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GIESSEN

Time-like
Baryon Form Factors

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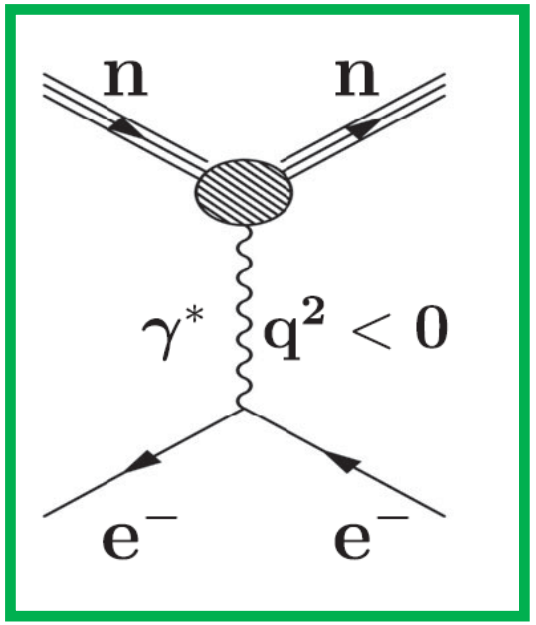
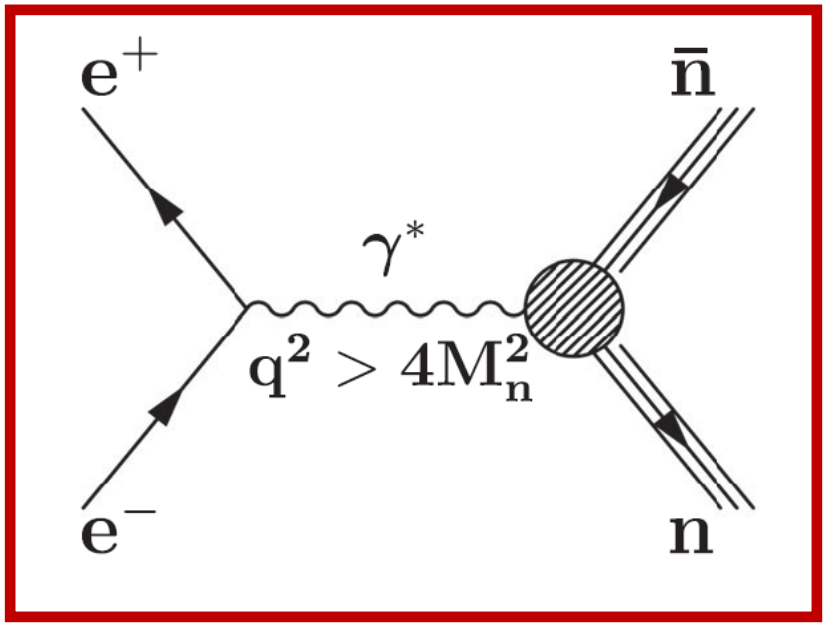
Recap on Baryon Form Factors

Space-like vs.Time-like Form Factors

$q^2=t$



Space-like:Scattering



Time-like: Production

$q^2=s$

Crossing Symmetry:

$e^- + N \rightarrow e'^- + N' \quad \leftrightarrow \quad e^- e^+ \rightarrow N\bar{N}$

Time-like Nucleon-Nucleon Electromagnetic Form Factors (EMFF) from Positron-Electron Annihilation

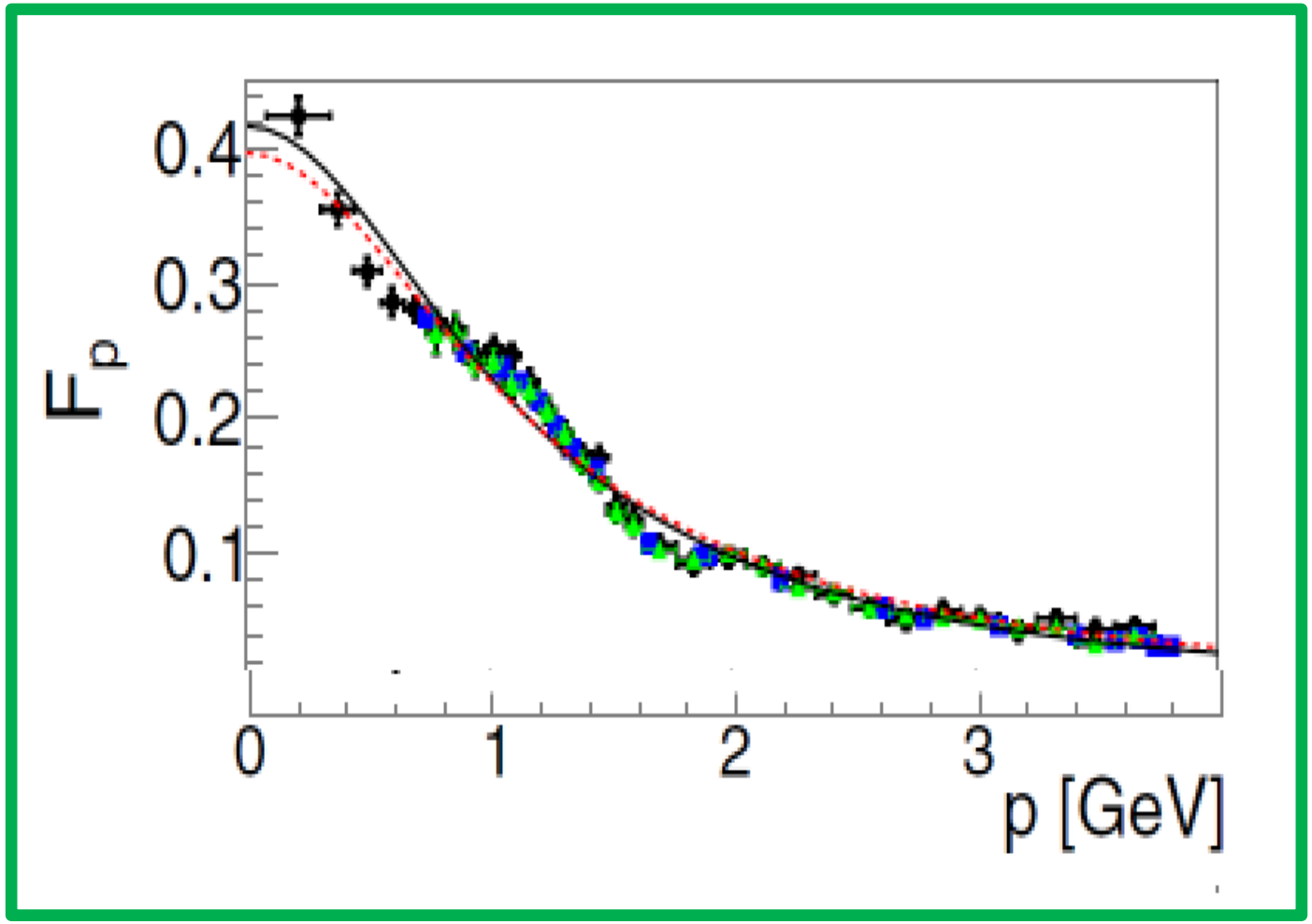
$$\sigma_{e^+e^-\to N\bar{N}} = \frac{4\pi\alpha^2\beta}{3s} C_N(s) \left[\left|G_M^N(q^2)\right|^2 + \frac{2M_N^2}{s} \left|G_E^N(q^2)\right|^2 \right],$$

- Dirac and Pauli Form Factors $F_1(s)$ and $F_2(s)$
- electric $G_E=F_1+\tau F_2$ and magnetic $G_M = F_1+F_2$ form factors ($\tau=s/4m^2$)
- G_E and G_M are complex valued functions of the 4-momentum transfer q^2

…and accordingly for other Octet Baryons, measured e.g. at the FENICE, BaBar, BESIII, and BELLE facilities (and *once upon the time* was planned for PANDA@FAIR).

Time-like Proton EMFF $G_{eff}=|G_p|=F_p$

World Data ~ 2020



PHYS. REV. C **103**, 035203 (2021), E. Tomasi-Gustafsson, A. Bianconi, S. Pacetti

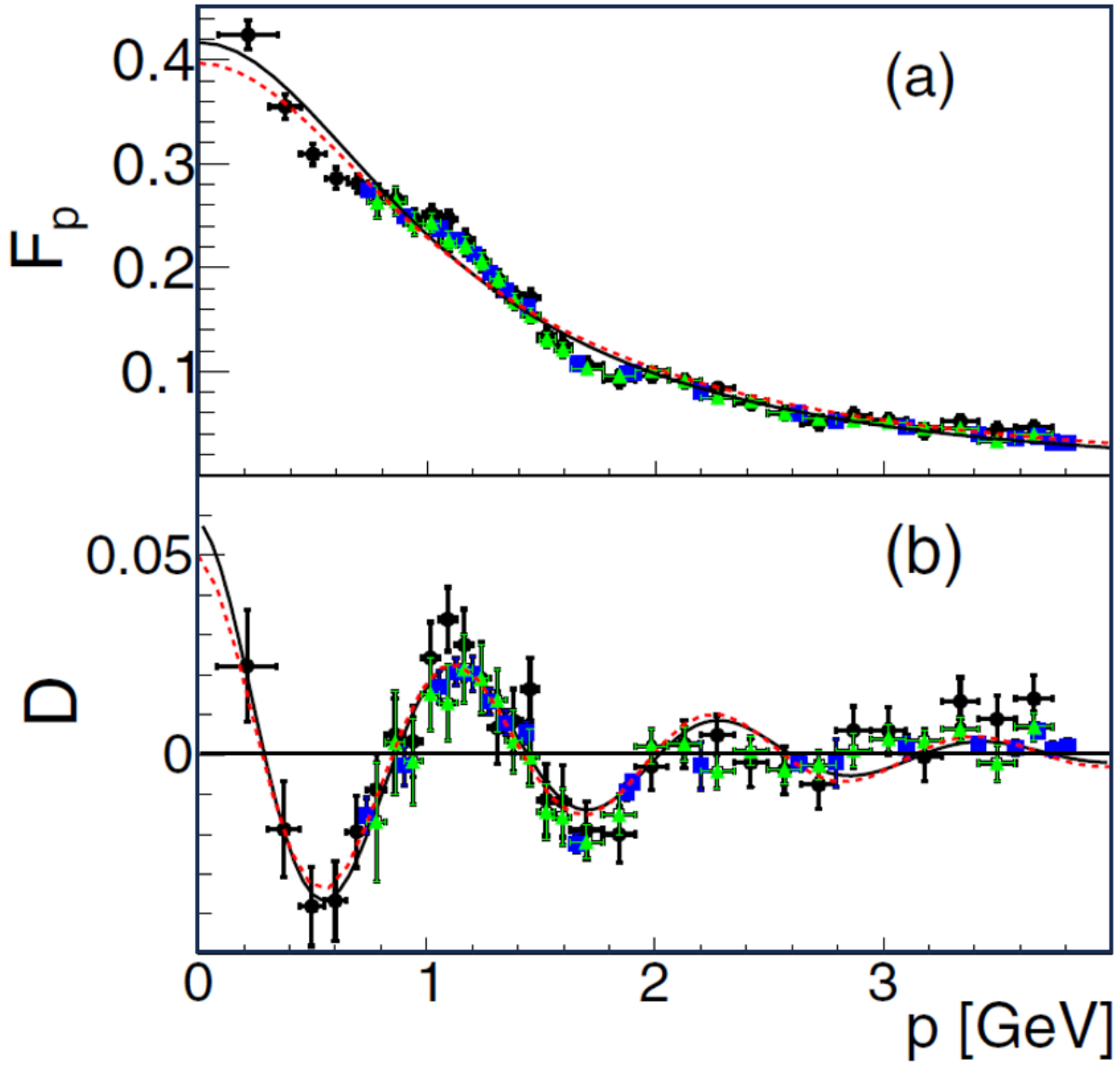
The Oscillation Mystery

Vector-Meson t-channel exchange?
Constituent Quark Model?
QCD/AdS correspondence?
Multi-meson intermediate states?

$$F_p^{\mathrm{fit}}(s) = F_{3p}(s) + F_{\mathrm{osc}}[p(s)].$$

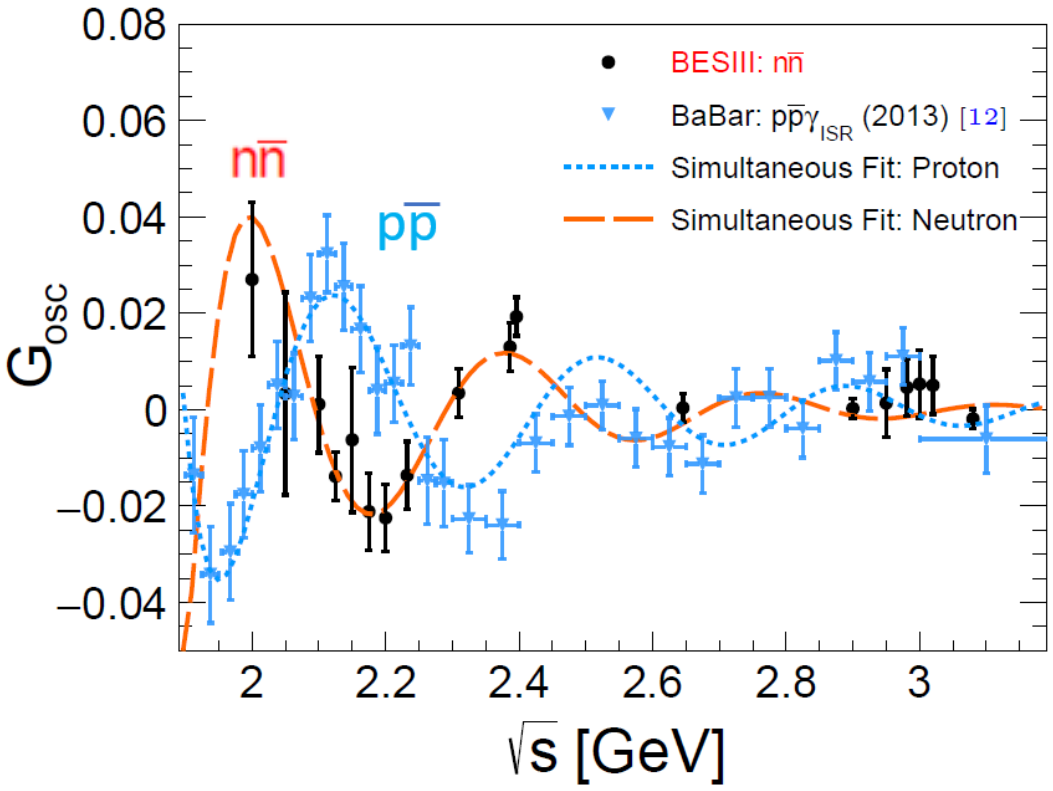
$$F_{3p}(s) = \frac{F_0}{\left(1+\frac{s}{m_a^2}\right)\left(1-\frac{s}{m_0^2}\right)^2},$$

$$F_{\mathrm{osc}}[p(s)] = Ae^{-Bp}\cos(Cp+D)\,.$$



PRL 114, 232301 (2015), PHYS. REV. C **103**, 035203 (2021),
E. Tomasi-Gustafsson, A. Bianconi, S. Pacetti

BESIII $e^+e^- \to n\bar{n}$ Measurements: Oscillatory Structures in Neutron and Proton EMFF



BaBar and BESIII e+e- Data
G_D subtracted
Ablikim et al., Nature Phys. 17 (2021) 1200

$$G_{osc}(q^2) = |G| - G_D$$

$$G_D(q^2) = \frac{\mathcal{A}_n}{\left(1 - \frac{q^2}{0.71\ (\mathrm{GeV}^2)}\right)^2}$$

Agenda

- **SU(2): Nucleon EMFF in Isospin Representation**
- **Oscilllation Pattern from Interfering Isospin Components**
- **Clues on Production Mechanism**
- **SU(3): EMFF of Σ Hyperons**
- **Summary and Outlook**

The Lanzhou-Giessen Approach

Xu Cao[1,2], Jian-Ping Dai[3], and H. Lenske:
- **Phys. Rev. D 105, L071503 (2022)**
- **Phys. Lett. B 846 (2023) 138192**

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Probing SU(2) Symmetry in Nucleon EMFF: Isospin Form Factors of the Nucleon

PHYSICAL REVIEW D **105,** L071503 (2022)

Timelike nucleon electromagnetic form factors: All about interference of isospin amplitudes

Xu Cao,[1,2,*] Jian-Ping Dai,[3,†] and Horst Lenske[4,‡]

Phenomenology:
Modelling the Form Factors

$$G_N^{eff}(q) = |G_N(q)| = G_N^D(q) + G_N^{res}(q)$$

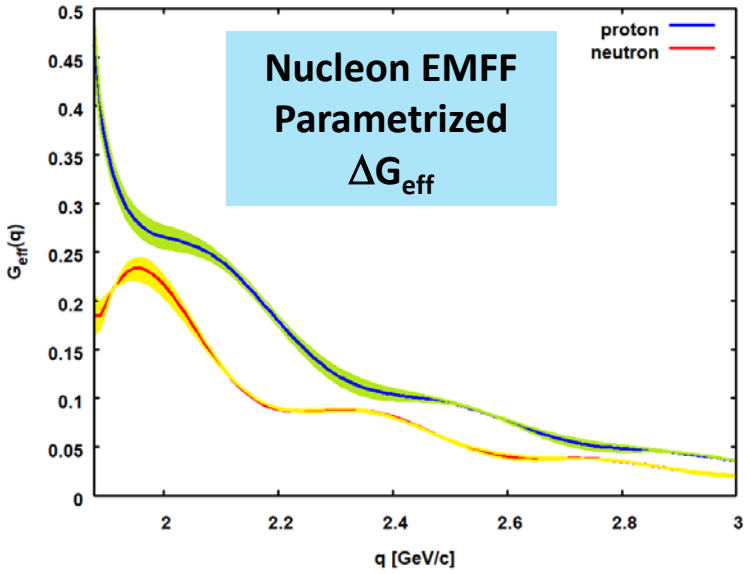
$$G_N^D(q) = \frac{A_N}{\left(1+\frac{q^2}{m_a^2}\right)\left(1-\frac{q^2}{m_D^2}\right)^2}$$

$$G_N^{res}(q) = B_N e^{-b_N p(q)} \cos(c_N p(q) + d_N)$$

$$p(q) = q\sqrt{\tau - 1}$$

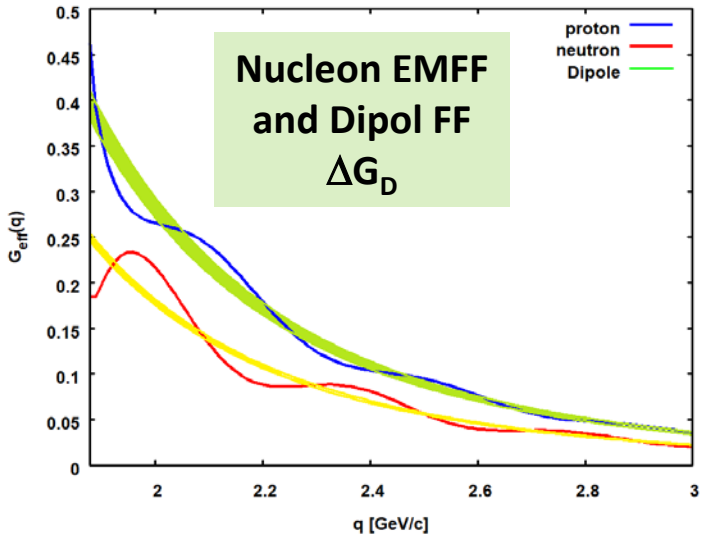
m_a=3.84 GeV, m_D=0.84 Gev, other parameter values see our Phys. Rev. D 105, L071503 (2022)

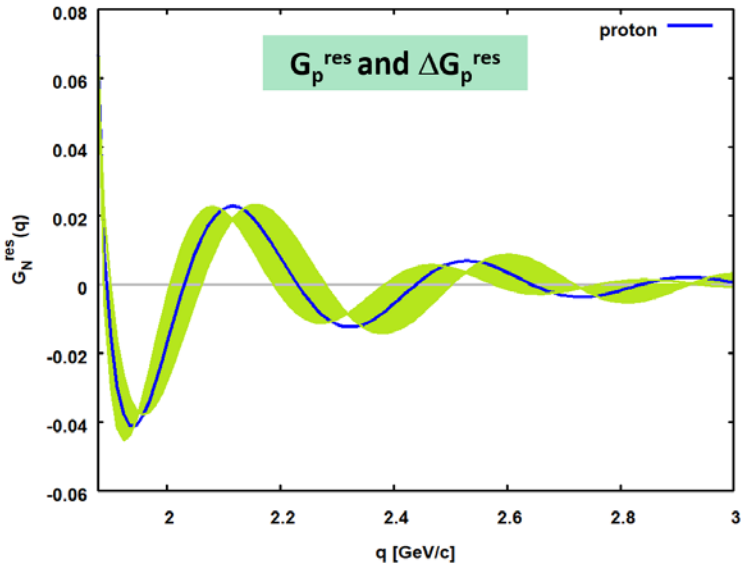
Momentum Structure of the Empirical Nucleon Form Factors



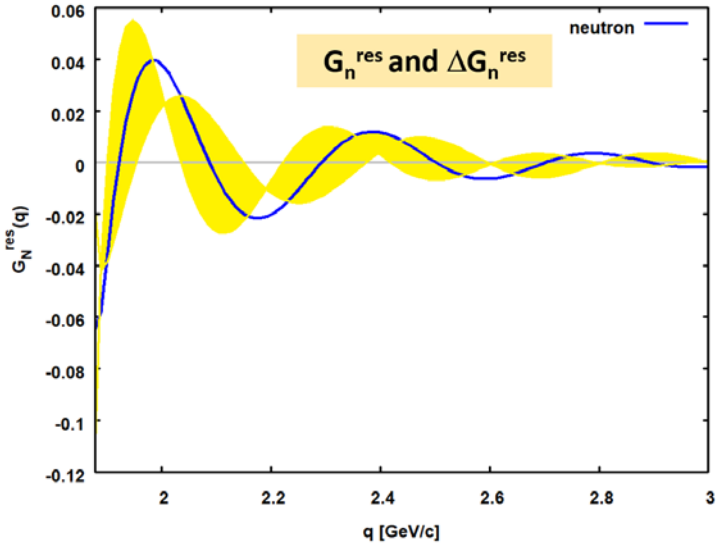
Modified Dipole plus damped oscillation:
$G_{eff}=G_D+G_{res}$

Clear Differences in $t_3=\pm 1/2$ Form Factors
Pronounced Isospin Effects!

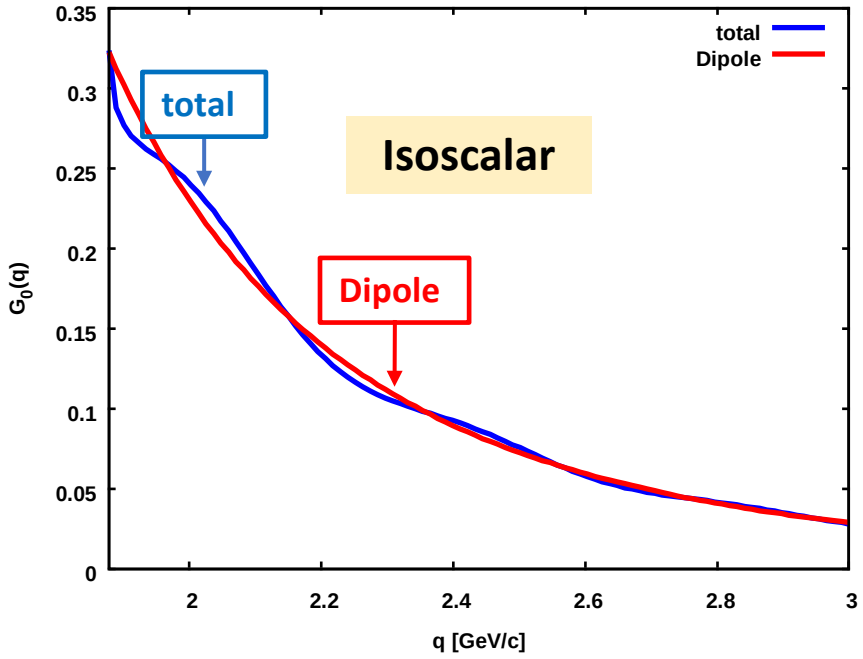




$$\frac{\sigma_n^D}{\sigma_p^0/C} = \left|\frac{G_n^D}{G_p^D}\right|^2 = 0.40 \pm 0.03$$

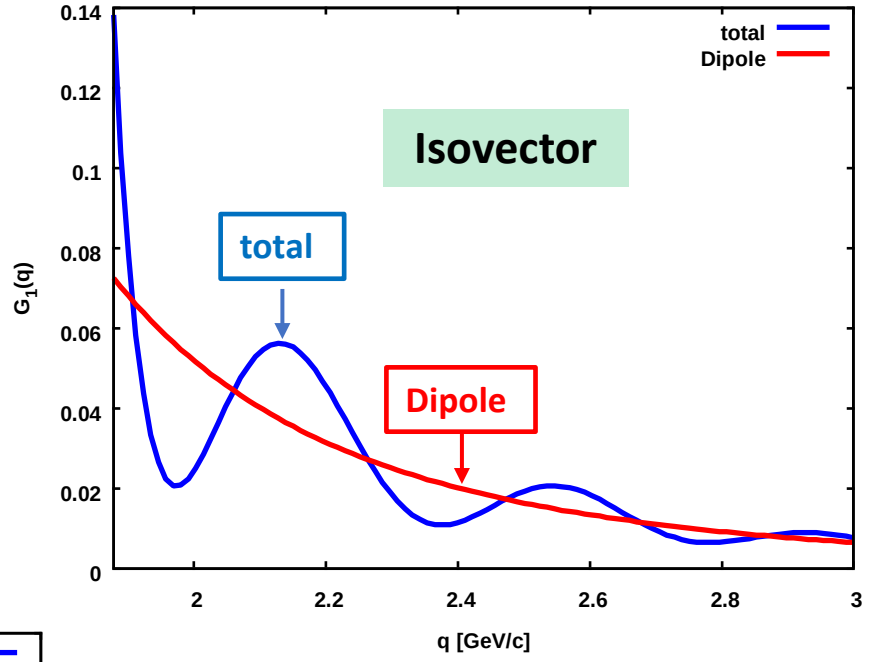


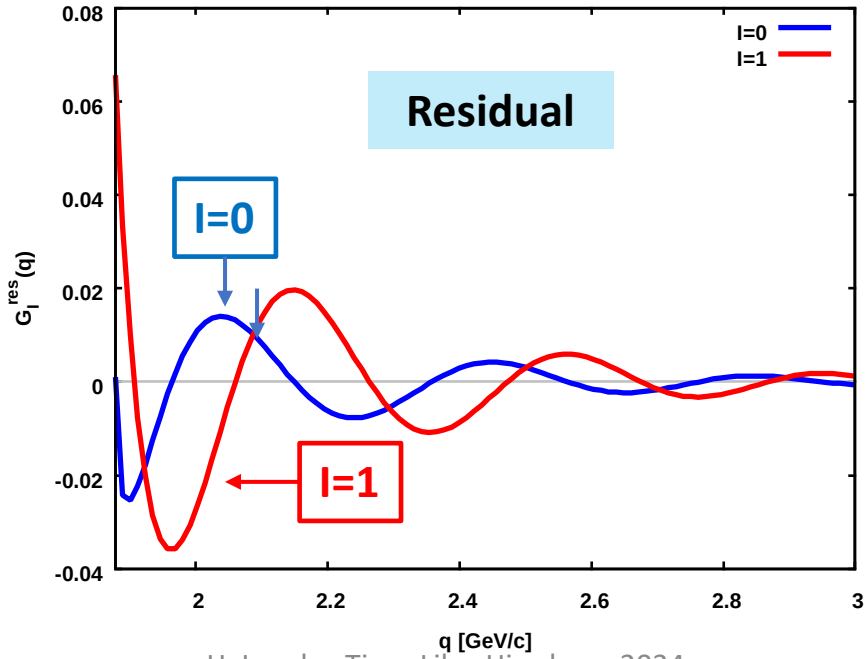
Isospin Form Factors of the Nucleon



$$\left|N\bar{N}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|00\right\rangle \pm \left|10\right\rangle\right)$$

$$G_{0,1}(q) = \frac{1}{2}\left(G_p(q) \pm G_n(q)\right)$$





Isospin Interference in Time-like Nucleon Form Factors

Complex Nucleon and Isospin Form Factors

Isoscalar (I=0) and Isovector (I=1) Form Factors:

$$G_{0,1}(q) = \frac{1}{2}\left(G_p(q) \pm G_n(q)\right)$$

Complex-valued Form Factors:

$$G_{p,n}(q) = e^{i\phi_{p,n}(q)} \left|G_{p,n}(q)\right| = G_0(q) \pm G_1(q)$$

$$G_{0,1}(q) = e^{i\phi_{0,1}(q)} \left|G_{0,1}(q)\right|$$

…extract information on phases $\phi_{p,n}$ from the oscillation petterns!

Effective Nucleon and Isospin Form Factors

$$\varphi = \frac{1}{2}(\phi_0 + \phi_1) \;\; ; \;\; \delta = \frac{1}{2}(\phi_0 - \phi_1) \;\; ; \;\; S = |G_0| + |G_1| \;\; ; \;\; D = |G_0| - |G_1|$$

$$G_p(q) = e^{i\phi_p(q)}\,|G_p(q)| = e^{i\varphi(q)}\left(e^{i\delta(q)}\,|G_0(q)| + e^{-i\delta(q)}\,|G_1(q)|\right)$$
$$G_n(q) = e^{i\phi_n(q)}\,|G_n(q)| = e^{i\varphi(q)}\left(e^{i\delta(q)}\,|G_0(q)| - e^{-i\delta(q)}\,|G_1(q)|\right)$$

G_{eff} ↔ Phases and Moduli of Isospin Form Factors

$$|G_p(q)| = |\cos(\delta)S(q) + i\sin(\delta)D(q)| = \sqrt{[\cos(\delta)S(q)]^2 + [\sin(\delta)D(q)]^2}$$
$$|G_n(q)| = |\cos(\delta)D(q) - i\sin(\delta)S(q)| = \sqrt{[\cos(\delta)D(q)]^2 + [\sin(\delta)S(q)]^2}$$

$$|G_p|^2 + |G_n|^2 = S^2 + D^2 = 2\left(|G_0|^2 + |G_1|^2\right)$$

The Residual Form Factor

$$G_{p,n} = \frac{I^D_{p,n} + I^{\mathrm{rsd}}_{p,n}}{\sqrt{2}} = \frac{I^D_1 \pm I^D_0}{\sqrt{2}} + \frac{I^{\mathrm{rsd}}_1 \pm I^{\mathrm{rsd}}_0}{\sqrt{2}}.$$

$$I^D_N = \sqrt{2} G^D_N e^{i\phi^D_N} \;\; ; \;\; I^{res}_N = \left| I^{res}_N \right| e^{i\phi^{res}_N}$$

$$\left|G_N\right|^2 - \left(G^D_N\right)^2 = G^{res}_N \left(2G^D_N + G^{res}_N\right) = \frac{1}{2}\left|I^{res}_N\right|^2 + \sqrt{2} G^D_N \left| I^{res}_N \right| \cos(\phi^D_N - \phi^{res}_N)$$

…up to order $(I^{res}/G^D)^2$:

$$G^{res}_N(q) \approx \sqrt{2}\,|\,I^{res}_N(q)\,|\cos(\phi^D_N(q) - \phi^{res}_N(q))$$

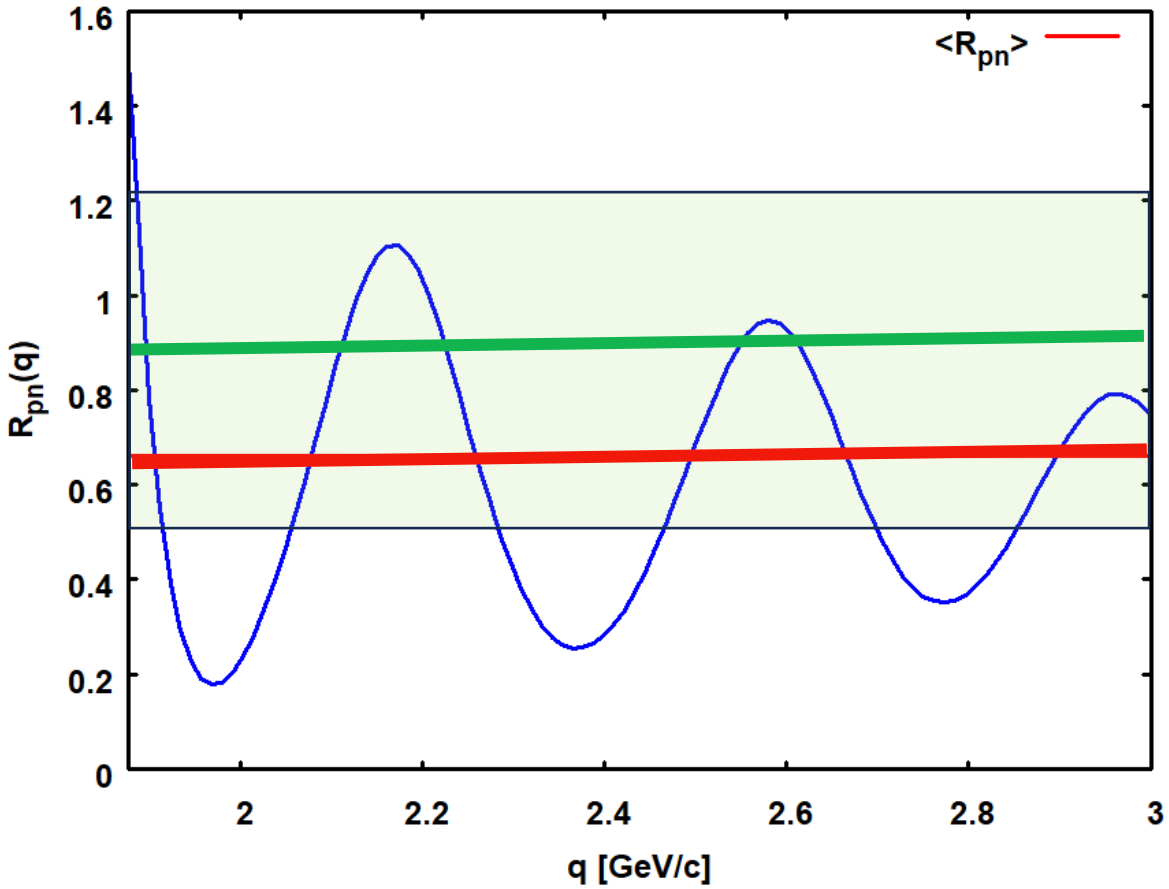
Relating Empirical and Isospin Model Parameters ($p(q)=q(\tau-1)^{1/2}$):

$$A_N e^{-b_N p(q)} = \sqrt{2}\,|\,I^{res}_N(q)\,| \quad ; \quad \cos(c_N p(q) + d_N) = \cos(\phi^D_N(q) - \phi^{res}_N(q))$$

Glimpses on the Physics Behind

Nucleon Form Factors and Isospin

$$R_{pn}(q) = \left|\frac{G_p(q)}{G_n(q)}\right| - 1 \approx \left|\frac{I_0^{res}(q) + I_1^{res}(q)}{I_0^{res}(q) - I_1^{res}(q)}\right|$$



$\langle R_{pn} \rangle_{|exp} = 0.88 \pm 0.35$

$\langle R_{pn} \rangle_{|theo} \cong 0.66...$

Hints on Production Scenario

$$R_{pn}(q^2) = \left|\frac{G_p(q^2)}{G_n(q^2)}\right| - 1 \approx \left|\frac{I_0^{res}(q^2) + I_1^{res}(q^2)}{I_0^{res}(q^2) - I_1^{res}(q^2)}\right| \approx \frac{2}{3}$$

Two Solutions:

Isoscalar Dominance:

$$\left(\frac{I_1^{res}}{I_0^{res}}\right)_{|theo} = -\frac{1}{5}$$

$$\left(\frac{I_1^{res}}{I_0^{res}}\right)_{|exp.} = -0.06 \text{ and } -15.67$$

$$e^+e^- \to \phi^*(2170) \to N\bar{N}$$

Isovector Dominance:

$$\left(\frac{I_1^{res}}{I_0^{res}}\right)_{|theo} = -5$$

e.g. $e^+e^- \to$ * $\bar{}$

Extension to SU(3)
Time-like Hyperon Form Factors

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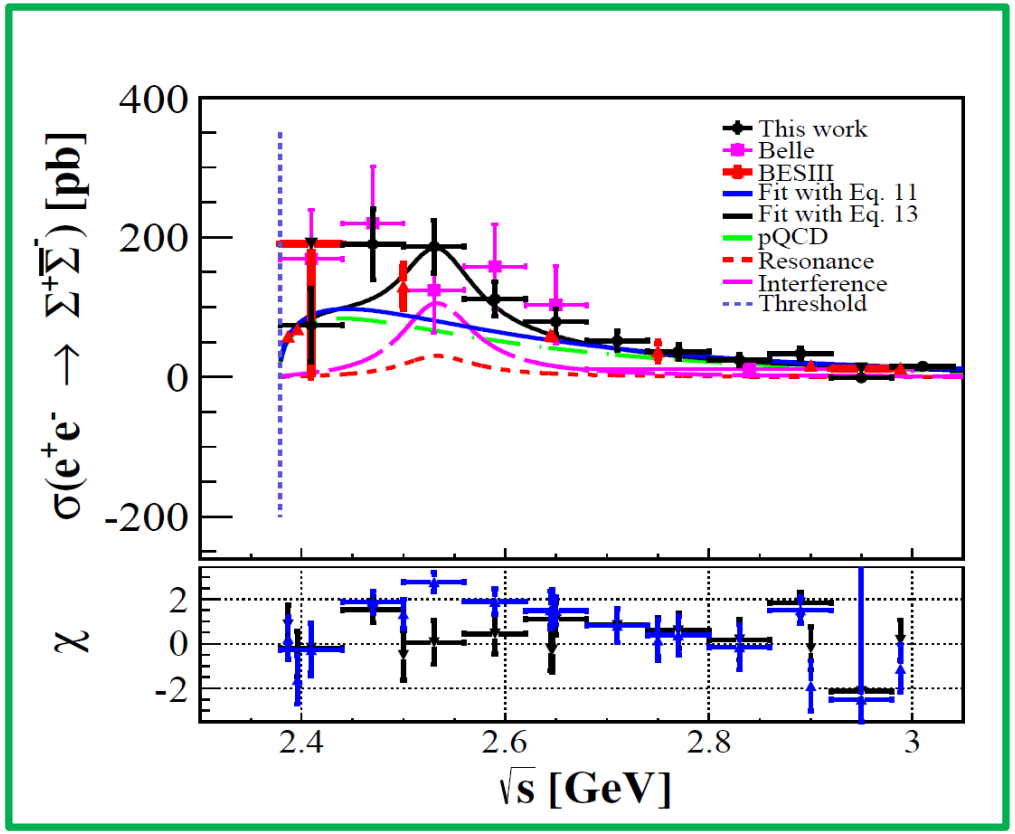
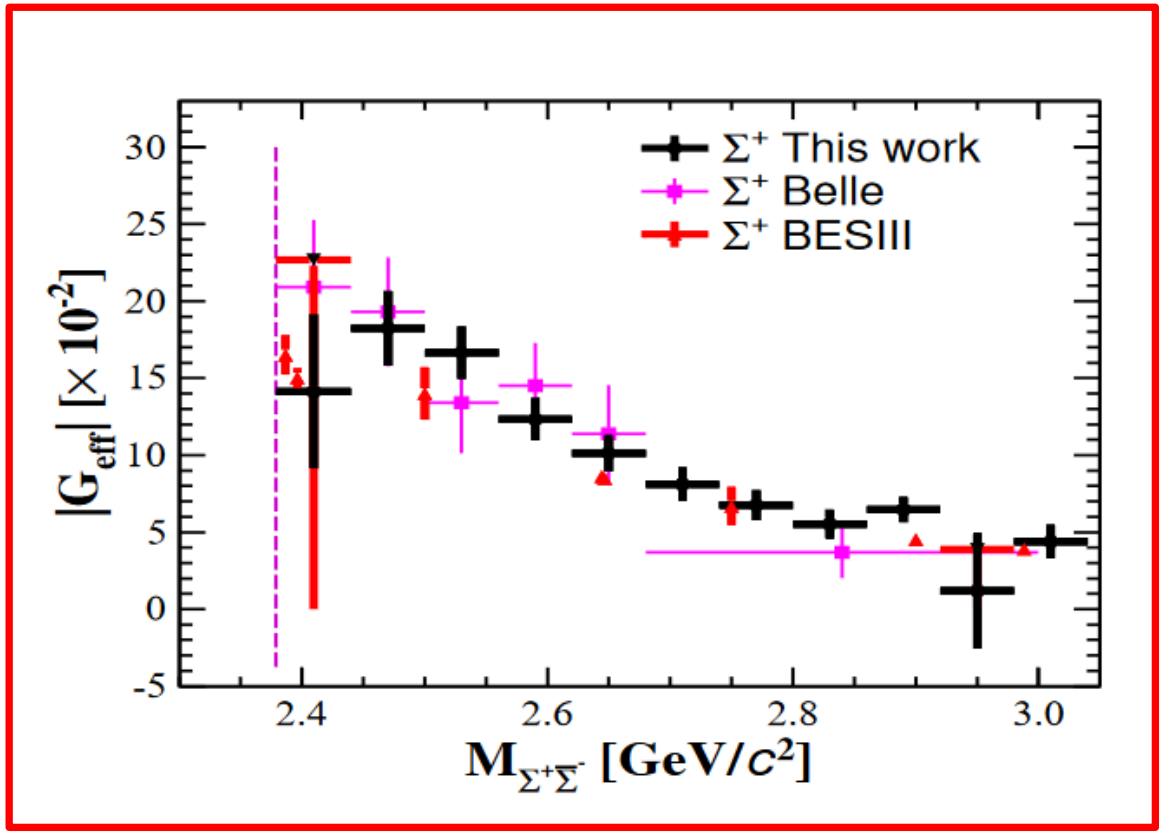
Data driven isospin analysis of timelike octet baryons electromagnetic form factors and charmonium decay into baryon-anti-baryon

Jian-Ping Dai [a], Xu Cao [b,c,d,*], Horst Lenske [e]

Most Recent BESIII Results on $e^+e^- \rightarrow \Sigma^+\bar{\Sigma}^-$

- Ablikim et a, 2312.12719 [hep-ex]





$$\sigma_{Y\bar{Y}}(s) = \frac{4\pi\alpha^2 C\beta}{3s}\left[|G_{\rm M}(s)|^2 + \frac{1}{2\tau}|G_{\rm E}(s)|^2\right],$$

$$|G_{\rm eff}(s)| = \sqrt{\frac{2\tau|G_{\rm M}(s)|^2 + |G_{\rm E}(s)|^2}{2\tau+1}}$$

Hyperon EMFF

- **Scarce Data of larger uncertainty, S=-1 only**
- **Σ-Hyperons: Isospin Triplet → I=0,1,2**
- **e+e- reactions populate only the I=0,1 components**
- **Three amplitudes and form factors:**

$e^+e^- \rightarrow$

$$G_+ = \frac{1}{\sqrt{2}} I_1 + \frac{1}{\sqrt{3}} I_0 \qquad \Sigma^+\overline{\Sigma}^-$$

$$G_- = \frac{1}{\sqrt{2}} I_1 - \frac{1}{\sqrt{3}} I_0 \qquad \Sigma^-\overline{\Sigma}^+$$

$$G_0 = \frac{1}{\sqrt{3}} I_0 \qquad \Sigma^0\overline{\Sigma}^0$$

Constraints from $\Sigma^{\pm}$ on Σ^0 EMF

$$G_{\pm,0} = |G_{\pm,0}|e^{i\phi_{\pm,0}}$$

Using the Isospin Form Factors Relations:

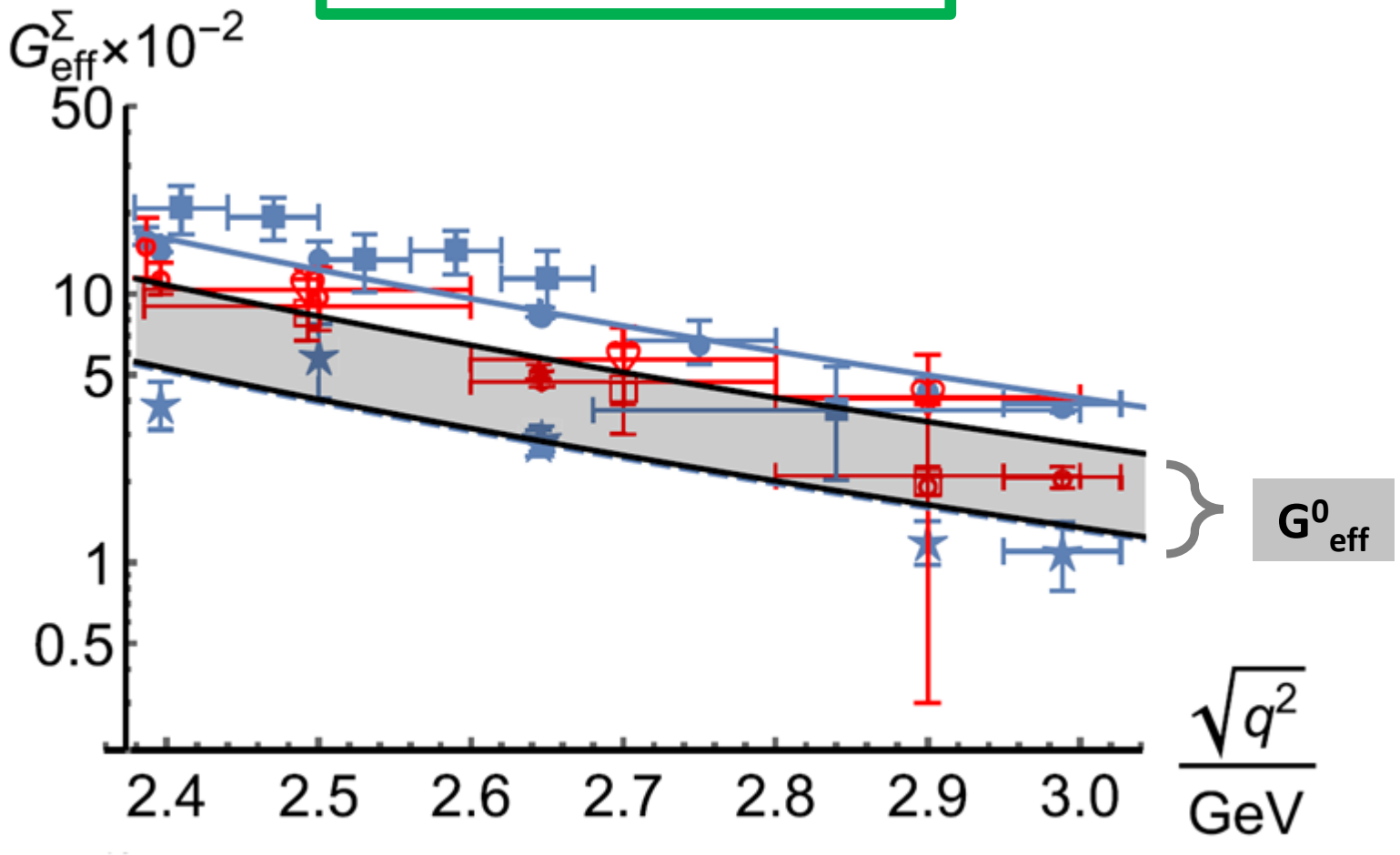
$$4\,|G_0|^2 = |G_+|^2 + |G_-|^2 - 2\,|G_+||G_-|\cos(\phi_+ - \phi_-)$$

→ Limits on the Σ^0 Form Factor:

$$||G_+| - |G_-|| \leq 2|G_0| \leq |G_+| + |G_-|$$

Σ Hyperon EMFF
World Data Set from BaBar, BELLE, BesIII

$$||G_+| - |G_-|| \leq 2|G_0| \leq |G_+| + |G_-|$$



Phys. Lett. B 846 (2023) 138192

Summary and Outlook

- **Isospin Structure of Proton and Neutron EMFF**
- **Oscillation Pattern: Interference of Isoscalar and Isovector Components**
- **EMFF Isospin Structure and Production Mechanism**
- **First Steps into Octet Sector: EMFF of Σ Hyperons**
- **Where do the differences in $G_n \leftrightarrow G_p$ /$G_0 \leftrightarrow G_1$ come from?**
- **In Progress: Quark-Hadron Duality**

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