Properties of Four-Quark states from functional methods

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17. January, 2024

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HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

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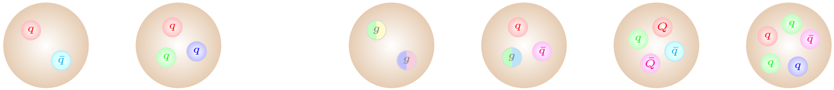
Motivation

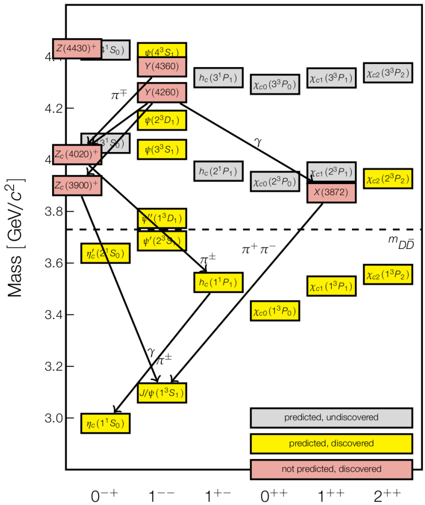
Conventional Hadrons: Exotic Hadrons:



Mesons Baryons Glueballs Hybrids Four-quark states Pentaquarks

- Powerful toolkit to classify conventional hadrons: Quark Model (QM).
- Lot of particles measured that do not fit into the QM picture, i.e., exotic hadrons.
- A few examples:
  - Light scalar mesons: σ, κ, a_0, f_0
  - Exotic XYZ-states: $X(3872)$, $X(3915)$, $Z_c(3900)$, $Z_c(4430)$, $\psi(4230)$

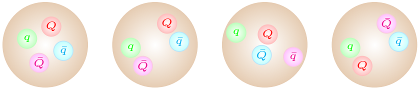
Tetraquark candidates in the charmonium region



Wolfgang Gradl, BESIII, St Goar 2015

Many unexpected states found by Belle, BARBAR, BES, LHCb, ...

Internal structure??

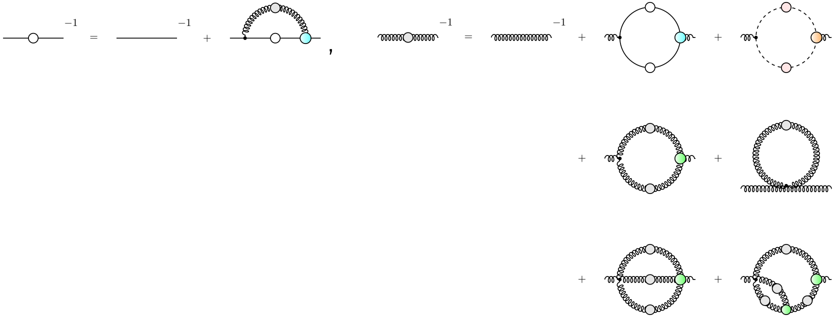


Compact tetraquark | Meson molecule | Hadro quarkonium | Diquark Antidiquark

Related to details of underlying QCD forces between quarks

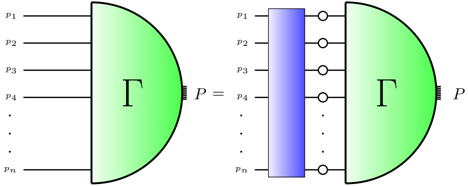
Functional Framework

- Non-perturbative, fully relativistic framework.
- To compute the properties of bound states, use combination of:
    - DSEs: The QCD quantum equations of motion,



Functional Framework

- Non-perturbative, fully relativistic framework.
- To compute the properties of bound states, use combination of:
  - DSEs: The QCD quantum equations of motion,
  - Hadronic bound state equations: BSEs, Faddeev eqs. .



Eigenvalue equation

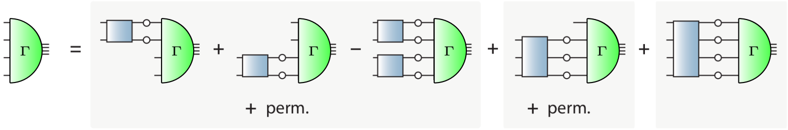
$$\lambda(P^2)\,\Gamma^{(n)} = K^{(n)}G^{(n)}\,\Gamma^{(n)}$$

with $\lambda(P^2 = -M^2) = 1$

Four-quark BSE

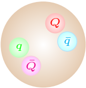
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, Fischer, PLB 718 (2012) 545-549
Eichmann, Fischer, Heupel, PLB 753 (2016) 282-287

Exact equation:

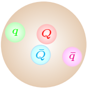


Two-body interactions

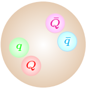
Three- and four-body interactions



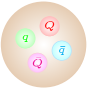
Meson molecule



Hadro quarkonium

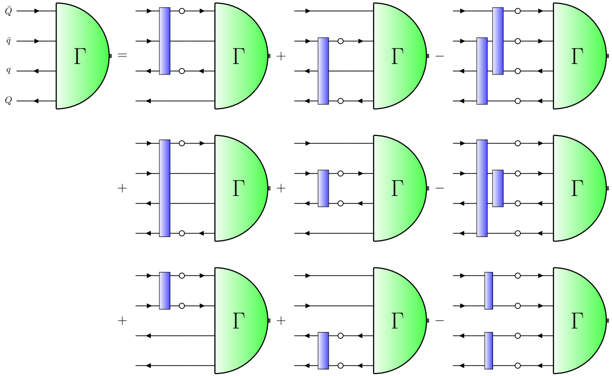


Diquark Antidiquark



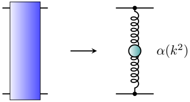
Compact tetraquark

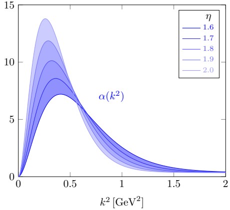
The Four-quark BSE:



$$\Gamma(k,q,p,P) = \sum_i f_i \underbrace{(\ldots)}_{\text{Lorentz-invariants}} \tau_i(k,q,p,P) \otimes \Gamma_C \otimes \Gamma_F$$

Calculations are done in the *Rainbow-Ladder truncation*:



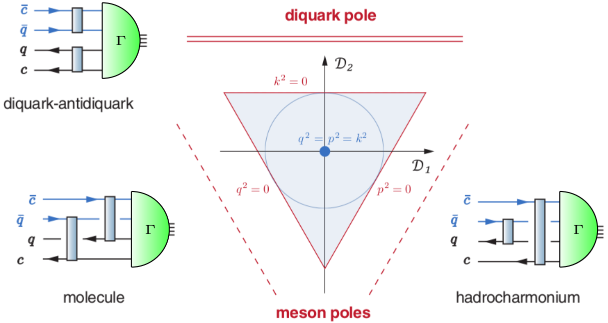


Maris, Tandy, PRC 60 (1999)
Qin et al., PRC 84 (2011)

- Can cast the Lorentz-invariants into multiplets of S_4:

Eichmann, Fischer, Heupel, Phys.Lett.B 753 (2016) 282-287

  - One singlet: S_0
  - One doublet: $D = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$
  - Two triples: T_0, T_1 $\rightarrow$ subleading
- Dressing functions: $f_i(S_0, D)$
- Poles dynamically generated in D
- "Physical basis": put poles in externally: $f_i(S_0, D) \rightarrow f_i(S_0) \cdot P_{ab} \cdot P_{cd}$



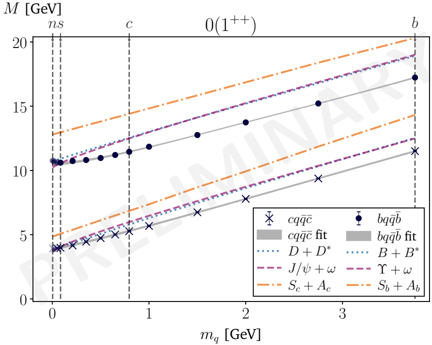
Eichmann, Fischer, Heupel, Santowsky, Wallbott,Few Body Syst. 61 (2020) 4, 38

Physical amplitude

- Structure of the Amplitude Γ is determined according to the quantum numbers of the state in question, the quark content and the physical decay channels.
- Example for the $X(3872) \quad (I(J^{PC}) = 0(1^{++}))$:
  - Meson-molecule: $D\bar{D}^*$
  - Hadro-charmonium: $J/\psi\,\omega$
  - Diquark-Antidiquark: $S_c A_c$

  $\Rightarrow \Gamma = f_{D\bar{D}^*} \cdot \tau_{D\bar{D}^*} + f_{J/\psi\omega} \cdot \tau_{J/\psi\omega} + f_{S_c A_c} \cdot \tau_{S_c A_c}$
- No assumptions of a dominant substructure needed!
- The internal two-body pole structures introduce decay thresholds into the equation.

Example quark mass evolution for the $0(1^{++})$ state



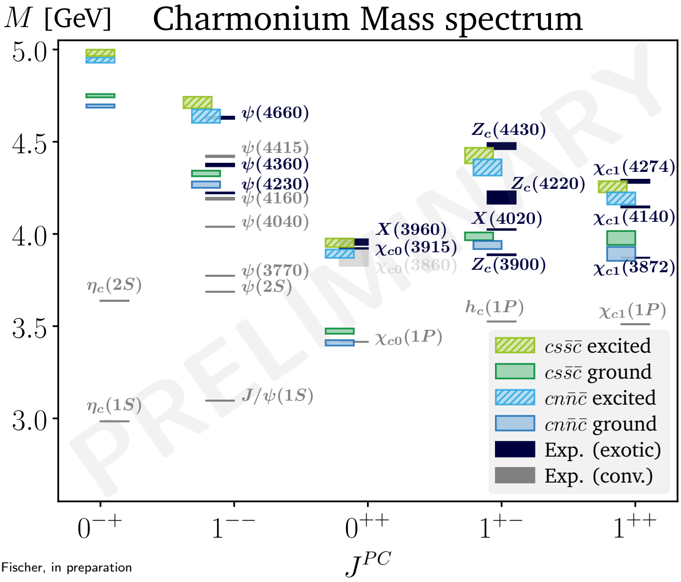
JH, Eichmann, Fischer, in preparation

Thresholds: Hidden-charm Hidden-bottom

$DD^*,\ J/\psi\omega,\ S_cA_c$ $BB^*,\ \Upsilon\omega,\ S_bA_b$

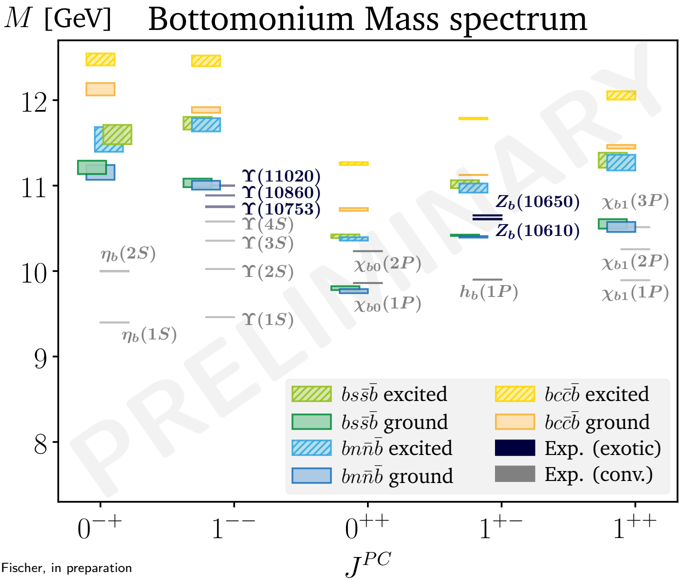
- $M_{1^{++}}^{cq\bar{q}\bar{c}} = 3.89 \pm 0.04$ GeV $\rightarrow$ $X(3872)$
- $M_{1^{++}}^{bq\bar{q}\bar{b}} = 10.52 \pm 0.06$ GeV $\rightarrow$ $(W_{b1}\,?)$

Hidden-charm mass spectrum



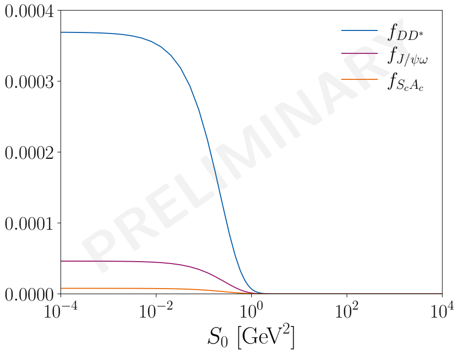
JH, Eichmann, Fischer, in preparation

Hidden-bottom mass spectrum



JH, Eichmann, Fischer, in preparation

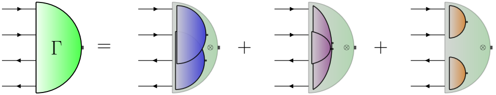
Amplitude for the $0(1^{++})$ state



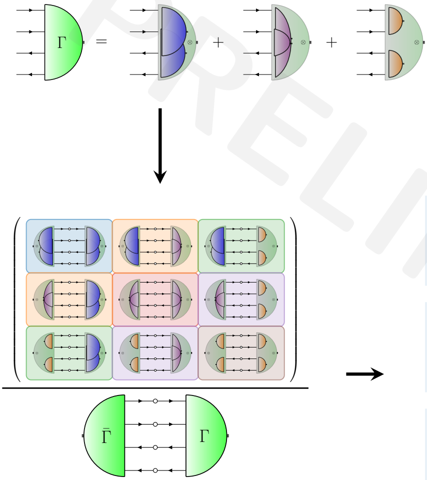
JH, Eichmann, Fischer, in preparation

Physical basis:

$$\Gamma = f_{D\bar{D}^*} \cdot \tau_{D\bar{D}^*} + f_{J/\psi\omega} \cdot \tau_{J/\psi\omega} + f_{S_cA_c} \cdot \tau_{S_cA_c}$$

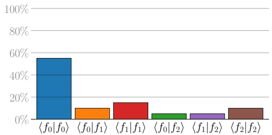


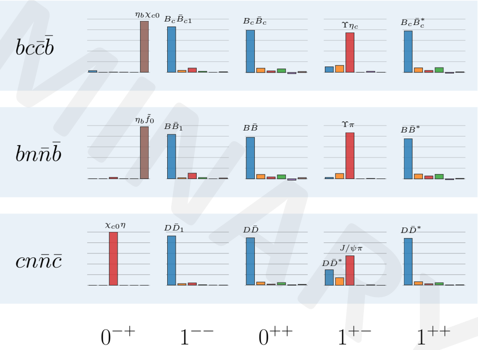
Dominant subcluster



($\tilde{f}_0 = f_0(1370)$)

E.g., $\chi_{c1}(3872)$:
$f_0 = f_{D\bar{D}^*}$, $f_1 = f_{J/\psi\omega}$, $f_2 = f_{S_c A_c}$





JH, Eichmann, Fischer, in preparation

Hidden-flavour ground state masses

	$I(J^{PC})$	Physical components	GS Mass	Exp.
hidden charm	$0(0^{++})$	$D\bar{D}$, $J/\psi\omega$, S_cS_c	$3.41(2)$	–
$(cq\bar{q}\bar{c})$	$0(1^{++})$	$D\bar{D}^*$, $J/\psi\omega$, S_cA_c	$3.89(4)$	$\chi_{c1}(3872)$
	$1(1^{+-})$	$D\bar{D}^*$, $J/\psi\pi$, S_cA_c	$3.94(2)$	$Z_c(3900)$
	$0(1^{--})$	$D\bar{D}_1$, $\chi_{c0}\omega$, $J/\psi\sigma$	$4.27(2)$	$\psi(4230)$
	$0(0^{-+})$	$D\bar{D}_0$, $\chi_{c0}\eta$, $\eta_c f_0(1370)$	$4.69(1)$	–
hidden bottom	$0(0^{++})$	$B\bar{B}$, $\Upsilon\omega$, S_bS_b	$9.77(2)$	$(W_{b0}?)$
$(bq\bar{q}\bar{b})$	$0(1^{++})$	$B\bar{B}^*$, $\Upsilon\omega$, S_bA_b	$10.52(6)$	$(W_{b1}?)$
	$1(1^{+-})$	$B\bar{B}^*$, $\Upsilon\pi$, S_bA_b	$10.40(1)$	$Z_b(10610)$
	$0(1^{--})$	$B\bar{B}_1$, $\chi_{b0}\omega$, $\Upsilon\sigma$	$11.01(5)$	$\Upsilon(10753)$
	$0(0^{-+})$	$B\bar{B}_0$, $\chi_{b0}\eta$, $\eta_b f_0(1370)$	$11.16(9)$	–

PRELIMINARY

JH, Eichmann, Fischer, in preparation,
Wallbott, Eichmann, Fischer, Phys.Rev.D 100 (2019) 1, 014033, Wallbott Eichmann, Fischer, Phys.Rev.D 102 (2020) 5, 051501

Open-flavour ground state masses

	$I(J^{PC})$	Physical components	GS Mass	Exp.
open charm	$1(0^+)$	DD, D^*D^*, A_cA_c	$3.39(1)$	–
$(cc\bar{q}\bar{q})$	$0(1^+)$	DD^*, D^*D^*, S_cA_c	$3.79(1)$	T_{cc}^+
	$1(1^+)$	DD^*, A_cA_c	$4.25(2)$	–
open bottom	$1(0^+)$	BB, B^*B^*, A_bA_b	$9.60(1)$	-
$(bb\bar{q}\bar{q})$	$0(1^+)$	BB^*, B^*B^*, S_bA_b	$10.14(2)$	$(T_{bb}^+?)$
	$1(1^+)$	BB^*, A_bA_b	$11.0(2)$	–

JH, Eichmann, Fischer, in preparation
Wallbott, Eichmann, Fischer, Phys.Rev.D 100 (2019) 1, 014033,
Wallbott Eichmann, Fischer, Phys.Rev.D 102 (2020) 5, 051501

Lattice QCD, e.g.,
Leskovec, Meinel, Pflaumer, Wagner, Phys. Rev. D 100, 014503

Summary:

- DSE/BSE framework is a good tool to qualitatively analyse the charm and bottom four-quark state region.
- New results for the the 1^{--} and 0^{-+} four-quark states.
- Analysed the norm contributions as a means to investigate the internal structure.

Outlook:

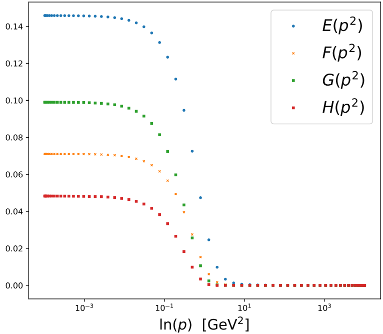
- Open-flavour states like
  - $I(1^+)$ and $I(0^+)$ with: $bc\bar{q}\bar{q}$, $cs\bar{q}\bar{q}$, $bs\bar{q}\bar{q}$, $cq\bar{q}\bar{q}$, $bq\bar{q}\bar{q}$, $sq\bar{q}\bar{q}$
  - $\frac{1}{2}(1^+)$ with $bb\bar{q}\bar{s}$
- Include the two-body quarkonium mixing.

Backup slides

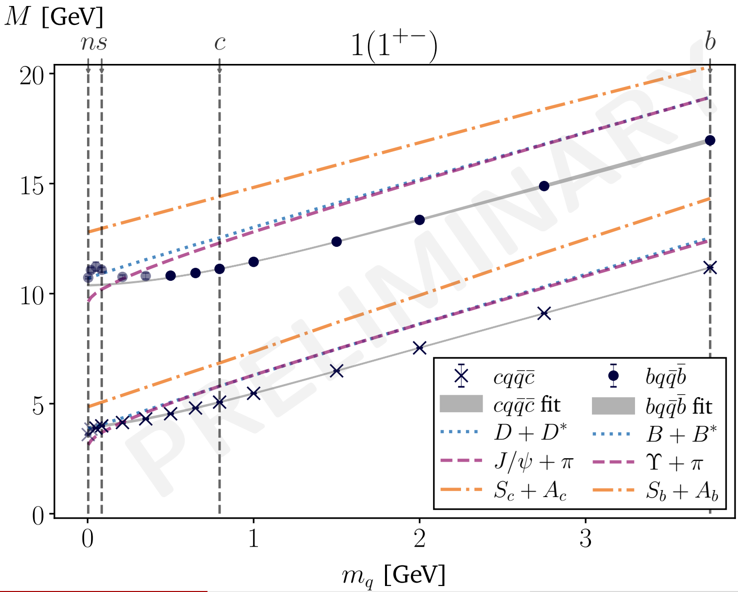
Pion BSE

Pion Bethe-Salpeter amplitude is given by:

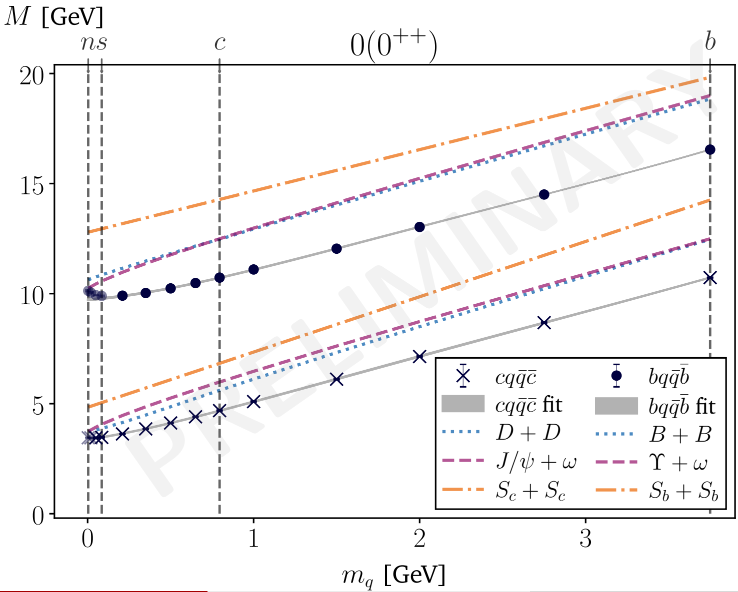
$$\Gamma_{\text{pion}}(p^2) = E(p^2)\cdot\tau_1(p,P) + F(p^2)\cdot\tau_2(p,P) + G(p^2)\cdot\tau_3(p,P) + H(p^2)\cdot\tau_4(p,P)$$



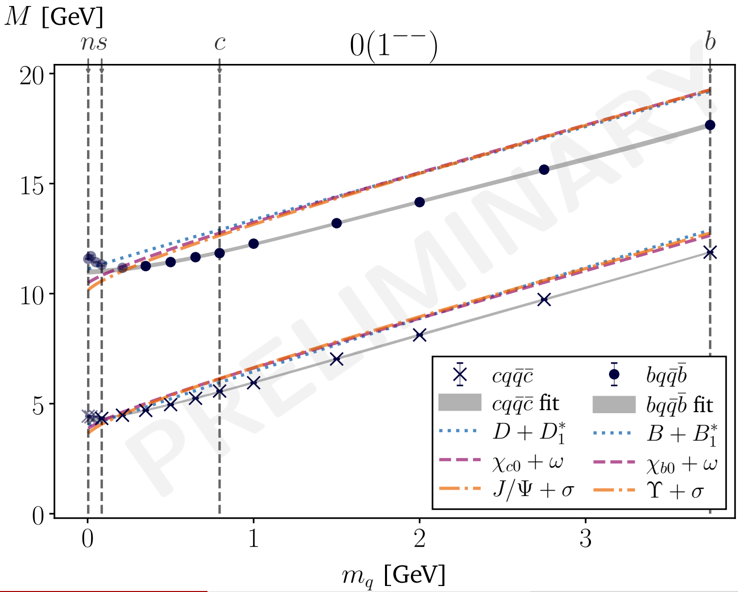
Quark mass evolution 1^{+-}



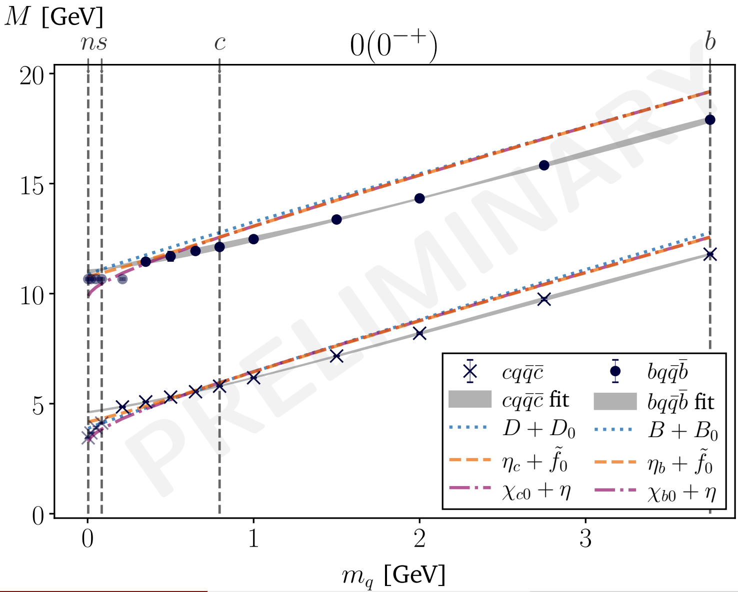
Quark mass evolution 0^{++}



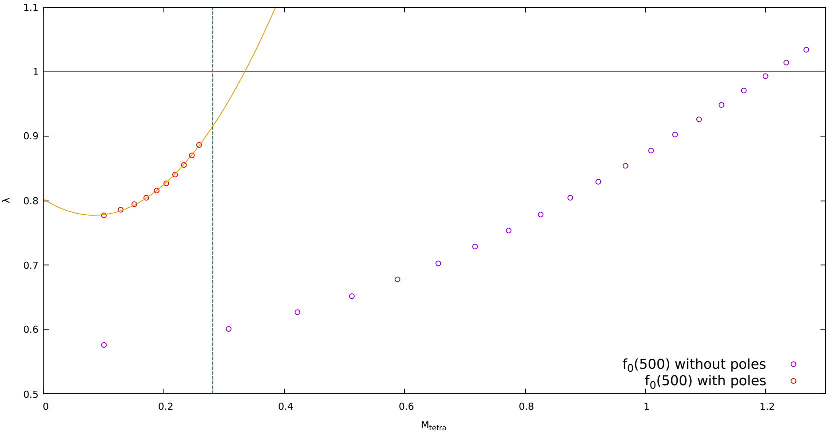
Quark mass evolution 1^{--}



Quark mass evolution 0^{-+}

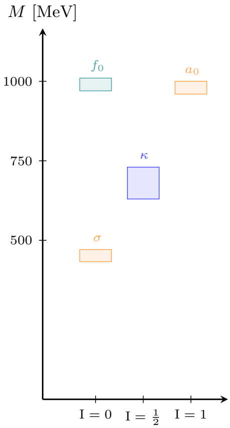


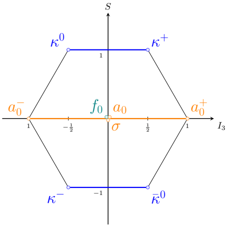
Eigenvalue curve



Light scalar mesons

The light scalar (0^{++}) mesons is an example where the Quark Model yields wrong predictions:





$f_0(980)$ $s\bar{s}$

$\kappa(700)$ $u\bar{s}$, $d\bar{s}$

$\left.\begin{matrix} a_0(980) \\ \sigma(500) \end{matrix}\right\} u\bar{u},\ d\bar{d},\ u\bar{d}$

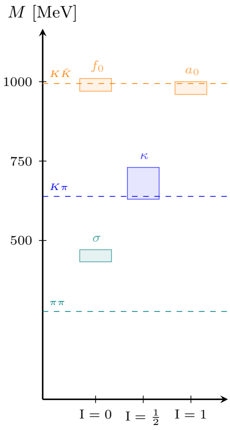
- Why are a_0, f_0 almost mass degenerate?
- Why are the decay widths so different?

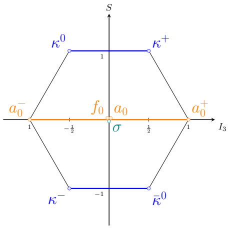
  $\Gamma(\sigma, \kappa) \approx 550\,\mathrm{MeV}$

  $\Gamma(a_0, f_0) \approx 50 - 100\,\mathrm{MeV}$
- Why are they so light?

Light scalar mesons

Suppose they were tetraquarks: Jaffe 1977; Close, Tornqvist 2002; Maiani, Polosa, Riquer 2004





$f_0(980)$, $a_0(980)$ } $us\bar{u}\bar{s}, \ldots$

$\kappa(700)$ $us\bar{u}\bar{d}, \ldots$

$\sigma(500)$ $ud\bar{u}\bar{d}$

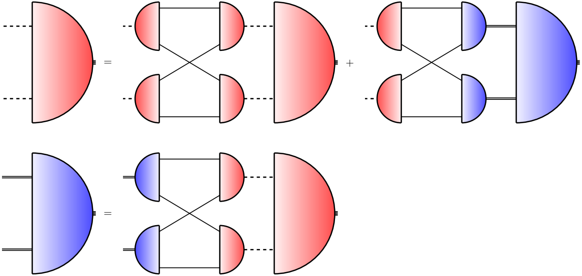
- Explains mass ordering and decay widths: a_0, f_0 couple to $K\bar{K}$, σ, κ large decay widths
- Non-$q\bar{q}$ nature of σ is supported by dispersive analyses, unitarized ChPT, large N_C, extended linear σ models, quark models

Pelaez, Phys.Rept. 658 (2016)

2-body approach

- Assume dominant 2-body forces → simplify the 4-body BSE to get the **2-body approach**.

  Santowsky, Fischer, Eur.Phys.J.C 82 (2022) 4, 313
  Santowsky, Eichmann, Fischer, Wallbott, Williams, Phys.Rev.D 102 (2020) 5, 056014



- It is a coupled system of meson-meson and diquark-antidiquark components which interact via quark exchange.
- This approach is close in spirit to an effective field theory description.