Heavy exotic mesons from lattice QCD

"Hirschegg 2024 – Strong interaction physics of heavy flavors"

Marc Wagner

Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik

mwagner@th.physik.uni-frankfurt.de

http://itp.uni-frankfurt.de/∼mwagner/

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Introductory remarks

- In this talk only heavy exotic mesons:
  - tetraquarks $\bar{b}\bar{c}qq$,
  - tetraquarks $\bar{b}\bar{b}qq$.

  (light quarks $q \in \{u, d, s\}$; in contrast to previous talks given at this meeting, tetraquark refers to any four-quark state, i.e. includes both mesonic molecules and diquark-antidiquark pairs).

- Lattice QCD $=$ numerical QCD.
  - Lattice QCD is not a model, there are no approximations.
  - Results are full and rigorous QCD results.
    $\rightarrow$ Lattice QCD simulations can be seen as computer experiments (based on QCD).
  - The investigation of exotic mesons in lattice QCD is technically very difficult.
    $\rightarrow$ Even though we use lattice QCD, there are assumptions and simplifying approximations (as you will see during the talk) ...

Two types of approaches

- Two types of approaches, when studying **heavy** exotic mesons with lattice QCD:
  - **Full lattice QCD computations of eigenvalues of the finite-volume QCD Hamiltonian**:
    - Masses of stable hadrons correspond to energy eigenvalues at infinite volume (for shallow bound states, it might be difficult to study this limit).
    - Masses and decay widths of resonances can be calculated from the volume dependence of the energy eigenvalues (difficult).

    $\rightarrow$ **Part 1 of this talk ($\bar{b}\bar{c}ud$).**
  - **Born-Oppenheimer approximation** (a 2-step procedure):

    (1) Compute the potential $V(r)$ of the two heavy quarks (approximated as static quarks) in the presence of two light quarks and/or gluons using lattice QCD.
    $\rightarrow$ full QCD results

    (2) Use standard techniques from quantum mechanics and $V(r)$ to study the dynamics of two heavy quarks (Schrödinger equation, scattering theory, etc.).
    $\rightarrow$ an approximation

    (+) Provides physical insights.

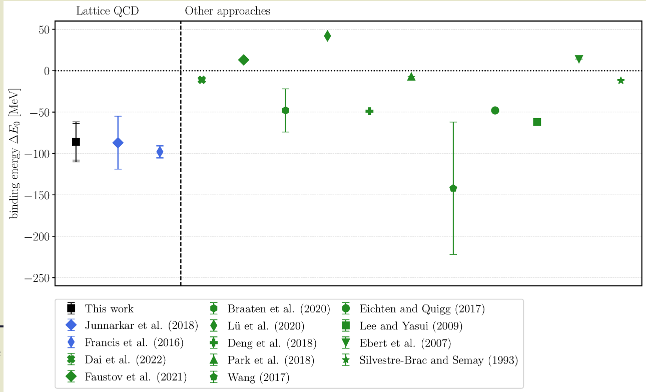
    (−) An approximation.

    $\rightarrow$ **Part 2 of this talk ($\bar{b}\bar{b}ud$).**

Part 1:
Full lattice QCD computations of eigenvalues of the QCD Hamiltonian

$\bar{b}\bar{b}ud$, $I(J^P) = 0(1^+)$ and $\bar{b}\bar{b}us$, $J^P = 1^+$

- QCD-stable $\bar{b}\bar{b}ud$ tetraquark, $I(J^P) = 0(1^+)$, $\approx 130\,\text{MeV}$ below the BB^* threshold.
- QCD-stable $\bar{b}\bar{b}us$ tetraquark, $J^P = 1^+$, $\approx 90\,\text{MeV}$ below the BB_s^* threshold.
- Lattice QCD results from independent groups consistent within statistical errors.
  [A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. Lett. **118**, 142001 (2017) [arXiv:1607.05214]] ($\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$)
  [P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D **99**, 034507 (2019) [arXiv:1810.12285]] ($\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$)
  [L. Leskovec, S. Meinel, M. Pflaumer, M.W., Phys. Rev. D **100**, 014503 (2019) [arXiv:1904.04197]] ($\bar{b}\bar{b}ud$)
  [P. Mohanta, S. Basak, Phys. Rev. D **102**, 094516 (2020) [arXiv:2008.11146]] ($\bar{b}\bar{b}ud$)
  [S. Meinel, M. Pflaumer, M.W., Phys. Rev. D **106**, 034507 (2022) [arXiv:2205.13982]] ($\bar{b}\bar{b}us$)
  [R. J. Hudspith, D. Mohler, Phys. Rev. D **107**, 114510 (2023) [arXiv:2303.17295]] ($\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$)
  [T. Aoki, S. Aoki, T. Inoue, [arXiv:2306.03565]] ($\bar{b}\bar{b}ud$)
- Strong discrepancies between non-lattice QCD results.
- A recent full lattice QCD study of the $\bar{b}\bar{b}ud$ and $\bar{b}\bar{b}us$ systems will be presented in the next talk by R. J. Hudspith.



$\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$ and $0(1^+)$ (1)

- T_{cc} ($\bar{c}\bar{c}ud$ with $I(J^P) = 0(1^+)$): slightly below the DD^* threshold, almost QCD-stable. (experiment)
- $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$: ≈ 100 MeV below the DD^* threshold, QCD-stable. (lattice QCD)
- What about $\bar{b}\bar{c}ud$ with $I(J^P) = 0(1^+)$ (and also $I(J^P) = 0(0^+)$)?
  - Physics might be somewhat different, because of non-identical heavy quark flavors.
  - Existing lattice studies contradictory or inconclusive.
    [A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. D **99**, 054505 (2019) [arXiv:1810.10550]] (hints for a bound state)
    [R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis, K. Maltman, Phys. Rev. D **102**, 114506 (2020) [arXiv:2006.14294]] (previous hints disappeared)
    [S. Meinel, M. Pflaumer, M.W., Phys. Rev. D **106**, 034507 (2022) [arXiv:2205.13982]] (no evidence for a bound state, a shallow bound state could not be ruled out)
    [M. Padmanath, A. Radhakrishnan, N. Mathur, [arXiv:2307.14128]] (bound state ≈ 43 MeV below the BD^* threshold via Lüscher's method)
  - Expected to be close to the B^*D threshold.
    $\rightarrow$ Lattice QCD studies technically difficult.

$\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$ and $0(1^+)$ (2)

- In the following a summary of
  [C. Alexandrou, J. Finkenrath, T. Leontiou, S. Meinel, M. Pflaumer, M.W., [arXiv:2312.02925]].
  - $\bar{b}\bar{c}ud$ systems with $I(J^P) = 0(1^+)$ and $I(J^P) = 0(0^+)$.
  - Different lattice setup and substantially more advanced methods compared to previous work.
    $\rightarrow$ Local and scattering operators at the source and at the sink of correlation functions.
    $\rightarrow$ Application of Lüscher's method to multiple excited states.
    $\rightarrow$ Reliable determination of the energy dependence of B-D and B^*-D S-wave scattering amplitudes.

Lattice setup

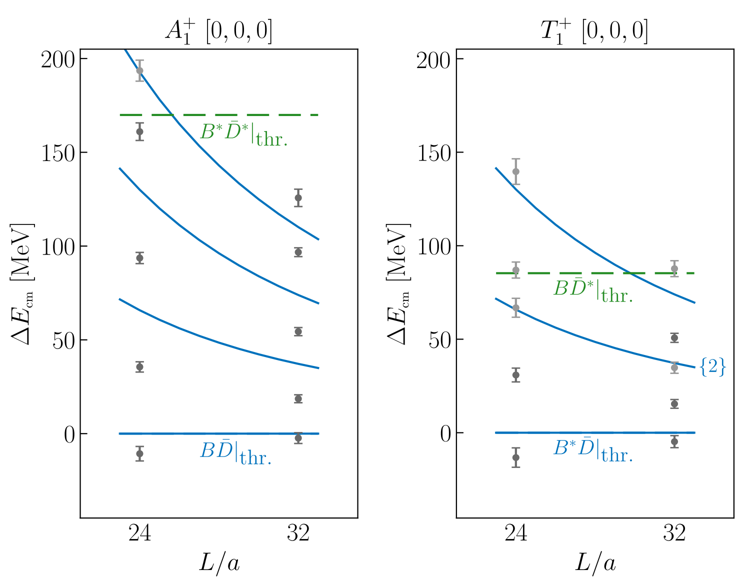
- Gauge link configurations generated with $N_f = 2 + 1 + 1$ flavors of highly improved staggered (HISQ) quarks by the MILC collaboration.
  [A. Bazavov *et al.* [MILC], Phys. Rev. D **87**, 054505 (2013) [arXiv:1212.4768]]
  - Two ensembles, which differ in the spatial volume:
    - $a \approx 0.12\,\text{fm}$.
    - $24^3 \times 64$, i.e. spatial lattice extent $\approx 2.9\,\text{fm}$,
      $32^3 \times 64$, i.e. spatial lattice extent $\approx 3.8\,\text{fm}$.
    - Pion mass $m_\pi \approx 220\,\text{MeV}$.
- Mixed-action setup tested and used by the PNDME collaboration for nucleon-structure computations.
  [T. Bhattacharya *et al.* [PNDME], Phys. Rev. D **92**, 094511 (2015) [arXiv:1506.06411]]
  [R. Gupta, Y. C. Jang, B. Yoon, H. W. Lin, V. Cirigliano, T. Bhattacharya, Phys. Rev. D **98**, 034503 (2018) [arXiv:1806.09006]]
  - Clover-improved Wilson action with HYP-smeared gauge links for the valence light and charm quarks.

$\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$ and $0(1^+)$ (3)

- Black and gray data points:
  Lowest five finite-volume energy levels as functions of the spatial lattice extent L.
  - First lattice QCD study of $\bar{b}\bar{c}ud$ using both local operators ("tetraquark structure") and scattering operators ("meson-meson structure") at the source and at the sink.
  - Such a set of operators might be necessary to get correct and precise results for the low-lying finite-volume energy levels.

- Blue curves:
  Noninteracting $B^{(*)}$-D energy levels, $E = E_{B^{(*)}}(\mathbf{p}^2) + E_D(\mathbf{p}^2)$ with momenta $\mathbf{p}$ satisfying periodic boundary conditions.

- Significant downward shift of finite-volume energy levels compared to noninteracting energy levels ("a larger number of energy levels").
  $\rightarrow$ A hint for the existence of a pole in the scattering amplitude, i.e. a shallow bound state or a resonance.



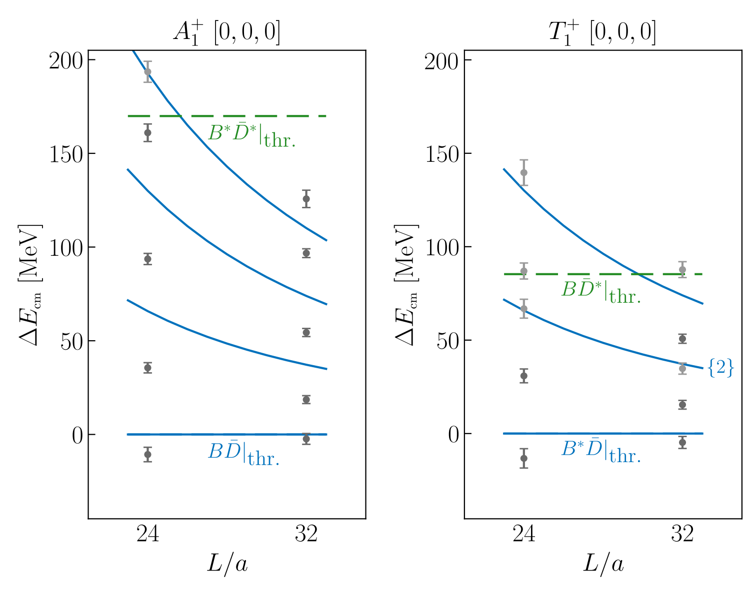
$\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$ and $0(1^+)$ (4)

- Rigorously investigate the existence of bound states or resonances by mapping the finite-volume energy levels E_n to infinite-volume S-wave $B^{(*)}$-D scattering phase shifts,

$$\cot \delta_0(k_n) = \frac{2 Z_{00}(1; (k_n L / 2\pi)^2)}{\pi^{1/2} k_n L}$$

(Lüscher's method).

  - Z_{00}: generalized zeta function.
  - k_n: scattering momenta associated with energy levels E_n, calculated via $E_n = E_{B^{(*)}}(k_n^2) + E_D(k_n^2)$.
  - Single-channel, single-partial-wave approach:
    → Only extract the phase shifts for energy levels below the B^*-D^* $(J = 0)$ and B-D^* $(J = 1)$ thresholds.
    → For $J = 1$ exclude the second excitation, because it is strongly D-wave dominated.
    (use black points, exclude gray points)



$\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$ and $0(1^+)$ (5)

- Blue data points:
  Infinite-volume S-wave $B^{(*)}$-D scattering phase shifts.
  $\rightarrow$ Data points / Lüscher's method valid above the left-hand cut associated with two-pion exchange and below the next threshold (B^*-D^* for $J = 0$ and B-D^* for $J = 1$).

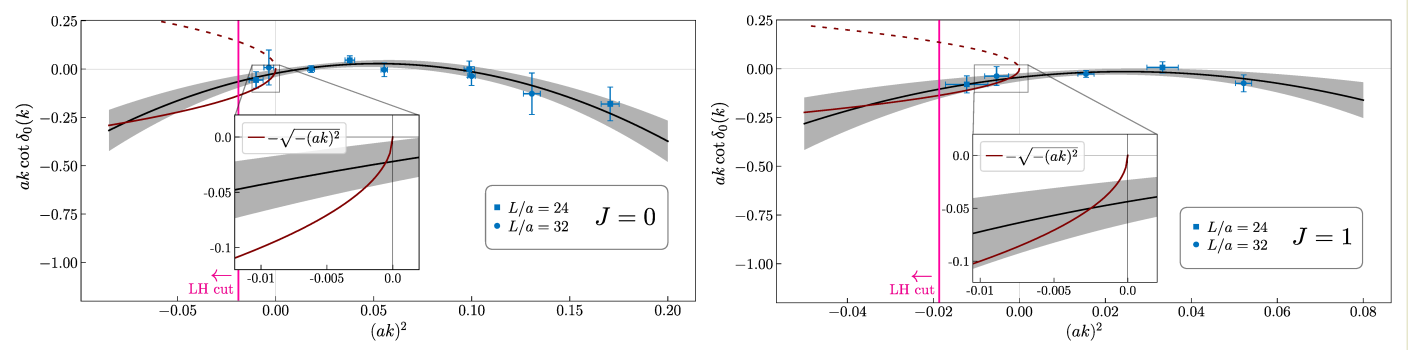
- Black curve:
  Effective-range expansion (ERE) fit,

$$k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + b_0 k^4 .$$

- S-wave scattering amplitude:

$$T_0(k) = \frac{1}{\cot \delta_0(k) - i},$$

  i.e. poles for $k \cot \delta_0(k) = ik$.



$\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$ and $0(1^+)$ (6)

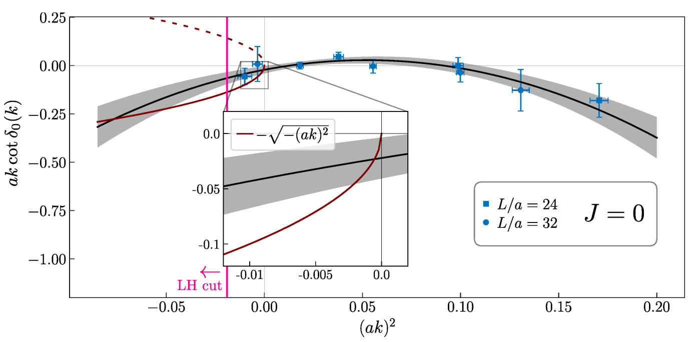
Bound states (1)

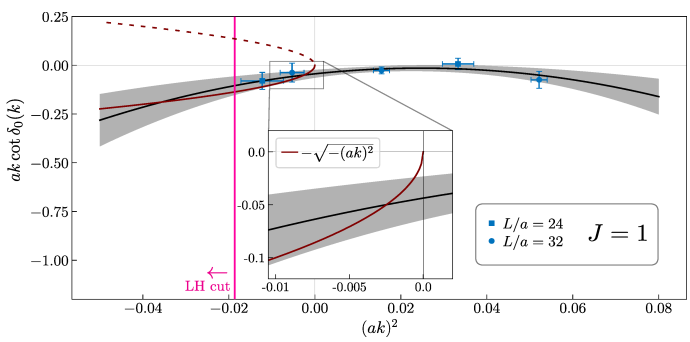
- Condition for poles in the scattering amplitude $k \cot \delta_0(k) = ik = \pm\sqrt{-k^2}$.
- For real energies, i.e. real k^2, the right-hand-side ik is real for $k^2 \leq 0$ (plotted in red); intersections with $k \cot \delta_0(k)$ correspond to poles below threshold, i.e. indicate bound states.

→ A bound state for $J = 0$ at $-0.5(0.9)$ MeV.

→ A bound state for $J = 1$ at $-2.4(2.9)$ MeV.

- Downward fluctuations of $k \cot \delta_0(k)$ by $\gtrsim 3\sigma$ for $J = 0$ and $\gtrsim 1\sigma$ for $J = 1$ would lead to a disappearance of the corresponding pole/bound state.



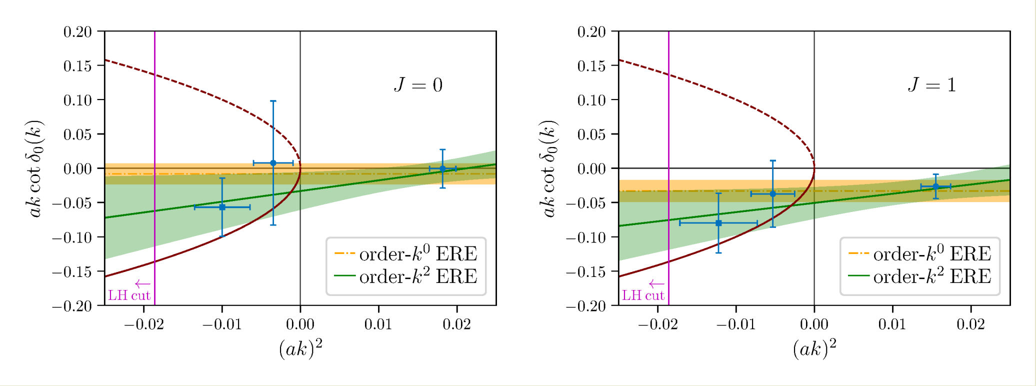


$\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$ and $0(1^+)$ (7)

Bound states (2)

- Additional test of our prediction of shallow bound states:
  - ERE fits of order k^0 and order k^2 using only the three data points closest to threshold.

  $\rightarrow$ Consistent results on the existence of shallow bound states and their masses.



$\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$ and $0(1^+)$ (8)

Resonances

- Poles in the scattering amplitude with real part of the energy above threshold, i.e. Re$(k^2) > 0$, and negative imaginary part indicate resonances.

$\rightarrow$ A resonance for $J = 0$ at $138(13)$ MeV, decay width $229(35)$ MeV.

$\rightarrow$ A resonance for $J = 1$ at $67(24)$ MeV, decay width $132(32)$ MeV.

- **These results on resonances should be treated with caution:**
  The resonance poles lie outside the radius of convergence of the ERE, which is limited by the presence of a left-hand cut associated with two-pion exchange (position of the cut ≈ 18 MeV below threshold for both $J = 0$ and $J = 1$).

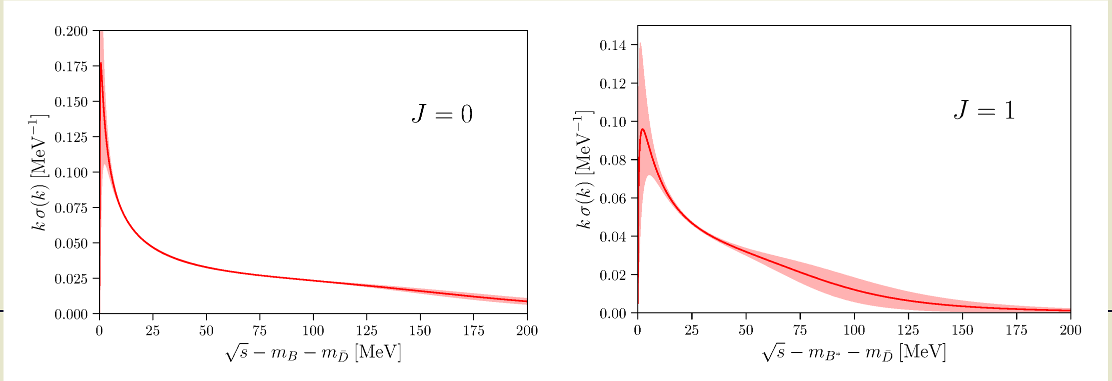
$\bar{b}\bar{c}ud$, $I(J^P) = 0(0^+)$ and $0(1^+)$ (9)

- S-wave cross section,

$$\sigma(k) = \frac{4\pi}{k^2}|T_0(k)|^2 \quad , \quad T_0(k) = \frac{1}{\cot\delta_0(k) - i}$$

with the ERE fit $k\cot\delta_0(k) = 1/a_0 + (r_0/2)k^2 + b_0 k^4$.

- scattering rate = flux $\times\ \sigma(k) \propto k\sigma(k)$ (for nonrelativistic k).
- Sharp enhancements in the scattering rates close to the thresholds, because of the shallow bound states.
- At higher energies still enhanced, because of the broad resonances.

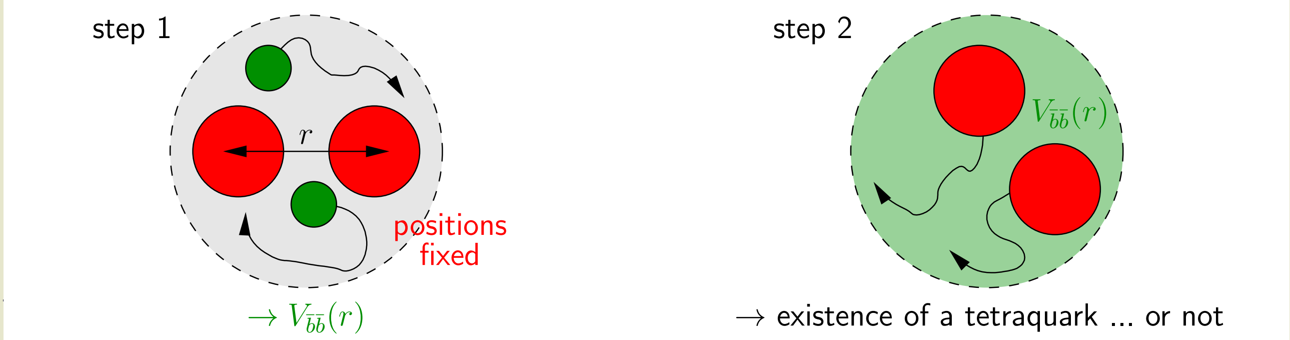


Part 2:
Born-Oppenheimer approximation

Basic idea: lattice QCD + BO

- Start with $\bar{b}\bar{b}qq$.
- $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$ is the bottom counterpart of the experimentally observed T_{cc}.
  [R. Aaij *et al.* [LHCb], Nature Phys. **18**, 751-754 (2022) [arXiv:2109.01038]].
- Study such $\bar{b}\bar{b}qq$ tetraquarks in two steps:
  (1) **Compute potentials of the two static quarks $\bar{b}\bar{b}$ in the presence of two lighter quarks qq ($q \in \{u, d, s\}$) using lattice QCD.**
  (2) **Check, whether these potentials are sufficiently attractive to host bound states or resonances** ($\rightarrow$ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

(1) + (2) $\rightarrow$ Born-Oppenheimer approximation.

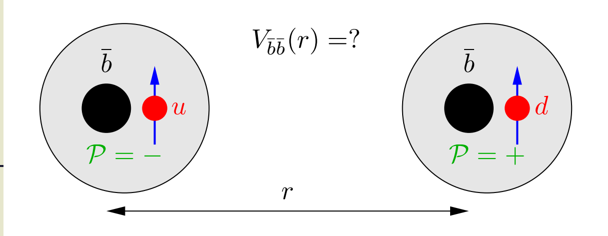


$\bar{b}\bar{b}qq$ / BB potentials (1)

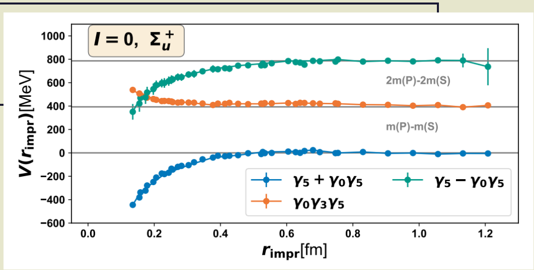
- At large $\bar{b}\bar{b}$ separation r, the four quarks form two static-light mesons $B = \bar{b}q$ and $B = \bar{b}q$.
- Potentials of static quarks are independent of the heavy spins.
- Consider only trial states/operators with vanishing orbital angular momentum, i.e. consider
  - pseudoscalar/vector mesons ($j^P = (1/2)^-$, PDG: B, B^*),
  - scalar/pseudovector mesons ($j^P = (1/2)^+$, PDG: B_0^*, B_1^*),

  which are among the lightest static-light mesons (j: spin of the light degrees of freedom).
- Compute and study the dependence of $\bar{b}\bar{b}$ potentials in the presence of qq on
  - the "light" quark flavors $q \in \{u, d, s\}$ (isospin, flavor),
  - the "light" quark spins (the static quark spin is irrelevant),
  - the types of the mesons B, B^* and/or B_0^*, B_1^* (parity).

  $\rightarrow$ Many different channels: attractive as well as repulsive, different asymptotic values ...



$\bar{b}\bar{b}qq$ / BB **potentials (2)**



- Rotational symmetry broken by static quarks $\bar{b}\bar{b}$.
- Symmetries and quantum numbers:
  - $|j_z| \equiv \Lambda$: rotations around the separation axis (e.g. z axis).
  - $P \equiv \eta$: parity.
  - $P_x \equiv \epsilon$: reflection along an axis perpendicular to the separation axis (e.g. x axis).
- To determine $\bar{b}\bar{b}$ potentials $V_{\bar{b}\bar{b};I,\Lambda_\eta^\epsilon}(r)$, compute temporal correlation functions

$$\langle\Omega|\mathcal{O}^\dagger_{BB,\Gamma}(t)\mathcal{O}_{BB,\Gamma}(0)|\Omega\rangle \propto_{t\rightarrow\infty} e^{-V_{\bar{b}\bar{b};I,\Lambda_\eta^\epsilon}(r)t}$$

of operators

$$\mathcal{O}_{BB,\Gamma} \quad = \quad 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)q^a_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)q^b_B(+\mathbf{r}/2)\Big).$$

  - $C = \gamma_0\gamma_2$ (charge conjugation matrix).
  - $qq \in \{ud - du\,,\, uu\,,\, dd\,,\, ud + du\}$ (isospin I, I_z).
  - Γ is an arbitrary combination of γ matrices (spin Λ, parity η, ϵ).
  - $\tilde{\Gamma} \in \{(1 - \gamma_0)\gamma_5\,,\, (1 - \gamma_0)\gamma_j\}$ (irrelevant).

Lattice setup

- Majority of published results computed on ETMC gauge link ensembles:
  - $N_f = 2$ dynamical quark flavors.
  - Lattice spacing $a \approx 0.079\,\text{fm}$.
  - $24^3 \times 48$, i.e. spatial lattice extent $\approx 1.9\,\text{fm}$.
  - Three different pion masses $m_\pi \approx 340\,\text{MeV}$, $m_\pi \approx 480\,\text{MeV}$, $m_\pi \approx 650\,\text{MeV}$.

  [R. Baron *et al.* [ETM Collaboration], JHEP **1008**, 097 (2010) [arXiv:0911.5061]

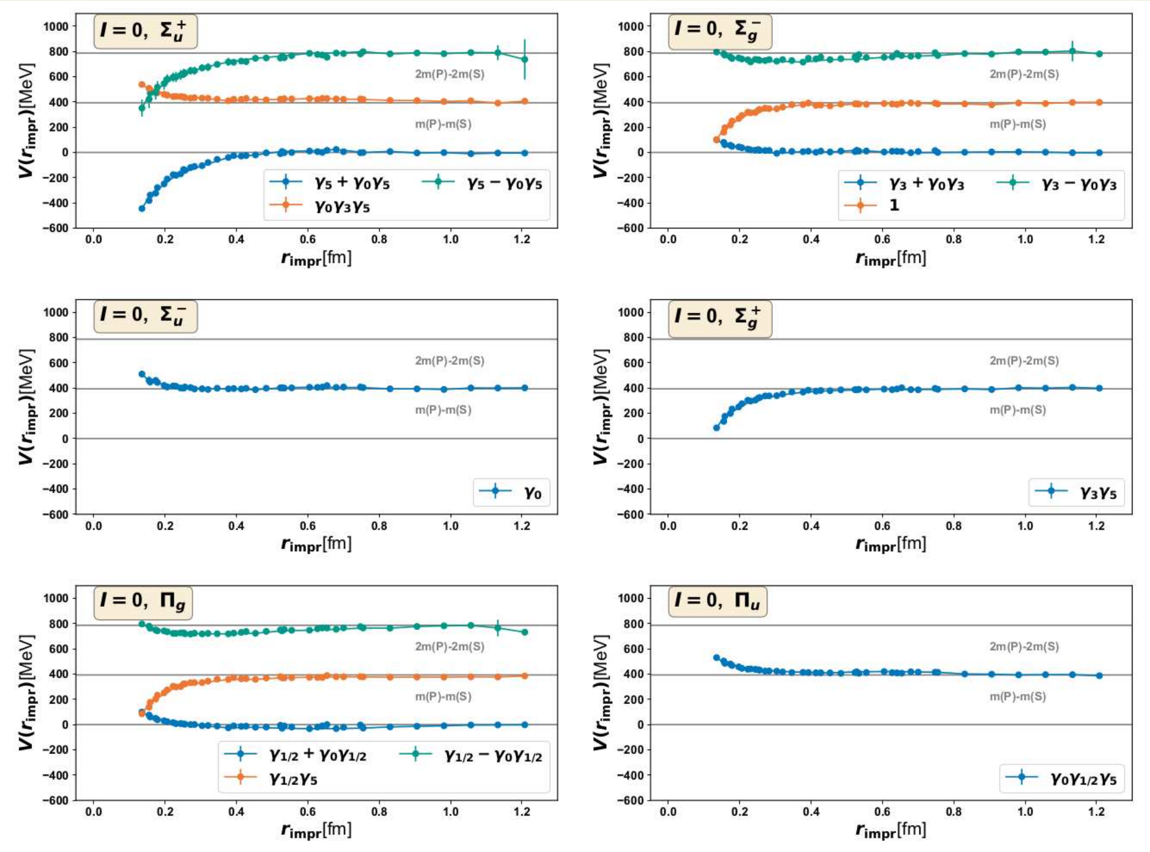
- Recent results (not yet published) computed on CLS gauge link ensembles:
  - $N_f = 2$ dynamical quark flavors.
  - Lattice spacing $a \approx 0.0749\,\text{fm}$.
  - $32^3 \times 64$, i.e. spatial lattice extent $\approx 2.42\,\text{fm}$.
  - Pion mass $m_\pi \approx 331\,\text{MeV}$.

  [P. Fritzsch, F. Knechtli, B. Leder, M. Marinkovic, S. Schaefer, R. Sommer, F. Virotta, Nucl. Phys. B **865**, 397-429 (2012) [arXiv:1205.5380]]

  [G. P. Engel, L. Giusti, S. Lottini and R. Sommer, Phys. Rev. D **91**, no. 5, 054505 (2015) [arXiv:1411.6386]]

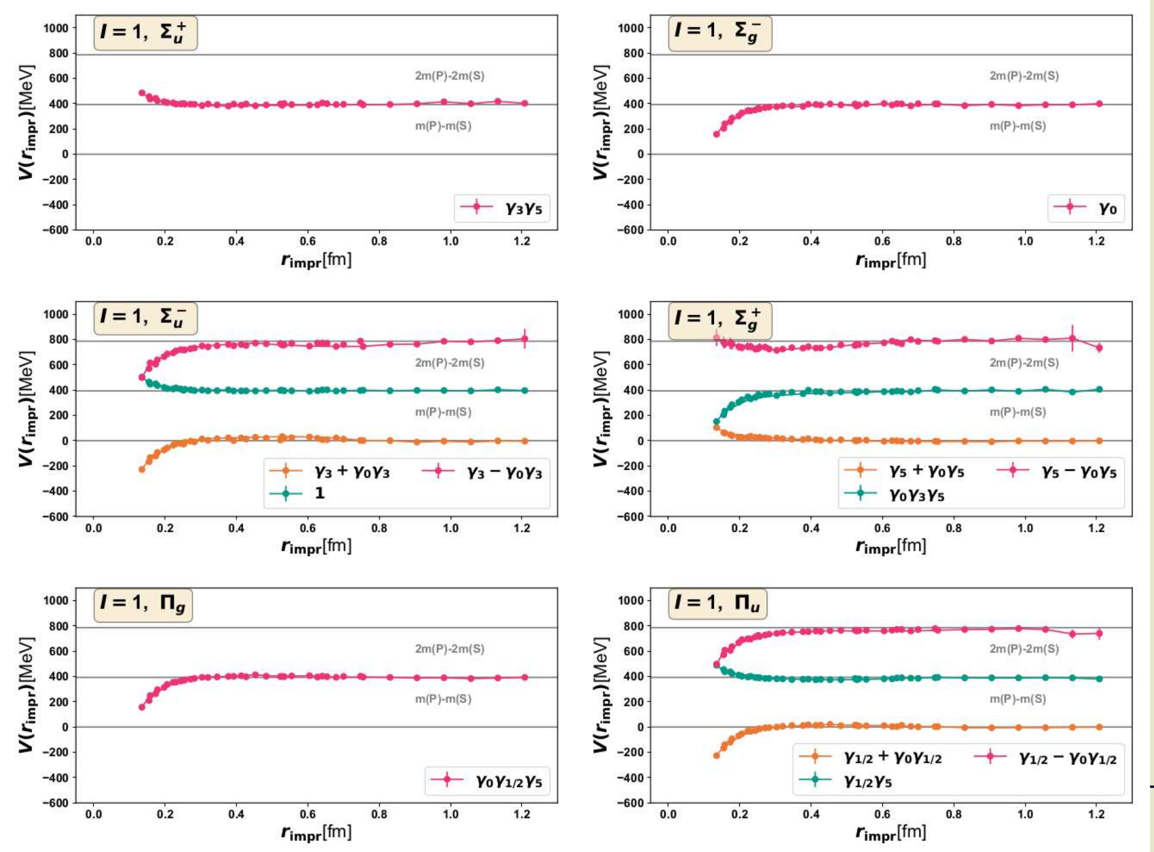
$\bar{b}\bar{b}qq$ / BB potentials (3)

[P. Bicudo, M. Marinkovic, L. Müller, M.W. unpublished ongoing work]



$\bar{b}\bar{b}qq$ / BB potentials (4)

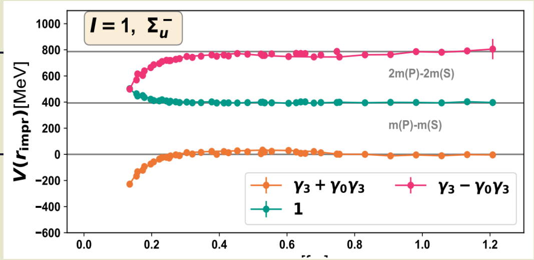
[P. Bicudo, M. Marinkovic, L. Müller, M.W. unpublished ongoing work]



$\bar{b}\bar{b}qq$ / BB potentials (5) to (8)

- **Why are there three different asymtotic values?**
  - They correspond to $B^{(*)}B^{(*)}$ potentials, to $B^{(*)}B^*_{0,1}$ potentials and $B^*_{0,1}B^*_{0,1}$ potentials.
- **Why are certain channels attractive and others repulsive?**
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ $\rightarrow$ attractive $\bar{b}\bar{b}qq$ / BB potentials.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ $\rightarrow$ repulsive $\bar{b}\bar{b}qq$ / BB potentials.
  - Because of the Pauli principle and "1-gluon exchange" at small r.
- **24 different (i.e. non-degenerate) $\bar{b}\bar{b}qq$ / BB potentials.**

$\bar{b}\bar{b}qq$ / BB potentials (5)



Why are there three different asymtotic values?

- Differences $\approx 400\,\text{MeV}$, approximately the mass difference of $B^{(*)}$ $(P = -)$ and $B^*_{0,1}$ $(P = +)$.
- Suggests that the three different asymtotic values correspond to $B^{(*)}B^{(*)}$ potentials, to $B^{(*)}B^*_{0,1}$ potentials and $B^*_{0,1}B^*_{0,1}$ potentials.
- Can be checked and confirmed, by rewriting the $\bar{b}\bar{b}qq$ creation operators in terms of meson-meson creation operators (Fierz transformation).
- Example: $qq = uu$, $\Gamma = (\gamma_3 + \gamma_0\gamma_3)$ (attractive, lowest asymptotic value),

$$\Big(C(\gamma_3 + \gamma_0\gamma_3)\Big)_{AB}\Big(\bar{Q}_C(-\mathbf{r}/2)q_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q_B(+\mathbf{r}/2)\Big) \quad \propto$$
$$\propto \quad (B^{(*)})_\uparrow(B^{(*)})_\downarrow + (B^{(*)})_\downarrow(B^{(*)})_\uparrow.$$

- Example: $qq = uu$, $\Gamma = 1$ (repulsive, medium asymptotic value),

$$\Big(C1\Big)_{AB}\Big(\bar{Q}_C(-\mathbf{r}/2)q_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q_B(+\mathbf{r}/2)\Big) \quad \propto$$
$$\propto \quad (B^{(*)})_\uparrow(B^*_{0,1})_\downarrow - (B^{(*)})_\downarrow(B^*_{0,1})_\uparrow + (B^*_{0,1})_\uparrow(B^{(*)})_\downarrow - (B^*_{0,1})_\downarrow(B^{(*)})_\uparrow.$$

$\bar{b}\bar{b}qq$ / BB potentials (6)

Why are certain channels attractive and others repulsive? (1)

- Fermionic wave functions must be antisymmetric (Pauli principle); in quantum field theory/QCD automatically realized.
- qq isospin: $I = 0$ antisymmetric, $I = 1$ symmetric.
- qq angular momentum/spin: $j = 0$ antisymmetric, $j = 1$ symmetric.
- qq color:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$: must be antisymmetric, i.e., a triplet $\bar{3}$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$: must be symmetric, i.e., a sextet 6.
- The four quarks $\bar{b}\bar{b}qq$ must form a color singlet:
  - qq in a color triplet $\bar{3}$ $\rightarrow$ static quarks $\bar{b}\bar{b}$ also in a triplet 3.
  - qq in a color sextet 6 $\rightarrow$ static quarks $\bar{b}\bar{b}$ also in a sextet $\bar{6}$.

$\bar{b}\bar{b}qq$ / BB potentials (7)

Why are certain channels attractive and others repulsive? (2)

- Assumption: attractive/repulsive behavior of $\bar{b}\bar{b}$ at small separations r is mainly due to 1-gluon exchange,
  - color triplet 3 is attractive, $V_{\bar{b}\bar{b}}(r) = -2\alpha_s/3r$,
  - color sextet $\bar{6}$ is repulsive, $V_{\bar{b}\bar{b}}(r) = +\alpha_s/3r$

  (easy to calculate in LO perturbation theory).

- Summary:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ $\rightarrow$ attractive $\bar{b}\bar{b}$ potential $V_{\bar{b}\bar{b}}(r)$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ $\rightarrow$ repulsive $\bar{b}\bar{b}$ potential $V_{\bar{b}\bar{b}}(r)$.

- Expectation consistent with the obtained lattice results.

- **Pauli principle and assuming "1-gluon exchange" at small r explains, why certain channels are attractive and others repulsive.**

$\bar{b}\bar{b}qq$ / BB potentials (8)

- Summary of $\bar{b}\bar{b}qq$ / BB potentials:

$B^{(*)}B^{(*)}$ potentials:	attractive:	$1 \oplus 3 \oplus 6$	(10 states).
	repulsive:	$1 \oplus 3 \oplus 2$	(6 states).
$B^{(*)}B^*_{0,1}$ potentials:	attractive:	$1 \oplus 1 \oplus 3 \oplus 3 \oplus 2 \oplus 6$	(16 states).
	repulsive:	$1 \oplus 1 \oplus 3 \oplus 3 \oplus 2 \oplus 6$	(16 states).
$B^*_{0,1}B^*_{0,1}$ potentials:	attractive:	$1 \oplus 3 \oplus 6$	(10 states).
	repulsive:	$1 \oplus 3 \oplus 2$	(6 states).

  - 2-fold degeneracy due to spin $j_z = \pm 1$.
  - 3-fold degeneracy due to isospin $I = 1$, $I_z = -1, 0, +1$.

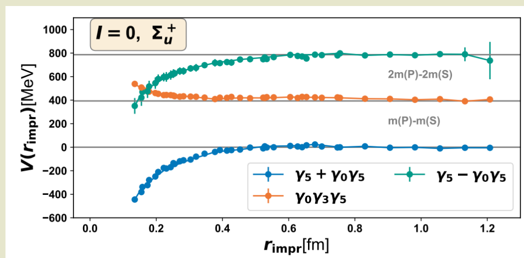
$\rightarrow 24$ **different** $\bar{b}\bar{b}qq$ **/** BB **potentials.**

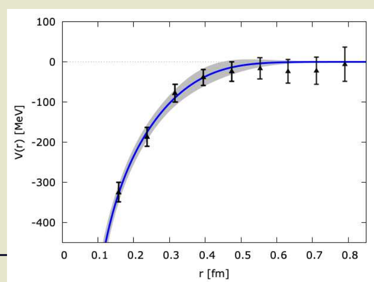
Stable $\bar{b}\bar{b}qq$ tetraquarks (2)

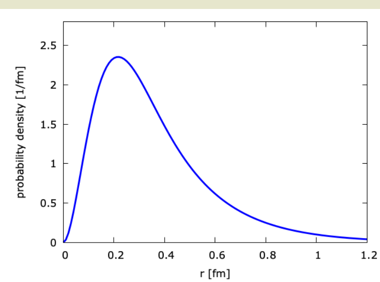
- The most attractive potential of a $B^{(*)}B^{(*)}$ meson pair has $I = 0, \Lambda_\eta^\epsilon = \Sigma_u^+$.
- Parameterize lattice results by

$$V_{\bar{b}\bar{b};0,\Sigma_u^+}(r) \quad = \quad -\frac{\alpha}{r}\exp\bigg(-\bigg(\frac{r}{d}\bigg)^p\bigg) + V_0$$

(1-gluon exchange at small r; color screening at large r).
[P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]







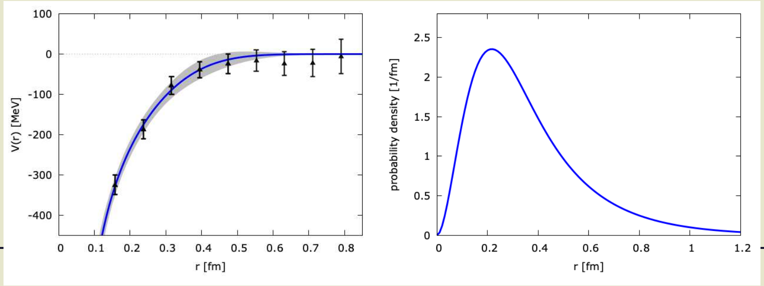
Stable $\bar{b}\bar{b}qq$ tetraquarks (2)

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ / BB potentials,

$$\left(\frac{1}{m_b}\left(-\frac{d^2}{dr^2}+\frac{L(L+1)}{r^2}\right)+V_{\bar{b}\bar{b};0,\Sigma_u^+}(r)-2m_B\right)R(r) \quad = \quad ER(r).$$

- Possibly existing bound states, i.e. $E < 0$, indicate QCD-stable $\bar{b}\bar{b}qq$ tetraquarks.
- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}\bar{b}$:
  - Binding energy $E = -90^{+43}_{-36}$ MeV with respect to the BB^* threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.

[P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]



Further $\bar{b}\bar{b}qq$ results (1)

- **Are there further QCD-stable $\bar{b}\bar{b}qq$ tetraquarks with other $I(J^P)$ and light flavor quantum numbers?**

  $\rightarrow$ No, not for $qq = ud$ (both $I = 0, 1$), not for $qq = ss$.
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]

  $\rightarrow$ $\bar{b}\bar{b}us$ was not investigated.

  – Strong evidence from full QCD computations that a QCD-stable $\bar{b}\bar{b}us$ tetraquark exists (see part 2 of this talk).

- **Effect of heavy quark spins:**

  – Expected to be $\mathcal{O}(m_{B^*} - m_B) = \mathcal{O}(45\,\text{MeV})$.

  – Previously ignored (potentials of static quarks are independent of the heavy spins).

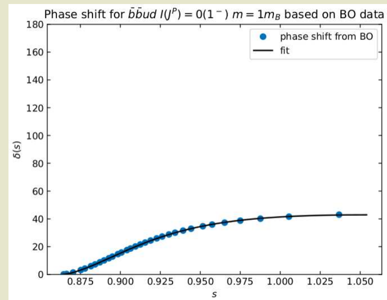
  – In [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]] included in a crude phenomenological way via a BB^* and a B^*B^* coupled channel Schrödinger equation with the experimental mass difference $m_{B^*} - m_B$ as input.

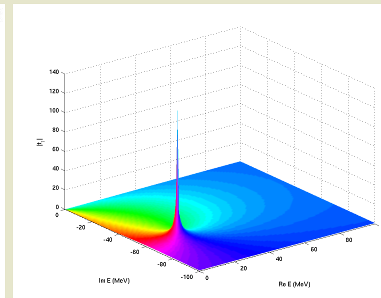
  $\rightarrow$ Binding energy reduced from around $90\,\text{MeV}$ to $59\,\text{MeV}$.

  $\rightarrow$ Physical reason: the previously discussed attractive potential does not only correspond to a lighter BB^* pair, but has also a heavier B^*B^* contribution.

Further $\bar{b}\bar{b}qq$ results (2)

- **Are there $\bar{b}\bar{b}qq$ tetraquark resonances?**
  - In [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]] resonances studied via standard scattering theory from quantum mechanics textbooks.





$\rightarrow$ Heavy quark spins ignored.

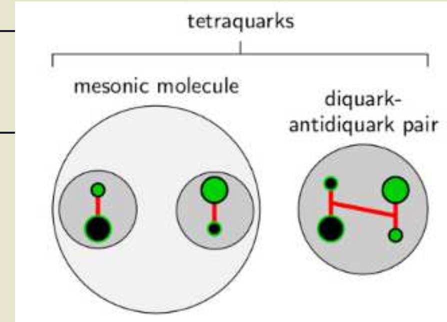
$\rightarrow$ Indication for $\bar{b}\bar{b}ud$ tetraquark resonance with $I(J^P) = 0(1^-)$ found, $E = 17^{+4}_{-4}$ MeV above the BB threshold, decay width $\Gamma = 112^{+90}_{-103}$ MeV.

  - In [J. Hoffmann, A. Zimermmane-Santos and M.W., PoS **LATTICE2022**, 262 (2023) [arXiv:2211.15765]] heavy quark spins included.

$\rightarrow$ $\bar{b}\bar{b}ud$ resonance not anymore existent.

$\rightarrow$ Physical reason: the relevant attractive potential does not only correspond to a lighter BB pair, but has also a heavier B^*B^* contribution.

Further $\bar{b}\bar{b}qq$ results (3)



- Structure of the QCD-stable $\bar{b}\bar{b}ud$ tetraquark with $I(J^P) = 0(1^+)$: meson-meson (BB) versus diquark-antidiquark (Dd).
  - Use not just one but two operators,

$$\mathcal{O}_{BB,\Gamma} = 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)q^a_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)q^b_B(+\mathbf{r}/2)\Big)$$

$$\mathcal{O}_{Dd,\Gamma} = -N_{Dd}\epsilon^{abc}\Big(q^b_A(\mathbf{z})(\mathcal{C}\Gamma)_{AB}q^c_B(\mathbf{z})\Big)$$

$$\epsilon^{ade}\Big(\bar{Q}^f_C(-\mathbf{r}/2)U^{fd}(-\mathbf{r}/2;\mathbf{z})(\mathcal{C}\tilde{\Gamma})_{CD}\bar{Q}^g_D(+\mathbf{r}/2)U^{ge}(+\mathbf{r}/2;\mathbf{z})\Big),$$

  compare the contribution of each operator to the $\bar{b}\bar{b}$ potential $V_{\bar{b}\bar{b};0,\Sigma_u^+}(r)$.
  [P. Bicudo, A. Peters, S. Velten, M.W., Phys. Rev. D **103**, 114506 (2021) [arXiv:2101.00723]]

→ $r \lesssim 0.2$ fm: Clear diquark-antidiquark dominance.

→ $0.5\,\text{fm} \lesssim r$: Essentially a meson-meson system.

→ Integrate over t to estimate the composition of the tetraquark: $\%BB \approx 60\%$, $\%Dd \approx 40\%$.