Heavy flavour spectroscopy from lattice QCD

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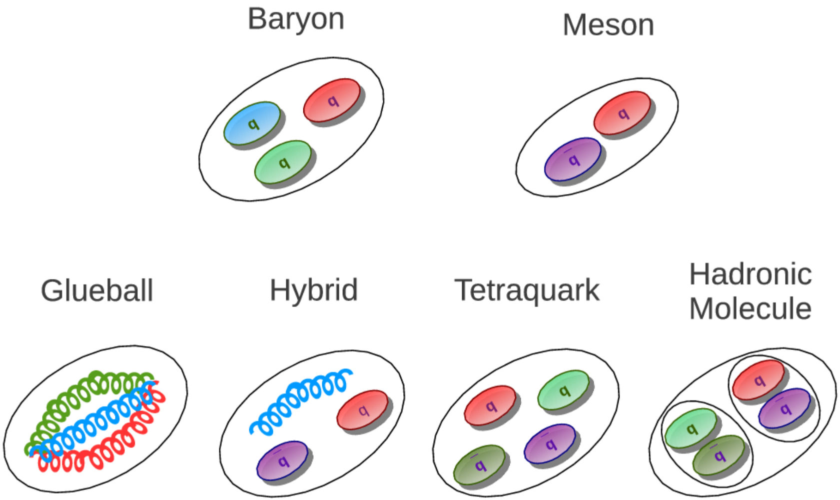
Trinity College Dublin

Strong Interaction Physics of Heavy Flavours, Hirschegg January 2024

QCD ADMITS A RICH AND EXOTIC SPECTRUM



WHY LATTICE QCD?

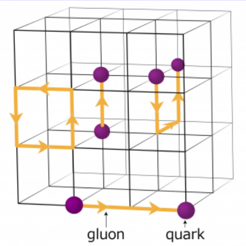
- A systematically-improvable non-perturbative formulation of QCD
  - Well-defined theory with the lattice a UV regulator
- Arbitrary precision is in principle possible
  - conceptual and practical complications can make this challenging!
- Facilitates numerical simulation
  - MCMC approach drawing from methods in statistical physics systems
- Starts from first principles - i.e. from the QCD Lagrangian
  - inputs are quark mass(es) and the coupling - can explore mass dependence and coupling dependence.

Practicalities of a lattice calculation
for spectroscopy

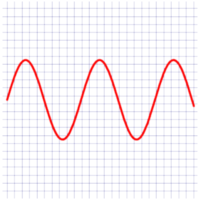
A LATTICE QCD PRIMER

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu} D_{\mu} - m\right)\psi - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

- Gluon fields are SU(3) matrices - links of a hypercube.
  $A_{\mu}(x) \rightarrow U_{\mu}(x) = \mathcal{P} e^{ig\int_{x}^{x+e_{\mu}} dz_{\mu} A_{\mu}(z)}$
- Quark fields $\psi(x)$ on sites with color, flavor, Dirac indices. Fermion discretisation - Wilson, Staggered, Overlap, . . .
- Derivatives → finite differences:
  $\nabla^{\mathrm{fwd}}_{\mu}\psi(x) = \frac{1}{a}\left[U_{\mu}(x)\psi(x+a\hat{\mu}) - \psi(x)\right]$



Lattice acts as UV and IR regulator:



- typical spacing:
  $0.04\mathrm{fm} \lesssim a \lesssim 0.2\mathrm{fm}$
  $(1\mathrm{GeV} \lesssim a^{-1} \lesssim 5\mathrm{Gev})$
- typical length: $2\mathrm{fm} \lesssim L \lesssim 6\mathrm{fm}$.
- (UV) $am_q \ll 1$; (IR) $M_{\pi} L \geq 4$

- Solve the QCD path integral on a finite lattice with spacing $a \neq 0$ with Monte Carlo in a Euclidean space-time metric (no useful importance sampling weight in Minkowski space).

  Observables determined from (Euclidean) path integrals of the QCD action

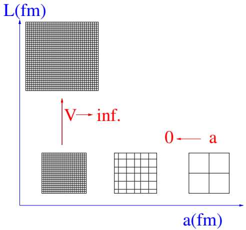
$$\langle \mathcal{O} \rangle = 1/Z \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \, \mathcal{O}[U, \bar{\psi}, \psi] e^{-S[U,\bar{\psi},\psi]}$$

- In principle the finite temporal extent implies a finite temperature e.g

$$S_f[\bar{\psi}, \psi, A_\mu] = \int_0^{1/T} d\tau \int_V d^3x \sum_f \bar{\psi}_f(x) \left(\gamma_\mu D_\mu + m_f\right) \psi_f(x)$$

- Change the temporal extent to change $T = 1/aN_t$

THE COMPROMISES AND CONSEQUENCES



1. **Working in a finite box at finite grid spacing**
   - Recover continuum QCD by extrapolation.
     Part of the error budget.
2. **Simulating at physical quark masses**
   - Computational and complexity cost of physical light and heavy quarks.
     Physical light & heavy quarks in reach. Mass dependence is a tool!
3. **Breaking symmetry**

   - Lorentz symmetry broken at $a \neq 0$: $SO(4)$ rotation group to rotation group of a hypercube. Identify states according to this symmetry.
4. **Euclidean time**
   - Energies via $C(t) \sim e^{-E_n t}$. No direct access to scattering matrix elements.
     Lüscher formalism and generalisations allow indirect access via finite volume.

CORRELATORS IN LATTICE EUCLIDEAN FIELD THEORY

- Physical observables $\mathcal{O}$ are determined from

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{O} e^{-S_{QCD}}$$

- Analytically integrate Grassman fields $(\Psi, \bar{\Psi}) \rightarrow$

$$\langle \mathcal{O} \rangle \overset{N_f=2}{=} \frac{1}{Z} \int \mathcal{D}U \det M^2 \mathcal{O} e^{-S_G}$$

  Calculated by importance sampling of gauge fields and averaging over ensembles.

- Simulate N_{cfg} samples of the field configuration, then

$$\langle \mathcal{O} \rangle = \lim_{N_{cfg} \rightarrow \infty} \frac{1}{N_{cfg}} \sum_{i=1}^{N_{cfg}} \mathcal{O}_i[U_i]$$

- Correlation functions have a (improvable!) statistically uncertainty $\sim 1/\sqrt{N_{cfg}}$.

A RECIPE FOR (MESON) SPECTROSCOPY

- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ e.g. from *distilled* fields [Peardon, PRD80 (2009) 054506].
- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

- Ground state mass from fits to $e^{-E_n t}$
- Beyond ground state: Solve generalised eigenvalue problem

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

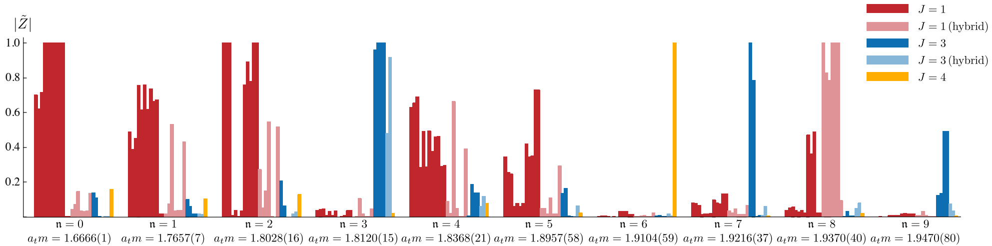
- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlator
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

- Results shown here for anisotropic lattices $a_t \ll a_s$.

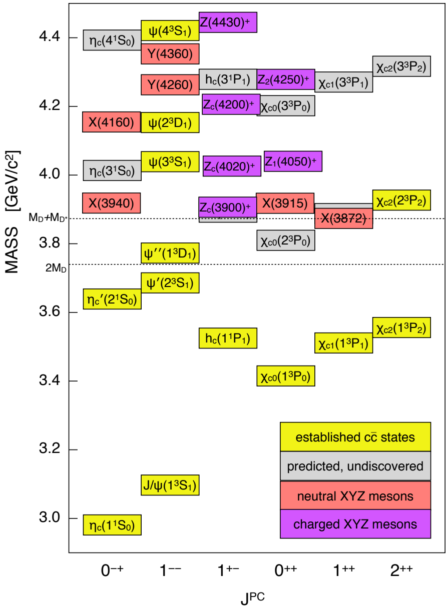
- operators of definite J^{PC} are subduced into relevant lattice irrep
- a subduced irrep carries a “memory” of continuum spin J from which it was subdduced - it **overlaps** predominantly with states of this J.

J	0	1	2	3	4
A_1	✓				✓
A_2				✓	
E			✓		✓
T_1		✓		✓	✓
T_2			✓	✓	✓

- Use overlaps, $Z = \langle 0|\Phi|k\rangle$, to identify continuum spins
- For $J \geq 2$, look for agreement between irreps
- Example (Υ): T_1^{--} irrep, with **Spin 1**, **Spin 3** and **Spin 4**.



RICH INTERPLAY OF THEORY AND EXPERIMENT



Below thresholds: "gold-plated" quantities

- Methods tested and validated.
- High statistics, improved actions - precision.
- Different actions in agreement.
- Comprehensive error budget

Above thresholds: high spin, exotic states & resonances

- Incomplete error budgets.
- Relatively heavy pions ← changing.

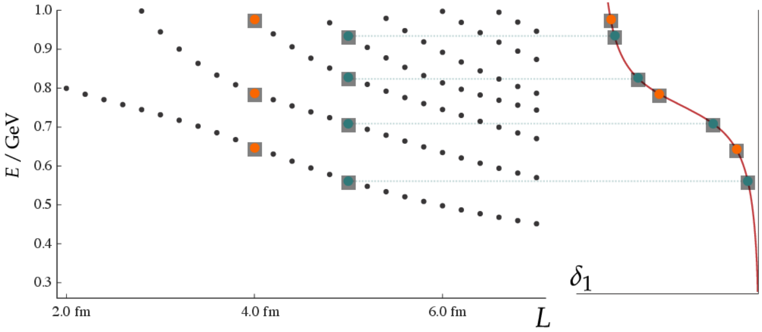
new ideas crucial

- Distillation - quark propagation enabling isoscalars, precision spectroscopy ...
- Framework for scattering & coupled channels.

BEYOND SIMPLE BOUND STATES: RESONANCES IN A EUCLIDEAN THEORY

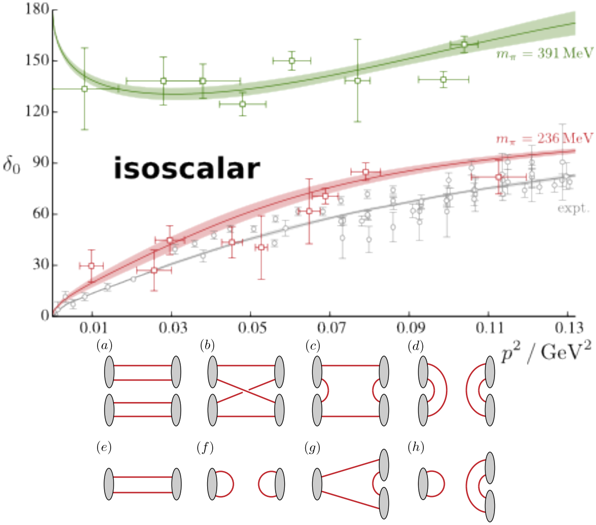
The problem: Lose direct access to scattering in a Euclidean QFT.
The solution: On lattice volumes extract the spectrum. Lüscher formalism (1991) allows to deduce phase shift information.

$$\det\left[\cot\delta(E_n^*) + \cot\phi(E_n, \vec{P}, L)\right] = 0$$



The more distinct spectrum points the better the phase shift picture

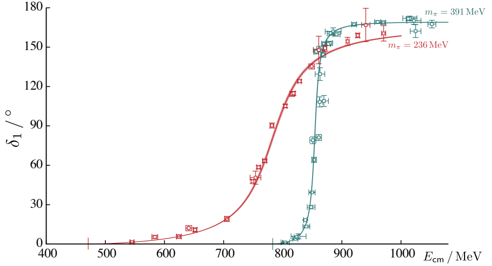
EXAMPLE: IN $\pi\pi$ SCATTERING



σ evolves from bound-state below $\pi\pi$ threshold at heavier mass to broad resonance at lighter mass [1607.05900].

$$\det\left[\mathbf{1} + i\rho(E)\cdot \mathbf{t}(E)\cdot(\mathbf{1} + i\mathcal{M}(E,L))\right] = 0.$$

ρ **resonance**

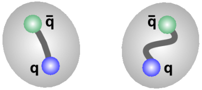


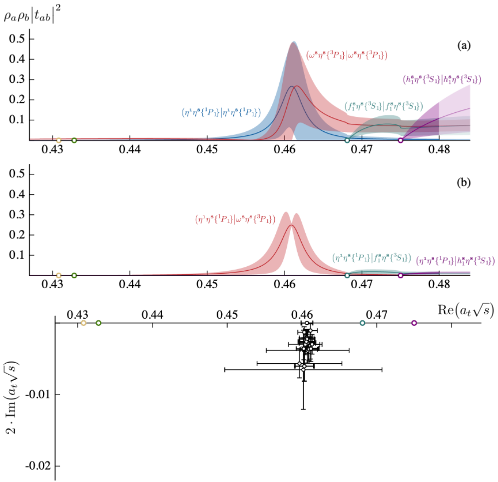
Lattice as a tool to study mass-dependence! [1507.02599]

EXAMPLE: EXOTIC $\pi_1(1^{-+})$ EXOTIC HYBRID MESON

A. J. Woss, et al, arXiv:2009.10034

- Calculation of the scattering amplitudes in exotic $J^{PC} = 1^{-+}$, $m_\pi \sim 700$ MeV, SU(3) point, 8 coupled channels.
- Narrow π_1 resonance found at heavy pion masses
- Crude extrapolation based on couplings suggests a broad resonance at lighter pion masses





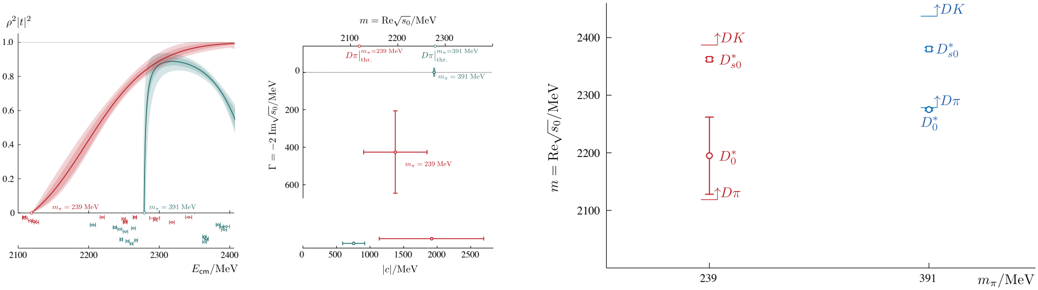
Towards a phenomenology of hybrid decays, starting from QCD

Heavy-light mesons

Scattering in D, D_s

Exploring B, B_s, B_c mesons

HEAVY-LIGHT MESON SCATTERING



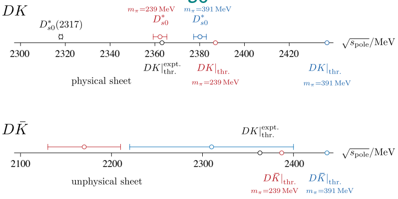
- A synthesis of results in $D\pi$, DK, $D\bar{K}$ yield information on D_0^*, $D_{s0}^*(2317)$

$D\pi$: D_0^*

$m = (2196 \pm 64)MeV$

$\Gamma = (425 \pm 224)MeV$

DK : $D_{s0}^*(2317)$



[2008.06432, 2102.04973]

BEYOND CHARM: B QUARKS AND ANISOTROPIC LATTICES

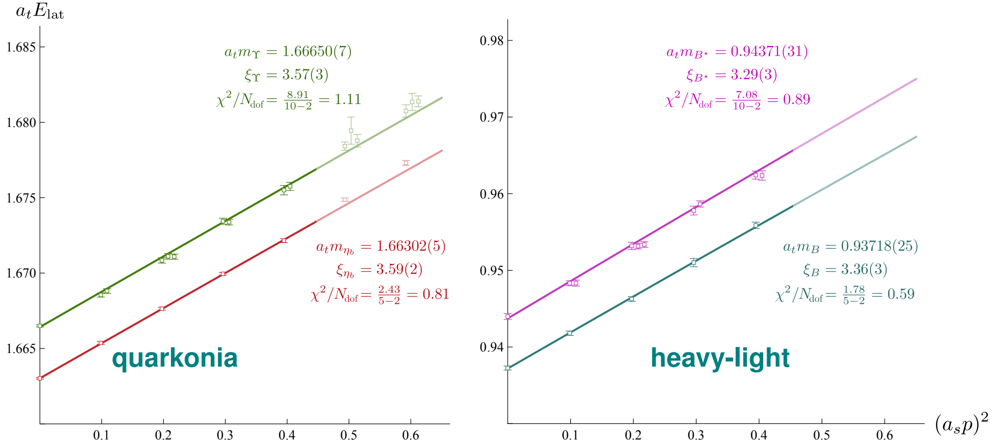
- Not a new story. Hashimoto, Onogi, Kronfeld et al investigated the practicalities in early 2000s. [PRD64 114503(2001); PRD64 074501 (2001); PRD66 014509 (2002)].
- Investigate discretisation effects via dispersion relations

$$(a_t E)^2 = (a_t M)^2 + \left(\frac{1}{\xi_M}\right)^2 (a_s p)^2$$

- Strategy: Mass-dependent parameter tuning, determine dispersion relations in heavy and heavy-light mesons.
- Explore $\mathcal{O}((a_s p)^2)$ discretisation effects in heavy sector [NPB (proc suppl) 47 (1996); NPB (proc suppl) 53 (1997)] and $M_1 \stackrel{?}{=} M_2$.
- Now: improvements on earlier studies include stout smeared spatial links and mass-dependent anisotropy tuning.
- Advantage: from charm to bottom in same framework.

THE LATTICE DISPERSION RELATION - NOW FOR B QUARKS

- Tune $M^{latt}_{\eta_b} = M^{expt}_{\eta_b}$ *and* ξ (from slope of dispersion) for η_b (red line).



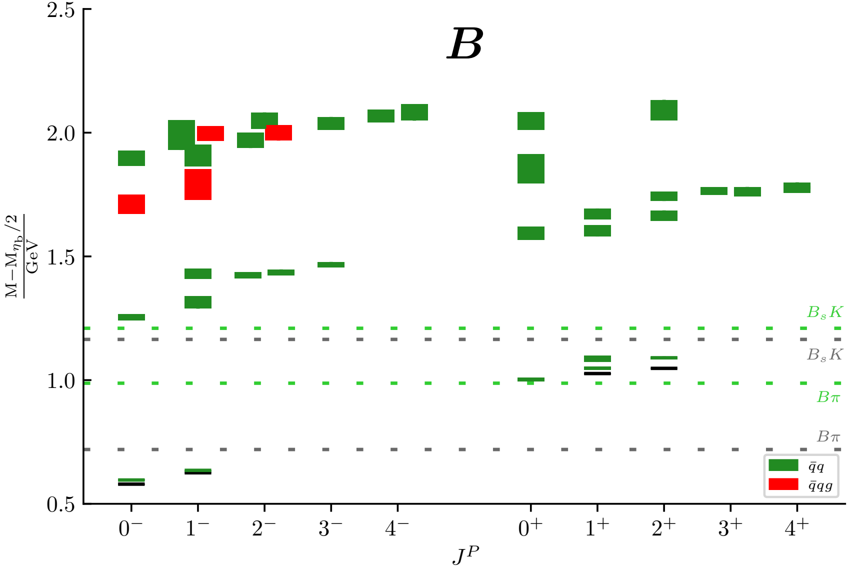
- As for charm, no deviation from relativistic dispersion relations for Υ, B, $B^\star$;

Exploring B, B_s, B_c mesons

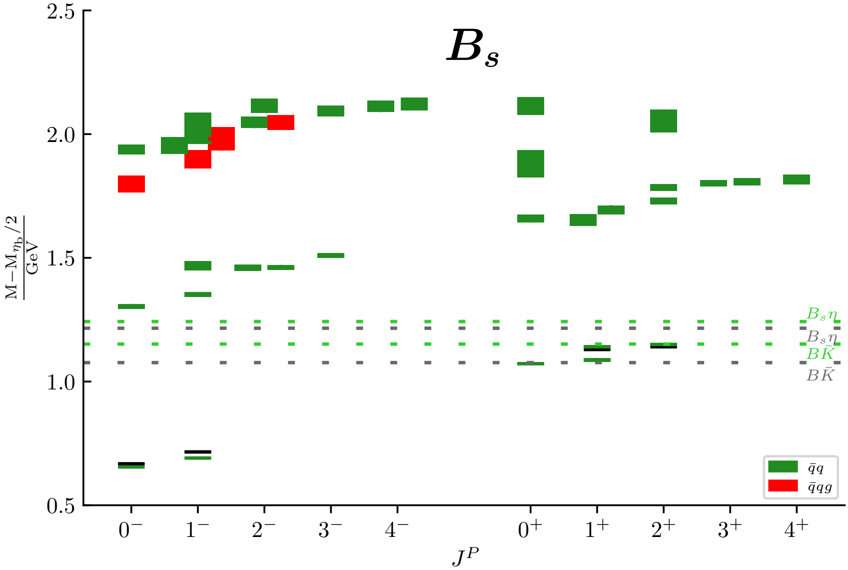
Caveat Emptor

- Spectra determined from single-hadron operators.
- Physics of multi-hadron states appears to need relevant operators
- Relatively heavy ($\sim$400MeV) pions
- Single lattice spacing

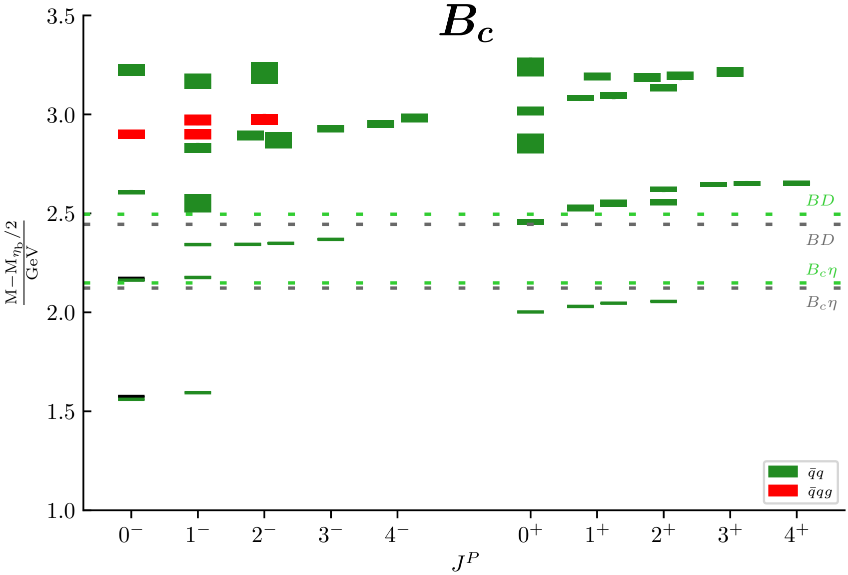
THE B MESON SPECTRUM



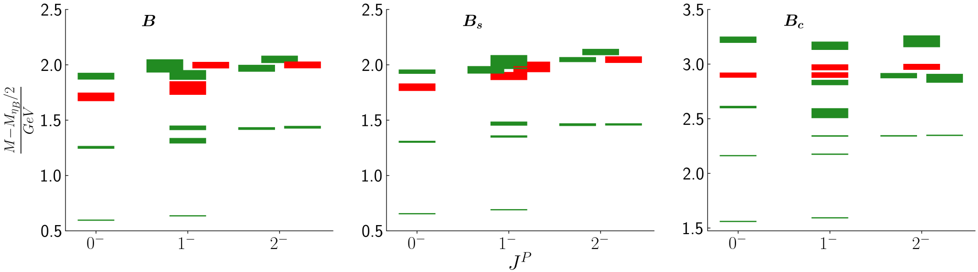
THE B_s MESON SPECTRUM

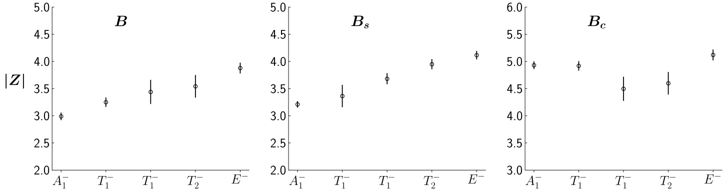


THE B_c SPECTRUM



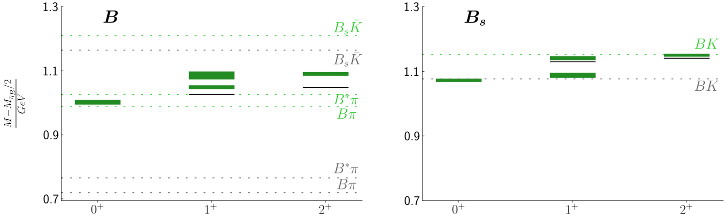
HYBRIDS: LIGHTEST SUPERMULTIPLETS



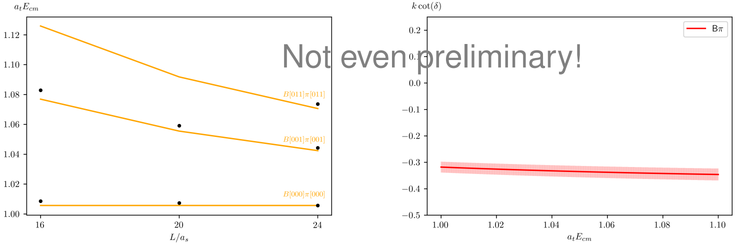


- Evidence of lightest hybrid supermultiplets.
- 1^-: mixtures of spin-singlet, spin-triplet.

NEXT STEPS ...

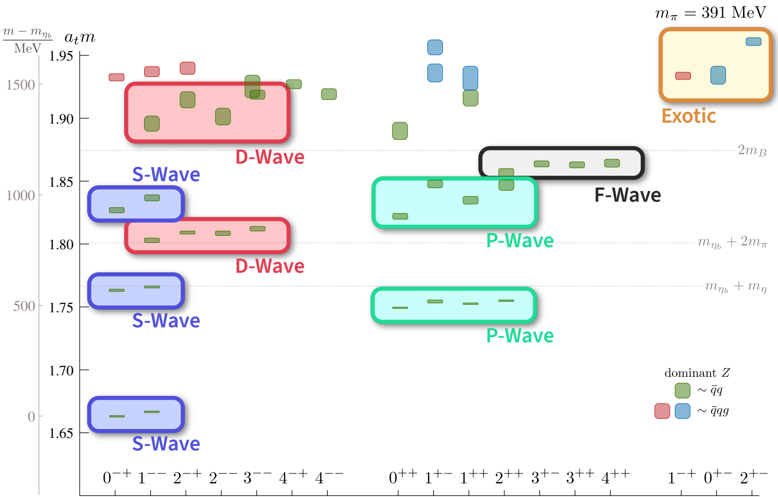


- A number of near-threshold states in positive parity B and Bs spectrum
- Elastic $B\pi$, A_1^+, $I = 3/2$: weakly repulsive interaction. $I = 1/2$ underway.



Not even preliminary!

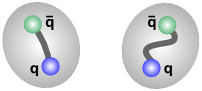
EXPLORATORY STUDY OF BOTTOMONIUM

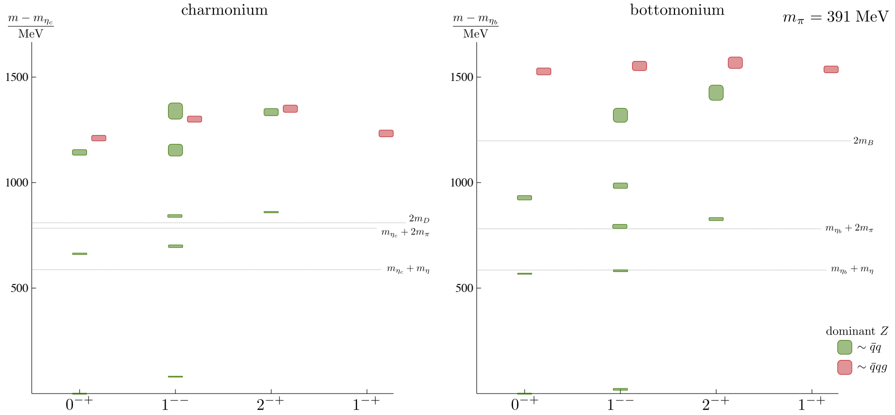


2008.02656

Systematic uncertainties remain: $q\bar{q}$ and $q\bar{q}g$ operators only; heavy pions, finite lattice spacing.

HYBRID MESONS (EXOTIC & NON-EXOTIC)





- Spin exotic and non-exotic hybrids determined.
- **Hybrids emerge in same pattern & energy scale in mesons and baryons, light and heavy - a common phenomenology of QCD hybrids.**

Radiative transitions in charmonium

- can calculate form factors and Q^2 dependence
- only stable states considered so far (in charmonium)

RADIATIVE TRANSITIONS $h \longrightarrow h'\gamma$ IN CHARMONIUM

$$\langle h'_{J'}(\lambda', \vec{p}')|j^\mu|h_J(\lambda, \vec{p})\rangle = \sum_i K_i^\mu[h'_{J'}; h_J]F_i(Q^2), \quad Q^2 = -(E_h - E_{h'})^2 + |\vec{p} - \vec{p}'|^2$$

- Radiative transitions offer a complementary window on hadronic properties.
- From form factors: infer qualitative and quantitative information
  - The partial radiative width for $h \to h'\gamma$ related to the multipole form factors

$$\Gamma(h \longrightarrow h'\gamma) = \frac{1}{4\pi(2J+1)} \frac{|\vec{q}|}{m^2} \sum_k |F_k(0)|^2,$$

  - Charge radius

$$\langle r^2 \rangle = -6 \left. \frac{dF(Q^2)}{dQ^2} \right|_{Q^2=0}$$

- Resonant transitions for light mesons $\rho \longrightarrow \pi\gamma^*$ in 1604.03530.

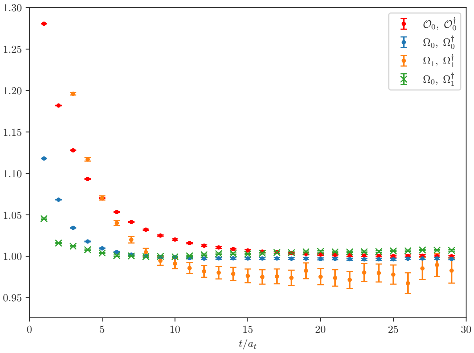
LATTICE RADIATIVE TRANSITIONS IN CHARMONIUM

- Recent calculation using HadSpec methods extended to charm
- The lattice three-point correlation function:

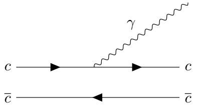
$$C^{\mu}_{\mathfrak{n}_f \mathfrak{n}_i}(\Delta t, t) = \langle 0 | \Omega_{\mathfrak{n}_f}(\Delta t)\, j^{\mu}(t)\, \Omega^{\dagger}_{\mathfrak{n}_i}(0) | 0 \rangle$$

$$e^{E_{\mathfrak{n}_f}(\Delta t - t)}\, e^{E_{\mathfrak{n}_i} t}\, C^{\mu}_{\mathfrak{n}_f \mathfrak{n}_i} = \langle \mathfrak{n}_f |\, j^{\mu}(0)\, | \mathfrak{n}_i \rangle + f_{\mathfrak{n}'_f}\, e^{-\delta E_f (\Delta t - t)} + f_{\mathfrak{n}'_i}\, e^{-\delta E_i t} + \ldots$$

  yields the transition matrix element ...
- Excited state contributions must be controlled - construct optimised operators built from GEVP outputs in two-point functions: $\Omega^{\dagger}_n = \sqrt{2E_n} e^{-E_n t_0/2} \sum_i v_i^{(n)} \mathcal{O}^{\dagger}_i$

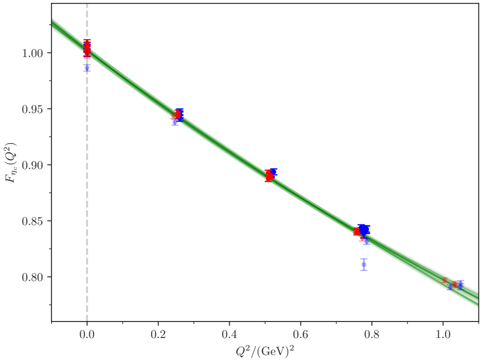


FORM FACTORS OF THE η_c

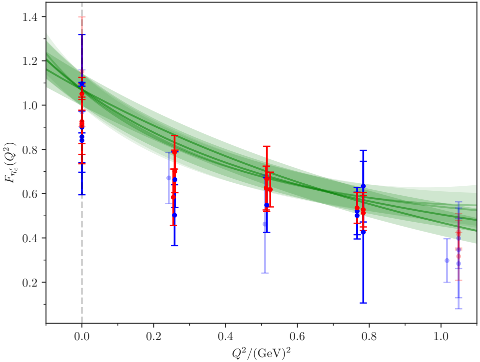


- Not a physical process: current only coupled to c quark
- Parameterised by single form factor: a sandbox for a first look.

η_c



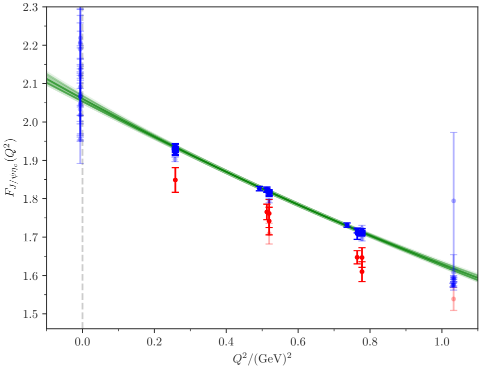
η_c'



- Determine $F(Q^2 = 0)$ and Q^2 dependence.

THE $J/\Psi \longrightarrow \eta_c$ TRANSITION

$$\langle \eta_c(p')|j^\mu(0)|J/\psi(\lambda,\vec{p})\rangle = \epsilon^{\mu\nu\rho\sigma} p'_\nu p_\rho \epsilon_\sigma(\lambda,\vec{p}) \frac{2}{m_{J/\psi}+m_{\eta_c}} F_{J/\psi\,\eta_c}(Q^2)$$



- yields radiative partial width

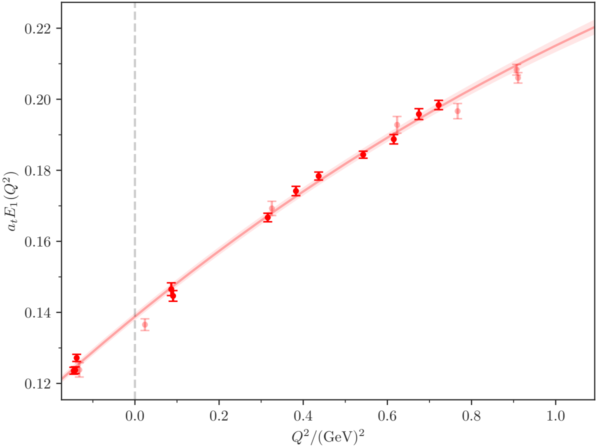
$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{64\alpha}{27}\frac{|\vec{q}|^3}{(m_{J/\psi}+m_{\eta_c})^2}|F_{J/\psi\,\eta_c}(0)|^2$$

strong dependence on hyperfine splitting via q^2

- Consider $|F(Q^2=0)|$ directly: 1.83-2.07.

THE $\chi_{c0} \longrightarrow J/\Psi$ TRANSITION

- Kinematic decomposition includes multiple form factors: electric-dipole E_1, longitudinal C_1
- Only E_1 contributes to partial width (first determination with 2+1 flavours)



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Many challenges in heavy sector remain

- ♦ Include disconnected diagrams and mixing with lighter states.
- ♦ Improving existing calculations - understanding the effect of lighter light quarks on thresholds etc, simulations at multiple and larger volumes and multiple spacings.
- ♦ Handling the large number of (open) channels relevant in heavy hadron physics
- ♦♦ A formalism and implementation for three-particle decays, left-hand cuts, . . . [talks tomorrow]

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  - First steps in exotic and excited b-physics and radiative transitions in charmonium.

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Thanks for listening!