The doubly-charmed tetraquark in finite and infinite volume

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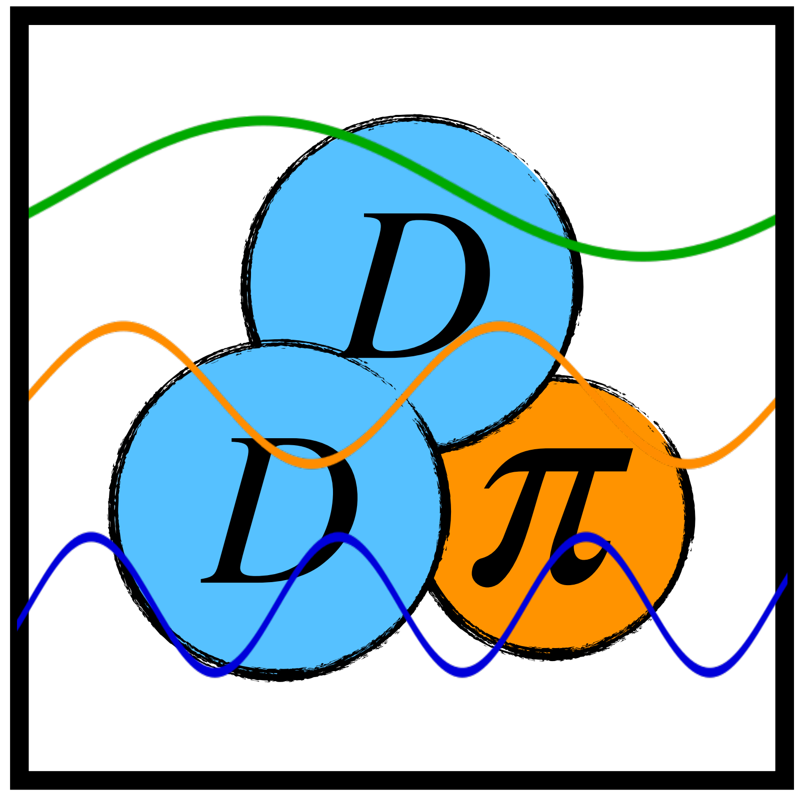
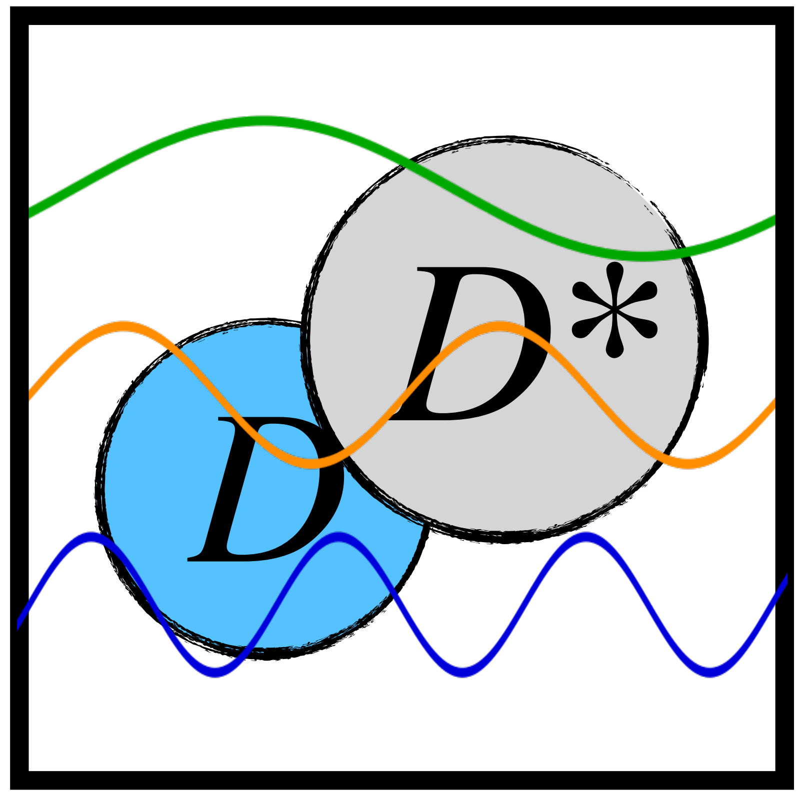
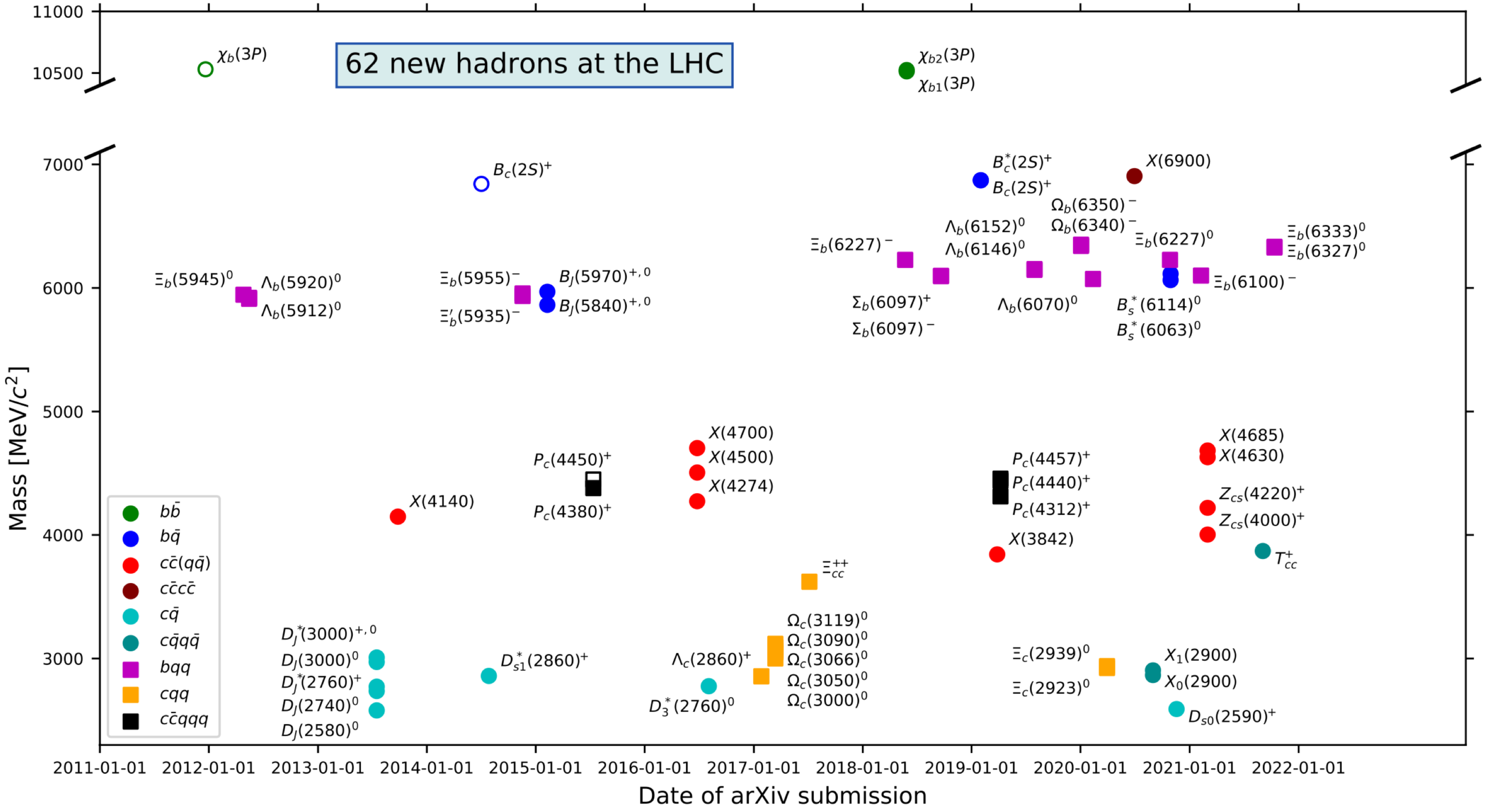
Hirschegg
Jan 17th





The Hadron Spectrum

A growing hadron spectrum still requires first-principles understanding



[LHCb, arXiv:2206.15233]

(+ Babar, Belle, COMPASS)

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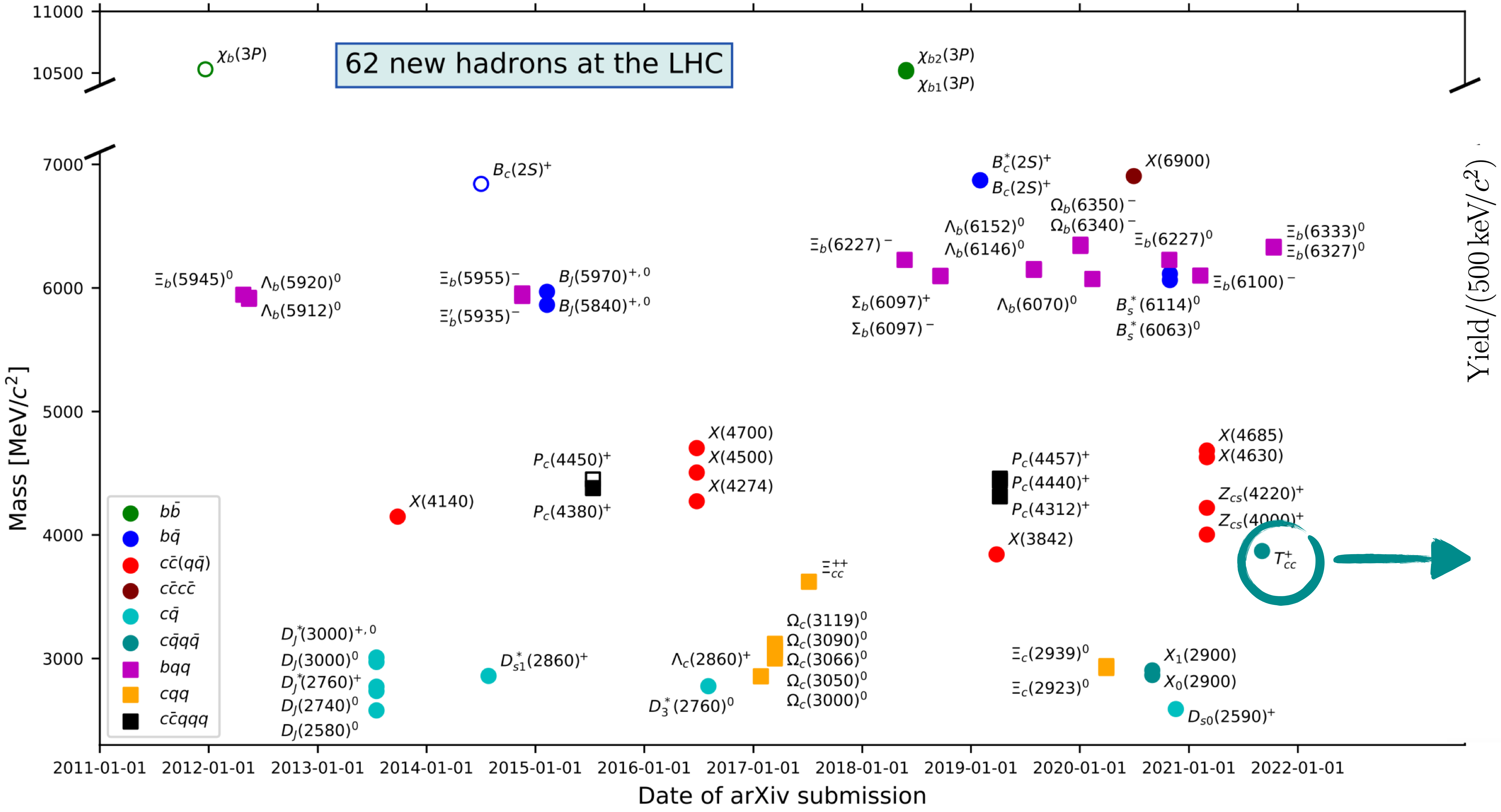

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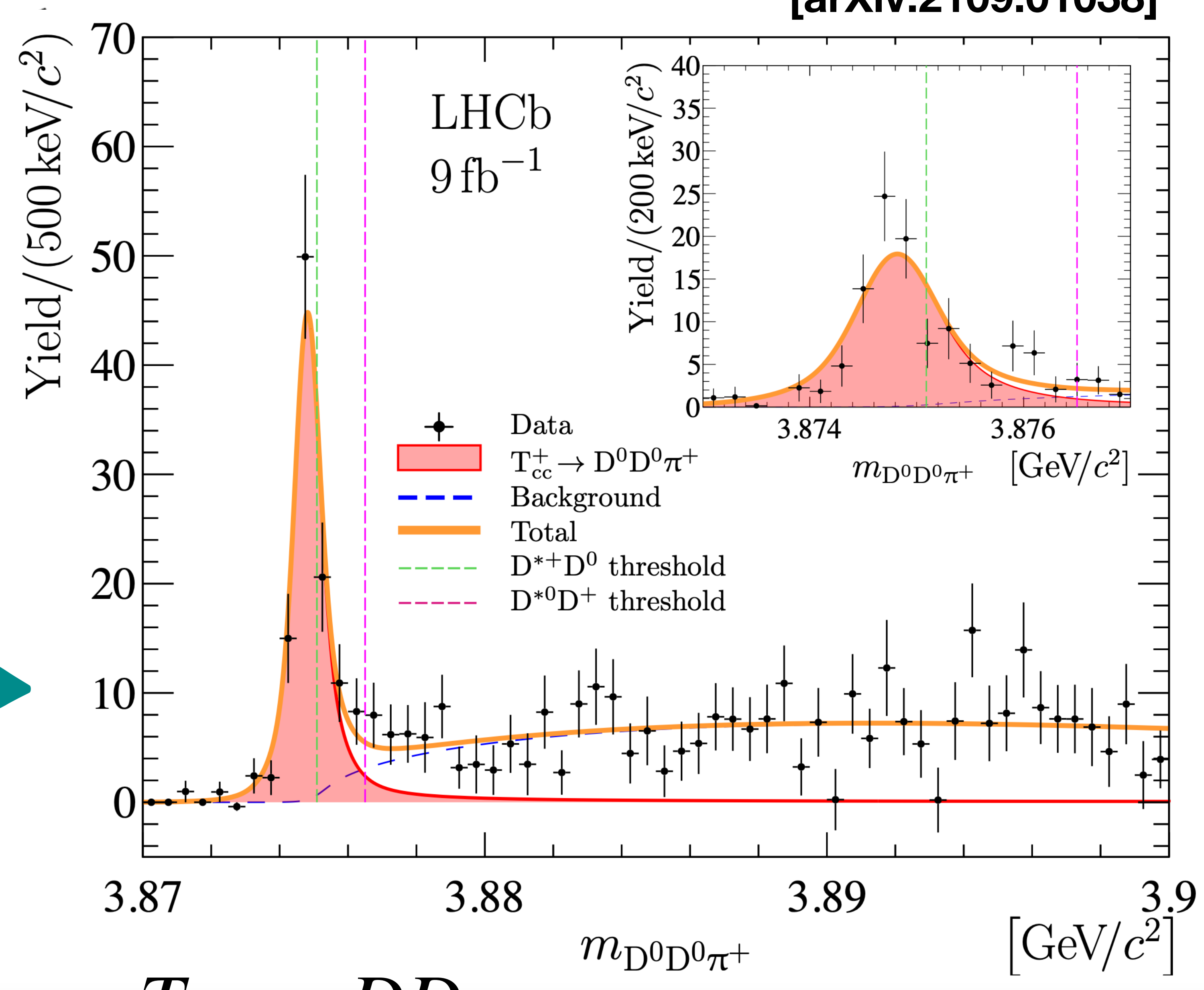
[arXiv:2109.01038]



$T_{cc} \to DD\pi$

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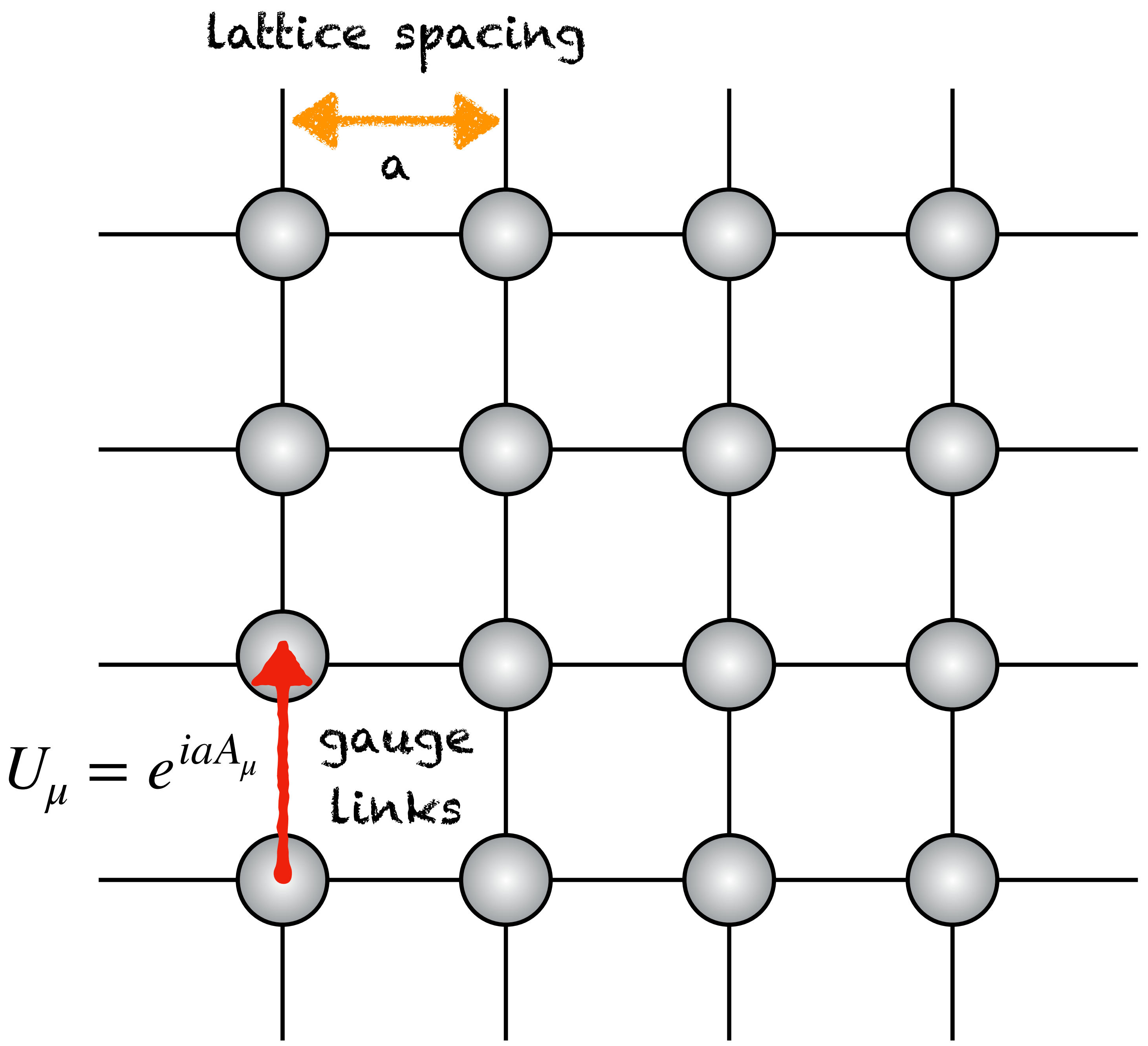
Lattice QCD

- Lattice QCD is a first-principles numerical approach to the strong interaction

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \frac{1}{\mathcal{Z}}\int D\psi D\bar{\psi} DA\, \mathcal{O}(t)\mathcal{O}(0)e^{-S_E(\psi,\bar{\psi},A_\mu)}$$

Euclidean action



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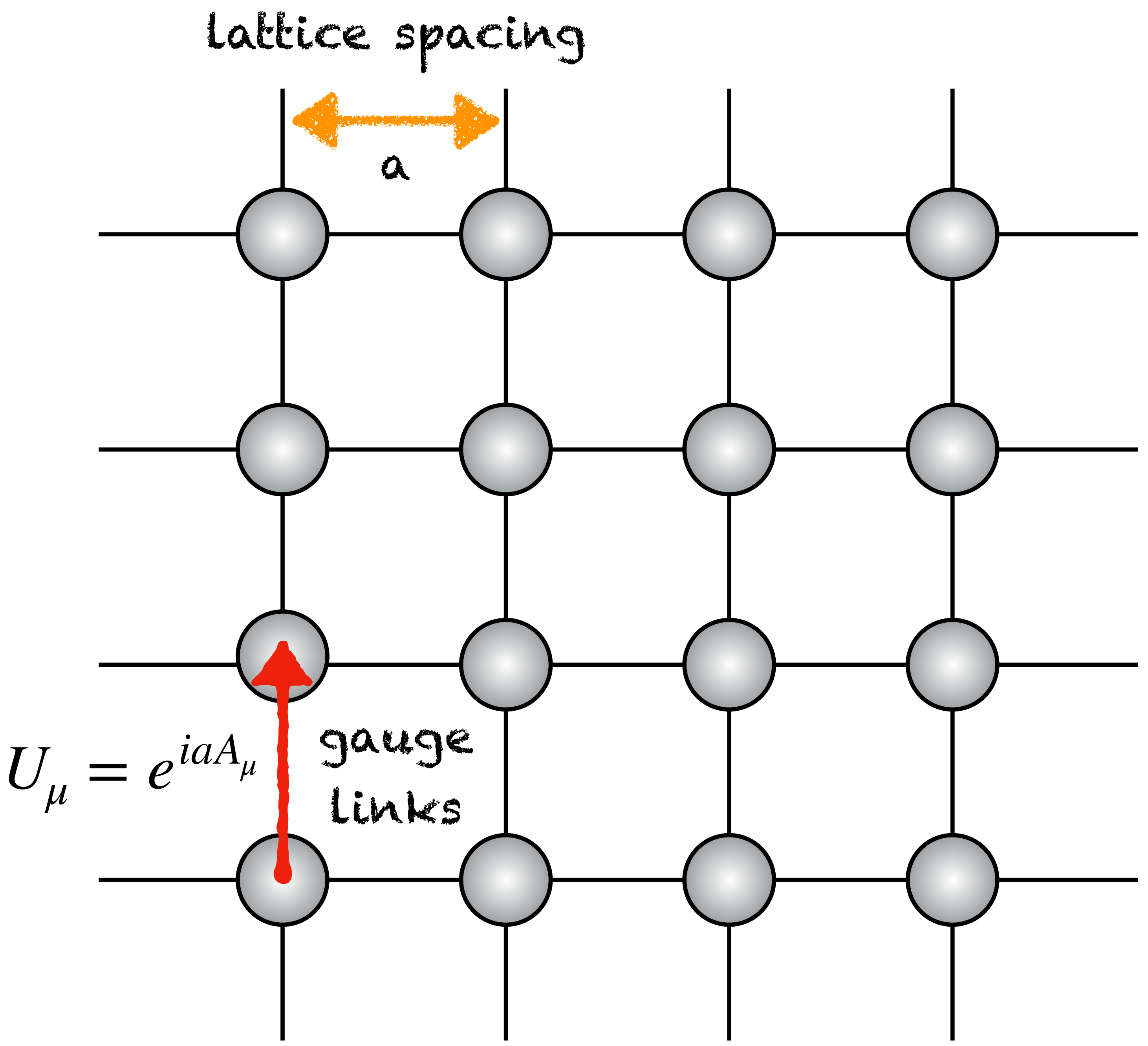

Can we obtain resonance properties from Euclidean correlation functions?

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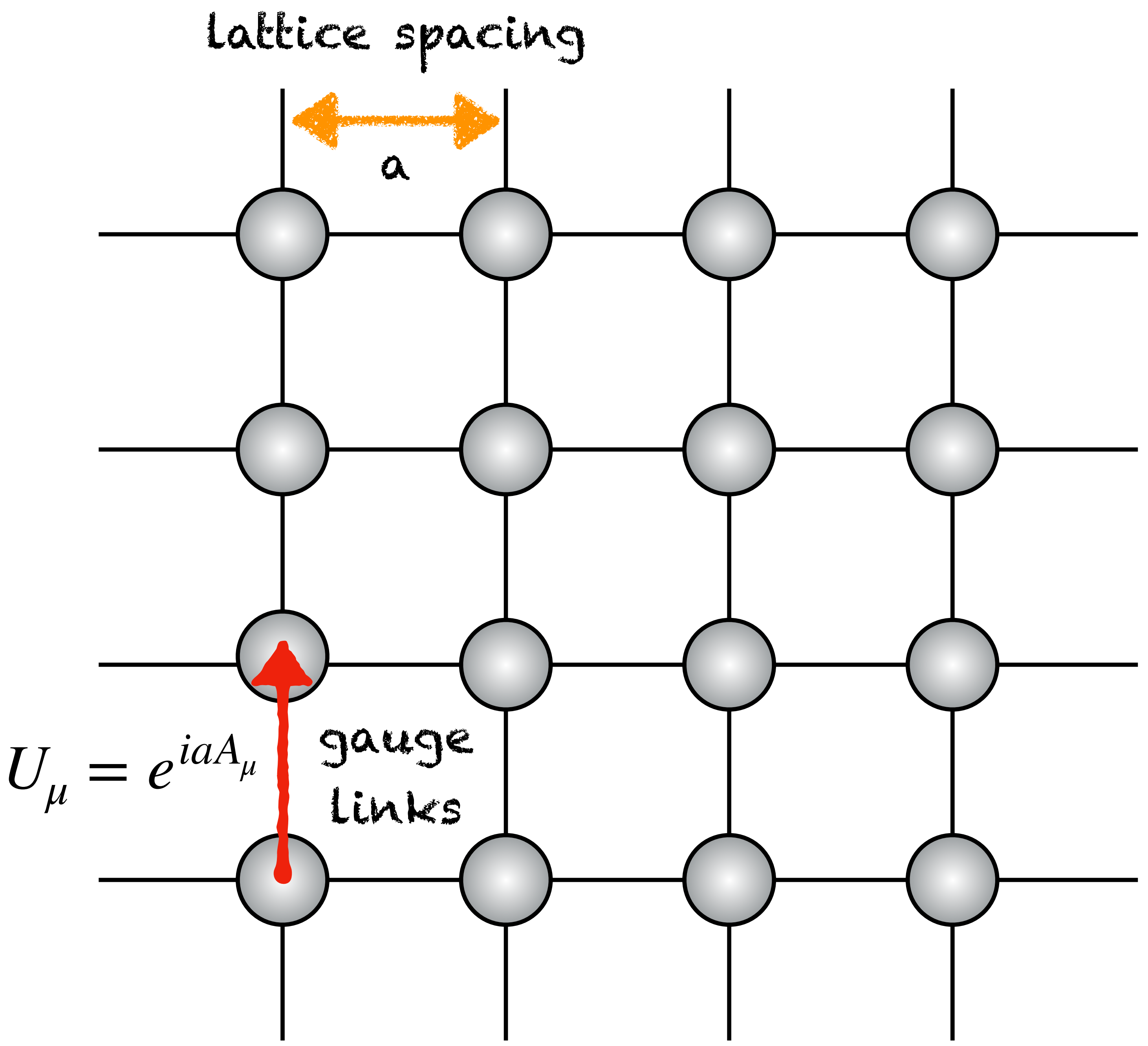


Can we obtain resonance properties from Euclidean correlation functions?

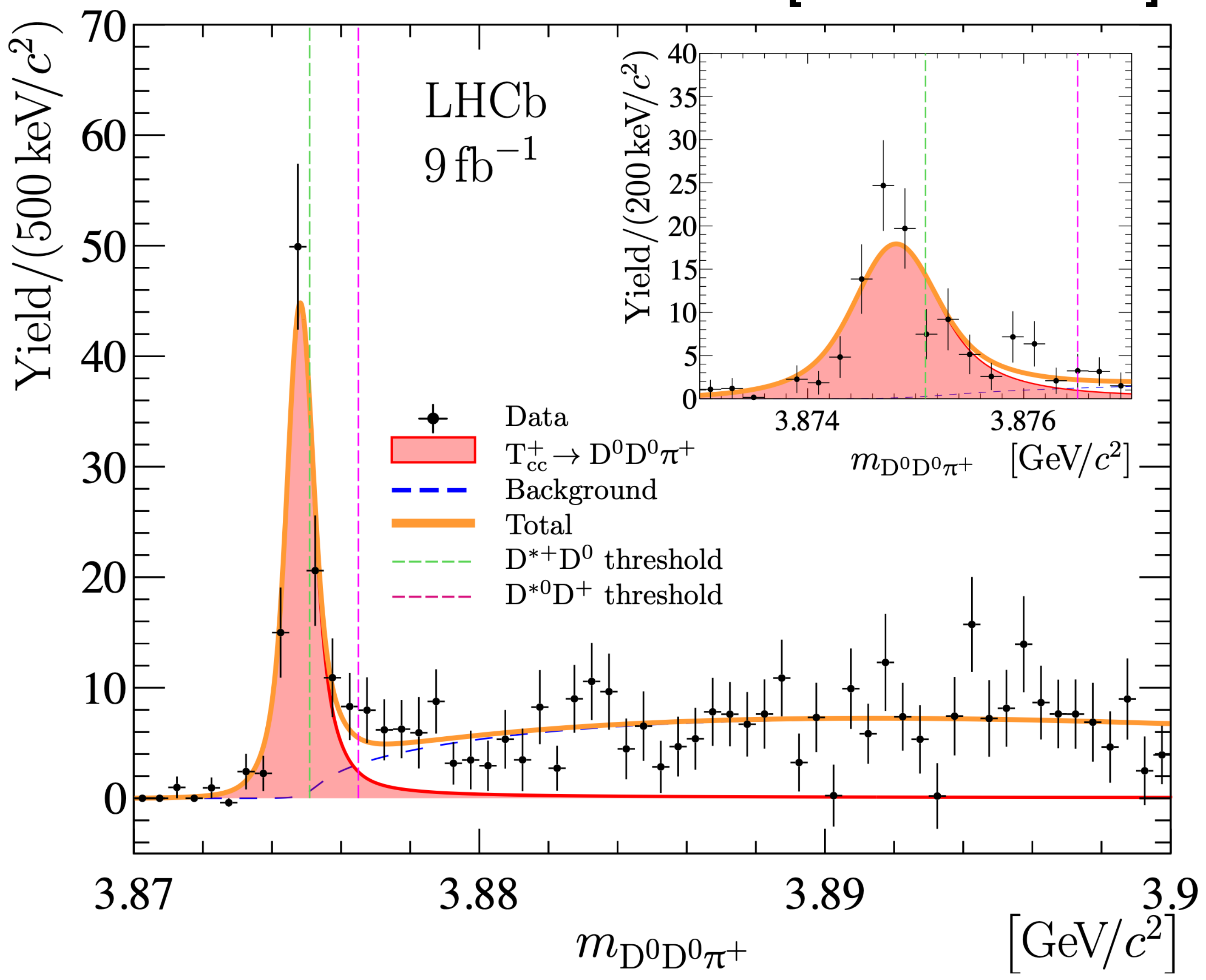
→ Yes, but not that simple!

Infinite vs finite volume

Experiments

- **Asymptotic states**
- **Direct access to scattering amplitudes**

[arXiv:2109.01038]



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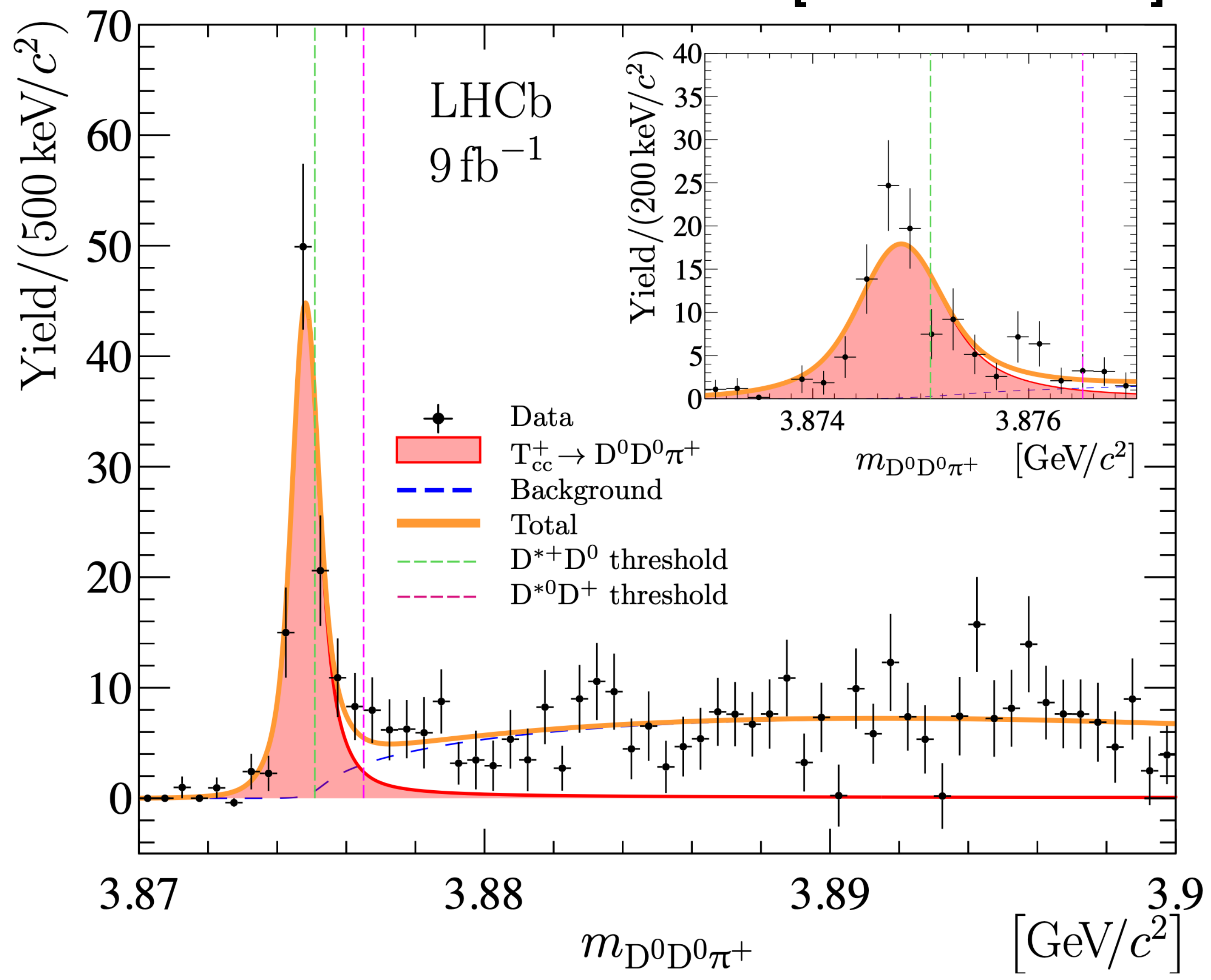
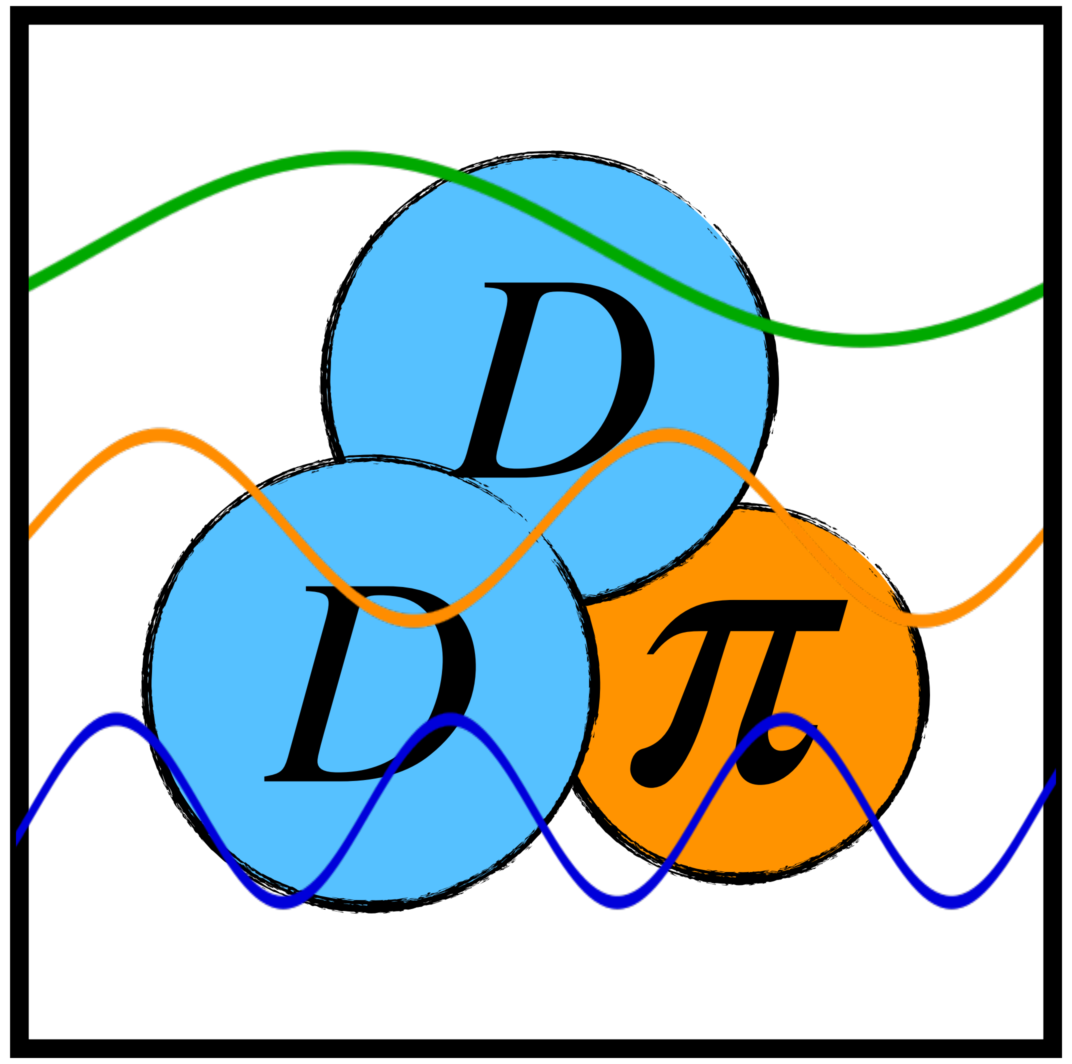


Lattice QCD

- Euclidean time
- Stationary states in a box



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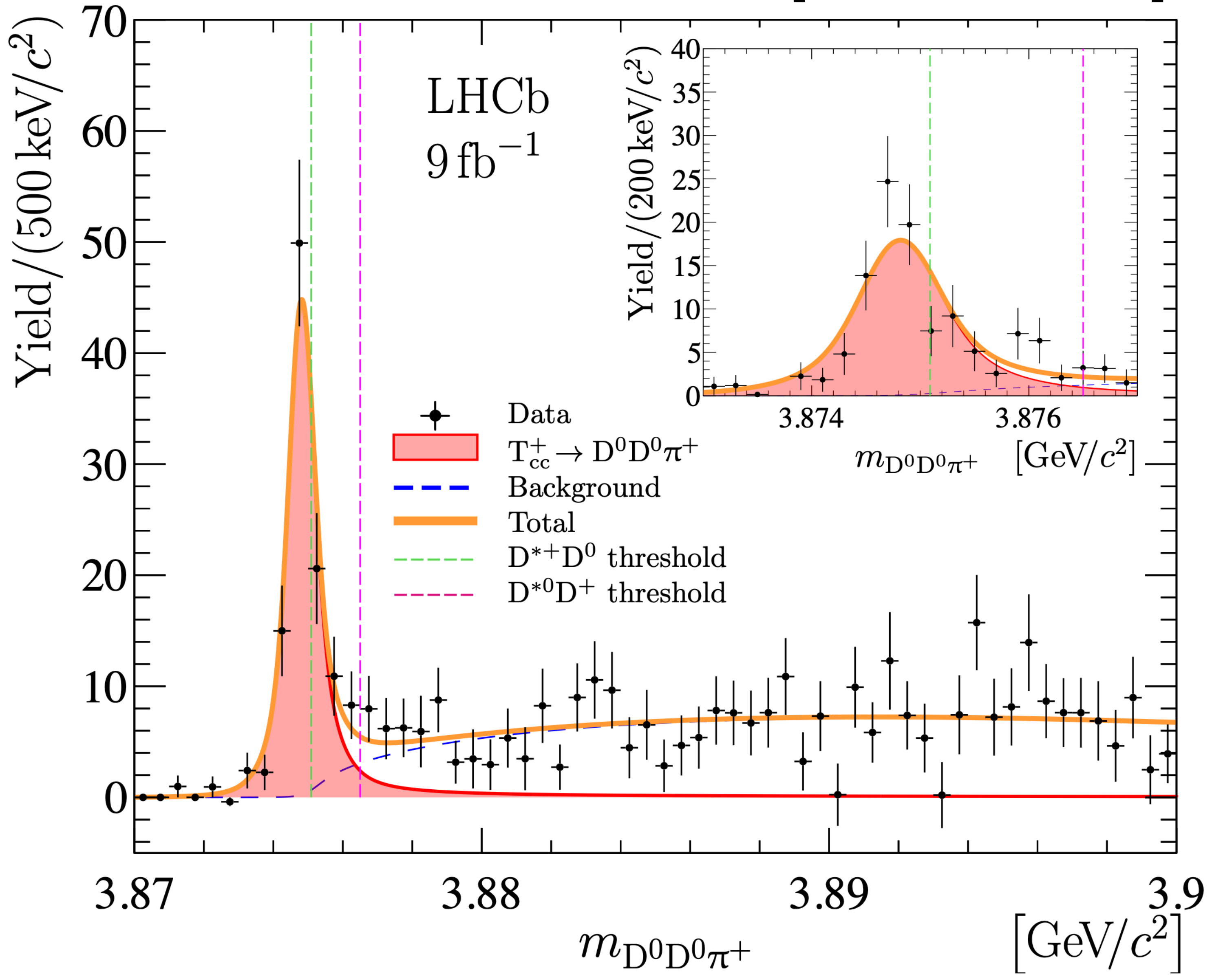

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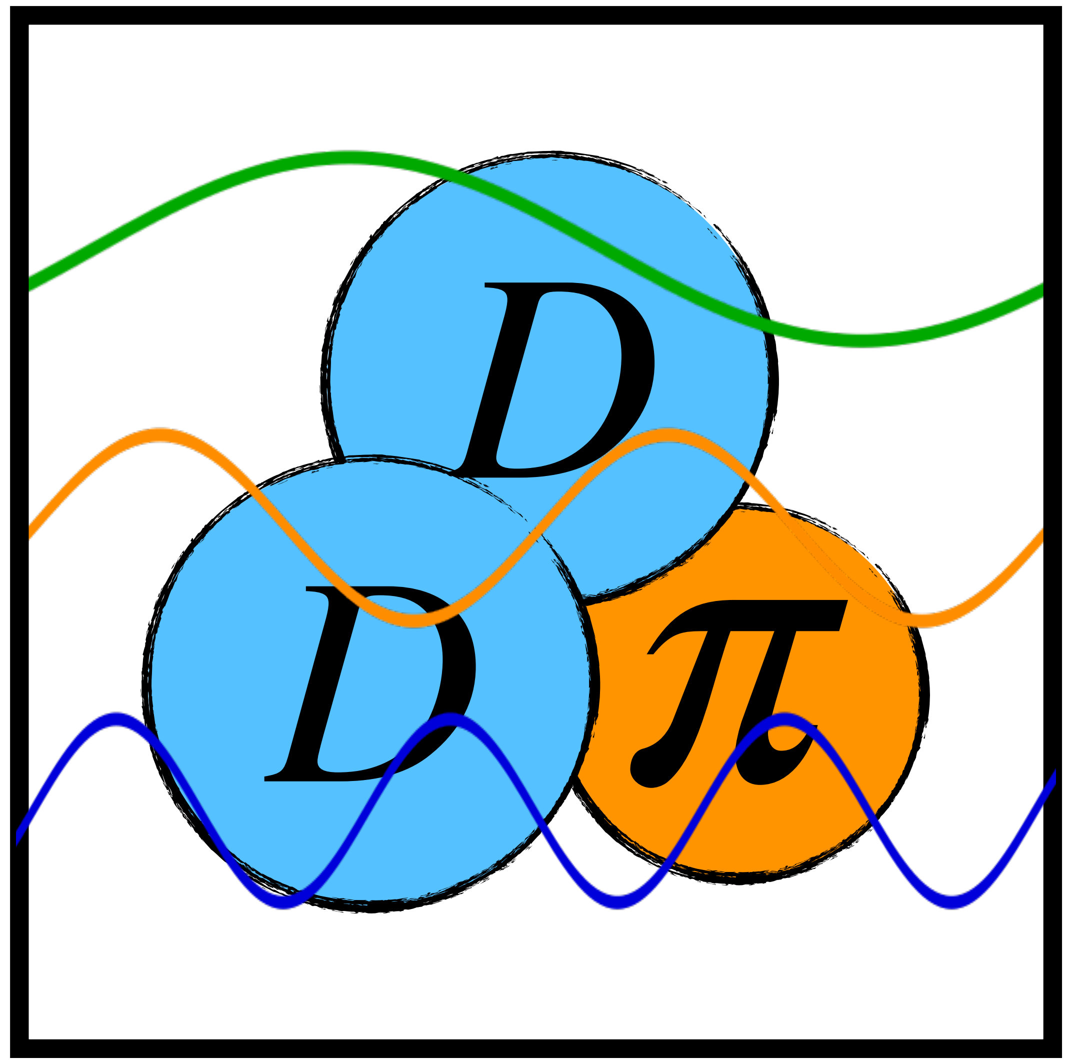
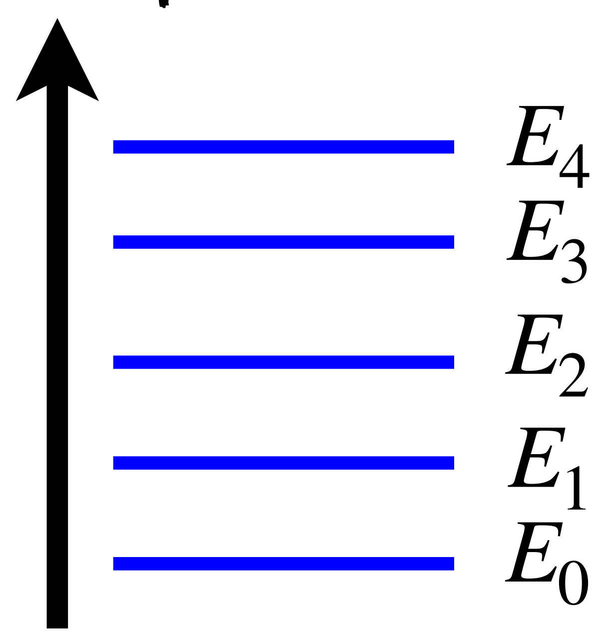


Finite-volume formalism?
[Lüscher, 1986]

Need to include 3-body effects!

The general strategy

Spectrum



From Euclidean
Correlation functions

Quantization condition

$$\det\left[F_n^{-1} + \mathcal{K}_n\right] = 0$$

A bridge between:
finite volume ⟶ infinite volume

Infinite-volume scattering quantities

K matrix	Phase shift	Amplitude
$\mathcal{K}$	δ_ℓ	$\mathcal{M}$

Resonance poles

$$\mathcal{M}^{-1}(E_{\text{pole}}) = 0$$

Outline

1. Three-particles in a finite volume
2. Finite-volume formalism for the T_{cc}
3. Infinite-volume three-body dynamics of the T_{cc}

Three particles in a finite volume

Why three particles?

- The two-body formalism is restricted to few interesting resonances
  - **Exotics:** $T_{cc} \to DD^*, DD\pi$
  - **Roper:** $N(1440) \to \Delta\pi \to N\pi\pi$

Resonance	$I_{\pi\pi\pi}$	J^P
$\omega(782)$	0	1^-
$h_1(1170)$	0	1^+
$\omega_3(1670)$	0	3^-
$\pi(1300)$	1	0^-
$a_1(1260)$	1	1^+
$\pi_1(1400)$	1	1^-
$\pi_2(1670)$	1	2^-
$a_2(1320)$	1	2^+
$a_4(1970)$	1	4^+

(with $\geq 3\pi$ decay modes)

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- CP violation: $K \to 3\pi, \quad K^0 \leftrightarrow 3\pi \leftrightarrow \overline{K}^0$
- ☑ Major developments in the three-particle finite-volume formalism

[Hansen, Sharpe, PRD 2014 & 2015], [Hammer, Pang, Rusetsky, JHEP 2017] x 2
[Mai, Döring, EPJA 2017]
[...]
[Blanton, FRL, Sharpe, JHEP 2019], [Hansen, FRL, Sharpe, JHEP 2020]
[Hansen, FRL, Sharpe, JHEP 2021], [Blanton, FRL, Sharpe, JHEP 2022]
[Hansen, FRL, Sharpe, arXiv:2401.06609]

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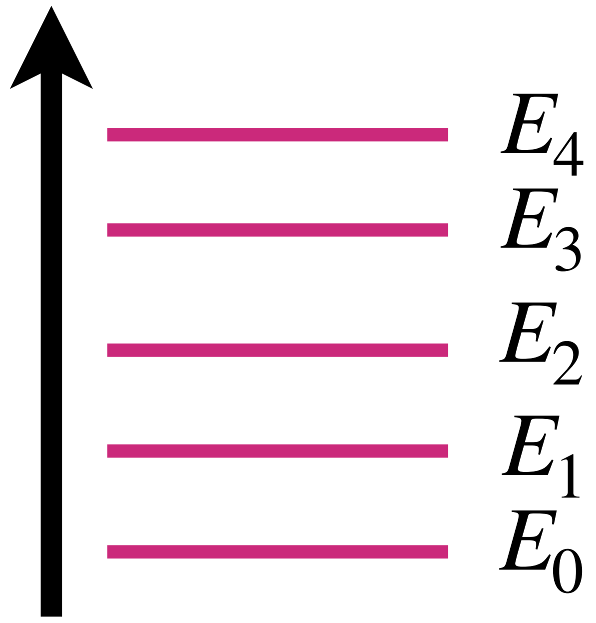
Relativistic three-particle formalism for identical particles [Hansen, Sharpe, PRD 2014 & 2015]

2π Spectrum



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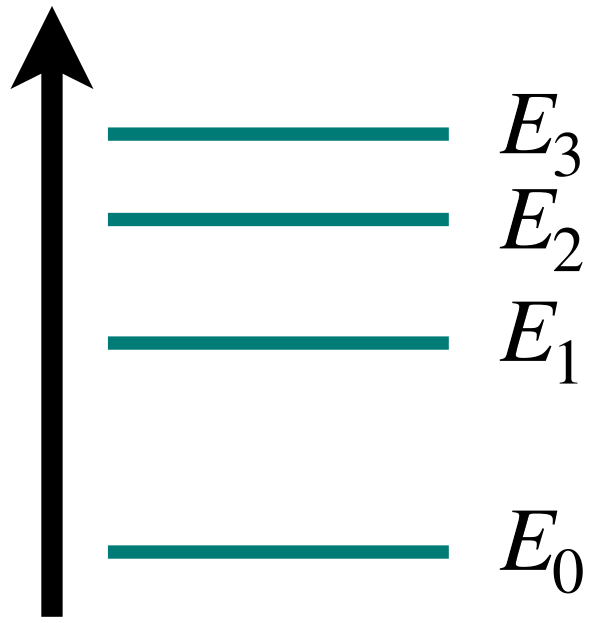

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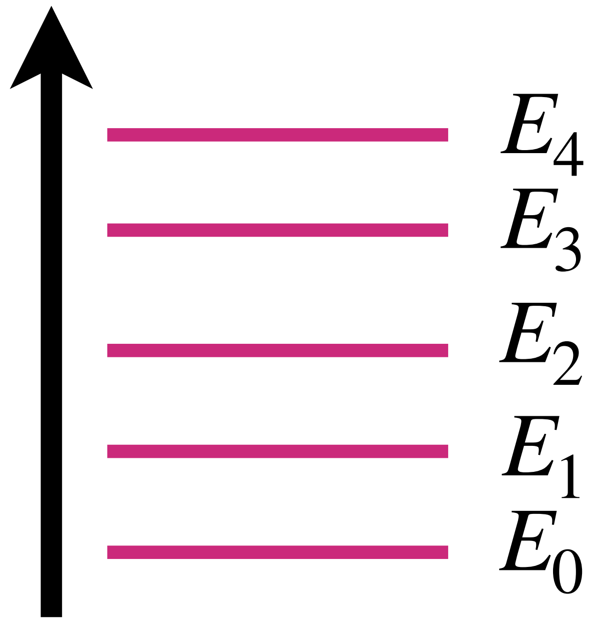

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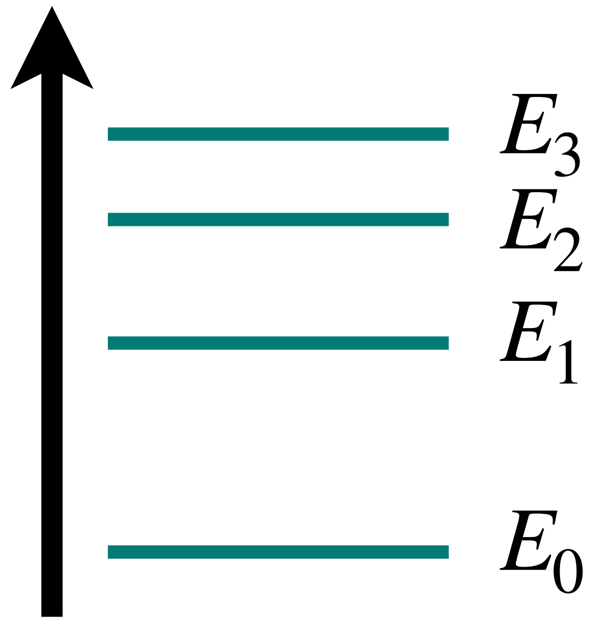

Quantization conditions

$$\det_{\ell m}\left[\mathcal{K}_2 + F_2^{-1}\right] = 0$$

$$\det_{k\ell m}\left[\mathcal{K}_{df,3} + F_3^{-1}\right] = 0$$

Matrix indices describe three on-shell particles

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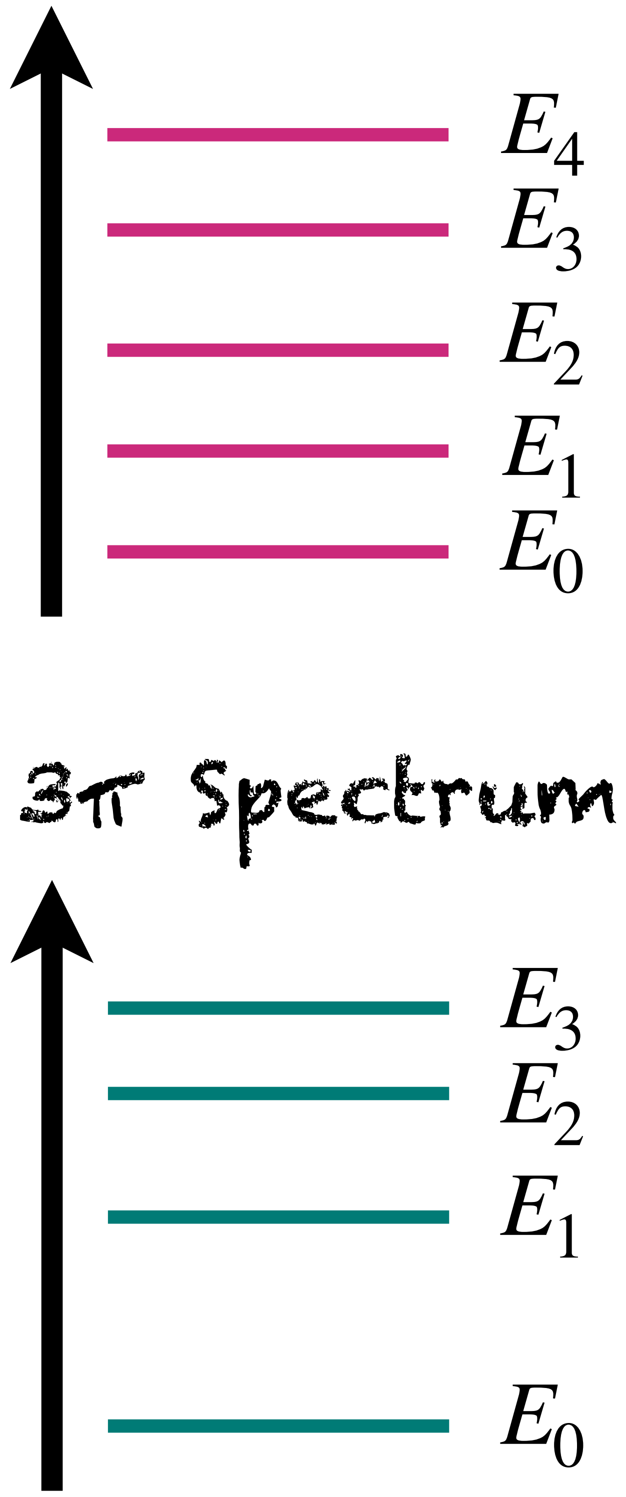

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Fit

K-matrices

$$\mathcal{K}_2$$

$$\mathcal{K}_{df,3}$$

Parametrize:

$$\mathcal{K}_2 = c_0 + c_1 k^2 + \ldots$$

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_{\text{df},3}^{\text{iso},0} + \mathcal{K}_{\text{df},3}^{\text{iso},1}\left(\frac{s-9m^2}{9m^2}\right) + \ldots$$

[Blanton, FRL, Sharpe, JHEP 2019]

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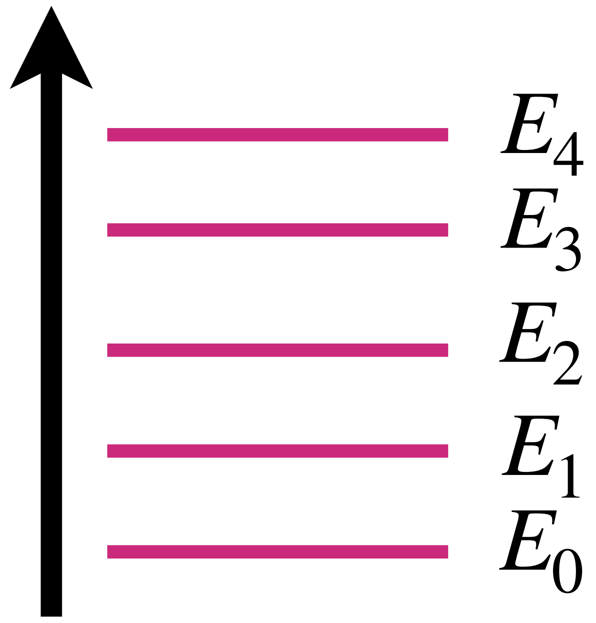

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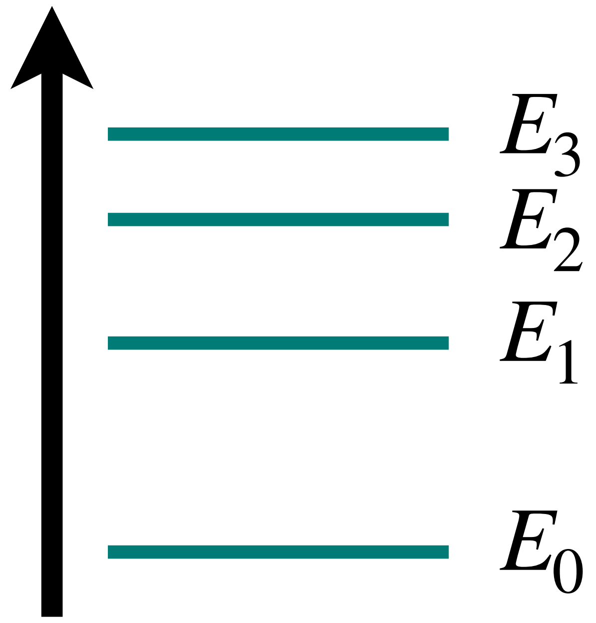

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Unitarity relations

Integral equations

[Briceño et al., PRD 2018]
[Hansen et al., PRL 2021]
[Jackura et al., PRD 2021]
[Dawid et al., 2303.04394]

Scattering amplitudes

$$\mathcal{M}_2$$

$$\mathcal{M}_3$$

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Relativistic three-particle formalism for identical particles

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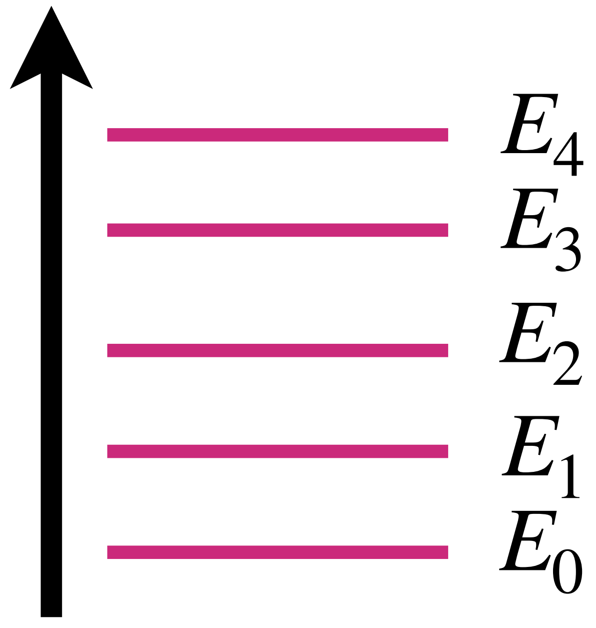

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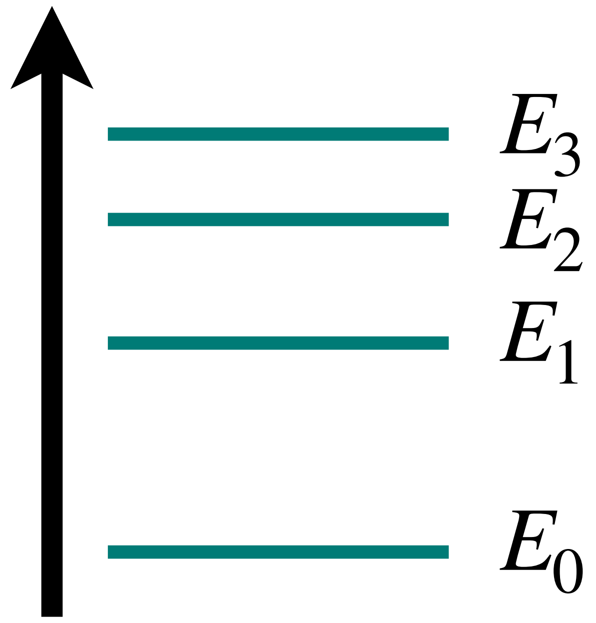

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Scattering amplitudes

$\mathcal{M}_2$

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↕

Experiments & resonance properties

Three-meson systems

- **Three-particle formalism applied to weakly-interacting (non-resonant) systems:** $\pi^+\pi^+\pi^+, \; \pi^+\pi^+K^+$

  [Blanton … FRL … et al., PRL 2020 & JHEP 2021], [Draper … FRL … et al., JHEP 2023], [Fischer … FRL … et al, EPJC 2021]

  [Alexandrou et al, Brett et al, Culver et al, Hansen et al, Mai et al]

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[Alexandrou et al, Brett et al, Culver et al, Hansen et al, Mai et al]

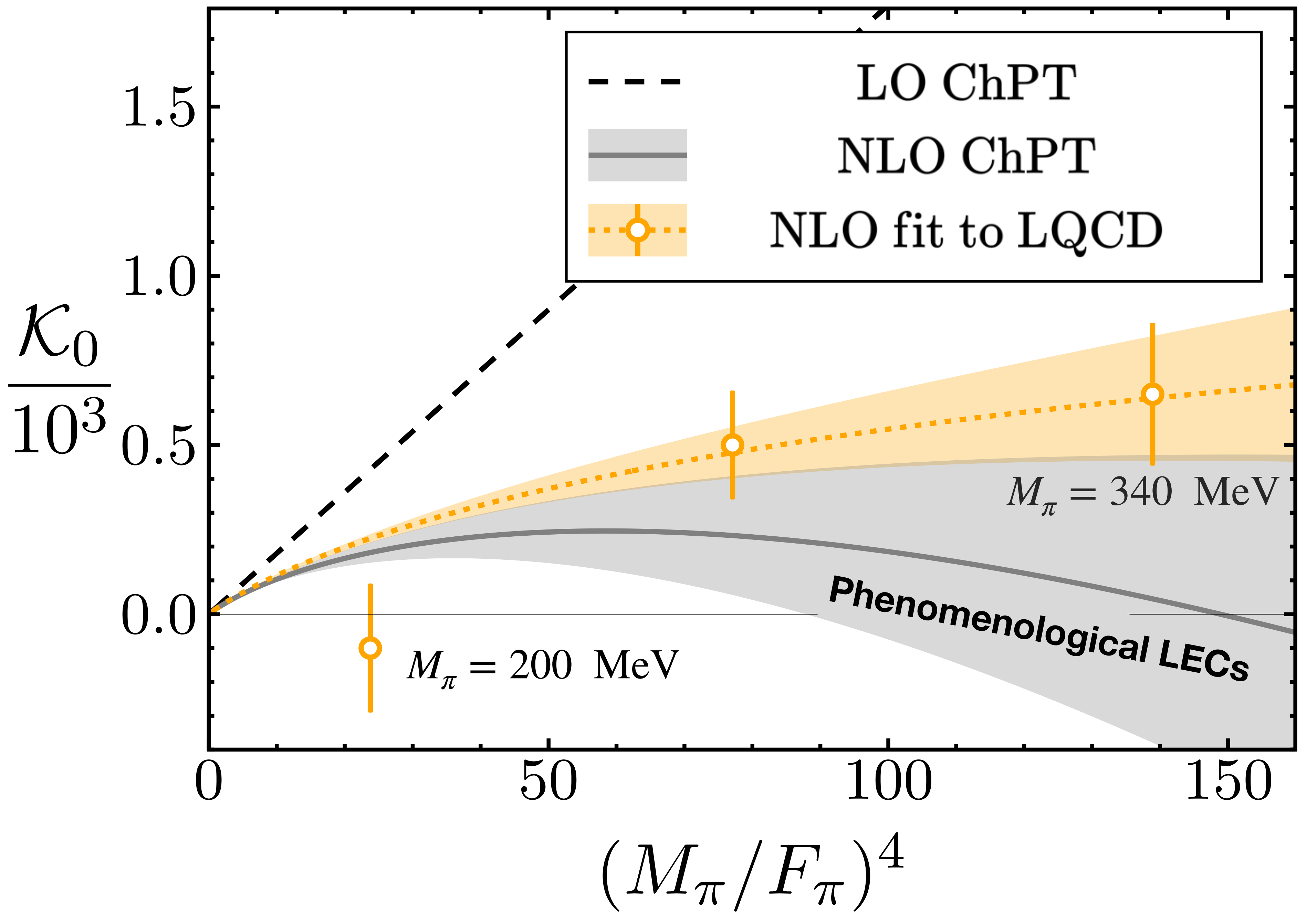
Example: $\pi^+\pi^+\pi^+$ scattering

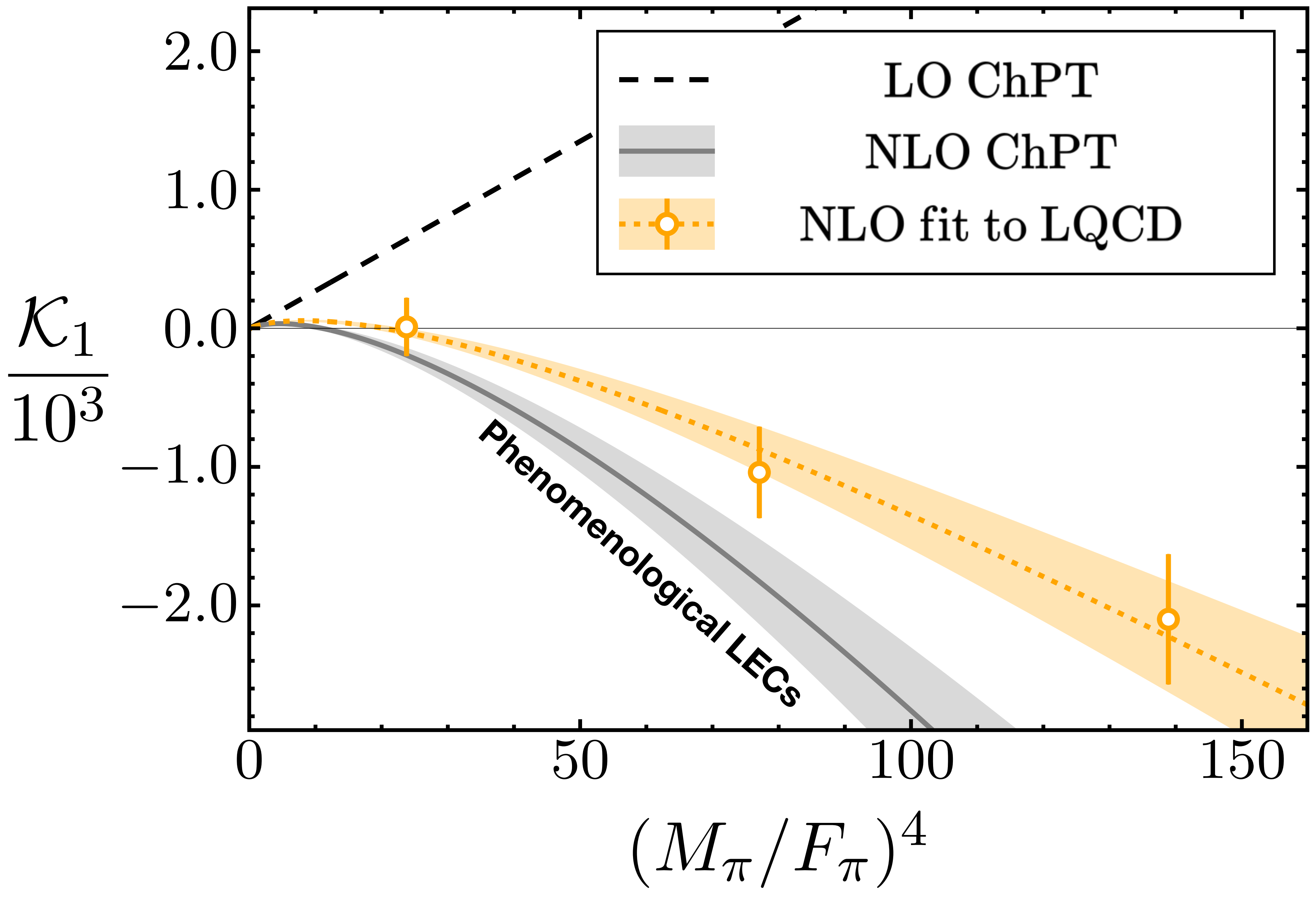
Lattice data: [Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe, JHEP 2021]

NLO ChPT: [Baeza-Ballesteros, Bijnens, Husek ,FRL, Sharpe, Sjö, JHEP 2023]

parametrized by the three-particle K-matrix

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_0 + \mathcal{K}_1\left(\frac{s-9M_\pi^2}{9M_\pi^2}\right) + \cdots$$





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Nondegenerate systems

- Relevant three-body systems involve nonidentical particles → $T_{cc} \to DD\pi$

- First step: RFT formalism for three different scalars
  [Blanton, Sharpe, PRD 2021] x 2, [Hansen, FRL, Sharpe, JHEP 2021]

  e.g. $\pi^+\pi^0\pi^-$, $K^+K^+\pi^+$, $D_s^+D^0\pi^-$

$$\det_{k,\ell,m,\mathbf{f}}[1 - \mathbf{K}_{\mathrm{df},3}(E^\star)\mathbf{F}_3(E,\boldsymbol{P},L)] = 0$$

Implementation: github.com/ferolo2/QC3_release

determinant runs over an additional "flavor" index

$\pi^+\pi^+K^+$ Lattice QCD spectrum

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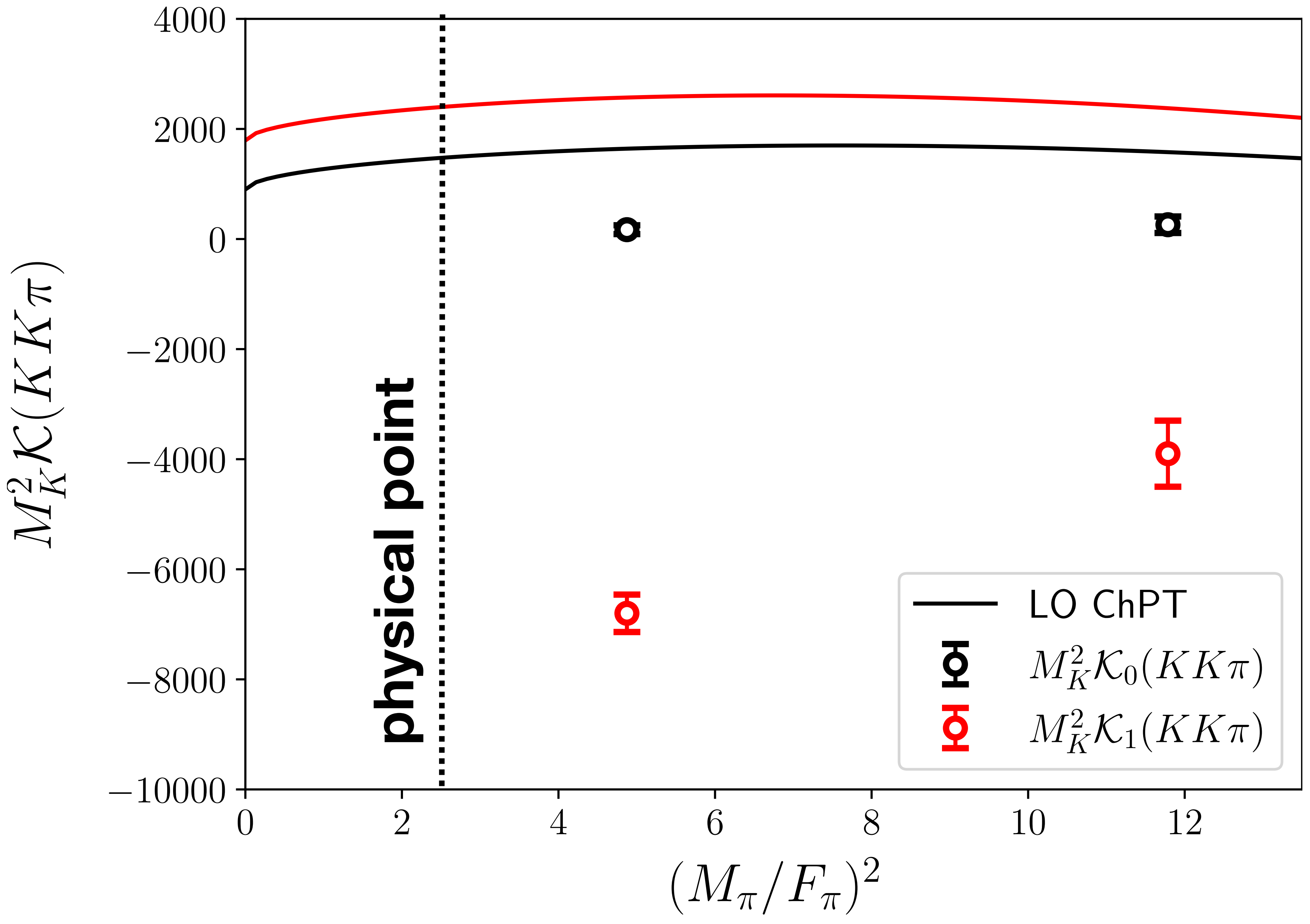
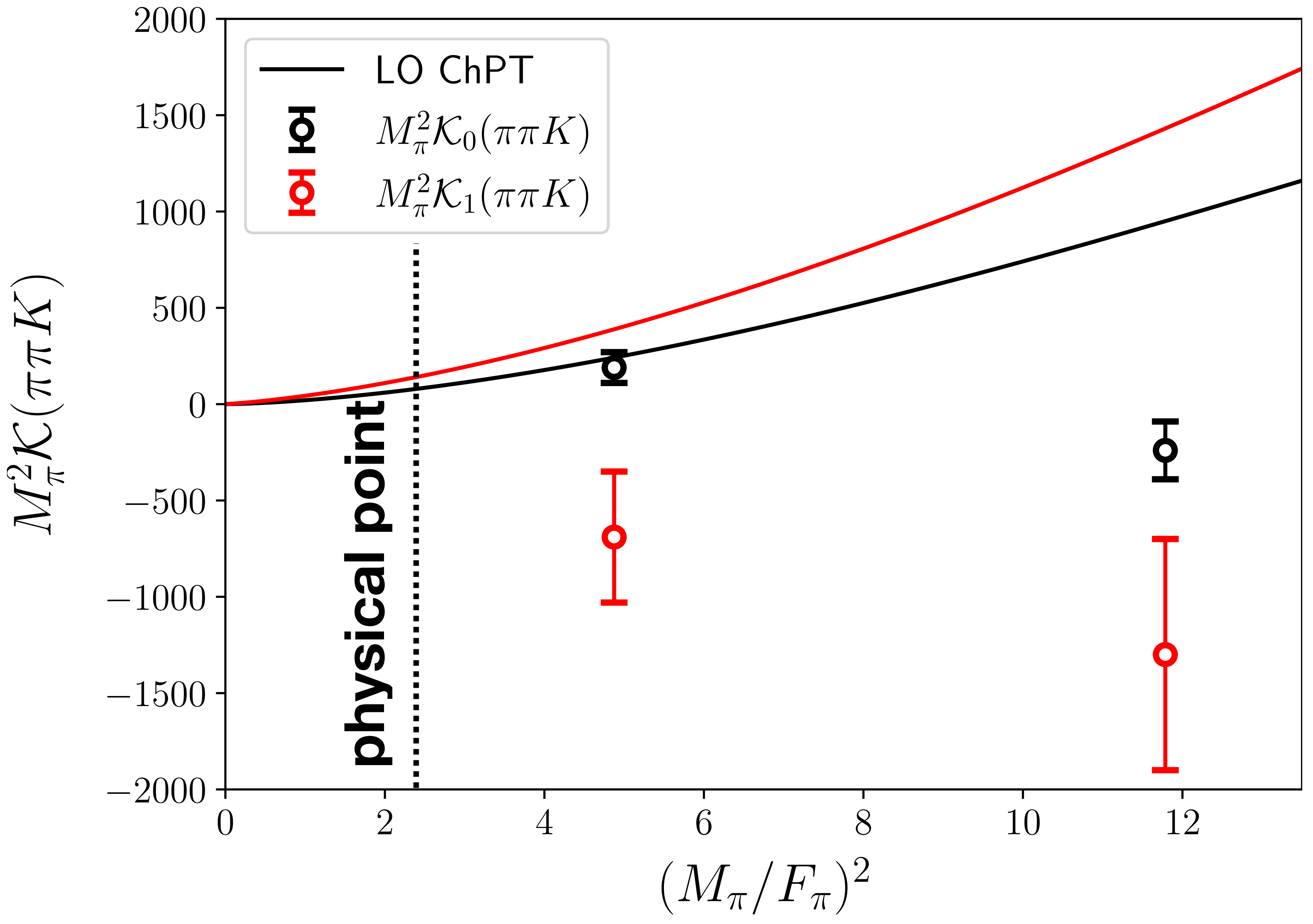
Selected $\mathcal{K}_{df,3}$ results

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_0 + \mathcal{K}_1 \left(\frac{s - s_{\text{th}}}{s_{\text{th}}} \right) + \cdots$$





- Statistically significant different from zero
- Disagreement with LO ChPT. NLO effects?

Finite-volume formalism for the Tcc

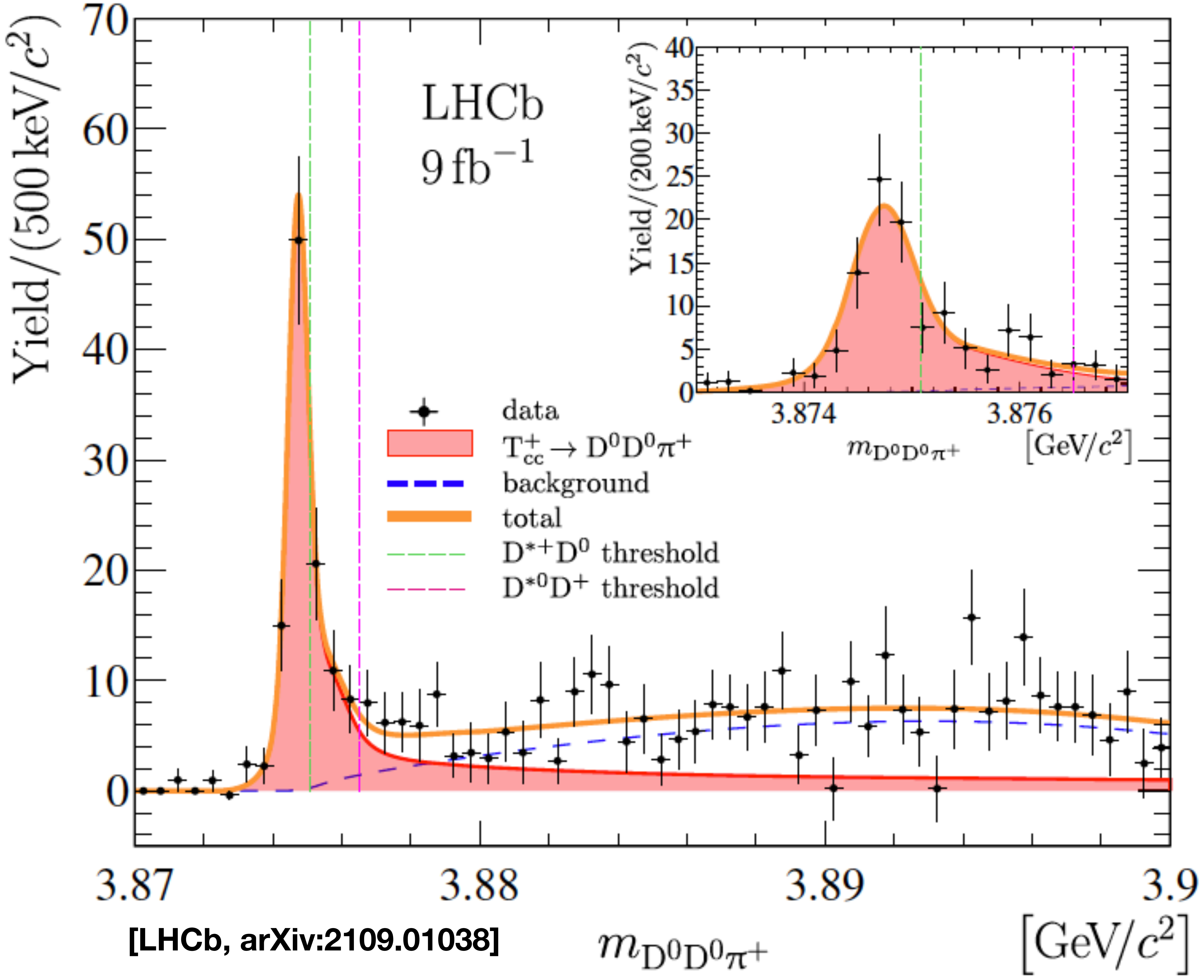
Doubly-charmed tetraquark

A gold-plated exotic for three-particle spectroscopy

Incorporating $DD\pi$ effects and left-hand cuts in lattice QCD studies of the $T_{cc}(3875)^+$

Maxwell T. Hansen[a], Fernando Romero-López[b], and Stephen R. Sharpe[c]

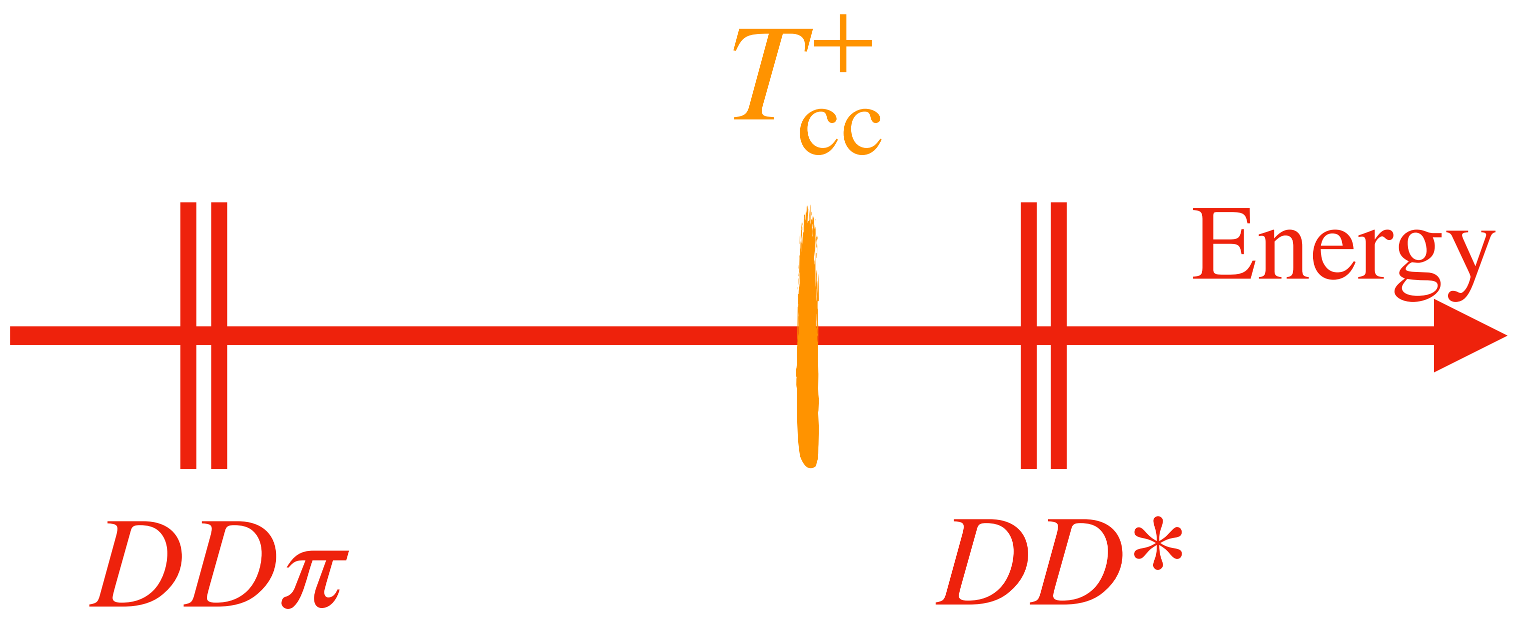
[arXiv:2401.06609]



[LHCb, arXiv:2109.01038]

Doubly-charmed tetraquark

Real world:



- **For physical quark masses is a three-body resonance**

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  need three-body formalism!

Doubly-charmed tetraquark

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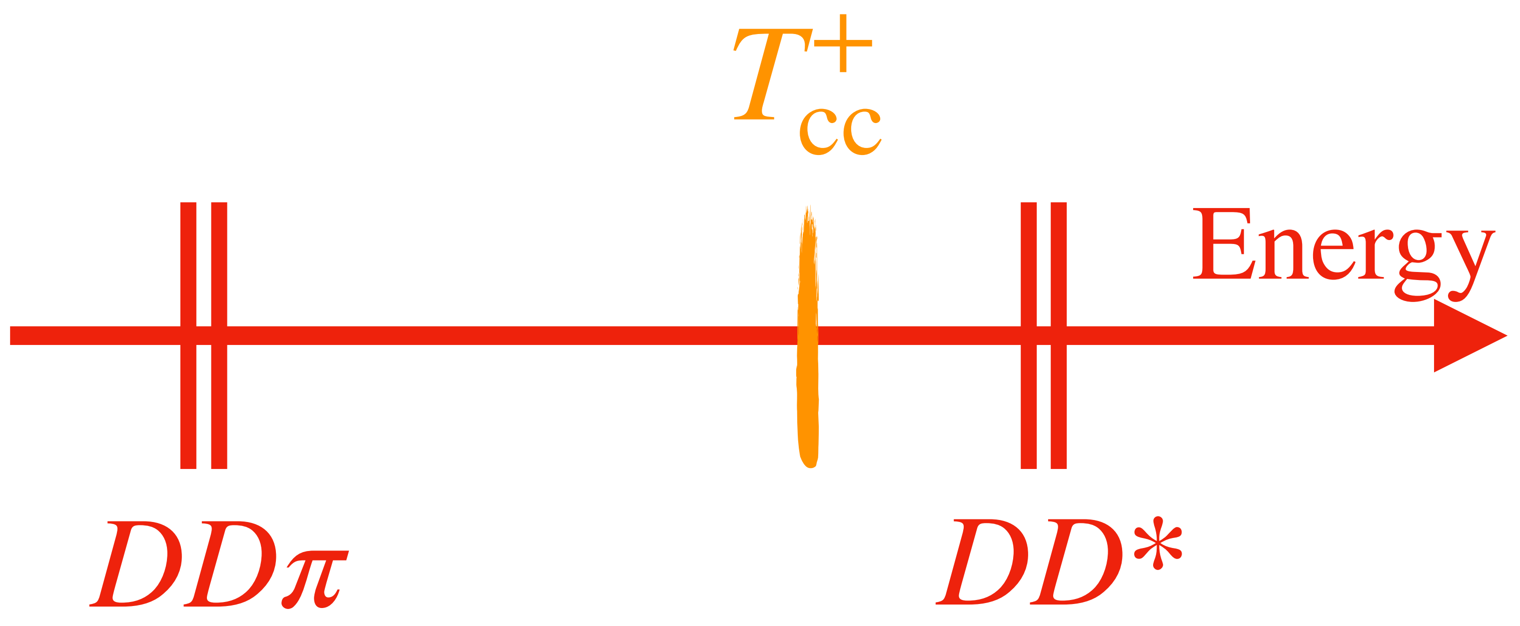

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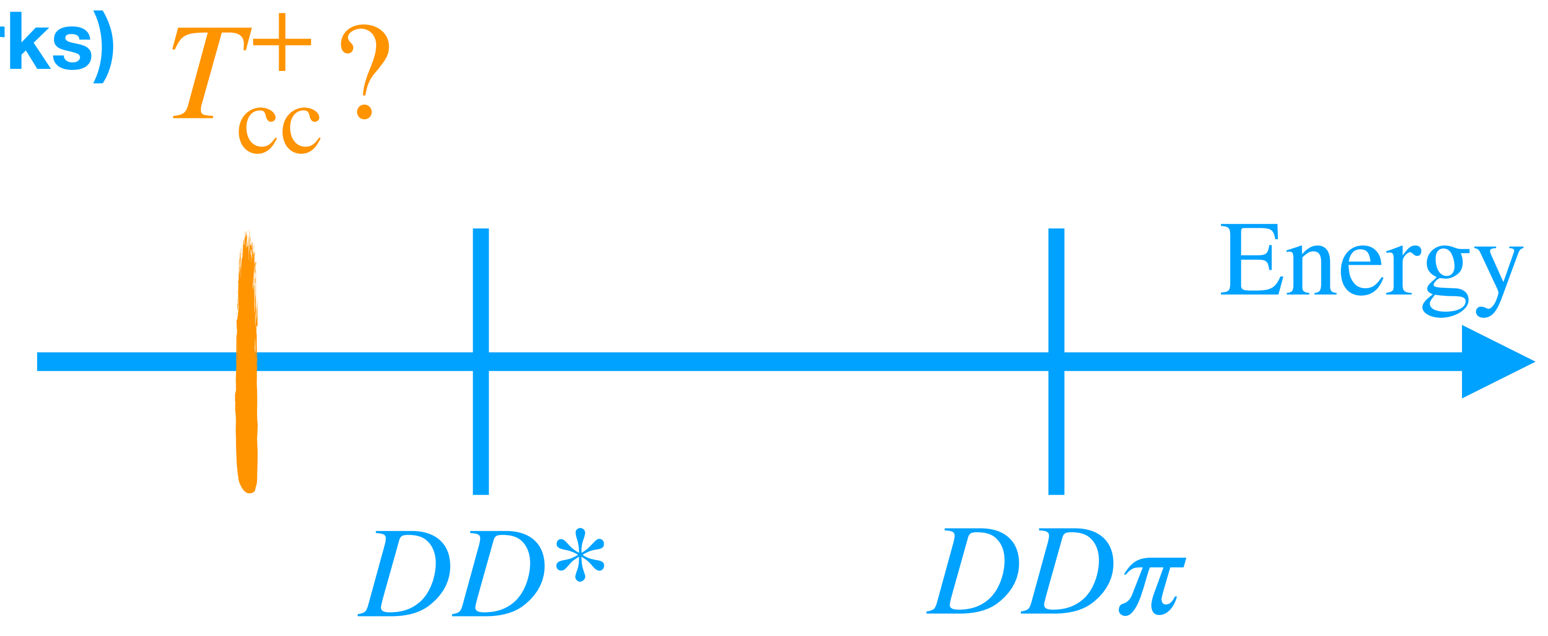
N_f=2+1+1 QCD (heavier quarks)



- **Stable D* at slightly heavier-than-physical quark mases**
  $T_{cc} \to DD^*$?

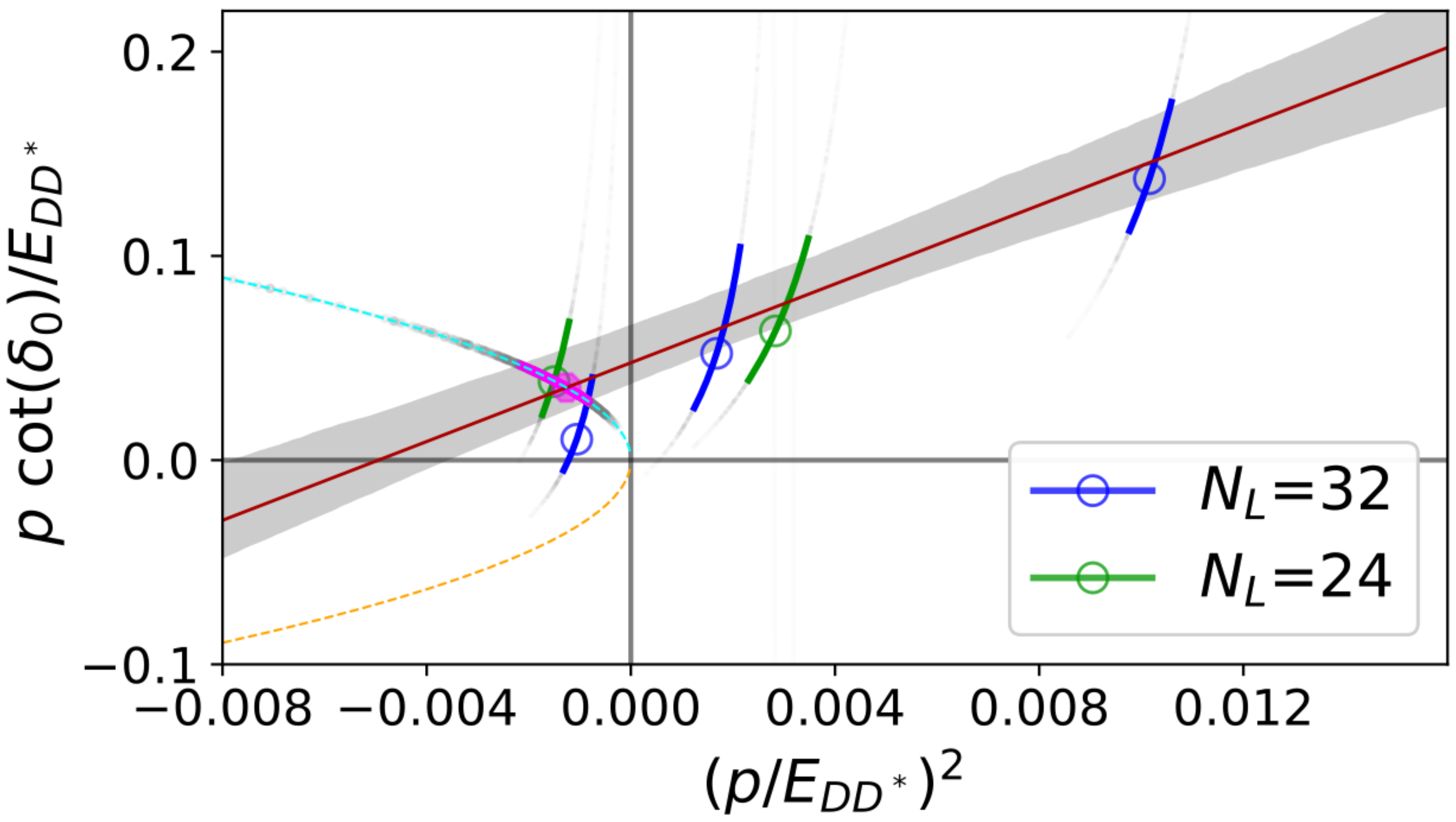
suitable for the two-body Lüscher formalism?

D-D* scattering

- Several work study the T_{cc} channel in this setup
  [Chen et al., 2206.06185] [Lyu et al. (HALQCD), 2302.04505]
  [Padmanath & Prelovsek, 2202.10110]

  - Signature of virtual bound state?

[Padmanath, Prelovsek, arXiv:2202.10110]



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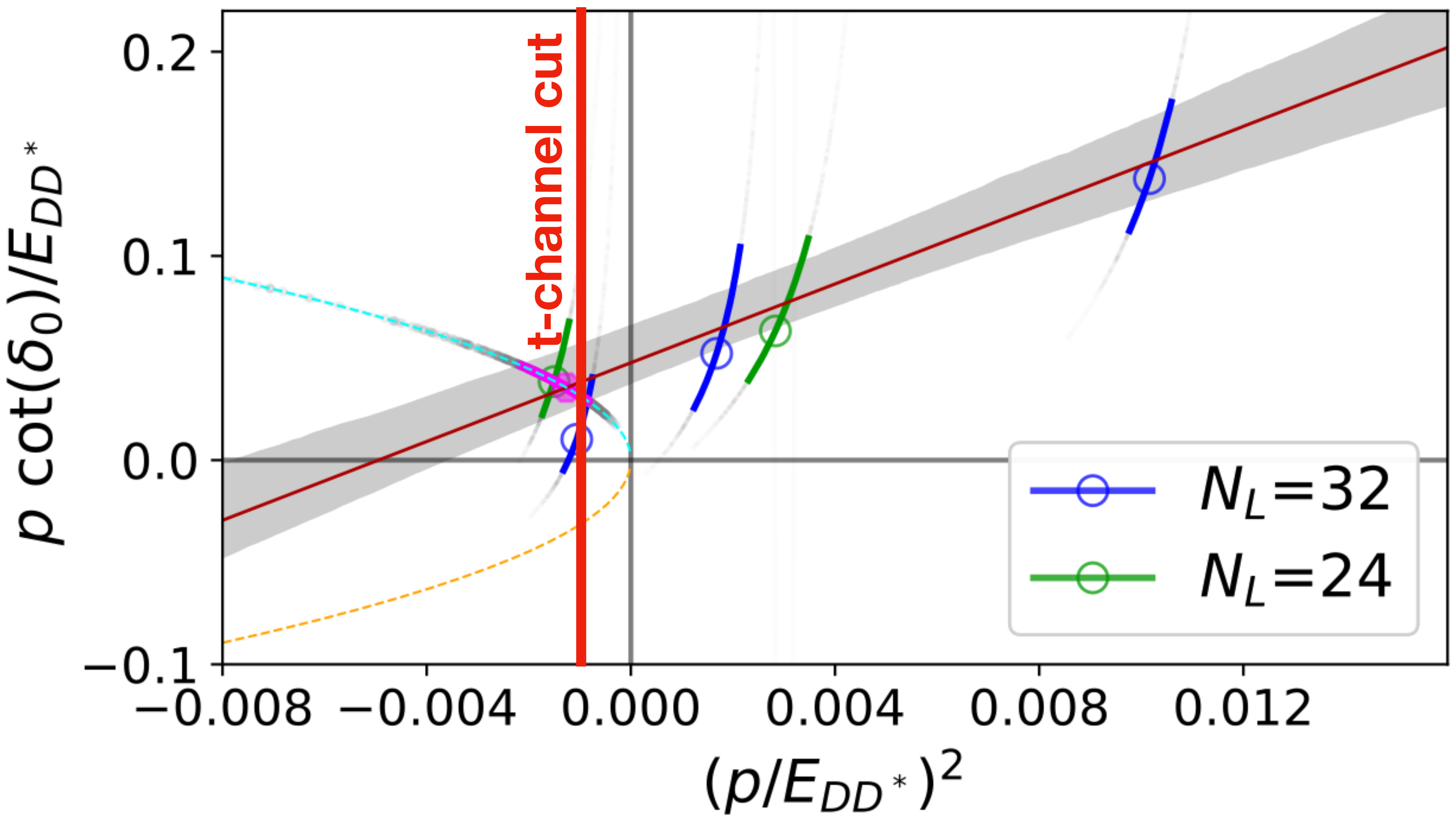
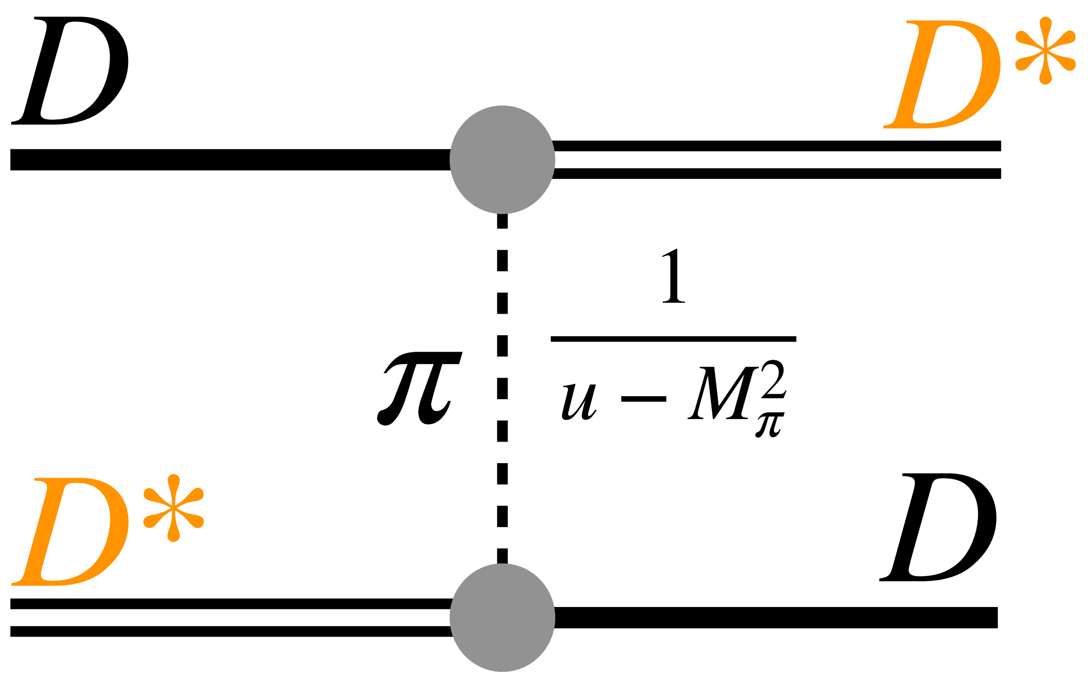
  - Signature of virtual bound state?
  - But two-particle formalism breaks down
    i.e. complex phase shift

[Padmanath, Prelovsek, arXiv:2202.10110]



! one-pion exchange creates non analytic behavior:
see also [Du et al, 2303.09441]



$$u = M_\pi^2, \quad t = 0, \quad s - s_{th} = -M_\pi^2 + (M_D - M_{D^*})^2$$

just 8 MeV below threshold!

The three-body solution

Three-body dynamics of a particle + dimer system as a solution for the left-hand cut problem

The three-body solution

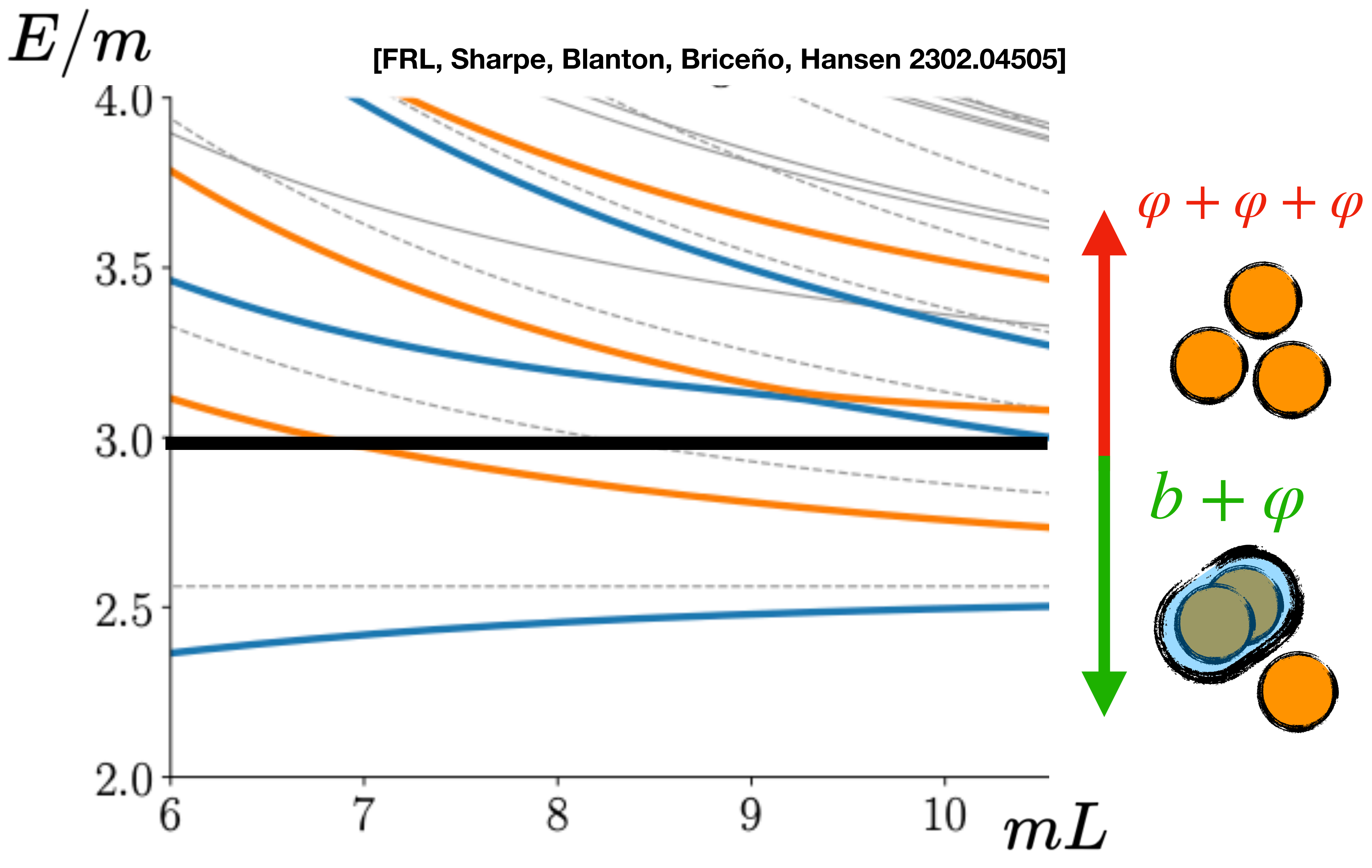
Three-body dynamics of a particle + dimer system as a solution for the left-hand cut problem

- Consider a toy model with a two-body bound state:

$$\mathcal{M}_2 \propto \frac{1}{k\cot\delta - ik} \qquad k\cot\delta = -\frac{1}{|a|}$$

$$m_b = 2m_\varphi\sqrt{1-\frac{1}{(m_\varphi a)^2}}$$

- Below the 3-body threshold, effective “particle-dimer”



[FRL et al 2302.04505] [Jackura et al 2010.09820]
[Dawid, Islam, Briceño, 2303.04394] [Pefkou et al (in prep)]

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$$\mathcal{M}_2 \propto \frac{1}{k\cot\delta - ik} \qquad k\cot\delta = -\frac{1}{|a|}$$

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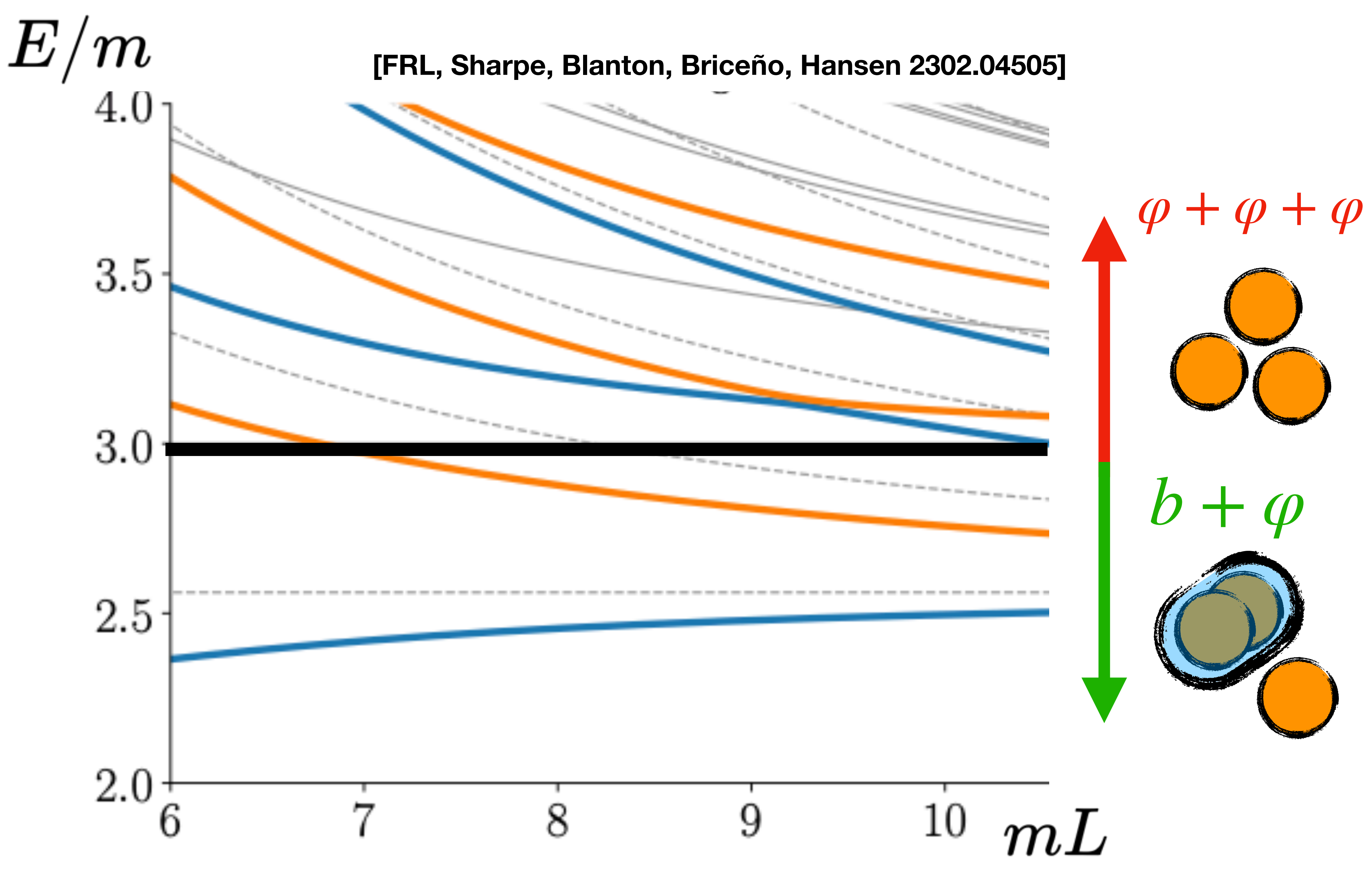
- Below the 3-body threshold, effective “particle-dimer”

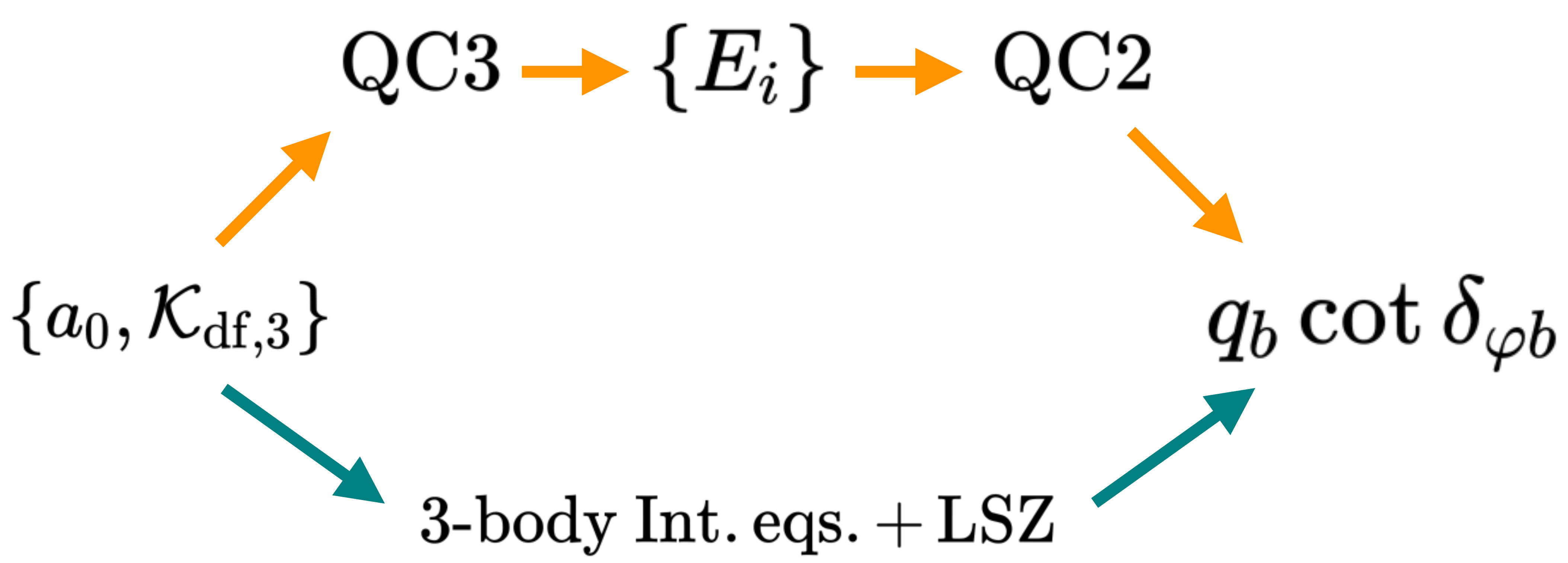
- Same idea for the T_{cc}
  - D^* as a $D\pi$ bound state
  - But system of non degenerate particles

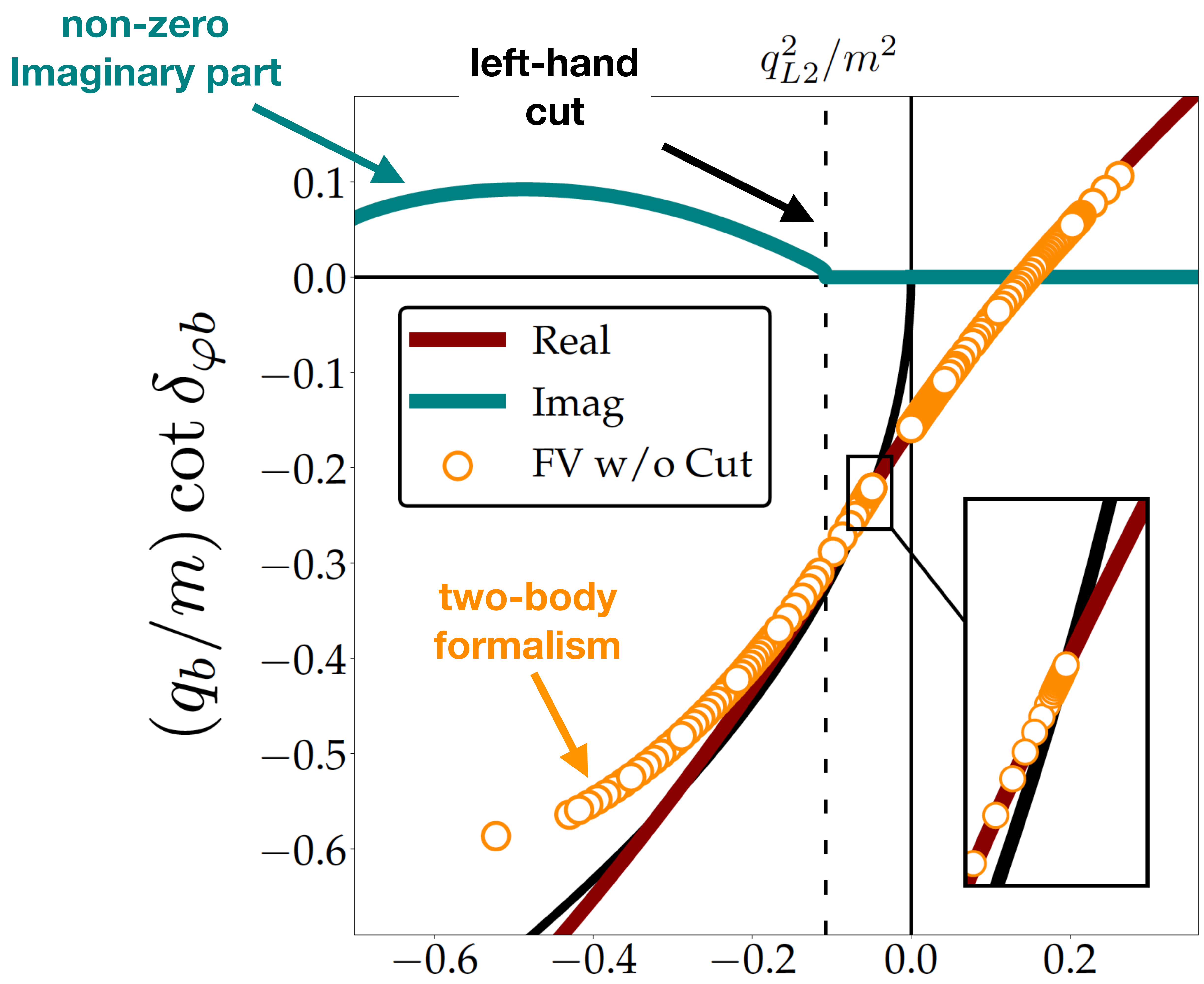


[FRL et al 2302.04505] [Jackura et al 2010.09820]
[Dawid, Islam, Briceño, 2303.04394] [Pefkou et al (in prep)]

The breakdown of QC2

An example of the breakdown of the two-body formalism is obtained by comparing the two- and three-particle formalisms



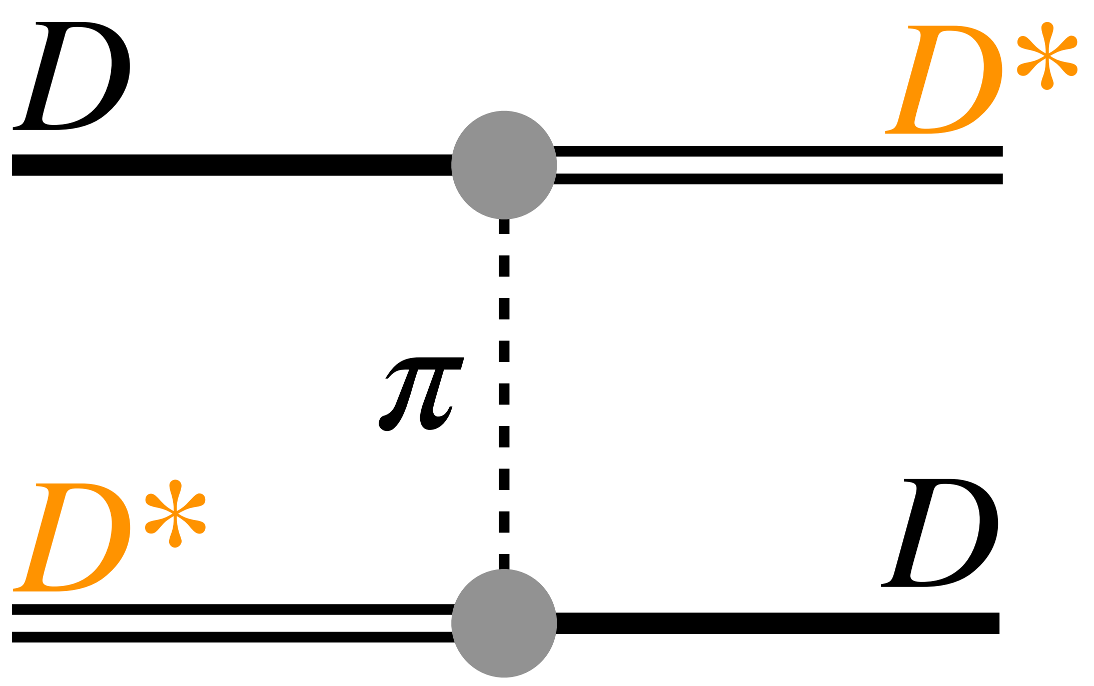


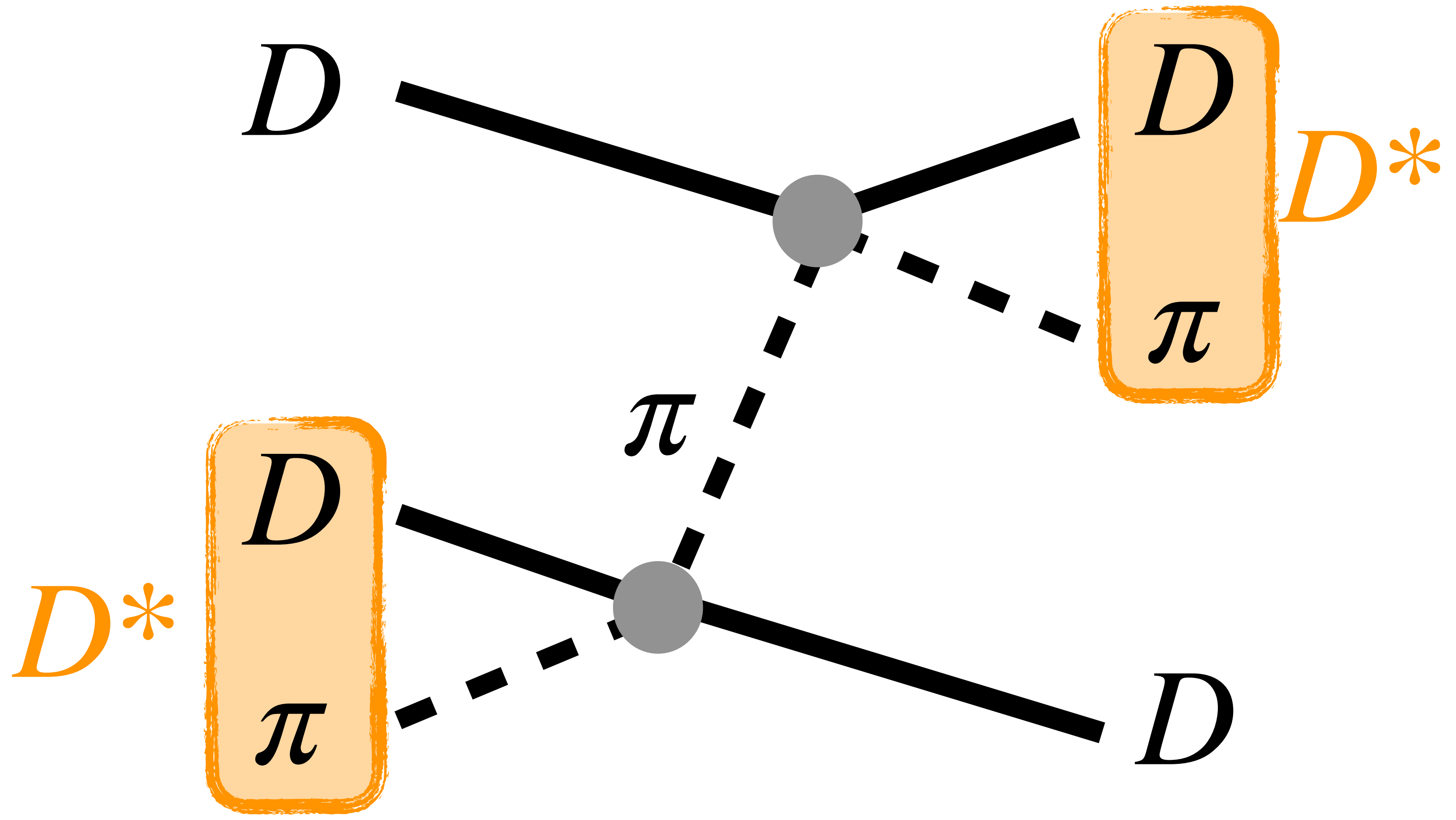
[Dawid, Islam, Briceño, 2303.04394]

Overcoming left-hand cuts

- How this solves the left-hand cut problem:
  - Finite-volume effects from one-pion exchange naturally incorporated

see alternative approaches: [Raposo, Hansen, arXiv:2311.18793]
[Meng et al, arXiv:2312.01930]





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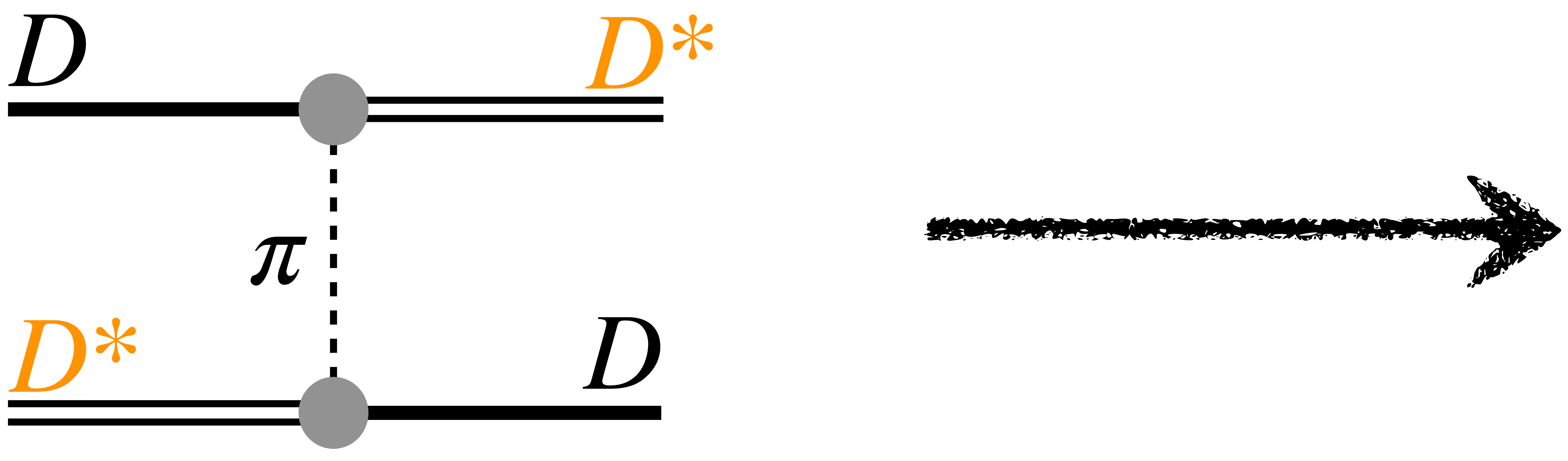
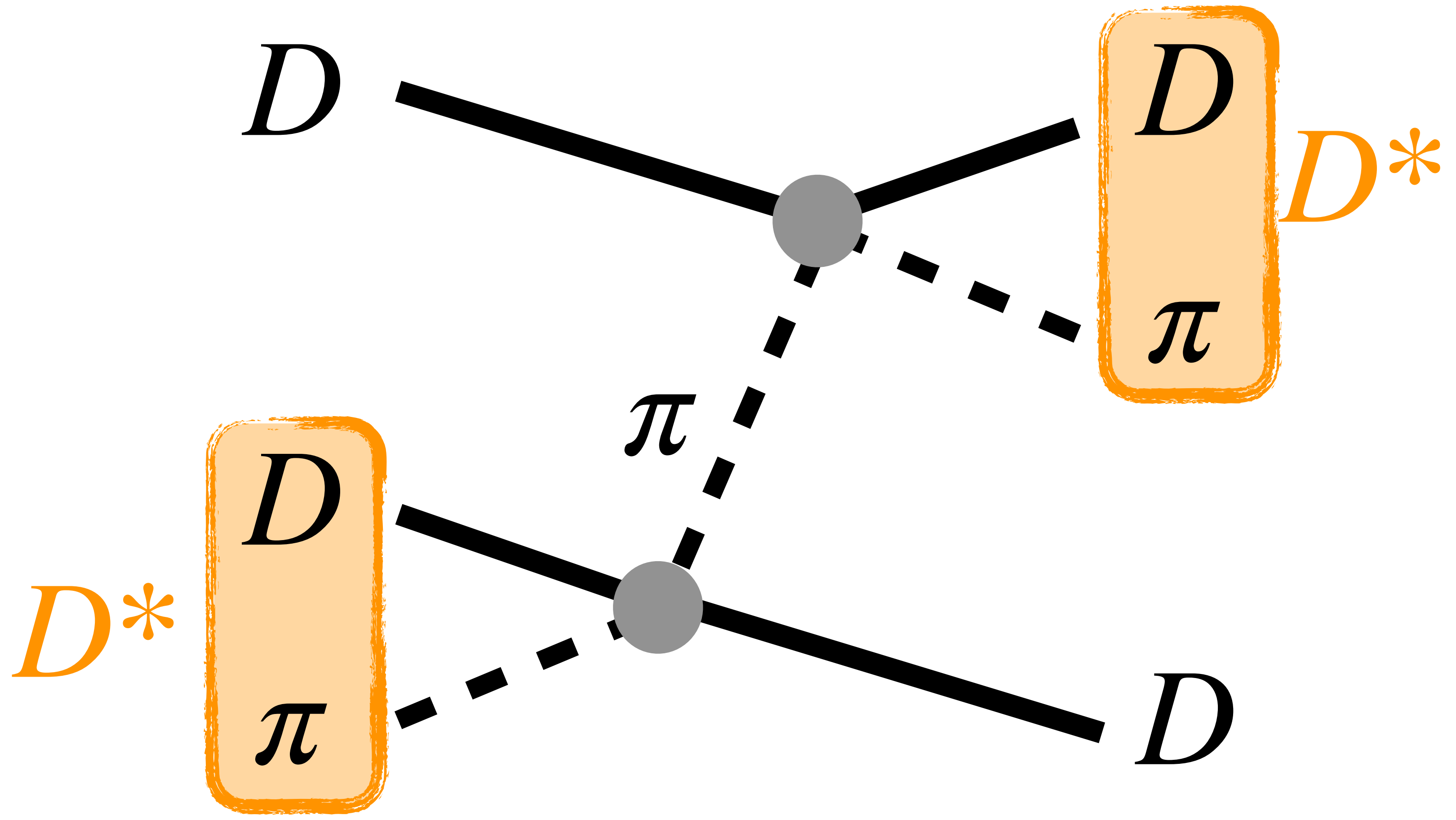
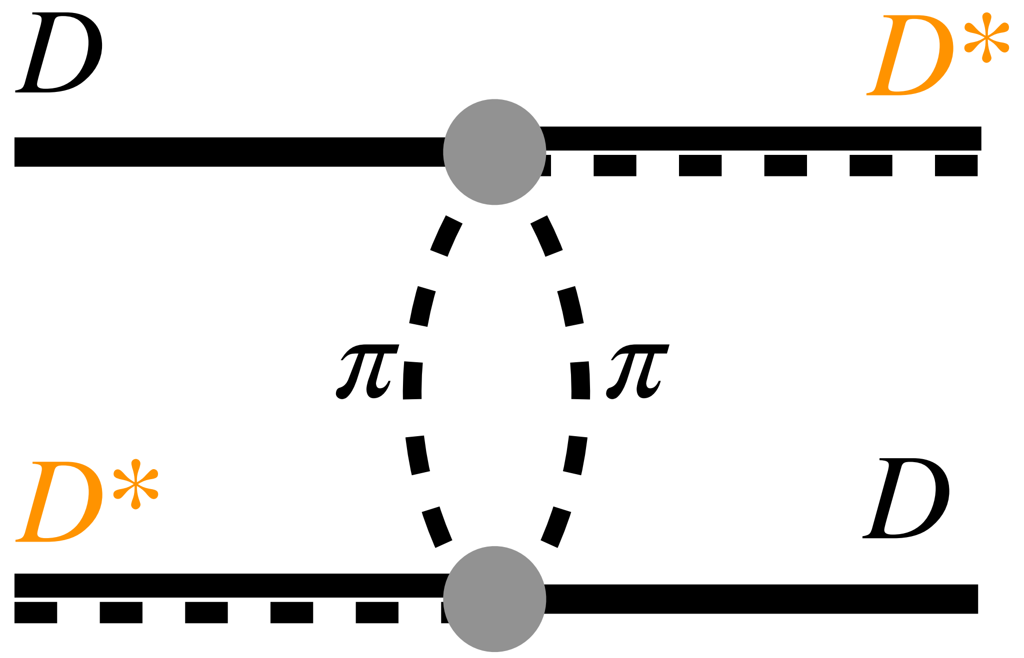




  - Range of applicability limited by two-pion exchange

21 MeV below one-pion lhc!

[Padmanath, Prelovsek, arXiv:2202.10110]



The Formalism

[Hansen, FRL, Sharpe, arXiv:2401.06609]

- The doubly-charmed tetraquark arises in coupled $DD\pi$ scattering

$$D^0 D^0 \pi^+ \quad D^+ D^0 \pi^0 \quad D^0 D^+ \pi^-$$

- Three isospin channels $\mathbf{1/2} \otimes \mathbf{1/2} \otimes \mathbf{1} \longrightarrow \mathbf{2} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{0}$

$T_{\rm cc}$

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- Formalism factorizes in three-meson isospin: $$\prod_{I\in\{0,1,2\}} \det_{i,k,\ell,m} \left[1 + \widehat{\mathcal{K}}^{[I]}_{\rm df,3} \widehat{F}^{[I]}_3\right] = 0$$

- Isospin 0 and 2 equations are formally identical to those for $K^+K^+\pi^+$ systems
[Blanton, Sharpe (2105.12094)]
  - Already implemented and tested!
[Blanton, FRL, Sharpe (2111.12734)]
  - Need two-meson interactions →

$$\widehat{\mathcal{K}}^{[I=0]}_{2,L} = \mathrm{diag}\left(\overline{\mathcal{K}}^{D\pi,I=1/2}_{2,L}, \frac{1}{2}\overline{\mathcal{K}}^{DD,I=1}_{2,L}\right)$$

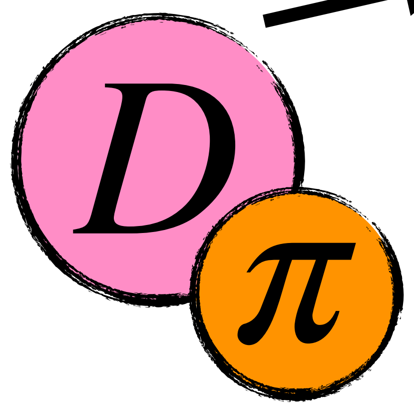
The strategy for the T_{cc}

[Hansen, FRL, Sharpe, arXiv:2401.06609]

Two-meson spectra

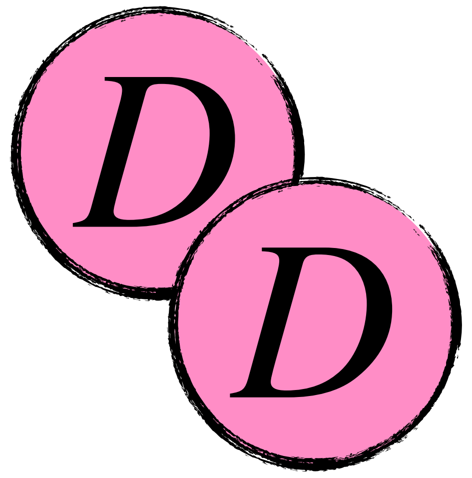
D^* as a bound state or resonance

$I_{D\pi} = 1/2$



s and p waves

$I_{DD} = 1$



s wave

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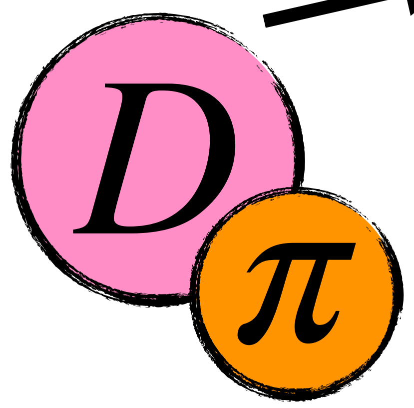

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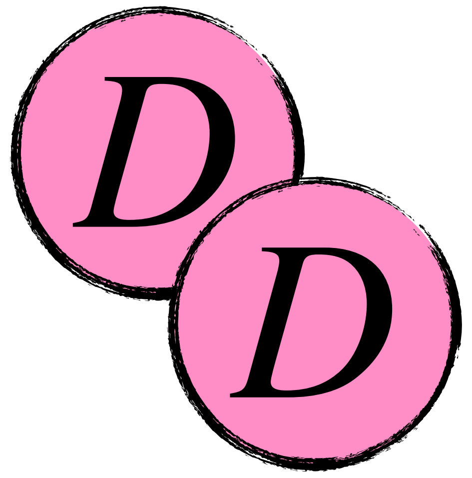
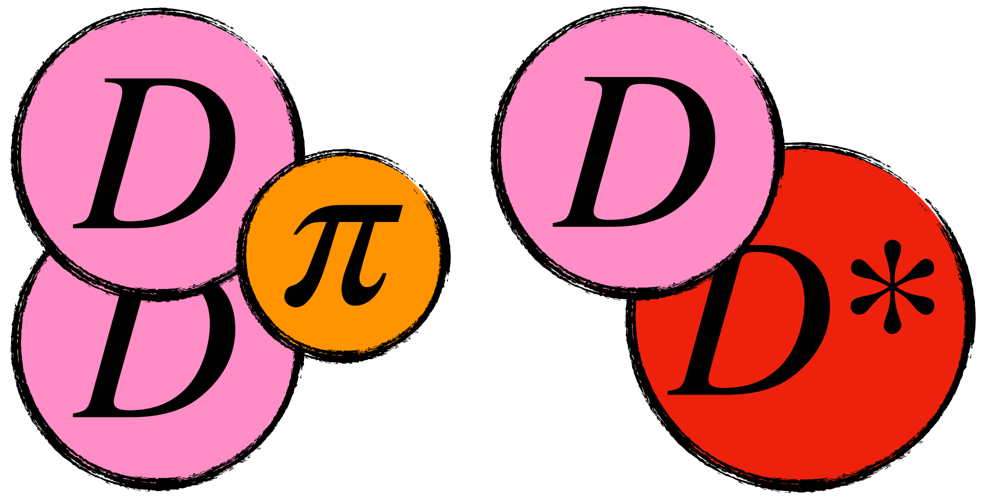


s wave

Three-meson spectrum



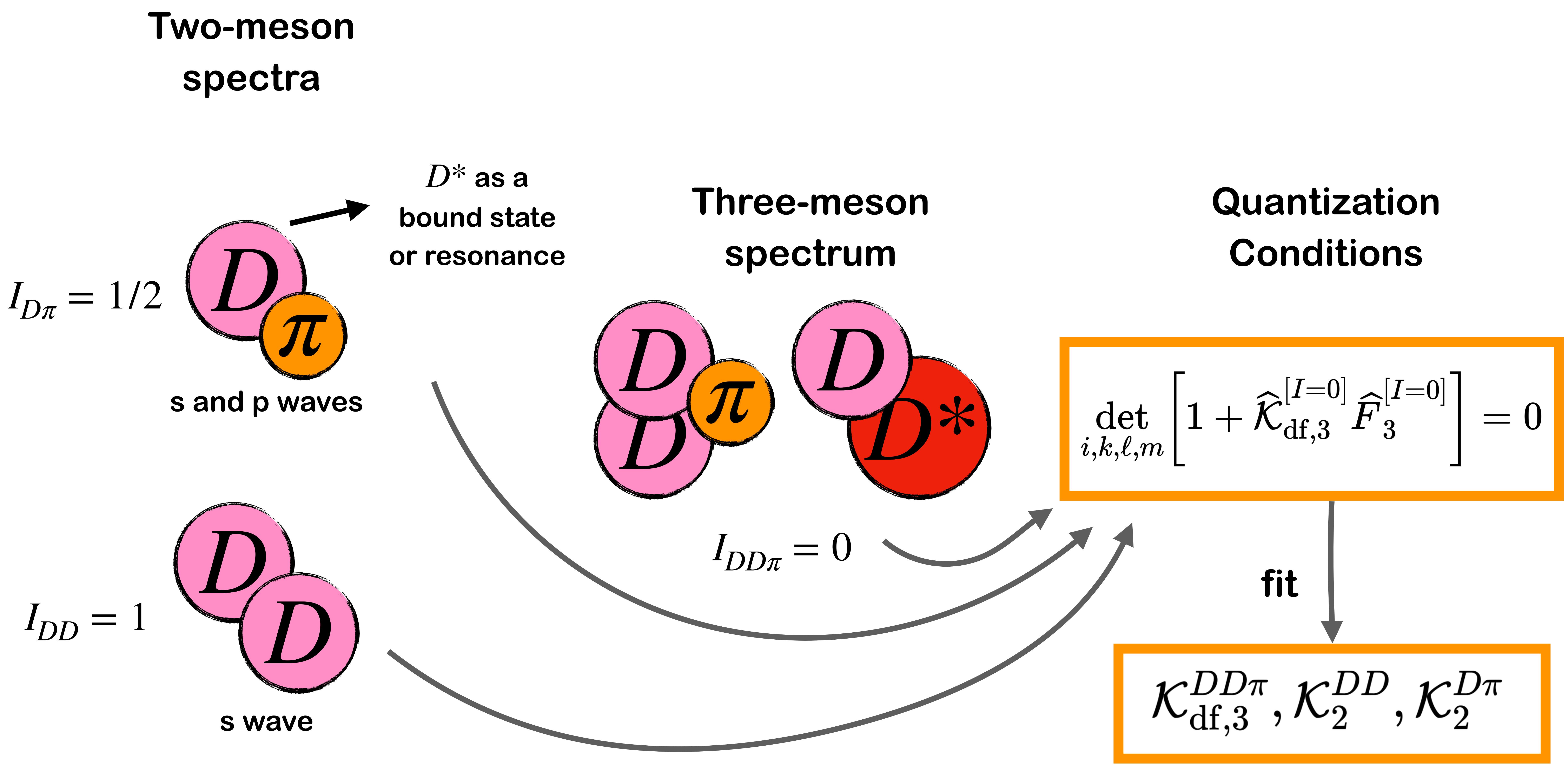
$I_{DD\pi} = 0$

Quantization Conditions

$$\det_{i,k,\ell,m}\left[1 + \widehat{\mathcal{K}}^{[I=0]}_{\mathrm{df},3}\widehat{F}^{[I=0]}_{3}\right] = 0$$

The strategy for the T_{cc}

[Hansen, FRL, Sharpe, arXiv:2401.06609]



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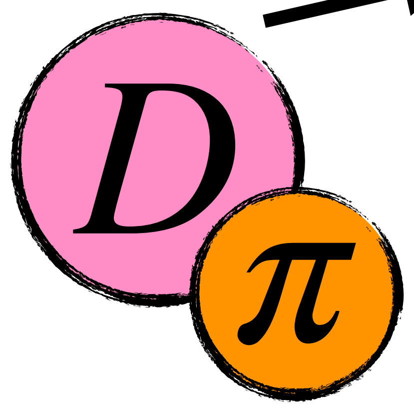

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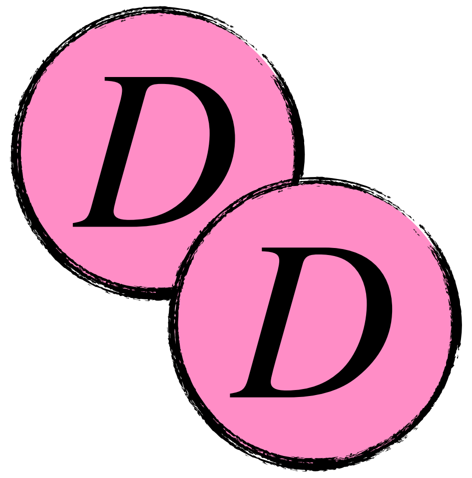

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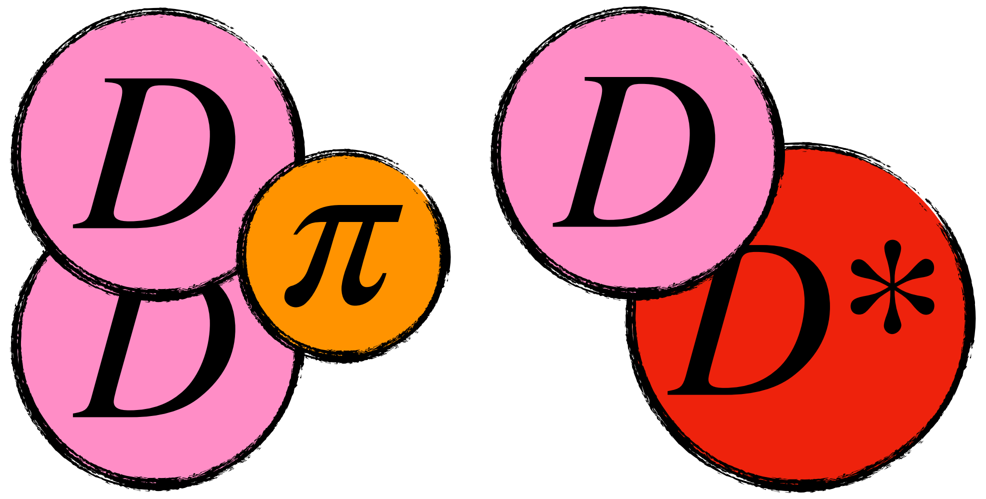

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Quantization Conditions

$$\det_{i,k,\ell,m}\left[1 + \widehat{\mathcal{K}}_{\mathrm{df},3}^{[I=0]} \widehat{F}_3^{[I=0]}\right] = 0$$

fit

$$\mathcal{K}_{\mathrm{df},3}^{DD\pi}, \mathcal{K}_2^{DD}, \mathcal{K}_2^{D\pi}$$

Tetraquark properties

$$\mathcal{M}_3 \sim \frac{-g^2}{s - M_{T_{cc}}^2}$$

Integral equations

[Dawid, FRL, Sharpe (in prep)]

Infinite-volume three-body dynamics of the T_{cc}

[Dawid, FRL, Sharpe (in prep)]

Integral equations

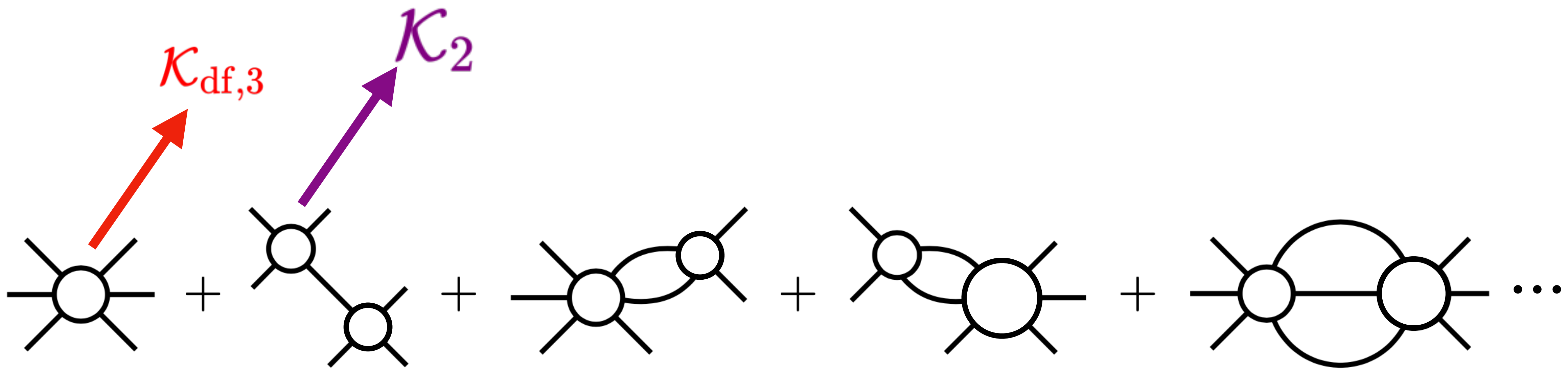
K-matrices

$\mathcal{K}_2$

$\mathcal{K}_{\mathrm{df},3}$

Integral equations →

Scattering amplitude

$\mathcal{M}_3 =$ 

- Adds infinite tower of rescattering processes
- Resulting scattering amplitude respects unitarity

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Integral equations →

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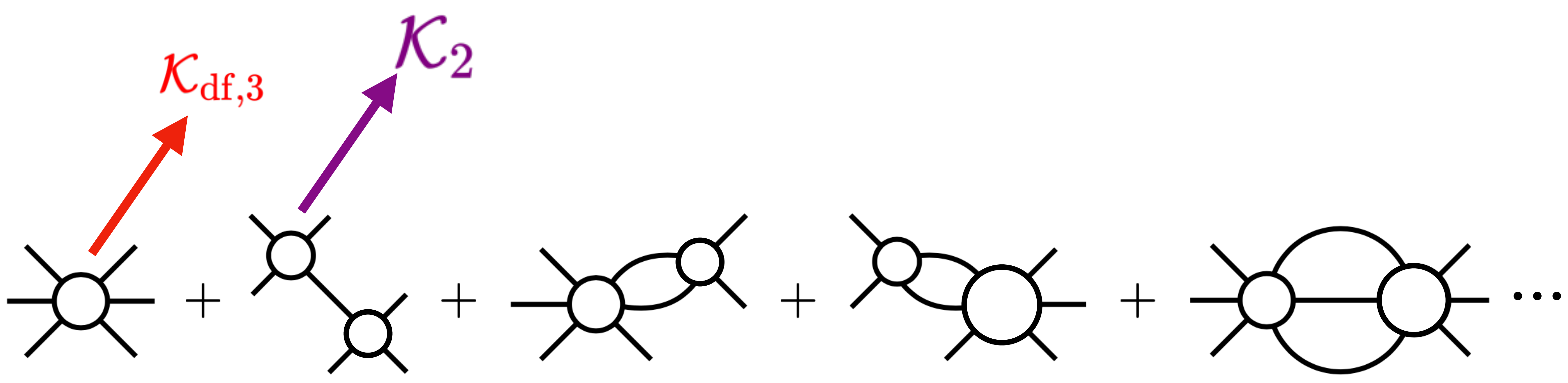


- Adds infinite tower of rescattering processes
- Resulting scattering amplitude respects unitarity

Key idea:

Integral equations can be used parametrize a unitary scattering amplitude using the K matrices, independently of lattice QCD!

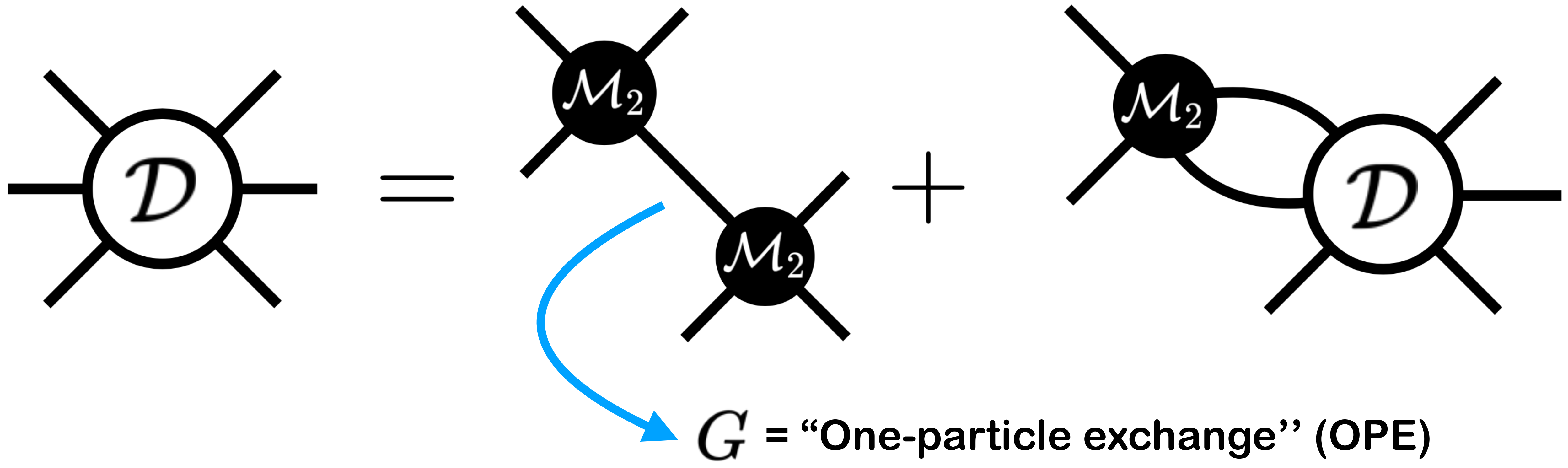
Fernando Romero-López, MIT

Ladder equation

- Consider a vanishing three-particle K matrix, $\mathcal{K}_{\mathrm{df},3} = 0$ (restriction can be lifted up later)

- The problem reduces to solving the ladder equation

$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \int \mathcal{M}_2 G \mathcal{D}$$



G = “One-particle exchange’’ (OPE)

Ladder equation

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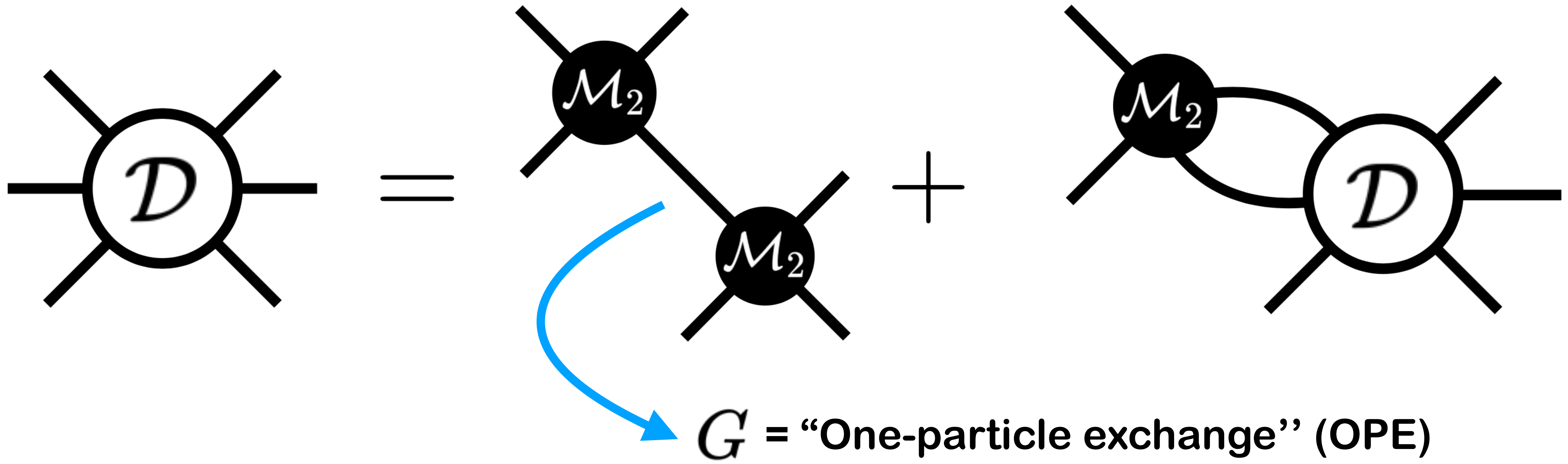
$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \int \mathcal{M}_2 G \mathcal{D} \quad \xrightarrow{\mathcal{D} = \mathcal{M}_2\, d\, \mathcal{M}_2} \quad d = -G - \int G \mathcal{M}_2 d$$



G = "One-particle exchange'' (OPE)

▷ Useful to define amputated integral equation

Dimer-particle scattering

- Assume the two-body subsystem has a bound state (dimer)

$$\mathcal{M}_2 \simeq \frac{-g^2}{s - m_b^2}$$

coupling (g^2), mass (m_b^2)

- The particle-dimer amplitude can be obtained using the LSZ reduction formula

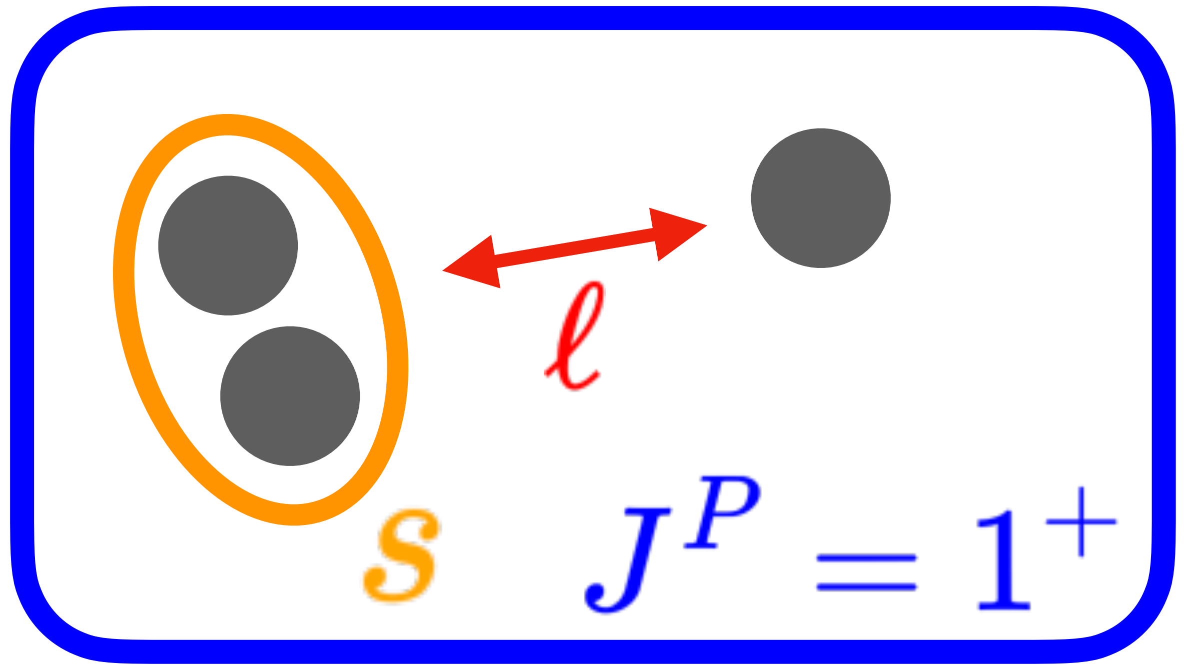
$$\mathcal{M}_{\varphi b} = \lim_{s_{2p}, s_{2k} \to m_b^2} (s_{2p} - m_b^2)\mathcal{D}(s,k,p)(s_{2k} - m_b^2) = g^2 d(s, q_{\varphi b}, q_{\varphi b})$$

- Given underlying K matrices, particle-dimer dynamics are determined through integral equations.

$\mathcal{M}_{DD^*}$ is fully determined by $DD,\ D\pi,\ DD\pi$ interactions!

$DD\pi$ system in a $J^P=1^+$, $I=0$

- A pair with "spin", and a relative angular momentum to the spectator
  - Note that (ℓ, s) can mix!
    - → Indeed, the T_{cc} has a "d-wave" component!



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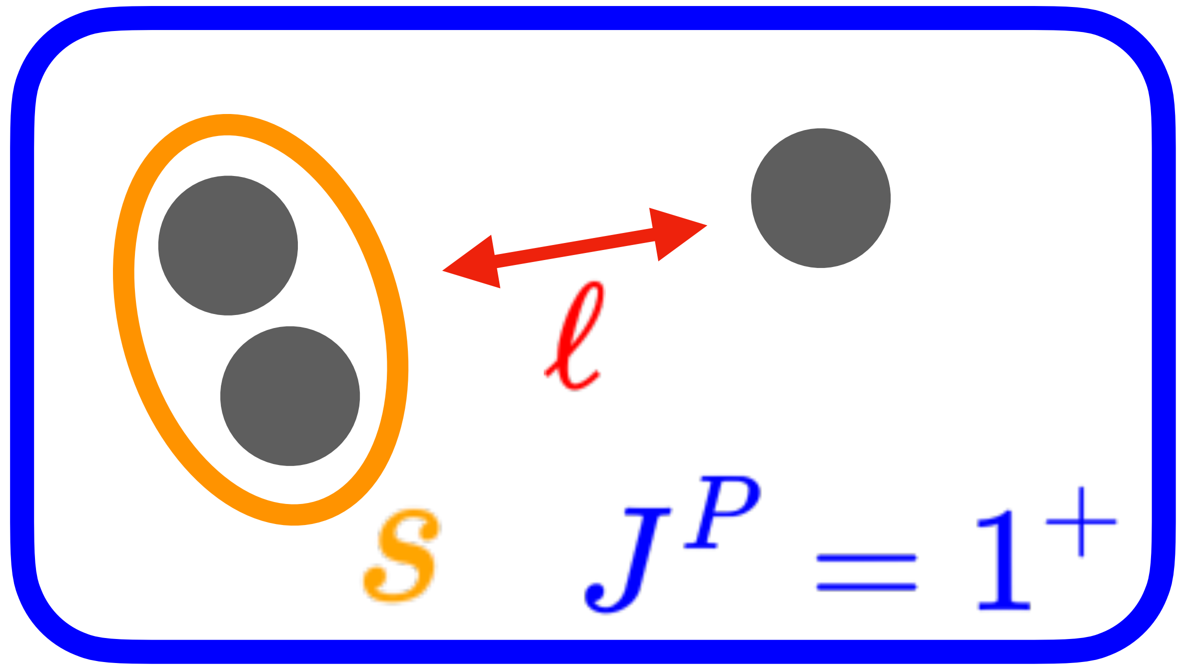

- Two choices of (pair + spectator), and various (ℓ, s) combinations:

$$(DD)_{I=1}\pi$$

$$(\ell = 1, s = 0)$$

$$(D\pi)_{I=1/2}D$$

$$(\ell = 1, s = 0)$$
$$(\ell = 0, s = 1)$$
$$(\ell = 2, s = 1)$$

Coupled integral equations

$$\mathcal{D}^{J^P=1^+} = \begin{pmatrix} \mathcal{D}^{1,1}_{10,10} & \mathcal{D}^{1,1}_{10,01} & \mathcal{D}^{1,1}_{10,21} & \mathcal{D}^{1,2}_{10,10} \\ \mathcal{D}^{1,1}_{01,10} & \mathcal{D}^{1,1}_{01,01} & \mathcal{D}^{1,1}_{01,21} & \mathcal{D}^{1,2}_{01,10} \\ \mathcal{D}^{1,1}_{21,10} & \mathcal{D}^{1,1}_{21,01} & \mathcal{D}^{1,1}_{21,21} & \mathcal{D}^{1,2}_{21,10} \\ \mathcal{D}^{2,1}_{10,10} & \mathcal{D}^{2,1}_{10,01} & \mathcal{D}^{2,1}_{10,21} & \mathcal{D}^{2,2}_{10,10} \end{pmatrix}$$

Note: $s \leq 1$

Solving integral equations

- Discretize integration in N steps and use linear algebra

$$d = -G - \int_0^{k_{\max}} dk\, K d \longrightarrow d_{kp} = -G_{kp} - \sum_r K_{kr} d_{rp} \longrightarrow \boxed{d_{kp} = -(1+K)^{-1}_{kr} G_{rp}}$$

(need $N \to \infty$ limit)

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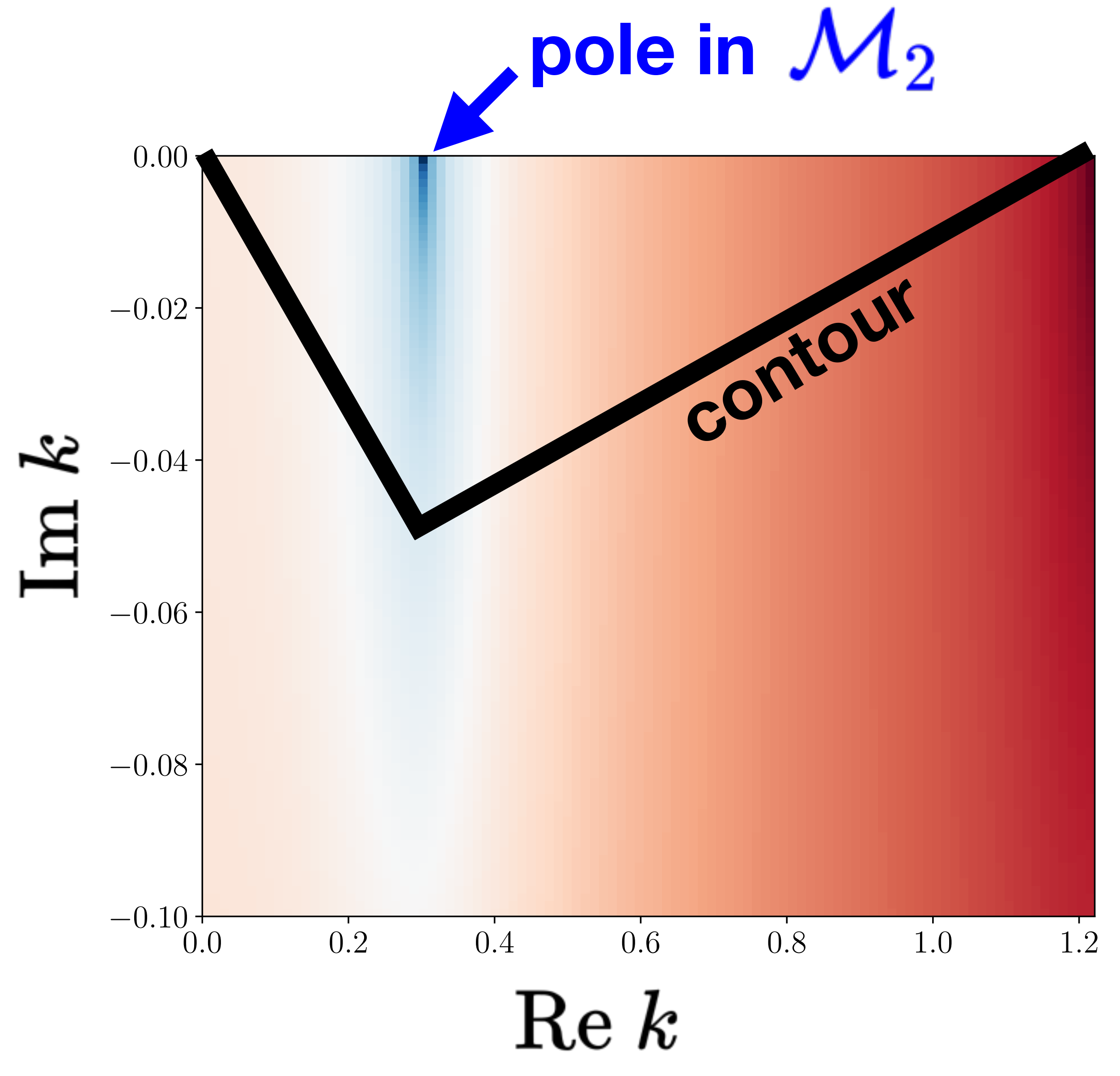
- Use Cauchy's theorem to deform the integration contour
  - Improves convergence
  - Sometimes necessary to avoid singularities



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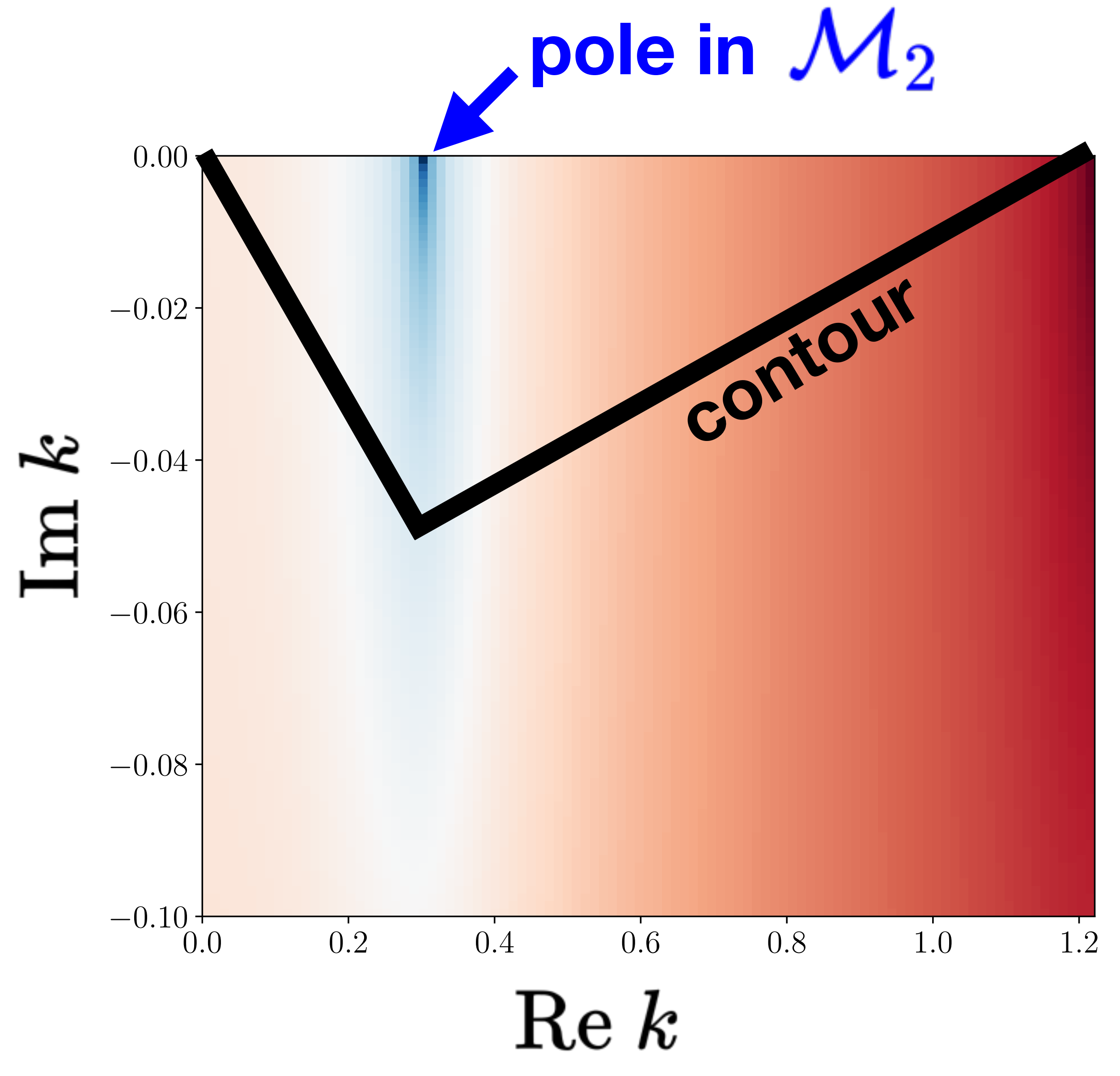


- Need partial-wave projected one-particle-exchange

$$G^{(ij)}(p,k)_{J;\ell' s';\ell s}$$

[Jackura, Briceño, arXiv:2312.00625]

A toy model for the T_{cc}

Parameters match heavier charm mass of [Padmanath, Prelovsek, arXiv:2202.10110]

- Choose meson masses:

$$\frac{M_\pi}{M_D} = 0.145$$

- Simple Dπ p-wave interactions to have a bound D*

$$\frac{k^3}{M_D^3}\cot\delta_1^{D\pi} = -\frac{1}{M_D a_1^{D\pi}} \longrightarrow M_D a_1^{D\pi} = -7.9^3 \longrightarrow \frac{M_{D^*}}{M_D} = 1.063$$

- Set Dπ s-wave, DD s-wave and three-body K matrix to zero

$$a_0^{DD} = a_0^{D\pi} = \mathcal{K}_{\mathrm{df},3} = 0$$

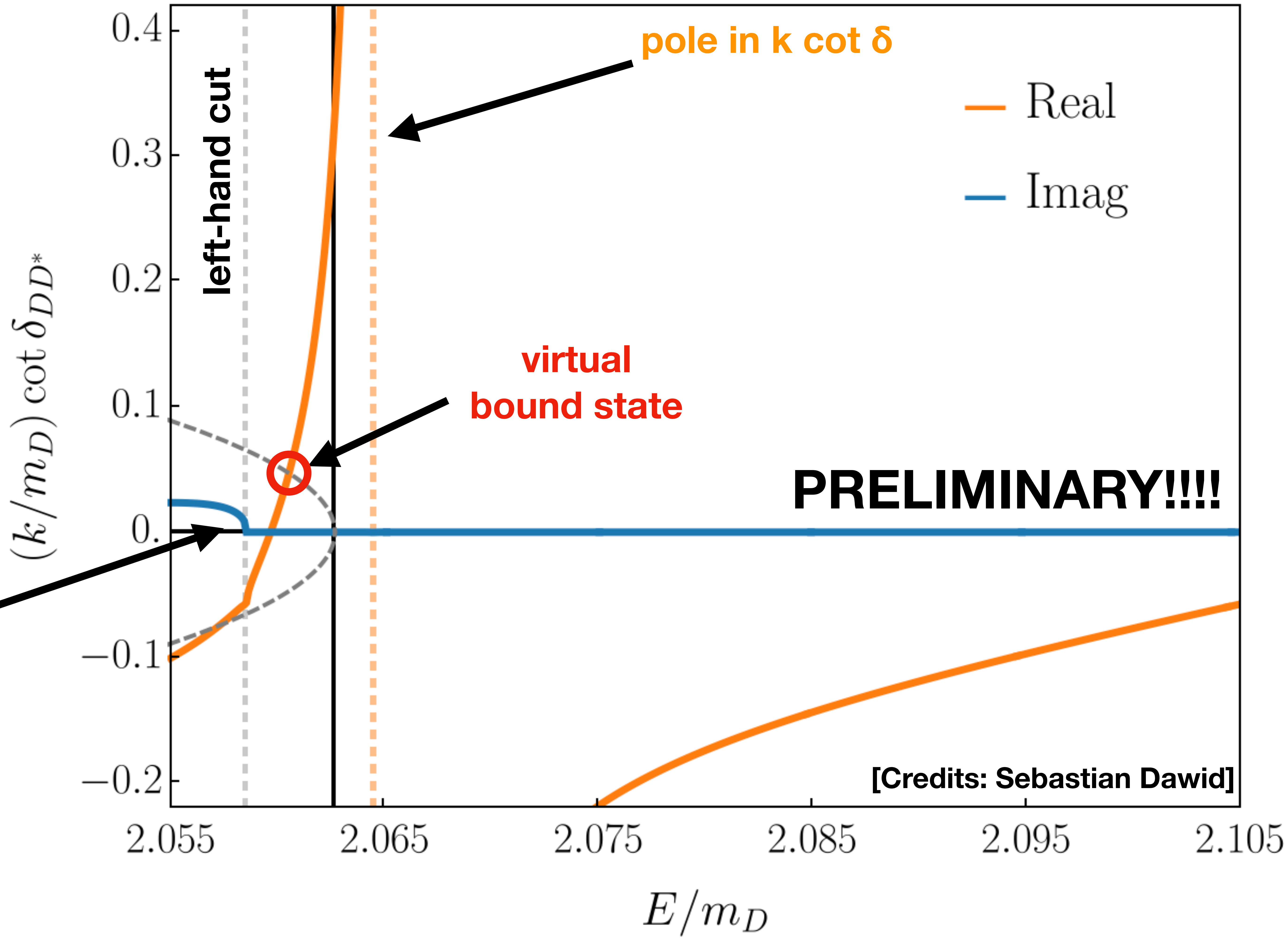
- Neglect DD* d-wave interactions

(ignore d-wave component of the T_{cc})

one-channel int. eq.

$$(D\pi)_{I=1/2}D$$
$$(\ell = 0, s = 1)$$

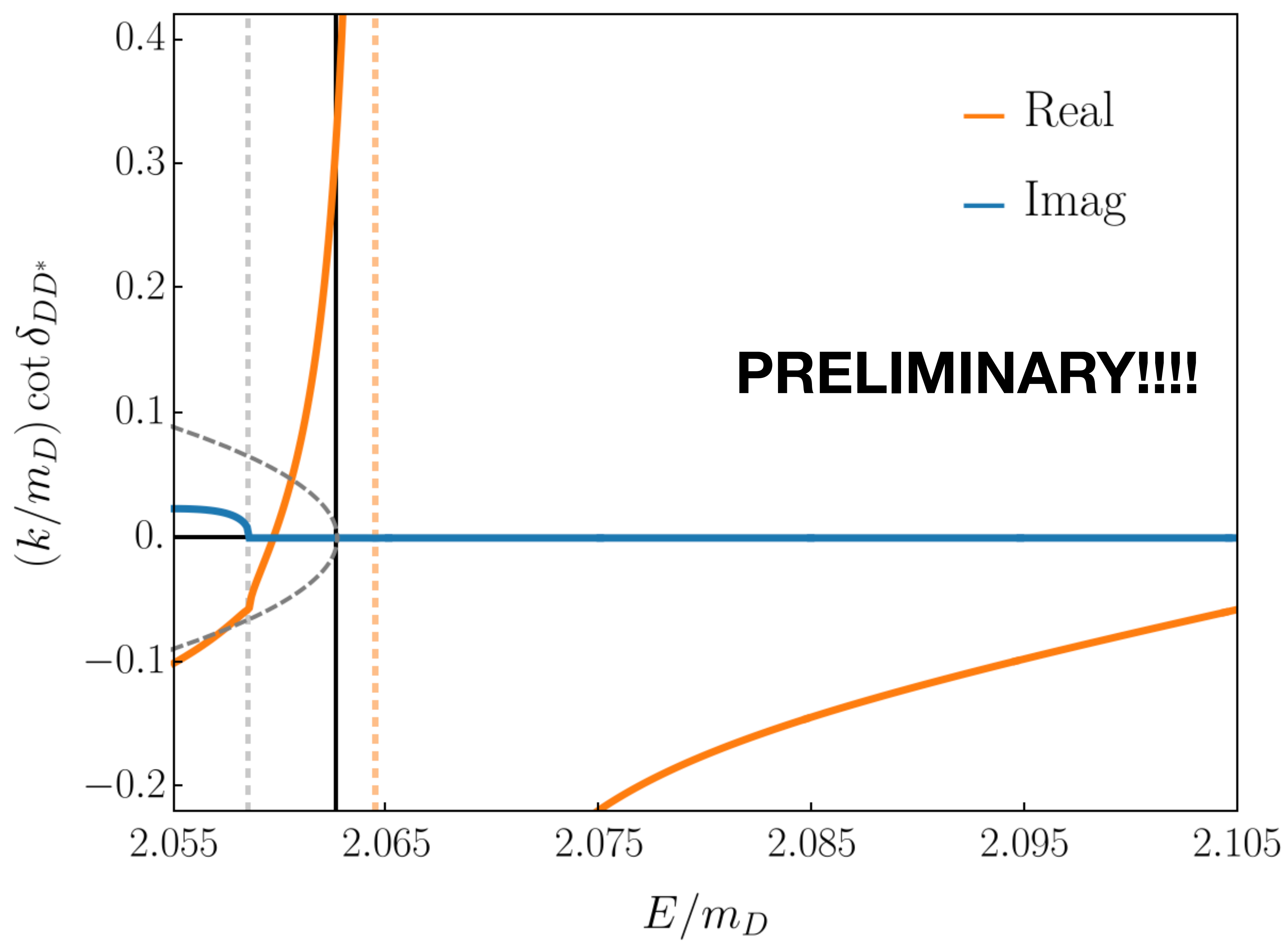
Results for D-D* scattering



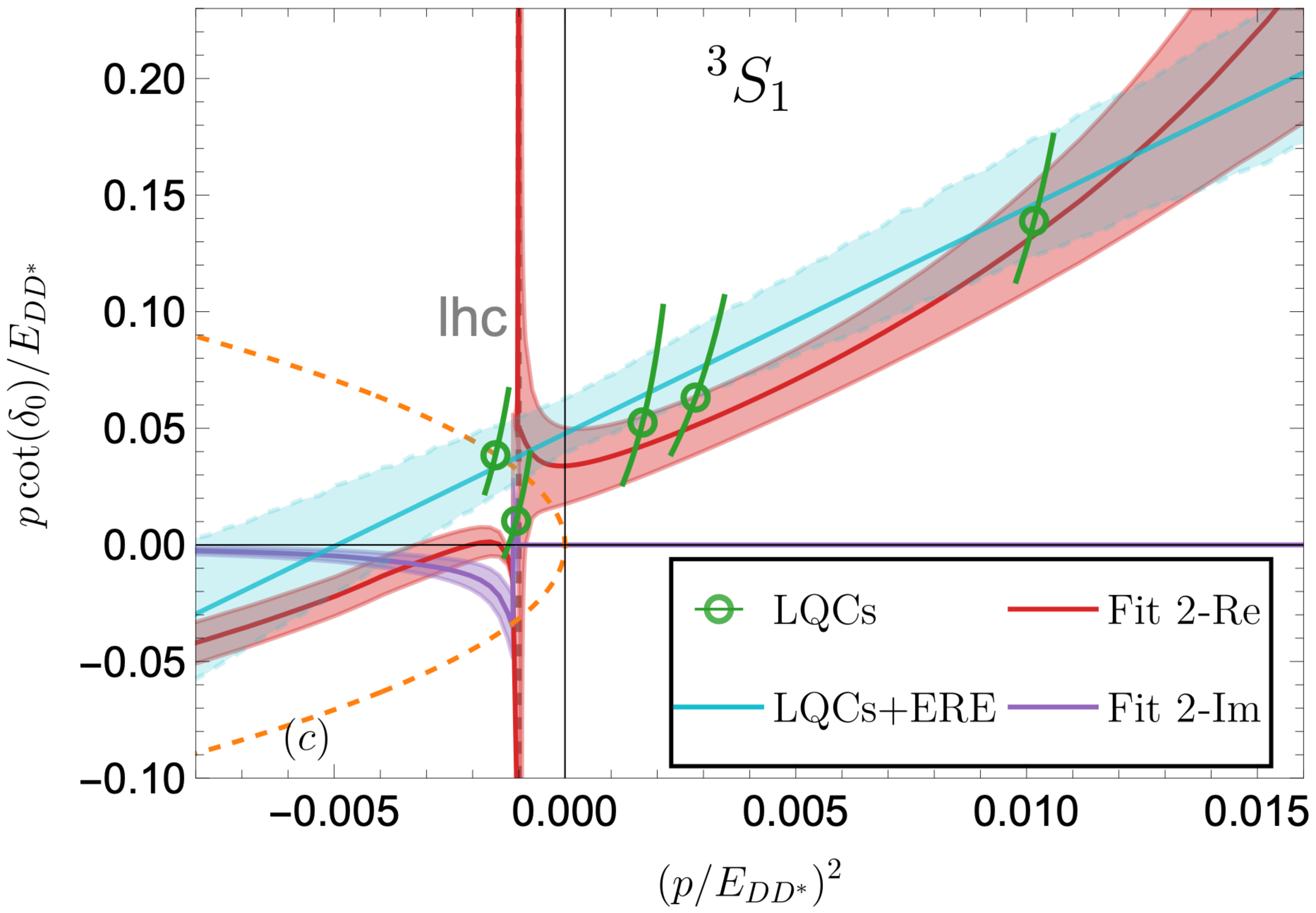
non-zero
imaginary part

Results for D-D* scattering

- Simple one-parameter model reproduces some qualitative features (virtual bound state)
- Important concerns about features of two body amplitudes: left-hand cut, pole in $k \cot \delta$.



[Credits: Sebastian Dawid]



[Padmanath, Prelovsek, arXiv:2202.10110]
[Meng et al, arXiv:2312.01930]

Next steps

- Solve full set of integral equations:
  - The T_{cc} has a d-wave component, how important is that?
  - How does s-wave DD and $D\pi$ scattering influence results?

$$\mathcal{D}^{J^P=1^+} = \begin{pmatrix} \mathcal{D}^{1,1}_{10,10} & \mathcal{D}^{1,1}_{10,01} & \mathcal{D}^{1,1}_{10,21} & \mathcal{D}^{1,2}_{10,10} \\ \mathcal{D}^{1,1}_{01,10} & \mathcal{D}^{1,1}_{01,01} & \mathcal{D}^{1,1}_{01,21} & \mathcal{D}^{1,2}_{01,10} \\ \mathcal{D}^{1,1}_{21,10} & \mathcal{D}^{1,1}_{21,01} & \mathcal{D}^{1,1}_{21,21} & \mathcal{D}^{1,2}_{21,10} \\ \mathcal{D}^{2,1}_{10,10} & \mathcal{D}^{2,1}_{10,01} & \mathcal{D}^{2,1}_{10,21} & \mathcal{D}^{2,2}_{10,10} \end{pmatrix}$$

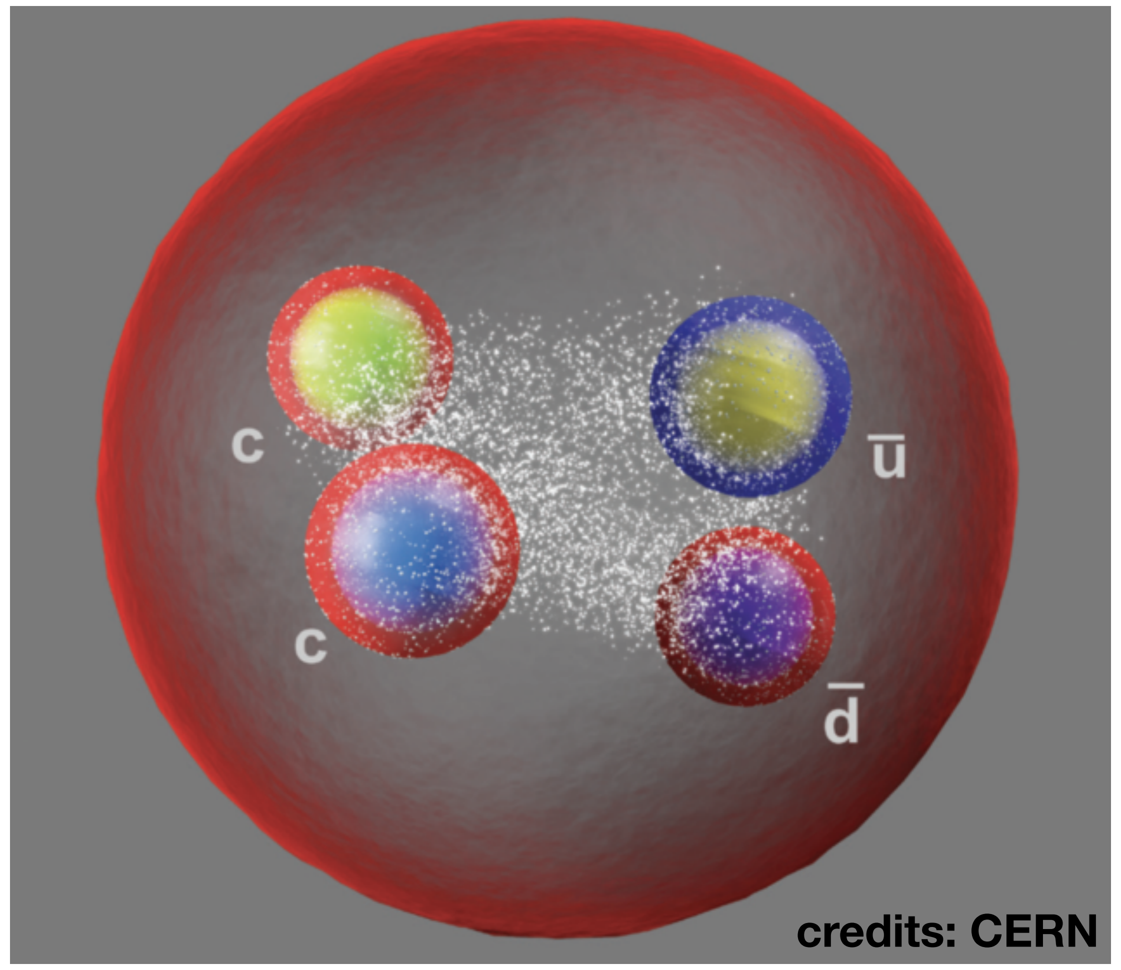
- Solve integral equations with three-particle K matrix
  - How much does $K_{df,3}$ matter?
- Explore the regime in which the tetraquark decays to three-particles.
  - Chiral dependence of the T_{cc}?
- Similar features in systems of B mesons. What about the T_{bb}?

[Hudspith, Mohler, arXiv:2303.17295]
[Meinel, Pflaumer, Wagner, arXiv:2205.13982]

Summary & Outlook

Summary and outlook

- ☑ Lattice QCD provides a first-principle tool to investigate the hadron spectrum
- ☑ First results for systems of three non degenerate particles
- ☑ The formalism for $DD\pi$ systems, allowing the study of the T_{cc}
- ☑ Integral equations could describe the T_{cc} using three-body dynamics
- ☐ Ready to analyze lattice data, but need DD and $D\pi$ interactions
- ☐ Need to explore more features of integral equations.



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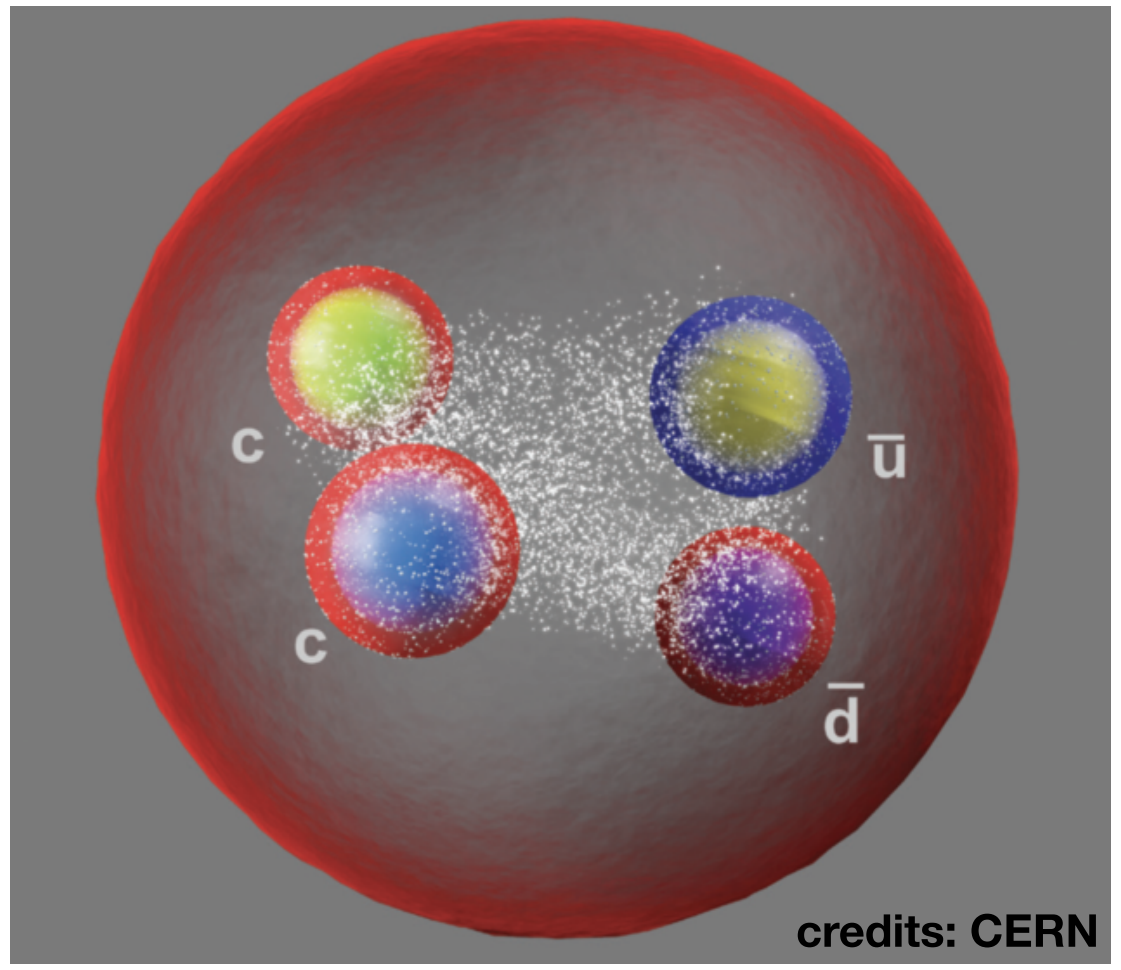


Thanks!

Back-up

Poles

$$\mathcal{M}_2(s) = \frac{16\pi\sqrt{s}}{k\cot\delta(k) - ik},$$

$$k = \pm\sqrt{k^2}$$

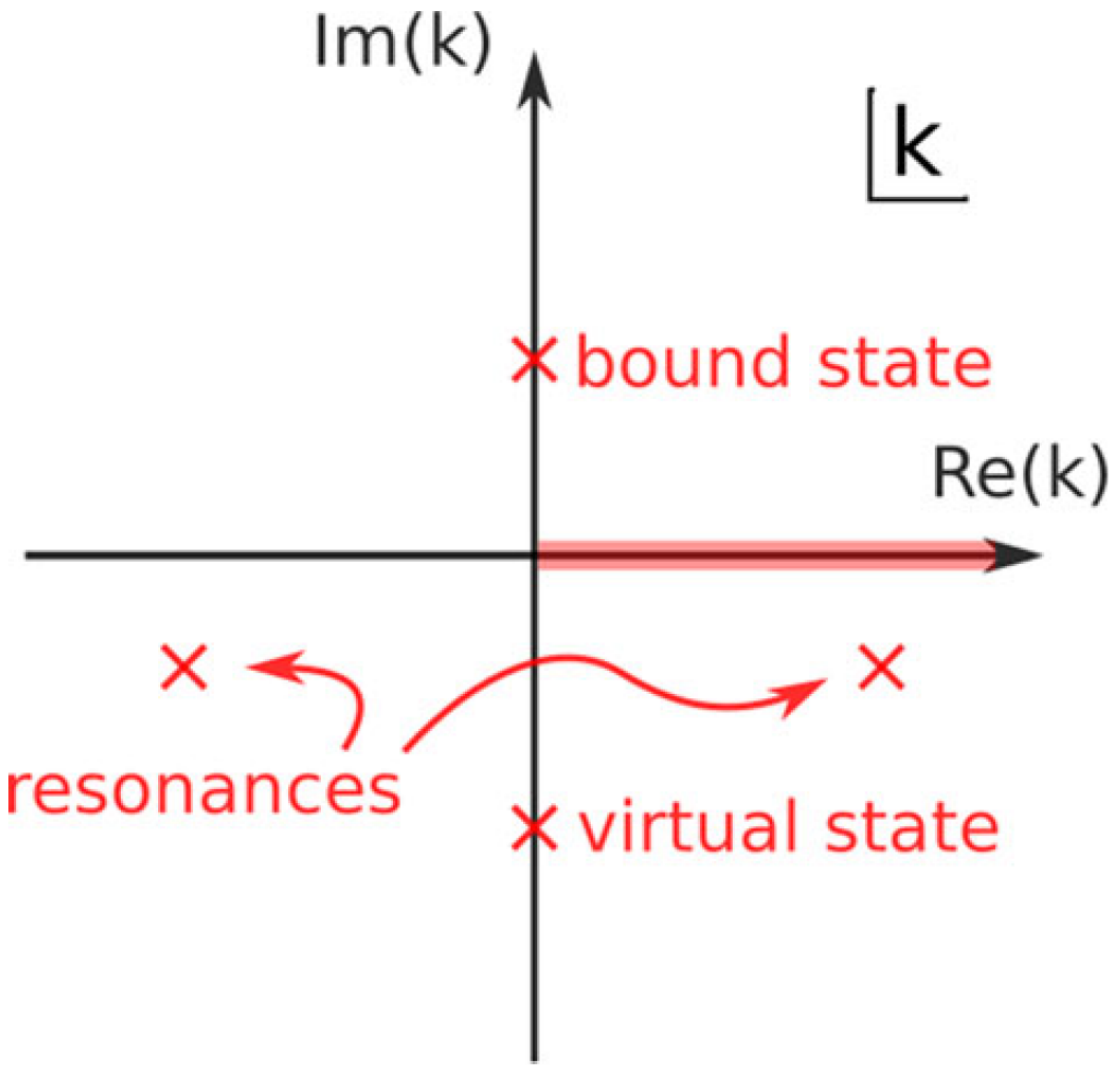


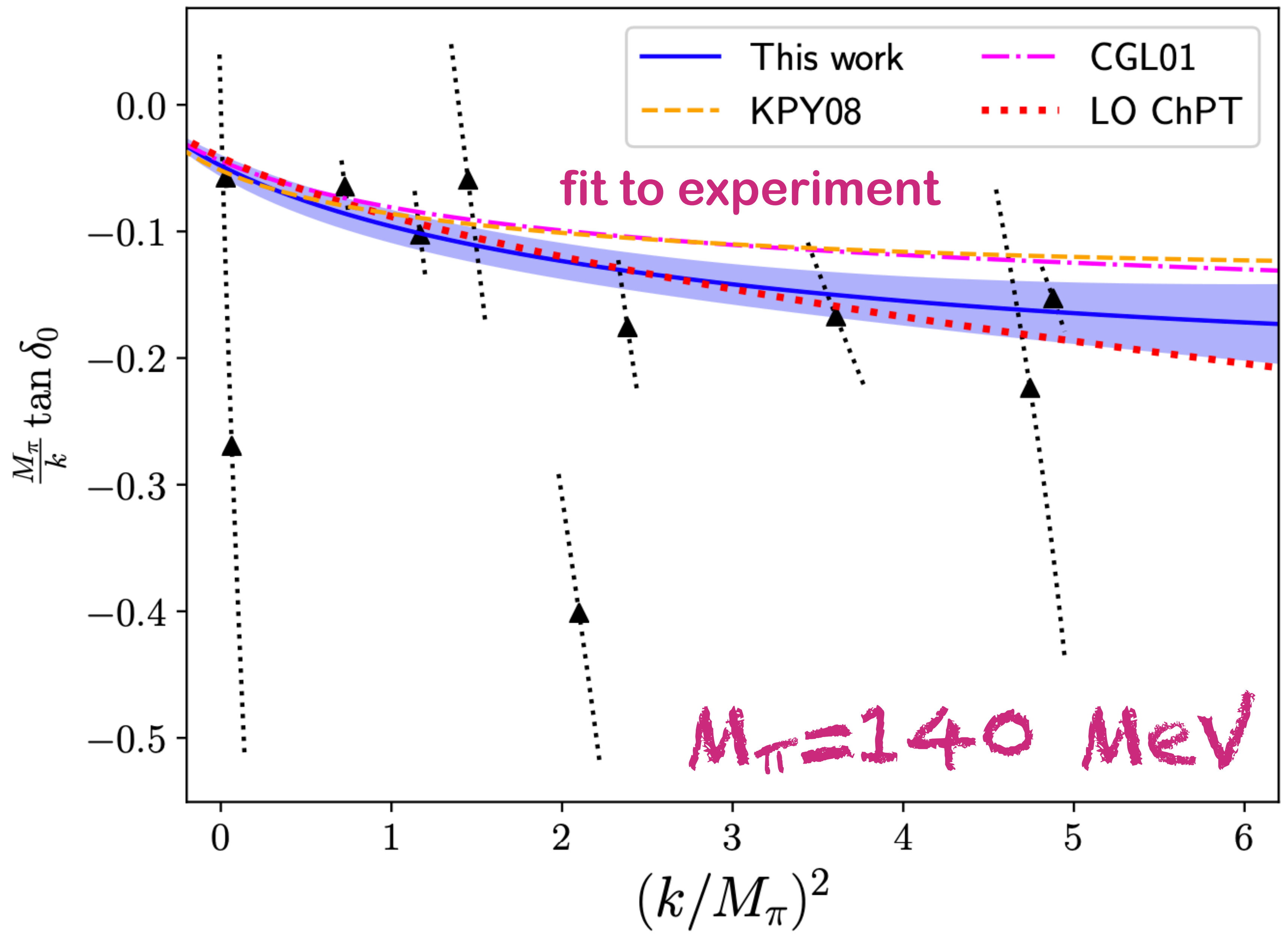
Fig. 1 Naming convention for the poles in the k-plane. The thick red line for positive real valued k marks the physical momenta in the scattering regime

[Matuschek et al, EPJA 2021]

Towards the physical point

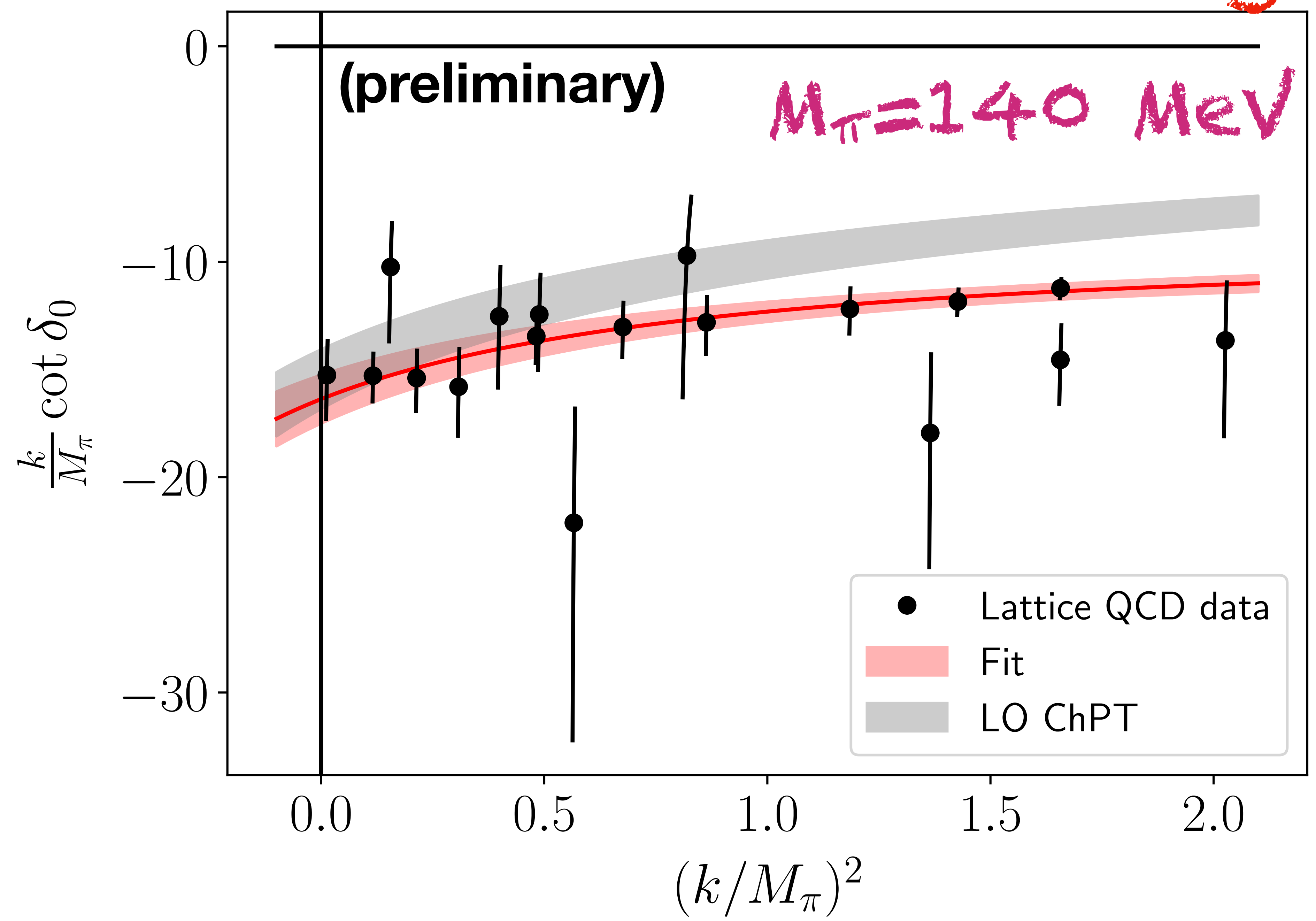
- Some systems already being studied at the **physical point**!

I=2 ππ scattering



[Fischer, Kostrzewa, Liu, FRL, Ueding, Urbach (ETMC), EPJC 2021]

I=3/2 πK scattering

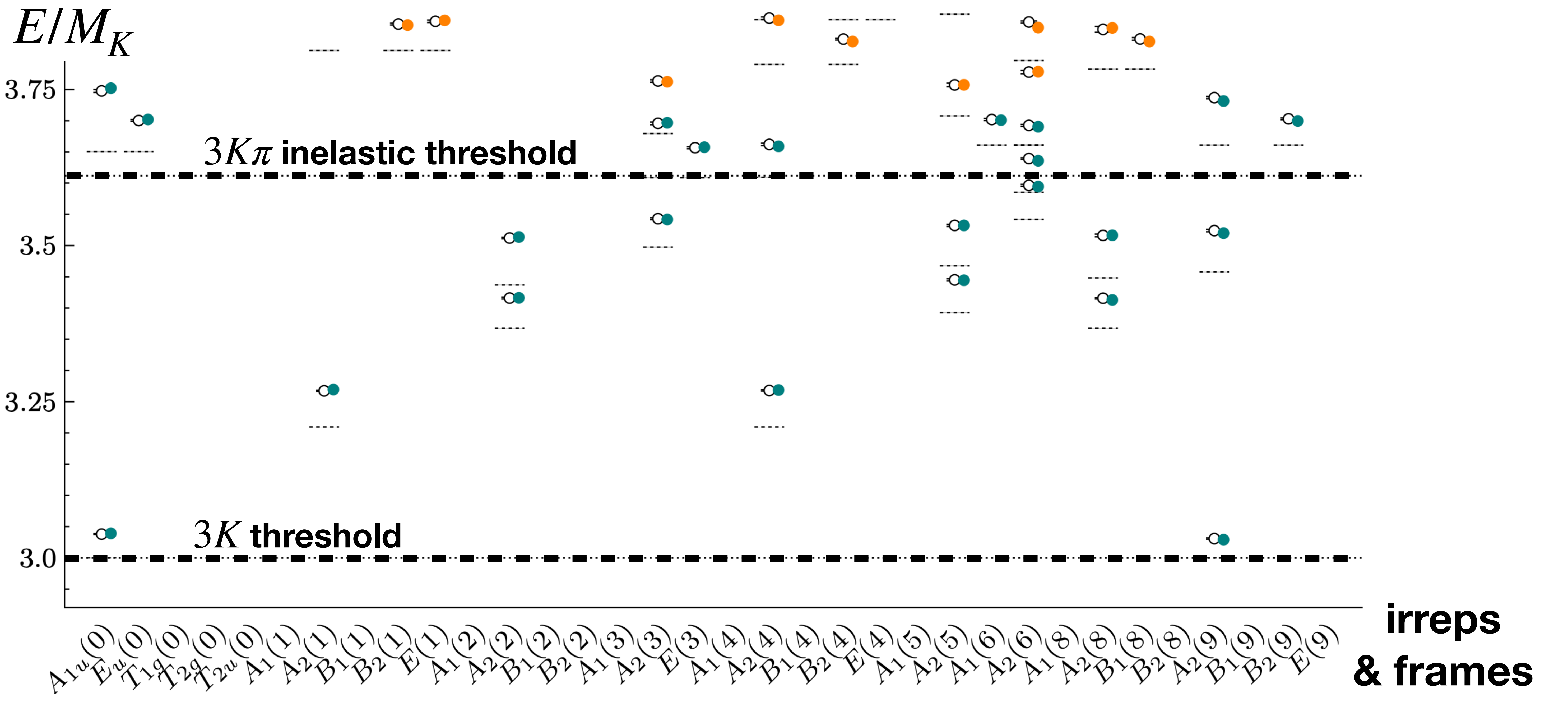


[Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe (in preparation)]

Applying the three-body formalism

- Three pions and three kaons at maximal isospin have been explored by different groups
  [Alexandrou et al, PRD 2020], [Blanton et al., PRL 2020 & JHEP 2021], [Brett et al, PRD 2021], [Culver et al, PRD 2021], [Fischer et al, EPJC 2021], [Hansen et al, PRL 2021], [Mai et a PRL 2019 & 2021]
- Requires large sets of energy levels obtained using variational techniques
  Using stochastic LapH method [Morningstar et al, PRD 2011]

$3K^+$ energy levels



[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe, JHEP 2021]

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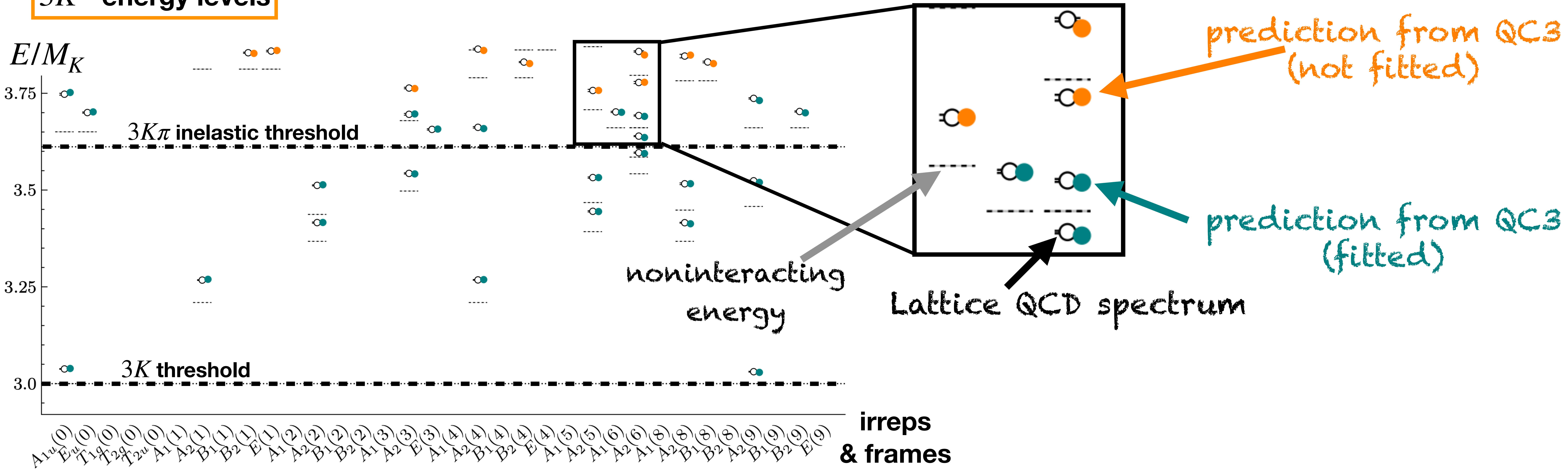
- **Requires large sets of energy levels obtained using variational techniques**
  Using stochastic LapH method [Morningstar et al, PRD 2011]

$3K^+$ **energy levels**



[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe, JHEP 2021]

Applying the three-body formalism

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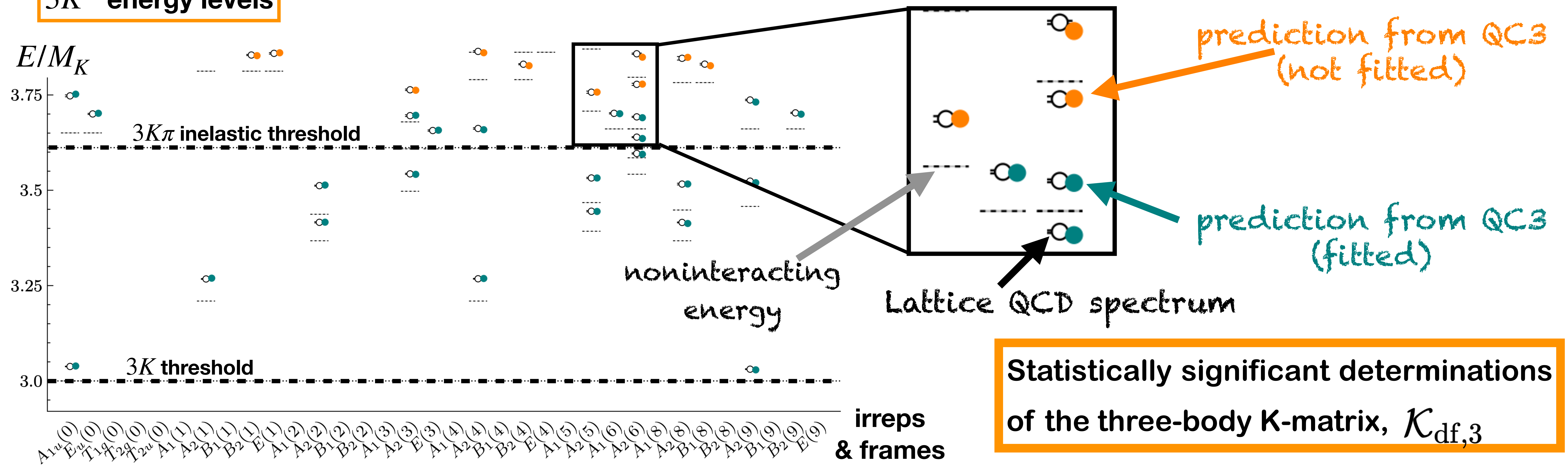

Statistically significant determinations of the three-body K-matrix, $\mathcal{K}_{\mathrm{df},3}$

[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe, JHEP 2021]

$\mathcal{K}_{df,3}$ from lattice QCD

Depend of CM energy | **Angular dependence**

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_{\text{df},3}^{\text{iso},0} + \mathcal{K}_{\text{df},3}^{\text{iso},1}\Delta + \mathcal{K}_{\text{df},3}^{\text{iso},2}\Delta^2 + \mathcal{K}_A\Delta_A + \mathcal{K}_B\Delta_B,$$

$$\Delta \equiv \frac{s-9m^2}{9m^2}$$

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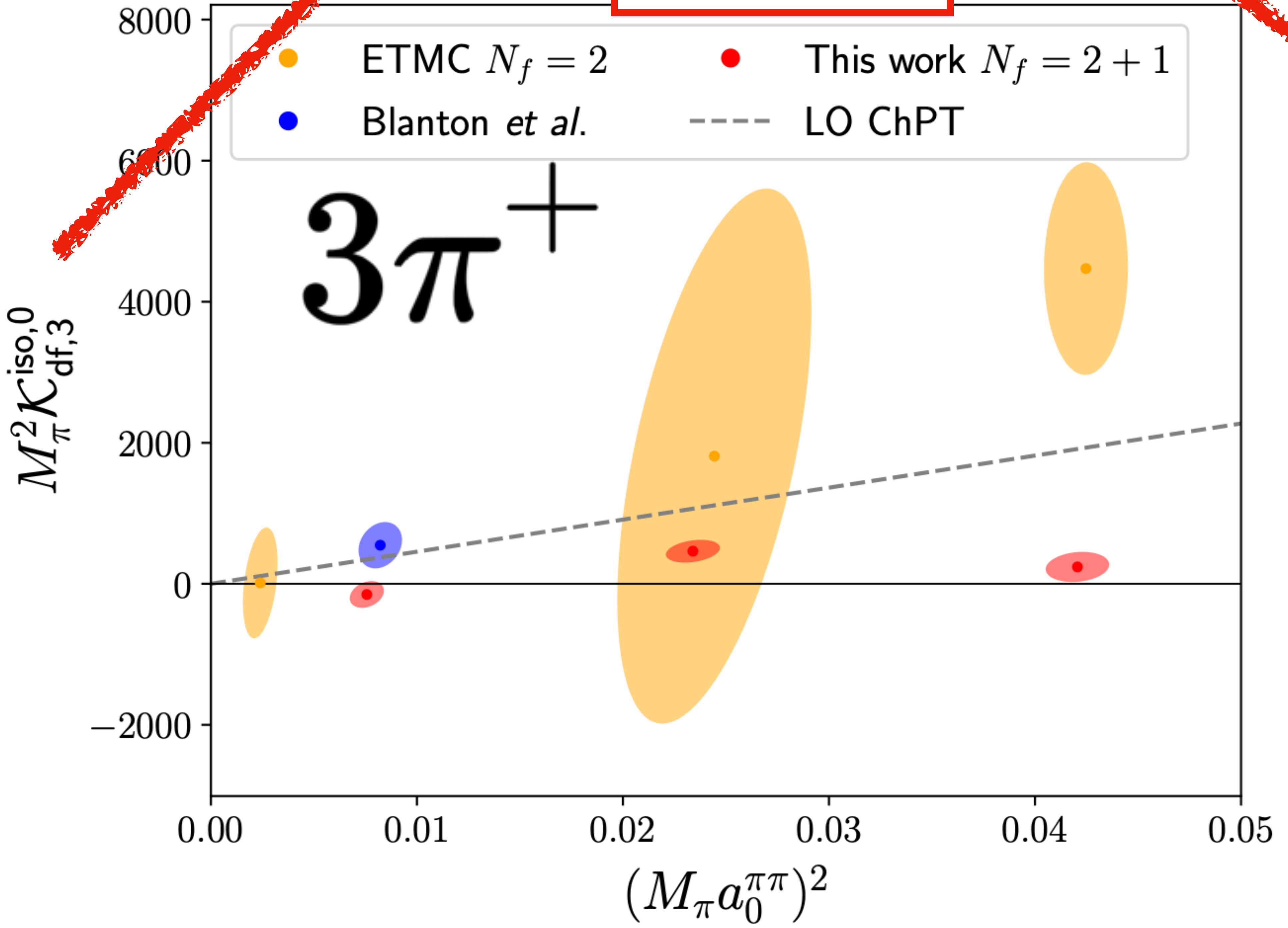

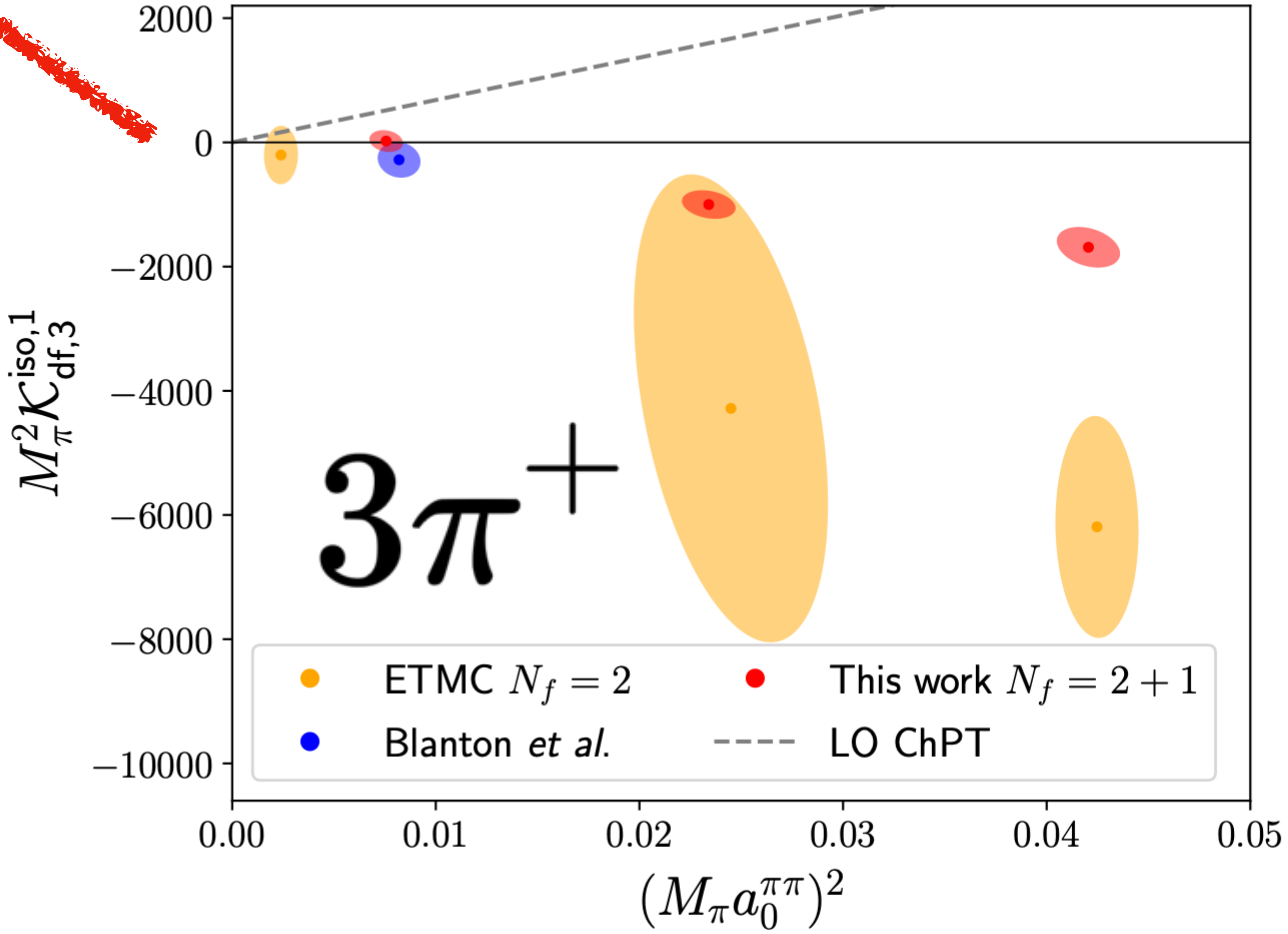

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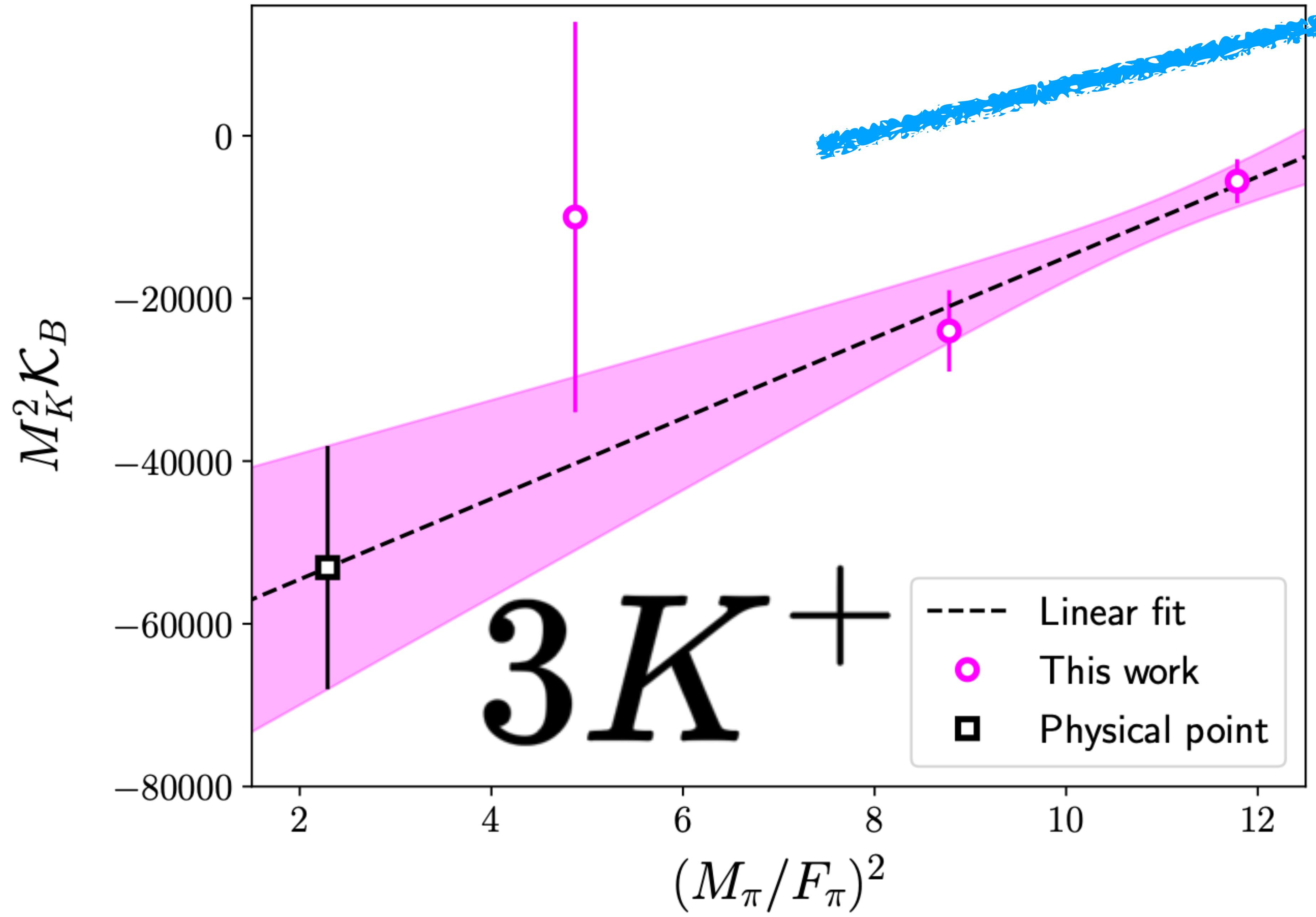

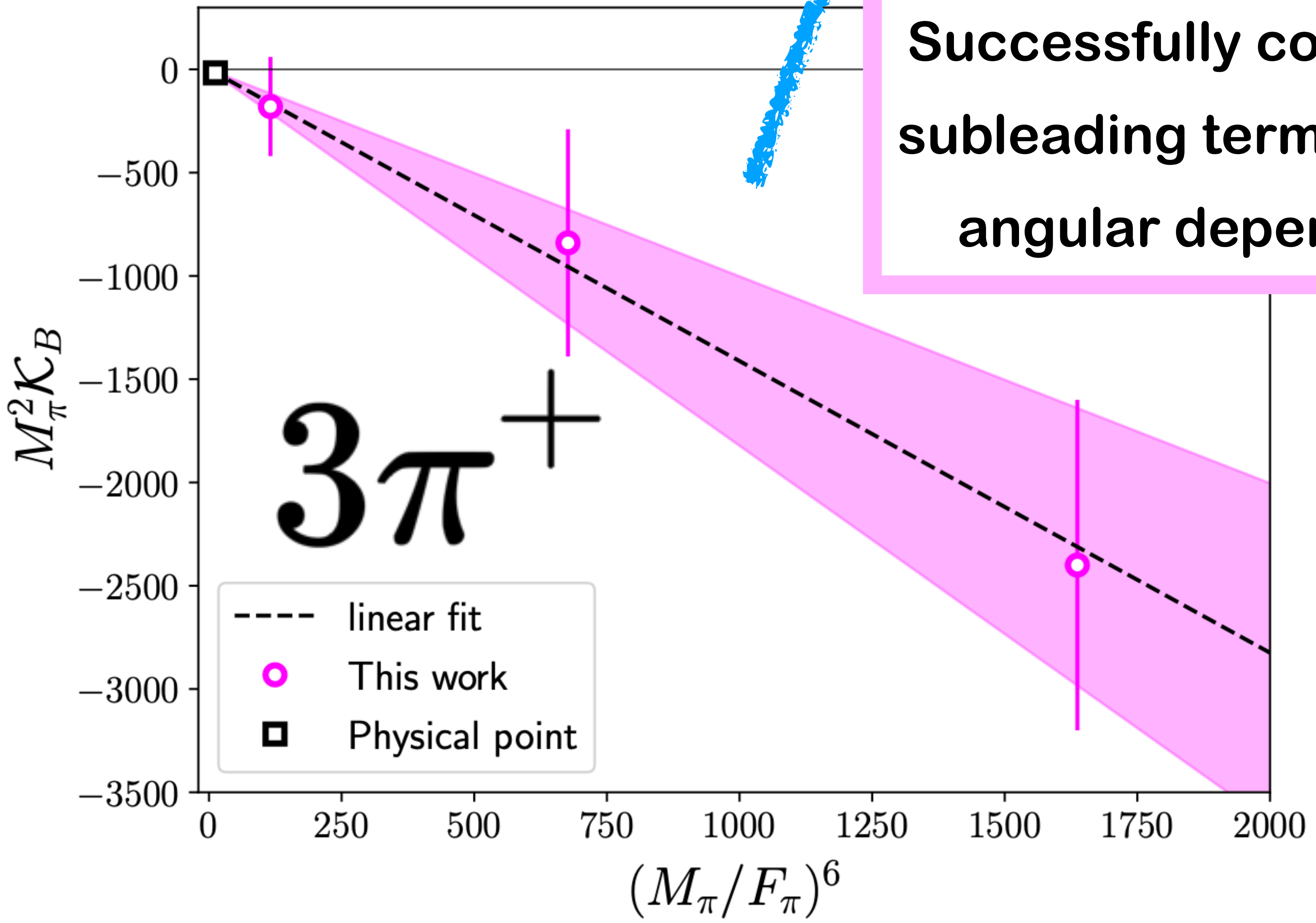

Successfully constrained subleading term including angular dependence!

[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe, JHEP 2021]

Nondegenerate systems

- Relevant three-body systems involve nonidentical particles ($\pi\pi N$)
- First step: formalism for three nonidentical scalars

  e.g. $\pi^+\pi^0\pi^-$, $K^+K^+\pi^+$, $D_s^+D^0\pi^-$

  [Blanton, Sharpe, PRD 2021] x 2, [Hansen, FRL, Sharpe, JHEP 2021], [Mai et al (GWQCD), PRL 2021]

$$\det_{k,\ell,m,\mathbf{f}}[1 - \mathbf{K}_{\mathrm{df},3}(E^\star)\mathbf{F}_3(E,\boldsymbol{P},L)] = 0$$

determinant runs over an additional "flavor" index

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Example:
$\pi^+\pi^+K^+$ scattering

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$$\Delta = \frac{s-M}{M^2} \qquad \tilde{t}_{22} = \frac{(p_2-p_2')^2}{M^2} \qquad \Delta_2 = \frac{(p_1+p_{1'})^2 - 4m_1^2}{M^2}$$

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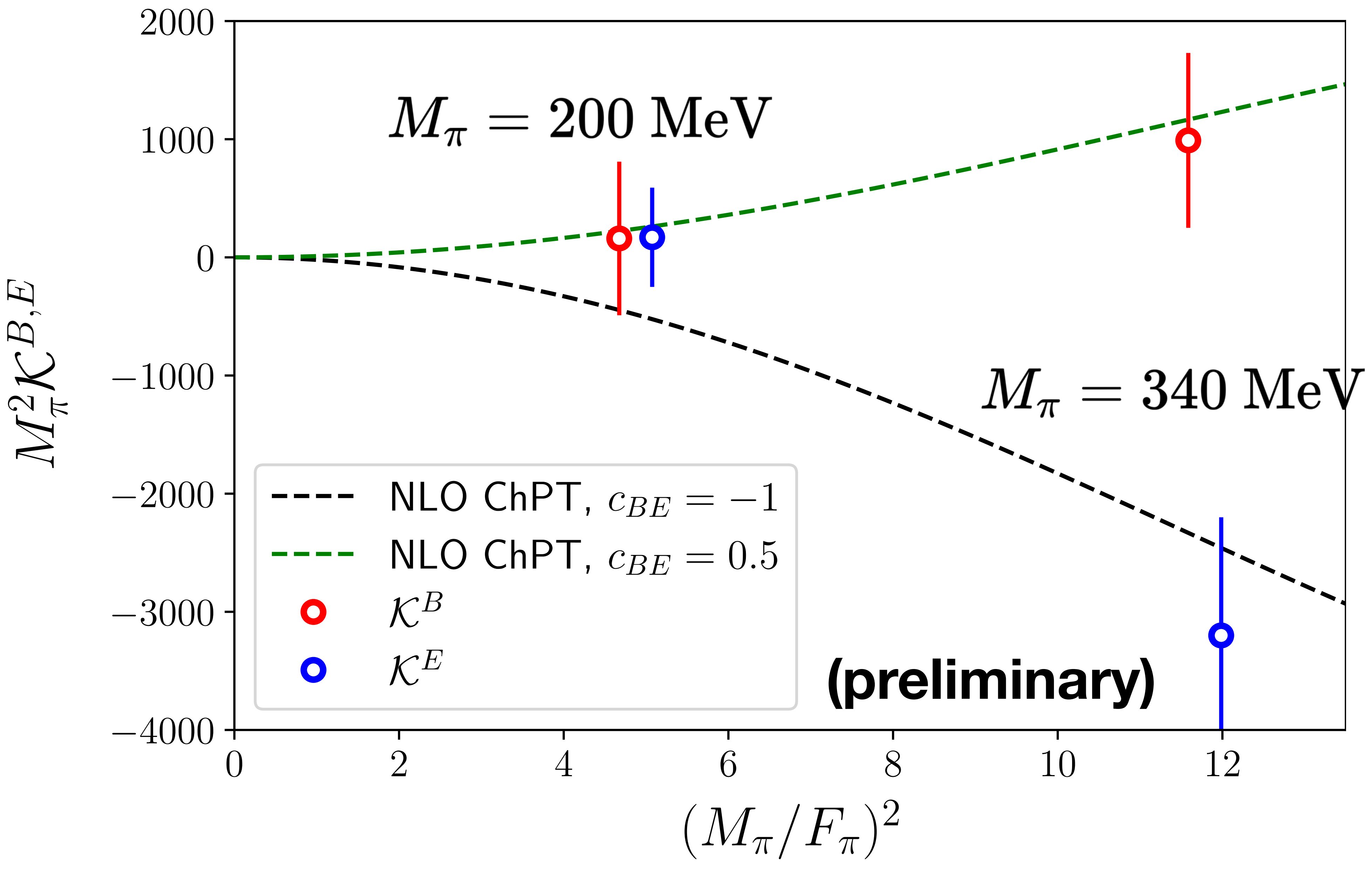
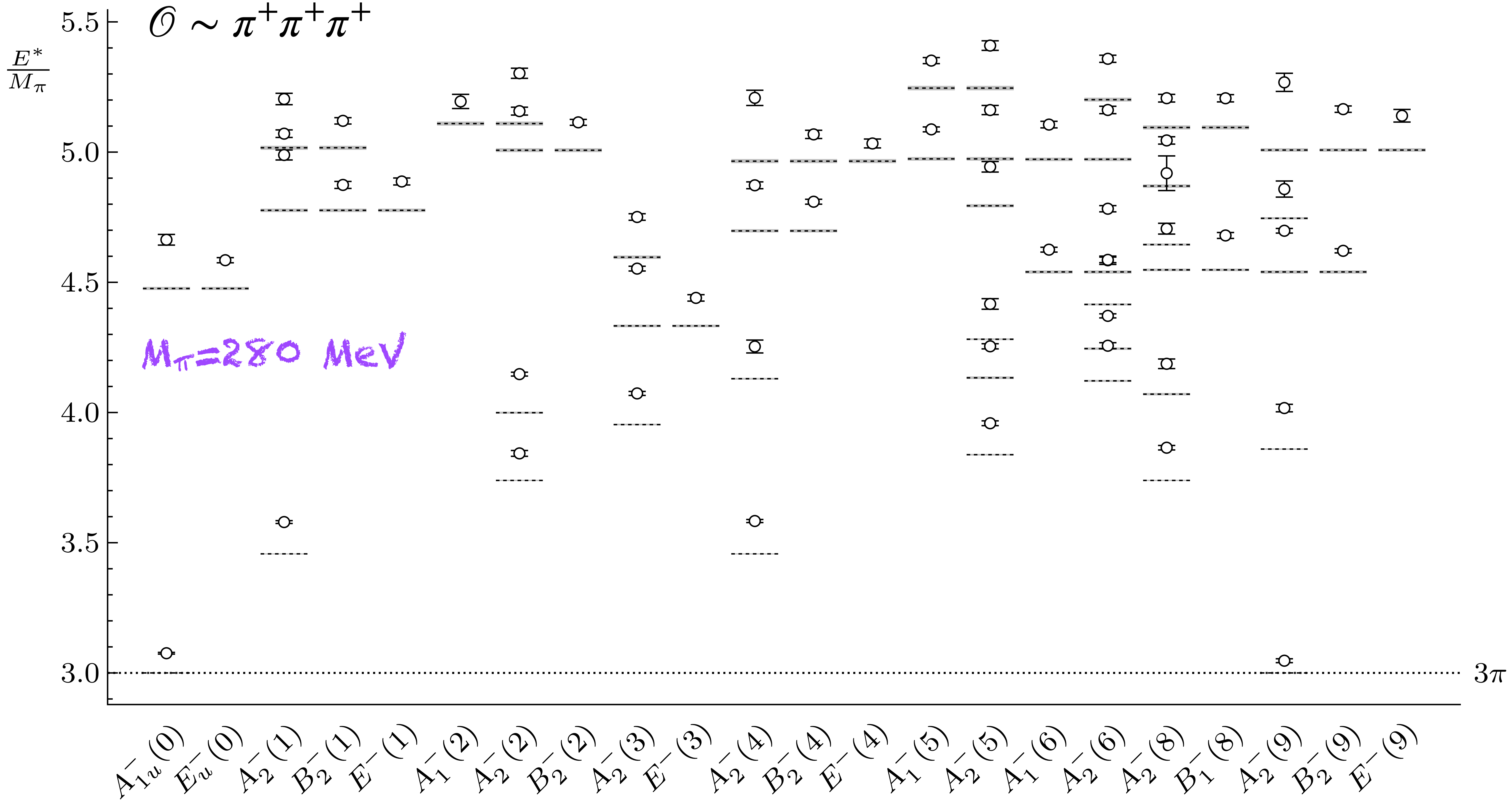
[Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe (in preparation)]

[Talk by S. Sharpe]



3π+ energy levels



[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe]

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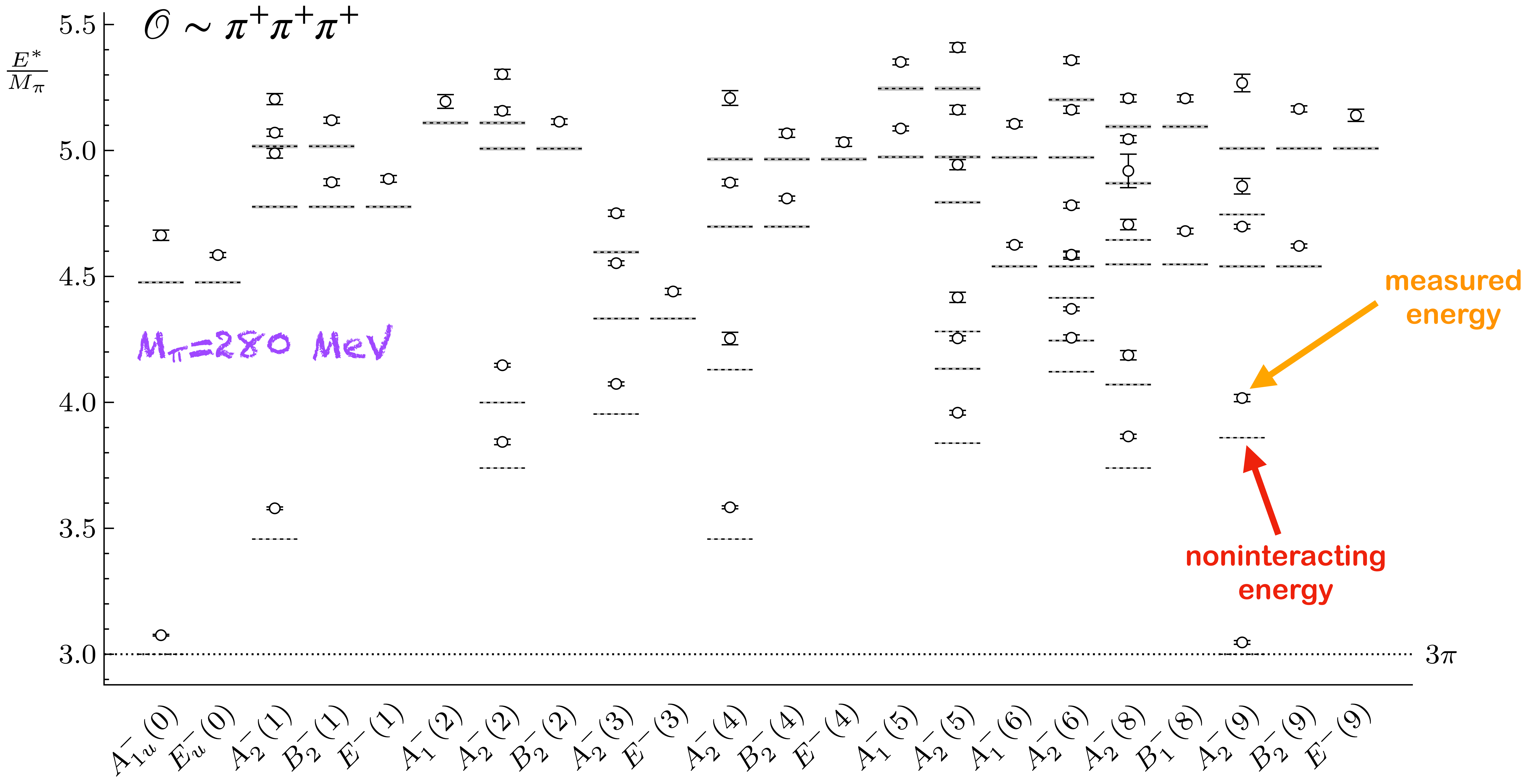

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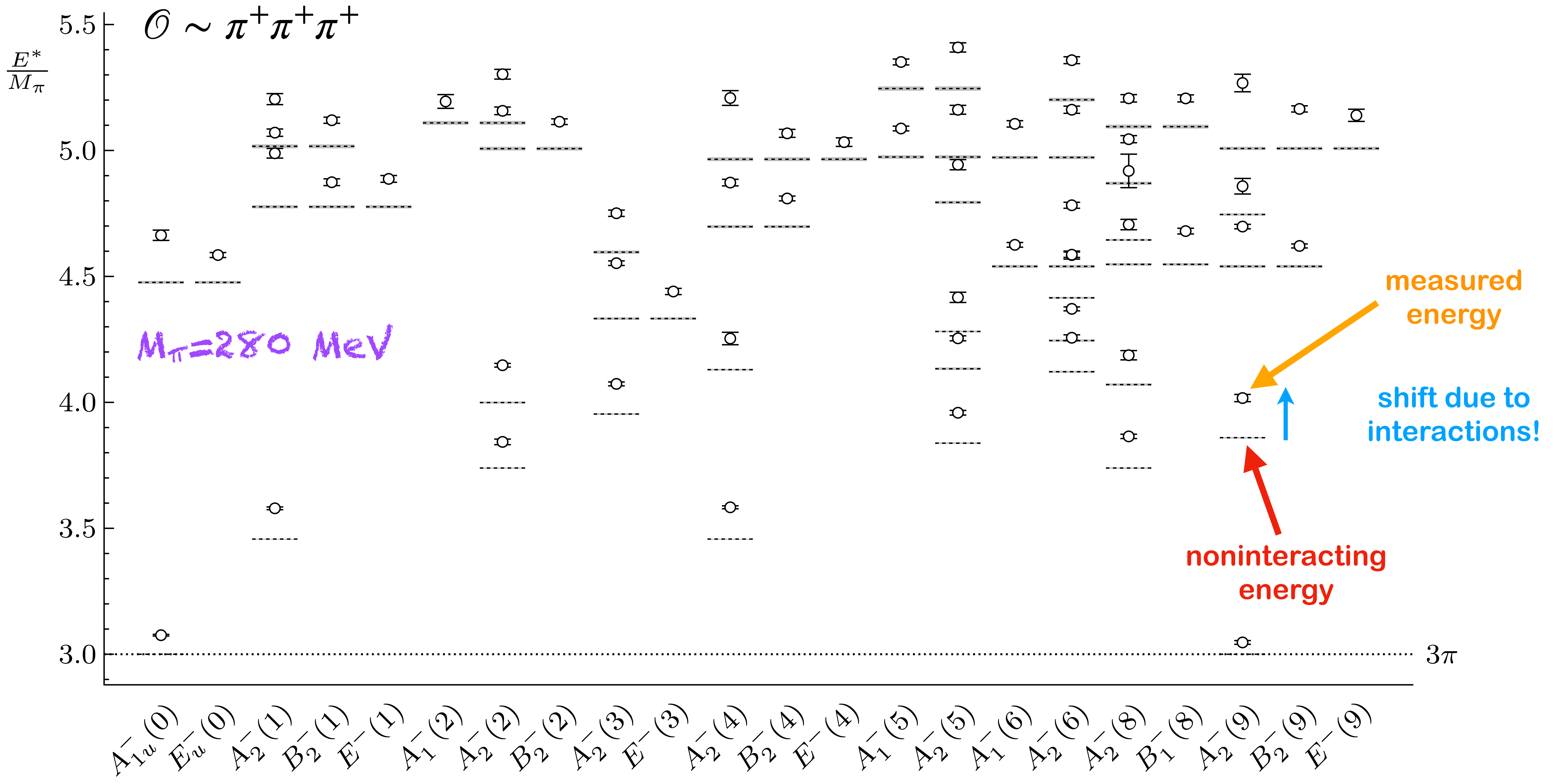

[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe]

Integral equations (RFT)

Final step

Physical 3->3 amplitude

$\mathcal{K}_2, \mathcal{K}_{df,3} \xrightarrow{\text{Integral equations}} \mathcal{M}_3$

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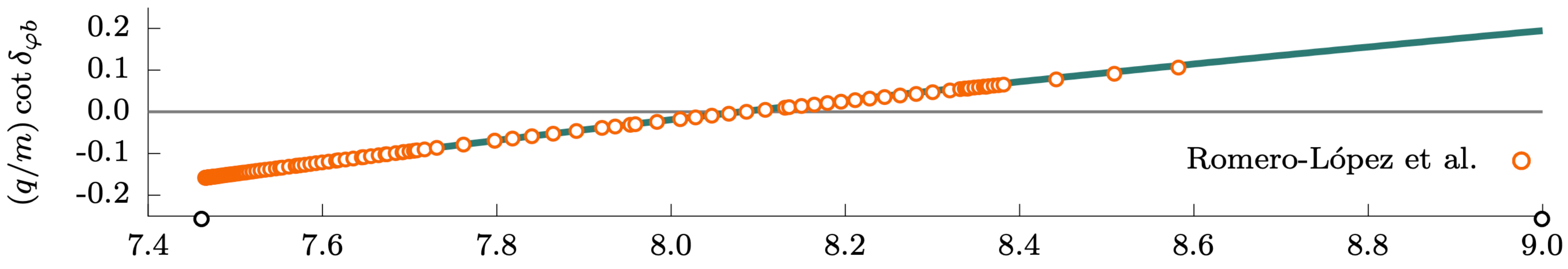
Particle-Dimer phase shift [Jackura et al.]



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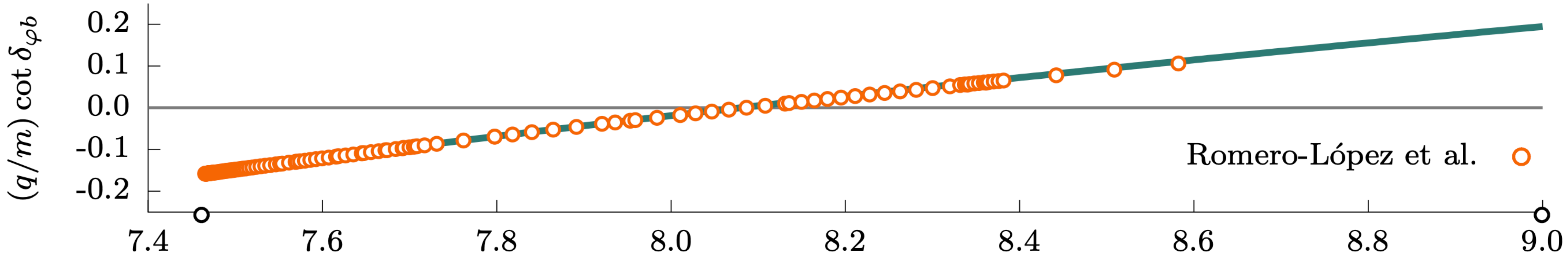
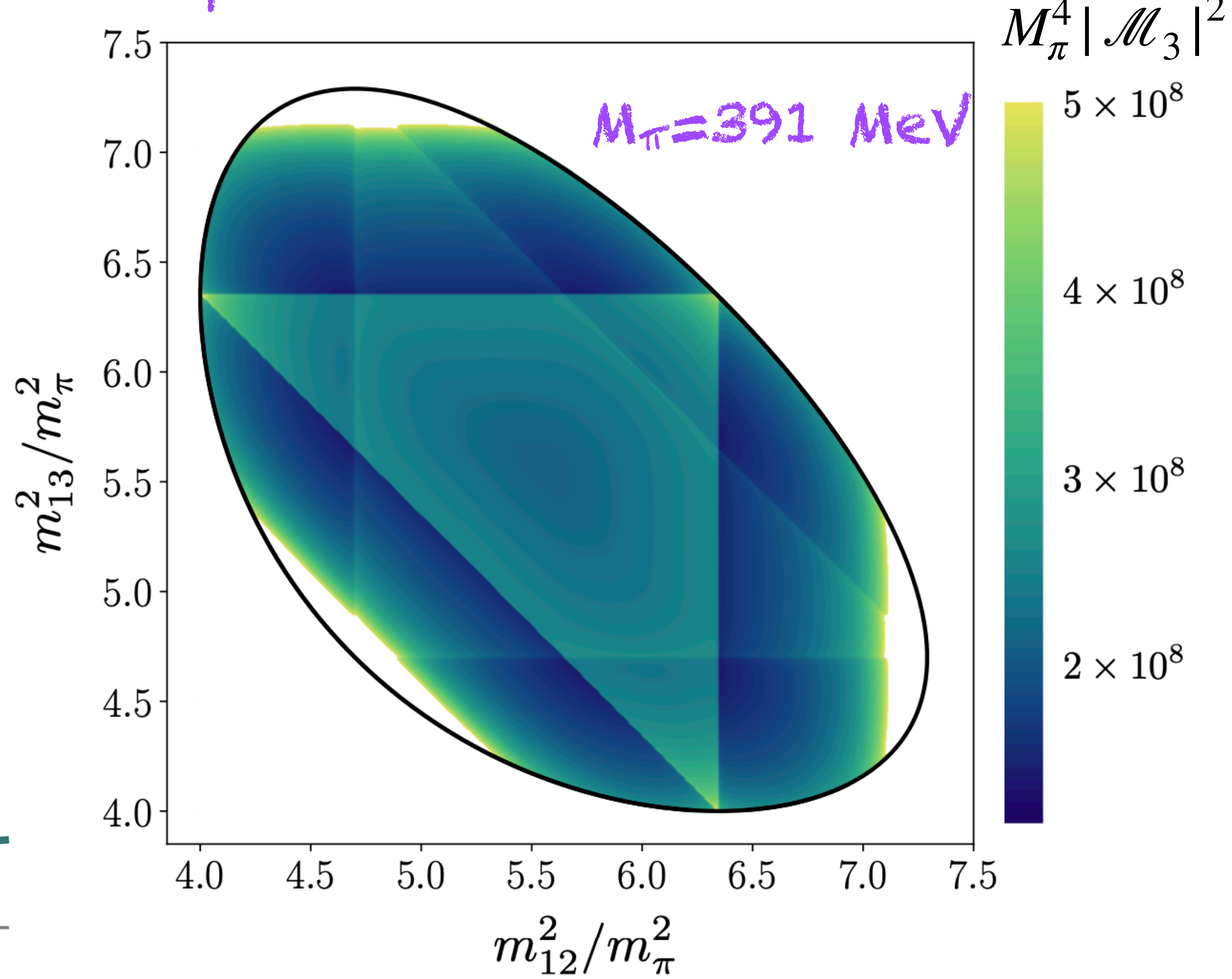


Dalitz plots from lattice QCD ($3\pi^+$)



[Hansen et al. (HadSpec)]

Quantization Condition

$$\det_{k\ell m}\left[\mathcal{K}_{\mathrm{df},3} + F_3^{-1}\right] = 0$$

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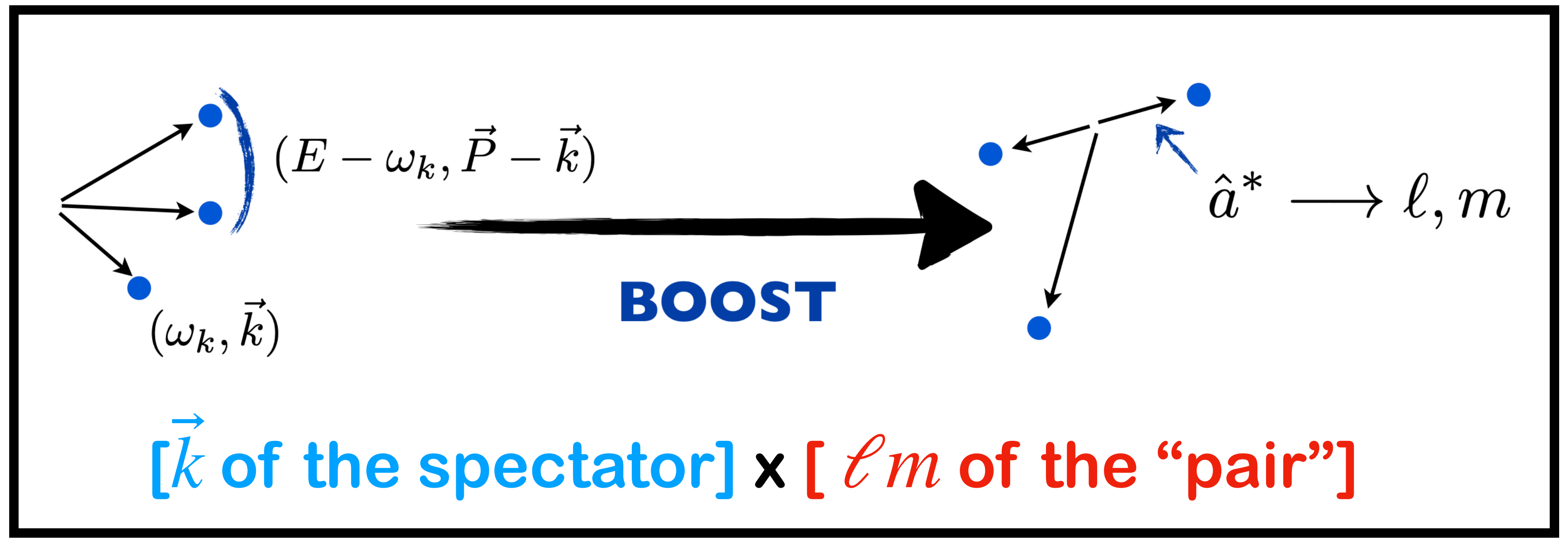

Finite-volume information & two-body interactions

$$F_3 = \frac{1}{L^3}\left[\frac{F}{3} - F\frac{1}{(\mathcal{K}_2)^{-1} + F + G}F\right]$$

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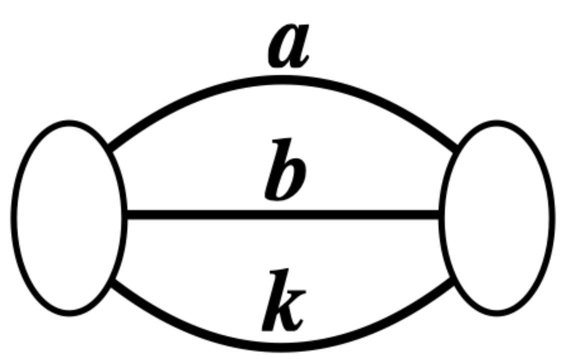
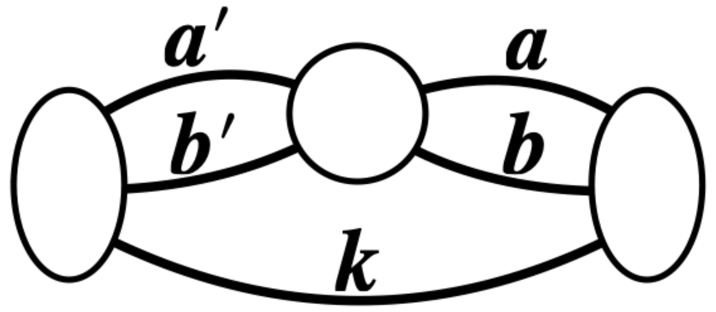
(a)



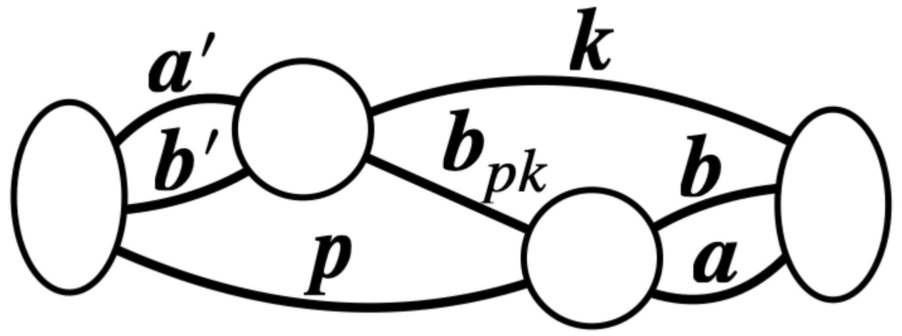
F

(b)



$\mathcal{K}_2$

(c)



G

$$F_{00}(q^2) \sim \left[\frac{1}{L^3}\sum_{\vec{k}} - \int\frac{d^3k}{(2\pi)^3}\right]\frac{1}{k^2-q^2}$$

$$\mathcal{K}_2^\ell = \frac{16\pi\sqrt{s}}{q^{2\ell+1}\cot\delta_\ell}$$

$$G_{p00;k00} \equiv \frac{1}{L^3}\frac{1}{2\omega_p}\frac{H(\vec{p})H(\vec{k})}{b_{pk}^2 - m^2}\frac{1}{2\omega_k}$$

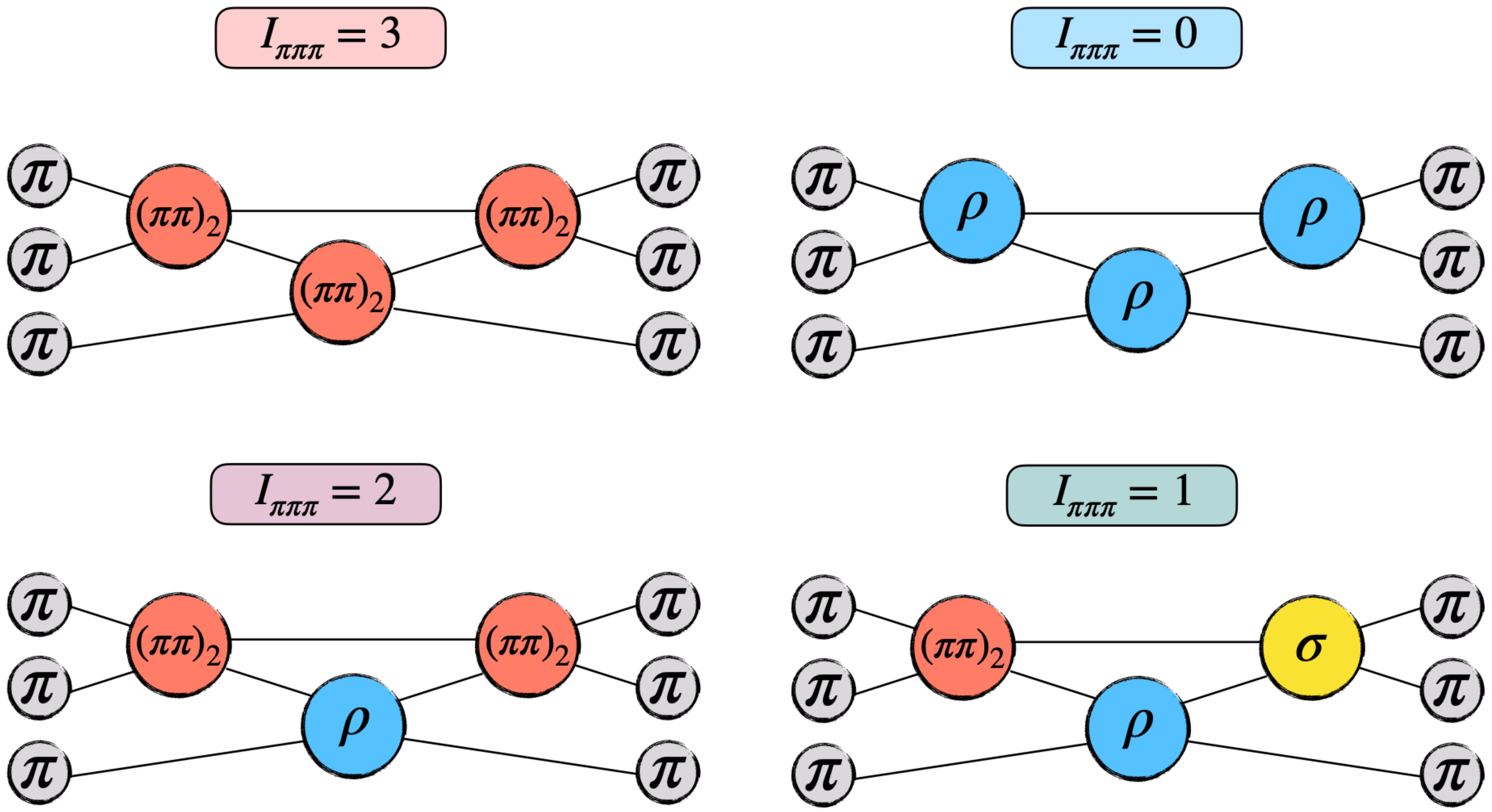
Including isospin

- Relevant three-body systems involve nonidentical particles ($\pi\pi N$)
- Let us consider mass-degenerate pions with different flavor

  e.g. $\pi^+\pi^0\pi^-$

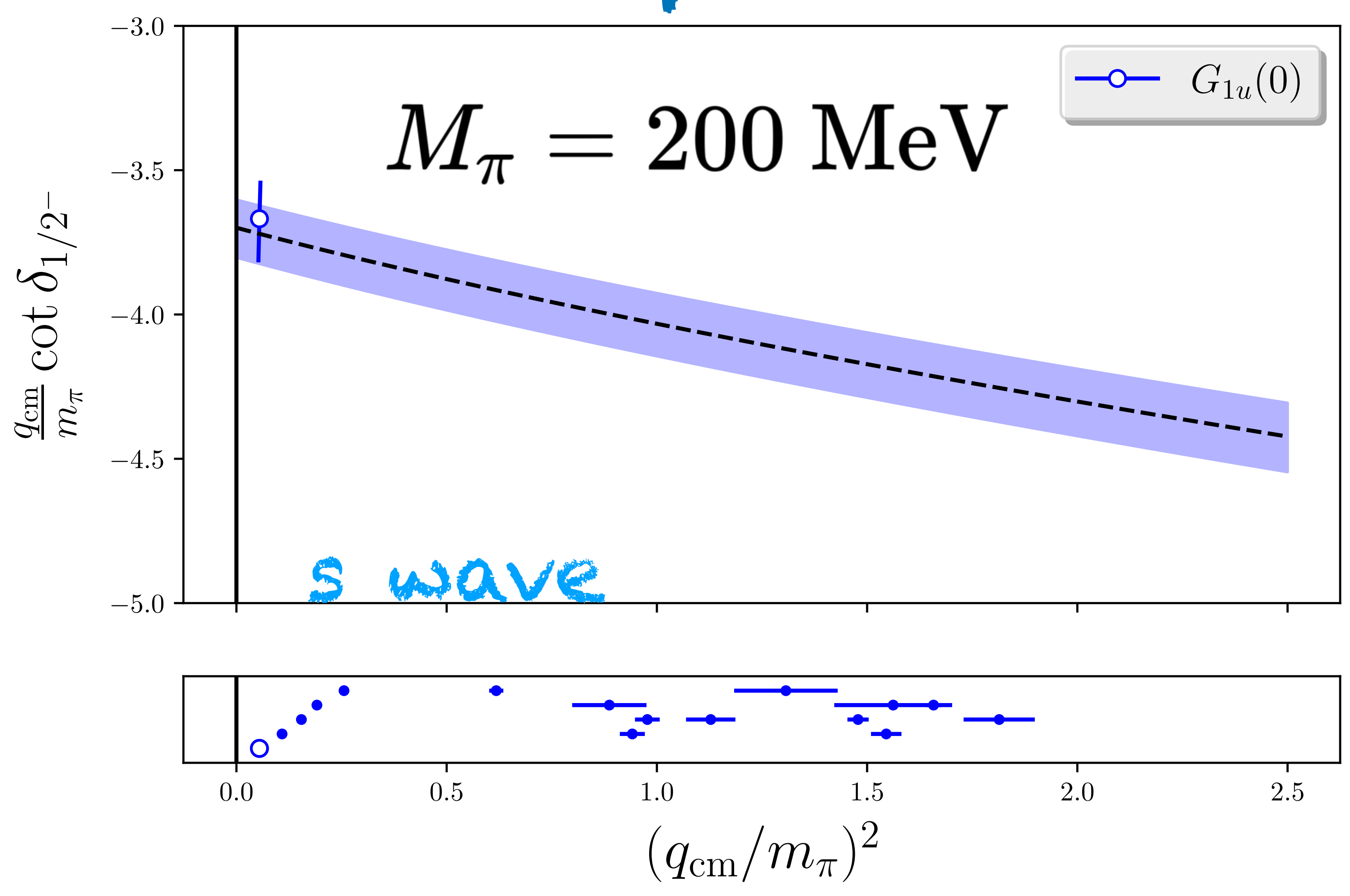
  [Hansen, FRL, Sharpe, JHEP 2020]

  - All pions have the same mass
  - Overall isospin is conserved
  - Presence of resonances
  - Example of multi-channel scattering



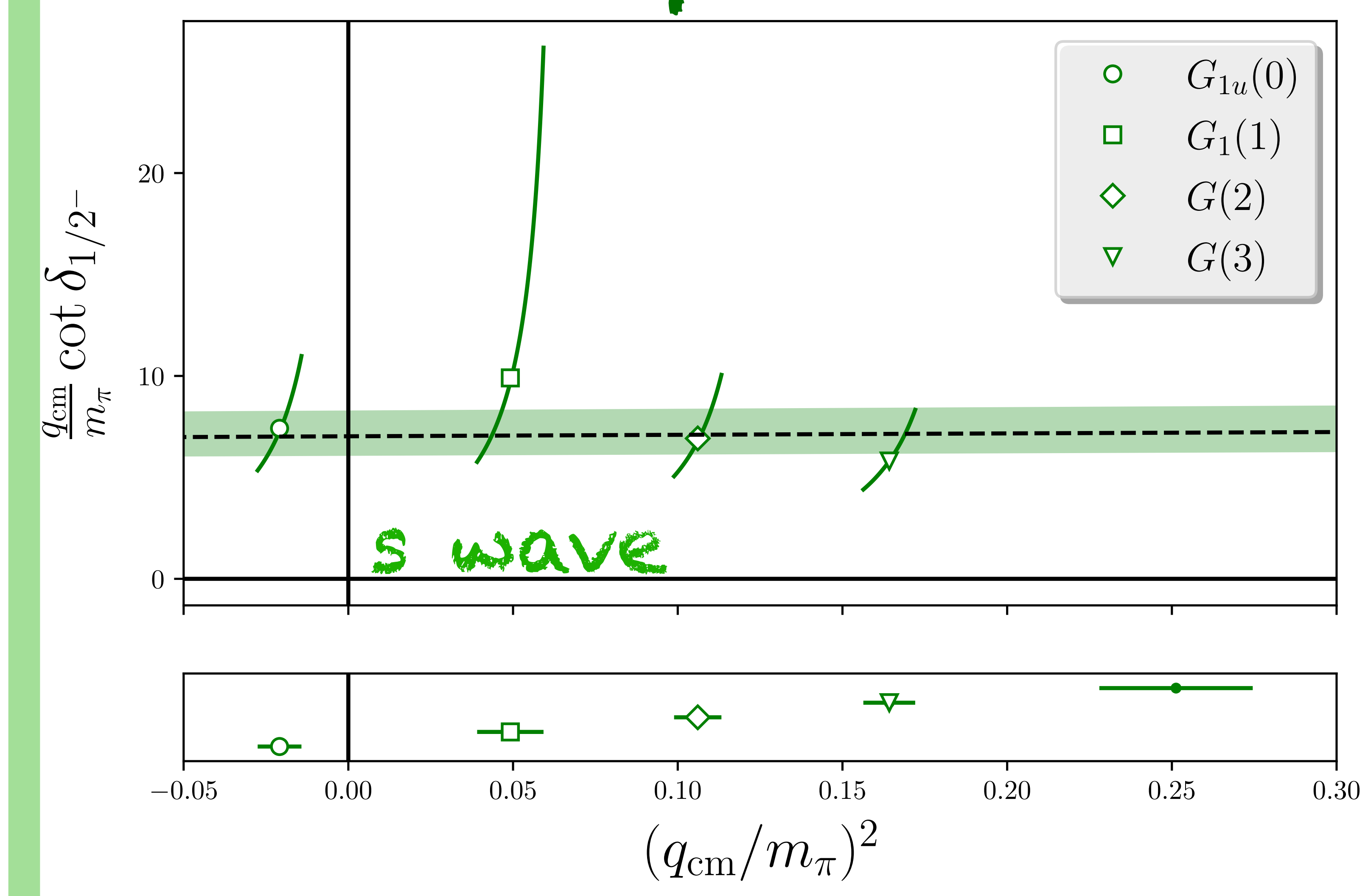
The πN scattering lengths

Isospin 3/2



$$m_\pi a_0^{3/2} = -0.2735(81)$$

Isospin 1/2



$$m_\pi a_0^{1/2} = 0.142(22)$$

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]

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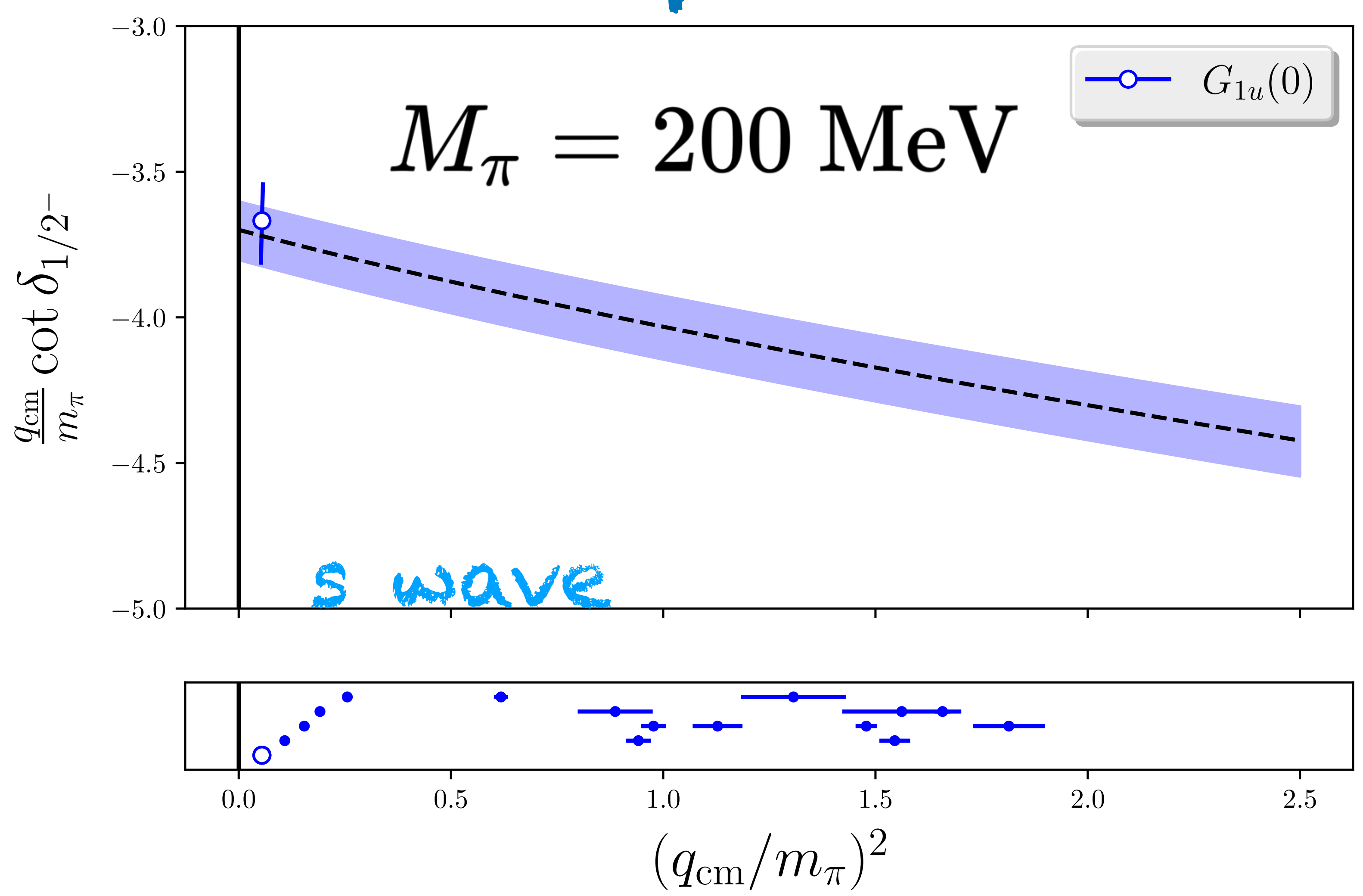
Isospin 3/2



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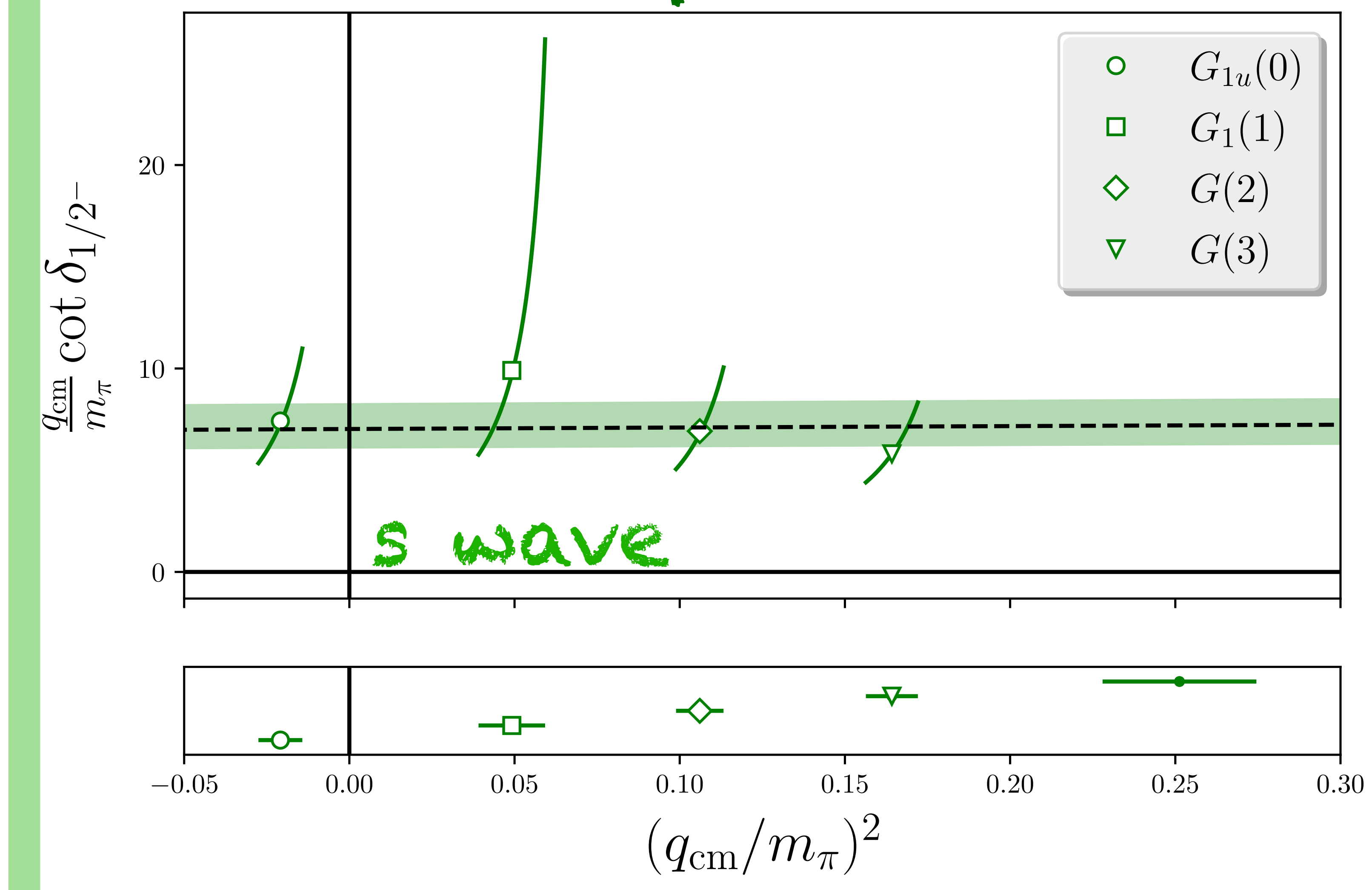

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Determination of scattering lengths closest to the physical point!

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]

The πN scattering lengths

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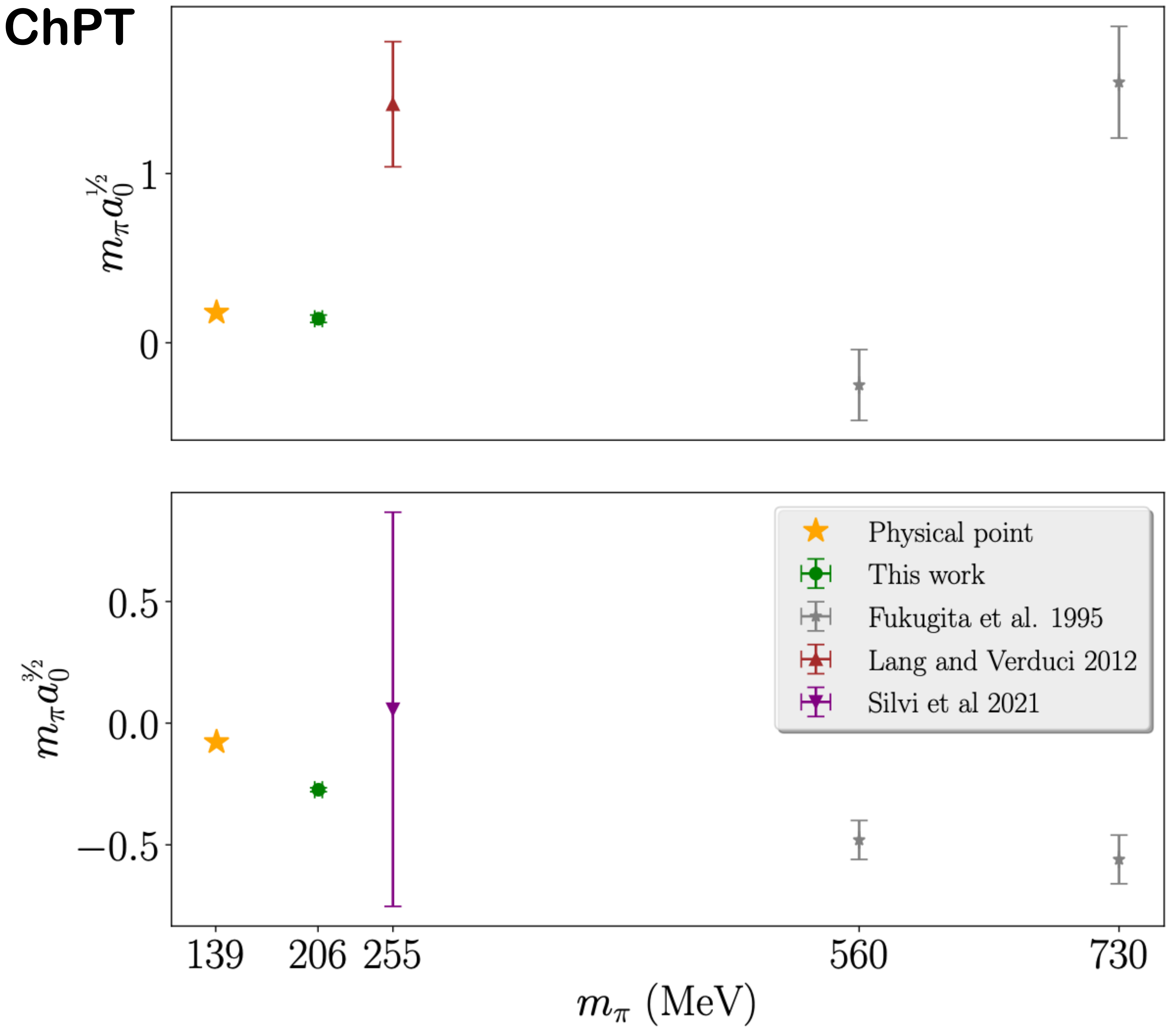
- Our results can be used to test the convergence of baryon ChPT

	m_π (MeV)	$m_\pi a_0^{1/2}$	$m_\pi a_0^{3/2}$
This work	200	0.142(22)	−0.2735(81)
LO χPT	200	0.321(04)(57)	−0.161(02)(28)



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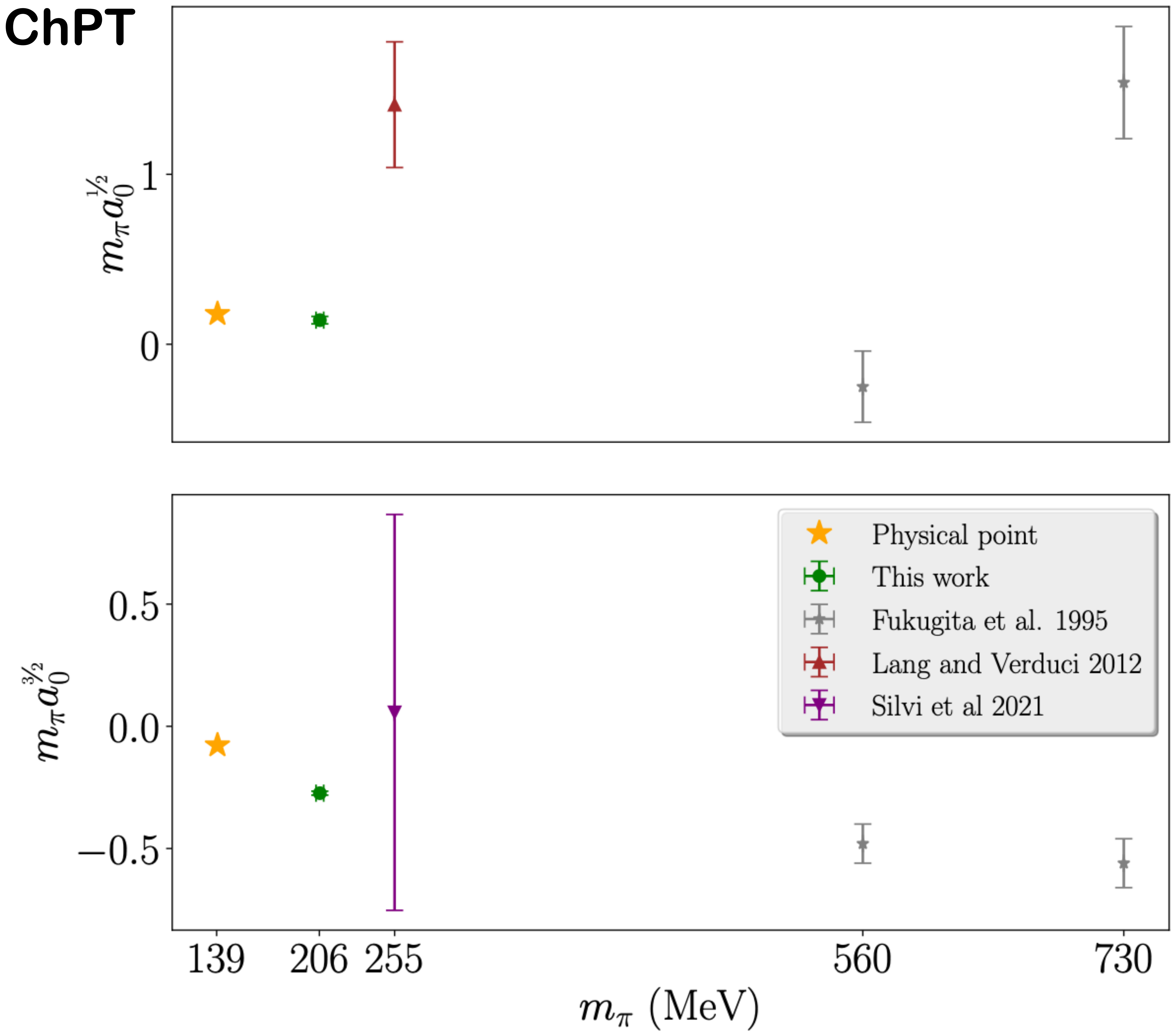
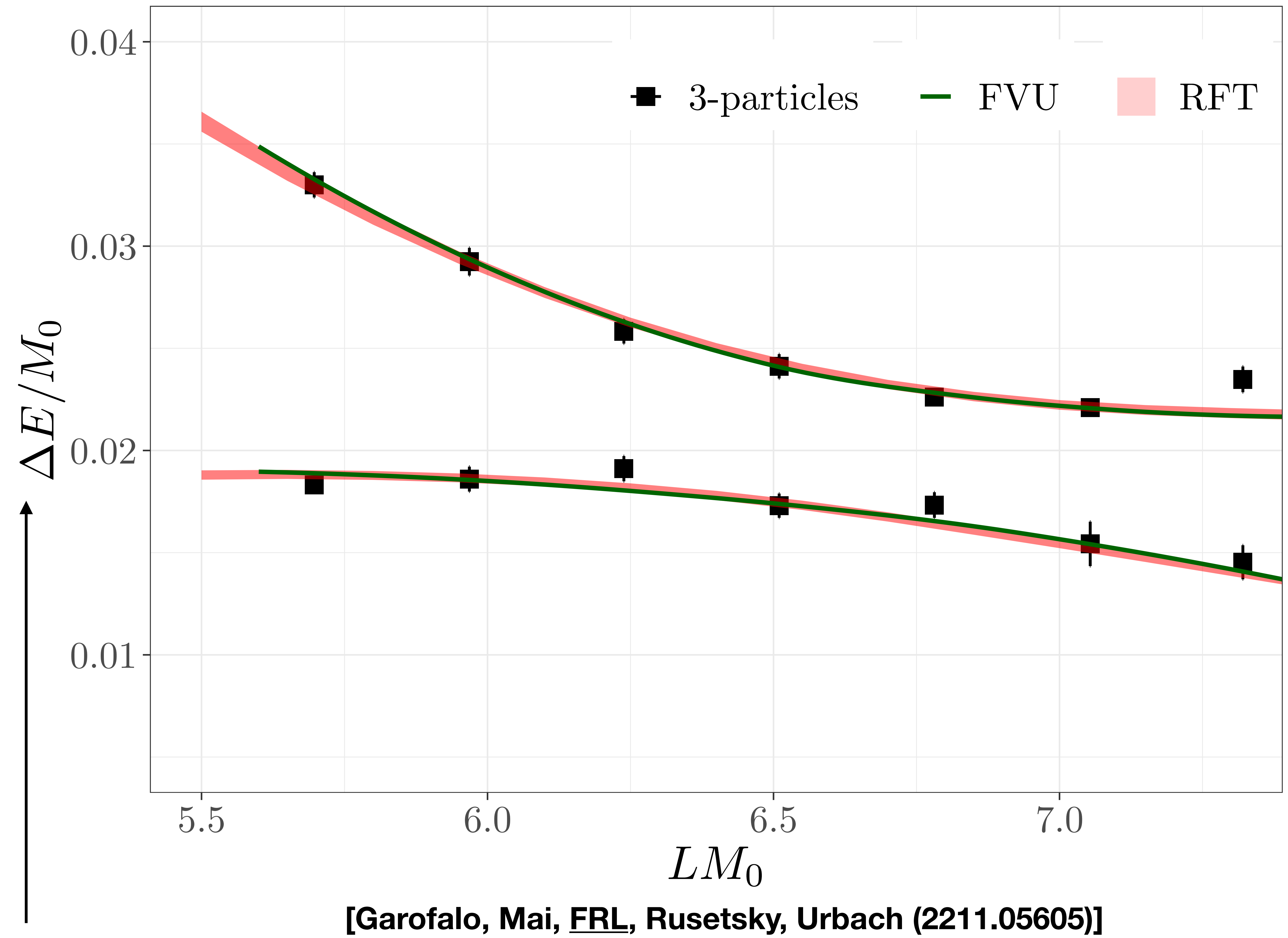
We find poor convergence at M_π = 200 MeV



- Additional values of the pion mass are needed!

Three-body spectrum



[Garofalo, Mai, FRL, Rusetsky, Urbach (2211.05605)]

$\Delta E = E - 3M_0$

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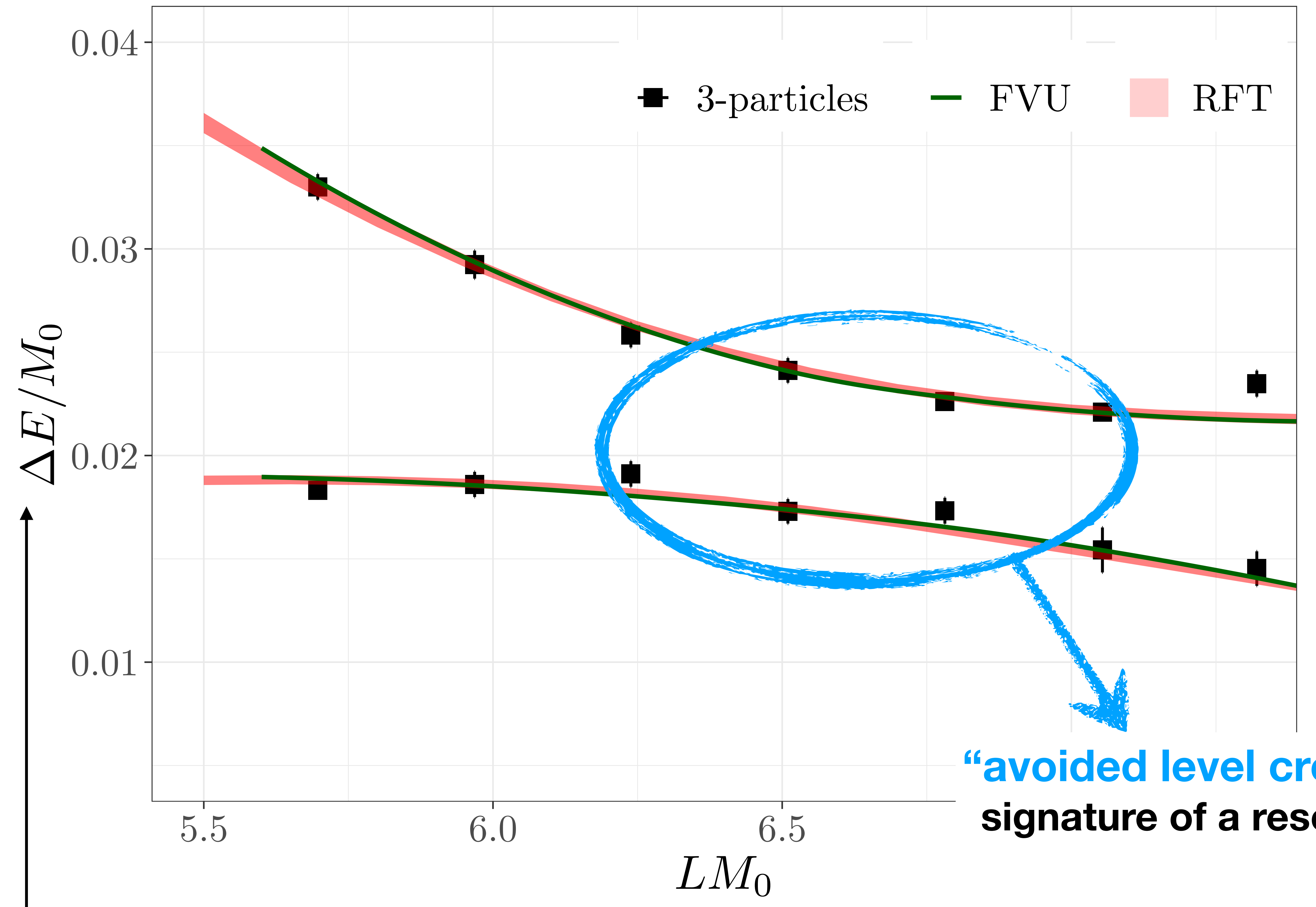

"avoided level crossing"
signature of a resonance

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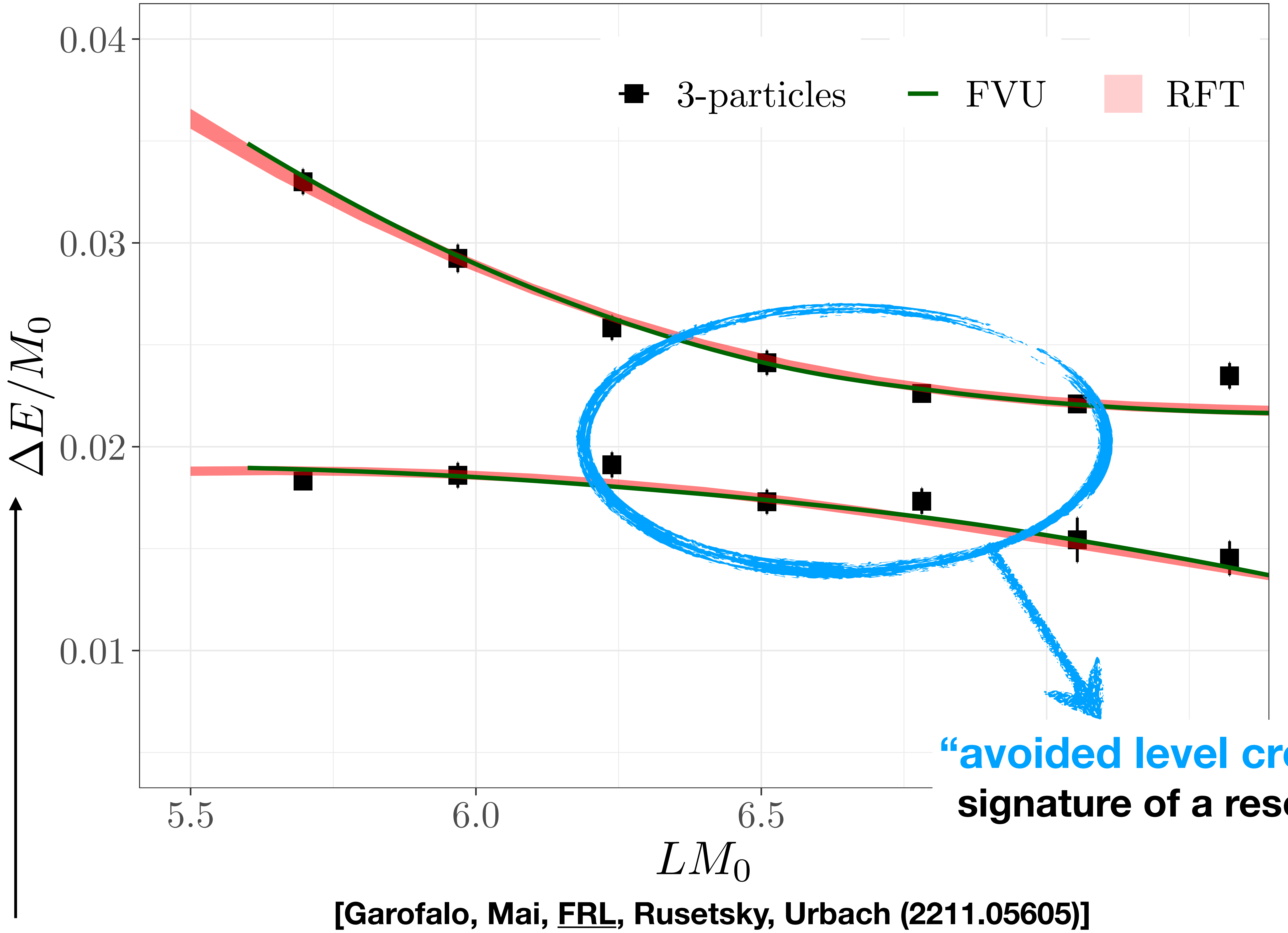

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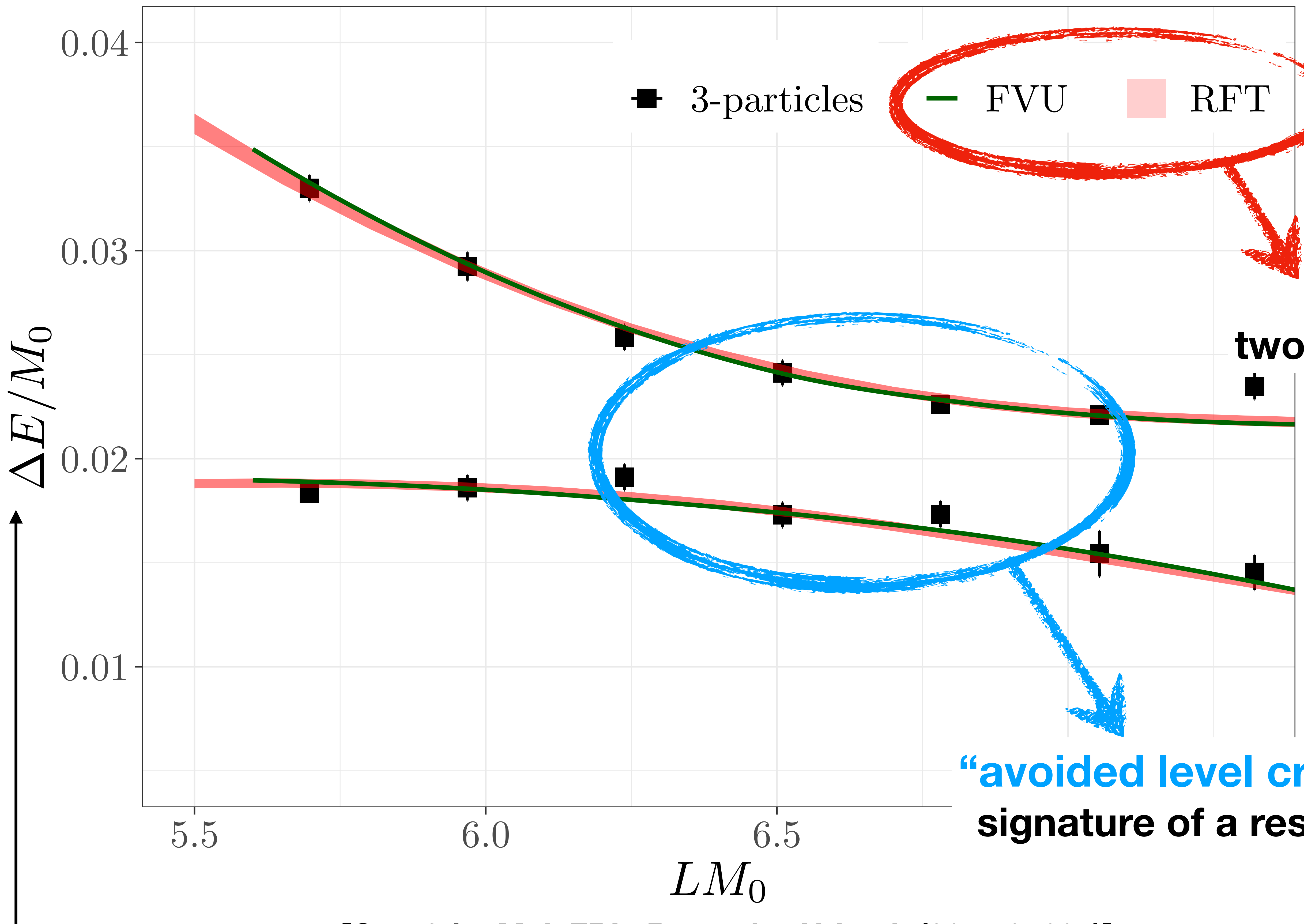

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[Garofalo, Mai, FRL, Rusetsky, Urbach (2211.05605)]

compare two approaches

Similar results

$\chi^2/dof \sim 1.3$

"avoided level crossing"
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Towards the QCD S-Matrix

The S-Matrix contains the physical information of the theory:

$$S_{ab}(E) \equiv \langle \text{out}|\mathcal{S}|\text{in}\rangle$$

Lattice QCD → QCD S-matrix

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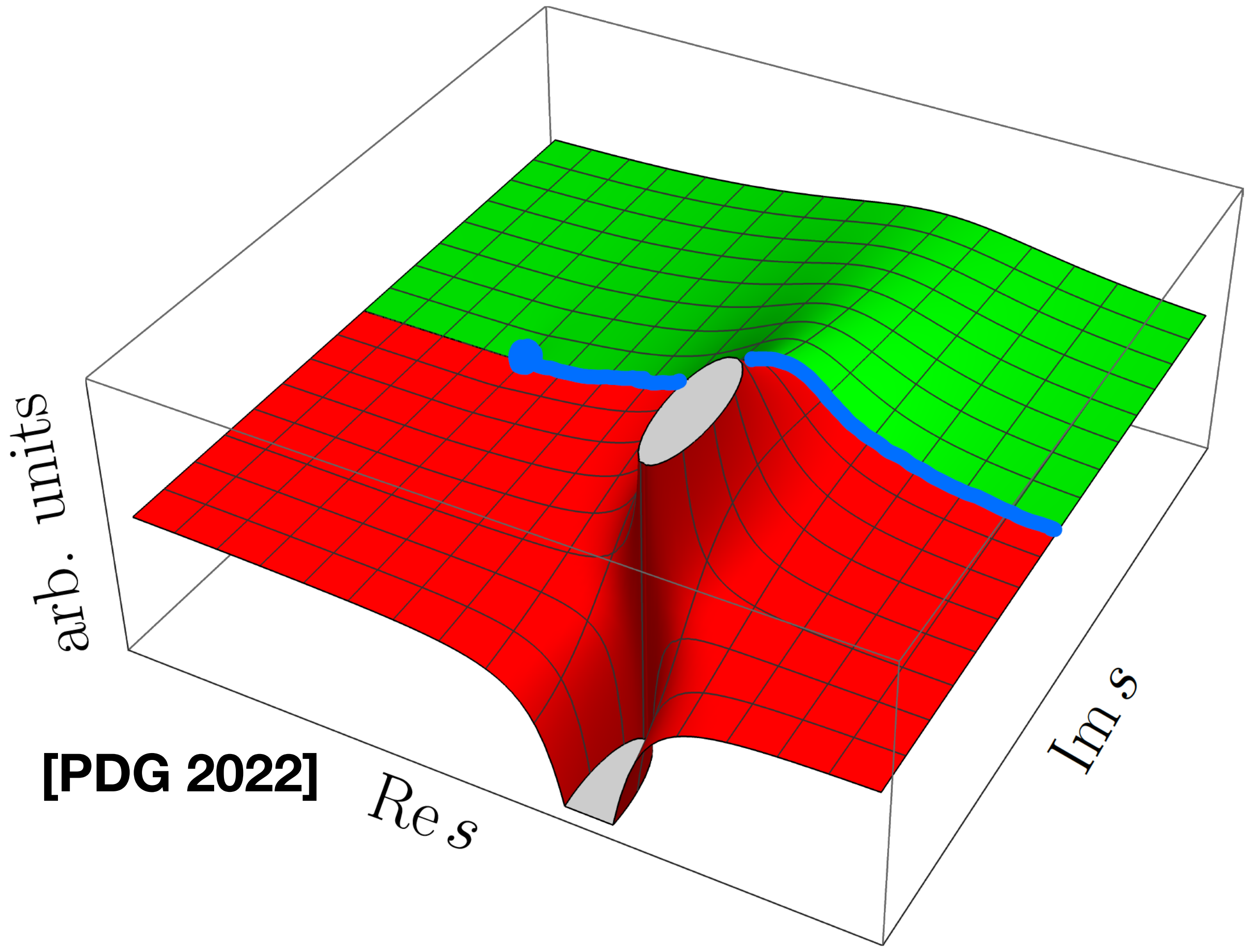
Lattice QCD → QCD S-matrix

- Resonances as poles in the S-matrix (or scattering amplitude)



[PDG 2022]

$$\sim \frac{g}{E^2 - E_R^2}$$

$$E_R = M_R - i\,\Gamma/2$$

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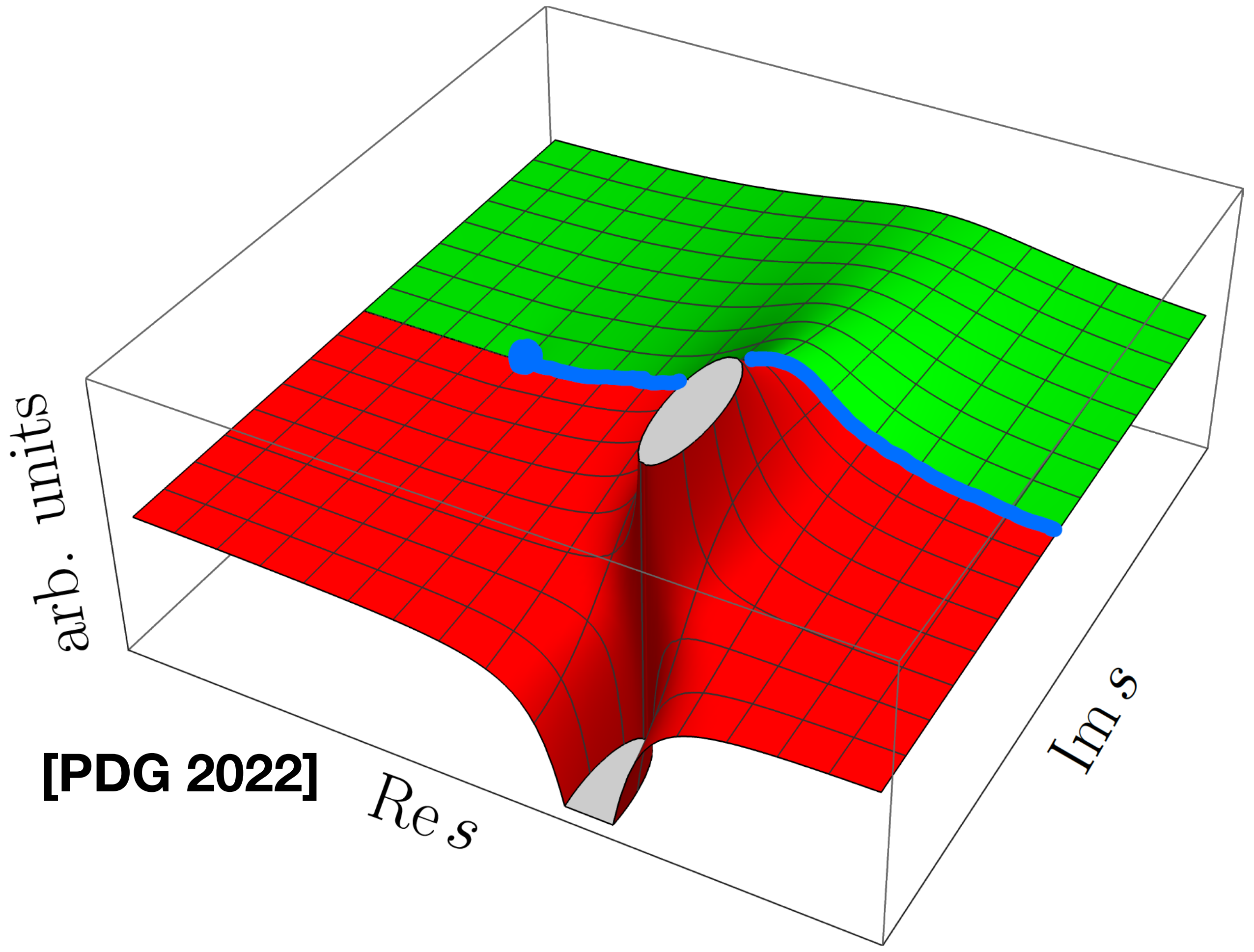

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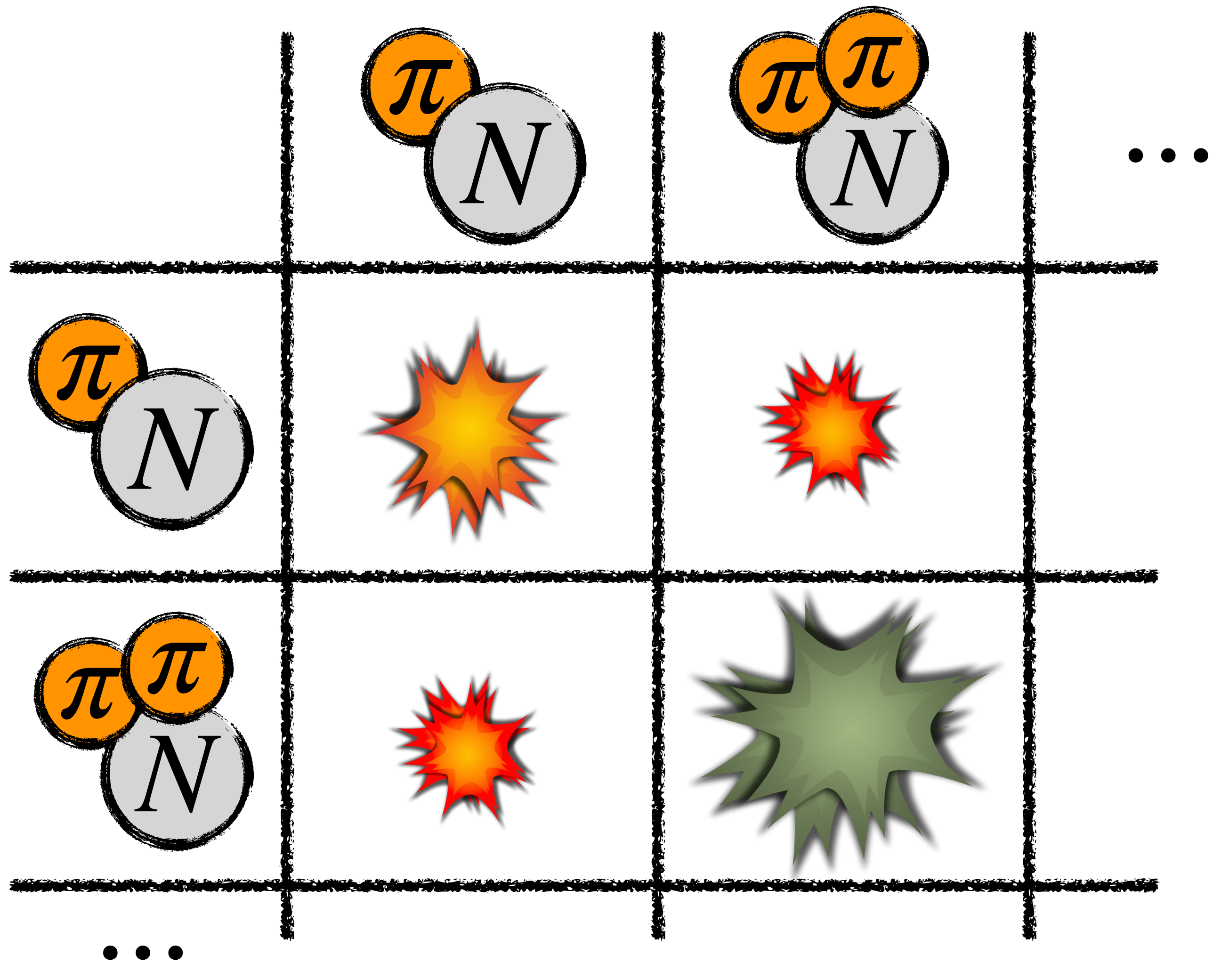

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Lattice QCD ⟶ QCD S-matrix

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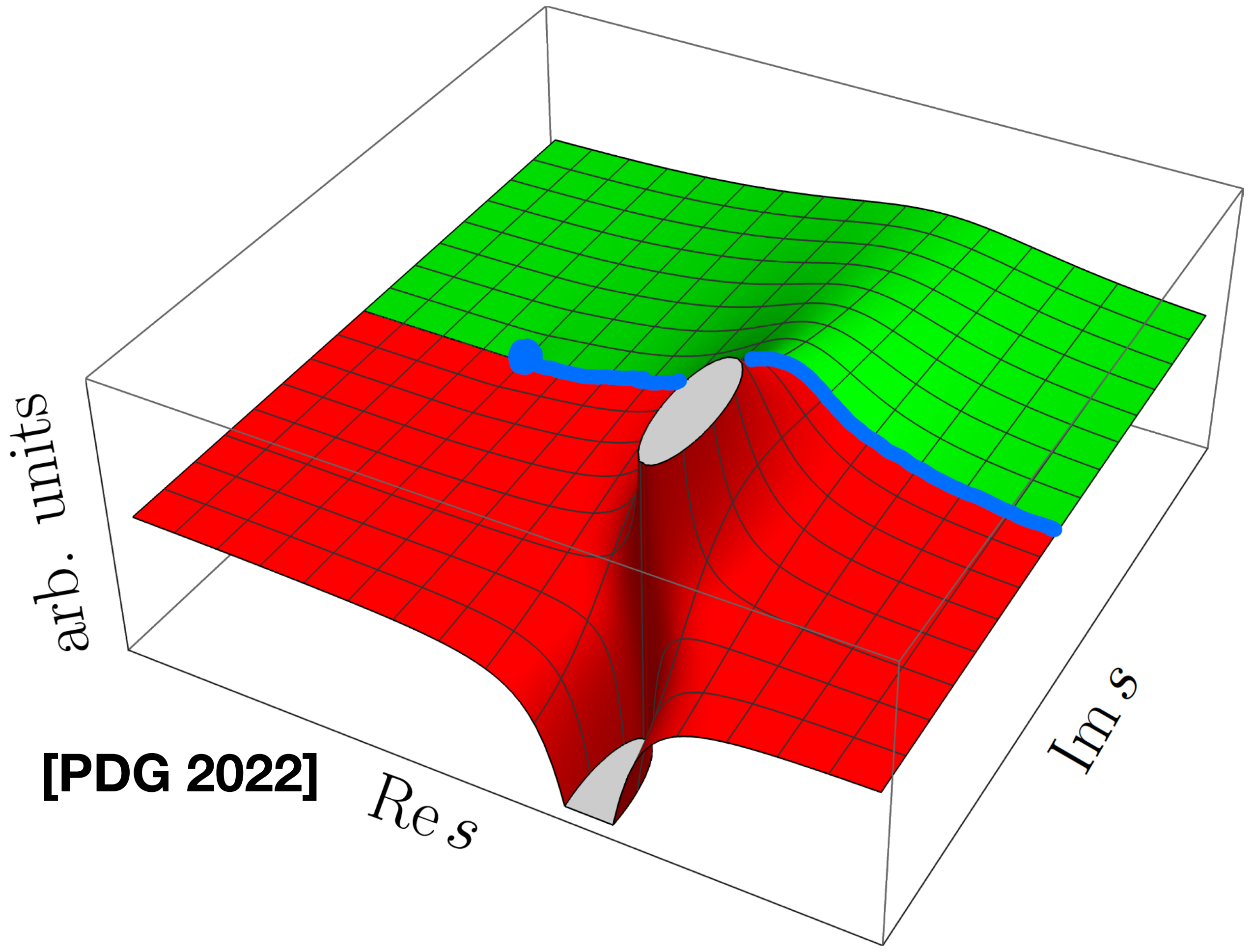

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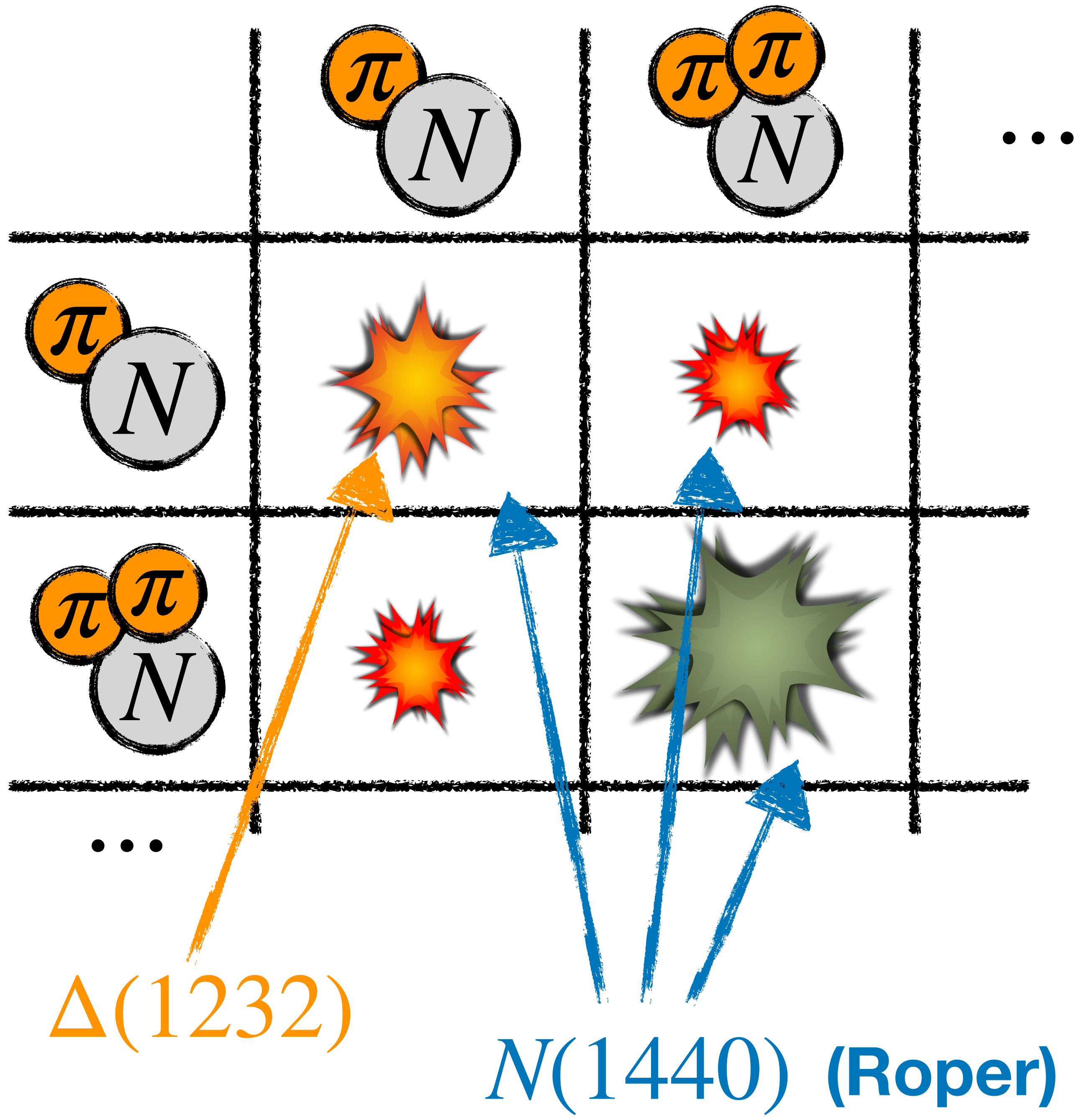
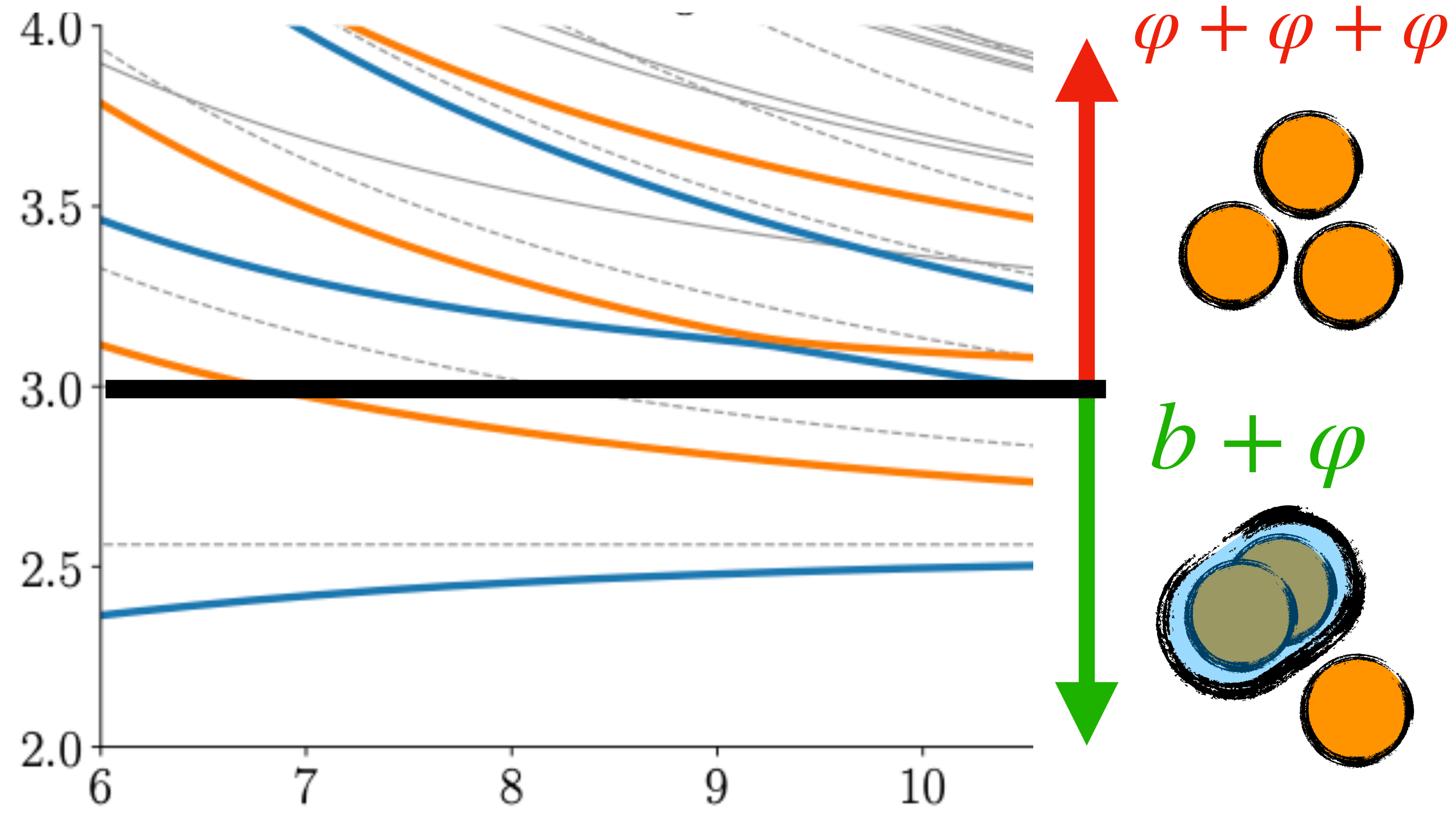


The three-body solution

- In the presence of a two-body bound state:
  - Below the three-particle threshold, effective "particle-dimer"

    [FRL et al 2302.04505] [Jackura et al 2010.09820]
    [Dawid, Islam, Briceño, 2303.04394] [Pefkou et al (in prep)]

[FRL, Sharpe, Blanton, Briceño, Hansen 2302.04505]



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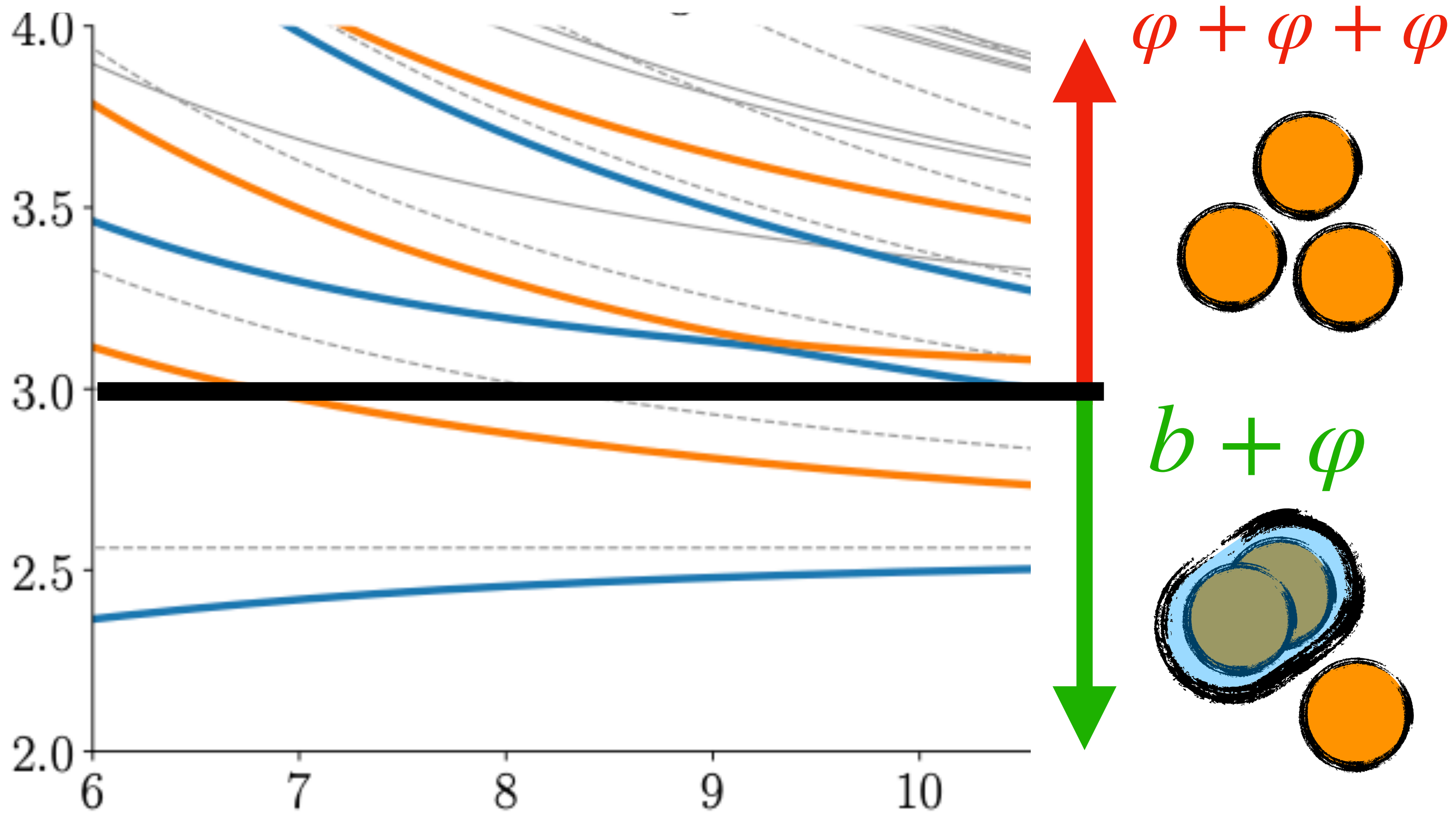
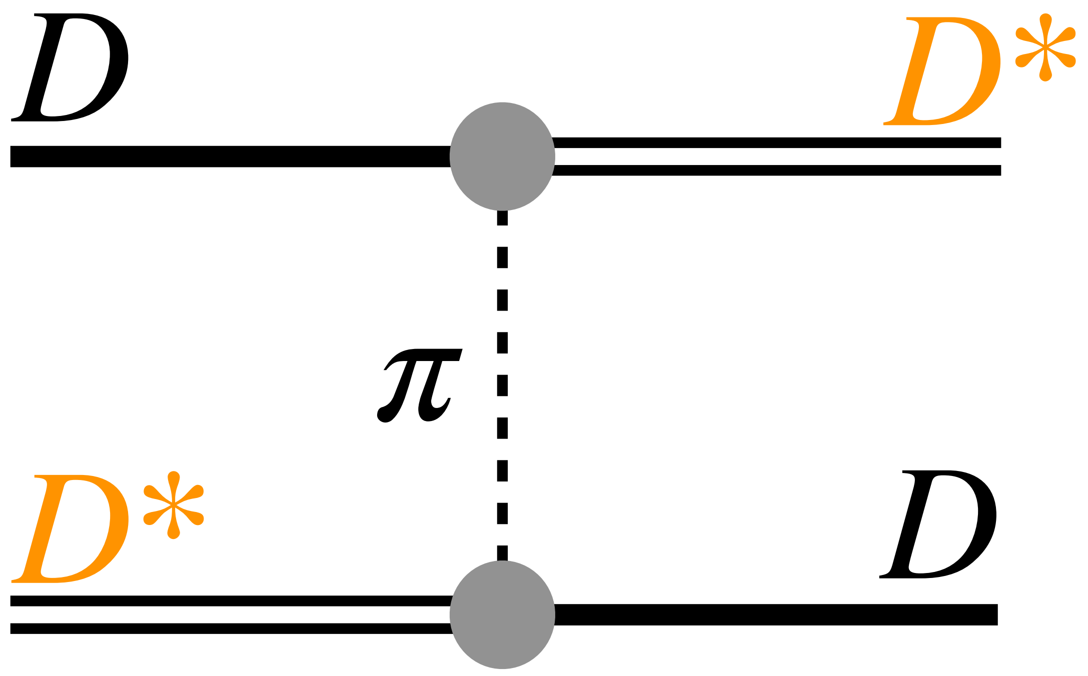
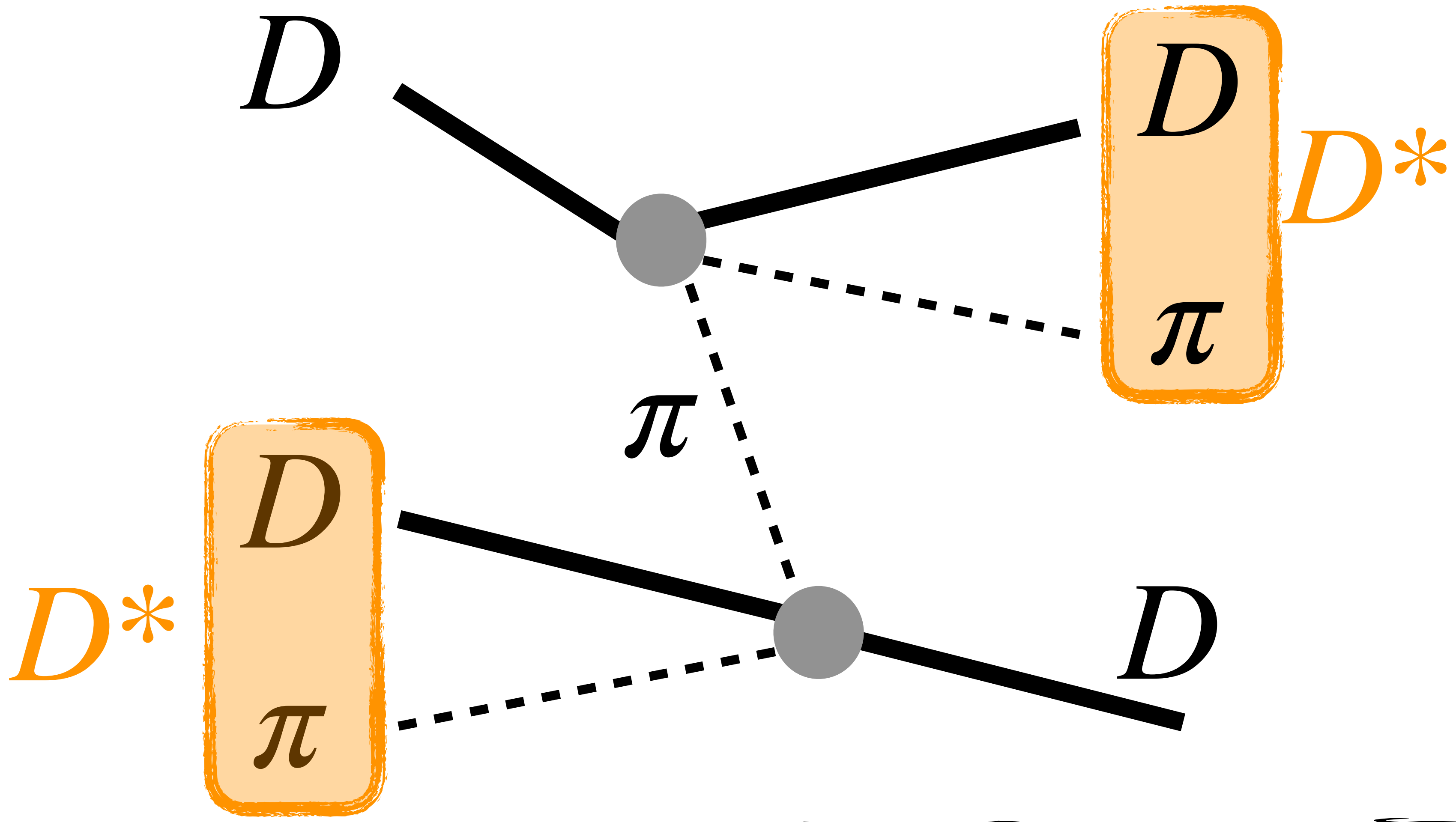


- This solves the left-hand cut problem:
  - Finite-volume effects from one-pion exchange naturally incorporated





Discretization effects

- So far, single- attice spacing calculations. What about discretization effects?

Wilson-ChPT = ChPT + discretization effects

$$M_{\pi K} a_0^{\pi K} = M_{\pi K} a_0^{\pi K}\Big|_{a=0} - \frac{(2w_6' + w_8')}{16\pi}$$

$$M_\pi^2 \mathcal{K}_{\text{df},3}^{\pi\pi K} = M_\pi^2 \mathcal{K}_{\text{df},3}^{\pi\pi K}\Big|_{a=0} - 6x_\pi^2\left(2w_6' + w_8'\right)$$

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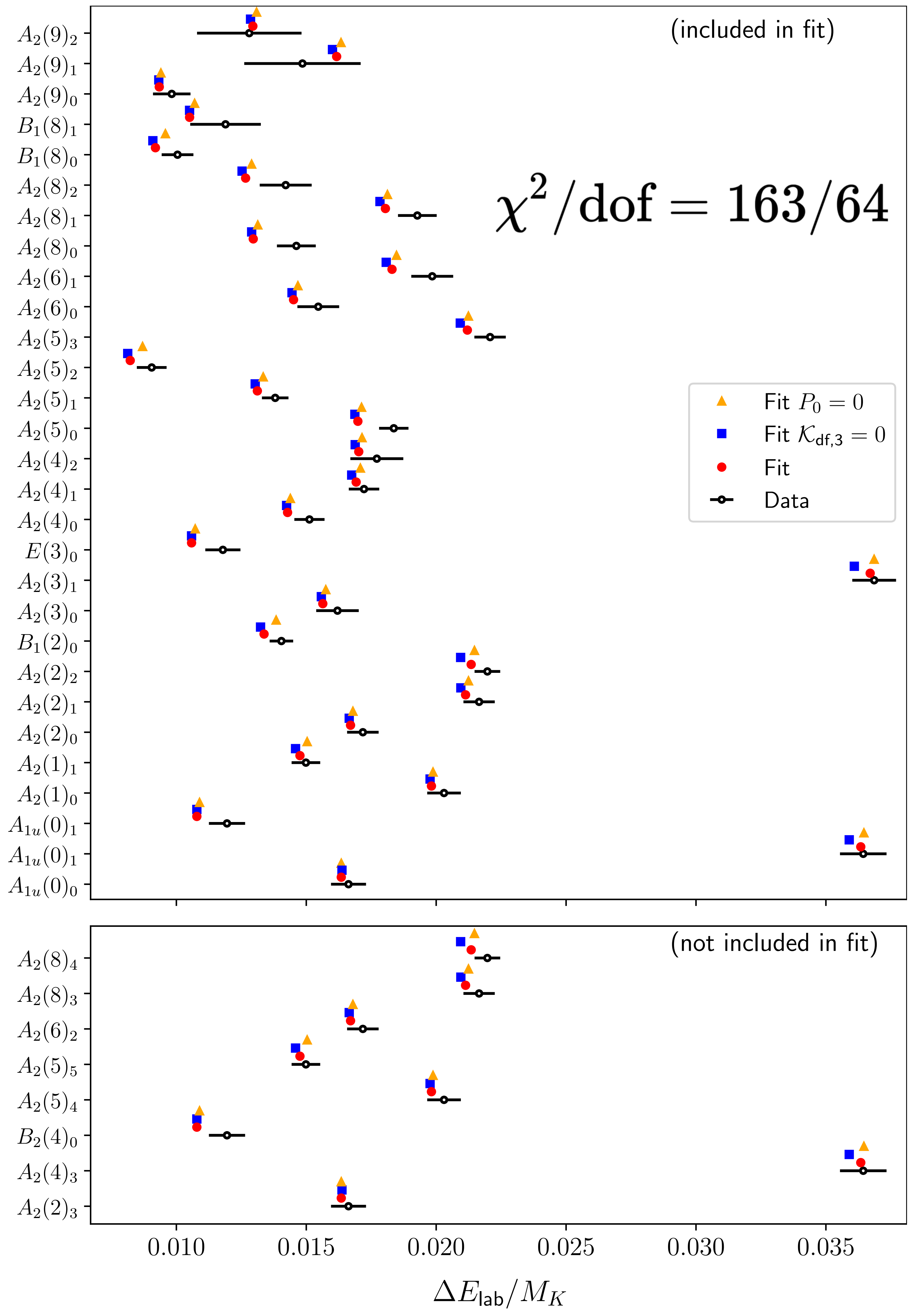
Ensemble	$\mathcal{K}_0$	$\delta_a(\mathcal{K}_0)$
$\pi\pi + \pi K + \pi\pi K$ fits		
D200	190(80)	4(7)
N203	−240(150)	10(16)
$KK + \pi K + KK\pi$ fits		
D200	170(270)	17(27)
N203	260(310)	14(23)

Visualization of fits



$KK\pi$, D200 $M_\pi = 200$ MeV

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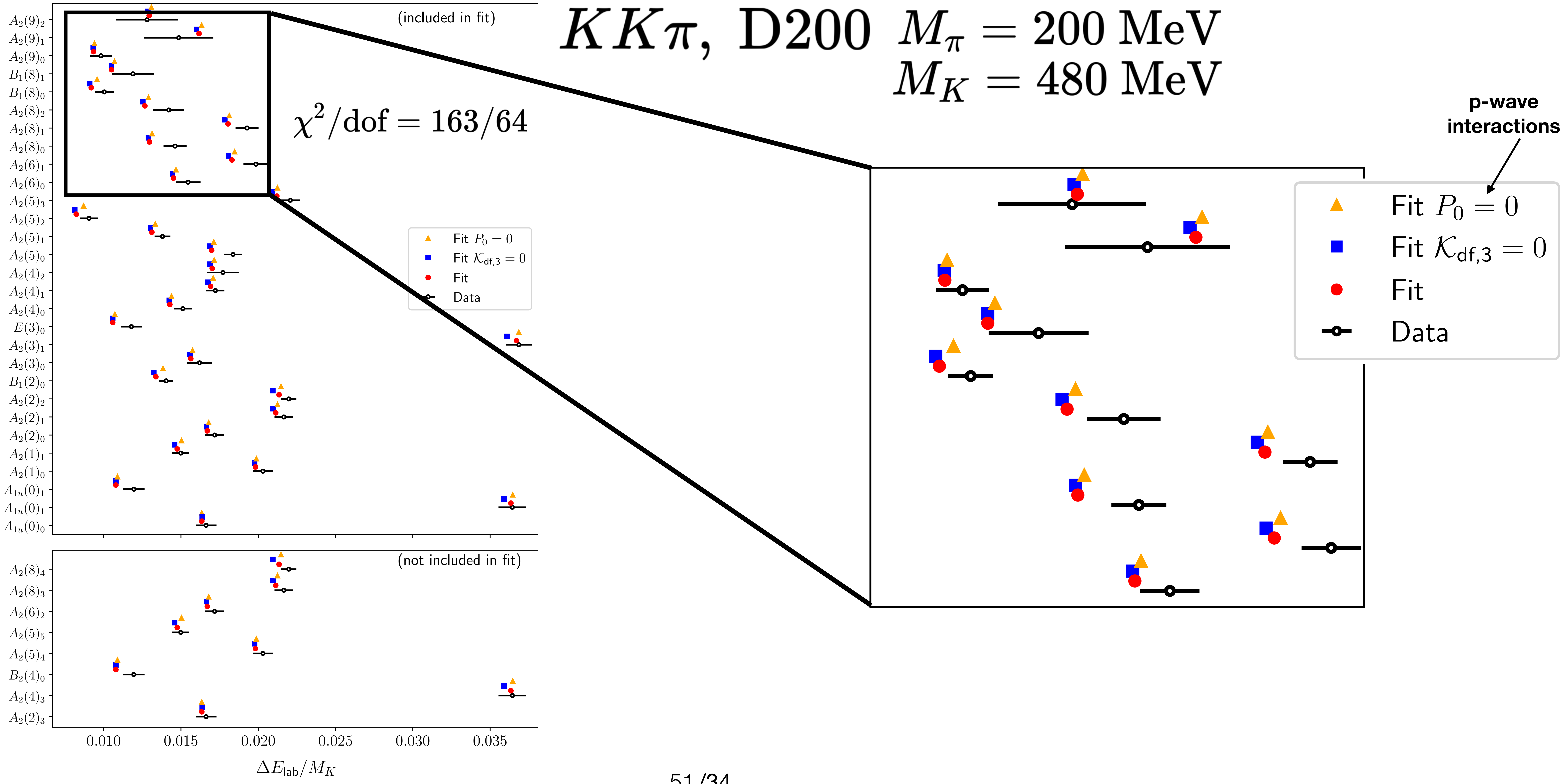


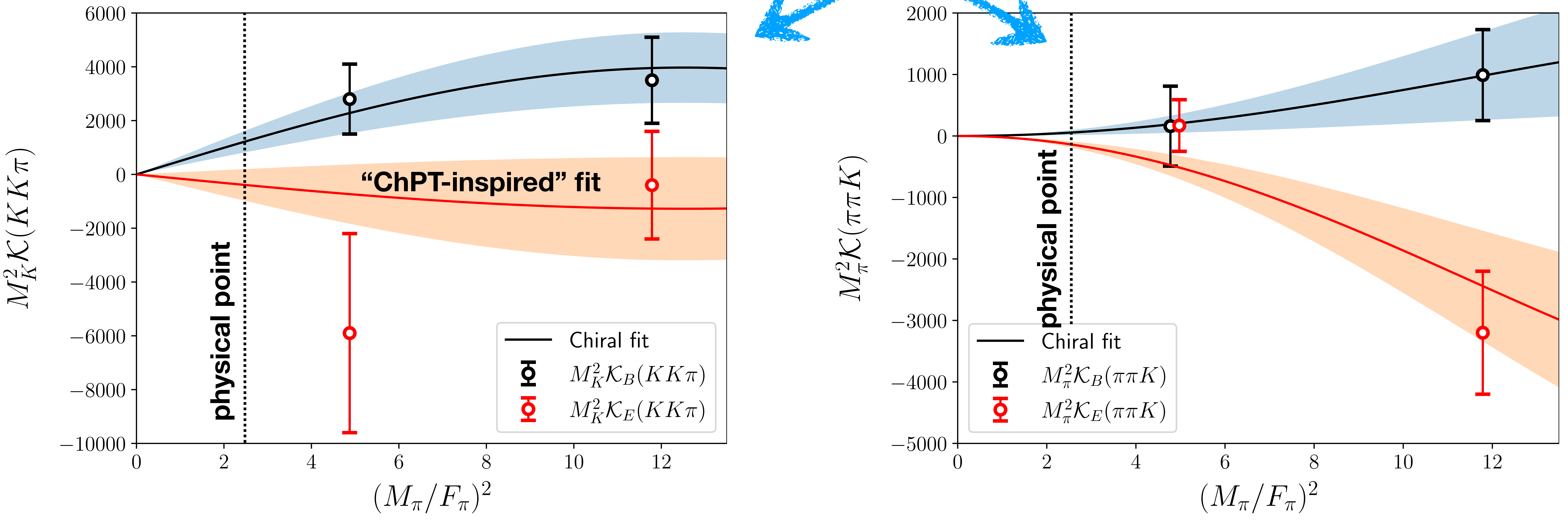
Fernando Romero-López, MIT

$\mathcal{K}_{df,3}$ results (II)

Depend on CM energy

Angular dependence

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_0 + \mathcal{K}_1\Delta + \mathcal{K}_B(\Delta_2 + \Delta_2') + \mathcal{K}_E\tilde{t}_{11}$$



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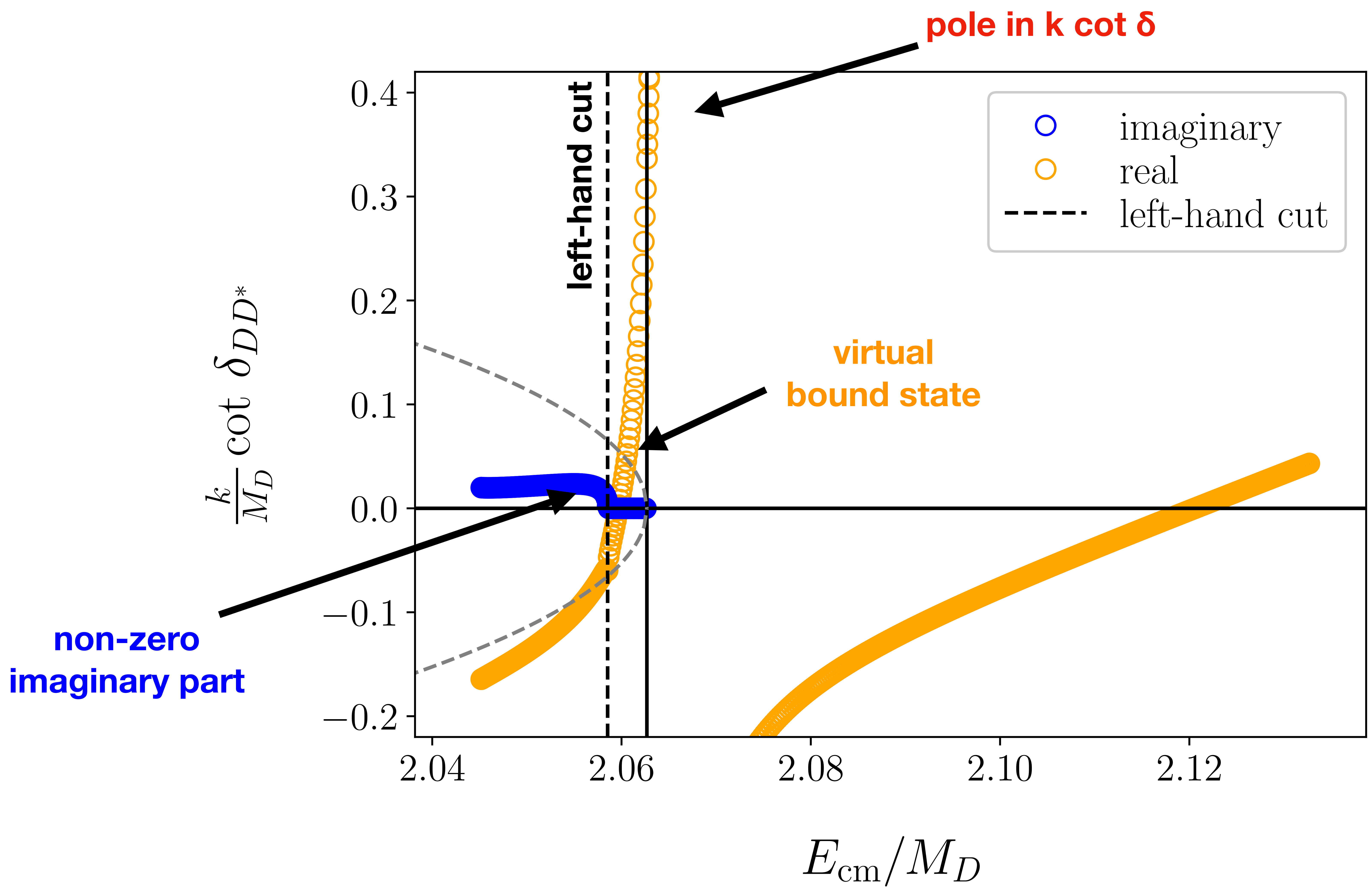
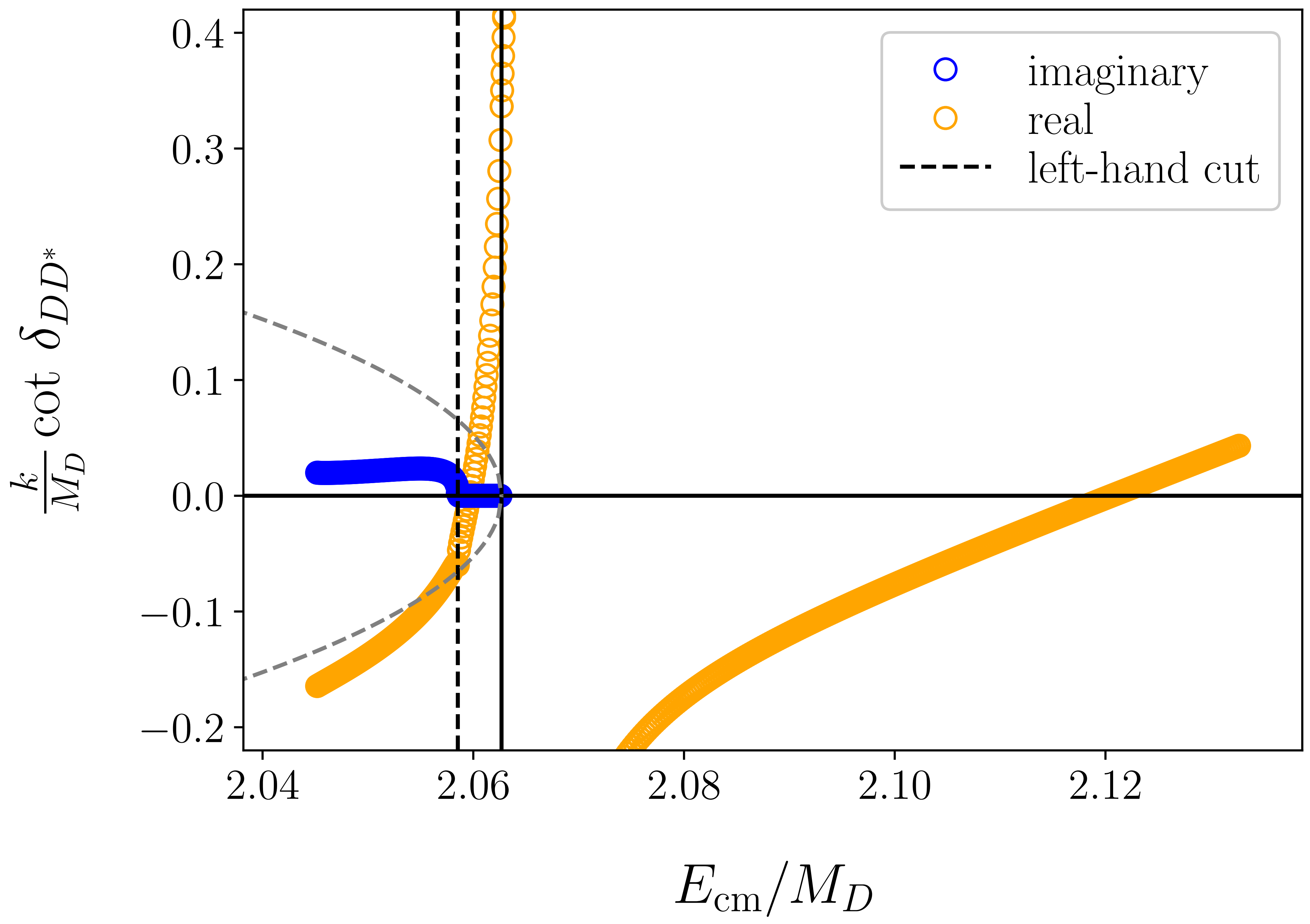
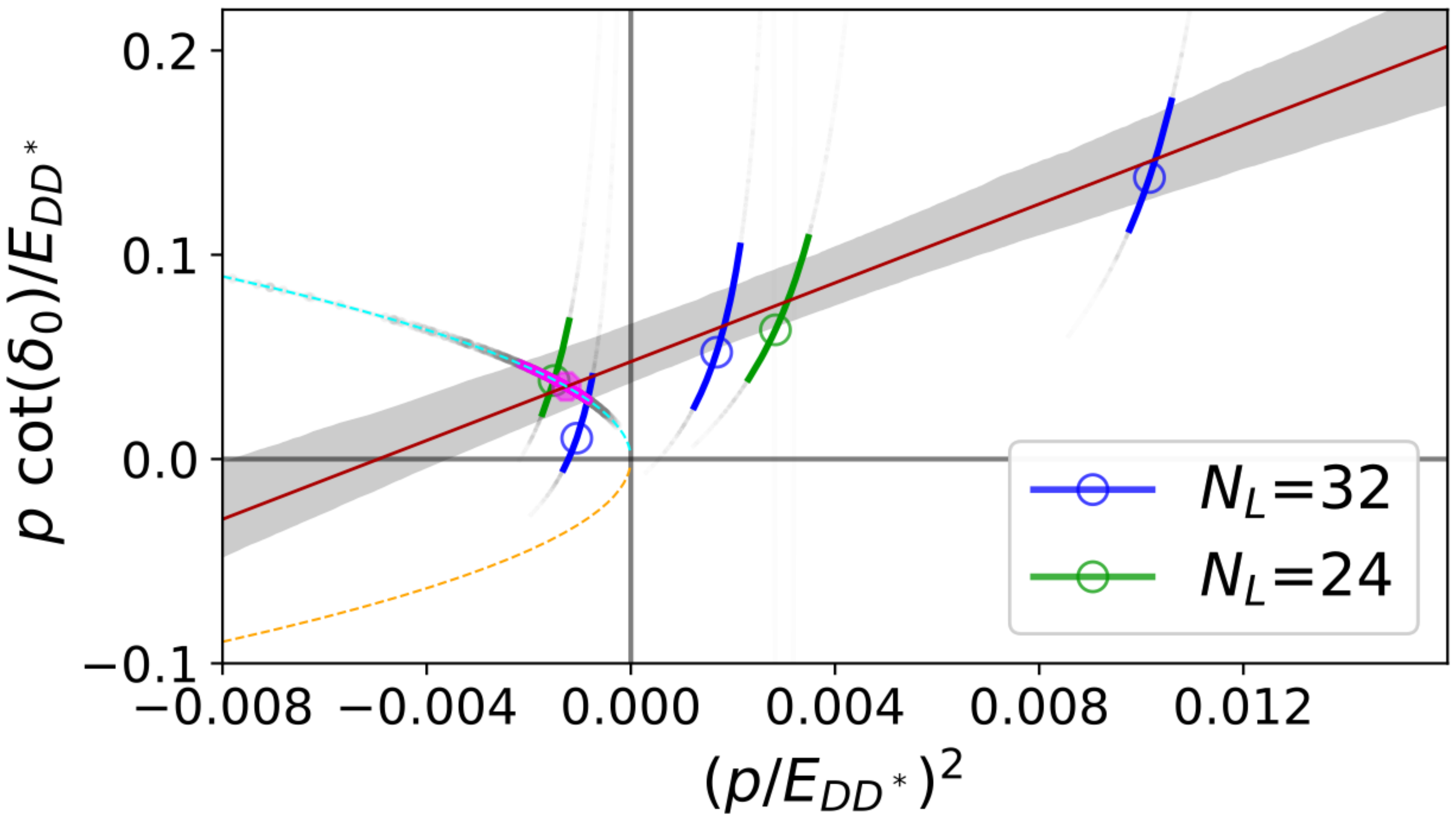
Results for D-D* scattering



Results for D-D* scattering

- Simple model reproduces some qualitative features (virtual bound state)
- Important concerns about features of two body amplitudes: left-hand cut, pole in $k \cot\delta$.





Nondegenerate systems

- Relevant three-body systems involve nonidentical particles → $T_{cc} \to DD\pi$
- First step: RFT formalism for three different scalars
  [Blanton, Sharpe, PRD 2021] x 2, [Hansen, FRL, Sharpe, JHEP 2021]

e.g. $\pi^+\pi^0\pi^-$, $K^+K^+\pi^+$, $D_s^+D^0\pi^-$

$$\det_{k,\ell,m,\mathbf{f}}[1 - \mathbf{K}_{\mathrm{df},3}(E^\star)\mathbf{F}_3(E, \boldsymbol{P}, L)] = 0$$

Implementation: github.com/ferolo2/QC3_release

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- Less symmetry in the K-matrix, leading to more coefficients in the expansion:

Example: $\pi^+\pi^+K^+$ scattering

$$\mathcal{K}_{\mathrm{df},3}=\mathcal{K}_0+\mathcal{K}_1\Delta+\mathcal{K}_B\left(\Delta_2+\Delta_2'\right)+\mathcal{K}_E\tilde{t}_{22}$$

$$\Delta=\frac{s-M}{M^2}\qquad \tilde{t}_{22}=\frac{(p_K-p_K')^2}{M^2}\qquad \Delta_2=\frac{(p_\pi+p_{\pi'})^2-4m_3^2}{M^2}$$

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Fitting "2+1" systems

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parameters in K-matrices

covariance matrix of lab-shifts

"predicted minus measured" lab-frame energy shifts

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- Energy shift is direct output of Lattice QCD