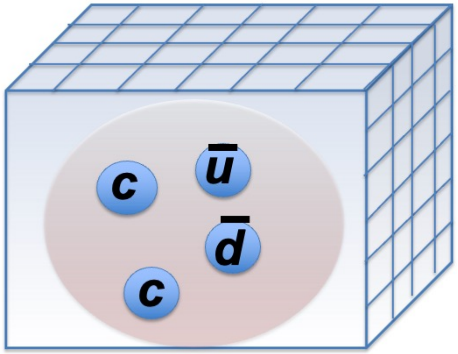
Doubly-charm tetraquark,

its quark mass dependence and left-hand cut

from lattice



Sasa Prelovsek

University of Ljubljana & Jozef Stefan Institute, Slovenia

Strong Interaction physics of heavy flavors

Hirschegg, Austria, January 2024

Plan: review lattice results on Tcc

- in collaboration with: Padmanath, S. Collins, A. Nefediev, E. Ortiz Pacheco, L. Leskovec
- from other collaborations
- reanalysis of lattice results

Outline

- T_{cc} needs to be established from DD* scattering amplitude T(E)

- T(E) from lattice input:

  - using effective range expansion

  - using Effective Field Theory that incorporates "left-hand cut"

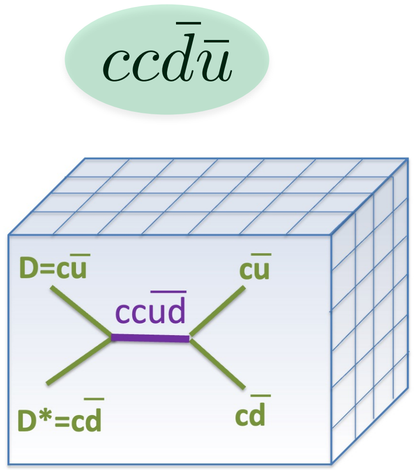
- consider dependence of pole positons on m_c , $m_{u,d}$
  to probe the nature of Tcc

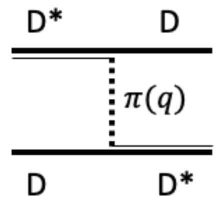
- effect of interpolators [cc][$\overline{ud}$]

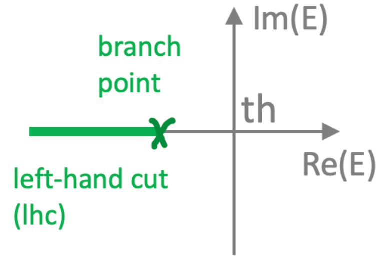
- comparing isospin I=0,1

- cotribution of various Wick contractions

T_{cc}



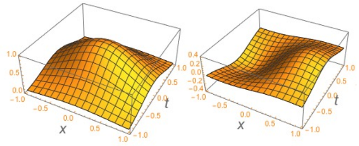




such cut appears near th. in Tcc, Tbb,... channels (at m_π^{lat})

Lattice QCD

$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$



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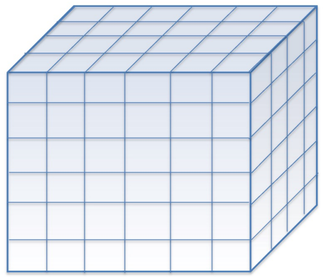


Main quantity extracted: finite-volume eigen-energies E_n $\qquad \hat{H}|n\rangle = E_n|n\rangle$

$\sum_n |n\rangle\langle n|$ $\qquad e^{-iE_n t_M}$

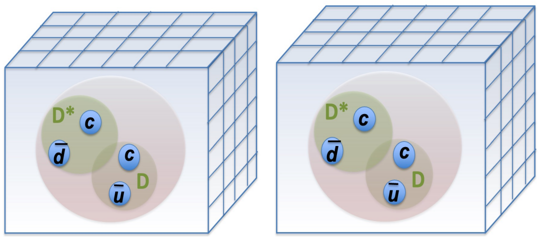
Eucledian time

$$C_{ij}^{2pt}(t) = \langle 0|\, \mathcal{O}_i(t)\ \mathcal{O}_j^+(0)\,|0\rangle = \sum_n \langle 0|\mathcal{O}_i|n\rangle\ e^{-E_n t_E}\langle n|\mathcal{O}_j^+|0\rangle$$

often “non-precision” studies:

single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

t, J^P, I $\qquad$ t=0, J^P, I

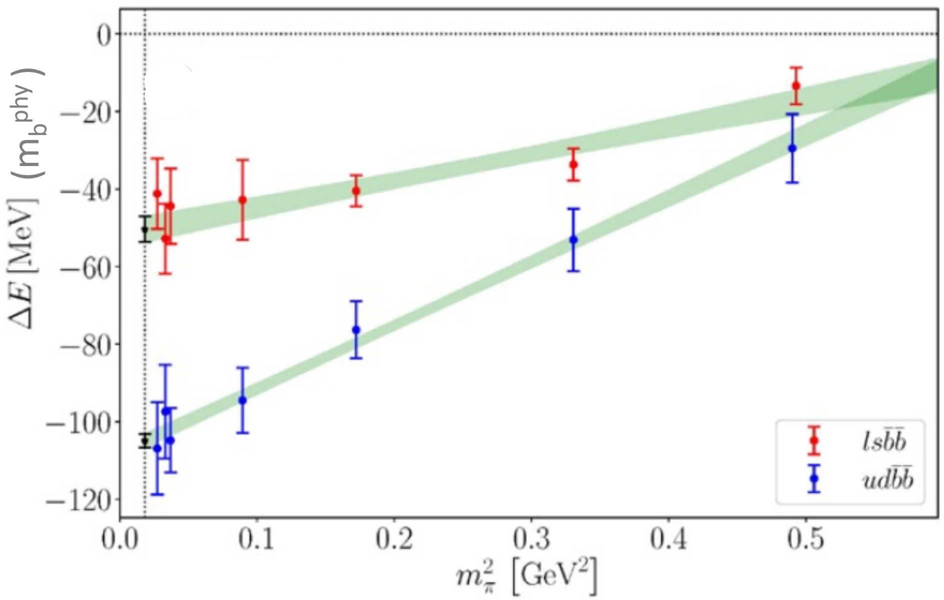


- for strongly stable state well below threshold : $\qquad E_n(P=0) = m$ $\qquad$ results on Tbb,... (Hudspith ..)
- resonances, near-threshold states (Luscher’s relation) $\qquad E_n^{cm} \to T(E_n^{cm})$ $\qquad$ this and a number of other talks
- static potentials: $\qquad E_n \to V(r)$ $\qquad$ Brambilla, Wagner,..

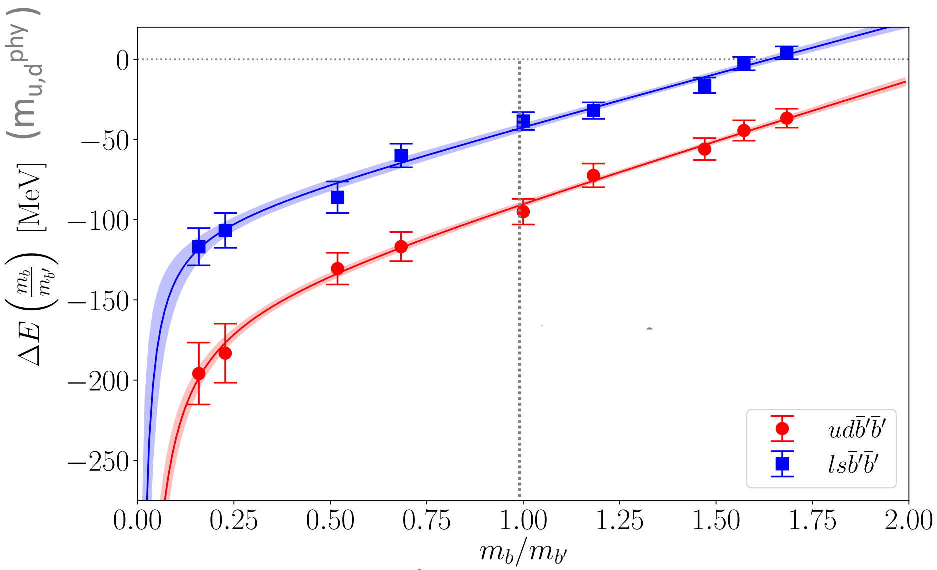
Doubly bottom tetraquarks

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$



$m_{u,d}$ increases →



$m_{b'}$ decreases →

Frances, Colquhoun, Lewis, Maltman, Hudspith
PoS LATTICE2021 (2022) 144

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$ The only ones expected significantly below strong-decay thresholds $BB^*_{(s)}$

Other $QQ'\bar{q}\bar{q}'$ and J^P

$bc\bar{q}\bar{q}',\ cc\bar{q}\bar{q}'$ q=u,d,s

talk by Wagner

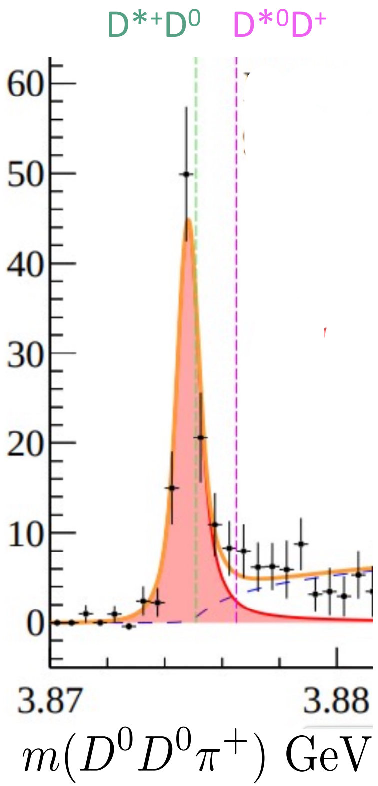
Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering T(E): much more challenging

$cc\bar{d}\bar{u}$ T_{cc}

The longest lived exotic hadron ever discovered

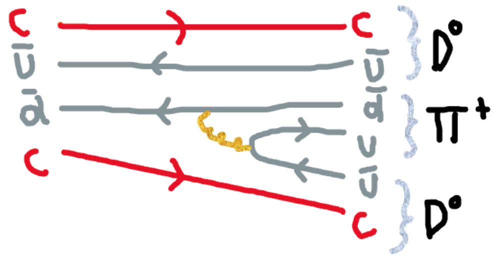
I=0, $J^P=1^+$ (most likely)



$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

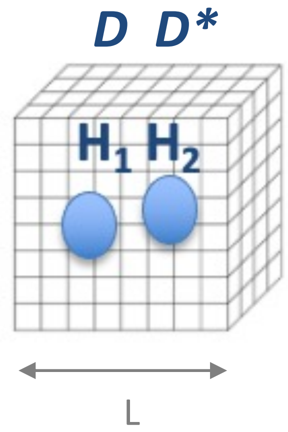
LHCb 2109.01038, 2109.01056, Nature Physics

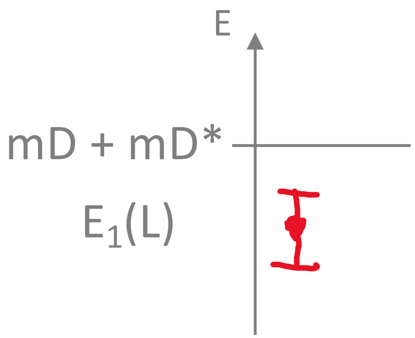


Omitting $D^* \to D\pi,\ T_{cc} \to DD\pi$

T_{cc} would-be a bound state

Why $m(T_{cc})$ can not be extracted as $m=E_1$?





- eigenstate with E_1 < mD + mD* could correspond to a essentially free pair of D(0) and D*(0)

  whose energy is slightly shifted down due to feeble attraction on a finite lattice

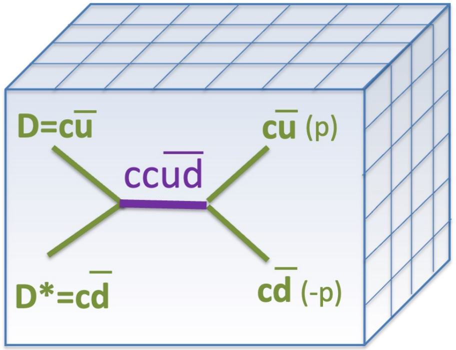
- E_1 < mD + mD* by itself does not imply there is a (virtual) bound state or resonance

- Scattering amplitude T(E) has to be extracted

- pole in T(E) indicates a presence of Tcc

applies for states near or above threshold

Extracting scattering amplitude : general strategy



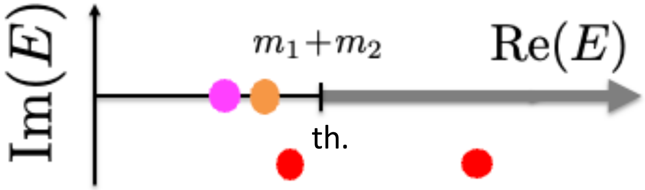
all available lattice studies of Tcc:

$$m_u = m_d$$

$$m_\pi > m_\pi^{phy}$$

$$D^* \not\to D\pi,\ T_{cc} \not\to DD\pi$$

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$



$$S(E) = e^{2i\delta(E)} = 1 + 2i\,\frac{2p}{E}\,T(E) \to \boxed{T \propto \frac{1}{p\cot\delta - ip}}$$

Virtual bound st. — $p = -i\,|p|$, sheet II

Bound st. — $p = i\,|p|$, sheet I

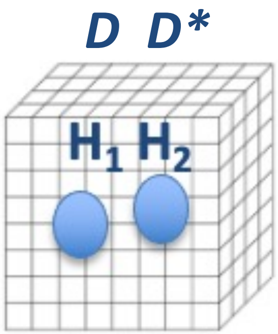
Resonances — sheet II

$p^2 < 0$

sheet I: Im(p)>0

sheet II: Im(p)<0

Scattering amplitude T(E) from lattice QCD



real E — for real E — for complex E

$$E \to T(E) \to T(E^c)$$

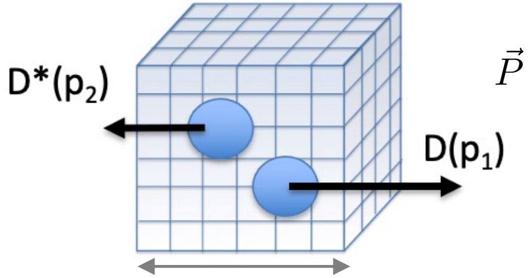
analytic relation:
Luscher 1991
generalizations by many authors

analytic contin.
to complex E

T_{cc}

$cc\bar{u}\bar{d}$

$I=0,\ J^P=1^+$



$\vec{P}=\vec{p}_1+\vec{p}_2$

L = 2.1 fm, 2.7 fm $m_\pi \approx 280$ MeV

CLS ensembles a~0.086 fm

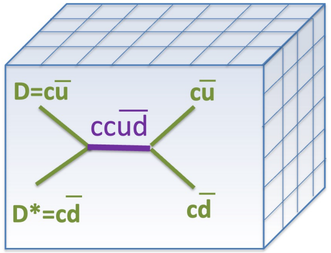
D D*

$$\mathcal{O}=(\bar{u}\gamma_5 c)_{\vec{p}_1}\ (\bar{d}\gamma_i c)_{\vec{p}_2}-(\vec{p}_1\leftrightarrow\vec{p}_2) \qquad \vec{p}_{1,2}=\vec{n}_{1,2}\ \frac{2\pi}{L}$$

$$(\bar{u}\gamma_5\gamma_t c)_{\vec{p}_1}\ (\bar{d}\gamma_i\gamma_t c)_{\vec{p}_2}$$

C evaluated using disillation method

O=[cc][ud] : effect discussed at the end of the talk



$D^*\not\to D\pi,\ T_{cc}\not\to DD\pi$

$DD\pi$ above analyzed region

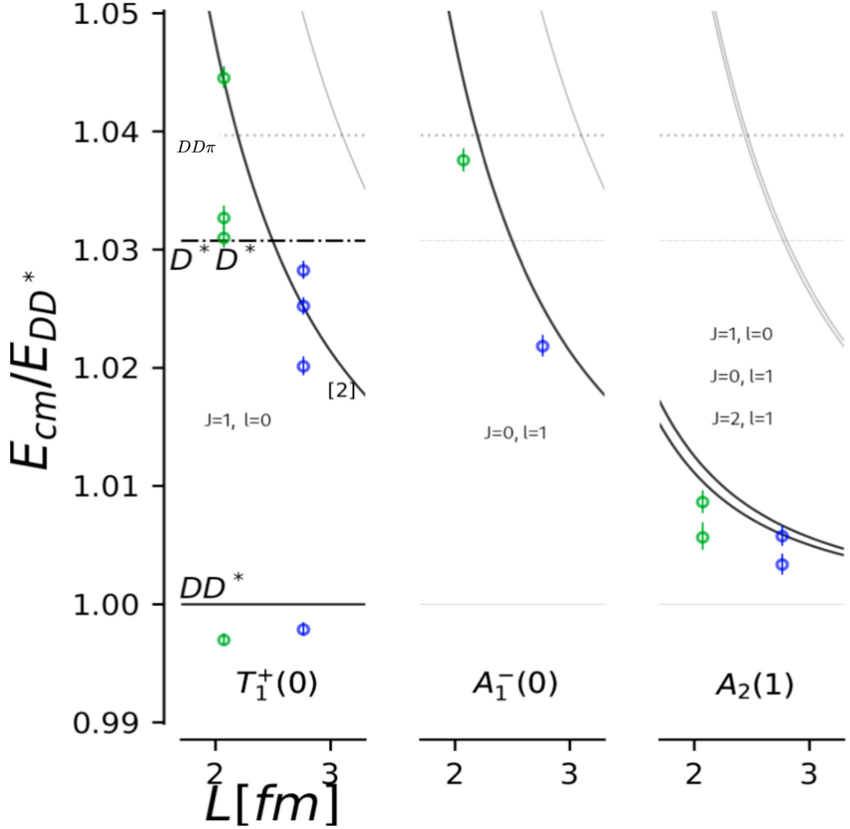
these applies to all available lattice studies of Tcc

E < $E^{non.int.}$ (lines) : $E^{n.i.}=\sqrt{m_D^2+\vec{p}_1^{\,2}}+\sqrt{m_{D^*}^2+\vec{p}_2^{\,2}}$, $\vec{p}_i=\vec{n}_i\frac{2\pi}{L}$

attractive interaction between D and D*

Padmanath & SP, 2202.10110, PRL 2022

$\vec{P}=0 \qquad \vec{P}=\frac{2\pi}{L}\ (0,0,1)$

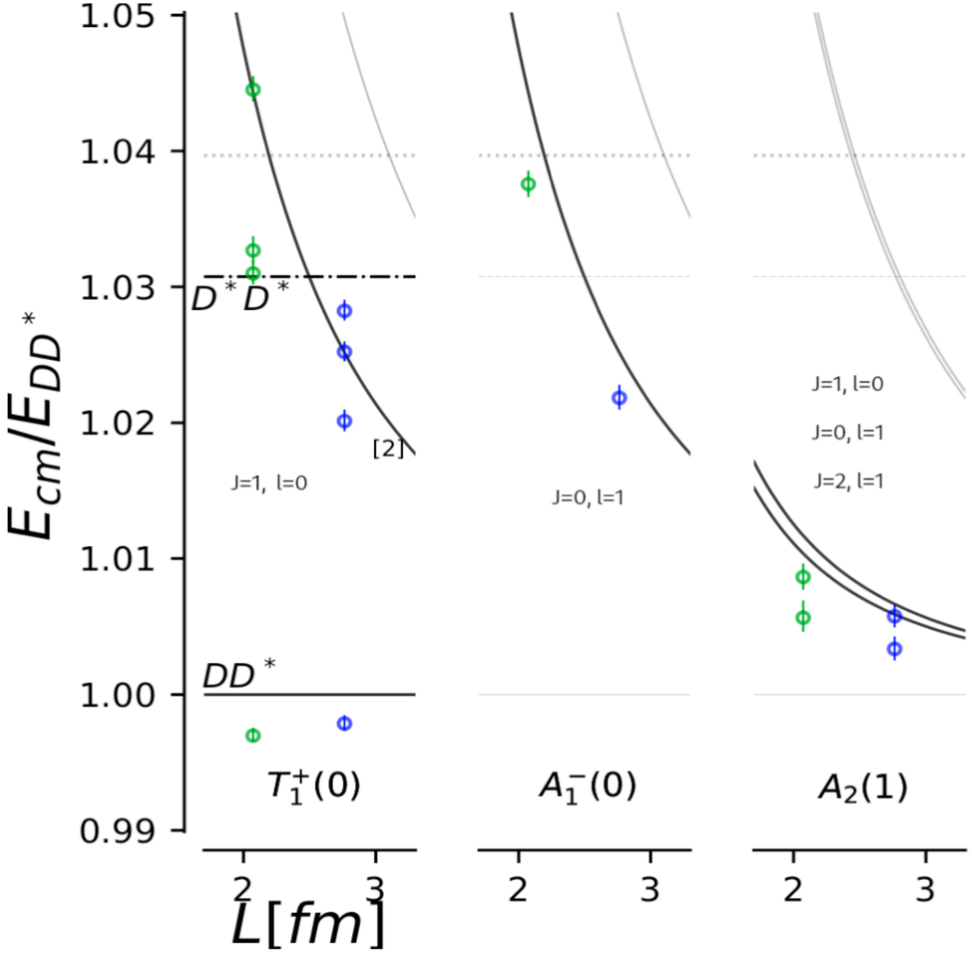


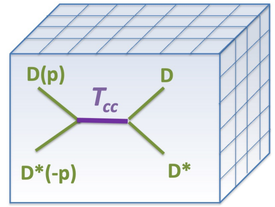
$E_{DD^*}\equiv m_D+m_{D^*}$

T_{cc} Eigen-energies and scattering amplitude

Padmanath, S.P.: 2202.10110, *PRL 2022*

at $m_\pi \approx 280\ MeV$



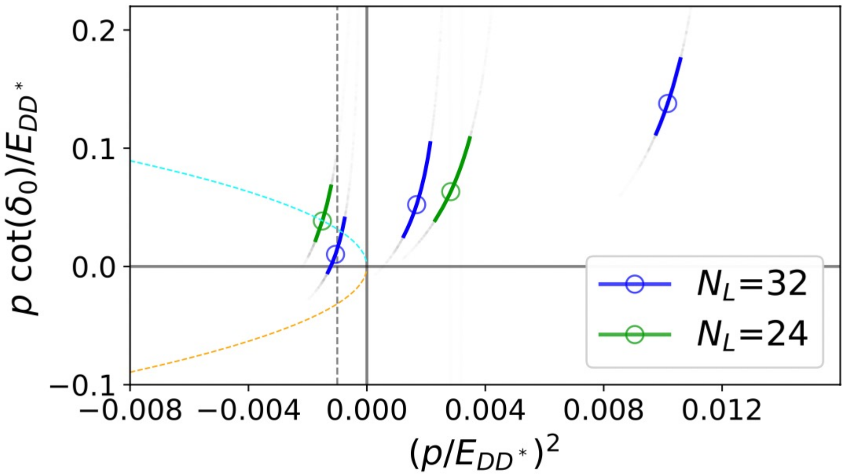


Luscher's relation

E -> T(E), $\delta(E)$

$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

partial wave l=0



$$E_{DD^*} \equiv m_D + m_{D^*}$$

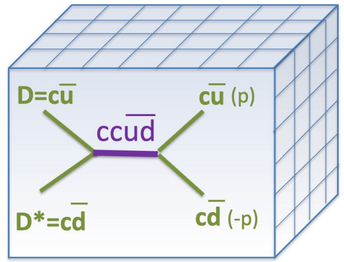
Analysis of T_{cc} lattice results assuming effective range expansion

$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2 + \ldots$$

(may not be valid assumption due to nearby left-hand cut,
taken into account in the second analysis discussed later)

T_{cc}: dependence on $m_{u/d}$

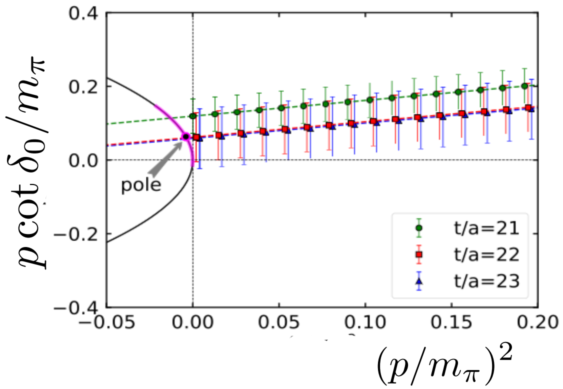
$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$$



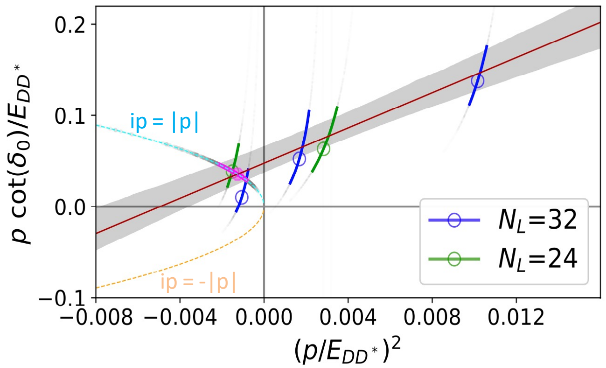
$m_{u,d}$

LHCb

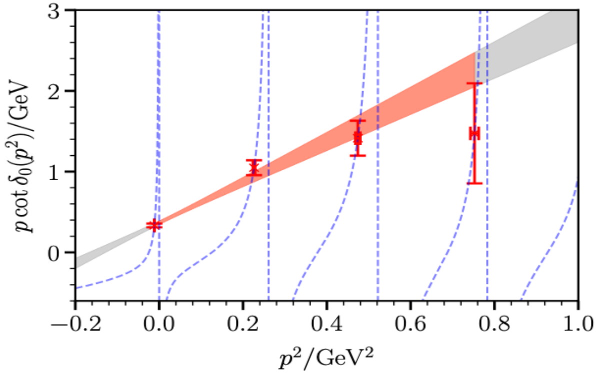
HALQCD method, 2302.04505, $m_\pi \approx$146 MeV



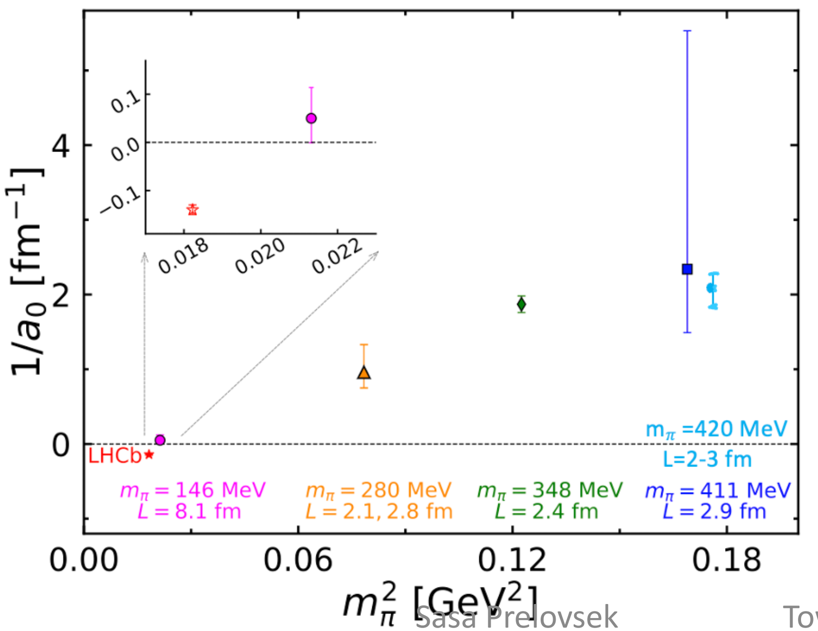
Padmanath, SP: 2202.10110, PRL, $m_\pi \approx$ 280 MeV



CLQCD 2206.06185, PLB, $m_\pi \approx$ 348 MeV

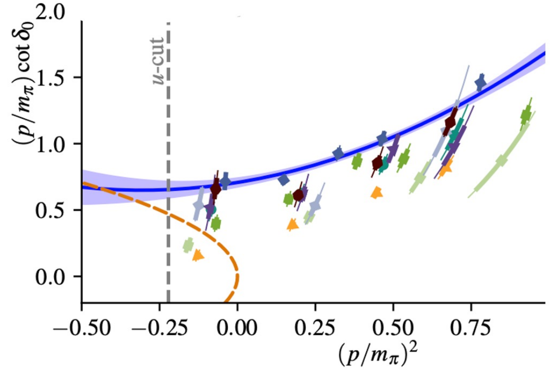


adapted from 2302.04505

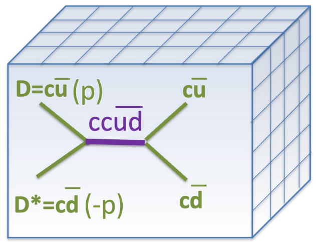


several a

Green et al, Mainz, Lat 2023, $m_\pi \approx$ 420 MeV



T_{cc}: dependence on $m_{u/d}$

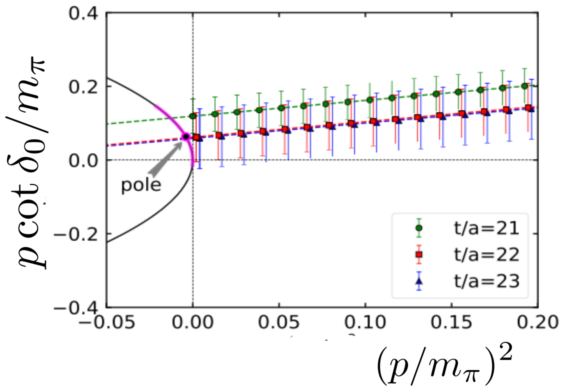


$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\ r_0\ p^2 \qquad T \propto \frac{1}{p \cot \delta - ip}$$

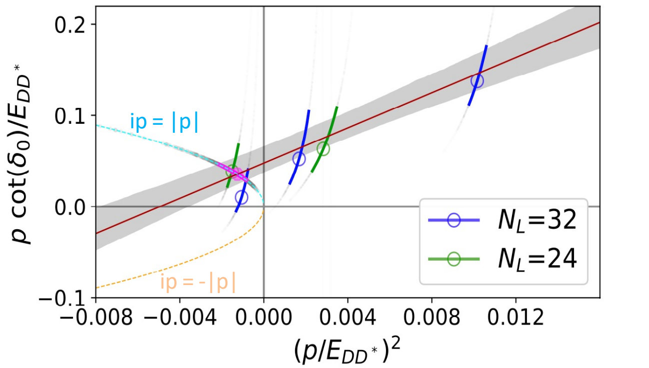
$m_{u,d}$ →

LHCb

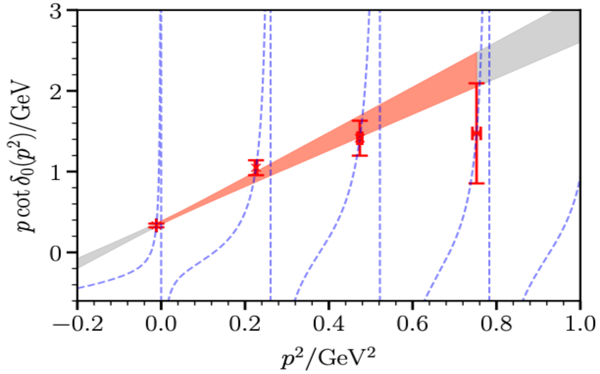
HALQCD method, 2302.04505, $m_\pi \approx 146$ MeV



Padmanath, SP: 2202.10110, PRL, $m_\pi \approx 280$ MeV



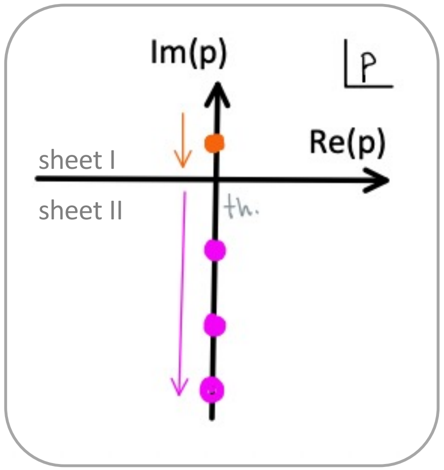
CLQCD 2206.06185, PLB, $m_\pi \approx 348$ MeV



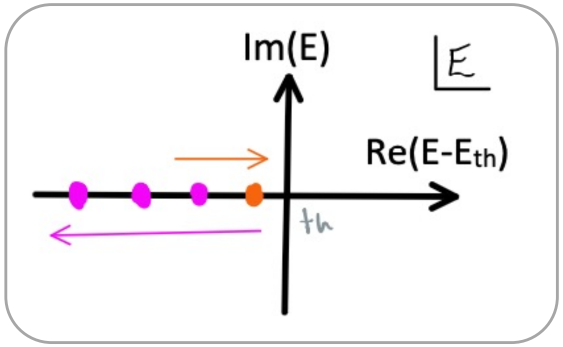
- 0.4 MeV   - 0.04 MeV   -10 MeV : binding energy

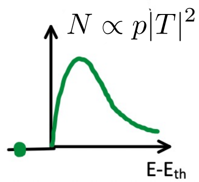
"bound" st.
at p = i |p|

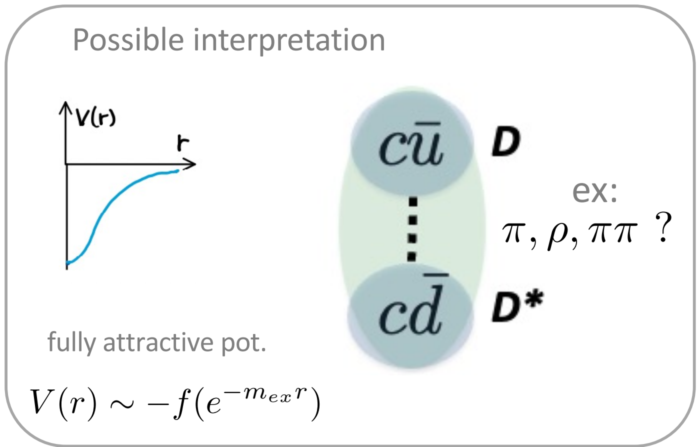
virtual st. pole
at p = - i |p|



arrow: increasing $m_{u,d}$







T_{cc}: dependence on m_c

$$m_r \simeq \frac{m_D m_{D^*}}{m_D + m_{D^*}}$$

increasing m_c and m_r

Collins, Nefediev, Padmanath, S.P.:

2401.xxxxx:

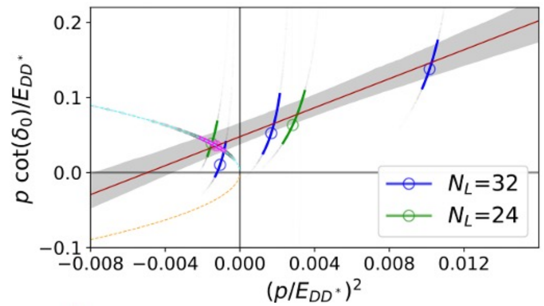
additional three values of $m_c > m_c^{phy}$

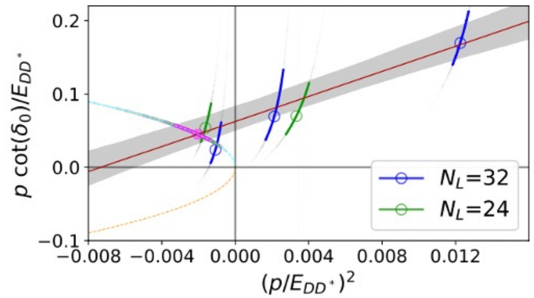
at fixed $m_\pi \approx 280$ MeV

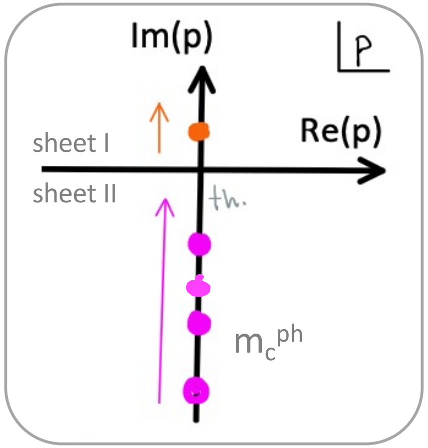
Tcc: Padmanath, S.P.: 2202.10110, *PRL*

	m_D [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
$m_c^{(h)}$	1927(1)	1.04(29)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.

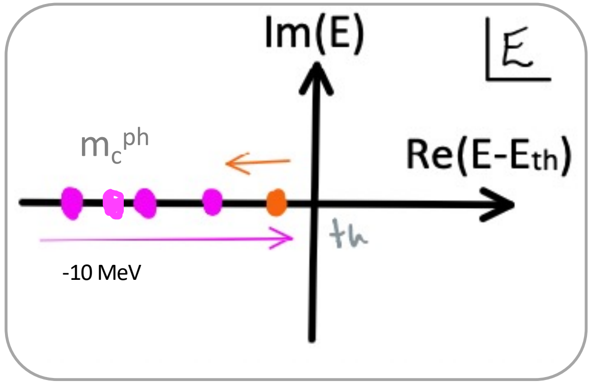
	m_D [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
$m_c^{(l)}$	1762(1)	0.86(0.22)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.

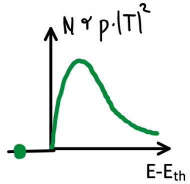




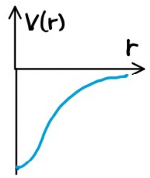


arrow: increasing m_c



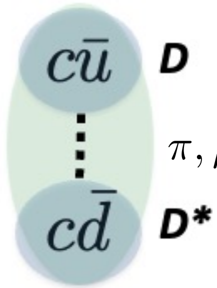


Possible interpretation $\hat{H}_{kin} = \frac{\hat{p}^2}{2\, m_{red}}$



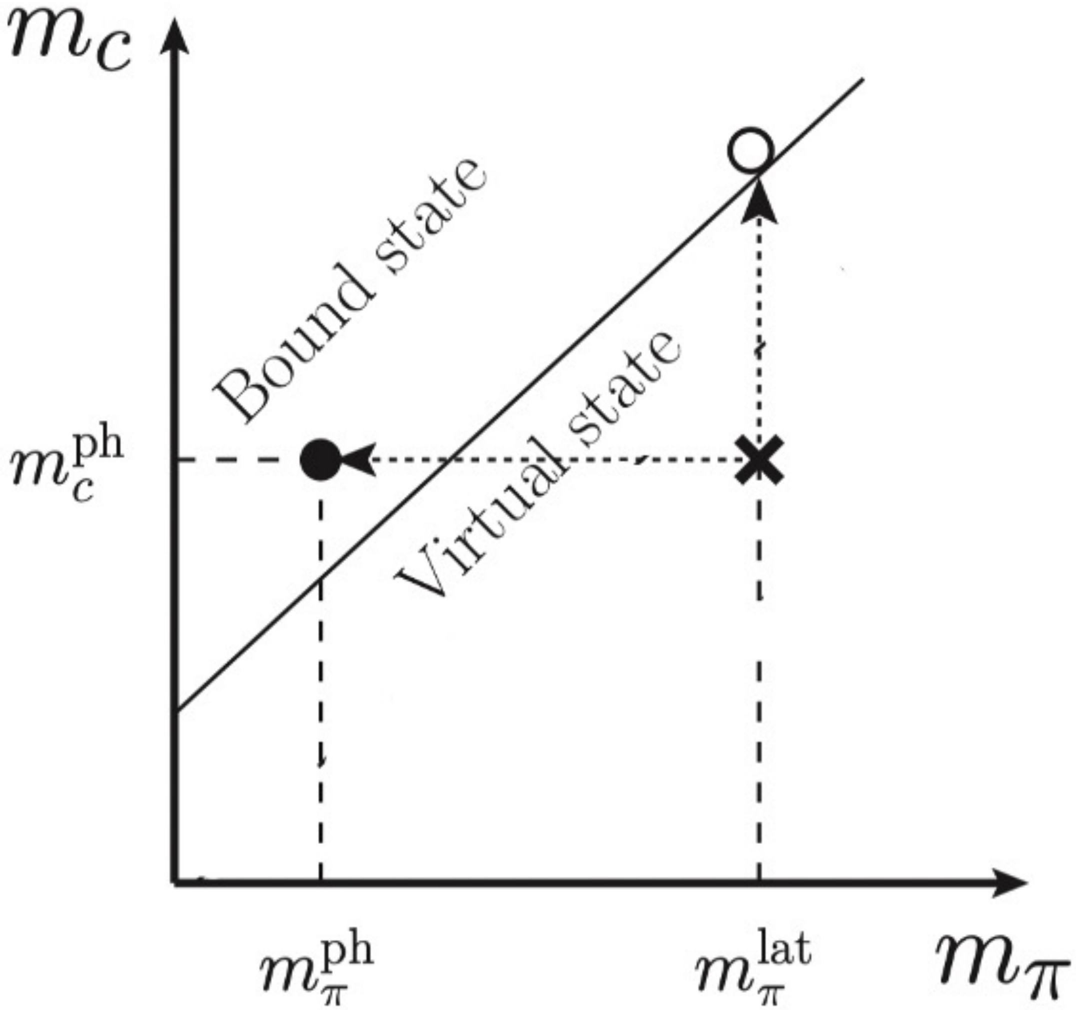
fully attractive V(r)

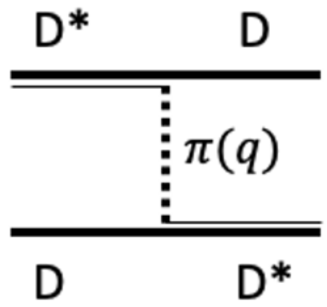
mostly independent on m_c



ex: $\pi, \rho, \pi\pi$?

Towards the nature of T_{cc} from lattice assuming effective range expansion

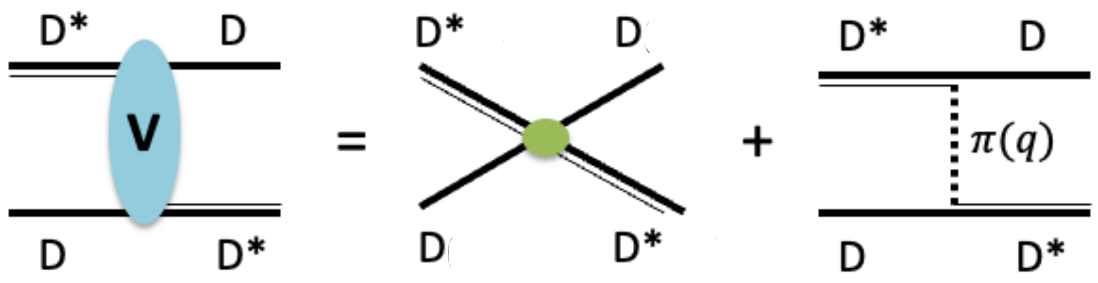


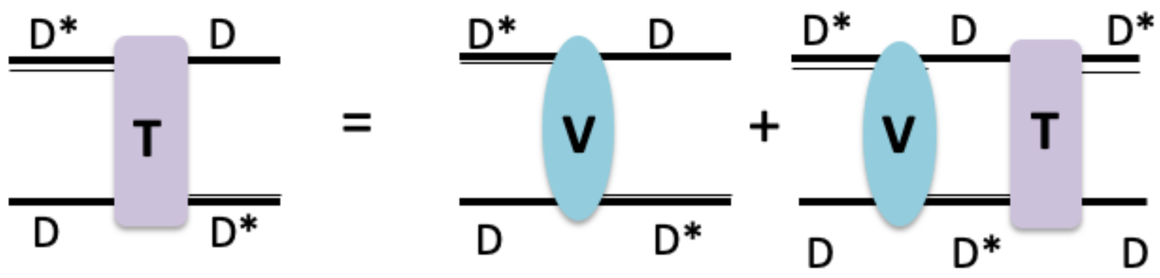


Analysis of T_{cc} lattice results based on Effective Field Theory

taking into account effect from left-hand cut

Tcc analysis based on Effective Field Theory





Du, Hanhart, Guo, Nefediev, Filin, et al, PRL 2023, 2303.09441

Collins, Nefediev, Padmanath, S.P.: 2401.xxxxx

Meng, Baru, Epelbaum et al., 2312.01930 (Luscher -> plane-wave approach)

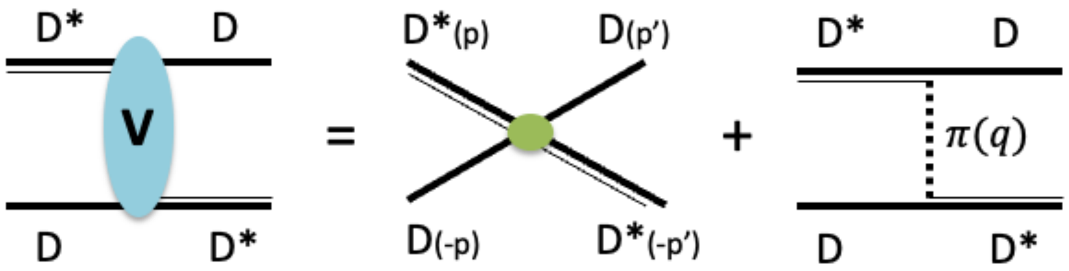
T_{cc} analysis based on Effective Field Theory

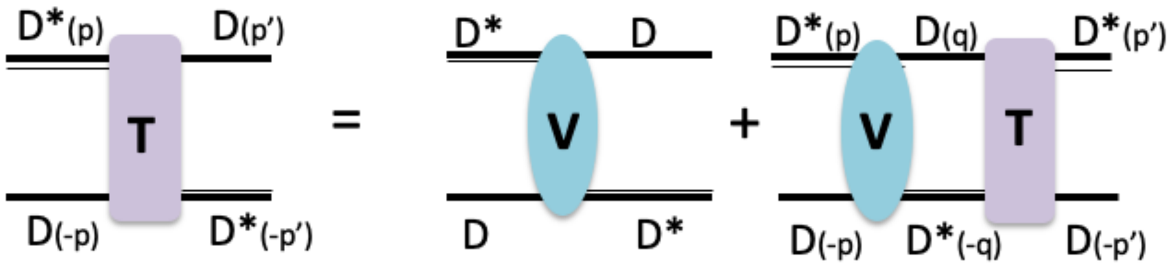
$c_{0,2}$ fitted from lat. data

significant short-distance attraction

$$V_{CT} = 2c_0 + 2c_2(p^2 + p'^2)$$

$$\frac{g_c}{2f_\pi}\vec{q}$$





$$T = V - VGT$$

$$T = \frac{1}{V^{-1} + G}$$

$$T(\boldsymbol{p}, \boldsymbol{p}'; E) \qquad V(\boldsymbol{p}, \boldsymbol{p}') \qquad - \int \frac{d^3q}{(2\pi)^3} V(\boldsymbol{p}, \boldsymbol{q}) G(\boldsymbol{q}; E) T(\boldsymbol{q}, \boldsymbol{p}'; E)$$

Du, Hanhart, Guo, Nefediev, Filin, et al, PRL 2023, 2303.09441

Collins, Nefediev, Padmanath, S.P.: 2401.xxxxx

Meng, Baru, Epelbaum et al., 2312.01930 (Luscher -> plane-wave approach)

thanks to Alexey Nefediev for plesant collaboration ...

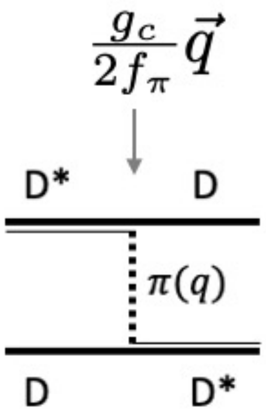
Pion exchange, left-hand cut etc

Scope of the meeting

- How important are the contributions from long-range pion exchange and how can they be addressed?

Heavy meson ChPT

$g_c(m_\pi),\ f_\pi(m_\pi)$



$$V_\pi(\boldsymbol{q}) = \frac{g_c^2}{4f_\pi^2}\frac{\boldsymbol{q}^2}{u - m_\pi^2}$$

$$= \frac{g_c^2}{4f_\pi^2}\left(-1 + \frac{\mu_\pi^2}{\boldsymbol{q}^2 + \mu_\pi^2}\right)$$

attraction at short distance — slight repulsion at long distance

$u = q^2 = q_0^2 - \mathbf{q}^2 \simeq (m_{D^*} - m_D)^2 - \mathbf{q}^2$

$\mu_\pi^2 = m_\pi^2 - (m_{D^*} - m_D)^2$

$\text{lat}: \mu_\pi^2 > 0$

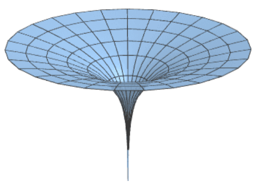
$\text{ph}: \mu_\pi^2 < 0$

on-shell: $|\boldsymbol{p}| = |\boldsymbol{p}'| = p$

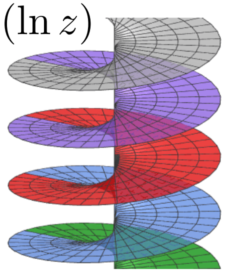
$$V_\pi^S(p,p) \propto \int V_\pi(\mathbf{q})\ d\cos\theta, \quad \mathbf{q}^2 = 2p^2(1-\cos\theta)$$

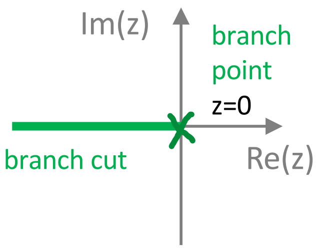
$$V_\pi^S(p,p) \propto \ln\left(1 + \frac{4p^2}{\mu_\pi^2}\right)$$

$\mathrm{Re}(\ln z)$

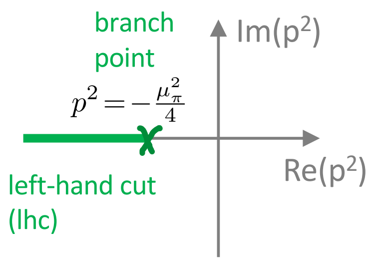


$\mathrm{Im}(\ln z)$





$\ln(z)$



$V_\pi(p^2),\ T_\pi(p^2)$

lhc slightly below DD*, BB*,... th.

T_{cc} analysis based on Effective Field Theory

$$\frac{g_c^2}{4f_\pi^2}\left(-1+\frac{\mu_\pi^2}{\boldsymbol{q}^2+\mu_\pi^2}\right)$$

attraction at short distance

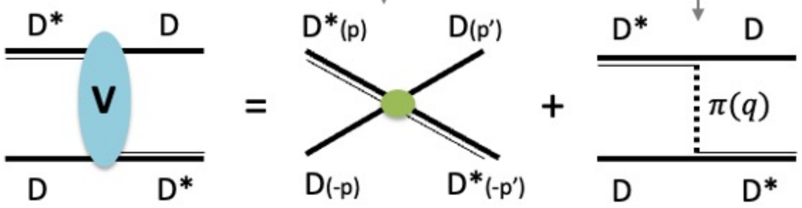
slight repulsion at long distance

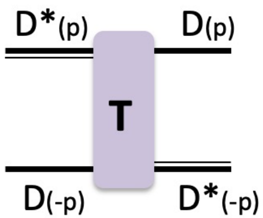
$c_{0,2}$ fitted from lat. data

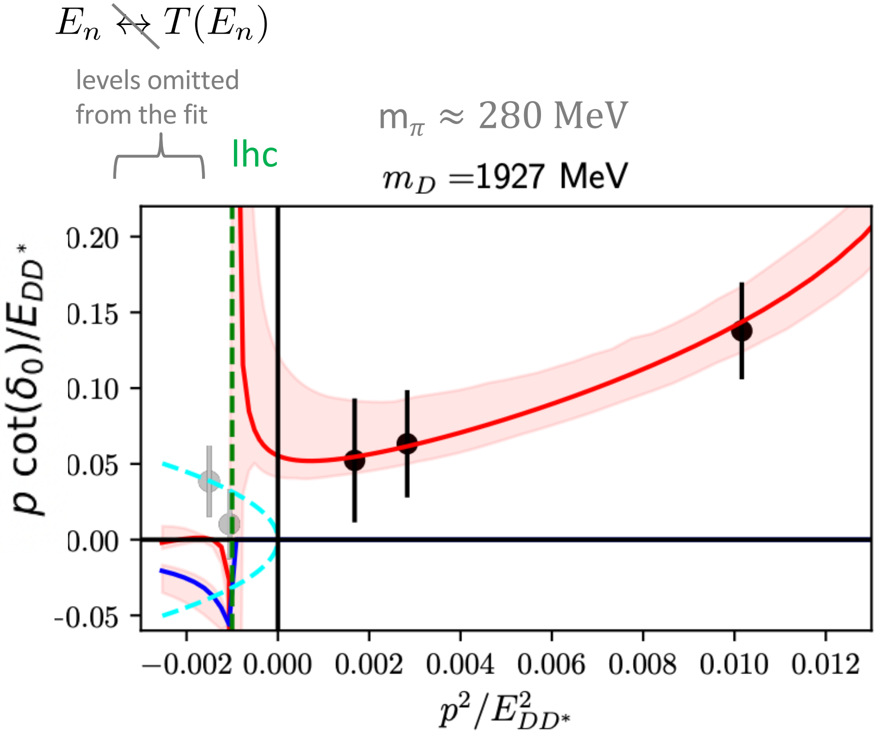
significant short-distance attraction

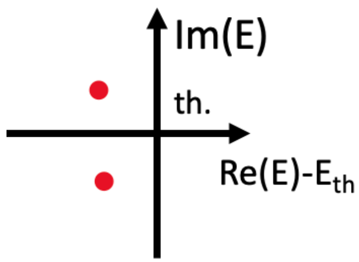
$$V_{CT} = 2c_0 + 2c_2(p^2+p'^2)$$

$$\frac{g_c}{2f_\pi}\vec{q}$$

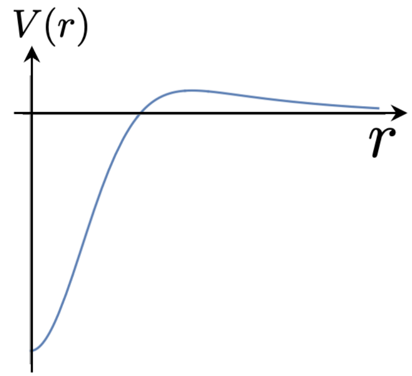








T(E): pair of resonance poles below th.



Du, Hanhart, Guo, Nefediev, Filin, et al, PRL 2023, 2303.09441

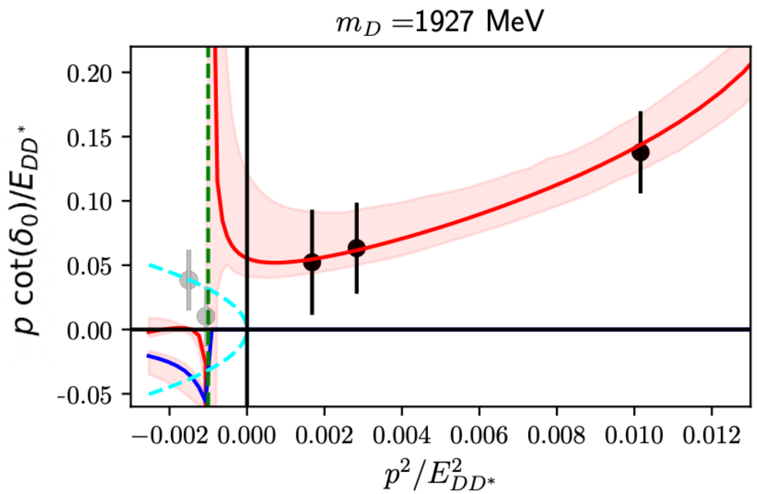
Collins, Nefediev, Padmanath, S.P.: 2401.xxxxx

T_{cc} : dependence on m_c

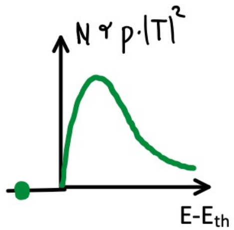
at fixed $m_\pi \approx 280$ MeV

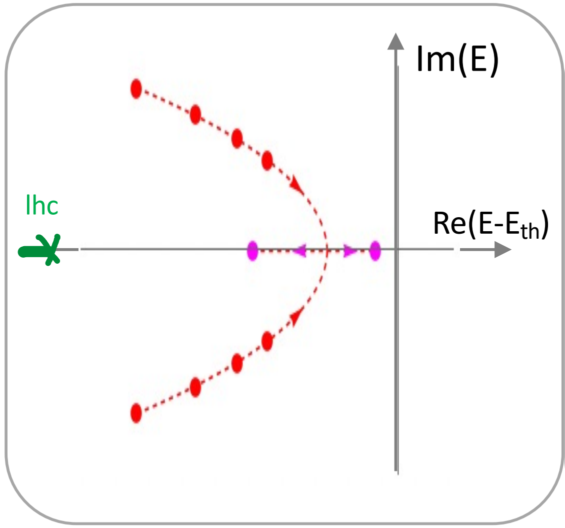
Collins, Nefediev, Padmanath, S.P.: 2401.xxxxx

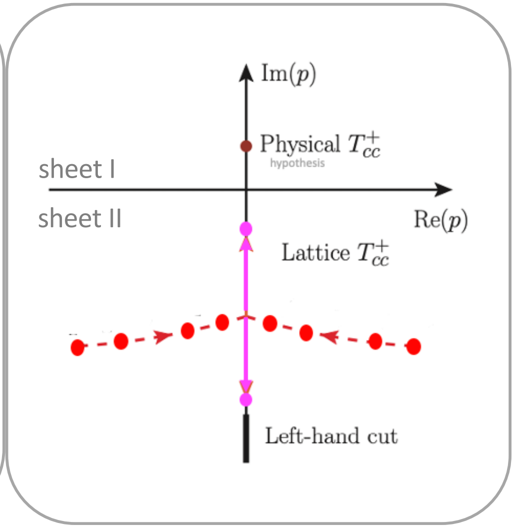
five values of m_c

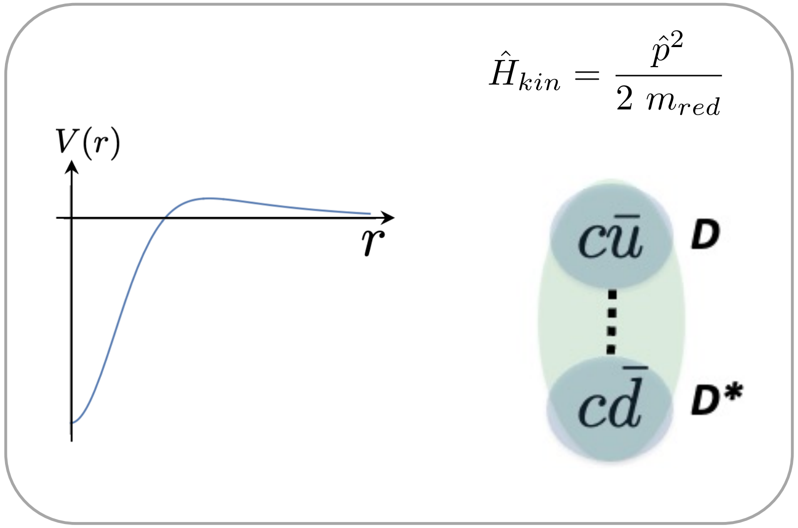


m_c









arrow: increasing m_c

.. resonance below threshold ...

Fully attractive potential in s-wave renders bound or virtual state, not a resonance.

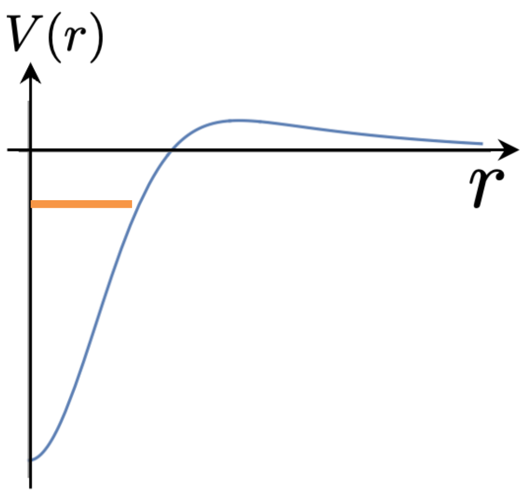
Barrier is needed for resonance above threshold.

to audience:

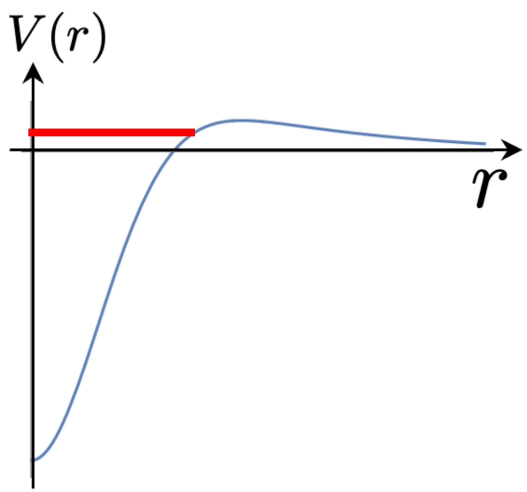
Interpretation of resonance below threshold ?

Examples in Nature? (κ and D_0* at $m_\pi > m_\pi^{ph}$)

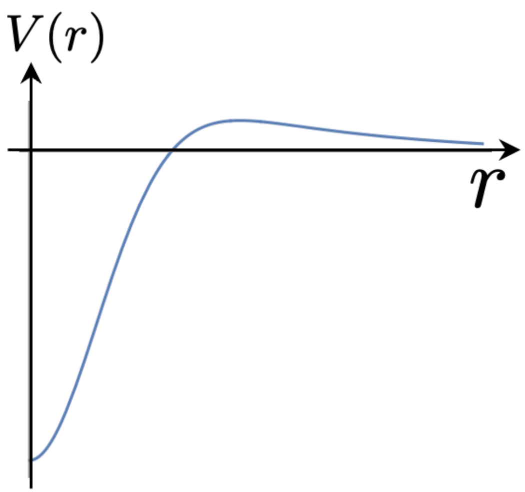
Features of V(r) that render resonance below threshold ?



bound state



resonance above th.

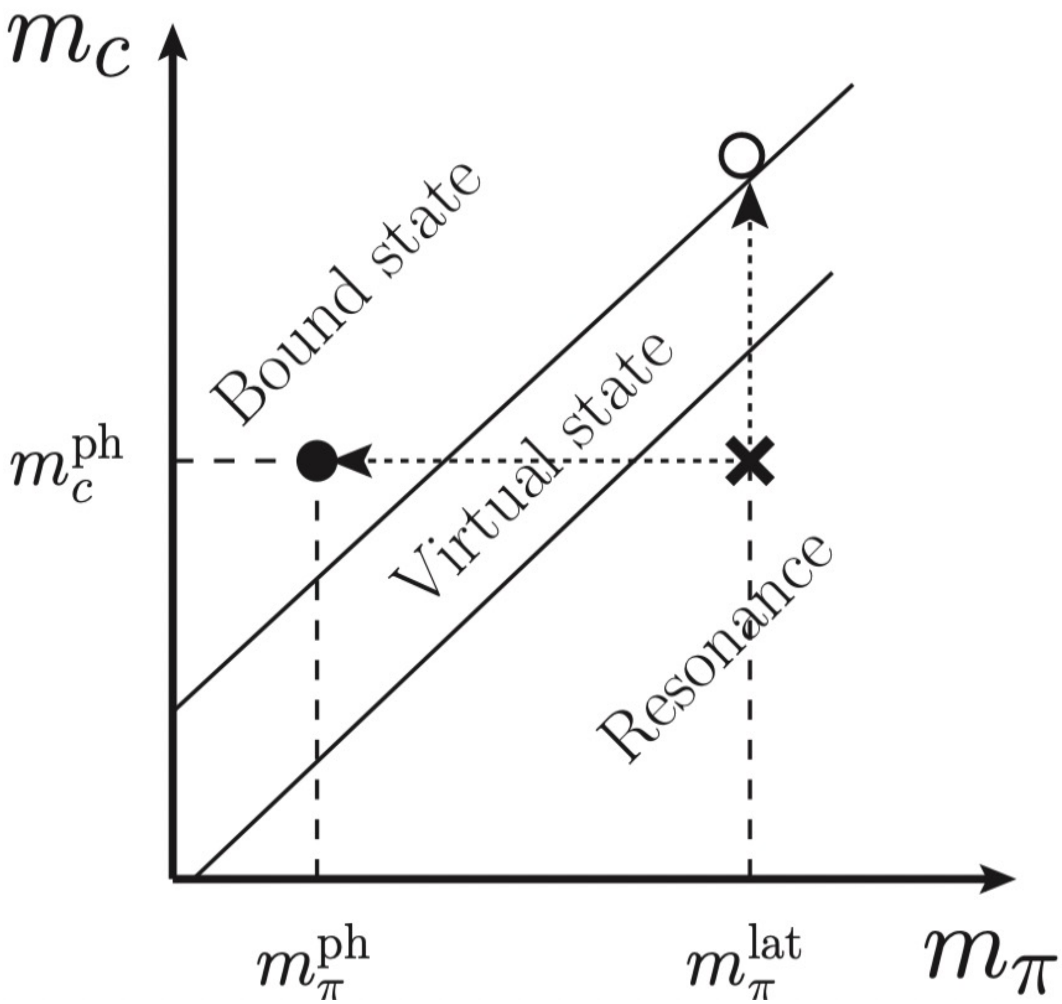


resonance below th. ?

Towards the nature of T_{cc} from lattice   based on EFT analysis

Scope of the meeting

What do we know about the structure of exotic hadrons with heavy quarks?



Collins, Nefediev, Padmanath, S.P.: 2401.xxxxx

Effect of [cc][ud] interpolators grows from c to b

distillation

DD* $O = (\bar{u}\gamma_5 c)_{\vec{p}}(\bar{d}\gamma_i c)_{-\vec{p}} - (u \leftrightarrow d) = \sum_{\vec{x}_1} \bar{u}^a_\alpha (\Gamma_1)_{\alpha\beta} e^{i\vec{p}_1\vec{x}_1} c^a_\beta \sum_{\vec{x}_2} \bar{d}^b_\delta (\Gamma_2)_{\delta\sigma} e^{i\vec{p}_2\vec{x}_2} c^b_\sigma \mp \{u \leftrightarrow d\}$ rank-2 tensors

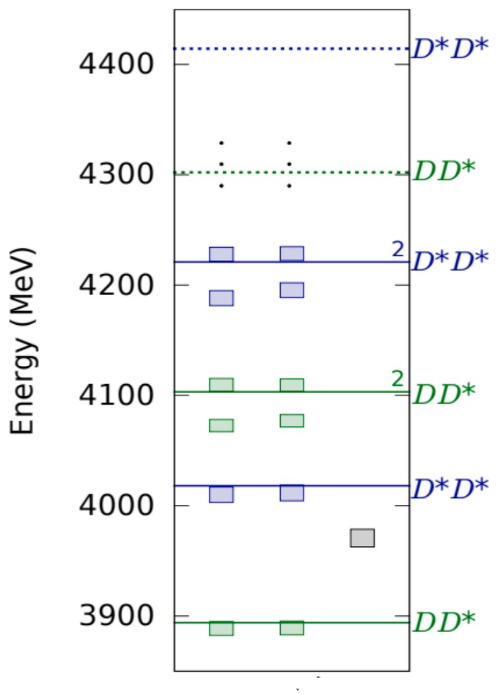
[cc][ud] $O = \sum_x [c(x) C\gamma_i c(x)][\bar{u}(x) C\gamma_5 \bar{d}(x)] e^{iPx} \, |_{P=0}$ rank-4 tensors (costly)

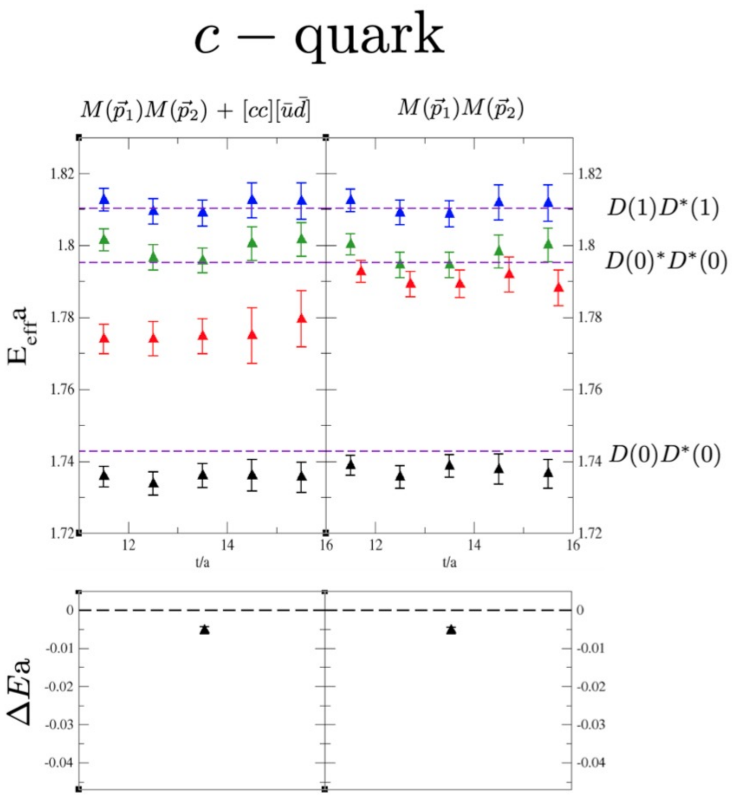
HadSpec, JHEP 11, 033 (2017), 1709.01417 $m_\pi \approx 400$ MeV

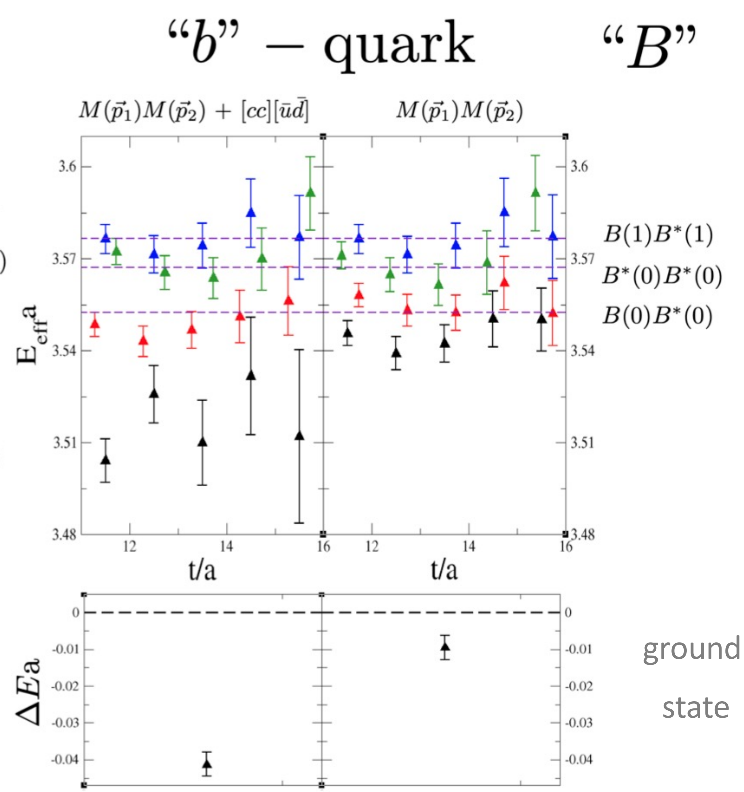
Emmanuel Ortiz Pacheco, Collins, Leskovec, Padmanath, SP. (Latt23, 2312.13441) $m_\pi \approx 280$ MeV

$m_Q \simeq m_c : m_D \simeq 1.931\,GeV \quad m_{D^*} \simeq 2.051\,GeV$

$m_Q \simeq m_{``b"} : m_{``B"} \simeq 4.042\,GeV \quad m_{``B^*"} \simeq 4.075\,GeV$



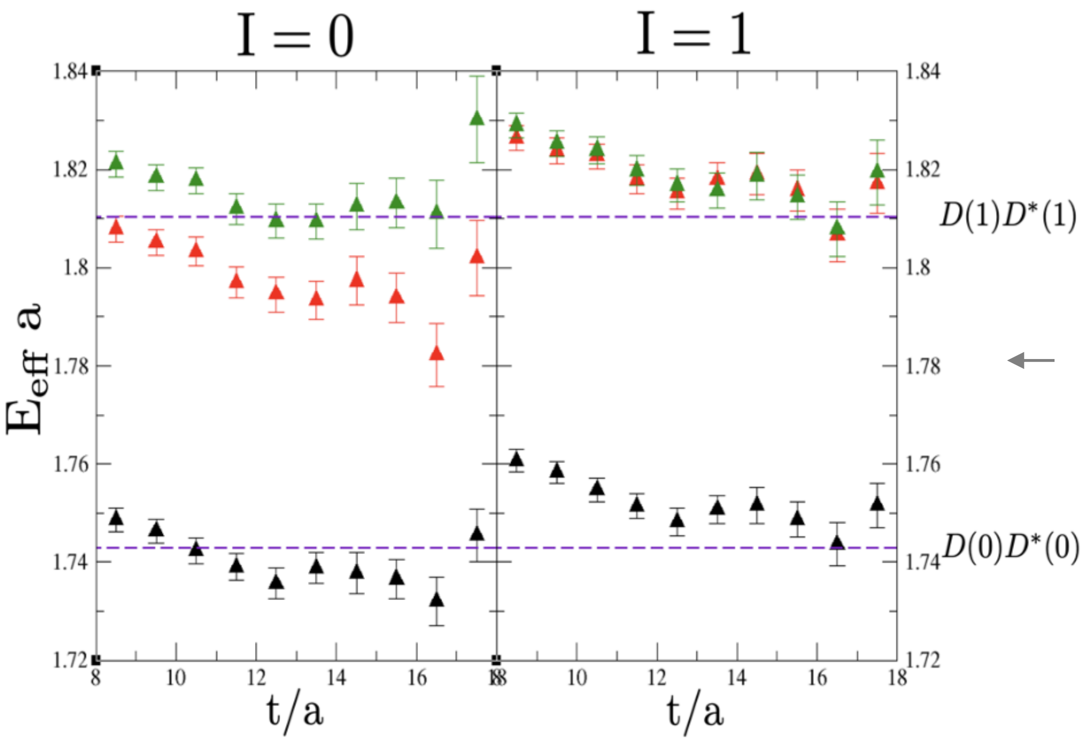




ground state

DD* scattering with $J^P=1^+$ and isospins $\mathrm{I} = 0, 1$

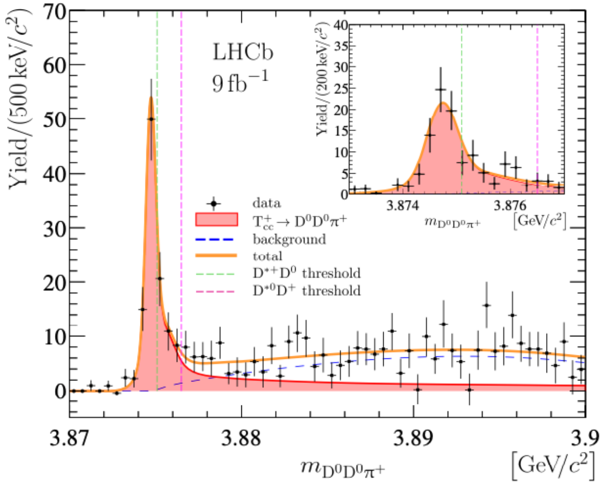
attraction repulsion

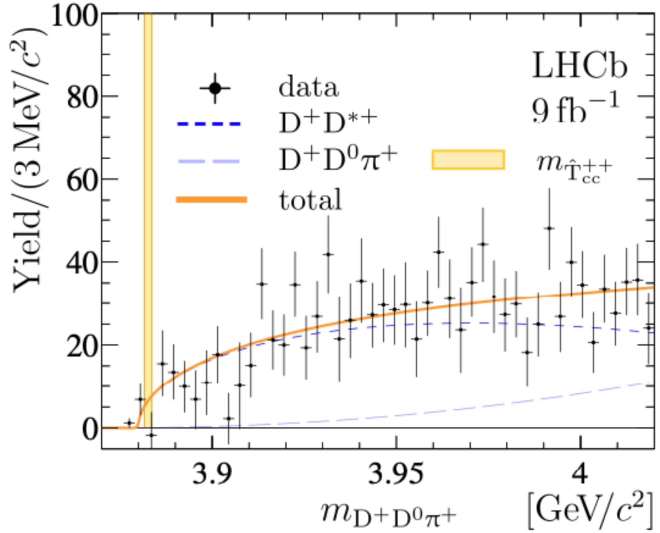


Emmanuel Ortiz Pacheco, Collins, Leskovec, Padmanath, SP (Lat23, 2312.13441) $m_\pi \approx 280$ MeV

agreement on conclusions with

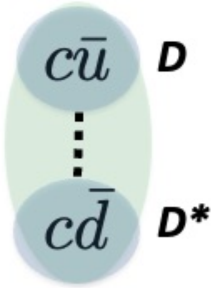
CLQCD 2206.06185, PLB,
$m_\pi \approx 348$ MeV





LHCb, Nature Commun. 13 (2022) 1, 3351

T_{cc} : Exchange of which particles drives the attraction?



$\pi, \rho, \pi\pi$?

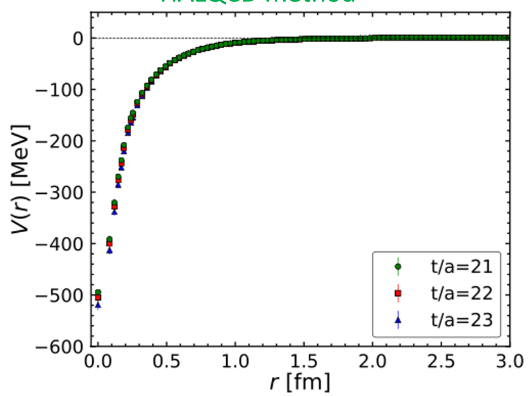
Is two-pion dominance understood in covariant chiral EFT [Li-Shen Geng et al. 2311.06569] ?

HALQCD, 2302.04505, $m_\pi \approx$146 MeV

$\pi, \rho, \pi\pi$?

$V(r) \simeq -\frac{e^{-2m_\pi r}}{r^2}$ r=1-2 fm

HALQCD method



CLQCD 2206.06185, PLB,
$m_\pi \approx$ 348 MeV

$\pi, \rho, \pi\pi$?

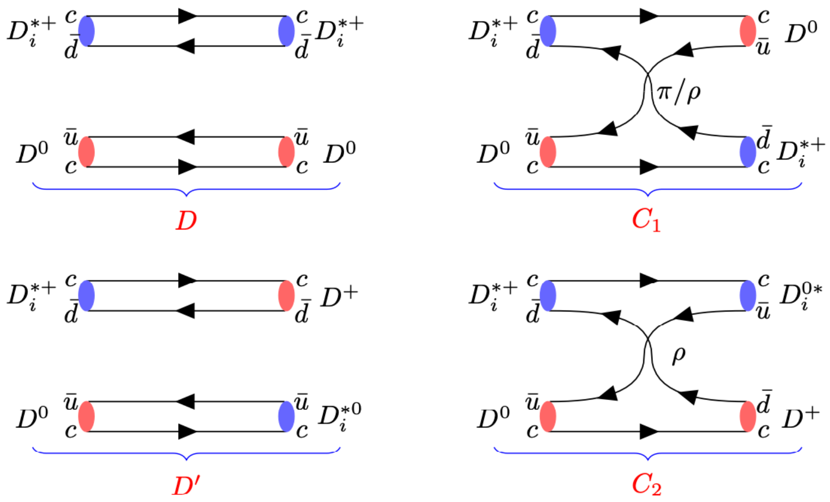
not excluded

considering contributions of various Wick contractions to I=0 and I=1 scattering (next slide)

T_{cc} : Exchange of which particles drives the attraction?

$$C_I^{DD^*}(t) = \mathcal{D} - \mathcal{C}_1 + (-)^{I+1}(\mathcal{D}' - \mathcal{C}_2).$$

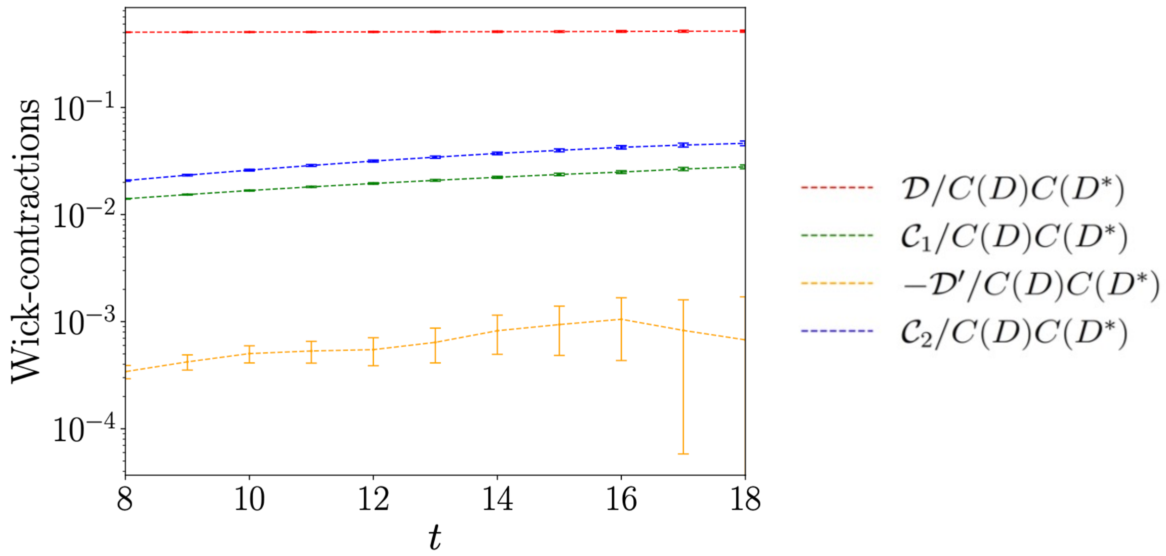
I=0 attractive, I=1: repulsive



C_2 drives attraction in I=0 channel

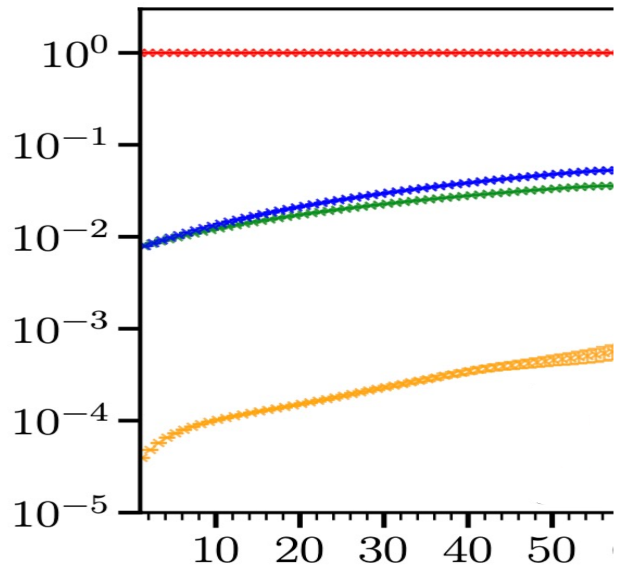
E. Pacheco, Collins, Leskovec, Padmanath, SP

(Lat23, 2312.13441) , $m_\pi \approx 280$ MeV



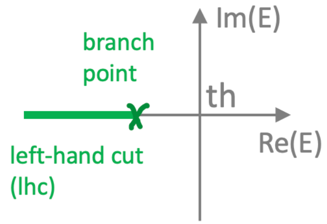
CLQCD 2206.06185, PLB,

$m_\pi \approx 348$ MeV



$$E_n \leftrightarrow T(E_n)$$

Scattering info from E_n below the left-hand cut

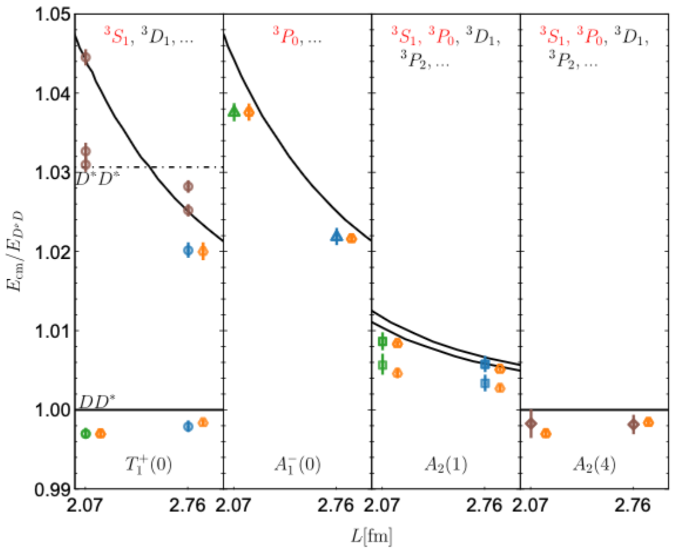


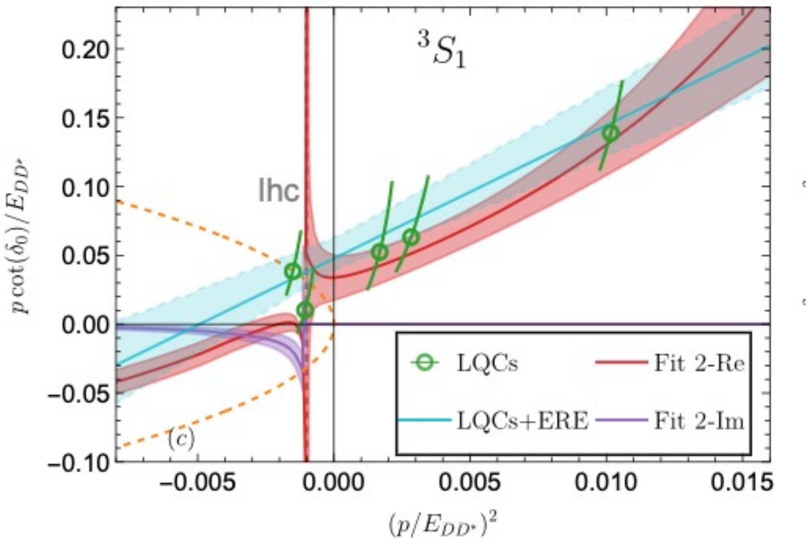
- **two-body formalism**

  - Luscher's approach with partial-wave basis -> plane-wave basis

Meng, Baru, Epelbaum et al., 2312.01930;
formalism: Meng, Epelbaum, 2108.02709;
lattice data: Padmanath, SP. 2202.10110, *PRL*

DD*





  - generalization of Luscher's formalism [Raposo, Hansen, 2311.18793]

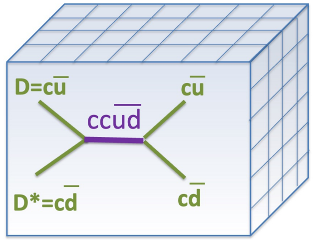
- **three-body formalism**

  - Romero-Lopez, Sharpe, Hansen, Draper, [2401.06609] this meeting, Lat23

  - Islam, Dawid, Briceno, Lattice 2023

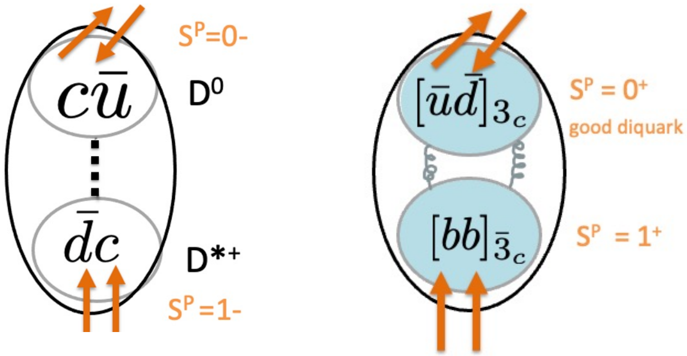
Conclusions

- Tcc is the longest-lived exotic hadron discovered in experiment



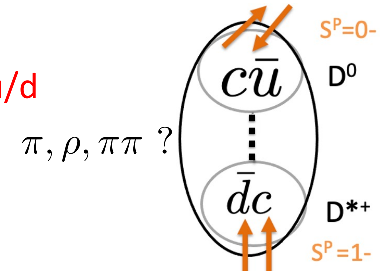
- lies near threshold -> has to be extracted from DD* scattering amplitude
- lattice studies find attraction
- attraction increases with decreasing pion mass
- attraction increases with increasing heavy quark mass

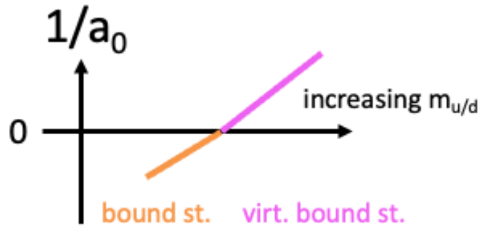
  this would (naively) imply that Tbc is more strongly bound than Tcc
- can quark-mass dependence be used to dissentangle which binding mechanism is dominant ?

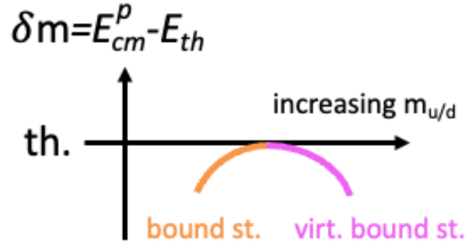


Backup

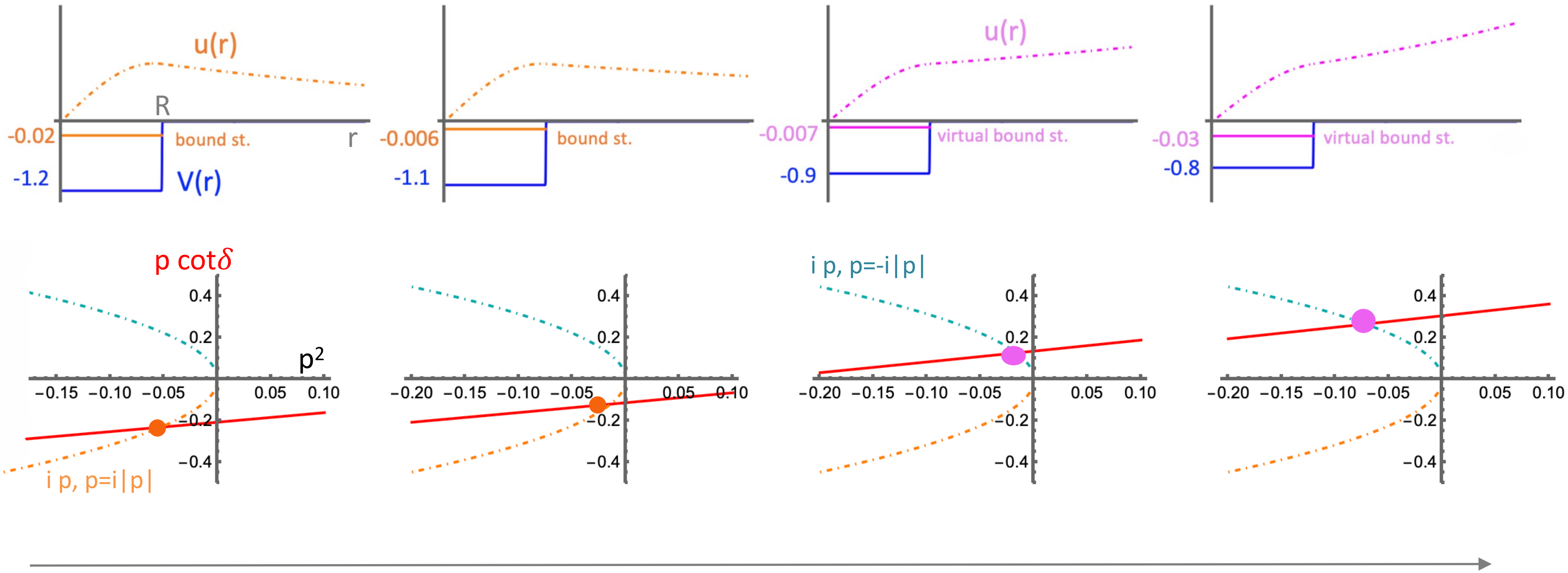
Dependence on $m_{u/d}$







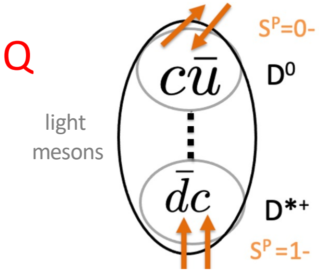
Square well potential (analogous conclusion for other fully attractive shapes)

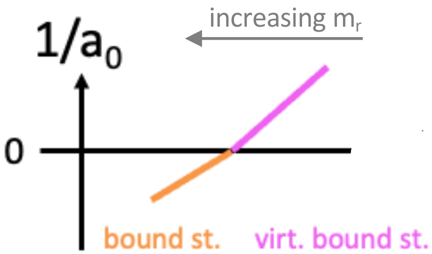


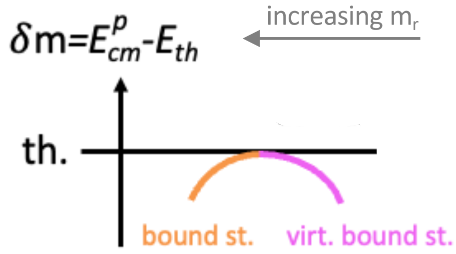
increasing $m_{u/d}$, decreasing attraction

Dependence on m_Q

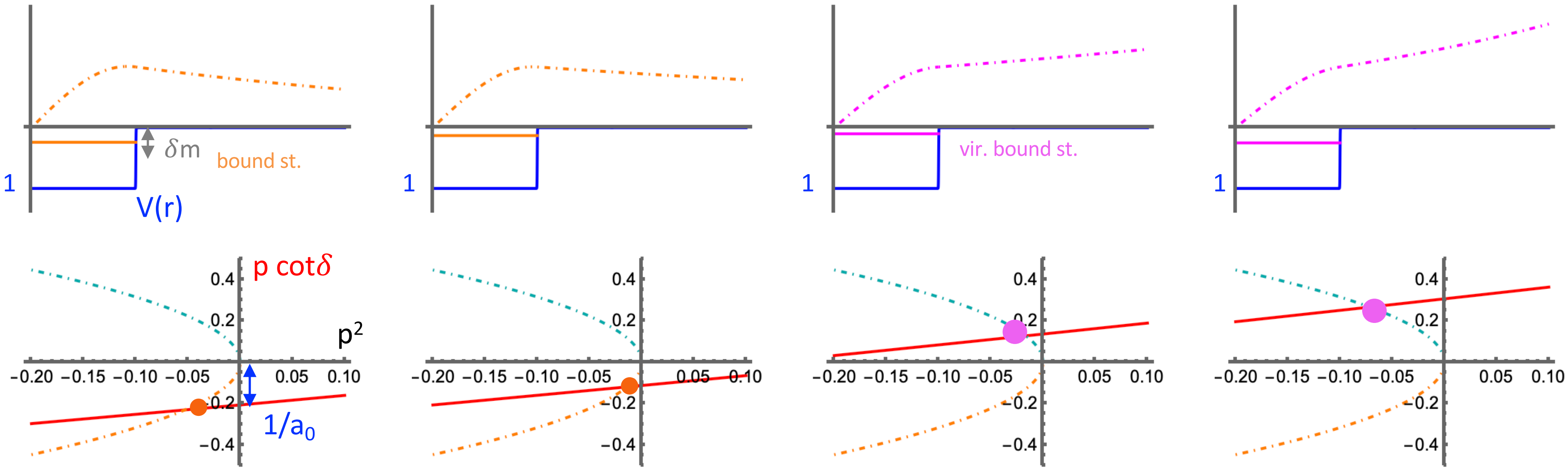
$$\hat{H}_{kin} = \frac{\hat{p}^2}{2\ m_{red}}$$







Square well potential (analogous conclusion for other fully attractive shapes)



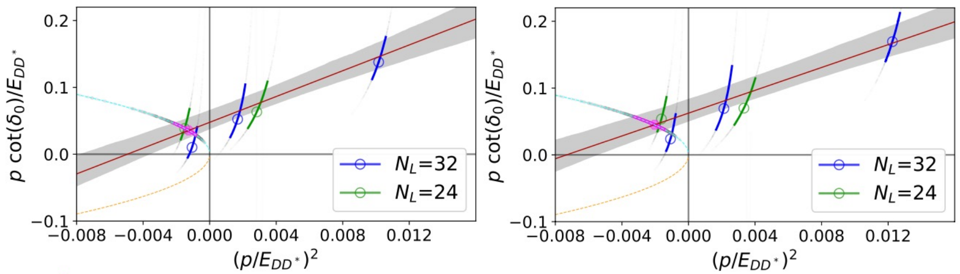
increasing m_c and m_r

Tcc: Padmanath, S.P.: 2202.10110, *PRL*

	m_D [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
$m_c^{(h)}$	1927(1)	1.04(29)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
$m_c^{(l)}$	1762(1)	0.86(0.22)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.

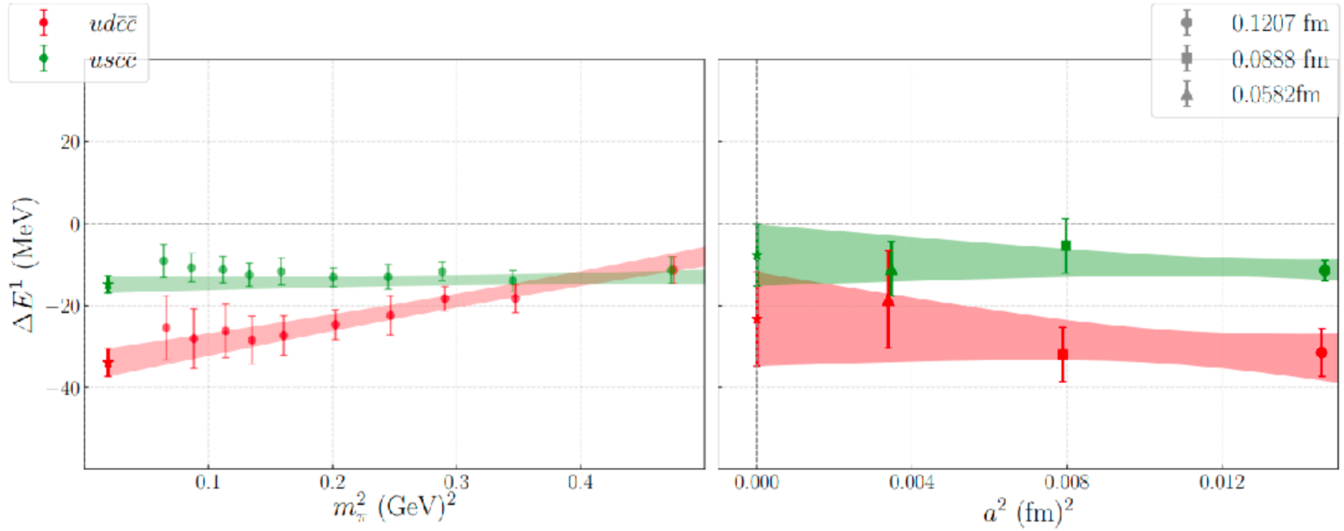
Collins, Nefediev, Padmanath, S.P.: 23xx.xxxxx:

further $m_c > m_c^{phy}$:



Previous lattice QCD study of T_{cc} channel

Junnarkar, Mathur, Padmanath, PRD 99, 034507 (2019), 1810.12285



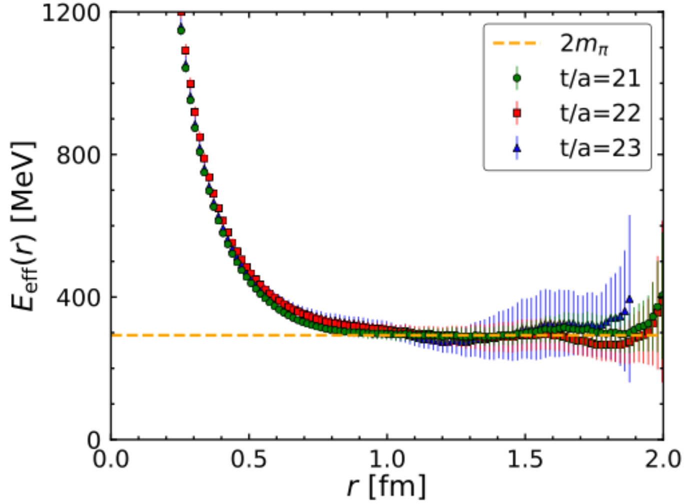
lowest finite-volume eigen-energy for P=0, $J^P=1^+$, I=0

- Study performed on LQCD ensembles with different lattice spacings. Single volume and only rest frame finite-volume irreps considered.
- Including a meson-meson and diquark-antidiquark interpolator. Diquark-antidiquark interpolators do not influence the low energy spectrum.
- The ground state energy subjected to chiral and continuum extrapolations.
- A finite-volume energy level 23(11) MeV below DD^* threshold. No rigorous scattering analysis and no pole structure determined.

HALQCD study of Tcc

Lyu, Aoki et al, 2302.04505

$$E_{\rm eff}(r) = -\frac{\ln[V(r)r^2/a_3]}{r}$$



$$R(\boldsymbol{r},t) = \sum_{\boldsymbol{x}} \langle 0|D^*(\boldsymbol{x}+\boldsymbol{r},t)D(\boldsymbol{x},t)\overline{\mathcal{J}}(0)|0\rangle \,/e^{-(m_{D^*}+m_D)t}$$

$$\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0 + O(\delta^2\partial_t^3)\right] R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t).$$

$$V(r) = R^{-1}(\boldsymbol{r},t)\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0\right] R(\boldsymbol{r},t).$$

$$V(r) \sim \frac{e^{-2m_\pi r}}{r^2} \quad r > 1 \text{ fm}$$

$$V_{\rm fit}^B(r;m_\pi) = \sum a_i e^{-(r/b_i)^2} + a_3(1-e^{-(r/b_3)^2})^n V_\pi^n$$

parameter set, $(a_1,a_2) = (-284(36), -201(60))$ in MeV, $a_3 = -45(12)$ MeV $\cdot$ fm^2, and $(b_1,b_2,b_3) = (0.15(2), 0.32(12), 0.49(24))$ in fm. Also, we find that