Elastic nucleon-pion scattering at $m_\pi = 200$ MeV from lattice QCD

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Summary

- $N\pi$ scattering
  - results
  - GEVP operator analysis
- $a_0(980)$, κ scattering results (if time)

Special thanks to my collaborators:

André Walker-Loud Danny Darvish
Amy Nicholson Pavlos Vranas
Fernando Romero-López
Colin Morningstar
Ben Hörz Andrew D. Hanlon John Bulava

These results are written up in

> J. Bulava et al., Lattice QCD studies of the Δ baryon resonance and the $K_0(700)$ and $a_0(980)$ meson resonances: the role of exotic operators in determining the finite-volume spectrum, *Proceedings of Science* doi:10.22323/1.453.0074.

$N\pi$ scattering

Motivations

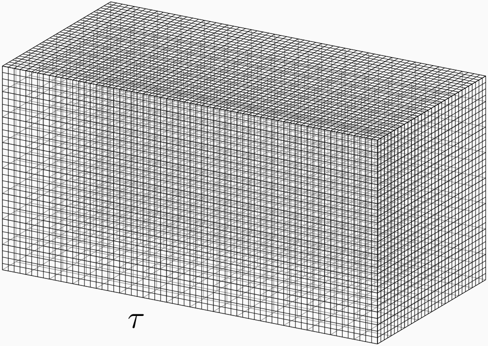
- $N\pi$ scattering
  - need this information for future experiments (DUNE)
  - sets us up for studying $N\pi\pi$ scattering
- at $m_\pi = 200$ MeV
  - study behavior near physical point
  - compare to and verify EFT predictions
- using stochastic LapH correlators
  - show high precision for baryon scattering
  - utilize larger lattice sizes

D200 Computational Details

- CLS Lattice
- Dim ($x^3 \times t$):

$$64^3 \times 128$$

- $a = 0.064fm$
- m_π = 200 MeV
- m_K = 480 MeV
- 2000 configurations
- open temporal boundary conditions
- $N_f = 2 + 1$



Computational Framework

1. Compute lattice configurations of fields

   quarks: $\psi^f, \bar{\psi}^f|_{f=u,d,s}$ gluons: $\mathcal{A}_\mu$

2. Create operators with the make-up and quantum numbers of the particles of interest

   $$\pi^+ = \bar{d}u$$

3. Construct matrices of two-point correlation functions within the channels of interest

   $$\langle 0|\pi\overline{\pi}|0\rangle, \langle 0|[N\pi][\overline{N\pi}]|0\rangle, \langle 0|\mathbf{\Delta}[\overline{N\pi}]|0\rangle...$$

4. Use GEVP and fitting method to extract the steady state energies of the channel

   $$\langle 0|\pi\overline{\pi}|0\rangle = \sum_{n=0}^{\infty} Ae^{-E_n t}$$

5. Fit to those energies using Lüscher formalism to calculate phase shifts and matrix elements

Notes on Operator/Correlator Construction

Operator Notes:

- Gluons $\rightarrow$ Stout smearing
- Quarks $\rightarrow$ LapH smearing

Correlator Notes:

- stochastic factorization $\rightarrow$ tensor contraction
- efficient algorithm $\rightarrow$ produce many different correlators

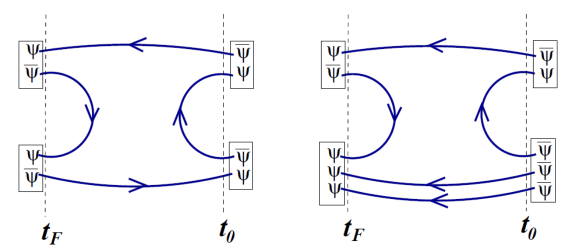


Image: C. Morningstar, J. Bulava, J. Foley, K. J. Juge, D. Lenkner, M. Peardon, and C. H. Wong Phys. Rev. D 83, 114505 – Published 3 June 2011

GEVP

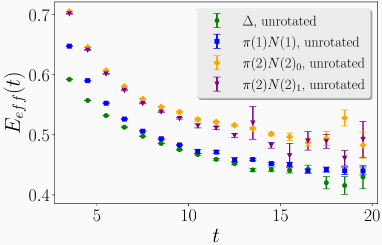
Correlation matrix elements in the same channel share the same FV energy levels

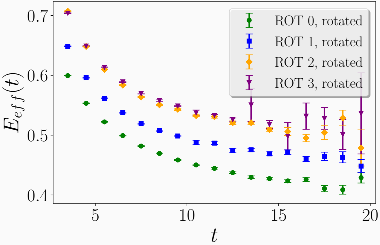
$$\langle 0|\mathcal{O}_i(t+t_0)\overline{\mathcal{O}}_j(t_0)|0\rangle = \sum_{n=0}^{\infty} Z_i^{(n)} Z_j^{(n)} e^{-E_n t}$$

Separate out by solving GEVP of $N \times N$ matrix and eigenvalues are

$$\lim_{t\to\infty} \lambda_n(t) \approx b_n e^{-E_n t}$$

Example ($N\pi$, $I = 3/2$, $H_g(0)$):





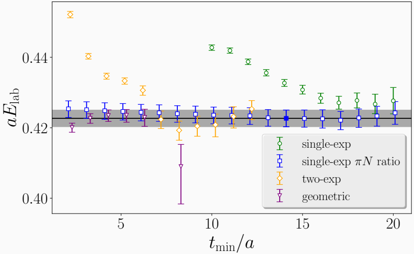
Finite-Volume Energy Spectrum

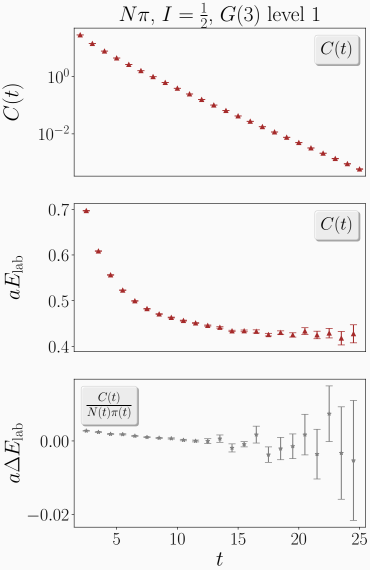
Fitting methods:

- single-exp: Ae^{-Et}
- double-exp: $Ae^{-Et}(1 + Re^{-D^2t})$
- geometric: $Ae^{-Et}/(1 - Re^{-Dt})$

Ratio:

$$R(t) = \frac{\lambda_n(t)}{C_1(t)C_2(t)}$$





Connect finite-volume to infinite-volume via Lüscher:

$$\det[\widetilde{K}^{-1}(E_{\mathrm{cm}}) - B^P(E_{\mathrm{cm}})] = 0$$

- truncate higher waves
- $\widetilde{K}$ - related to the usual scattering K-matrix
- B^P ('box matrix') - finite volume irreps
- only works for 2-2 scattering

NLO

$$\frac{q_{\mathrm{cm}}^3}{m_\pi^3}\cot\delta_{3/2^+} = \frac{6\pi\sqrt{s}}{m_\pi^3 g_{\Delta,\mathrm{BW}}^2}(m_\Delta^2 - s),$$

$J^P = 3/2^+$

Effective range expansion

$$\frac{q_{\mathrm{cm}}^{2\ell+1}}{m_\pi^{2\ell+1}}\cot\delta_{J^P}^I = \frac{\sqrt{s}}{m_\pi A_{J^P}^I},$$

$J^P = 1/2^-,\ 1/2^+,\ 3/2^-,\ 5/2^-$

Delta Resonances

$$N\pi \to N\pi$$

Correlation Matrix Information:

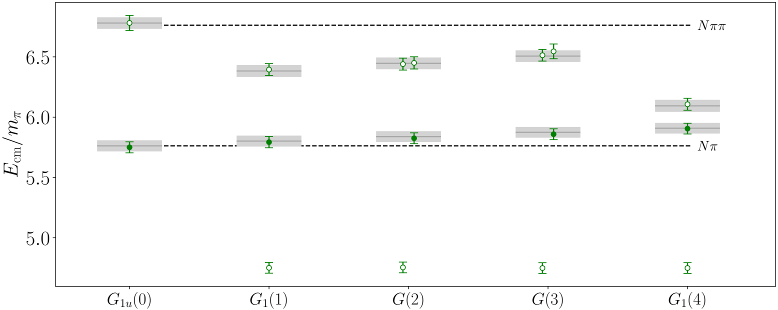
$a_{N\pi}^{I=1/2}$

- $I = 1/2$
- operators:
  - N
  - $N\pi$
- momenta: $\mathrm{d}^2 = 0, 1, 2, 3, 4$

$\boldsymbol{\Delta}(1232)$, $a_{N\pi}^{I=3/2}$

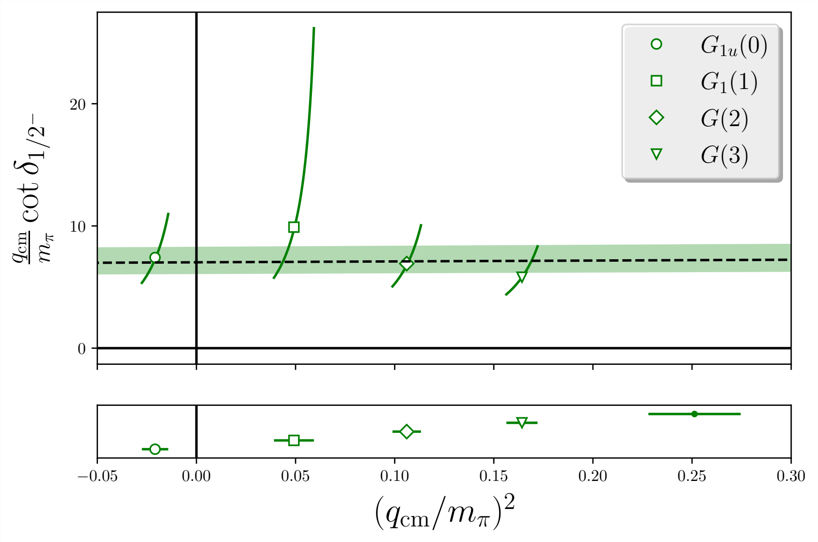
- $I = 3/2$
- operators:
  - $\boldsymbol{\Delta}$
  - $N\pi$
- momenta: $\mathrm{d}^2 = 0, 1, 2, 3, 4$

I=1/2 $N\pi$

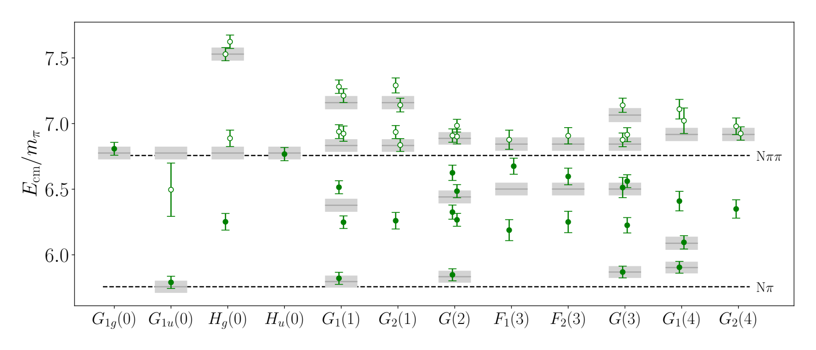


- Grey bands: noninteracting scattering levels (N, π correlators)
- Green dots: interacting levels ($N\pi$, N correlators)
- Filled green dots: levels used for constraining $a_{N\pi}^{I=1/2}$

Phase Shifts - I=1/2 $N\pi$

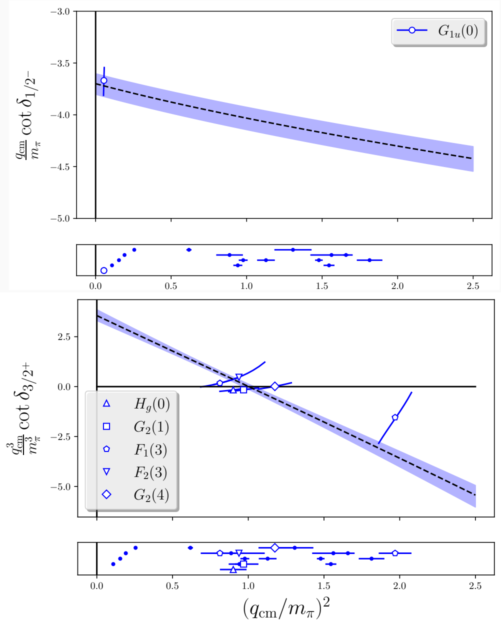


I=3/2 $N\pi$, Δ(1232)

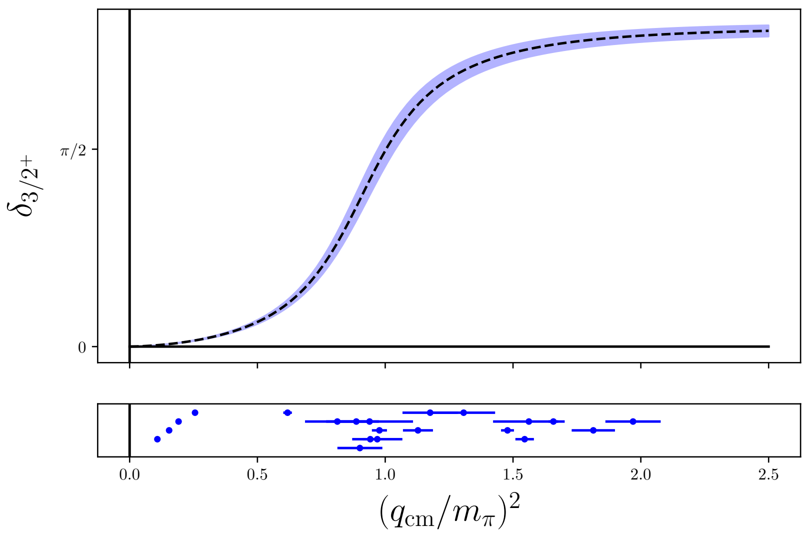


- Grey bands: noninteracting scattering levels (N, π correlators)
- Green dots: interacting levels ($N\pi$, Δ correlators)
- Filled green dots: levels used for calculating $a_{N\pi}^{I=3/2}$

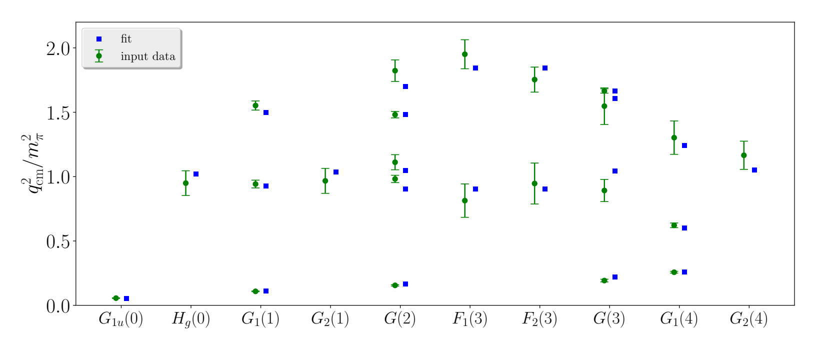
Phase Shifts - I=3/2 $N\pi$



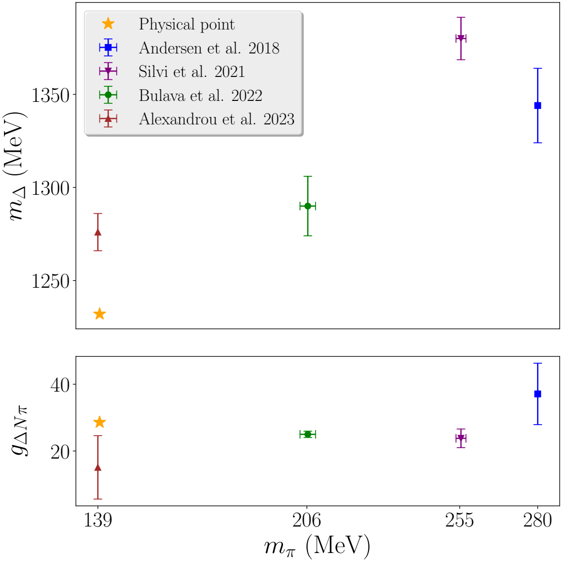
Phase Shifts - I=3/2 $N\pi$



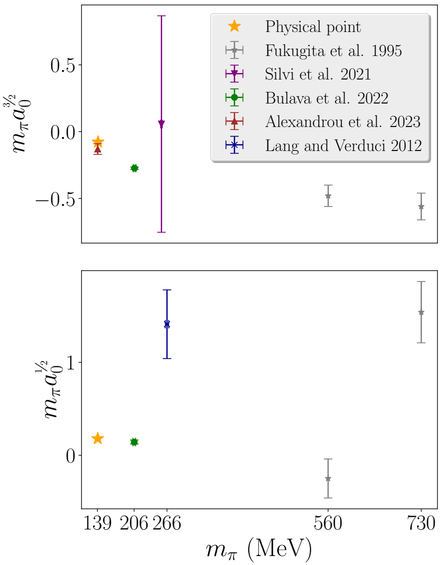
Reconstruct $I = 3/2$ spectrum



Delta Mass


Physical point
Andersen et al. 2018
Silvi et al. 2021
Bulava et al. 2022
Alexandrou et al. 2023
m_Δ (MeV)
1350
1300
1250
$g_{\Delta N\pi}$
40
20
139
206
255
280
m_π (MeV)

Scattering Lengths



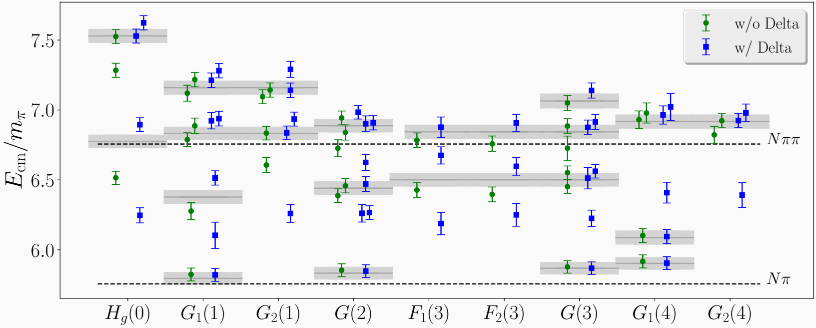
$\boldsymbol{\Delta}$ Operator's Impact

How important was the $\boldsymbol{\Delta}$ operator?

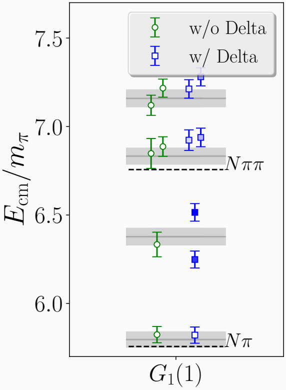
Reminder: used all variations of $N\pi$ and $\boldsymbol{\Delta}$ correlators in the $I = 3/2$ channel.

$$\langle 0|\mathcal{O}_i(t + t_0)\overline{\mathcal{O}}_j(t_0)|0\rangle = \sum_{n=0}^{\infty} Z_i^{(n)} Z_j^{(n)} e^{-E_n t}$$

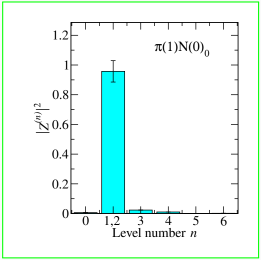
Δ Operator's Impact



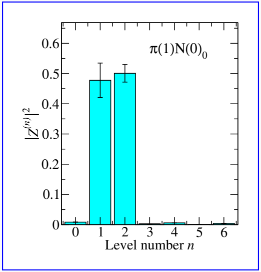
Δ Operator’s Impact

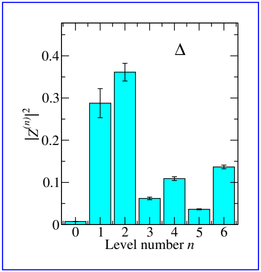


$N\pi$



$N\pi$, **Δ**





Conclusions and Future Work

Conclusions

- Extracted precise scattering information in the Δ channel
- needed Δ operators to extract accurate spectrum
- no evidence of the roper resonance found under the three-particle threshold

Future Work

- more statistics
- multiple-lattice spacing
- cutoff effects
- use 3-particle formalism for the roper channel

$a_0(980)$, κ scattering

Motivation

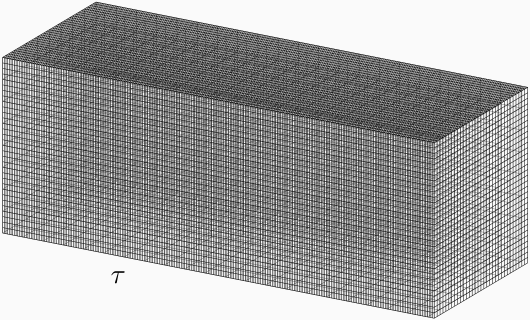
- $a_0(980)$ and κ (or $K_0^*(800)$) do not appear to align with naïve quark model structure
- previous study showed that in $a_0(980)$ channel that tetraquark-type operators produce levels in the two-particle scattering regime -> Alexandrou et al. 1711.09815
- Do we really need to calculate tetraquark operators?

Lattice #2 Computational Details

- Dim ($x^3 \times t$):

  $32^3 \times 256$

- $a_s = 0.11 fm$
- $a_t = 0.033 fm$
- m_π = 230 MeV
- m_K = 490 MeV
- 412 configurations
- periodic temporal boundary conditions
- $N_f = 2 + 1$



Tetraquark operators

Two coupled-channel scattering channels investigated:

$$K\pi, K\eta \to K\pi, K\eta$$

- resonance: κ
- $I = 1/2$
- operators:
  - K
  - $K\pi$
  - $K\eta$ $(\eta = u\bar{u} + d\bar{d})$
  - $K\phi$ $(\phi = s\bar{s})$
  - $\bar{s}u\bar{s}s$
- momentums: $\mathrm{d}^2 = 0$

$$K\bar{K}, \pi\eta \to K\bar{K}, \pi\eta$$

- resonance: $a_0(980)$
- $I = 1$
- operators:
  - π
  - $K\bar{K}$
  - $\pi\eta$ $(\eta = u\bar{u} + d\bar{d})$
  - $\pi\phi$ $(\phi = s\bar{s})$
  - $\bar{u}u\bar{d}u$
- momentums: $\mathrm{d}^2 = 0$

Tetraquark construction

- stout and sLapH smearing (Morningstar)
- particles have definite quantum numbers: isospin, parity, G-parity, octahedral little group
- constituents of tetraquarks do not have definite quantum numbers
- 100+ tetraquark operators investigated (Danny Darvish)

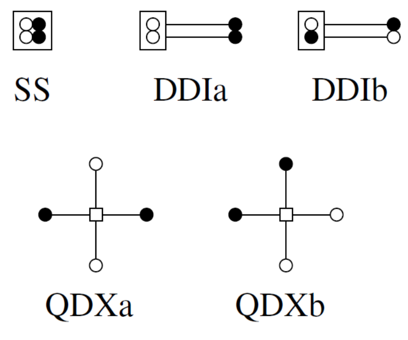
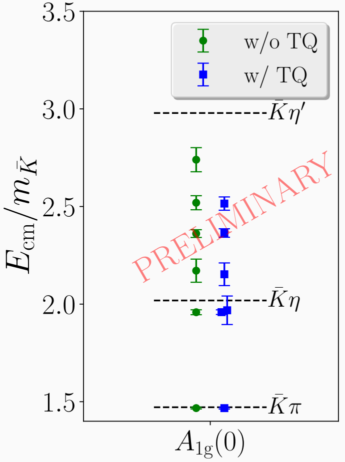


Diagram provided by Colin Morningstar

Meson-Meson Spectrums

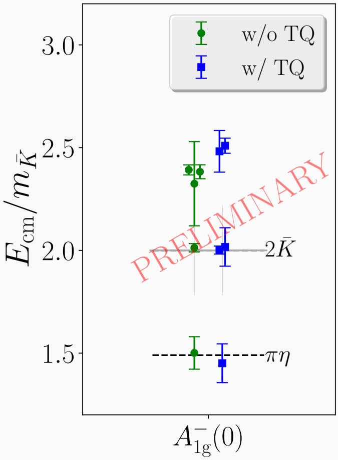
κ channel

TQ = $\bar{s}u\bar{s}s$

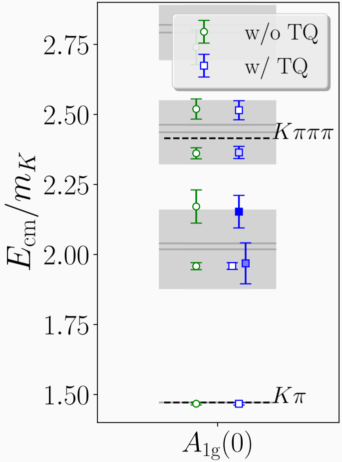


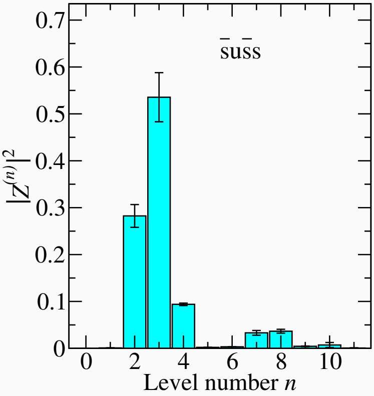
a_0 channel

TQ = $\bar{u}u\bar{d}u$

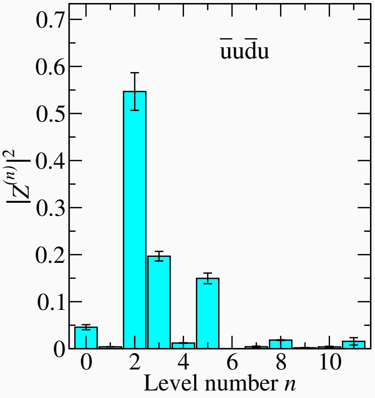


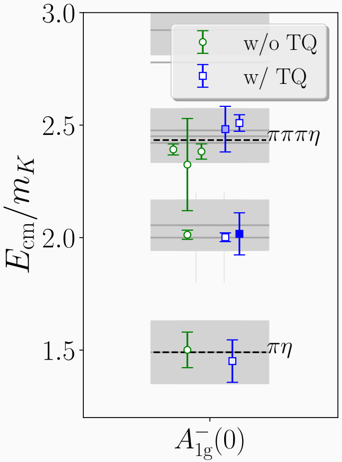
Operator overlaps





Operator overlaps

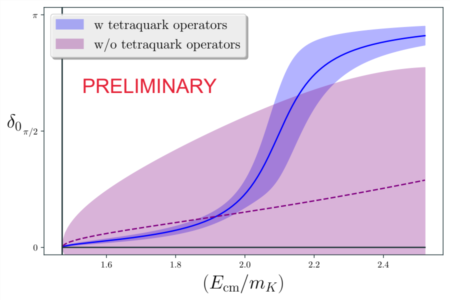




Amplitudes

$K\pi$–$K\eta$ Spectrum (κ channel) - Single channel s-wave

- Without tetraquark $\rightarrow$ no resonance (fit to 5 levels)
- With tetraquark $\rightarrow$ resonance at $\sim 2.1 m_K$ (fit to 5+TQ levels)
- Without tetraquark results similar to Briceño et al. 1904.03188



$K\bar{K}$–$\pi\eta$ Spectrum (a_0 channel) - Single channel s-wave

- Without tetraquark $\rightarrow$ no resonance (fit to 3 levels)
- With tetraquark $\rightarrow$ virtual bound state (fit to 2+TQ levels)

Conclusions and Future Work

Conclusions

- single meson and meson-meson operators are not enough to produce accurate spectrum
- tetraquark operators may be necessary for $a_0(980)$ and κ

Future Work

- increase statistics
- nonzero momentum irreps
- investigate cheaper operators
- develop methods to indicate that there are missing levels in the GEVP