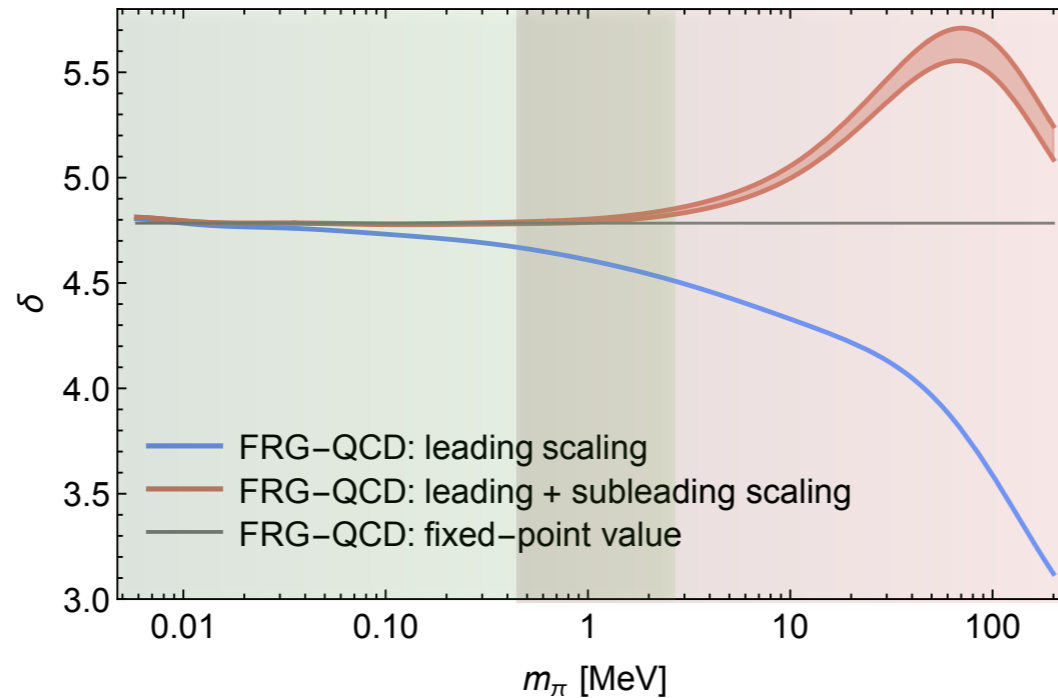
Small critical regime around pot. CEP

!!Great News!!

Location of CEP/New phase accessible via combination of precision measurements & computations

Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



Critical O(4) scaling

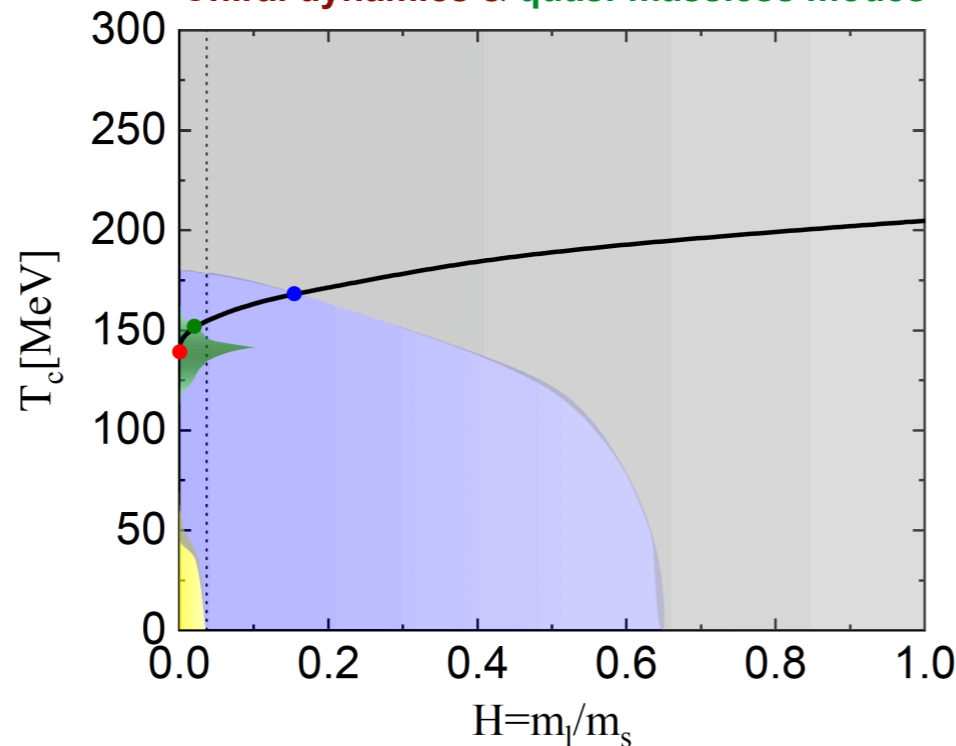
'chiral scaling'

Trivial $\Delta_l^{1+\delta}$ scaling

fQCD collaboration, in preparation

QM: Chen, Wen, WF, PRD 104 (2021) 054009

Chiral dynamics & quasi-massless modes



Critical scaling



'Non-critical chiral scaling'

Far away from the critical regime for $m_\pi \gtrsim 1$ MeV

$$\Delta_l(T, H) \approx \Delta_{l,\chi}(0) \left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H \right)$$



$$V_\chi(\Delta_l) \propto$$

$$\Delta_l^6$$

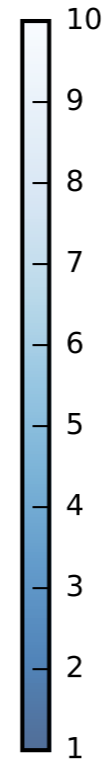
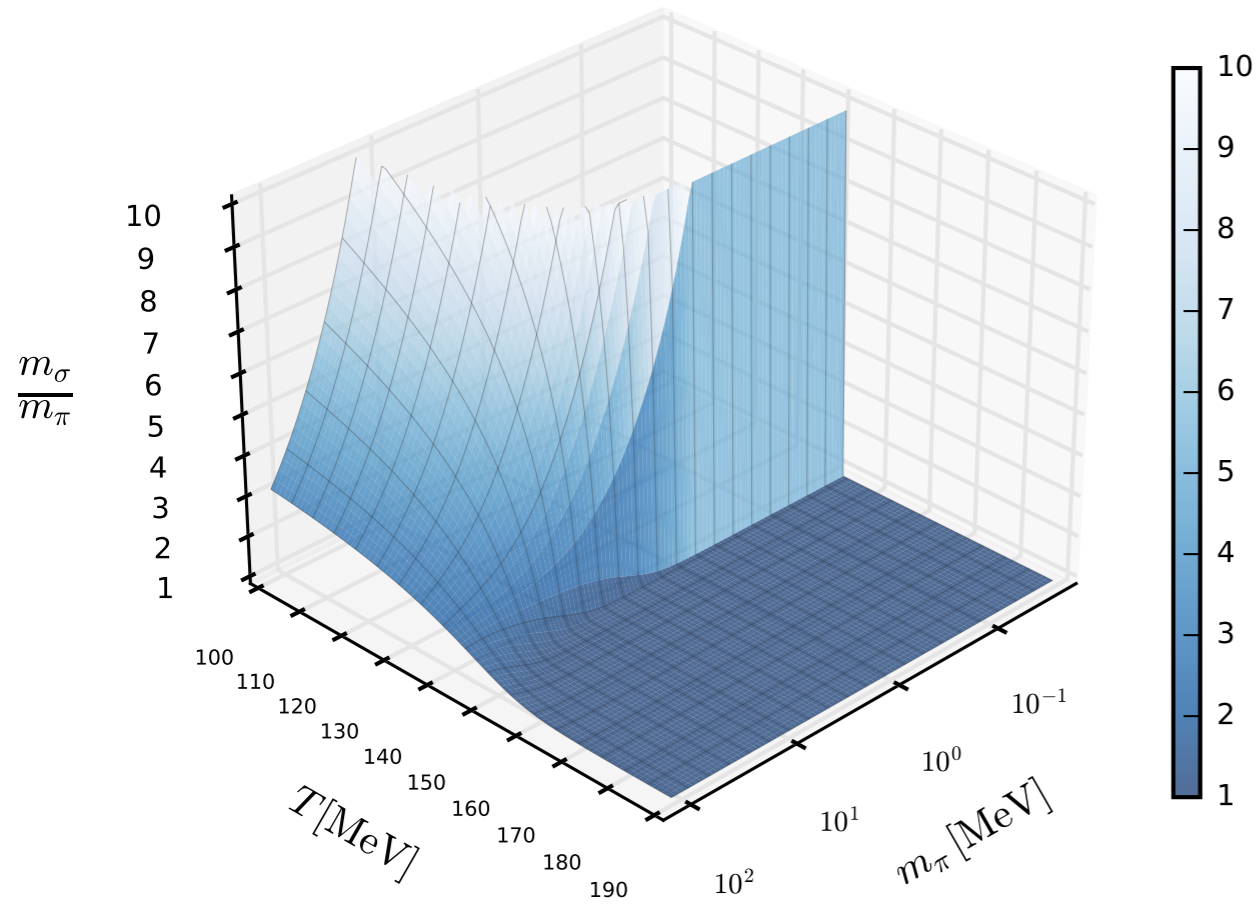
$$\Delta_l^4$$

$$\Delta_l^2$$

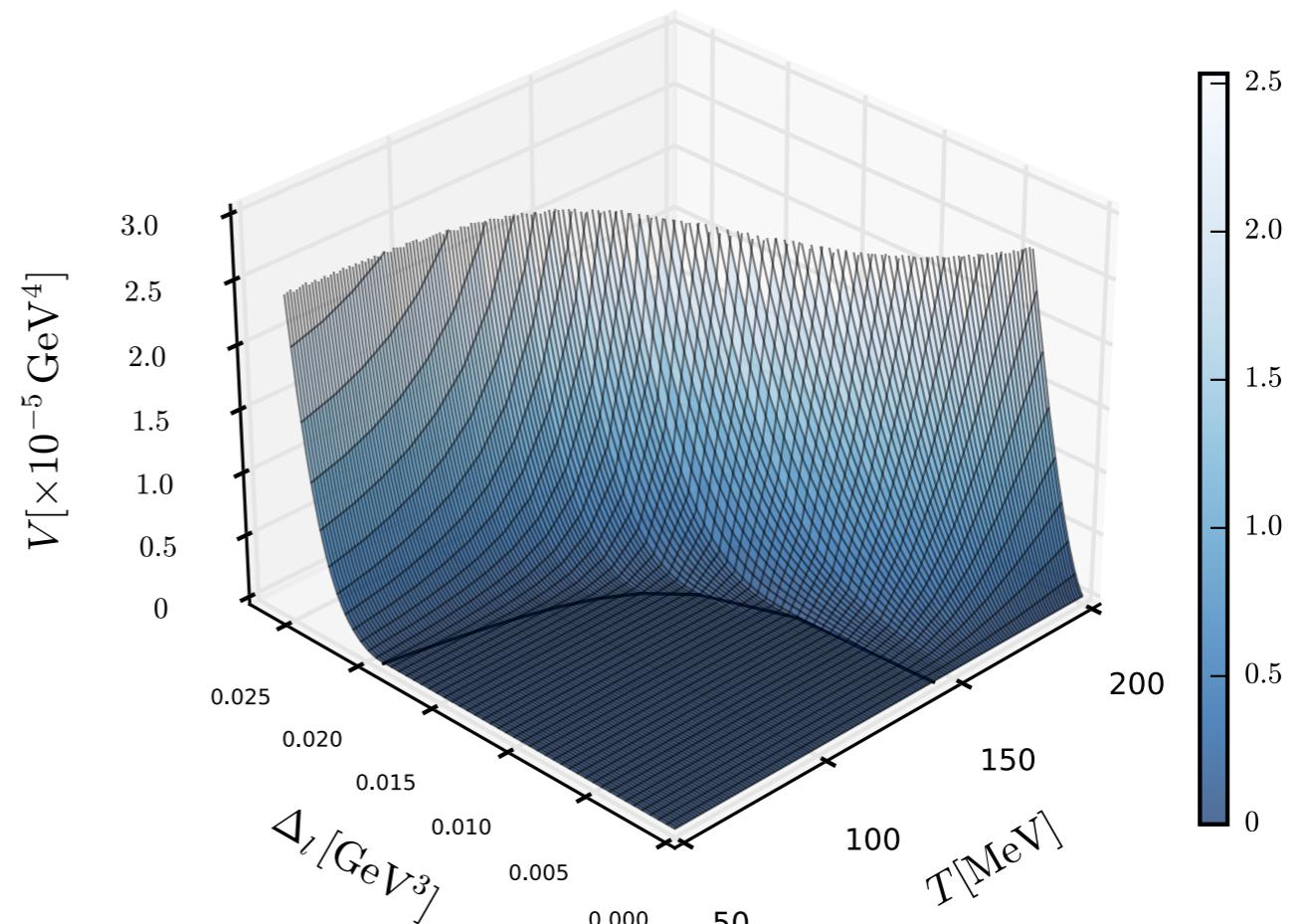
Gao, JMP, PRD 105 (2022) 094020

Chiral dynamics & quasi-massless modes

Measure: correlation length



Use for chiral dynamics
in heavy ion collisions

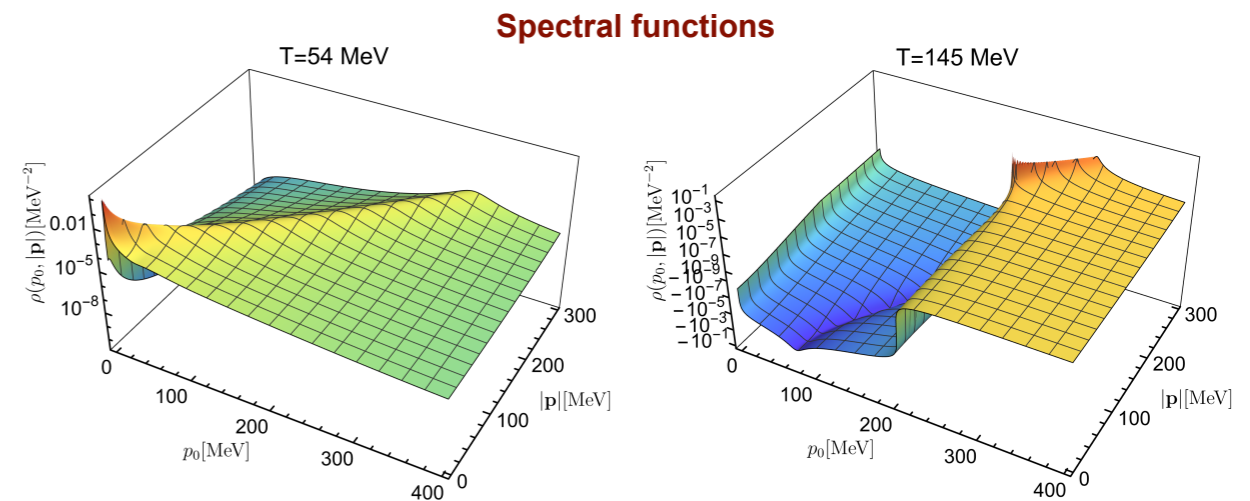
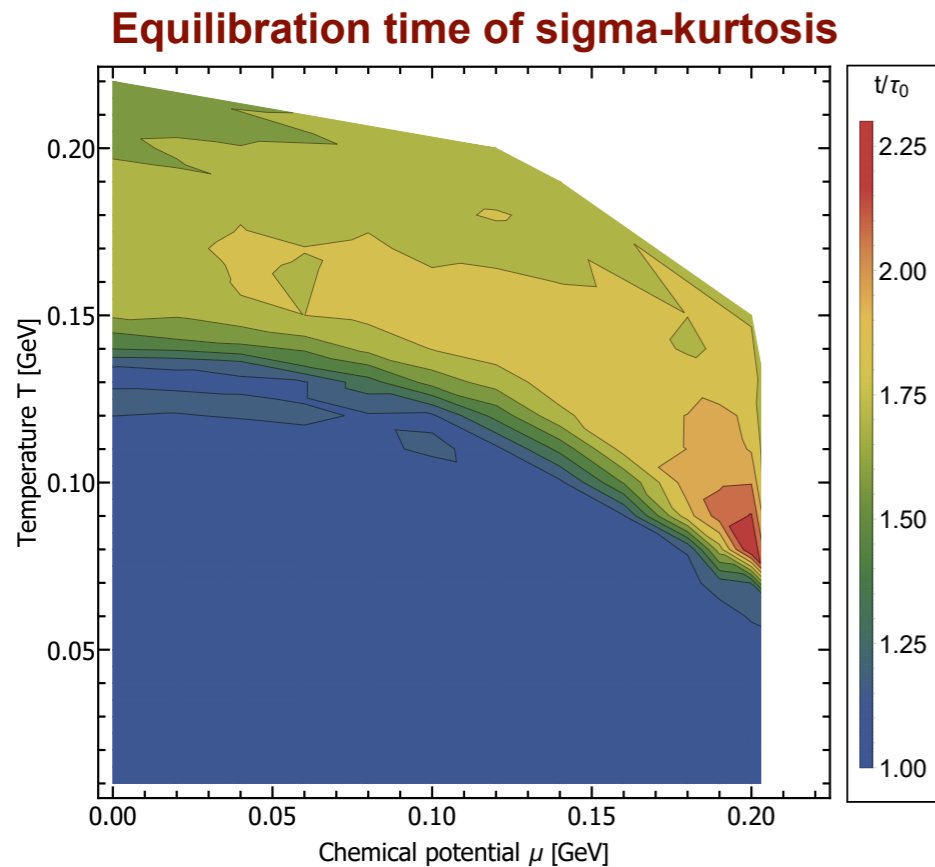


Dynamics and the size of the critical regime

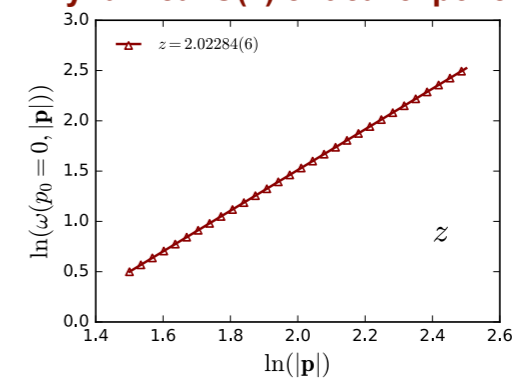
Showcases in linear sigma models

Transport with fRG spectral functions & effective potential

Dynamical universality



Dynamical O(4) critical exponent



$$\omega \propto (\vec{p}^2)^{\frac{z}{2}}$$

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871

Tan, Chen, Fu, SciPost Phys. 12 (2022) 026

QM: Roth, Schweitzer, Rieke, von Smekal, PRD 105 (2022) 116017

Outline

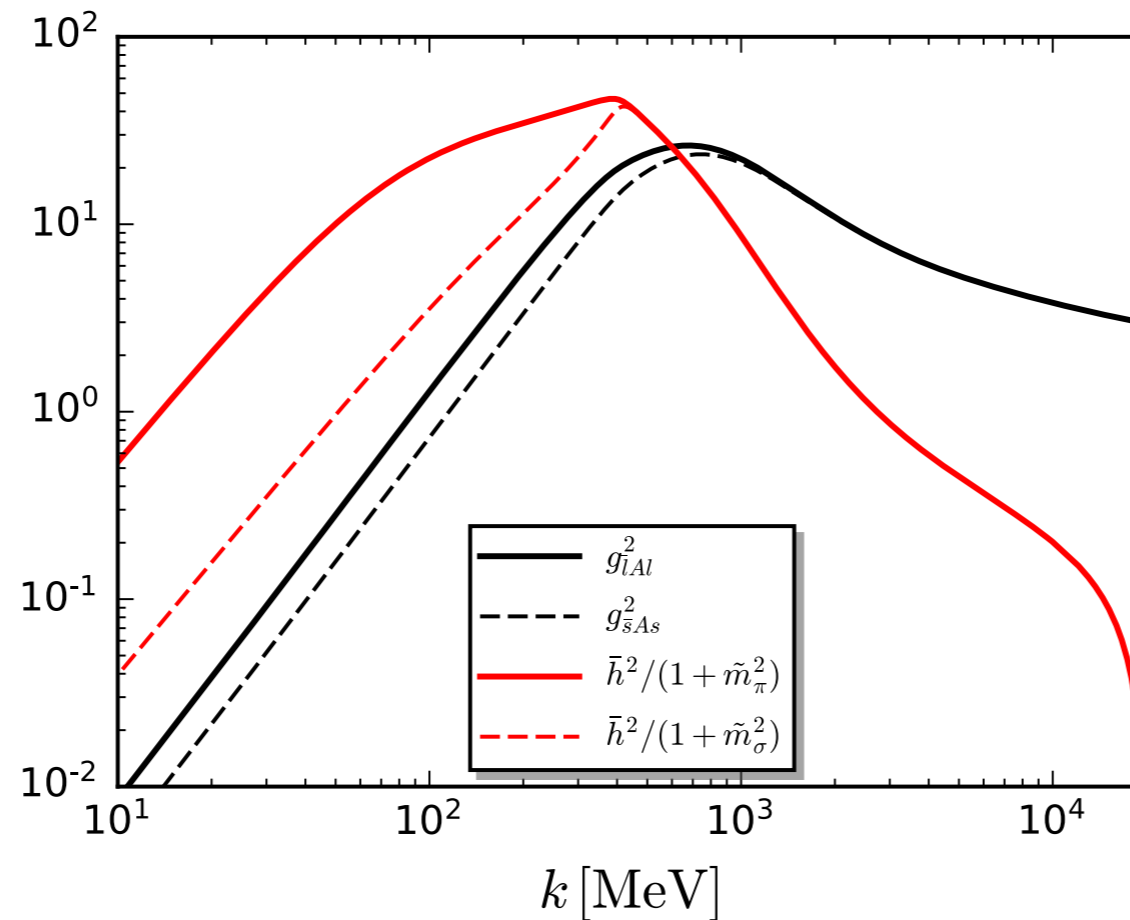
- QCD phase structure: Where do we stand?
- Chiral dynamics
- Fluctuations of conserved charges
- Summary & outlook

On the unreasonable effectiveness of low energy effective theories

fQCD

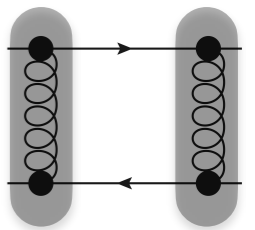
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange loop} - \text{dashed loop} - \text{solid loop} + \frac{1}{2} \text{blue loop} \right)$$

Sequential decoupling of gluon, quark, sigma, pion fluctuations



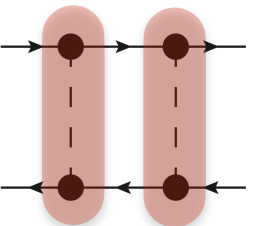
Fu, JMP, Rennecke, PRD 101, (2020) 054032

$$\frac{g_{IAI}^2}{g_{sAs}^2}$$



$$\frac{\bar{h}^2}{1 + \tilde{m}_\pi^2}$$

$$\frac{\bar{h}^2}{1 + \tilde{m}_\sigma^2}$$



Based on:

Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

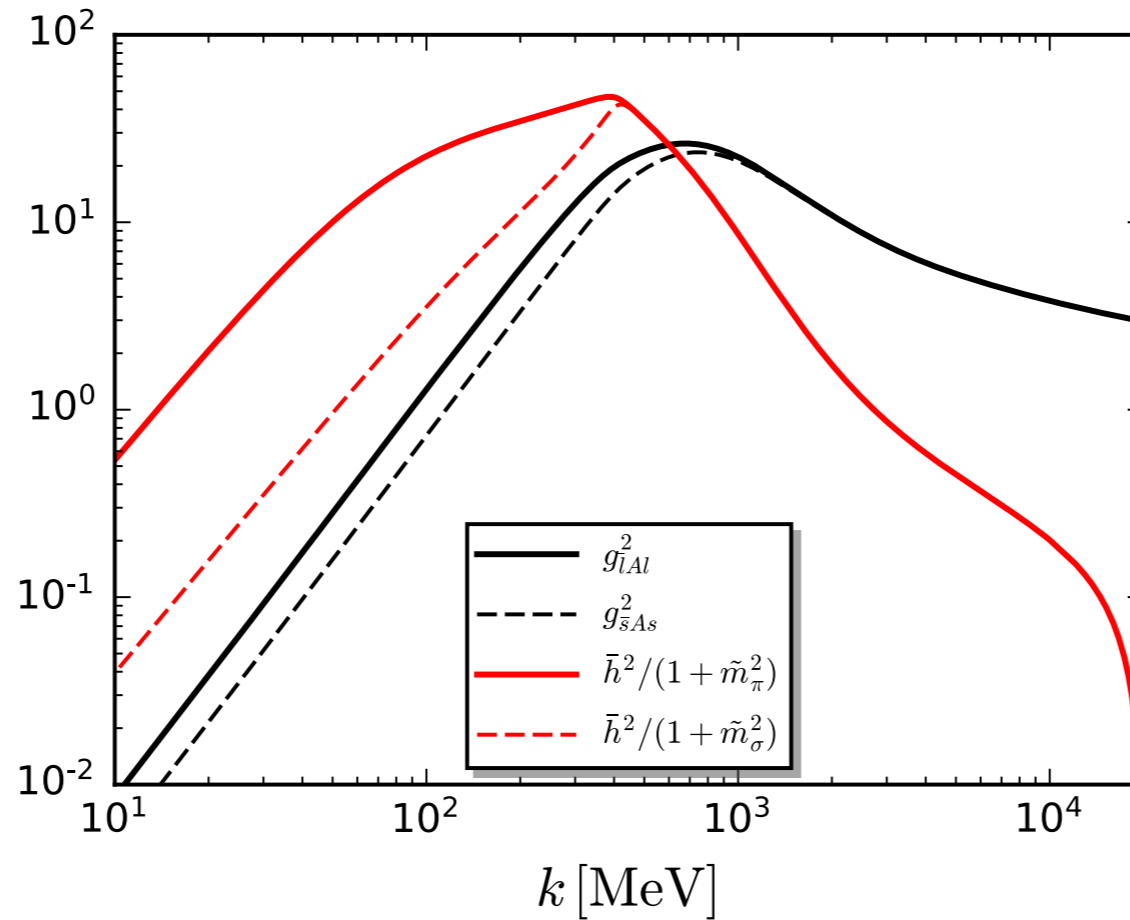
Rennecke, PRD 92 (2015) 076012

On the unreasonable effectiveness of low energy effective theories

fQCD

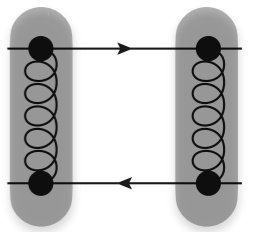
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{[diagram 1]} - \text{[diagram 2]} - \text{[diagram 3]} + \frac{1}{2} \text{[diagram 4]}$$

Sequential decoupling of gluon, quark, sigma, pion fluctuations



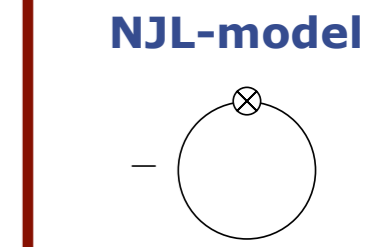
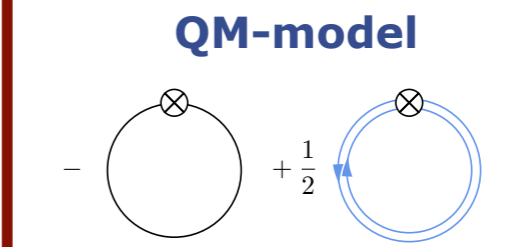
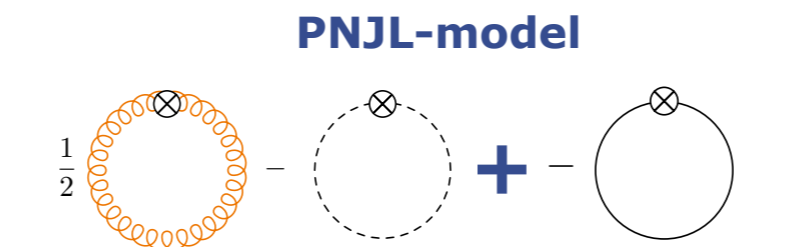
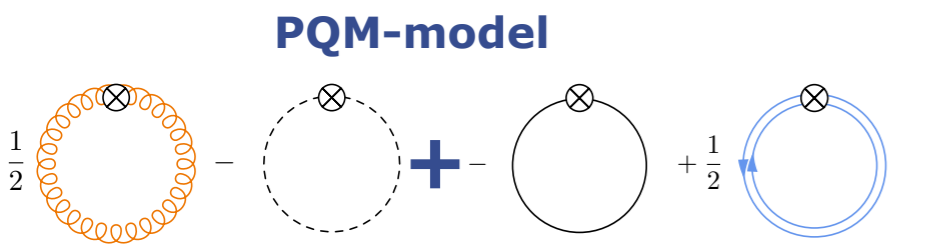
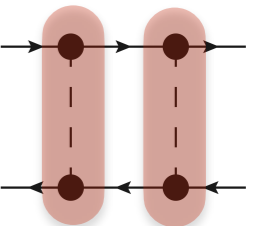
Fu, JMP, Rennecke, PRD 101, (2020) 054032

$$\frac{g_{IA}^2}{g_{sAs}^2}$$



$$\frac{\bar{h}^2}{1 + \bar{m}_\pi^2}$$

$$\frac{\bar{h}^2}{1 + \bar{m}_\sigma^2}$$

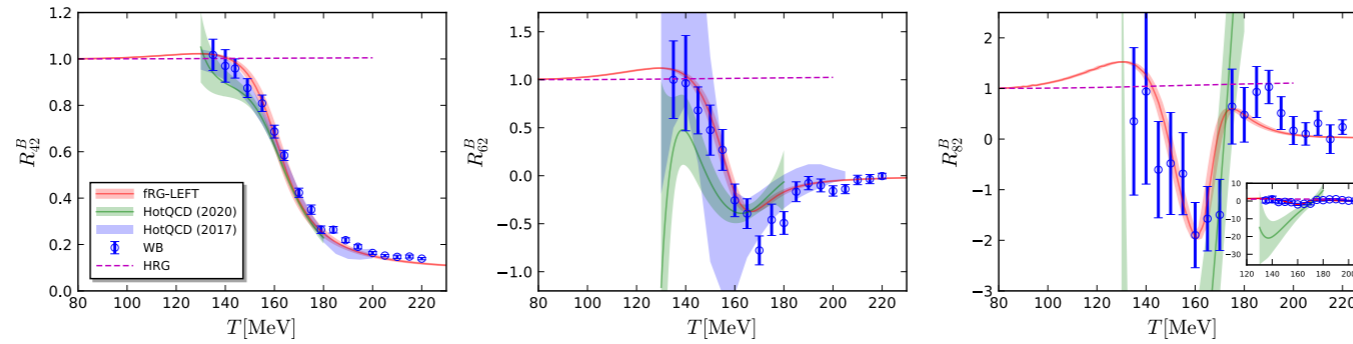


QCD-assisted low energy effective theories

Fluctuations of conserved charges

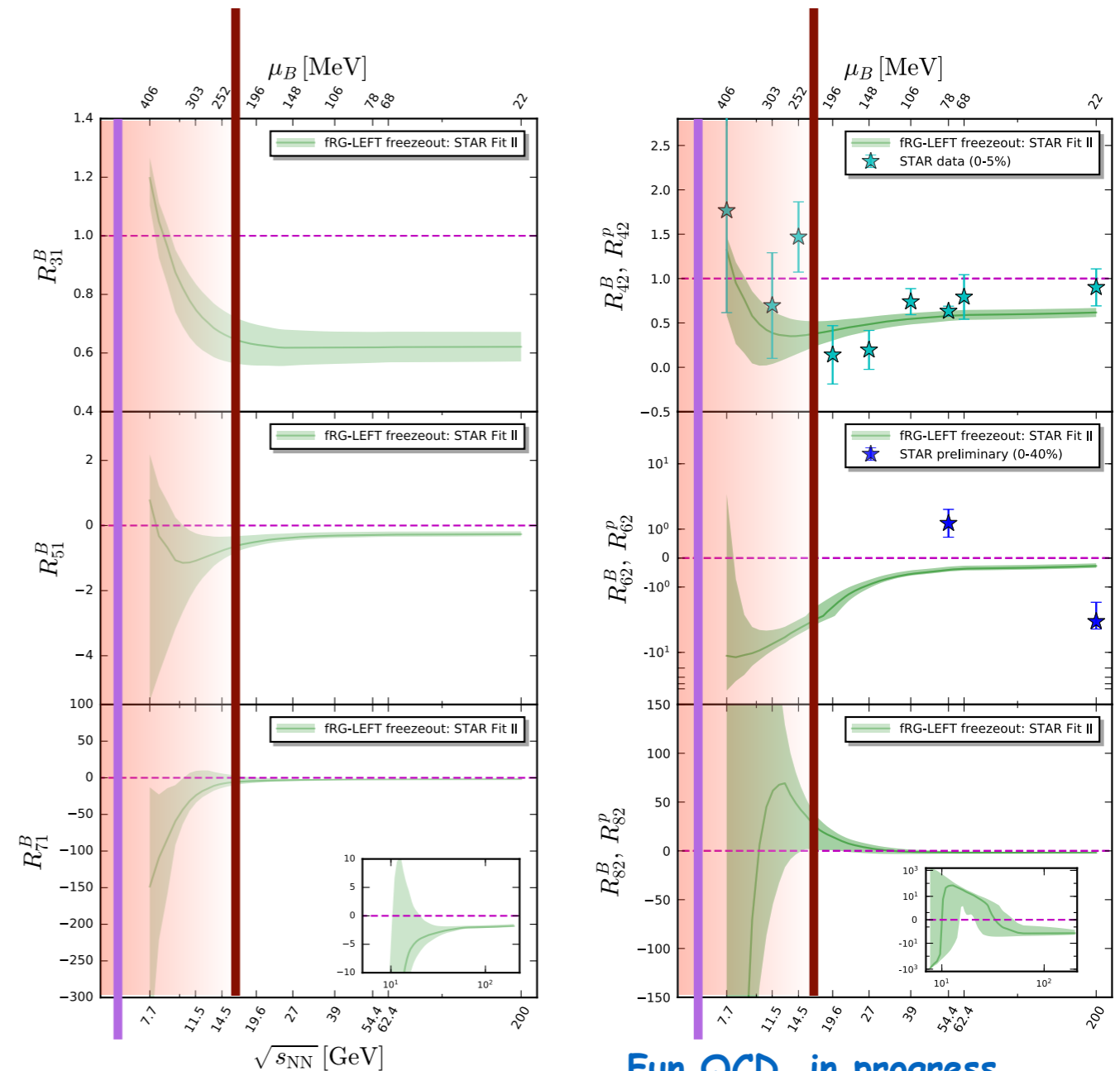
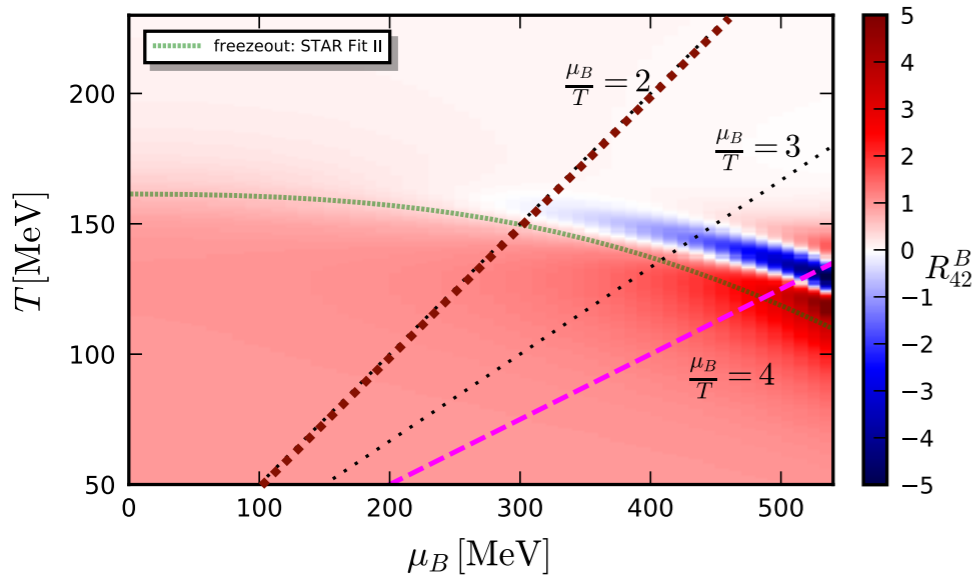
QCD-assisted LEFT

Benchmark at vanishing density



QCD-assisted LEFT

Freezeout curve

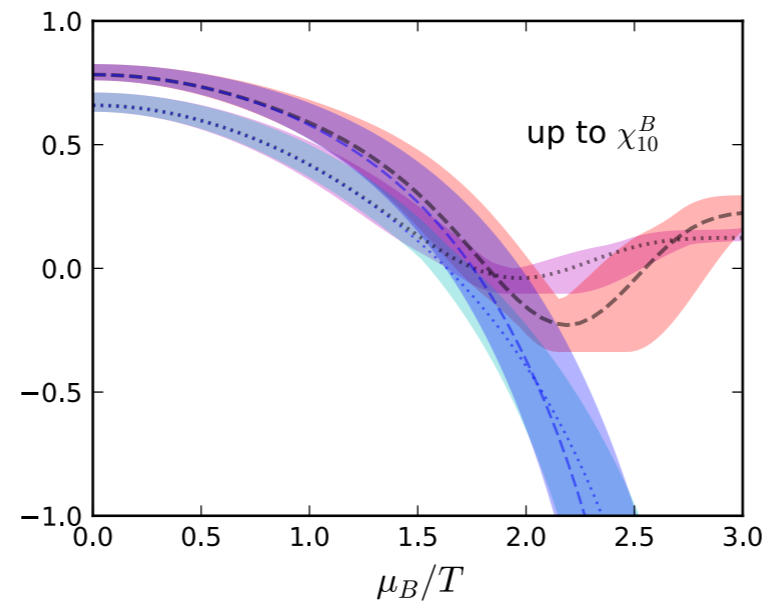
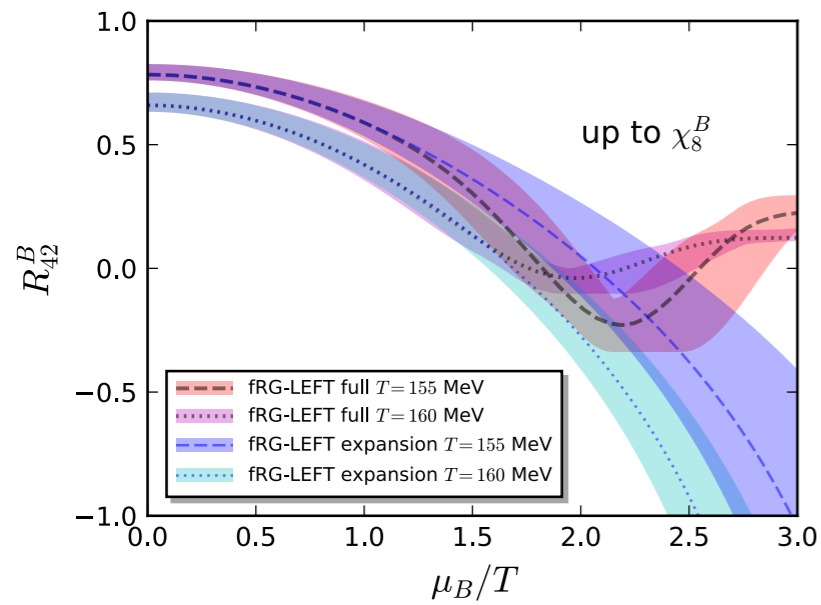


Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

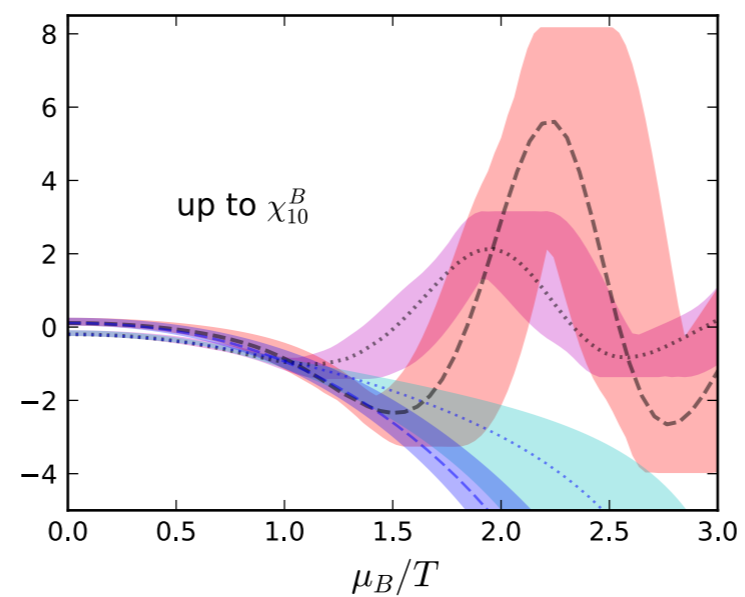
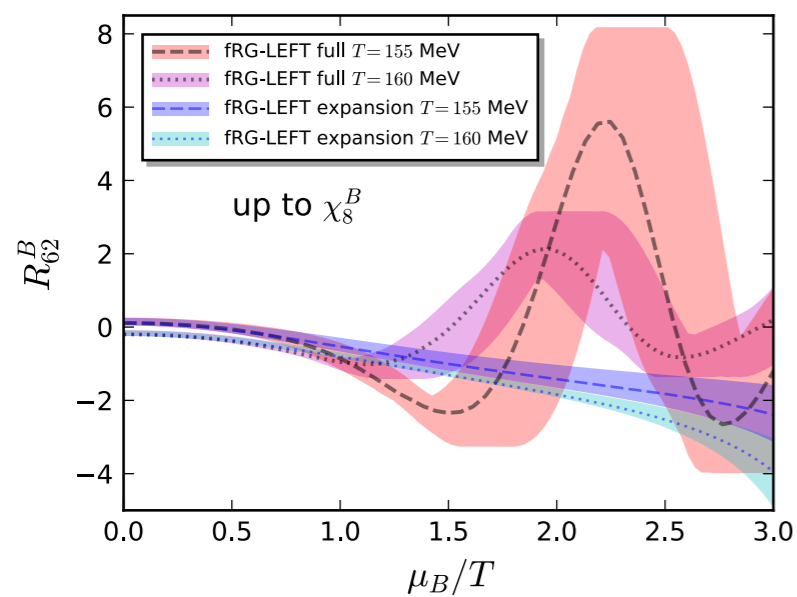
Fun QCD, in progress

Fluctuations of conserved charges

Fluctuations of conserved charges



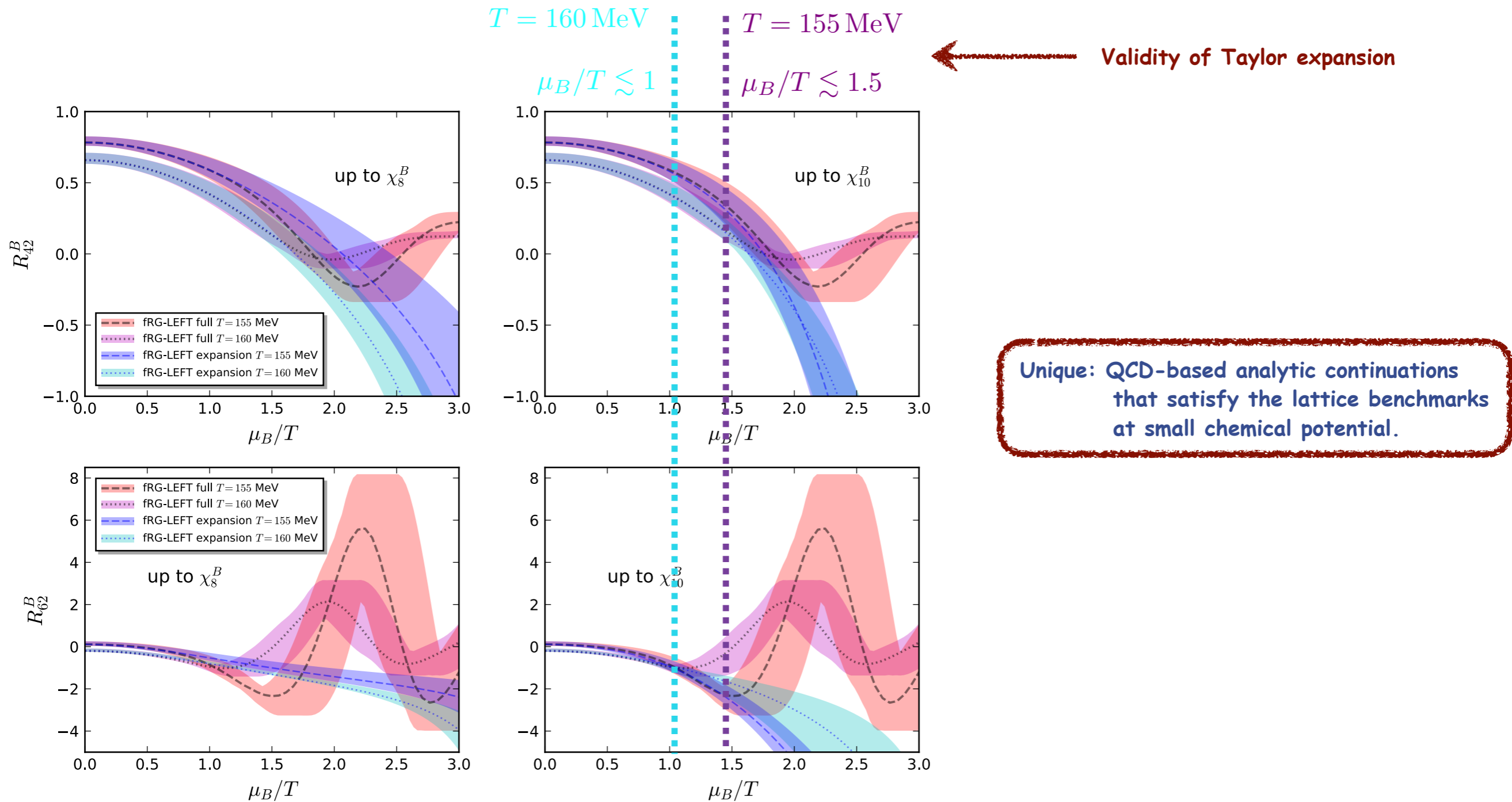
Unique: QCD-based analytic continuations that satisfy the lattice benchmarks at small chemical potential.



Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fluctuations of conserved charges

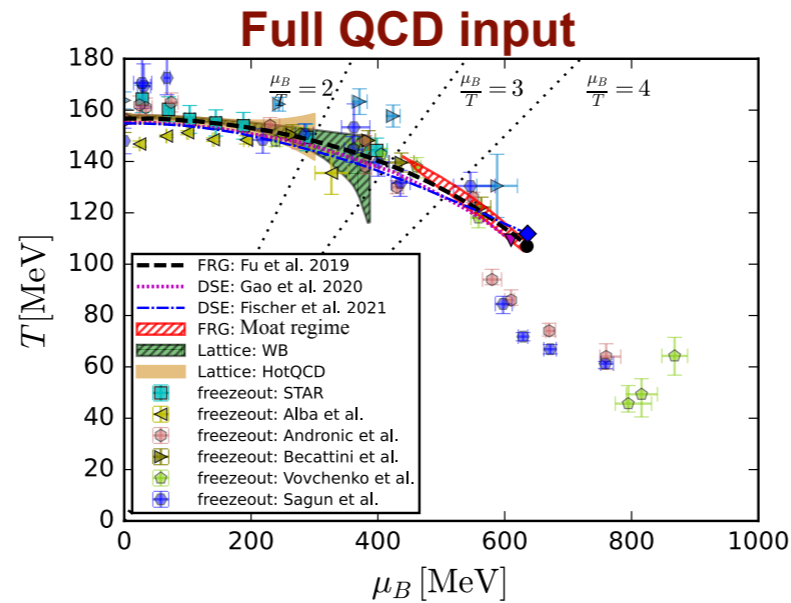
Fluctuations of conserved charges



Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

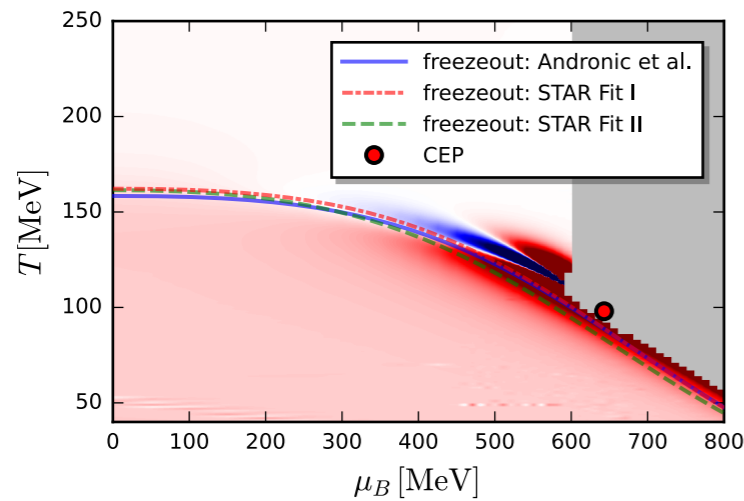
Fluctuations of conserved charges

Sneak preview towards QCD

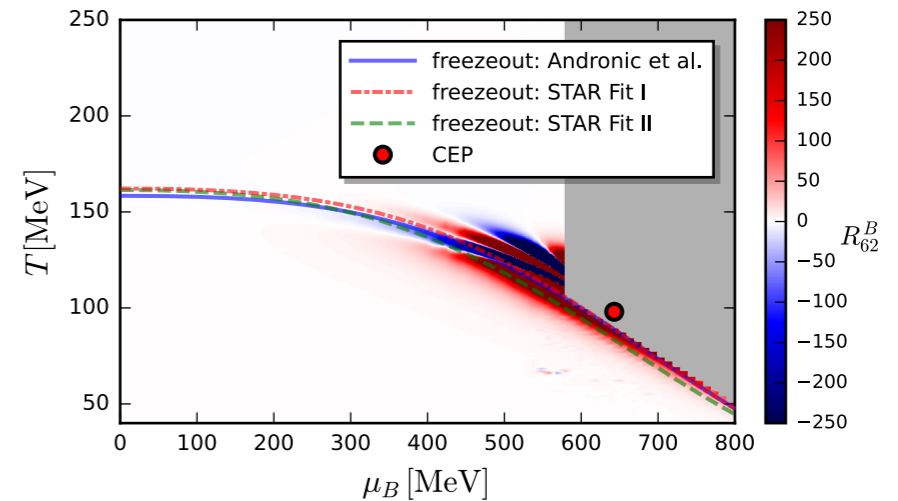


Towards QCD

'Use QCD flows for quark-meson couplings in low energy effective theory'



preliminary



Fluctuations of conserved charges

Canonical ensemble

