Tetraquarks with two b-quarks

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Introduction

- T_{bb} analog to the experimentally-discovered T_{cc}
- Long-standing pheno+lattice predictions of purely exotic deeply-bound tetraquark state $I(J^P) = 0(1^+)$, flavor $ud\bar{b}\bar{b}$ - A "Gold-Plated" hadron

Expected binding energy is very different!

$$\Delta \mathrm{T}_{cc} \approx -273(61) \text{ KeV} \qquad \Delta \mathrm{T}_{bb} \approx -110 \text{ MeV}$$

- Similar $I(J^P) = \frac{1}{2}(1^+)$, flavor $ls\bar{b}\bar{b}$
- Part of a family T_{bb}, T_{bbs}, T_{bc}, T_{cc} of bound tetraquarks?

Overview of most-recent setup

Pick CLS configurations (2+1 Clover-Wilson) at $\beta = 3.4$ lattice spacing ($a \approx 0.0864$ fm)

- Have 10 boxes that differ in quark mass and volume
- Small enough volumes with long chains that allow for high statistics - beating down the signal to noise
- (Sink) Smearing to improve ground-state determination (GF-Wall source)
- Can we tune lattice NRQCD to behave more continuum-like?

$O(v^4)$ NRQCD

Propagators from time evolution equation:

$$S(x,t+1) = \left(1 - \frac{H_I + H_D}{2}\right)\left(1 - \frac{H_0}{2n}\right)^n \tilde{U}_t(x,t)^\dagger \left(1 - \frac{H_0}{2n}\right)^n \left(1 - \frac{H_I + H_D}{2}\right) S(x,t)$$

$$\begin{aligned}
H_0 &= -\frac{1}{2aM_0}\Delta^2, \\
H_I &= \left(-c_1 \frac{1}{8(aM_0)^3} - c_6 \frac{1}{16n(aM_0)^2}\right)\left(\Delta^2\right)^2 \\
&\quad + c_2 \frac{i}{8(aM_0)^2}(\tilde{\Delta}\cdot\tilde{E} - \tilde{E}\cdot\tilde{\Delta}) + c_5 \frac{\Delta^4}{24(aM_0)} \\
H_D &= -c_3 \frac{1}{8(aM_0)^2}\sigma\cdot\left(\tilde{\Delta}\times\tilde{E} - \tilde{E}\times\tilde{\Delta}\right) - c_4 \frac{1}{2(aM_0)}\sigma\cdot\tilde{B}
\end{aligned}$$

Spin-dependent term c_4 controls S1 hyperfine. $c_3 \approx 1$. c_5, c_6 higher-order lattice corrections. $c_2 \propto 1P - 1S$ splitting.

Improving lattice NRQCD

Can we tune lattice NRQCD coefficients nonpeturbatively? **Yes!** using maching learning....

- In our most recent work Phys.Rev.D 107 (2023) 11, 114510 - RJH and DM we proposed a fully non-perturbative tuning of lattice NRQCD using machine learning
- Pure bottomonia tuning - due to additive mass we must only consider splittings
- 7-parameter tuning, bare mass aM_0 and corrections c_i
- tuning precision is around 1%

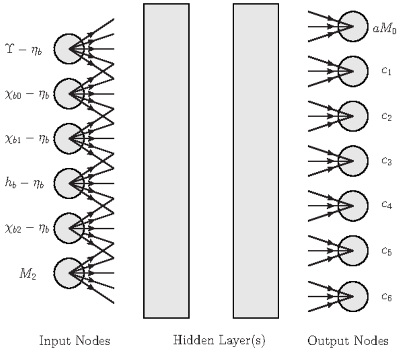


Figure: Schematic picture of our NRQCD setup

Excited bottomonium spectrum from our tuning

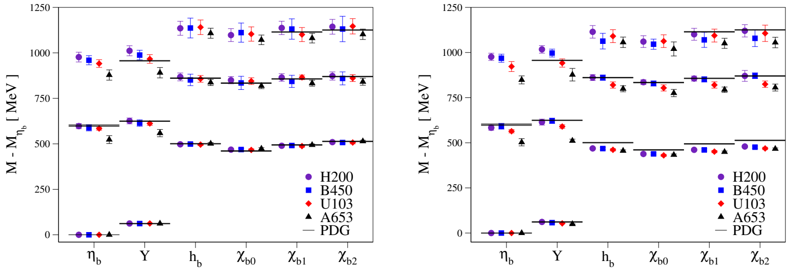


Figure: (Left) neural network tuning for excited bottomonia, (Right) tree-level tuning.

Better match to experiment, smaller cut-off effects between splittings, better hierarchy of states.

Example effective mass - Tbb

$$D = ({u_a}^T C\gamma_5 d_b)(\bar{b}_a C\gamma_i \bar{b}_b^T), \quad E = ({u_a}^T C\gamma_t\gamma_5 d_b)(\bar{b}_a C\gamma_i\gamma_t \bar{b}_b^T),$$
$$M = (\bar{b}\gamma_5 u)(\bar{b}\gamma_i d) - [u \leftrightarrow d], \quad N = (\bar{b}Iu)(\bar{b}\gamma_5\gamma_i d) - [u \leftrightarrow d].$$

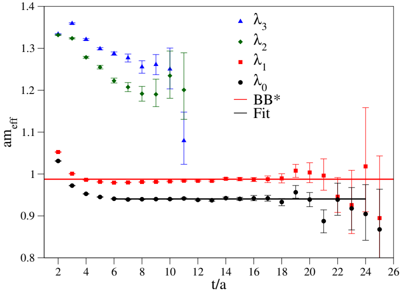


Figure: Example effective masses for our principle correlators

Combined mass and volume extrapolations

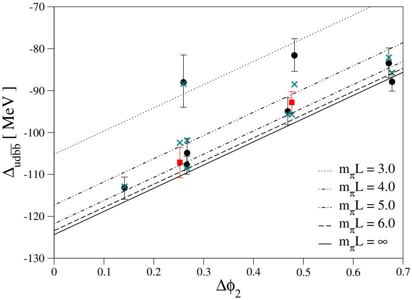


Figure: Mass and finite volume dependence of the binding energy of our T_{bb}

More volumes and pion masses show strong $e^{-m_\pi L}$ volume effects and deeper binding at lighter pion mass. −37 MeV from SU(3)$_f$-symmetric point to physical pion mass.

The problem(s) with tuning NRQCD - T_{bb}

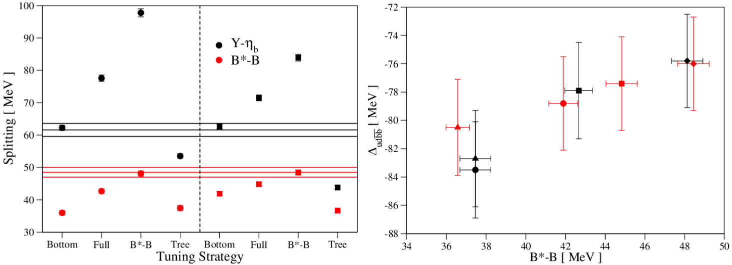
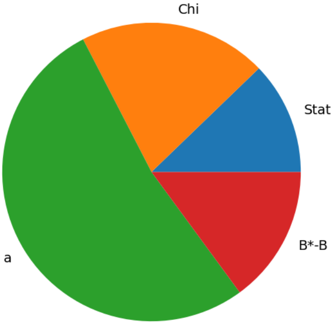


Figure: Alternative tuning strategies with/without B-mesons and higher-order terms (left). The effect of the $B^* - B$ splitting on the T_{bb} (right).

It appears **impossible** to simultaneously reproduce experimental $\Upsilon - \eta_b$ and $B^* - B$ splittings with same tuning parameters. Higher orders helps a little.
Shallower T_{bb} binding, with increased $B^* - B$ splitting.

A slice of the systematics pie



$$\Delta_{ud\bar{b}\bar{b}}(0,\infty,0) = -112.0(2.7)_{\text{Stat.}}(4.5)_{\chi}(11.6)_{a}(3.3)_{B^*-B}$$

Dominated by our lattice spacing uncertainty! Something we can do little about

The most recent T_{bb} picture

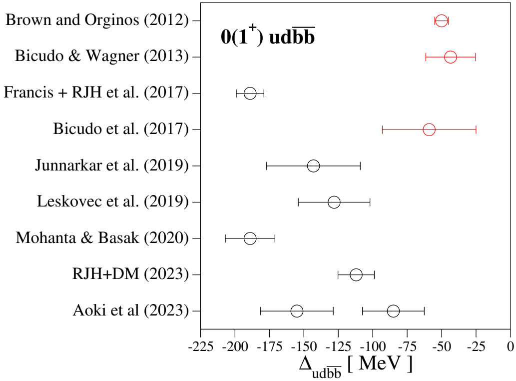


Figure: Review of lattice $0(1^+)$ T_{bb} determinations

Consensus is deeply bound, strong interaction stable, weakly decaying,

$$M_{bb} = 10.492 \text{ GeV}$$

The T_{bbs}

$$M = (\bar{b}\gamma_5 u)(\bar{b}\gamma_i s), \quad N = (\bar{b}Iu)(\bar{b}\gamma_5\gamma_i s)$$
$$O = (\bar{b}\gamma_5 s)(\bar{b}\gamma_i u), \quad P = (\bar{b}Is)(\bar{b}\gamma_5\gamma_i u)$$
$$Q = \epsilon_{ijk}(\bar{b}\gamma_j u)(\bar{b}\gamma_k s).$$

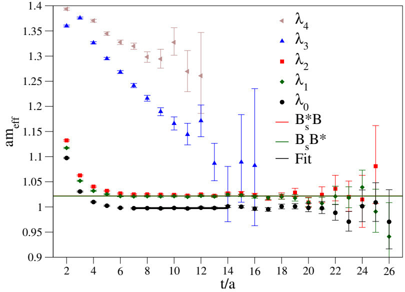


Figure: Exemplary principle correlators for our T_{bbs}

The T_{bbs}

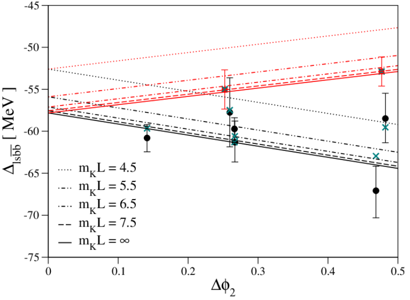


Figure: Combined chiral/infinite-volume fit for the T_{bbs}.

Large $e^{-m_K L}$ volume effects. Still consistent with light-diquark picture.
Shallower binding than other studies.

T_{bbs} review plot

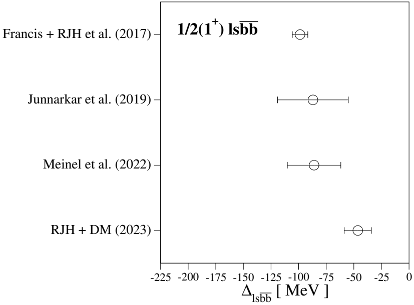


Figure: Review of the current lattice T_{bbs} determinations

Close/overlapping EM threshold $BB_s\gamma$, still possible that it is narrow and decays weakly

$$M_{bbs} = 10.645 \text{ GeV}$$

A brief aside - The B^*_{s0} and B_{s1}

The scalar and axial B_s-mesons

For the T_{bbs} we generated various B_s-mesons. Use naive operators $J^P = 0^+, 1^+$ (no scattering operators seemingly needed):

$$O_{B^*_{s0}} = (\bar{b} I s), \qquad O_{B_{s1}} = (\bar{b}\gamma_i\gamma_5 s)$$

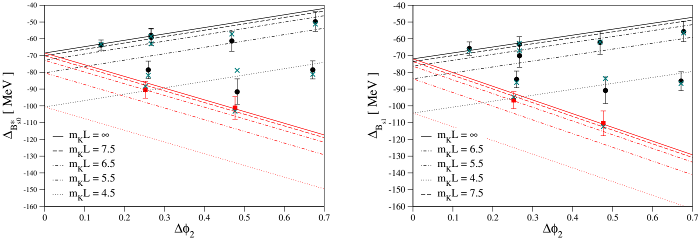


Figure: (Left) chiral and finite-volume dependence of B_{s0}, (Right) same for the B_{s1}.

Two different am_s trajectories give compatible results Phys.Rev.D 107 (2023) 11, 114510 - RJH and DM

The scalar and axial B_s-mesons in review

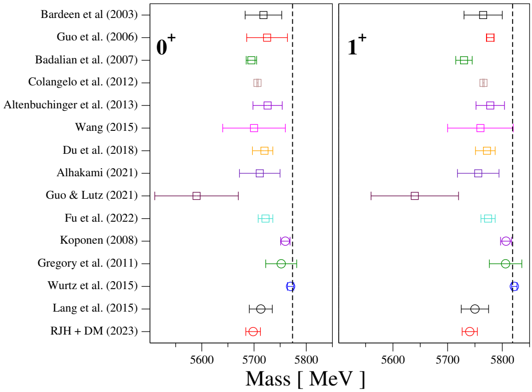


Figure: Phenomenological and lattice results for the B_{s0}^* and B_{s1} mesons.

Non quark-model predictions agree with us

Relativistic Heavy Quarks for the T_{bb}

Improving RHQ b-quarks

I think NRQCD for the T_{bb} and T_{bbs} is at an end. RHQ b-tuning using the "Tsukuba" action (Aoki-2003).

- Same idea as for charm in Phys.Rev.D 106 (2022): (RJH+DM)
- Learn the dependence of states on parameters
- Absolute scales included
- Fixed $c^2 = 1$ to ensure relativistic nature
- 5-parameter tuning
- see large variations from 1 of r_s, ν, c_E, c_B

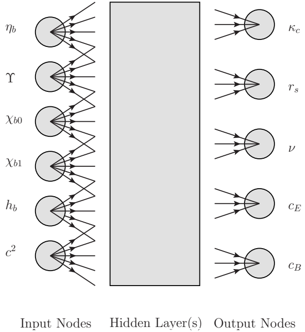


Figure: Schematic picture of our RHQ b-quark tuning

A technical issue:

Correlators can span **75** orders of magnitude!

- Cannot solve to a fixed residual as "convergence" is achieved too quickly
- Fixed number of CGNE iterations
- Distance preconditioning
- Hopping parameter expansion (HPE)
- As $a \to 0$ more iterations needed

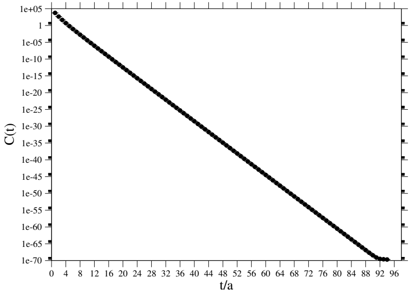


Figure: A J500 η_b correlator

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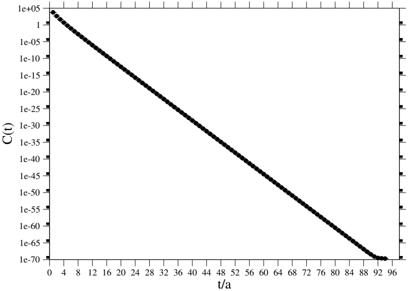


Figure: A J500 η_b correlator

Convergence of CG and HPE

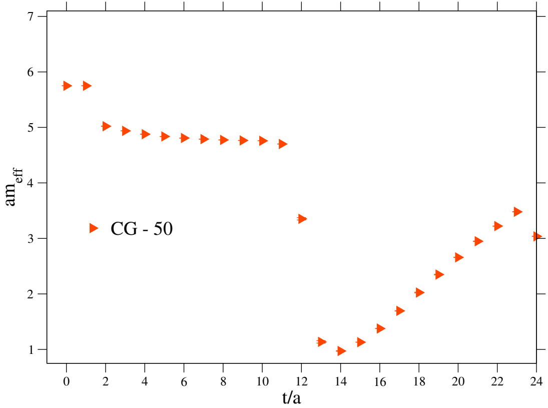
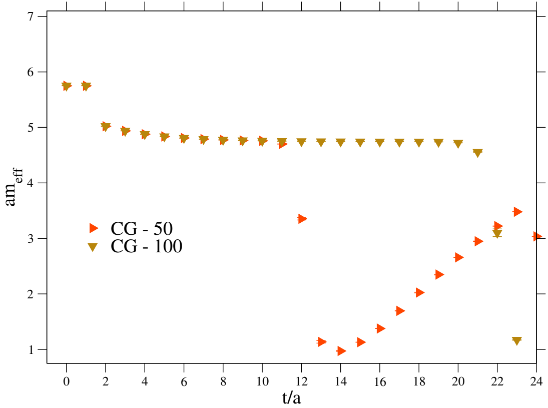


Figure: Effective mass convergence with increased number of iterations of HPE or CG on a small, coarse, box

Convergence of CG and HPE



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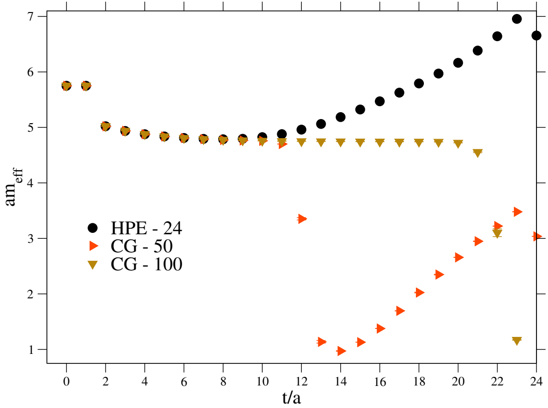

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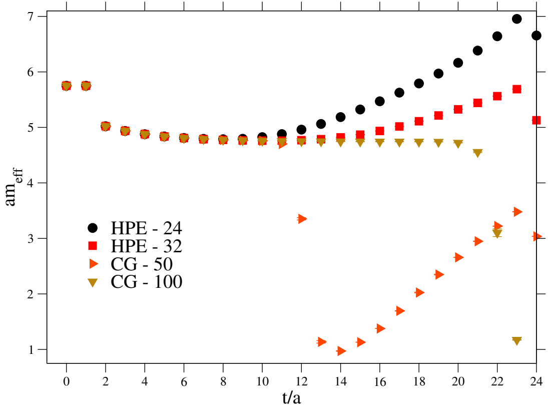

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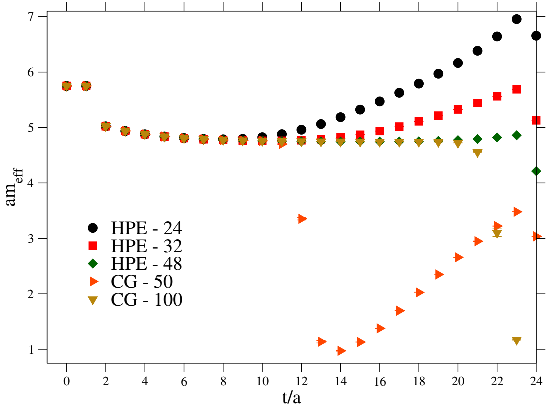

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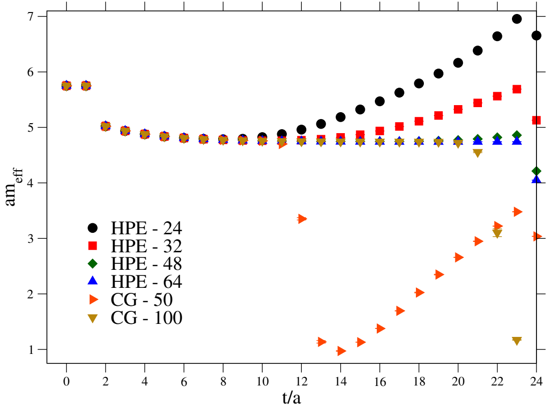

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Comparison of b and c parameters - c_E and c_B

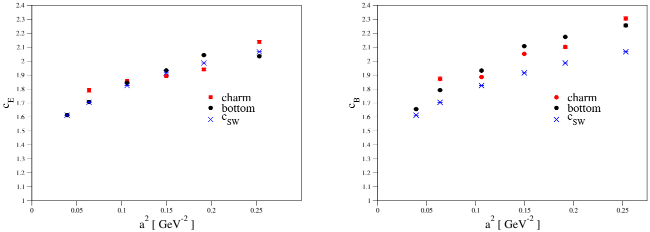

Figure: RHQ clover terms c_E and c_B for **bottom** and **charm**

As a rule of thumb: $c_E \approx c_{SW}$, $c_B > c_E$. No big difference between bottom and charm!

Comparison of b and c parameters - κ, r_s, ν

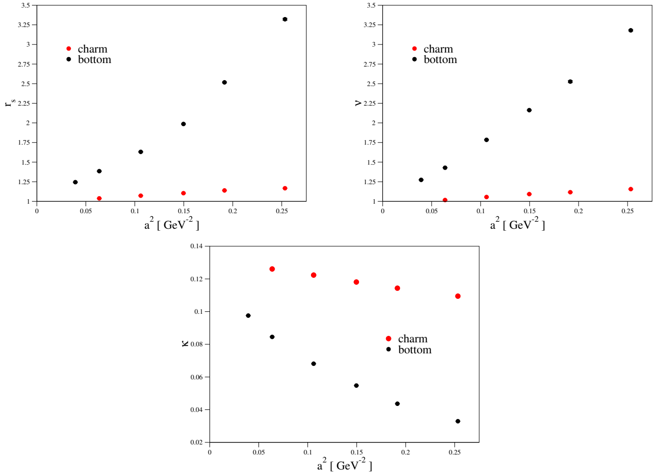

Figure: RHQ action terms r_s, ν, κ for **bottom** and **charm**

B-meson and bottomonium 1S splittings

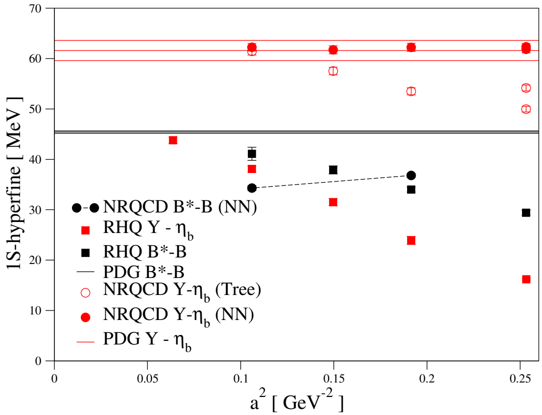

Figure: 1S hyperfine splittings from the nonperturbative NRQCD tuning and the RHQ action for bottomonia and b-mesons.

Preliminary RHQ T_{bb} at the symmetric point

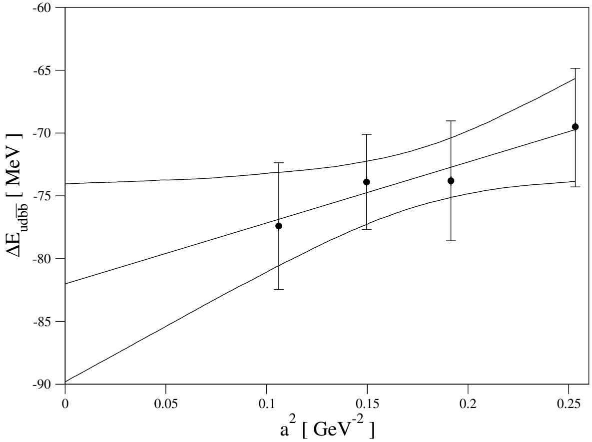

Figure: Preliminary continuum extrapolation at the $SU(3)_f$-symmetric point

Conclusions

- Nonperturbatively tuned NRQCD, but didn't alter T_{bb} picture
- Have we reached the systematic limit of NRQCD for T_{bb} and T_{bbs}?
- Nonperturbative RHQ b-quark is doable, several difficult hurdles that were needed to be addressed, for us the HPE is the way forward
- Seems that $SU(3)_f$-symmetric data is pointing to the right direction and continuum extrapolation appears mild
- Rough combination of m_π^2-dependence of NRQCD study suggests a binding energy of $\approx 115(10)$ MeV - Consistent with our previous study!