Strong interaction physics of heavy flavors (Hirschegg, January 2024)

Comparative Study of Quarkonium Transport in Hot QCD Matter

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EMMI

CRC-TR 211
Strong-interaction matter under extreme conditions

In the name of: A. Andronic*, P.B. Gossiaux*, P. Petreczky*, R. Rapp*, M. Strickland*, J.P. Blaizot, N. Brambilla, P. Braun-Munzinger, B. Chen, S. Delorme, X. Du, M. A. Escobedo, E. G. Ferreiro, A. Jaiswal, A. Rothkopf, T. Song, J. Stachel, P. Vander Griend, R. Vogt, B. Wu, J. Zhao, and X. Yao

* : Conveners

30 ans Subatech and Pays de la Loire

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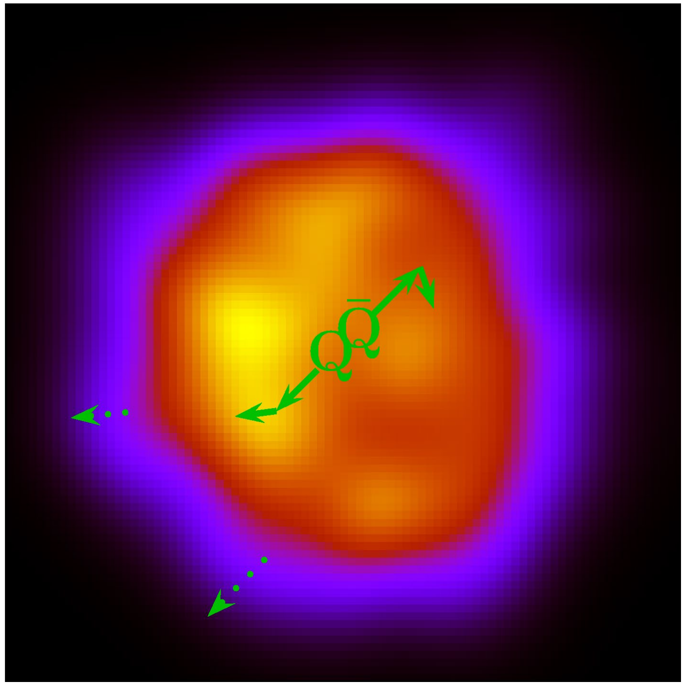
cnrs IN2P3

Nantes Université

STRONG 2020

Motivation and general introduction

What is a quarkonia… in a hot QGP medium ?



Answer may vary depending on how hot is the QGP, and how long you observe

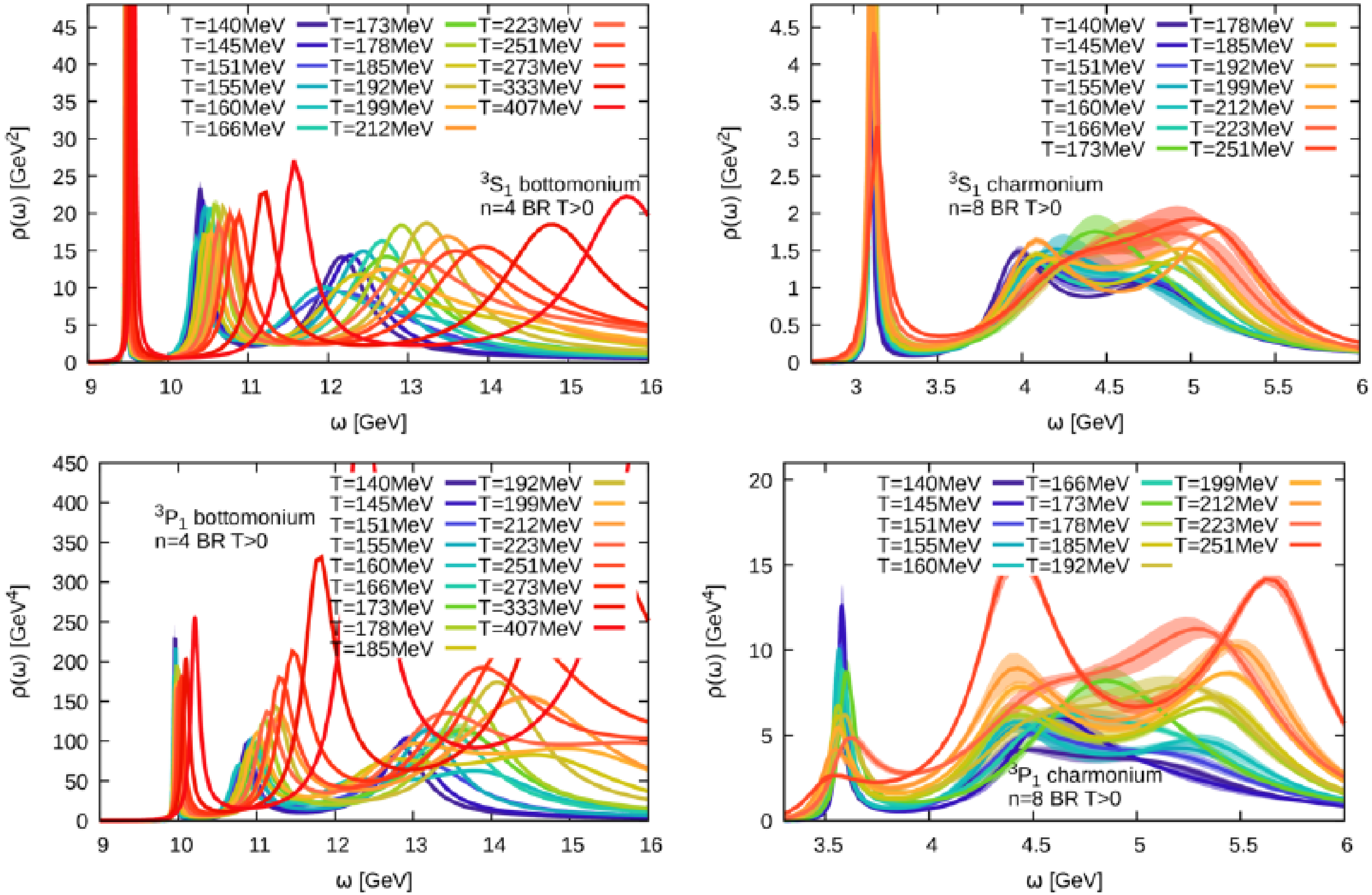
- Not to high T, not too long : Same as in vacuum (see Maxim's talk) + some external perturbation

- If not : probably better to speak a $Q\bar{Q}$ pair

- When is it legitimate to speak of a bound state ?... And deal with it as such in the transport theory. Answer may vary depending on the fundamental ingredients

lQCD perspective : spectral function

Kim et al, JHEP11(2018)088



Many such kind of results in the literature; see Sajid Ali's talk this morning

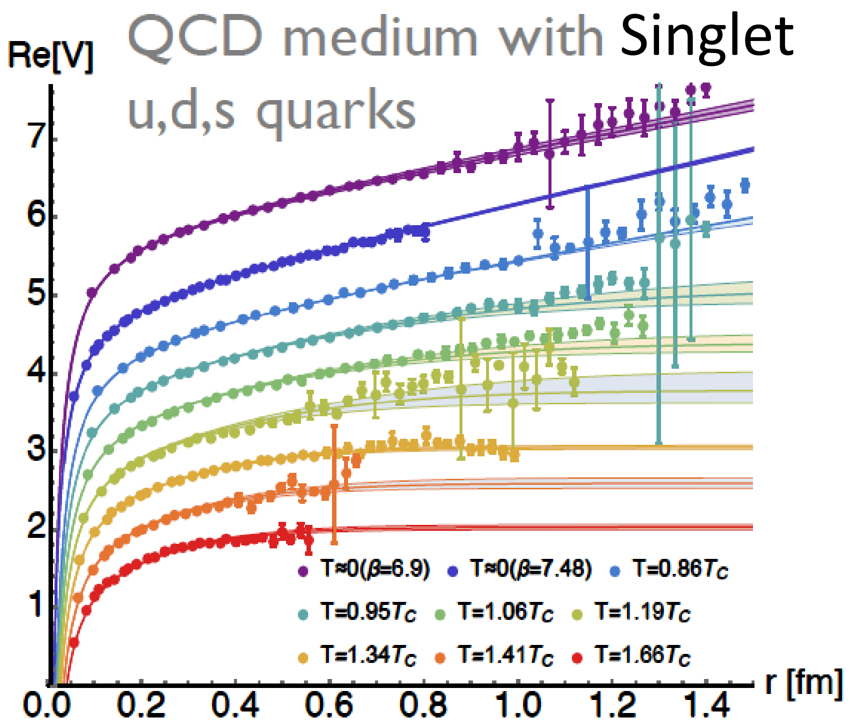
Rich structure : broadening and mass shift. What are the underlying "ingredients'" ?

The 3 pillars of quarkonia production in AA

↳ Implicitly in the pNRQD EFT.

Screening of the real potential

Protential (recent lQCD calculations)



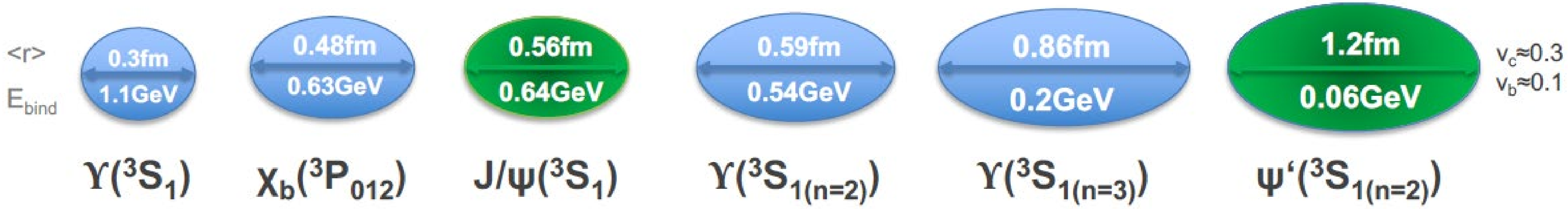
At T=0, well described by the Cornell shape:

$$V(r) = -\frac{\alpha}{r} + Kr$$

<u>Quarkonia scales</u>

- m_Q
- In vacuum: Binding energy / separation energy btwn levels: ΔE α m_Q g^4 (Coulomb part) => v α g^2
- Radius : (m_Q g^2) $^{-1}$
- For a linear potental $\hbar\omega_0 = \left(\frac{\hbar^2 K_l^2}{m_b/2}\right)^{\frac{1}{3}} \approx 0.504\ \mathrm{GeV}$

$$v \propto \left(\frac{K_l}{m_b^2}\right)^{\frac{1}{3}}$$



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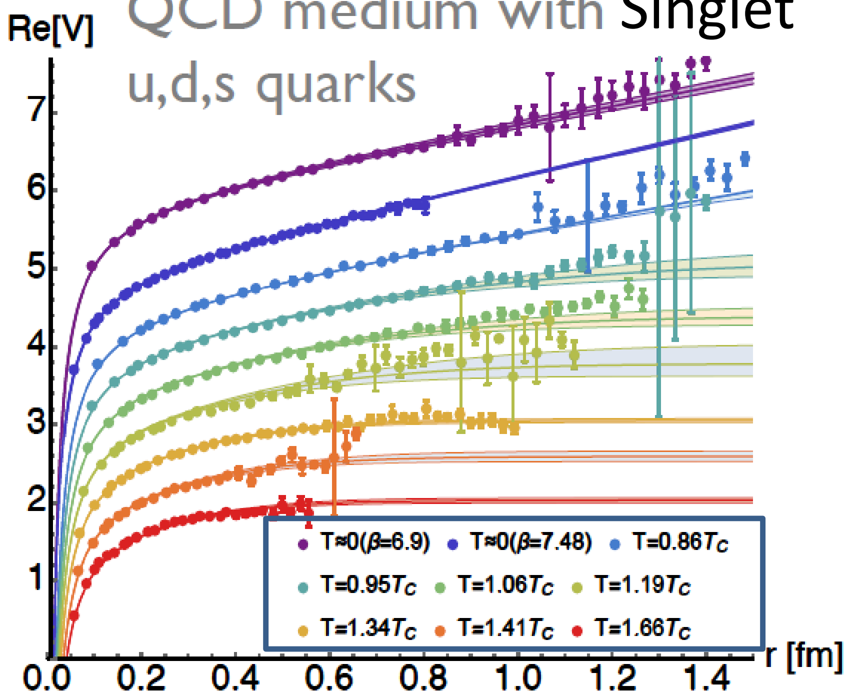

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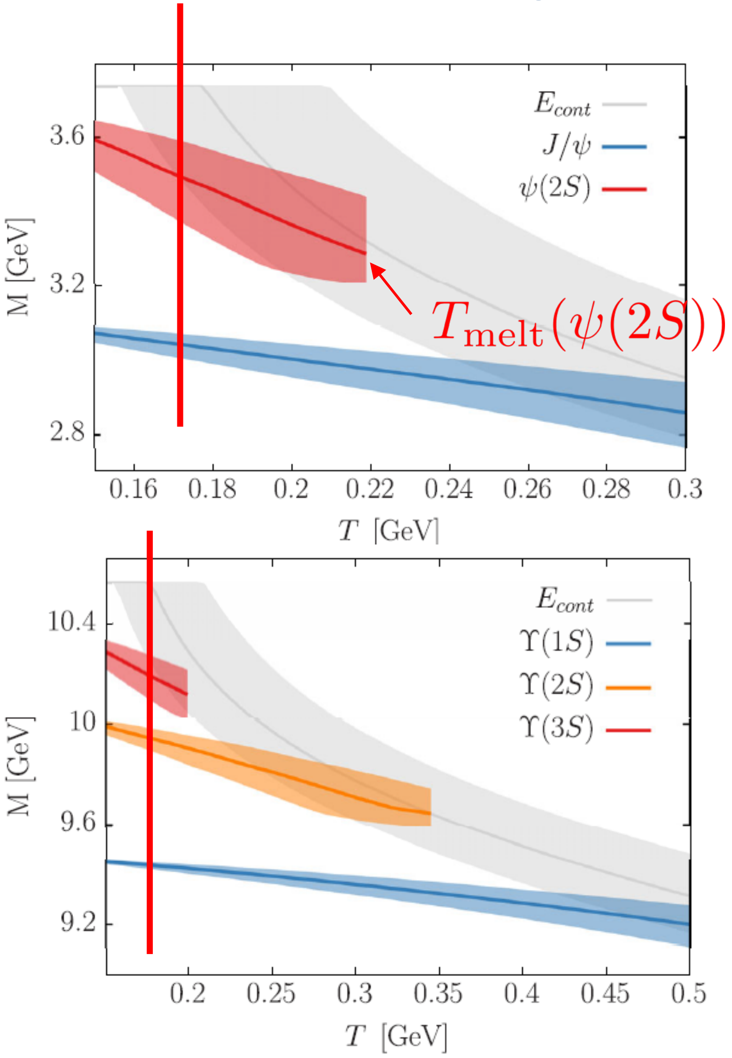
$$v \propto \left(\frac{K_l}{m_b^2}\right)^{\frac{1}{3}}$$

Compact and tightly bound states (at least for the lowest ones) => could survive QGP at low/mid T as well as to interactions with hadronic matter.

Screening of the real potential

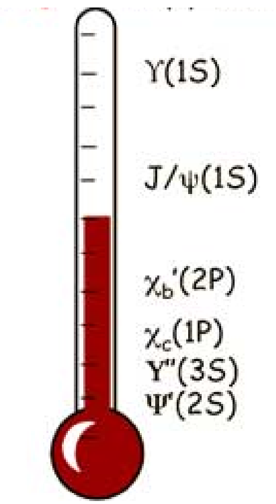
Recent In-medium spectrum (Lafferty and Rothkopf 2020)



<u>« all or nothing scenario»:</u>

- If $T_{early\ QGP} > T_{melt}$ => the state is not produced
- If $T_{early\ QGP} < T_{melt}$ => the state is produced like in pp

=> *SEQUENTIAL SUPPRESSION; Quarkonia as early QGP thermometer*



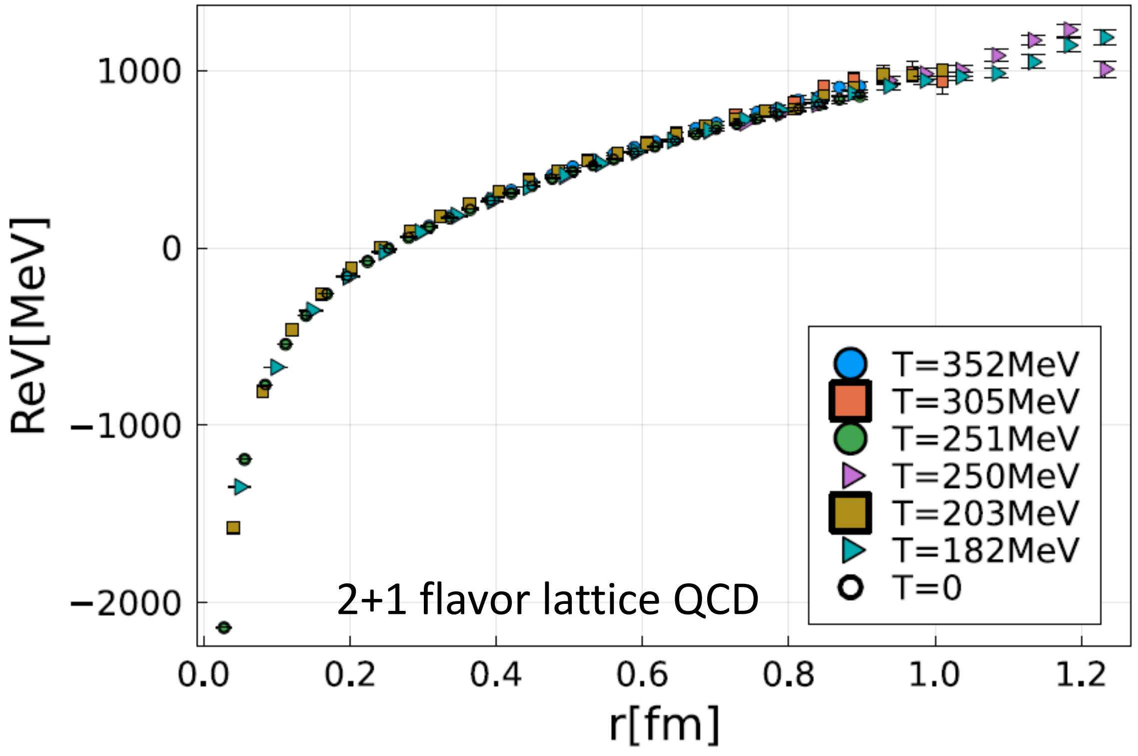
Most prominently : probing new state of matter in AA collision: Original idea by Matsui and Satz (86)…

… and advertized as a motivation in hundreds of talks (and papers) since then

David Lafferty, Alexander Rothkopf , Phys. Rev. D 101, 056010 (2020)

Screening of the real potential

Recent news : the real potential is not screened at temperatures reached in AA collisions !!!



Bazazov et al 2023 (Hot QCD collaboration)

How to define properly a “potential” on the lattice ?

Historically : thermodynamical potential like the free energy (in presence of a static dipole) or the total internal energy.

Modern approach : evaluate the Wilson loop and connect it to the r-dependent spectral density

$$W(\tau, r, T) = \int_{-\infty}^{+\infty} d\omega e^{-\omega\tau} \rho_r(\omega, T)$$

A “peak” contribution in the spectral density modelled as

$$\rho_r^{\text{peak}}(\omega, T) = \frac{1}{\pi} \text{Im} \frac{A_r(T)}{\omega - \text{Re}V(r, T) - i\Gamma(\omega, r, T)}$$

=> Lattice data then unfolded with this Ansatz.

Does not seems quite intuitive, may not be the end of the story

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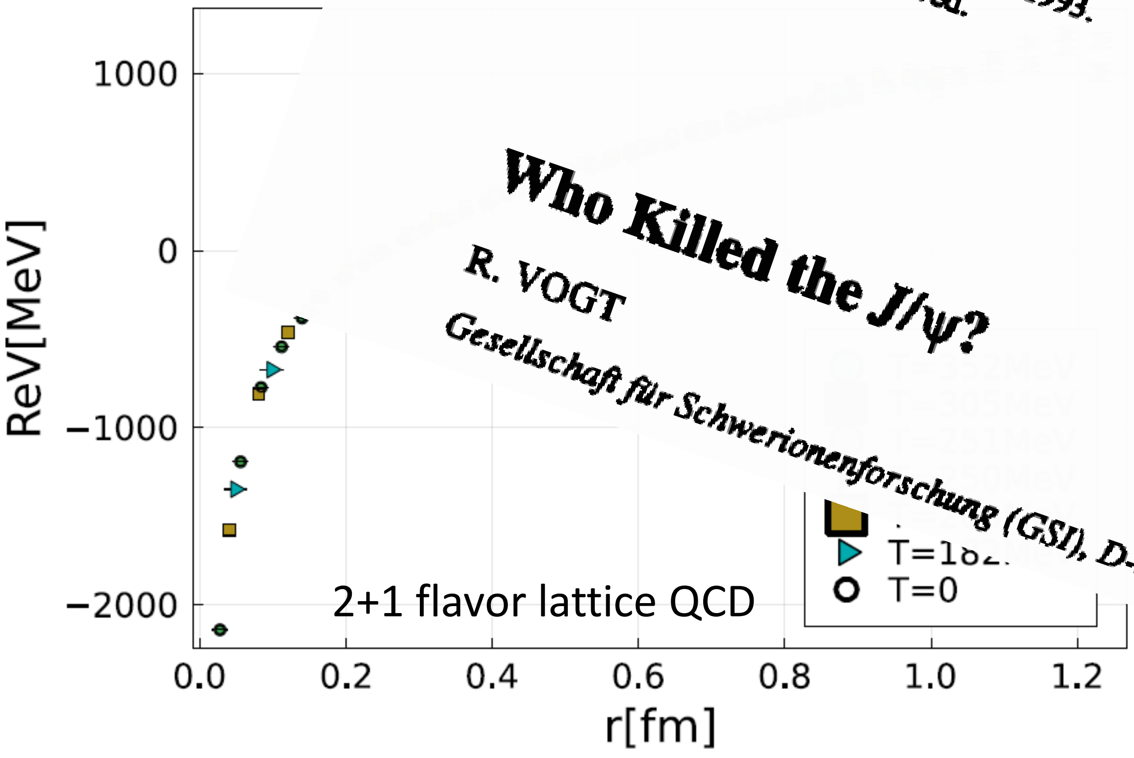

Bazazov et al 2023 (Hot QCD collaboration)

Prog. Part. Nucl. Phys., Vol. 30, pp. 405–406, 1993.

Who Killed the J/ψ?

R. VOGT
Gesellschaft für Schwerionenforschung (GSI), D-6100 Darmstadt 11, Germany

0146-6410/93 $24.00

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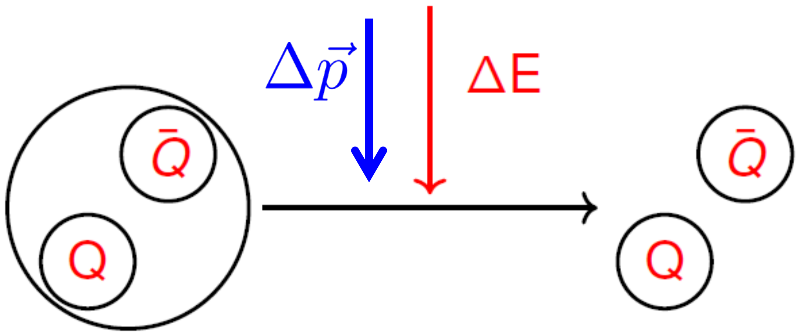
=> Lattice data then deconvoluted with this Ansatz.

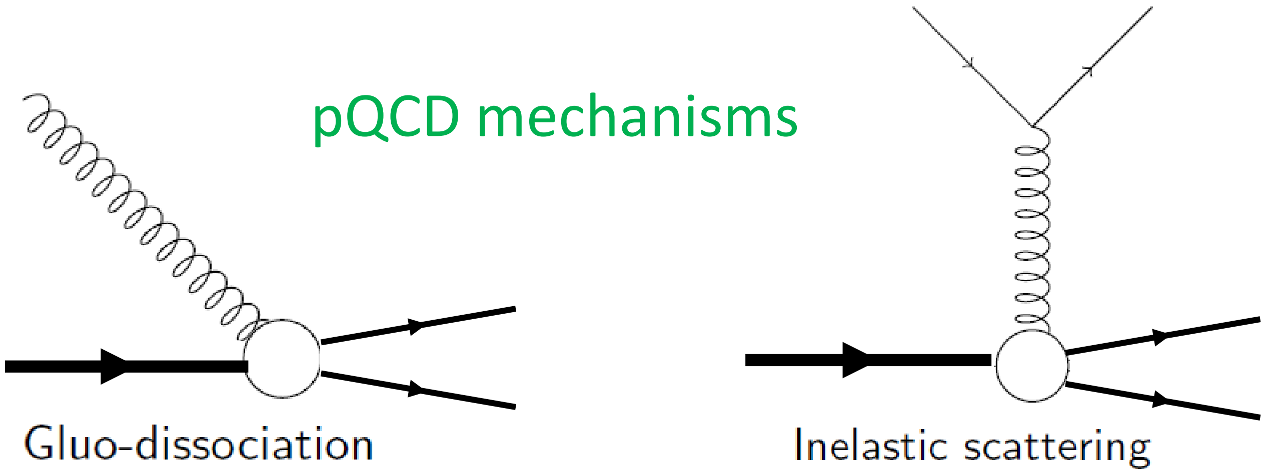
Does not seems quite intuitive, may not be the end of the story

Collisions with the QGP

- Besides arguments based on the Debye mass / screening, it was pointed out already in the 90's that interactions with partons in the QGP could lead to dissociation of bound states (whose spectral function thus acquire some width Γ corresponding to the dissociation rate)

- Energy-momentum exchange with the QGP (gluo-dissociation, q – quarkonia quasi elastic scattering)





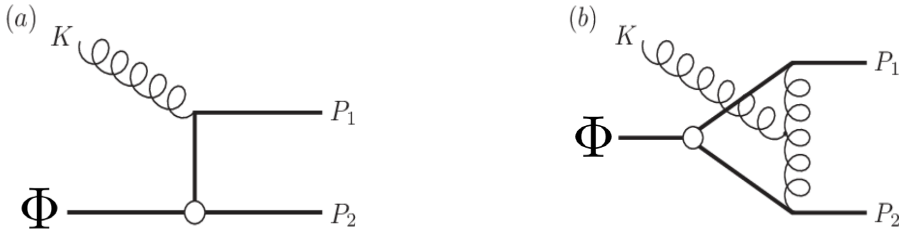
- => pair dissociation => **Suppression**

- ⬄ loss of probability of the quarkonia … Often described by some imaginary potential W in modern approaches

A central quantity: the decay rate Γ

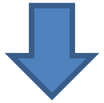
Many approaches

pQCD view (Bhanot & Peskin), later on consolidated by NRQCD (Brambilla & Vairo)



$$\Phi + g \to Q + \bar{Q}$$

Dissociation cross section σ



$$\Gamma_\Phi(T) = \langle \sigma n_g \rangle_T$$

Other mechanisms : $x + \Phi \to x + Q + \bar{Q}$

QFT/Lattice QCD

Time correlator

$$\mathcal{C}_>(t,\vec{r}) \approx \langle \psi(t,\tfrac{\vec{r}}{2})\bar{\psi}(t,-\tfrac{\vec{r}}{2})\psi(0,0)\bar{\psi}(0,0)\rangle$$

Satisfies Schroedinger equation with complex potential V+iW . Breakthrough by Laine et al. (2006)

$$\Gamma_\Phi(T) = -2\langle \Phi | W | \Phi \rangle$$

Concept better suited at it genuinely encodes the "in medium" propagation

=> Simple decay law : Prob survival $= \exp\left(-\int_{t_0}^{t_{\rm fin}} \Gamma(T(t))dt\right)$

A central quantity: the decay rate Γ

Recent lQCD calculations of W(r) = Im(V(r)) (at ω=0)

$$\rho_r^{\mathrm{peak}}(\omega, T) = \frac{1}{\pi}\mathrm{Im}\frac{A_r(T)}{\omega - \mathrm{Re}V(r,T) - i\Gamma(\omega, r, T)}$$

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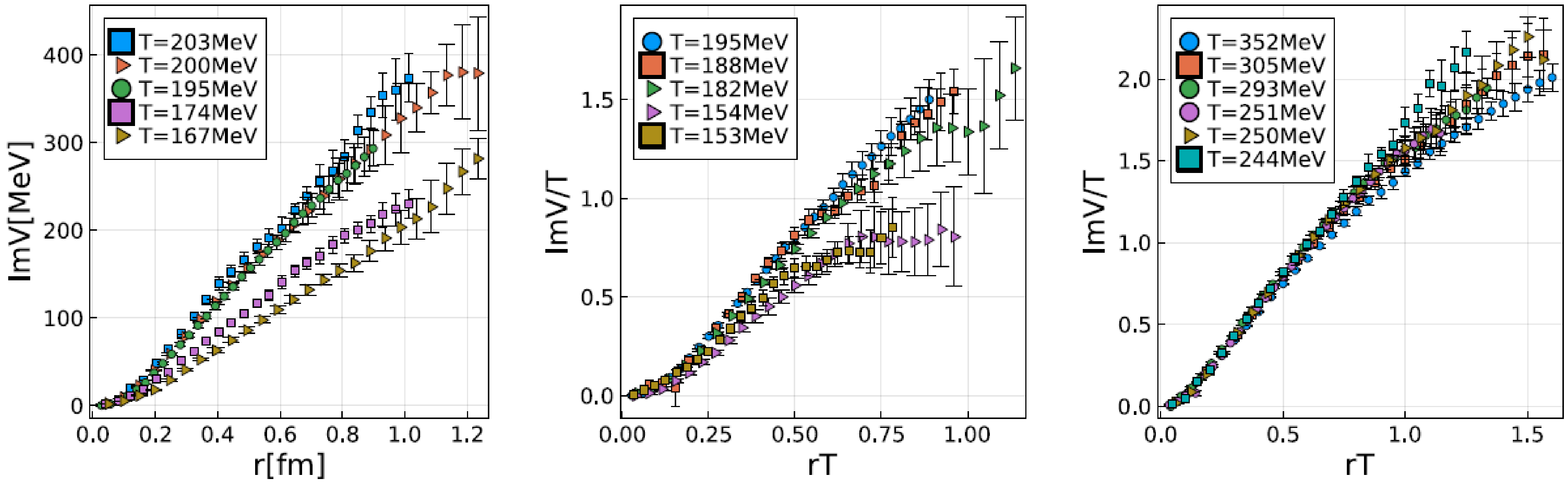
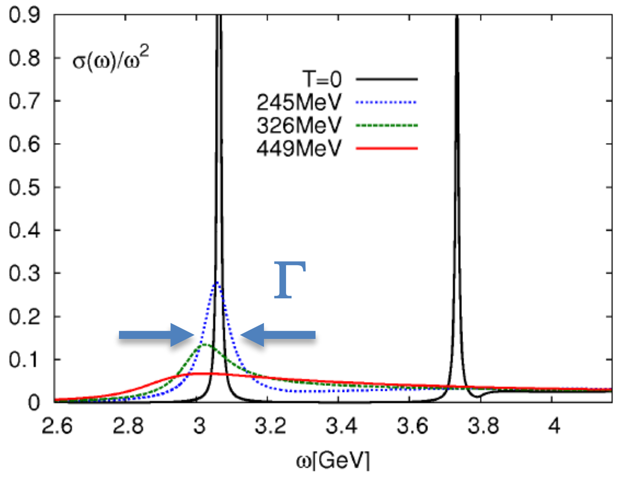


- Nice r T scaling
- Dipole structure at small r, no saturation seen at “large” r

Quarkonia at finite T

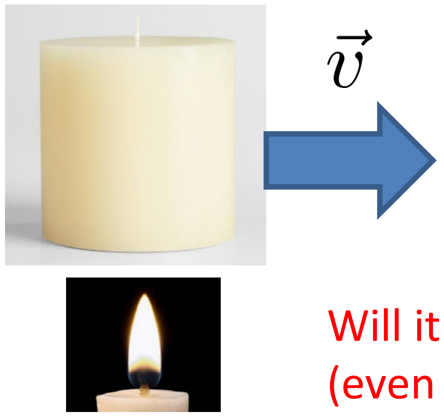
- Pheno: Yet, these pictures might still be compatible with the notion of sequential « suppression »…
- However, this notion has to be made more precise : (LQCD) spectral function lQCD



$$\rho(\omega,p,T)=\frac{1}{2\pi}\mathrm{Im}\int_{-\infty}^{\infty}dte^{i\omega t}\int d^3xe^{ipx}\langle[J(x,t),J(0,0)]\rangle_T$$

At T=245 MeV, ψ' has disappeared but J/ψ still surviving for ≈ $1/\Gamma$ ≈ a couple of fm/c … which needs to be compared with the local QGP cooling time τ_{cool} : Γ x τ_{cool} > 1 ⇔ suppressed

- N.B.: The opposite phenomenom might also be relevant: some state above the « melting » temperature can survive (for a short while < $1/\Gamma$) before getting lost definitively.
- Key question : do the quarkonia states (chemically) equilibrate with the QGP ?

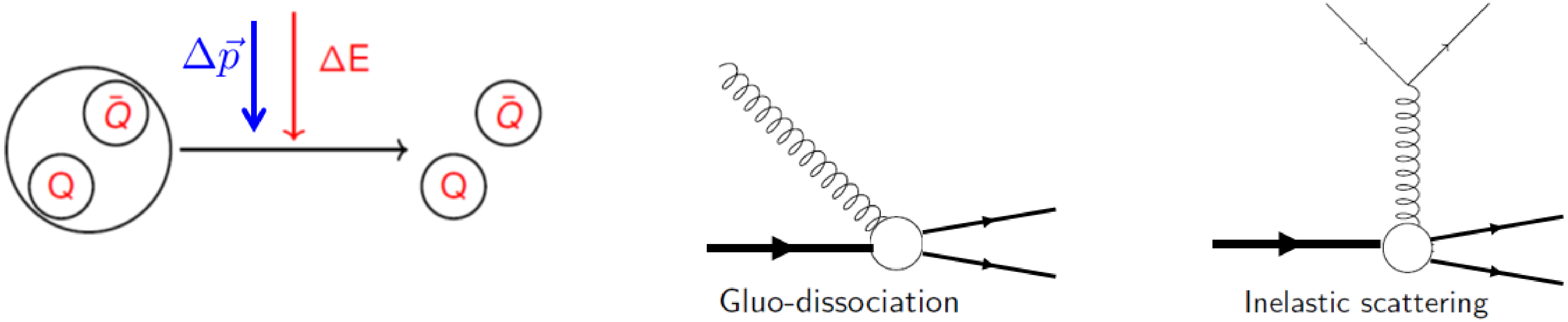


Will it melt (even party) ?

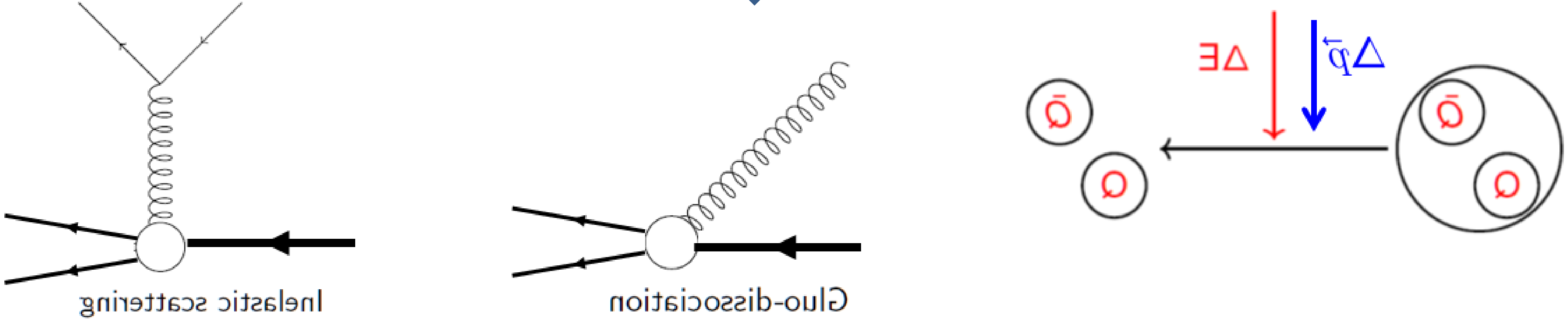
Modern era

Regeneration

Detailed balance :

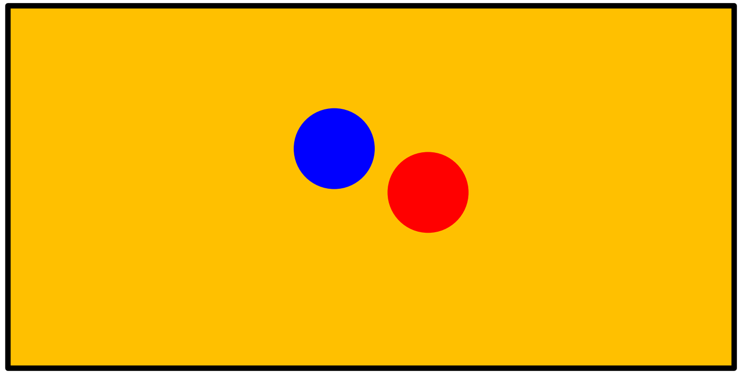


Reverse mechanisms

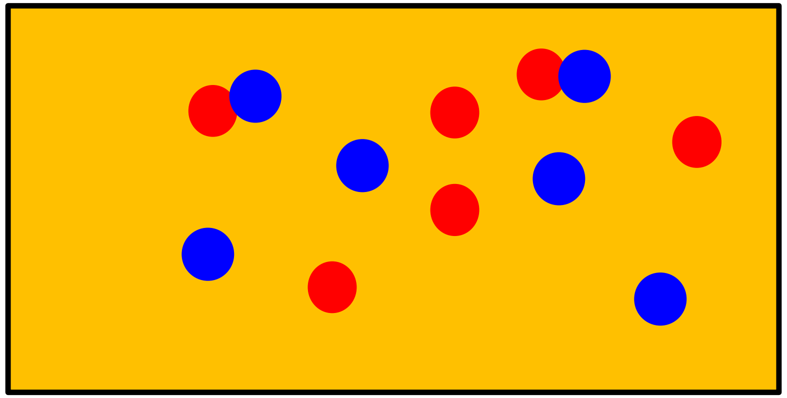


Regeneration: Dilute vs Dense

Bottomia

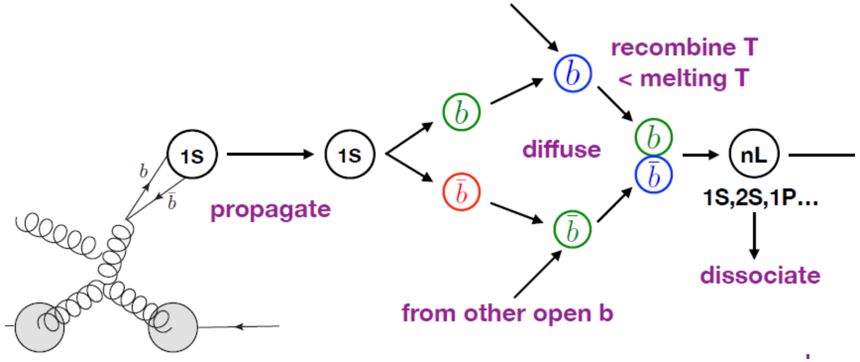


Charmonia



No exogenous recombination : only the b-bbar pairs which are initially close together will emerge as bottomia states

In some SC formalisms : intermediate regeneration



Yao, Mehen, Müller

Exogenous recombination : c & cbar initially far from each other may recombine and emerge as charmonia states

No full quantum treatment possible => need semi-classical approximation(s)

Key question : when does the recombination (dominantly) happen ? Crucial role of the binding force.

One extreme viewpoint : regeneration happens at the end of the QGP (Statistical Hadronization Model)

The present challenges for Quarkonium modelling in URHIC

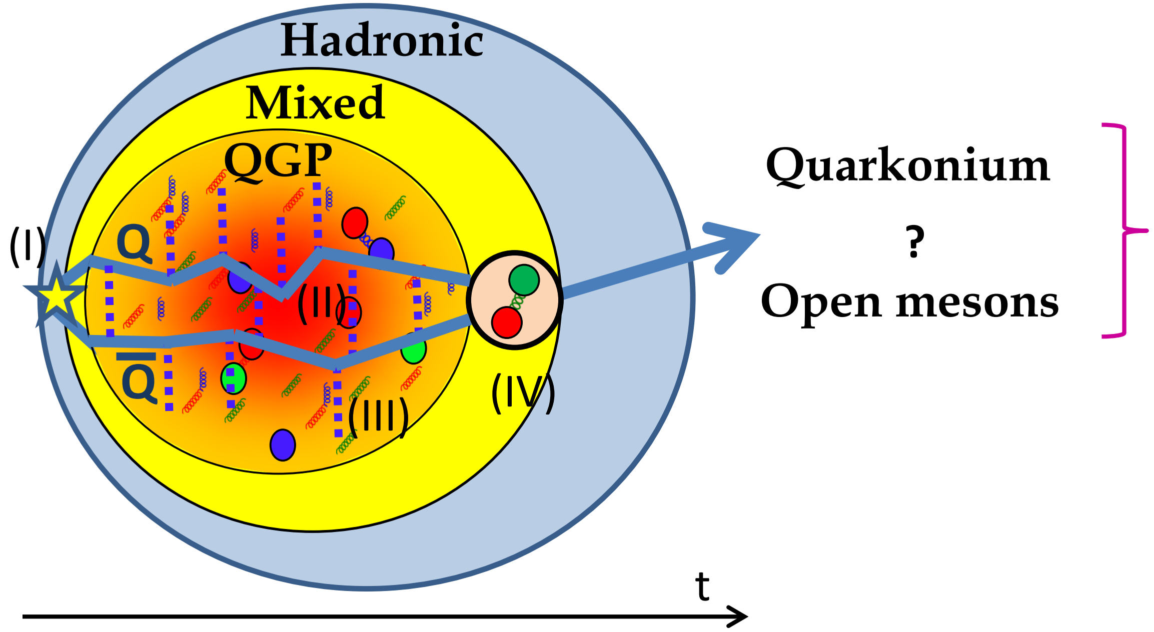
Meet the higher and higher precision of experimental data (already beyond the present model uncertainties)

Unravel the Q-Qbar interactions under the influence of the surrounding QGP and with the QGP

Develop a scheme able to deal with the evolution of one (or many) $Q\bar{Q}$ pair(s) in a QGP, fulfilling all fundamental principles (quantum features, gauge invariance, equilibration,…)

Need for lQCD constraints / inputs

The full scheme



Strictly speaking, only resolved at the end of the evolution

Beware of quantum coherences during the whole evolution !

Especially at early time…

In practice, what counts is the so-called decoherence time, not the "Heisenberg time"

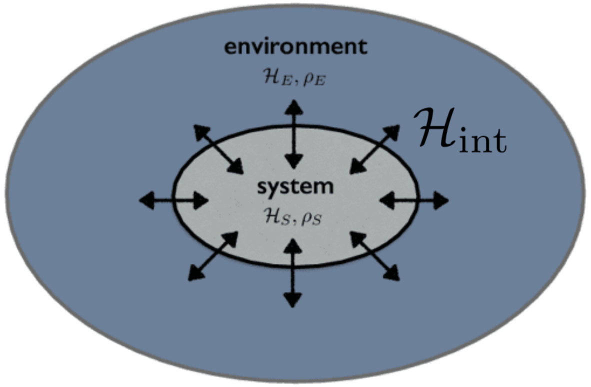
Complicated QFT problem (also due to the evolving nature of the QGP that mixes several scales)… only started to be addressed at face value recently

How to proceed ?

1) Initial state
2) (Screened) interaction between both HQ
3) Interactions with surrounding QGP partons
4) Projection on the final quarkonia

First incomplete QM treatments dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90's

Open Quantum Systems & Quantum Master Equations



Quite generally, system (Q-Qbar pair) builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}^{(0)}_{Q\bar{Q}} + \hat{H}_E + \hat{H}_{\rm int}$ with $\hat{H}_E = \hat{H}_{\rm QGP}$

Von Neumann equation for the total density operator ρ

$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H, \rho]$

System + environment (QGP)

$\rho(t=0) = \rho_{Q\bar{Q}} \otimes \rho_{QGP}$

Evolution of the total system

$\rho(t) = U(t,0)\left[\rho_{Q\bar{Q}} \otimes \rho_{QGP}\right] U(t,0)^\dagger$

Trace out QGP degrees of freedom =>
Reduced density operator $\rho_{Q\bar{Q}}$

System ($Q\bar{Q}$ pair)

$\rho_{Q\bar{Q}}(t=0)$

Evolution of the system

$\rho_{Q\bar{Q}}(t) = Tr_{QGP}\left[U(t,0)\rho(t=0)U(t,0)^\dagger\right]$

Can be formulated differentially ./. time :

$$\frac{\mathrm{d}\rho_{Q\bar{Q}}}{\mathrm{d}t} = \mathcal{L}[\rho_{Q\bar{Q}}]$$

Definition of $\mathcal{L}[\cdot]$

Open Quantum Systems & Quantum Master Equations

$\hat{\rho}_{Q\bar{Q}} = \sum_{\alpha,\beta} d_{\alpha,\beta} |\alpha\rangle\langle\beta|$

QME deal with the (coupled) evolution of probabilities ($d_{\alpha,\alpha}$) and coherences ($d_{\alpha,\beta\neq\alpha}$)

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$\frac{\mathrm{d}\rho_{Q\bar{Q}}}{\mathrm{d}t} = \mathcal{L}[\rho_{Q\bar{Q}}]$ →

Evolution of the system

$\rho_{Q\bar{Q}}(t) = Tr_{QGP}\left[U(t,0)\rho(t=0)U(t,0)^\dagger\right]$

However, $\mathcal{L}[\cdot]$ is generically a non local super-operator in time (linear map)

A special QME: The Lindblad Equation

There are many different QME… a special one :

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \boxed{\gamma_i}\left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

γ_i Characterize the coupling of the system (Q-Qbar) with the environment

$H_{Q\bar{Q}}$: $\{Q, \bar{Q}\}$ $\underbrace{\text{kinetics + Vacuum potential V}}_{\hat{H}^{(0)}_{Q\bar{Q}}}$ + Lamb shift / screening (every unitary term that is generated by tracing out the environment)

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

3 important conservation properties :

$\rho^\dagger_{Q\bar{Q}} = \rho_{Q\bar{Q}}$	$\mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$	$\langle\varphi\|\rho_{Q\bar{Q}}\|\varphi\rangle > 0, \forall\|\varphi\rangle$
(Hermiticity)	(Norm)	(Positivity)

… but in general, non unitary !!! (relaxation)

Nice feature : Can be brought to the form of a stochastic Schroedinger equation (quantum jump method : QTRAJ)

A special QME: The Lindblad Equation

Non unitary / dissipative evolution ≡ decoherence

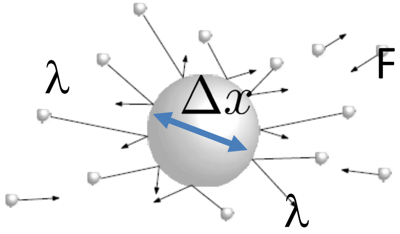
$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

Genuine transitions :
- ✓ Singlet <-> octet
- ✓ Octet <-> octet

Can be reshuffled into non Hermitic effective hamiltonian

$$\hat{H}_{Q\bar{Q},\mathrm{eff}} = \hat{H}_{Q\bar{Q}} - i\sum_j \gamma_j \frac{L_j L_j^\dagger}{2}$$ ≡ Dissociation width

For **infinitely massive single Q** and environment wave length λ >> wave packet size Δx:



Fluctuations from env. ⟺

$$\frac{\partial \rho_Q(x_Q, x'_Q)}{\partial t} = -F(x_Q - x'_Q)\rho_Q(x_Q, x'_Q)$$

Decoherence factor: $F \approx \kappa\,(x_Q - x'_Q)^2$

HQ momentum diffusion coefficient (adjoint)

In Q world: smaller objects live longer !

At 1rst order in 1/m_Q : recoil corrections ⟺ friction / dissipation

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

Pictorial summary

τ_E: environment autocorrelation time **τ_S: system intrinsic time scale** **τ_R: system relaxation time**

Subsystem + environment: von Neumann equation

↓ Trace out environment

Subsystem: non-unitary, time-irreversible evolution

↓ Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$

Redfield equation

$$\frac{\partial}{\partial t}\rho_{\rm I}(t) = -\frac{1}{\hbar^2}\sum_{m,n}\int_0^{\infty} d\tau\Big(C_{mn}(\tau)\Big[S_{m,{\rm I}}(t), S_{n,{\rm I}}(t-\tau)\rho_{\rm I}(t)\Big] - C^*_{mn}(\tau)\Big[S_{m,{\rm I}}(t), \rho_{\rm I}(t)S_{n,{\rm I}}(t-\tau)\Big]\Big)$$

Similar structure to the Linblad equation but with time delay effects

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Redfield equation

Smallest time scales wins it all !

$\tau_S \ll \tau_R$ Quantum Optical Regime → **Lindblad equation** — Eigenstates of the HQ Hamiltonian

$\tau_E \ll \tau_S$ Quantum Brownian Motion → **Lindblad equation** — Phase space densities

Not the same basis !

Wigner transform + gradient expansion

Boltzmann equation

Rate equations: ⇔ transport models

Semi-classical approx : density matrix ≈ diagonal

Fokker-Planck equation

Good method for many $c\bar{c}$ pairs

Two types of dynamical modelling

$m_D \ll E_{\rm bind}$ — Quantum Optical Regime

$m_D \sim E_{\rm bind}$

$m_D \gg E_{\rm bind}$ — Quantum Brownian Motion

- **Well identified resonances**
- Time long enough wrt quantum decoherence time (once we reach this regime)

Good description with transport models (TAMU, Tsinghua, Duke)

Central quantities :
2->2 and 2->3 Cross sections, decay rates

Equilibrium : exp(-E_n/T) (theorem)

SC Approx: rate equations

?

- Correlations growing with cooling QGP
- **Best described in position-momentum space**
- Time short wrt quantum decoherence time ?

Quantum Master Equations for **microscopic dof (QS and Qbars)**

Equilibrium / asympt* : some limiting cases

SC Approx: Fokker-Planck equations in position-momentum space

* Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these models is an important prerequisite !!!

QCD time scales

τ_E: environment autocorrelation time

$$\tau_E \approx \frac{1}{m_D} \approx \frac{1}{CT} \approx \frac{1}{T}$$ (C taken as close to unity)

τ_S: system intrinsic time scale

$$\tau_S \approx \underbrace{\frac{1}{\Delta E}} \approx \frac{1}{m_Q v^2}$$ with $v \approx \alpha_S$ … at the beginning of the evolution

Difference btwn energy levels

τ_R: system relaxation time

$$\Gamma = \tau_R^{-1} \sim 2\langle\psi|W\psi\rangle \approx \alpha_s T \times \Phi(m_D r) \approx \alpha_s T \times \Phi(\frac{CT}{m_Q \alpha_s})$$

At "small" T $\left(T \lesssim \frac{m_Q \alpha_S}{C}\right)$: dipole approximation : $\Gamma = \tau_R^{-1} \approx \frac{C^2 T^3}{\alpha_s m_Q^2}$

$$\frac{\tau_R}{\tau_E} = \frac{\alpha_s m_Q^2}{CT^2} \gg 1$$ And $$\frac{\tau_R}{\tau_S} = \frac{\alpha_s^3 m_Q^3}{C^2 T^3} \gg 1$$ for $T \lesssim m_Q \frac{\alpha_S}{C^{2/3}}$

Fine with the Markovian assumption

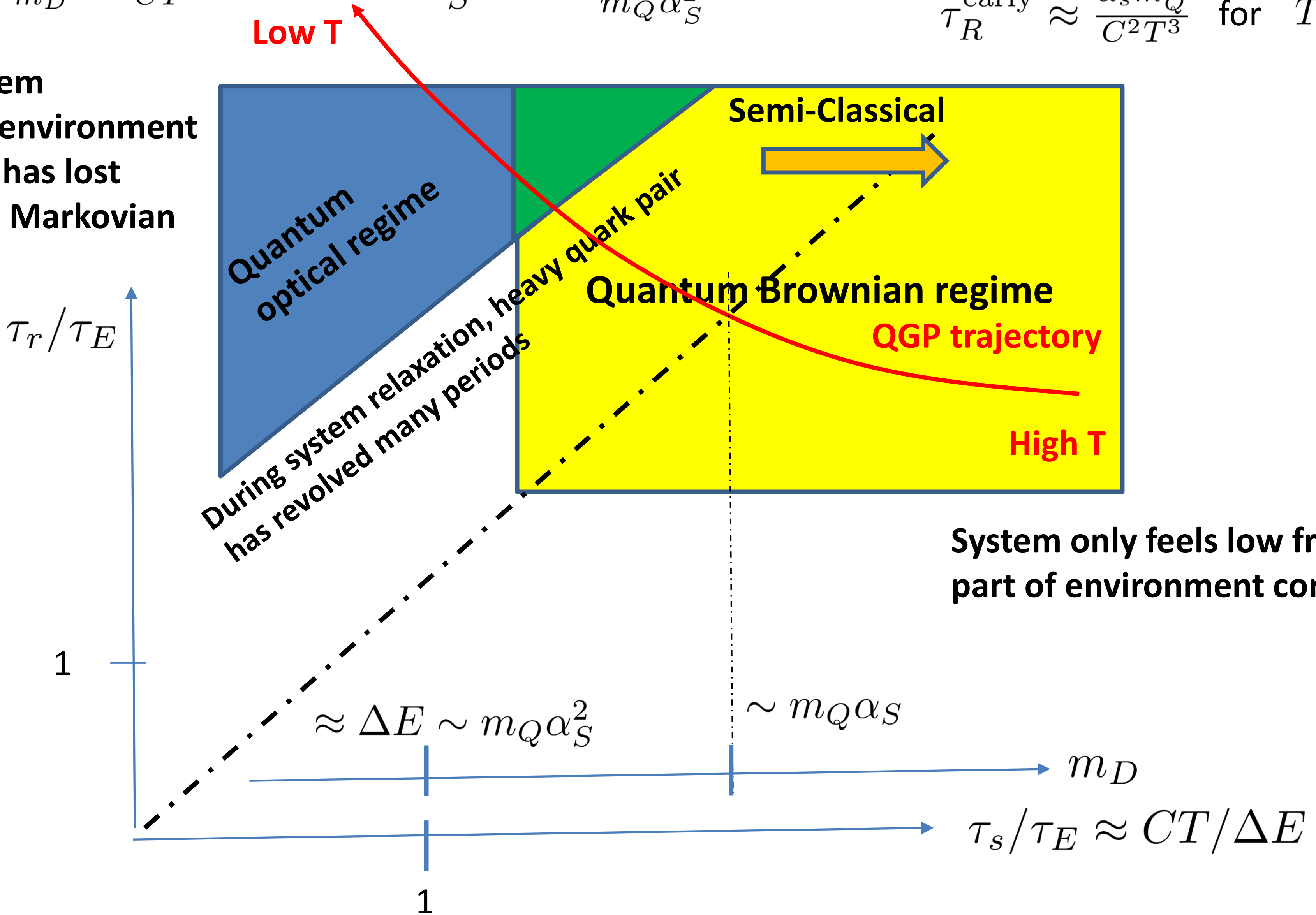
QCD time scales

$\tau_E \approx \frac{1}{m_D} = \frac{1}{CT}$

$\tau_S^{\text{early}} \approx \frac{1}{m_Q \alpha_S^2}$

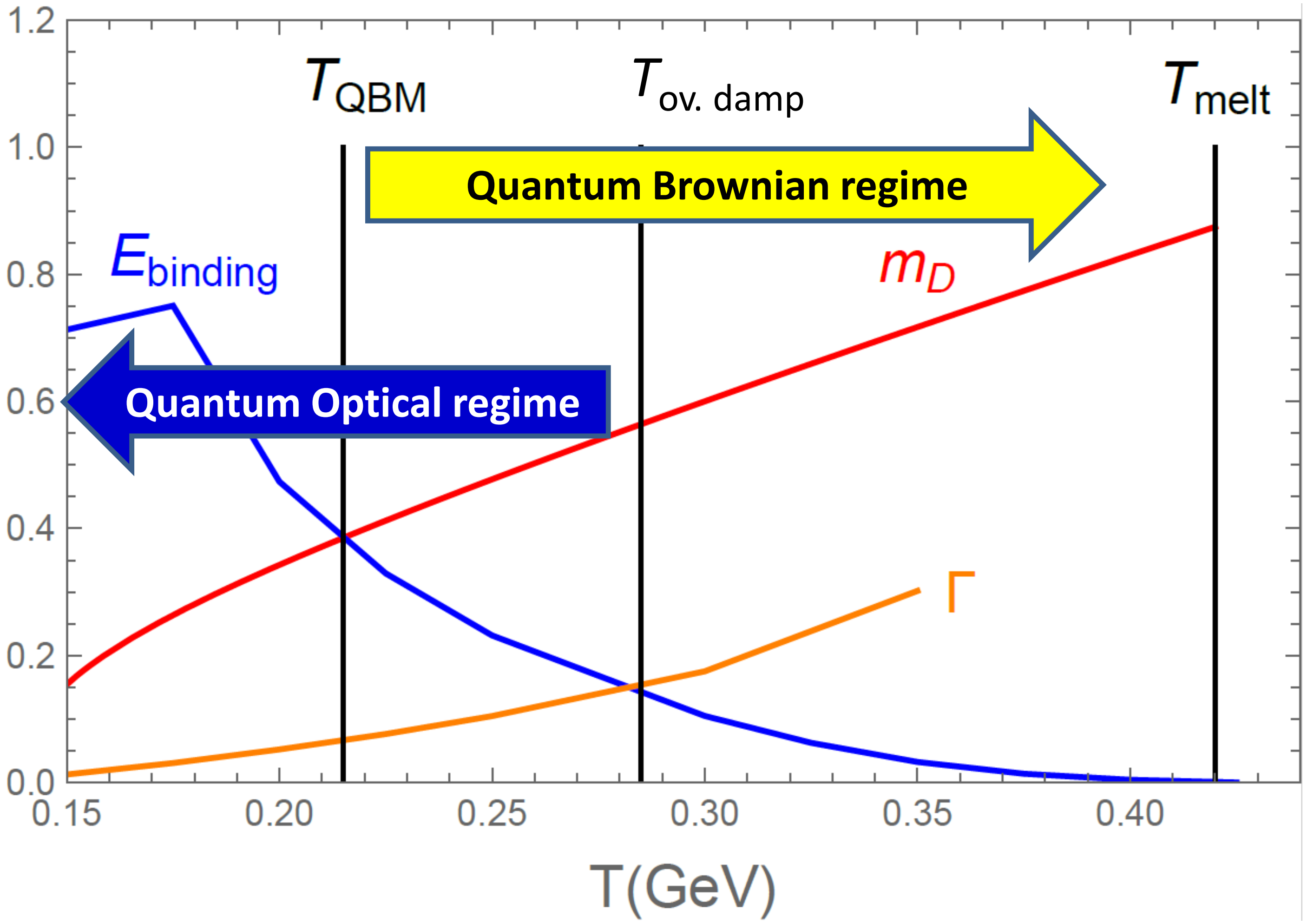
$\tau_R^{\text{early}} \approx \frac{\alpha_s m_Q^2}{C^2 T^3}$ for $T \lesssim \frac{m_Q \alpha_S}{C}$

During system relaxation, environment correlation has lost memory => Markovian process



System only feels low frequency part of environment correlation

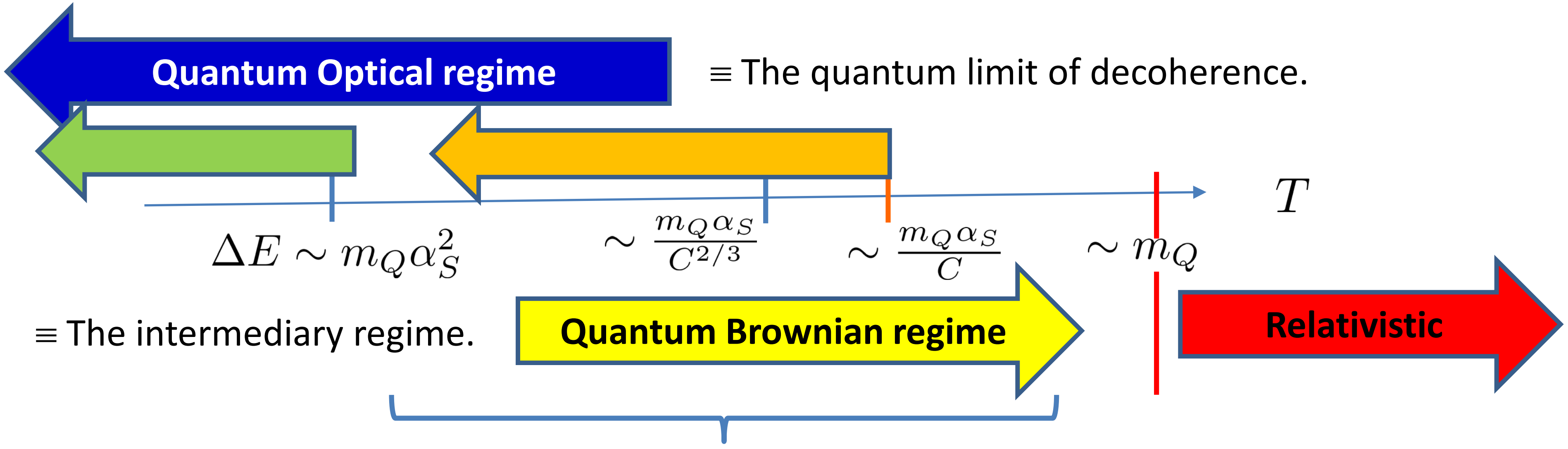
Two types of dynamical modelling



$c\bar{c}$ pair

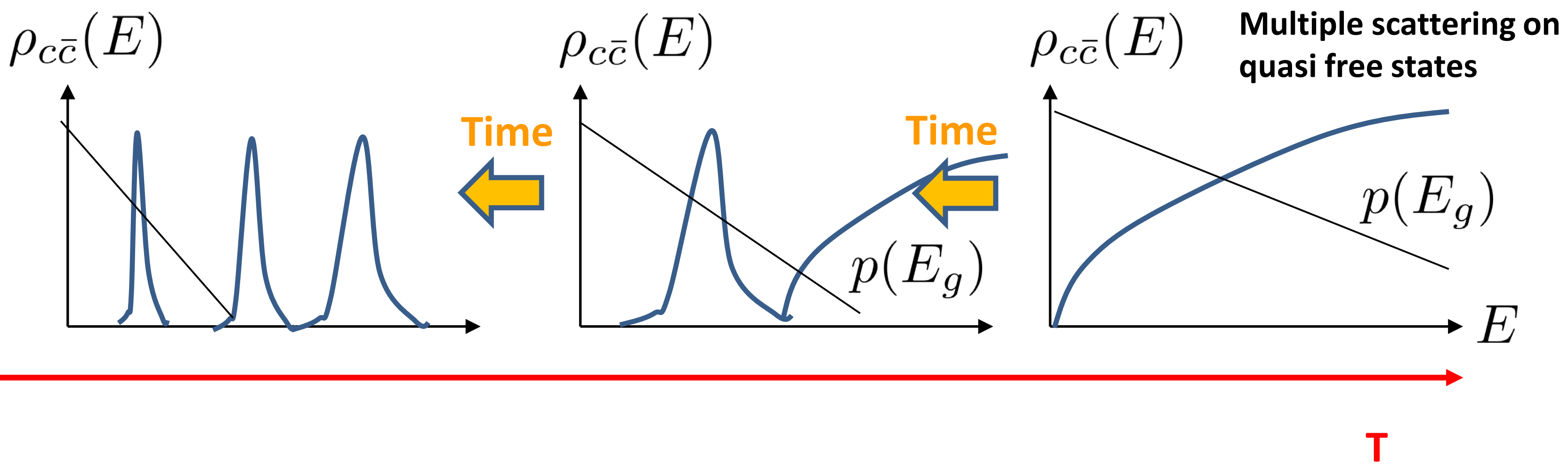
Numbers extracted from a specific potential model : Katz et al, Phys. Rev. D 101, 056010 (2020)

QCD Temperature scales



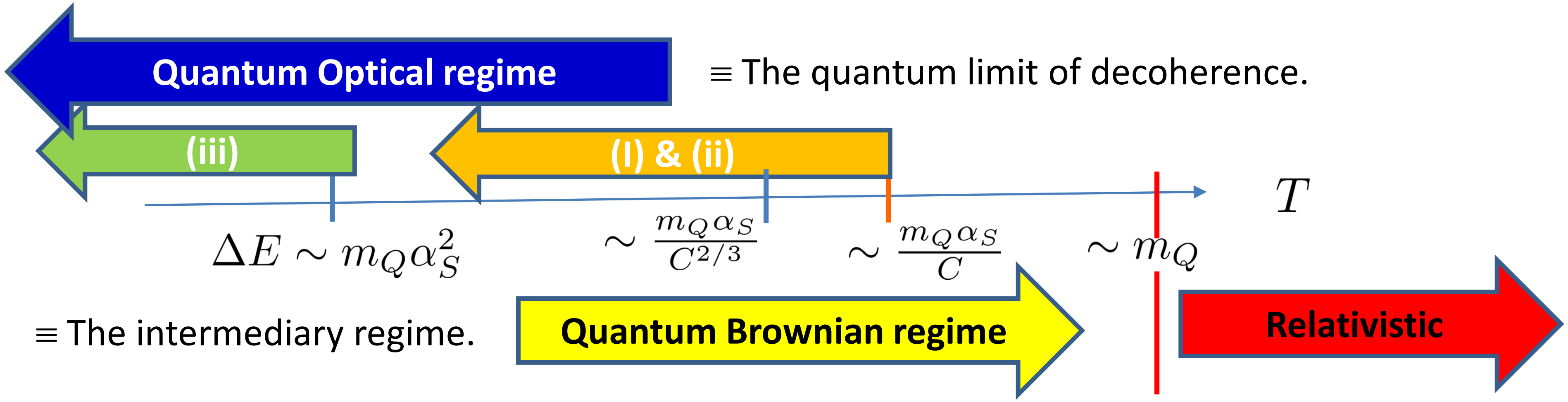
For these « large » temperatures, the Q-Qbar gain enough energy to overwhelm the real binding potential
=> larger distance => larger decoherence ….

dissociation of well identified levels by scarce "high-energy" modes (dilute medium => cross section ok)



In // : continuous evolution of the $Q\bar{Q}$ spectral function

QCD Temperature scales



Refined subregimes when playing with the scales of NRQCD / pNRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo, M Strickland et al, Yao, Müller and Mehen,…)

NRQCD: $Mv, \Lambda_{\rm QCD}, T \ll \mu_{\rm NR} \ll M$: most general scheme for markovian OQS !

pNRQCD:

(Singlet and octet quarkonium fields)

- (i) $1/r \gg T \sim m_D \gg E$: « strongly coupled » QME same as small dipole limit of NRQCD (applies for small time evolution) : **See talk by Tom Magorsch on Monday**
- (ii) $1/r \gg T \gg E \gg m_D$: « weakly coupled » : g T << T : essential contribution is gluo – dissociation from hard mode T : does not apply in QCD
- (iii) $1/r \gg T \sim E \gg m_D$: Quantum optical regime

Quantum Brownian Motion : The Blaizot-Escobedo QME

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2n^a_{\boldsymbol{x}}\mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q\dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

External "ingredients" : complex potential V + I W

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms** :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_4$

Application to QED-like and QCD for both cases of 1 body and 2 body densities…

Recent OQS implementations (single $Q\bar{Q}$ pair)

regime	SU3 ?	Dissipation ?	3D / 1D	Num method	year	remark	ref
NRQCD ⇔ QBM	No	No	1D	Stoch potential	2018		Kajimotoet al. , Phys. Rev. D 97, 014003 (2018), 1705.03365
	Yes	No	3D	Stoch potential	2020	Small dipole	R. Sharma et al Phys. Rev. D 101, 074004 (2020), 1912.07036
	Yes	No	3D	Stoch potential	2021		Y. Akamatsu, M. Asakawa, S. Kajimoto (2021), 2108.06921
	No	Yes	1D	Quantum state diffusion	2020		T. Miura, Y. Akamatsu et al, Phys. Rev. D 101, 034011 (2020), 1908.06293
	Yes ✓	Yes ✓	1D	Quantum state diffusion	**2021**		Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402
	No	Yes	1D	Direct resolution	**2021**		O. Ålund, Y. Akamatsu et al, Comput. Phys. 425, 109917 (2021), 2004.04406
	Yes ✓	Yes ✓	1D	Direct resolution	**2022**		S Delorme et al, https://inspirehep.net /literature/ 2026925
pNRQCD (i)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D96, 034021 (2017), 1612.07248
(i) Et (ii)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D 97, 074009 (2018), 1711.04515
(i)	Yes	No	Yes	Quantum jump	**2021**	See SQM 2021	N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & *Phys.Rev.D* 104 (2021) 9, 094049, 2107.06222
(i)	Yes ✓	Yes ✓	Yes ✓	Quantum jump	**2022**		N. Brambilla et al. 2205.10289
(iii)	Yes ✓	Yes ✓	Yes ✓	Boltzmann (?)	2019		Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027
NRQCD & « pNRQCD »	Yes	Yes	1D	Quantum state diffusion	**2022**		Miura et al. http://arxiv.org/abs/2205.15551v1
Other	No	Yes	1D	Stochastic Langevin Eq.	2016	Quadratic W	Katz and Gossiaux

(Year > 2015)

Not exhaustive

See as well table in 2111.15402v1

…

The present challenges for Quarkonium modelling in URHIC

Meet the higher and higher precision of experimental data (already beyond the present model uncertainties)

Unravel the Q-Qbar interactions under the influence of the surrounding QGP and with the QGP

Develop a scheme able to deal with the evolution of one (or many) $Q\bar{Q}$ pair(s) in a QGP, fulfilling all fundamental principles (quantum features, gauge invariance, equilibration,…)

Need for lQCD constraints / inputs

Ultimately, go beyond the “one team – one model” paradigm

the EMMI Rapid Reaction Task Force

Main focuses of EMMI RRTF (proposal 2019)

Most important issues :

i) the identification and model comparisons of transport parameters;
ii) the controlled implementation of constraints from lattice QCD;
iii) the significance of quantum transport treatments.

Binding energies, decay rates,…

5 key questions

1) To what extent are the currently employed **transport approaches** (mostly carried out in semi-classical approximations) **consistent** in their treatment of quarkonium dissociation and regeneration ?

2) What are the equilibrium limits of the transport approaches and how do the former compare to the results of the statistical hadronization model ?

3) What is the significance of the **effects on quantum transport** of the quarkonium wave packets, and what is needed to develop quantum transport into a realistic phenomenology ?

4) How can the abundant information from **lattice QCD** (quarkonium correlation functions, heavy quark free energies and susceptibilities, and the open heavy-flavor sector) be **systematically implemented into transport approaches** ?

5) What are the **ultimate model uncertainties**, and will those allow for conclusions on the fundamental question of the existence of hadronic correlations in a deconfined medium?

Several tasks and homeworks + 2 in-person meetings + one year of work for the 5 conveners

EMMI-RRTF Context and warnings

Follow up action of the OHF EMMI RRTF (2017)

Took place in 2019 + 2022 => comparisons from models “frozen” late 2022 => not the most recent developments (refer to other talks at this workshop, at QM 2023, or at upcoming SQM 2024)

Not all approaches could be included in the comparison… and will therefore not be covered in this talk (sorry for this)

Web links:

2019 : https://indico.gsi.de/event/9314/

2022 : https://indico.gsi.de/event/15946/

The list of models participating in the EMMI RRTF action

Illustration of freshly born transport models getting ready to compete with the older ones

The list of models participating in the EMMI RRTF action

Comover interaction (Santiago)

TAMU

Tsinghua

→ Transport models (AKA kinetic rate equations) for bound states

PHSD

Duke-MIT

Saclay

→ Transport models derived / inspired from open quantum system principles

Munich-KSU (see talk on Monday)

Nantes - Saclay

→ Linblad equation / quantum jump algorithm

Disclaimer : just meant to be a broad classification, not a hierarchy !

CNM
SHM

→ See presentationS of Anton Andronic

ID card of each model

In-medium "bound states"

In-medium potential
T-dependence of the HQ mass

Binding energies

QGP properties

Assumptions about the medium (degrees of freedom, etc.):

Ingredients of quarkonium – QGP interactions

-Nature of the coupling
-Type of transport

Reactions rates

Equilibrium limit ?

Range of applicability (according to the authors)

Quantum features

Constraints from lQCD

T-range / regeneration / …

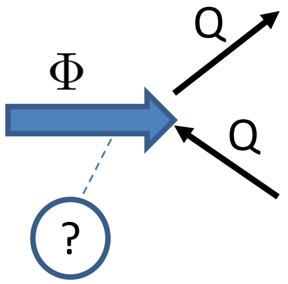
… (CNM / coupling to HF / pA / experimental span / …)

Comover interaction (Santiago): E. Ferreiro, JP. Lansberg

In-medium "bound states"

Not relevant, the model does not rely on individual HQ

Binding energies



Ingredients of quarkonium – QGP interactions

Boltzmann eqs. With tuned geometrical cross sections

$$\Gamma^{\mathrm{Q}}(T)=\int_{E^{\mathrm{Q}}_{\mathrm{thr}}}^{\infty} dE^{\mathrm{co}}\,\sigma^{\mathrm{Q}}_{\mathrm{geo}}\left(1-\frac{E^{\mathrm{Q}}_{\mathrm{thr}}}{E^{\mathrm{co}}}\right)^{n}\frac{\rho^{\mathrm{co}}}{e^{E^{\mathrm{co}}/T_{\mathrm{eff}}}-1}$$

QGP/medium properties

Agnostic for dof !
Teff≃ 200-300 MeV control the energy dependence

Equilibrium limit ?

As a limit of the Boltzmann equation

Quantum features

Not addressed

Constraints from lQCD

None

Range of applicability (according to the authors)

Correctly describes the quarkonium production in both proton-nucleus and nucleus-nucleus collisions at SPS, RHIC and LHC energies without the need to invoke any other phenomena.
Regeneration: Yes

TAMU : R. Rapp, M. He, X. Du,…

In-medium "bound states"

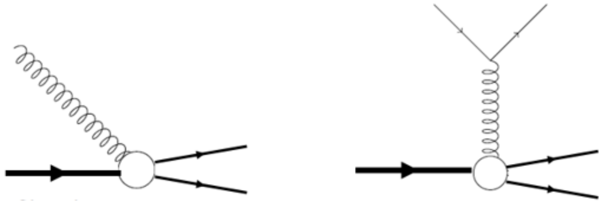
T-dep HQ masses and V – taken as the internal energy -- obtained from a thermodynamic T-matrix approach

QGP/medium properties

Massive quarks and gluons + hadronic phase for charmonia
Uniform fireball+ thermal blast-wave spectra for regeneration

Ingredients of quarkonium – QGP interactions

kinetic eqs. with inelastic reaction rate Γ + recombination; Γ dominated by "quasifree" dissociation processes, computed using perturbative diagrams with an effective but universal coupling constant αs ; interference term for bottomia



Gluo-dissociation + quasi-free el. Scattering cross sections

$$\frac{dN_{\mathcal{Q}}(\tau)}{d\tau} = -\Gamma_{\mathcal{Q}}(T(\tau))\left[N_{\mathcal{Q}}(\tau) - N_{\mathcal{Q}}^{\mathrm{eq}}(T(\tau))\right]$$

Equilibrium limit ?

N^{eq} Enforced through the rate equation; computed from relative chemical equilibrium with fugacity factor

Quantum features

Linear reduction of the reaction rate until Formation time

Constraints from lQCD

Several quantities compared with the T matrix approach : internal energy, quarkonium spectral functions, charm-quark susceptibilities

Range of applicability (according to the authors)

Both (LO) gluo-dissociation and (NLO) quasifree dissociation have been computed … this follows the expected applicability for temperature ranges, $E_B \gg T$ and $E_B \lesssim T$, respectively,…
Regeneration: Yes

Tsinghua collaboration : P. Zhuang, B. Chen, J. Zhao,…

In-medium "bound states"

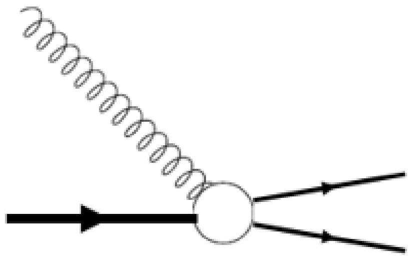
Screening of the potential through eff. constant in-medium binding E for U potential;
no Temp effect on HQ mass

QGP/medium properties

treated as an ideal gas consisting of massless u/d quarks.
MUSIC hydro

Ingredients of quarkonium – QGP interactions

Relativistic Boltzmann equation in Milne coordinates; dissociation due to gluo-dissociation (Peskin and Bhanot + geom scaling for the excited states)



Gluo-dissociation

$$\left[\cosh(y-\eta)\frac{\partial}{\partial\tau}+\frac{\sinh(y-\eta)}{\tau}\frac{\partial}{\partial\eta}+\mathbf{v}_T\cdot\nabla_T\right]f_\psi=-\alpha f_\psi+\beta$$

Equilibrium limit ?

Naturally through detailed balance

Quantum features

None

Constraints from lQCD

Internal energy used as a potential

Range of applicability (according to the authors)

Formation time of quarkonia not considered => mostly applies in the quantum optical limit. The charm quarks phase-space distribution is assumed to be kinetically thermalized
Regeneration: Yes (not for bottomia)

PHSD : T. Song, E. Bratkovskaya, J. Aichelin,…

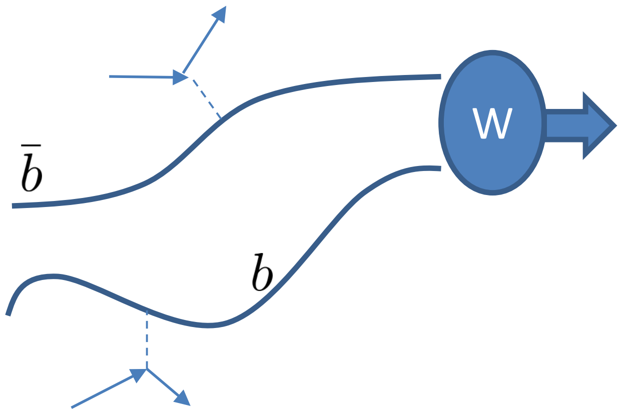
In-medium "bound states"

Screening of the potential through modelled with the free energy

QGP/medium properties

Massive quarks and gluons in the DQPM.

Ingredients of quarkonium – QGP interactions



Semi-classical propagation of a Q-Qbar pair under the individual rescattering with the QGP constituents (reduced rate) ⇔ quasi-free scattering before projection on the quarkonium state (Remler approach).
Reduced rate: dipolar nature of bottomia

Equilibrium limit ?

Remler formalism which the PHSD adopts tested in thermalized and thermalizing boxes

Quantum features

Quantum coherence between several pairs
(at the level of the Wigner density)

Constraints from lQCD

Free energy used as a potential; EOS of DQPM fitted to lQCD; spatial diffusion coefficient of heavy quark described with a factor 2

Range of applicability (according to the authors)

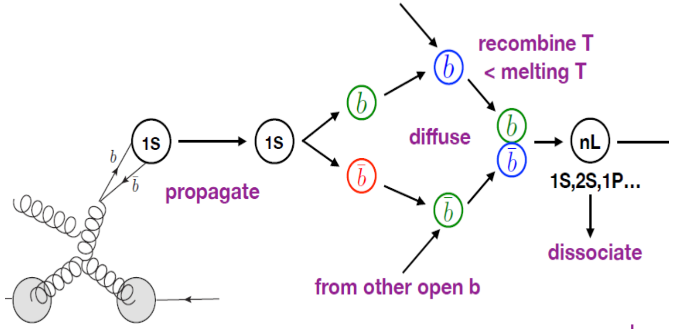
dynamics of heavy quarks is calculated resorting to semi-classical trajectories => needs quantum correction ass low T.
Regeneration: Yes (but not easy to distinguish from the "diagonal" contribution in the Remler approach)

Duke – MIT : X. Yao, T. Mehen, B. Müller,…

In-medium "bound states"

Unscreened potential with T-independent c-quark mass

Binding energies same as in vacuum



QGP/medium properties

Massless quarks and gluons
Medium evolution with VISHNU

Ingredients of quarkonium – QGP interactions

Boltz. eqs. derived by using pNRQCD
Including gluo and inelastic dissociation : dip. transition x chromoelectric distribution fct of the QGP

Equilibrium limit ?

As a limit of the Boltzmann equation

Quantum features

Working in the SC approximation for the recombination; 1rst quantum correction identified

Constraints from lQCD

None at the time of EMMI; Force-force correlator (see Jacopo's talk)

Range of applicability (according to the authors)

Quantum Optical Regime : $1/r \gg E \gg T \gg m_D > \Lambda_{QCD}$
Regeneration: Yes

Saclay : JP. Blaizot & MA. Escobedo (2nd approach)

In-medium "bound states"

2 examples of screened potentials:
Yukawa and Lafferty Rothkopf
T-independent c-quark mass

Binding energies same as in vacuum

QGP/medium properties

HTL
T evolution through Bjorken scenario

Ingredients of quarkonium – QGP interactions

Rate equation coupling the singlet and the octet sectors, (HTL one-gluon exchange approximation) deduced from the QOR with the secular approximation

Equilibrium limit ?

Yes, if one takes the energy gap into account in the reaction rate

$$\frac{\mathrm{d}p_n^s}{\mathrm{d}t} = 4g^2 C_F \sum_m \left(p_m^o - p_n^s e^{-\frac{E_m^o - E_n^s}{T}} \right) \int_q \Gamma^>(E_m^o - E_n^s, q) |\langle n^s | \mathcal{S}_{q\cdot r} | m^o \rangle|^2$$

Quantum features

Derived from OQS approach

Constraints from lQCD

input lattice data on the real part of the potential

Range of applicability (according to the authors)

Quantum Optical Regime : $\Gamma \ll \Delta E$
Regeneration: No "off diagonal" (dilute limit)

Munich-KSU: N. Brambilla, M. Strickland, A. Vairo, MA. Escobedo, P. Vander Griend,…)

In-medium "bound states"

pNRQCD in the strong coupling limit

QGP properties

3+1D dissipative hydrodynamics: relativistic quasiparticle anisotropic hydrodynamics (aHydroQP)

Ingredients of quarkonium – QGP interactions

Linblad equation trough the Quantum jump method (Qtraj) with NLO corrections in E/T

Equilibrium limit ?

ongoing investigation

Quantum feature

Fully quantum

Constraints from lQCD

κ_{adj} (diffusion coefficient in the adj. representation) and γ (mass shift)

Range of applicability (according to the authors)

$1/r \gg T \sim m_D \gg E \gg \Lambda_{QCD}$

No "off diagonal" (dilute limit)

See talk by Tom Magorsch on Monday

Nantes-Saclay: S. Delorme, PB. Gossiaux, T. Gousset,… and JP. Blaizot

In-medium “bound states”

T-dependent Q-mass
1D potential matched from the 3D and Lafferty-Rothkopf complex potential

QGP properties

Agnostic wrt degrees of freedom
T evolution modelled with EPOS4

Ingredients of quarkonium – QGP interactions

Blaizot-Escobedo QME with positivity preserving NNLO correction => Linblad-type equation

Equilibrium limit ?

Possible, but not addressed at the time of EMMI meetings

Quantum features

Fully quantum

Constraints from lQCD

Through the LR complex potential

Range of applicability (according to the authors)

$m_Q >> 1/r \sim T \sim m_D >> E >> \Lambda_{QCD}$

No “off diagonal” (dilute limit)

Results from the quantitative comparison

ID card of each model

In-medium "bound states"

In-medium potential
T-dependence of the HQ mass

Binding energies

QGP properties

Assumptions about the medium (degrees of freedom, etc.):

Ingredients of quarkonium – QGP interactions

-Nature of the coupling
-Type of transport

Reactions rates

Equilibrium limit ?

Range of applicability (according to the authors)

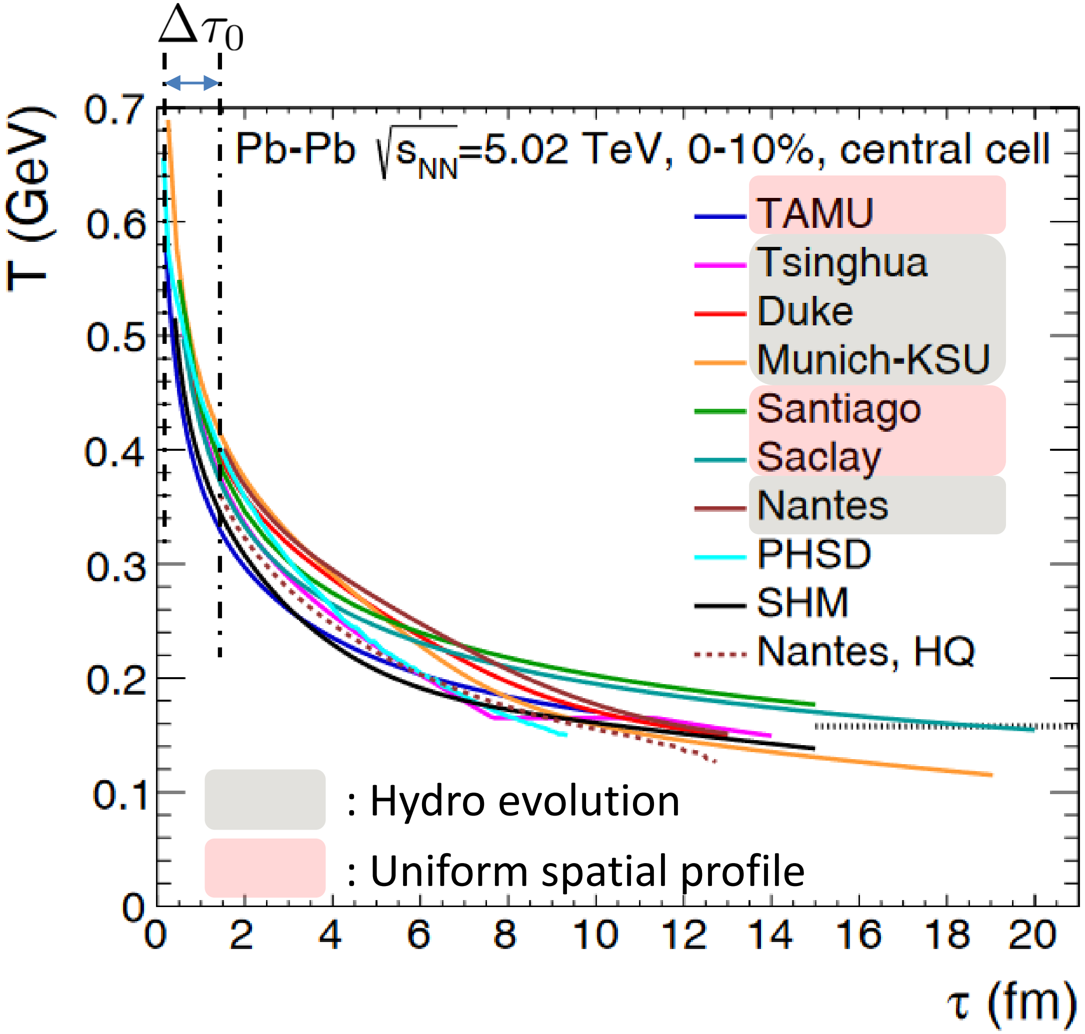
Quantum features

Constraints from lQCD

T-range / regeneration / …

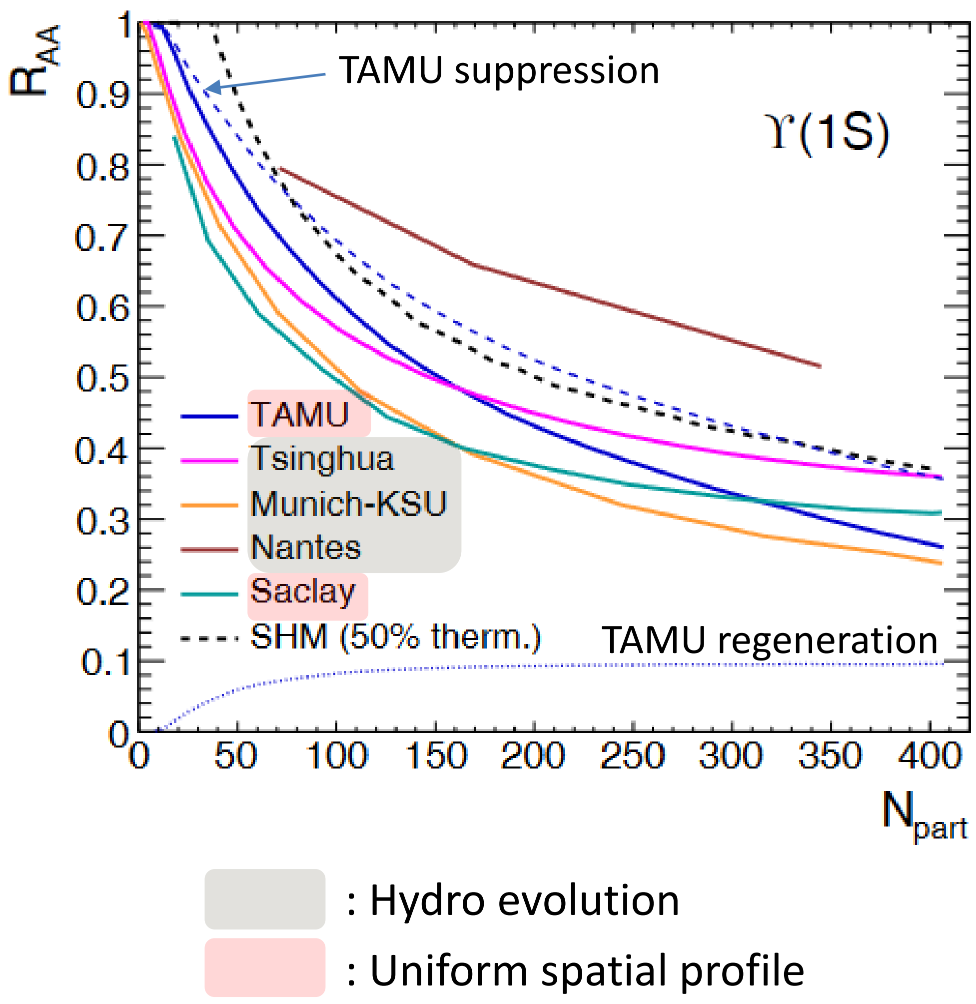
… (CNM / coupling to HF / pA / experimental span / …)

Medium evolution



- Good agreement between the 4 hydro codes, especially btwn Duke, Munich-KSU and Nantes (lQCD based EOS), while MUSIC drops faster.
- For TAMU, Santiago and Saclay, uniform averaged fireball => slightly lower T… As well as the "Nantes HQ" curve (T at the position of the b quarks) : consistent.
- For Santiago and Saclay : Only longitudinal expansion => stay longer above T_{pc}. May have some consequence for the excited states.
- PHSD : Only transport code based on quasi particle => extracting a T is less natural… fast drop after 4 fm/c.
- Large difference btwn the various thermalization times τ_0 : from 0.2 fm/c (Munich-KSU) to 1.5 fm/c (Nantes) !

R_{AA} of N_{part} for a common reaction rate



- Imposed linear increase of Γ_Υ from 0 at T=200 MeV to 0.2 GeV at T=600 MeV.
- Every group implements in its respective evolution model **from τ_0 on** (but no formation time effect, no CNM, no feed down).
- Noticeable outlier : Nantes ; main reason: Large thermalization time => lack of suppression.
- For the other 4 models : in reasonable agreement, roughly ordered by inversely wrt T hierarchy in previous slide.
- Other sources of discrepancy – especially for peripheral collisions – could be the difference in the thermalization time (smallest for Munich-KSU => largest slope around N_{part}=0).

"systematic error" : ≈ 10 %

ID card of each model

In-medium "bound states"

In-medium potential
T-dependence of the HQ mass

Binding energies

QGP properties

Assumptions about the medium (degrees of freedom, etc.):

Ingredients of quarkonium – QGP interactions

-Nature of the coupling
-Type of transport

Reactions rates

Equilibrium limit ?

Range of applicability (according to the authors)

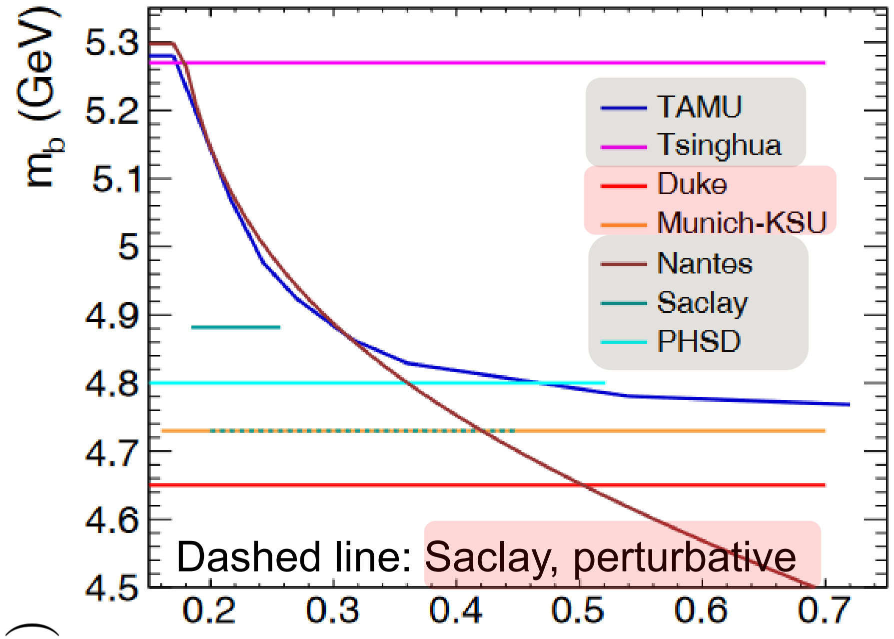
Quantum features

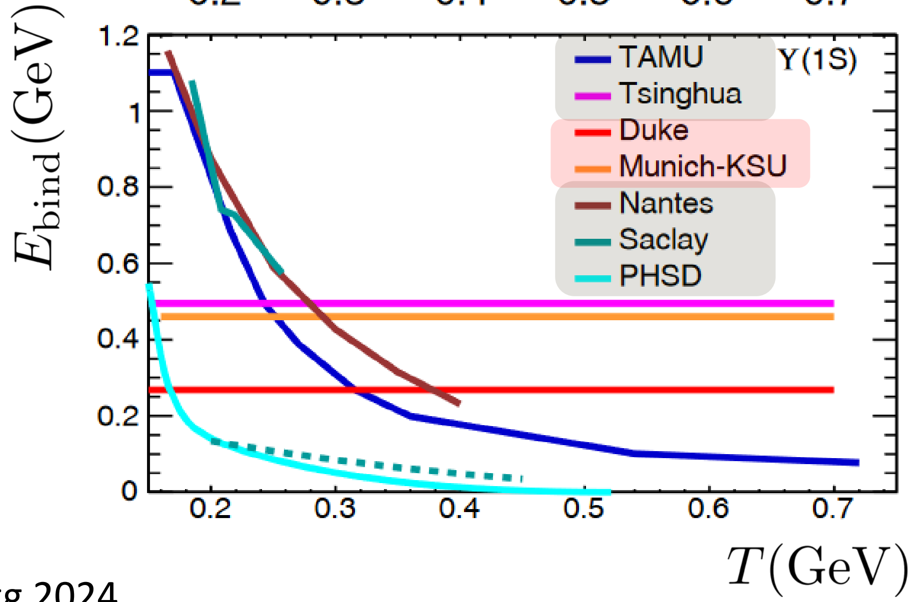
Constraints from lQCD

T-range / regeneration / …

… (CNM / coupling to HF / pA / experimental span / …)

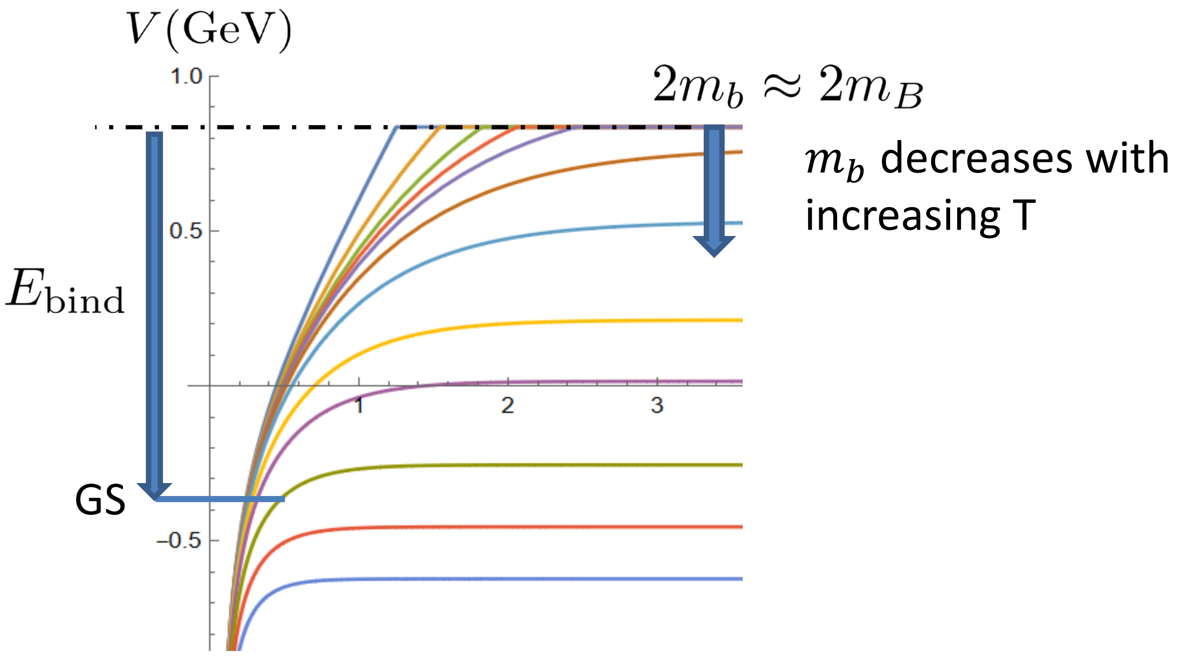
HQ-mass and binding energies





Convention for m_Q and binding energy E_b :

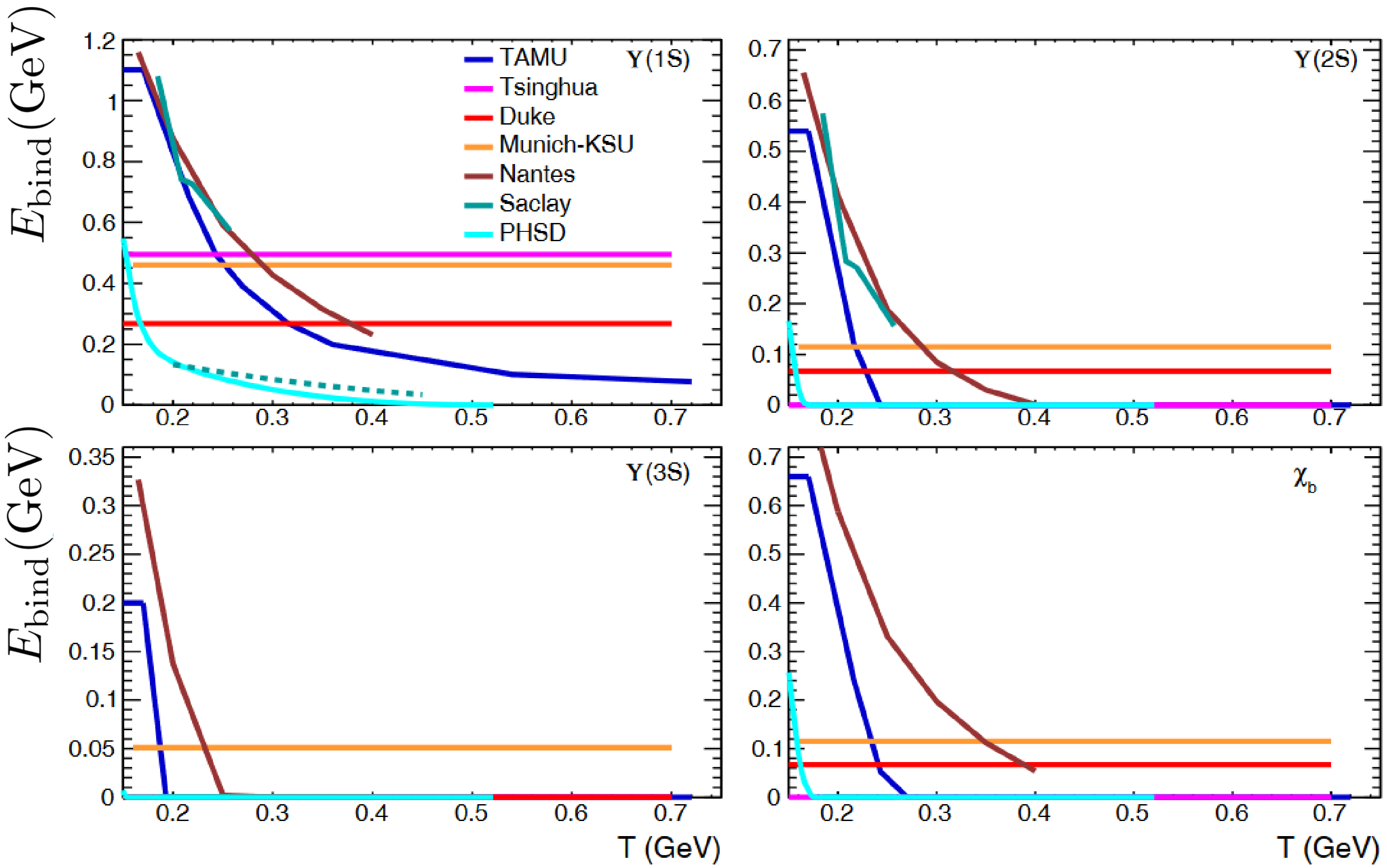
$$m_{Y(1S)} = 2m_b - E_{\rm bind} = 9.46\,{\rm GeV}$$



- Vacuum spectroscopy : Approaches which include long-range forces (TAMU, Tsinghua, Nantes, Saclay –non pert) generate a larger binding energy than approaches relying on Coulomb potential (Duke, Munich-KSU, Saclay pert)
- As the consequence, the m_b in the models vary by +/- 5%

HQ-mass and binding energies

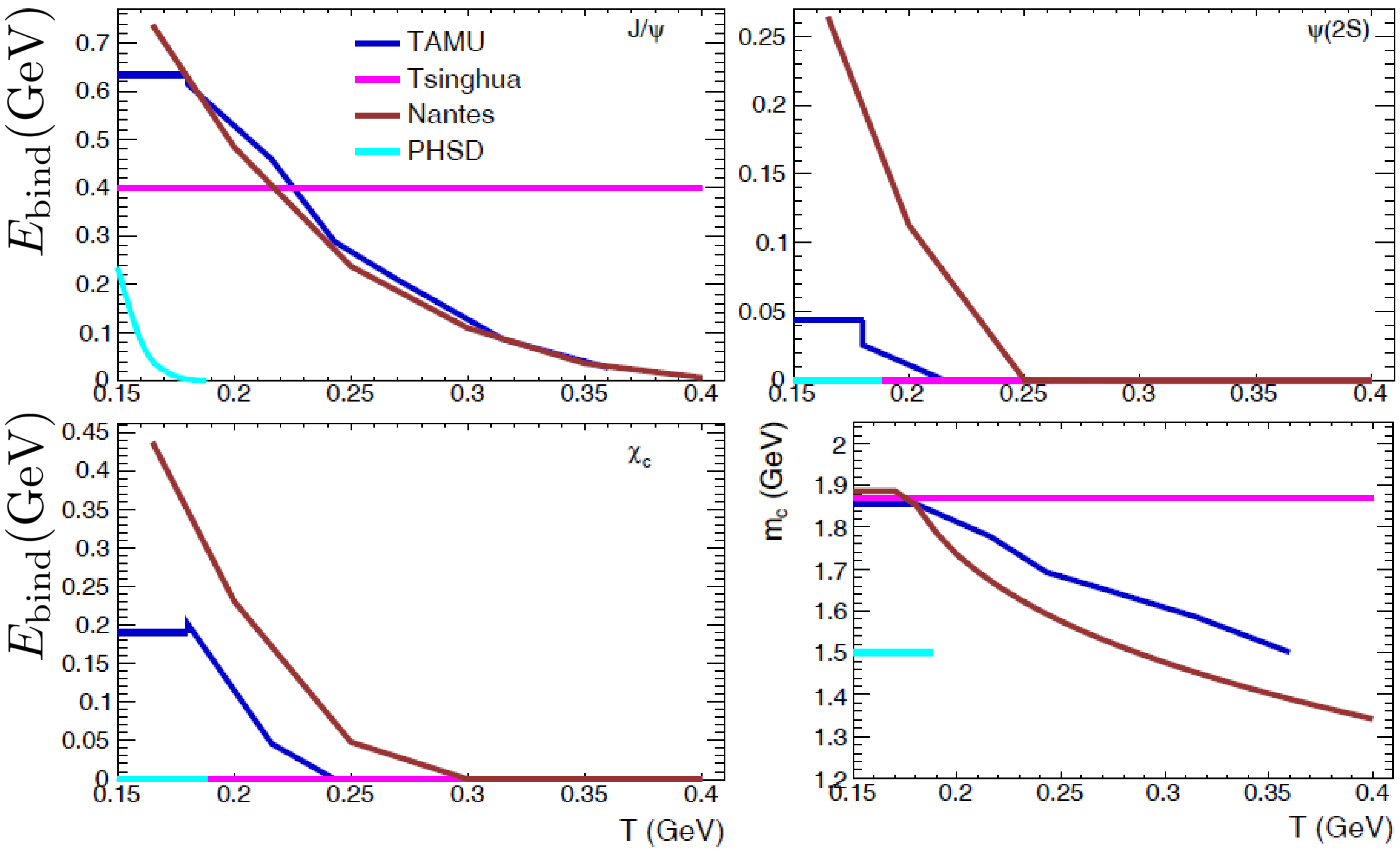
Bottomonia family



- Irrespective of the vacuum spectroscopy, the T-dependences of the binding energies differ quite a lot between models
- 3 groups :
  - TAMU, Nantes, Saclay NP : fast decrease of E_b with T
  - Tsinghua : Average E_b covering the T-range achieved in QGP
  - Duke , Saclay pert, Munich-KSU: weak/no dependence on T (for the 2 first, mainly targeted towards low T QOR).
- !!! E_b is a crucial ingredient for the reaction rates :
  - Acts as a threshold for the QGP spectral function;
  - Governs quarkonia size entering the dipolar transition amplitudes.

HQ-mass and binding energies

Charmonia family (less covered)



- Irrespective of the vacuum spectroscopy, the T-dependences of the binding energies differ quite a lot between models.
- Nantes has too large E_b at small T (not in the range of applicability of the model, but still…).
- !!! E_b is a crucial ingredient for the reaction rates :
  - Acts as a threshold for the QGP spectral function;
  - Governs quarkonia size entering the dipolar transition amplitudes.

ID card of each model

In-medium "bound states"

In-medium potential
T-dependence of the HQ mass

Binding energies

QGP properties

Assumptions about the medium (degrees of freedom, etc.):

Ingredients of quarkonium – QGP interactions

-Nature of the coupling
-Type of transport

Reactions rates

Equilibrium limit ?

Range of applicability (according to the authors)

Quantum features

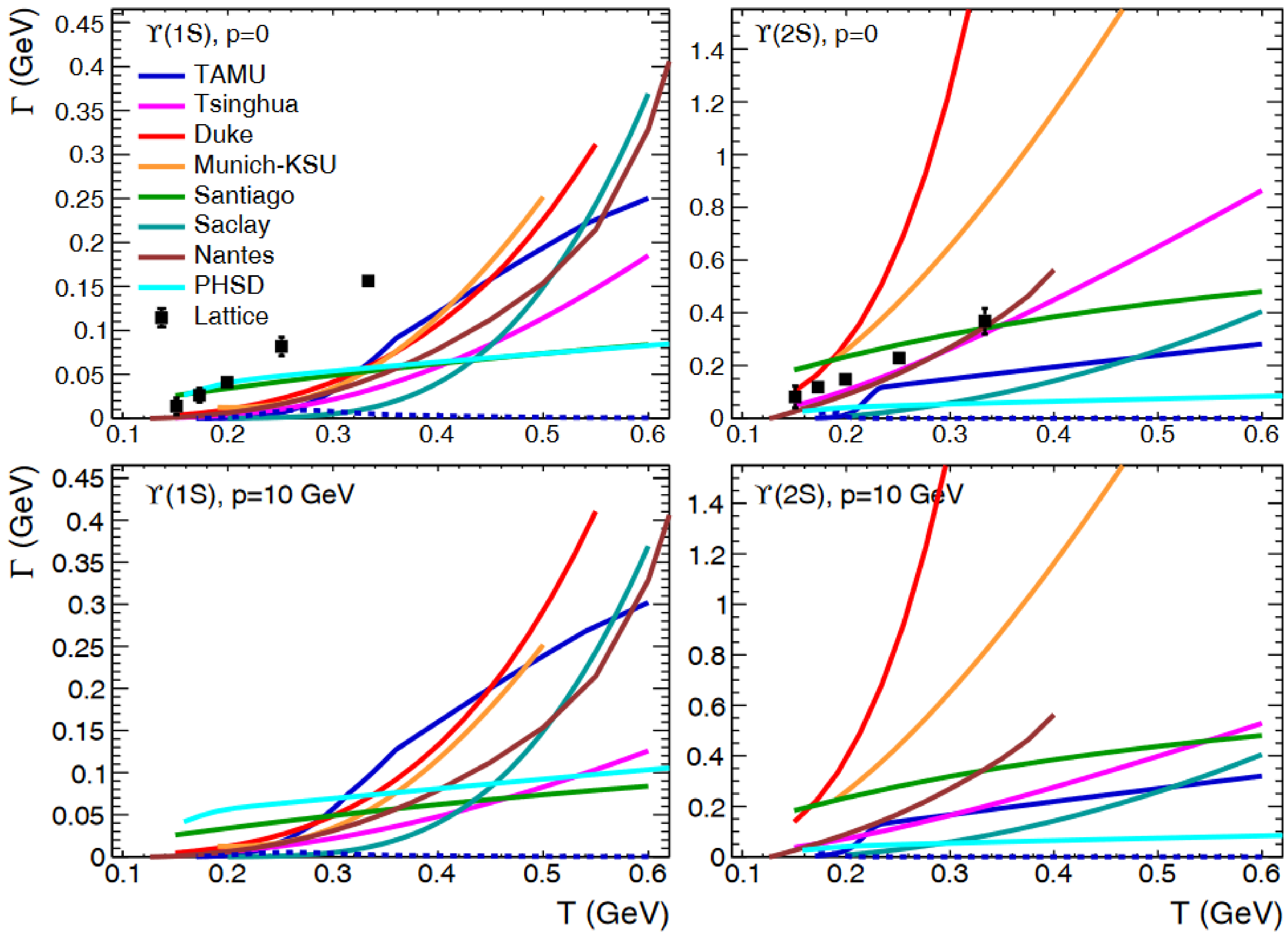
Constraints from lQCD

T-range / regeneration / …

… (CNM / coupling to HF / pA / experimental span / …)

Reaction rates of in-medium bound-states

Bottomonia family

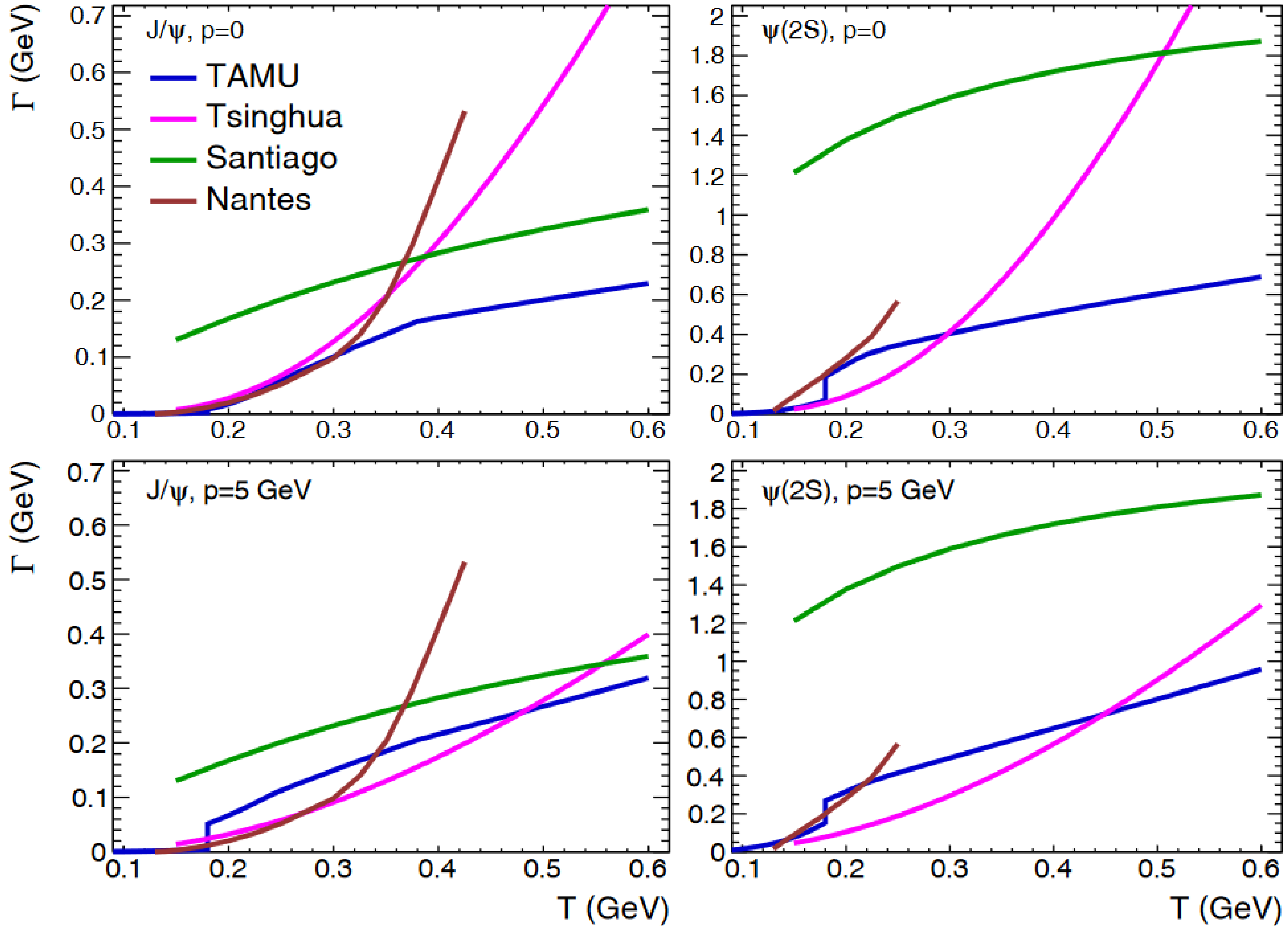


- Large overall spread for both values of p considered; larger for Y(2S).
- Different T-dependences. Models less based on microscopic modelling of Y (Santiago, PHSD) have the most “flatish” one.
- For Y(1S) : some convergence in the T-range [0.3;0.4] (most relevant interval), apart from Saclay (advocated as due to the E_b-dependent Γ, but other models include such effect as well) but with different mechanisms.
- Hints of common hierarchy with $E_b(T)$, but some exceptions (f.i. TAMU, which has the largest T-dep. of E_b does not has the fastest increase) => other effects.
- Duke and KSU-Munich (pNRQCD) in good agreement for Y(1S).

Insufficient constraints !

Reaction rates of in-medium bound-states

Charmonia family (less addressed)

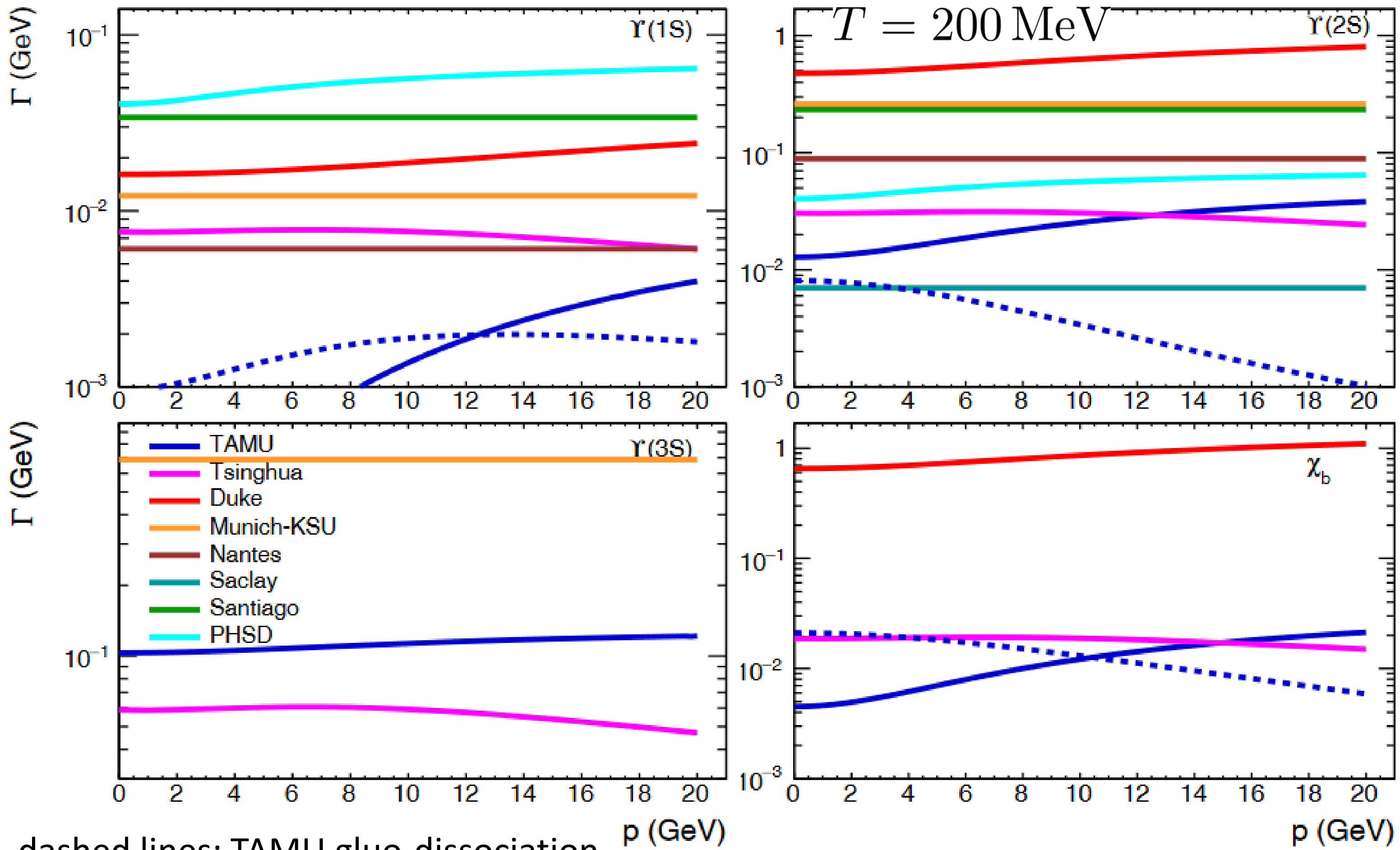


- Same trends as for the bottomonia; larger spread for ψ(2S).
- 3 “microscopic” models (Tsinghua, TAMU & Nantes) in good agreement for the J/ψ up to 350 MeV.
- Some overall reasonable agreement btwn Nantes and TAMU (similar ingredients) up to 350 MeV.
- For T > 350 MeV, largest increase by Nantes, up to the state melting, while other models prolongate beyond melting temperature (in TAMU, charmonia considered as a $c\bar{c}$ pair with quasi elastic scattering).
- Fast increase of Tsinghua as well due to the gluo-dissociation mechanism (peaked near E_b).
- Finite momentum => rate *increase* for TAMU while rate *decrease* for Tsinghua.

Momentum dependence of the reaction rates

Pivotal information to model the p_T spectrum of quarkonia, through dissociation but also for regeneration (detailed balance)

Bottomonia family

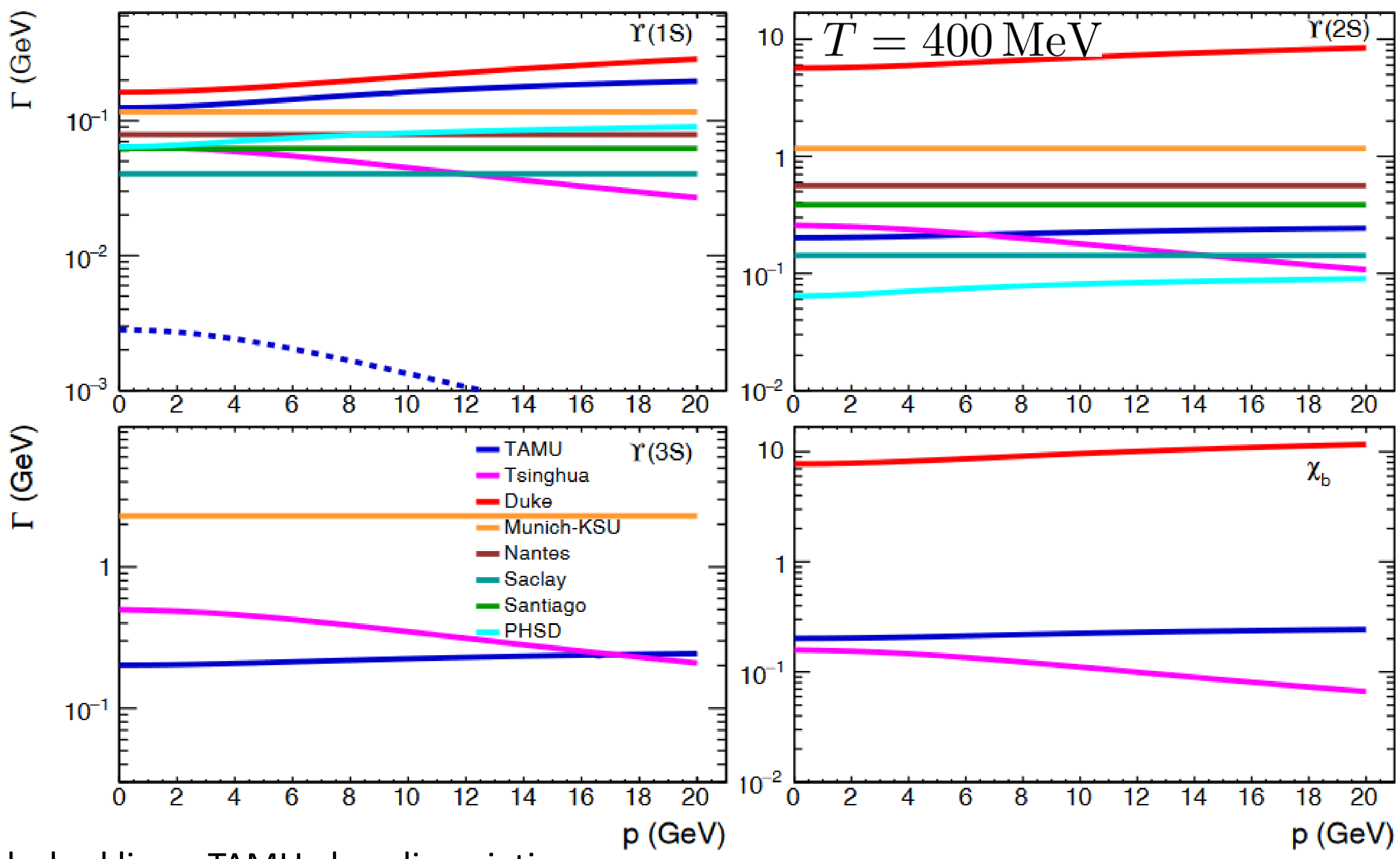


dashed lines: TAMU gluo-dissociation

- ϒ(1S) : Large spread, mitigated by the overall smallness of the rates.
- Hierarchy according to (inverse) binding energy.
- Nantes, Munich-KSU and comovers: no momentum dependence up to now.
- Increase with momentum for PHSD, Duke and even more for TAMU (opening up of reactions with available phase-space).
- … while slight reduction with finite momentum for Tsinghua due to the gluo-dissociation mechanism (similar to gluo-dissociation in TAMU).
- Same spread and trends for excited states.

Momentum dependence of the reaction rates

Bottomonia family



dashed lines: TAMU gluo-dissociation

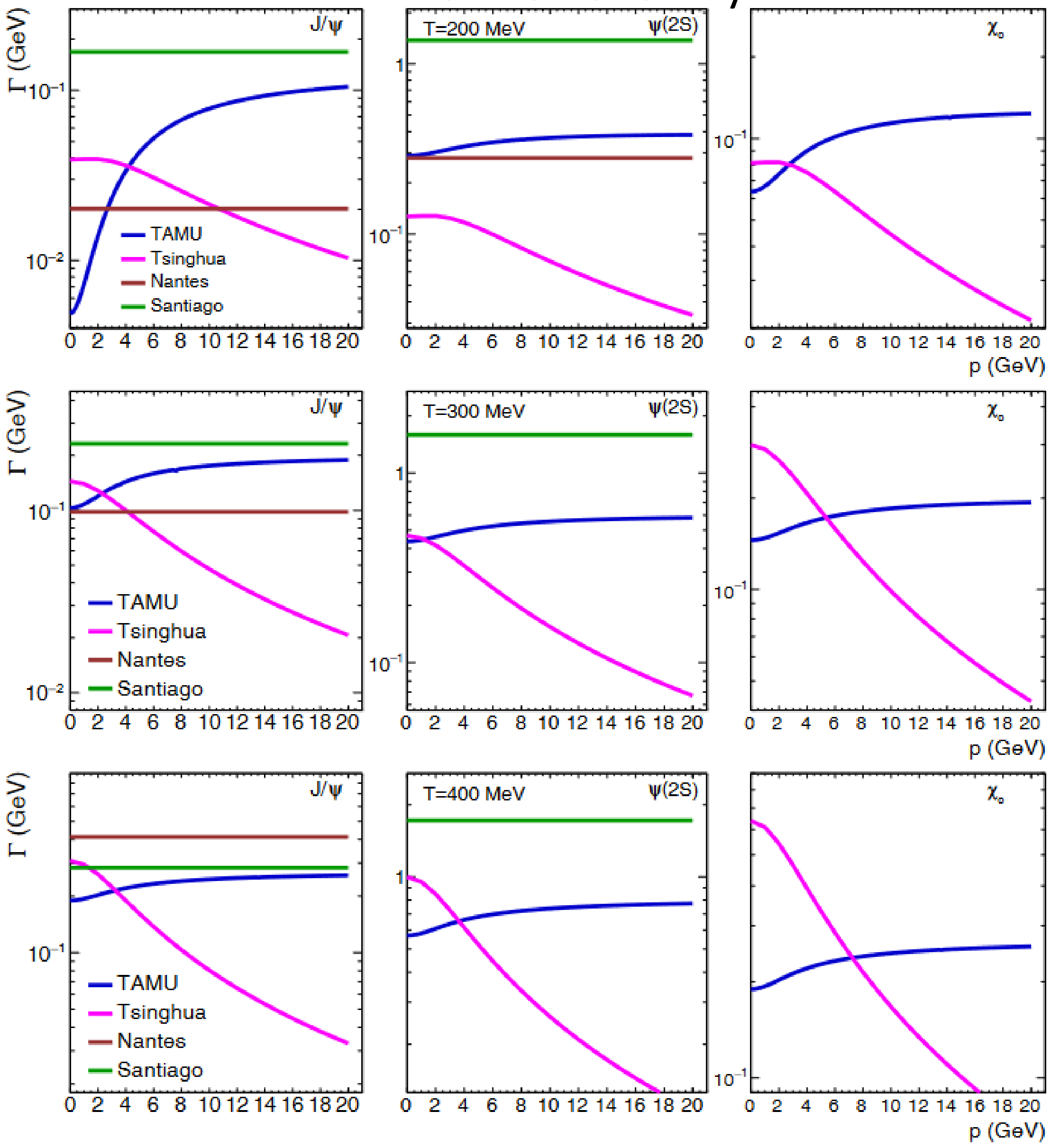
- Υ(1S) : reduced spread, as compared btwn the models as compared to T=200 MeV.
- Hierarchy according to (inverse) binding energy less clear for high temperature (being in the QBM regime).
- Rate probably underestimated for TAMU (perturbative diagrams used for the quasi-inelastic scattering).
- better addressed by pNRQCD due to the dipolar imaginary potential

$$W \simeq -\kappa r^2$$

  where k contains NP physics (see Jacopo's talk), although the small r regime may be questionable.
- Duke probably out of its range of applicability.

Momentum dependence of the reaction rates

Charmonia family



- Same analysis as the for the bottomonia.
- For TAMU : larger increase in the case of J/ψ at T=200 MeV (strongest effect of the binding energy).
- "Inversion" between TAMU and Tsinghua from low to high p_T even more pronounced than for the bottomonia family.

Insufficient constraints !

ID card of each model

In-medium "bound states"

In-medium potential
T-dependence of the HQ mass

Binding energies

QGP properties

Assumptions about the medium (degrees of freedom, etc.):

Ingredients of quarkonium – QGP interactions

-Nature of the coupling
-Type of transport

Reactions rates

Equilibrium limit ?

Quantum features

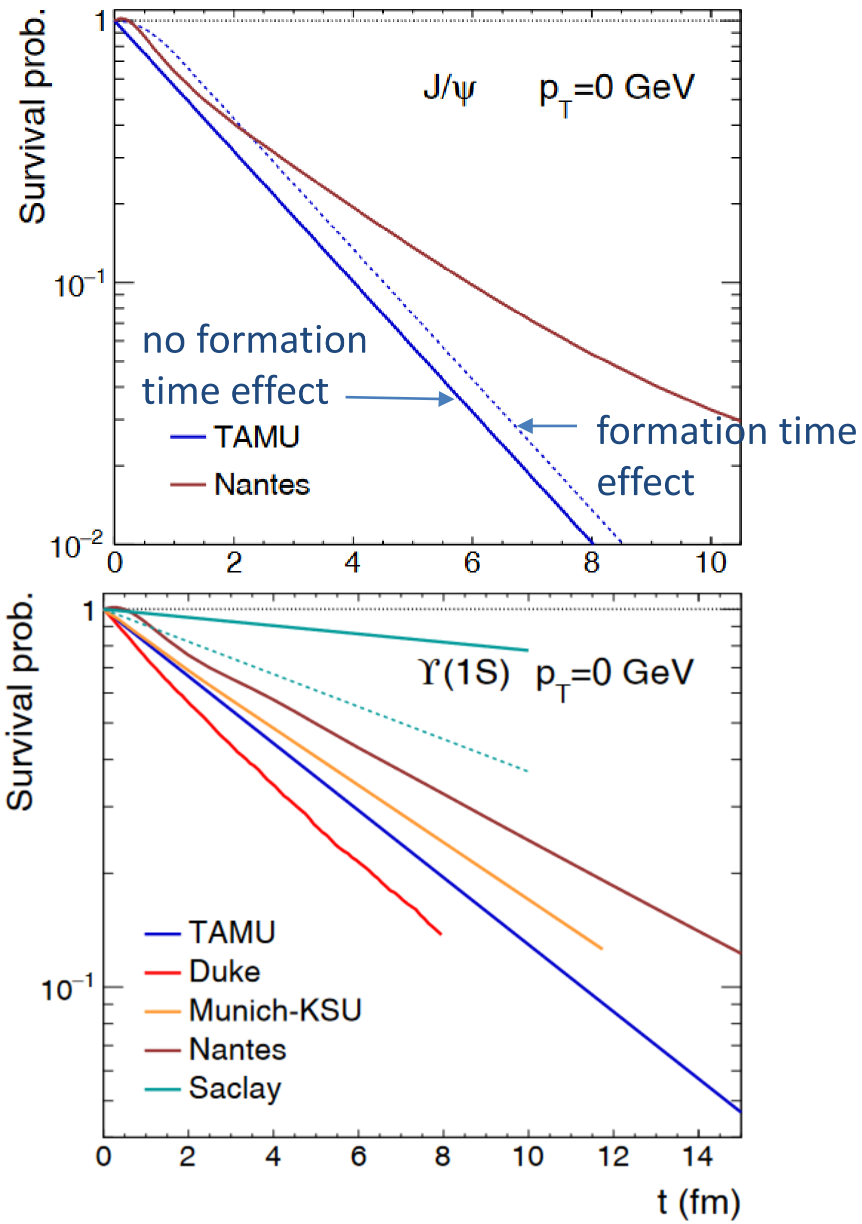
Constraints from lQCD

Range of applicability (according to the authors)

T-range / regeneration / …

… (CNM / coupling to HF / pA / experimental span / …)

Quarkonium Formation Time Effects and Quantum interference



- Task :
  - Start from ”realistic” initial Q-Qbar state (the one used in the respective dynamical model, usually reported as a ”**point-like initial state” in the O**QS and the ground state in semi-classical approaches).
  - Evolve in a QGP at fixed temperature T=300 MeV, neglecting regeneration.
- For J/ψ :
  - Semi-classical formation-time is compatible with the Nantes evolution (OQS) up to 2 fm/c. No solid conclusion can be reached for later time as no regeneration was included in TAMU while it cannot be removed from Nantes QME.
- For Υ(1S):
  - Duke, TAMU and Saclay start from a (in-medium) 1S state and observe an **exponential suppression** ruled by their decay rate
  - Munich-KSU starts from a compact state and observe a decay rate twice the imaginary part of the lowest eigenstate of the non-hermitian H_{eff}
  - Nantes starts from a compact state and observes a transient stage lasting for ≈ 0.5 fm/c, then decays exponentially.

OQS evolutions and SC evolutions seem similar for the 1S decay… to be pursued.

H

Conclusions: Back to the 5 key questions addressed in the EMMI RRTF

1) To what extent are the currently employed ***transport approaches*** *(mostly carried out in semi-classical approximations)* ***consistent*** *in their treatment of quarkonium dissociation and regeneration ?*

a) The various approaches rely on different hypothesis and employ rather different inputs as for the **crucial link between the in-medium binding energy and the reaction rate.**

b) All Semi-Classical approaches include regeneration in a way that accounts for regeneration and **multiple heavy-quark** pairs…

c) … while the current quantum-transport approaches, mainly mostly focus on bottomonia, only a single pair, and then **“diagonal” regeneration** is included.

d) Some models are sometimes used at the borderline of their range of applicability when compared to experimental data (personal opinion).

Conclusions: Back to the 5 key questions addressed in the EMMI RRTF

*2) What are the **equilibrium limits** of the transport approaches and how do the former compare to the results of the statistical hadronization model ?*

a) All Semi-classical approaches (Santiago, Tsinghua, TAMU, Duke PHSD and Saclay) admit **some equilibrium distribution.**

b) However, no explicit comparison to the SHM has been performed ☹ … but expected to be quantitatively different due to different HQ masses and binding energies.

c) Remains a subject of **investigation for quantum approaches** (ask in the discussion session if you wish)

Conclusions: Back to the 5 key questions addressed in the EMMI RRTF

*3) What is the significance of the **effects on quantum transport** of the quarkonium wave packets, and what is needed to develop quantum transport into a realistic phenomenology ?*

a) All the comparisons of suppression factors appear to confirm that these are **mostly relevant in the early stage of the evolution.**

b) **Long-time** behavior of suppression can be characterized by **exponential decays** that correspond to the pertinent reaction rate in semi classical approaches and the lowest eigenvalue in the quantum approaches.

c) … The specific comparison dedicated to quantum features should be extended, including the excited states.

Conclusions: Back to the 5 key questions addressed in the EMMI RRTF

*4) How can the abundant information from **lattice QCD** (quarkonium correlation functions, heavy quark free energies and susceptibilities, and the open heavy-flavor sector) be **systematically implemented into transport approaches** ?*

- Variously implemented, depending on the spread of the models…
  - Either in terms of **directly computed quantities**, i.e., transport coefficients (which, however, are restricted to vanishing 3-momentum); See Jacopo's talk…
  - …Or more **indirectly** by computing lQCD quantities (e.g., free energies or euclidean correlators) within a model approach to constrain its input quantities (like the potential or HQ masses); offers broader phenomenological flexibility as well as microscopic insights
- lQCD constraints should be implemented more systematically in the future

Conclusions: Back to the 5 key questions addressed in the EMMI RRTF

5) What are the ***ultimate model uncertainties****, and will those allow for conclusions on the fundamental question of the existence of hadronic correlations in a deconfined medium?*

Unravel the Q-Qbar interactions under the influence of the surrounding QGP and with the QGP

Develop a scheme able to deal with the evolution of one (or many) $Q\bar{Q}$ pair(s) in a QGP, fulfilling all fundamental principles (quantum features, gauge invariance, equilibration,…)

a) The EMMI RRTF has allowed to identify the numerous assumptions and implementations, which can be considered as the uncertainties affecting the field, on both the fundamental interactions AND the transport treatment.

b) Roadmap for improvement:
   - my personal opinion : DO NOT START with APPROXIMATE TREATMENTS. Approaches which are the most deeply rooted to QCD and to exact transport treatment should be appreciated and recognized (also by the experimental community).
   - more systematic implementation of lQCD constraints on the input quantities (such as the in-medium potential) on an equal footing across model approaches is desirable.
   - Then (or in //) : comparison of semiclassical to quantum transport approaches with the same microscopic input.

Overall Conclusions from EMMI RRTF on Quarkonia transport

Illustration of a crashed transport model practitionner

Good piece of work achieved, we'll do even better next time