RESONANT MULTI-HADRON SYSTEMS IN FINITE/INFINITE VOLUME

NSF PHY-2012289

DOE DE-SC0016582

DOE DE-SC0016583

DFG CRC 110

MAXIM MAI

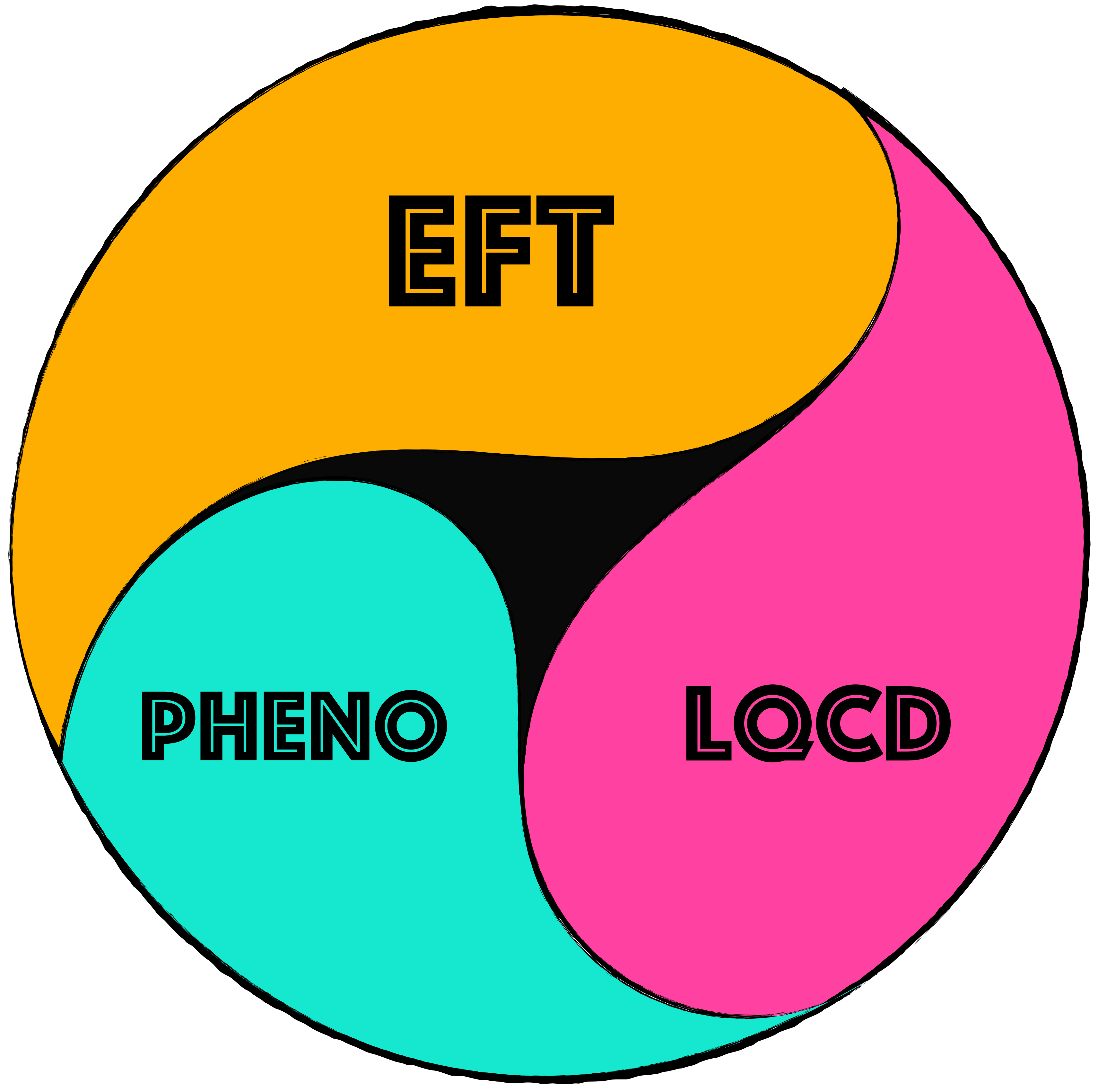
Hirschegg meetings 2024

Heisenberg-Programm

DFG Deutsche Forschungsgemeinschaft

OUTLINE

- **Introduction**
- **Two hadrons**
  - Chiral Unitary approach
  - N*(1535), N*(1650), Lambda 1405
- **Three hadrons**
  - Resonant systems in infinite volume
  - Resonant systems in finite volume
  - Theoretical extensions / studies
- **Summary and outlook**



HADRON SPECTRUM

Mostly excited states

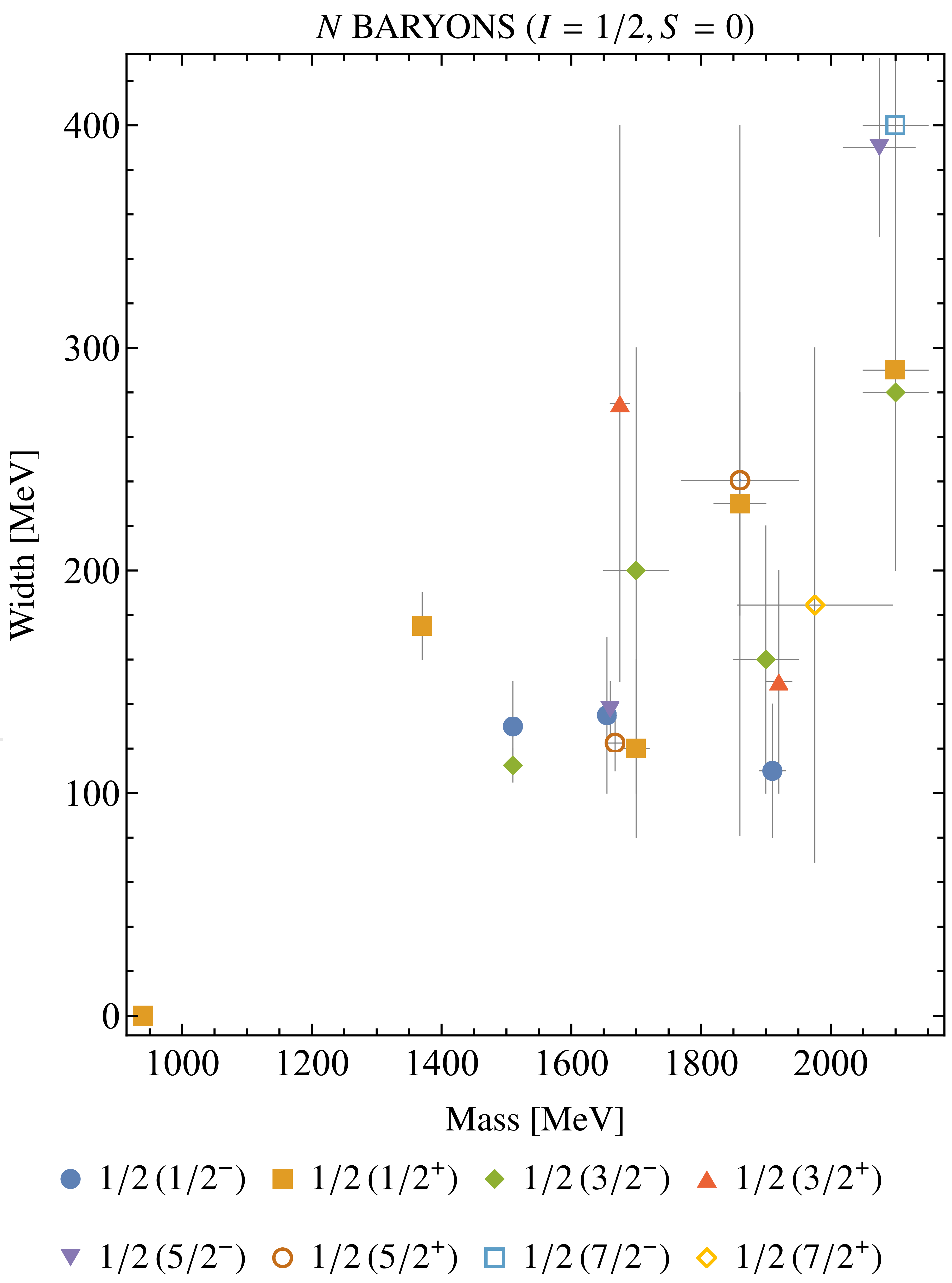
≈100 mesons & ≈50 baryons (****)

Key questions

🦅 ***"what is the pattern of these states?"***

🐸 ***"how are they formed?"***

- *Quark models*
- *Functional methods* [1]
- *Dynamical coupled-channel models*
- *Chiral EFT*
- *Lattice QCD* [2]



[1] Joshua Hoffer talk
[2] Sinead Ryan, Marc Wagner, Sasa Prelovsek, …

BIRD'S VIEW

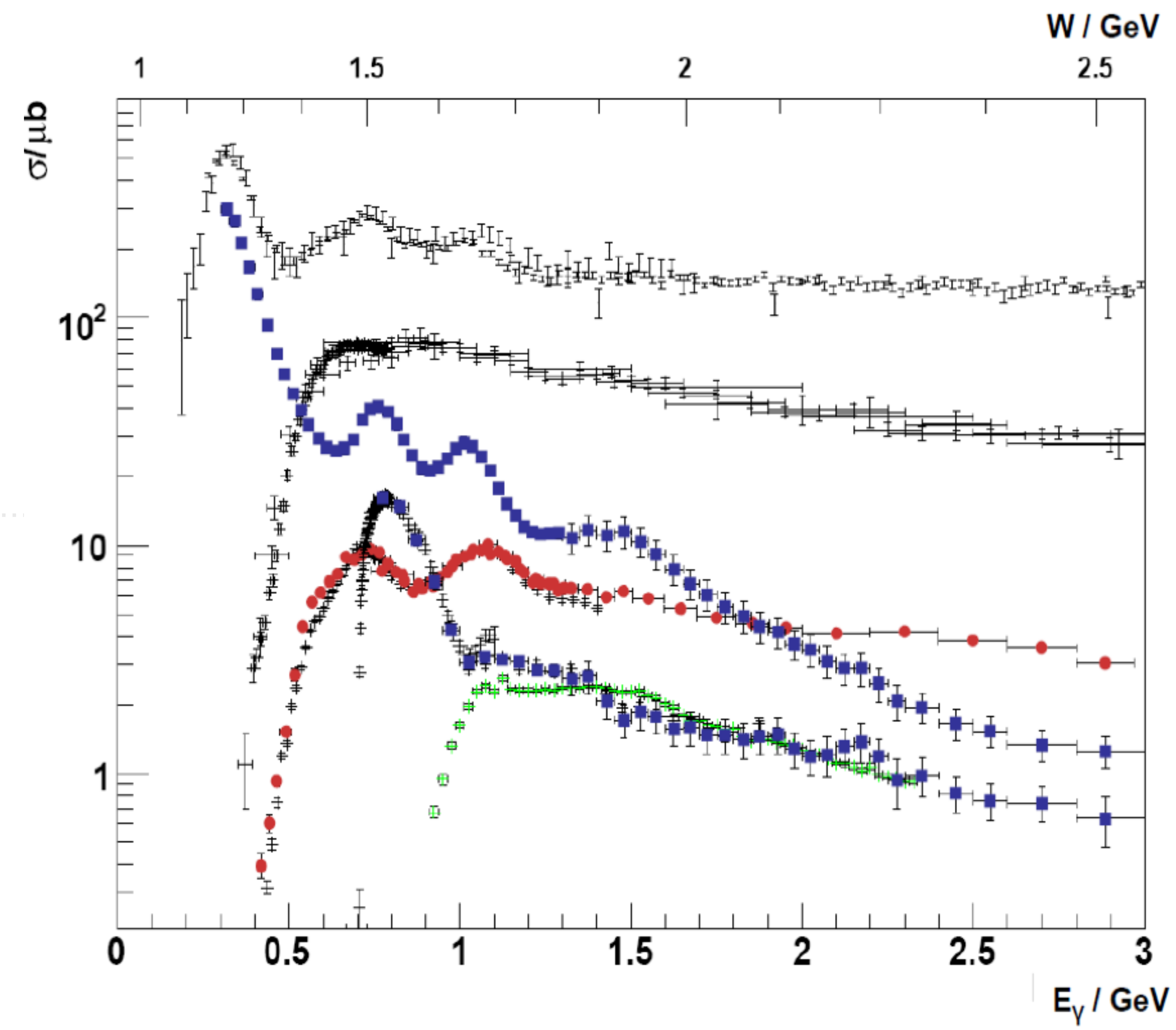
Physical input

- many experimental data & ongoing experiments[1] on

  $\gamma p \to \pi N, \pi\pi N, K\Lambda, ..$

Theory

- global analysis through dynamical models[2]
  - SAID/MAID/…
  - Jülich-Bonn-Washington[3] jbw.phys.gwu.edu/
    - ➡ gauge invariance/final-state unitarity/…
    - ➡ global description of the spectrum
    - ➡ new insights into formation of individual states



[1] MAMI/ELSA/JLAB/...
[2] Tiator/Sato/Workman/Arndt/Aznauryan/....
[3] [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201; Eur.Phys.J.A 59 (2023) 12

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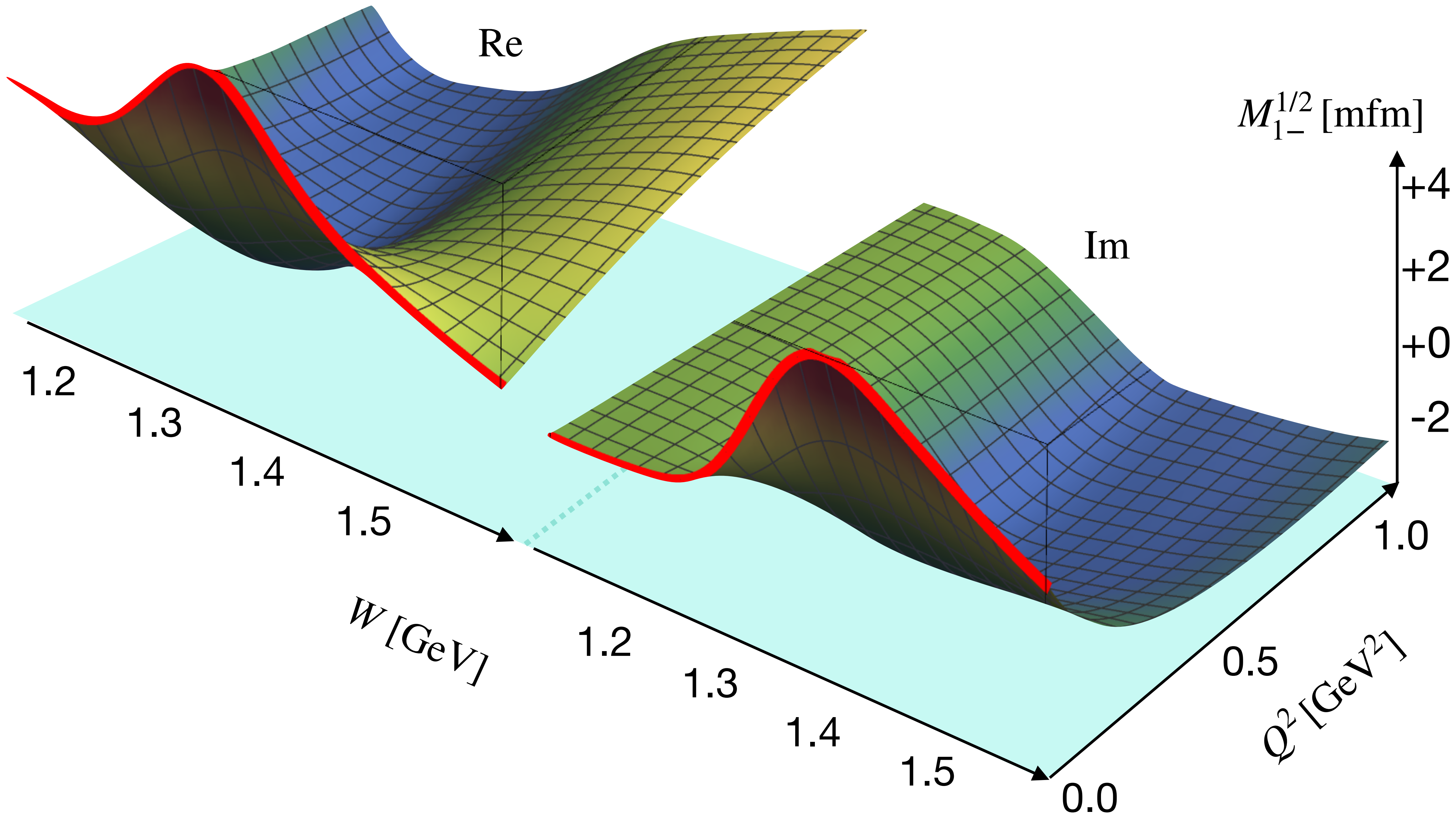
$N^*(1440)$ Roper [3]

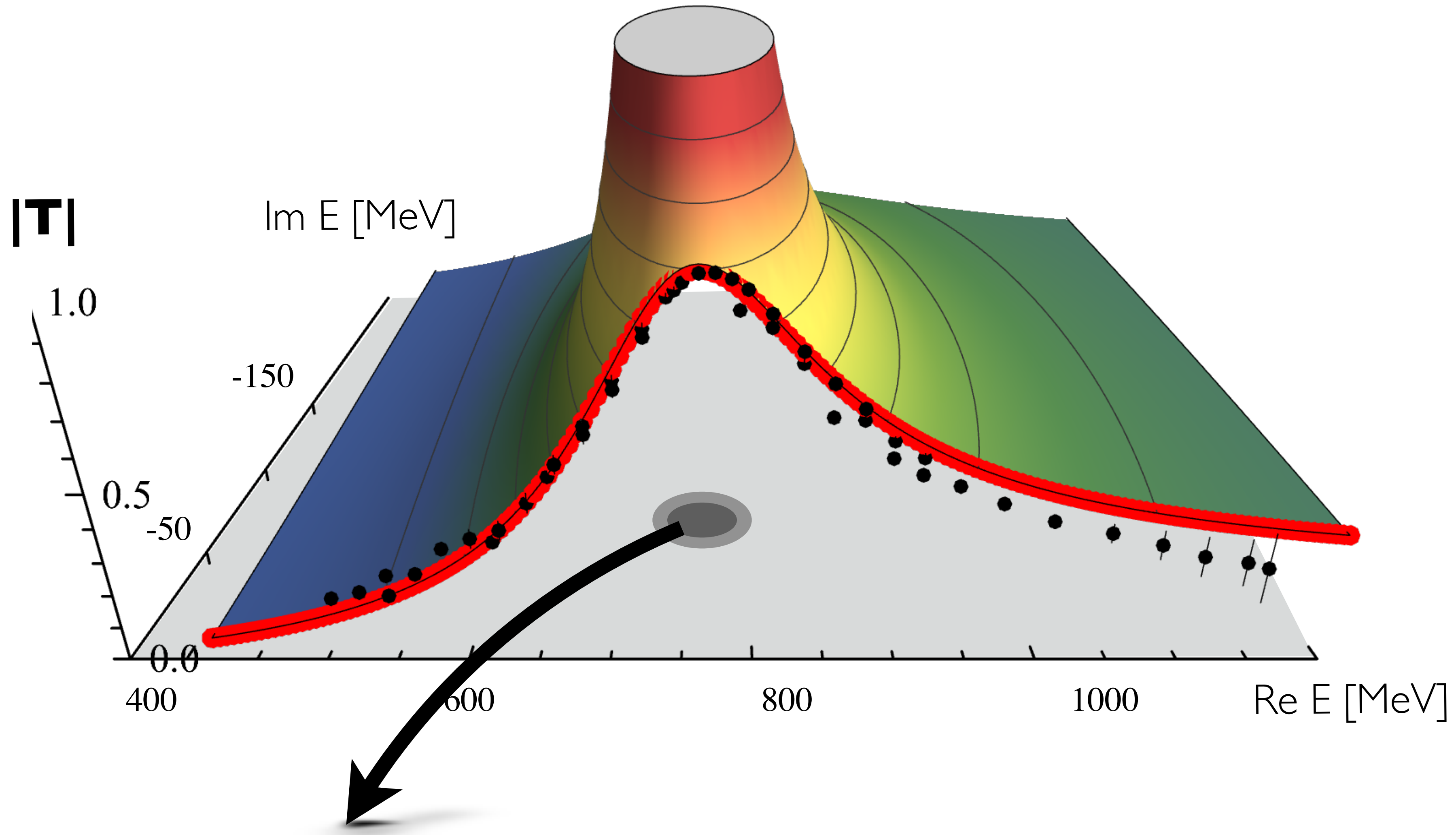


[1] MAMI/ELSA/JLAB/...
[2] Tiator/Sato/Workman/Arndt/Aznauryan/....
[3] [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201; Eur.Phys.J.A 59 (2023) 12

FROG'S VIEW

Universal parameters of resonances[1]

- pole positions on unphysical Riemann Sheets
- central quantity: ***transition amplitudes***
  - Constraints from S-matrix theory
    - Unitarity, Analyticity, Crossing symmetry, …
  - Constrains
    - Experiment
    - CHPT
    - Lattice QCD



$M^* = (750 - i60)$ MeV

Universal property of the ρ – meson

[1] **Review**: MM et al. ``Towards a theory of hadron resonances'' Phys. Rept. 1001 (2023)
[2] Figure Data: Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

FROG'S VIEW

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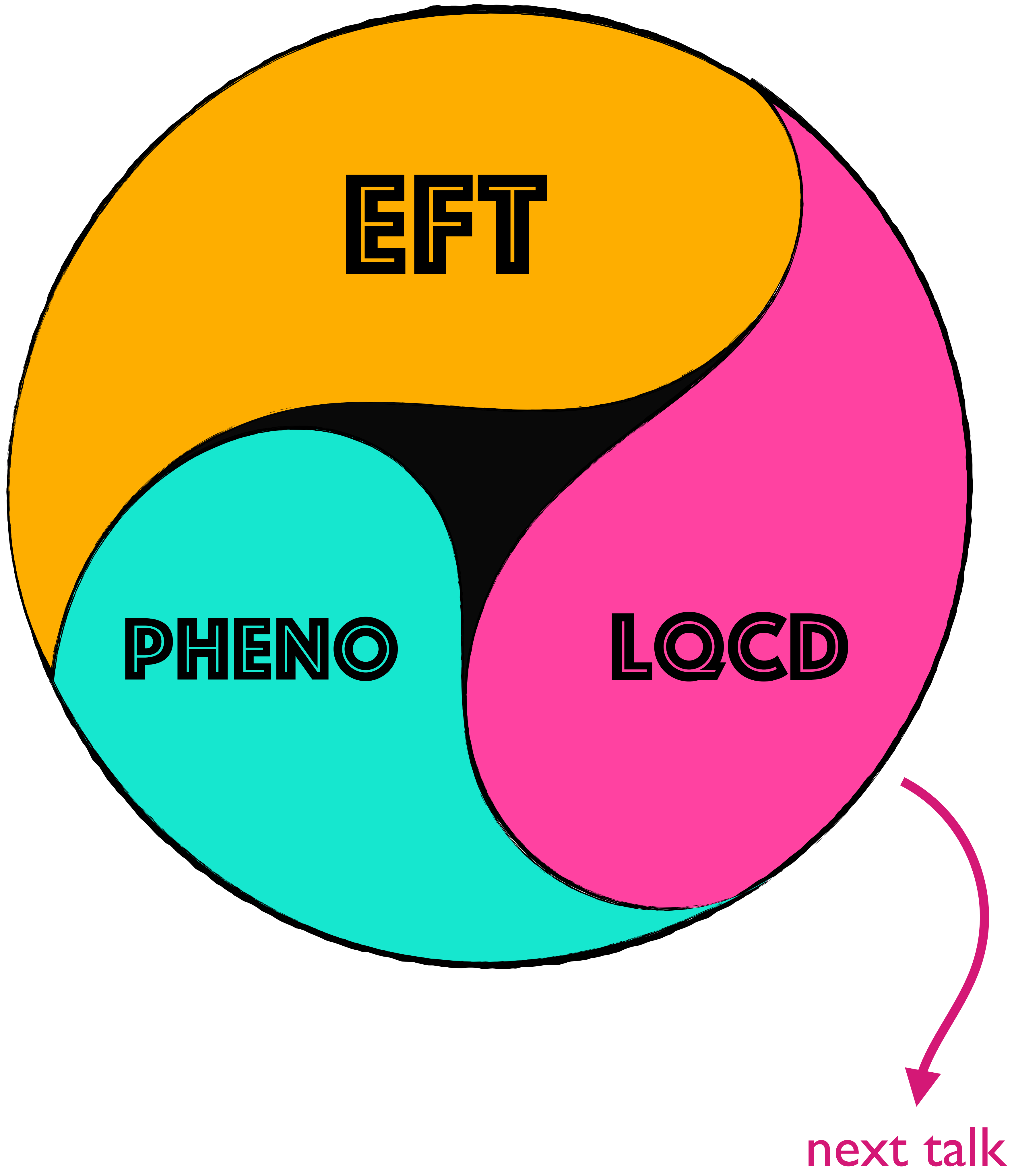
Tridge (Midland, MI/USA)

[1] **Review**: MM et al. ``Towards a theory of hadron resonances'' Phys. Rept. 1001 (2023)
[2] Figure Data: Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

TWO HADRONS

MESON-BARYON RESONANCES

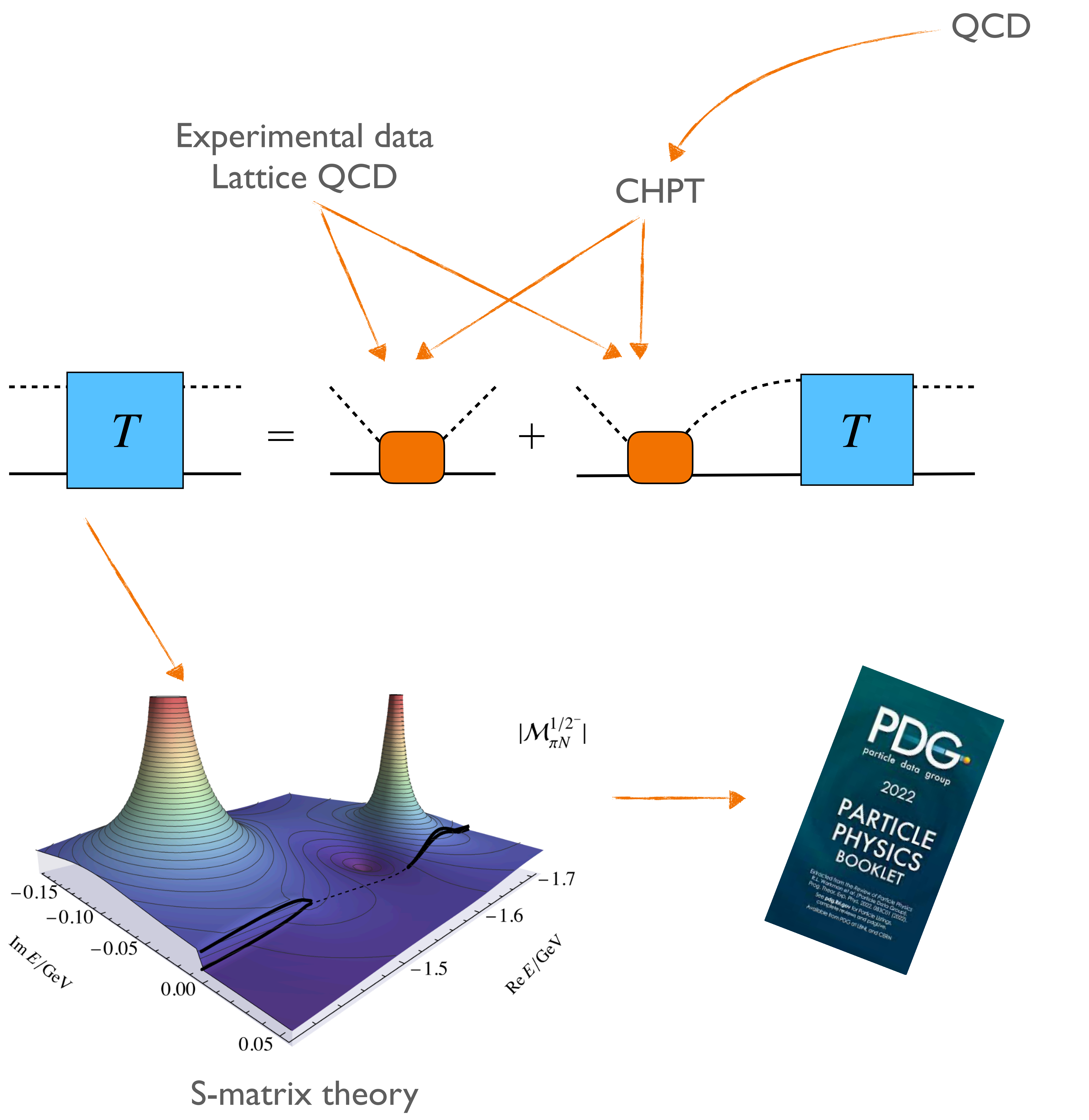
CHPT/UNITARITY/PHENOMENOLOGY



TRANSITION AMPLITUDE

Chiral unitary approach[1]

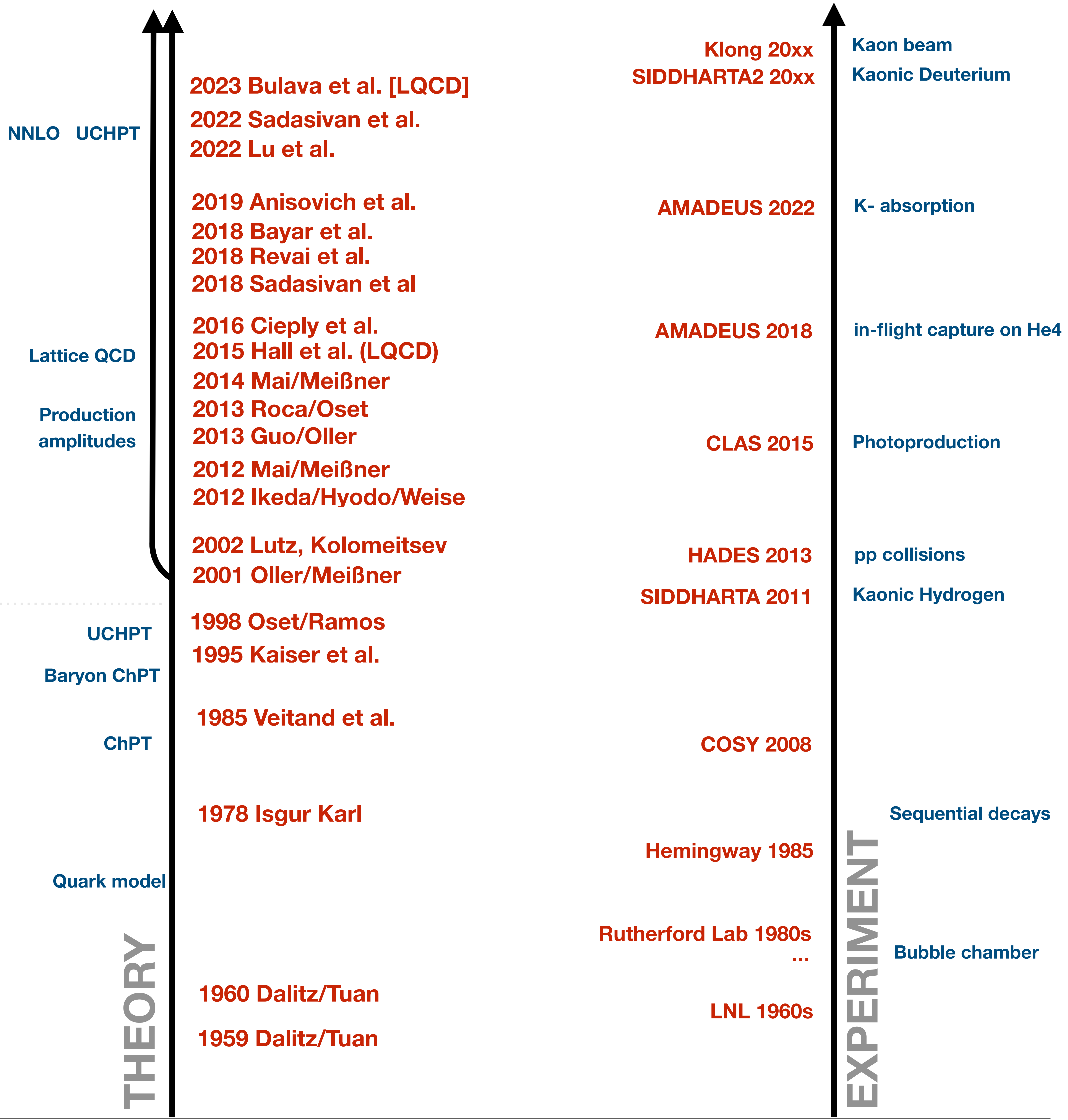
- Chiral Perturbation Theory (#QCD#EFT) dictates the form of the interaction at low energies
- Unitary scattering amplitude from the Bethe-Salpeter equation
  - Fit free parameters to experimental data / LQCD
  - Record complex pole positions
  - Many states can be explained[2]: $N^*(1535)$, $N^*(1650)$, …



[1] Weise/Kaiser/Meißner/Lutz/Oset/Oller/Ramos/Hyodo/Borasoy…
[2] Kaiser/Siegel/Weise Phys.Lett.B 362 (1995) Lutz/Soyeur Nucl.Phys.A 773 (2006); MM et al. Phys.Lett.B 697 (2011); …

THE ENIGMA OF THE $\Lambda(1405)$

- Long history of experimental and theoretical efforts[1]
- Second state predicted from UCHPT $\mathbf{\Lambda}(1380)$
  - no direct experimental verification
  - confirmed by many critical tests

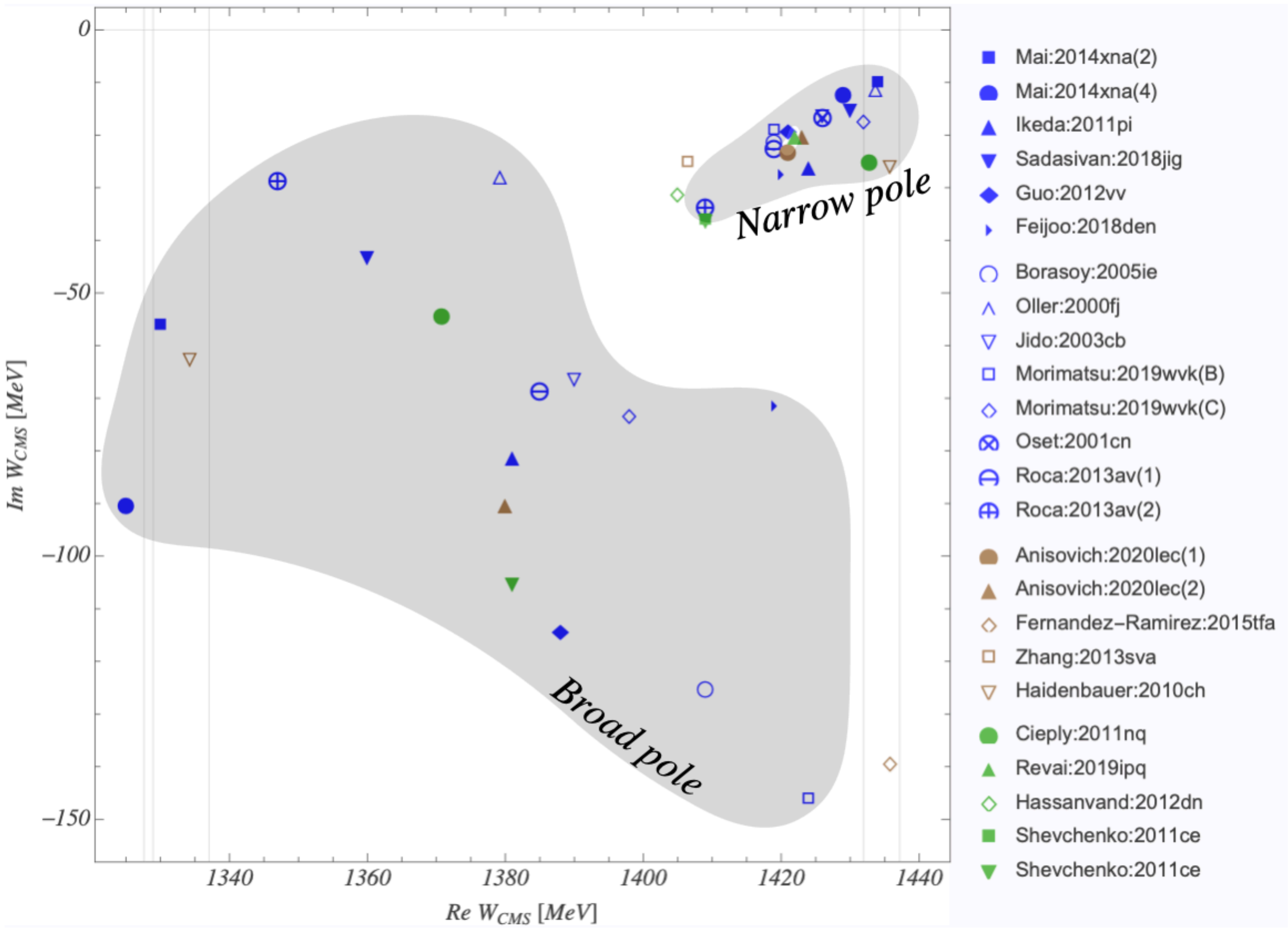


[1] MM EPJST 230 (2021) "Review of the $\Lambda(1405)$ A curious case of a strangeness resonance";

THE ENIGMA OF THE $\Lambda(1405)$

…

- Theory frontier: NNLO UCHPT determination[1]
- Consistently two poles, but the second pole is less well known
  - second pole below KbarN threshold
  - line-shape only through $\gamma p \to K\pi\Sigma$ [2]

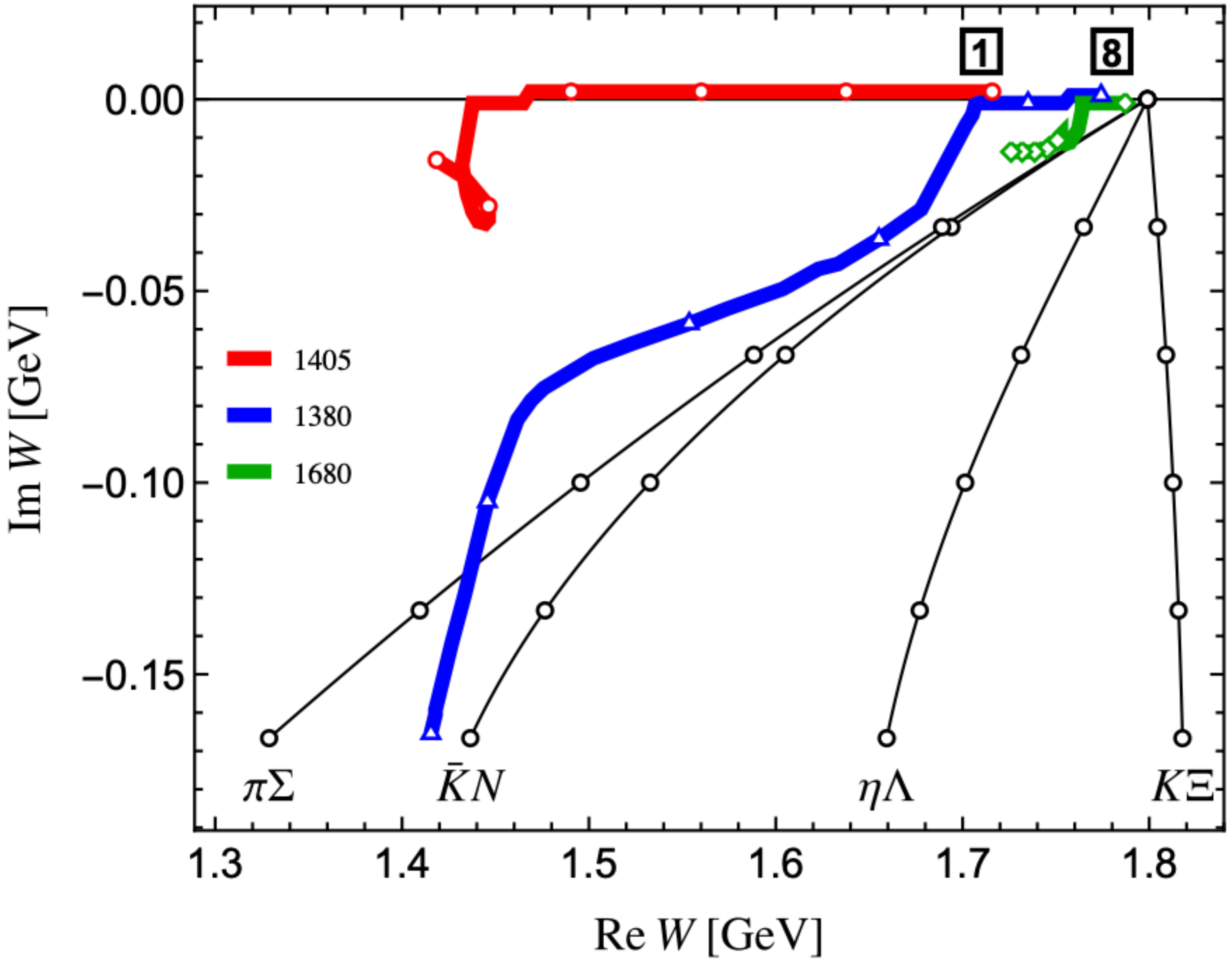


[1] Lu/Geng/Döring/MM Phys.Rev.Lett. 130 (2023)
[2] [CLAS] Moriya et al (2013)

UNPHYSICAL QUARK MASSES

CHPT is predestined to scan quark mass dependence

- SU(3) limit provides a simpler pole distribution
  - 1 singlet + 2 octet poles
  - LO/NLO "tracks" can differ[2]

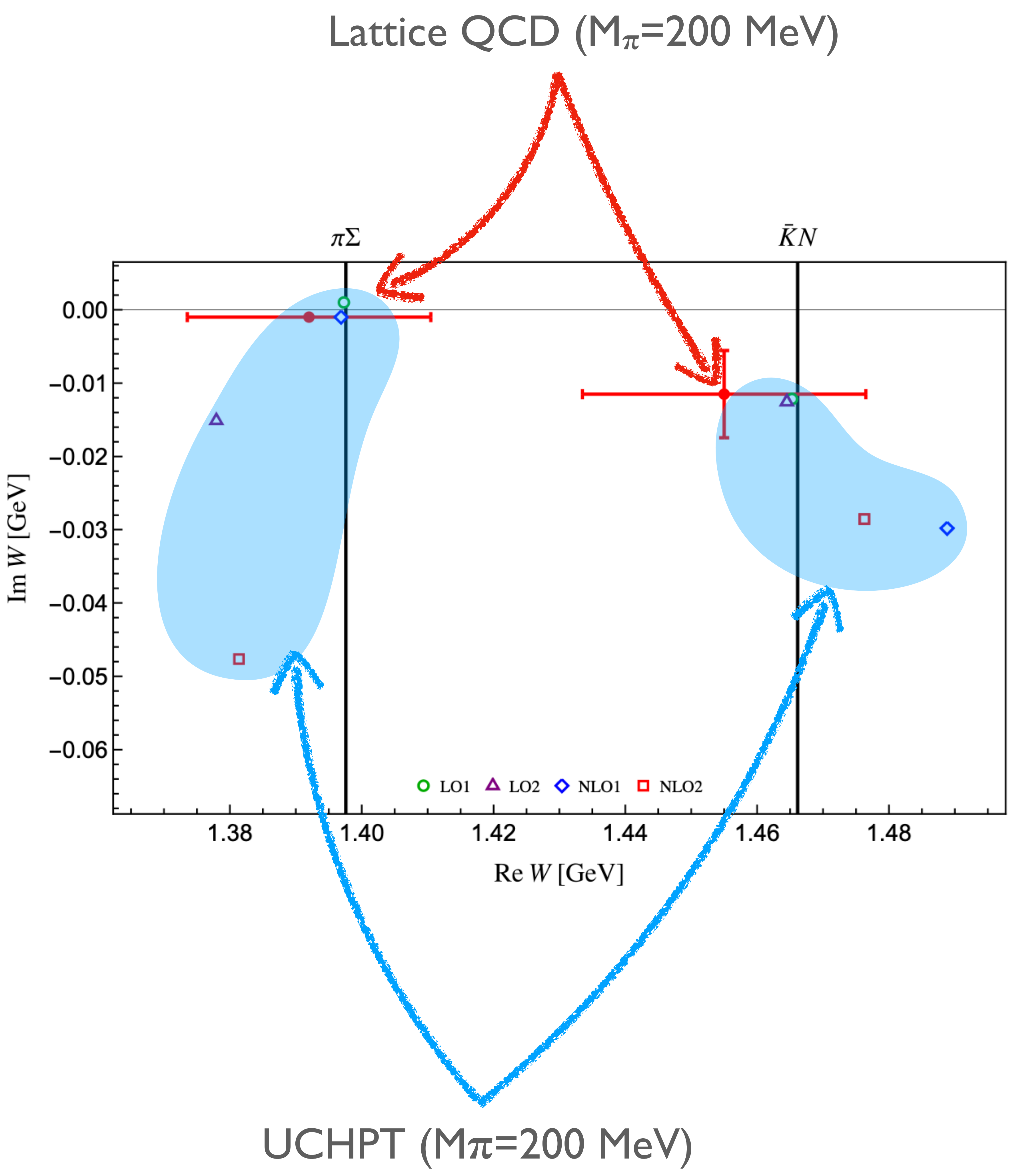


[1] Jido et al. Nucl.Phys.A 725 (2003); Bruns/Cieply, SU(3) Nucl. Phys. A 1019 (2022);
[2] Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)
[3] [BaSc] Bulava et al. 2307.10413; 2307.13471 — **NEXT TALK**

UNPHYSICAL QUARK MASSES

CHPT is predestined to scan quark mass dependence

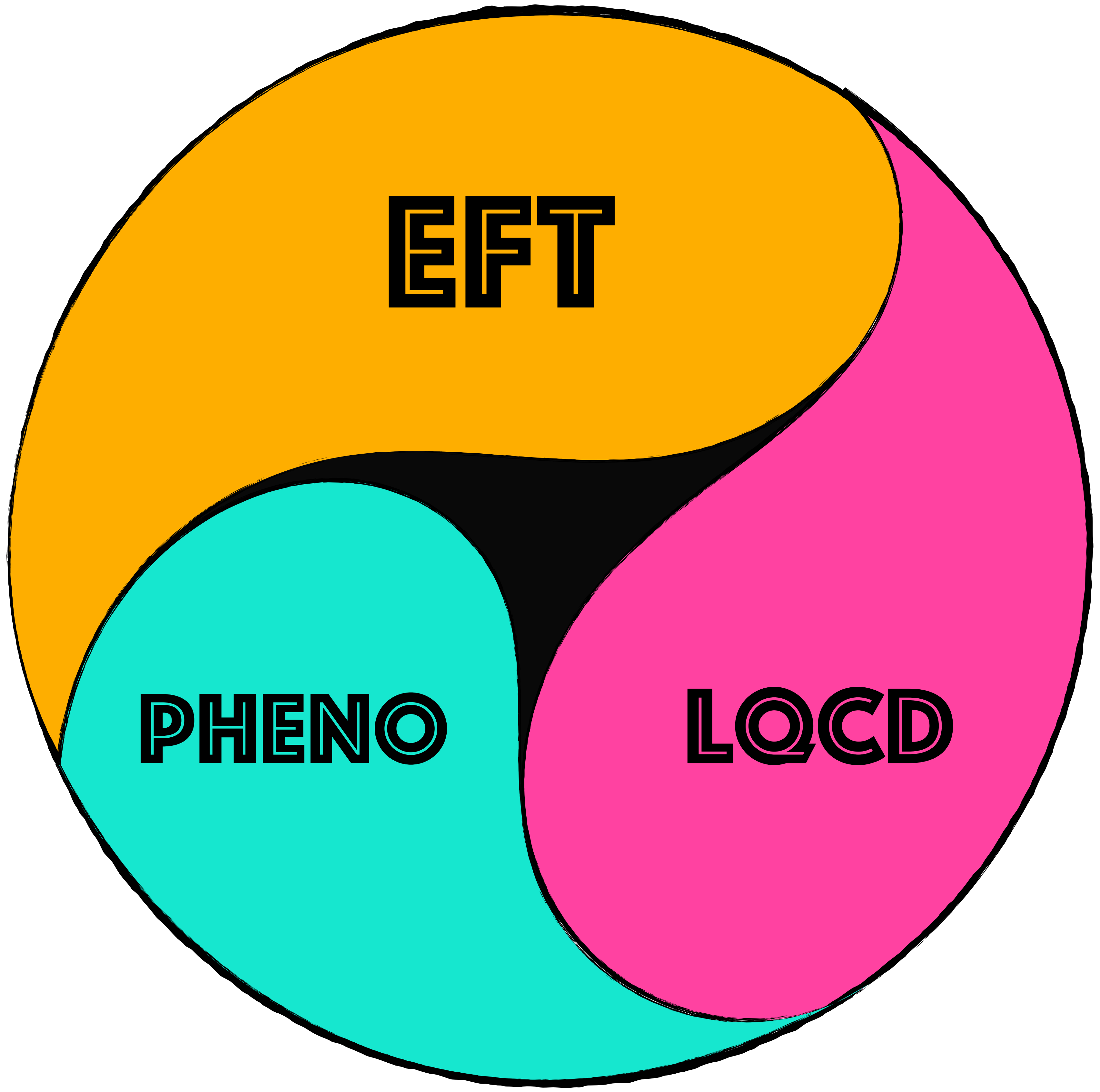
- Other trajectory >> BaSc setup[3]
  - Mpi = 200 MeV
  - **Outlook**: add LQCD results as constraints on UCHPT directly… [in progress]



[1] Jido et al. Nucl.Phys.A 725 (2003); Bruns/Cieply, SU(3) Nucl. Phys. A 1019 (2022);
[2] Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)
[3] [BaSc] Bulava et al. 2307.10413; 2307.13471 — **NEXT TALK**

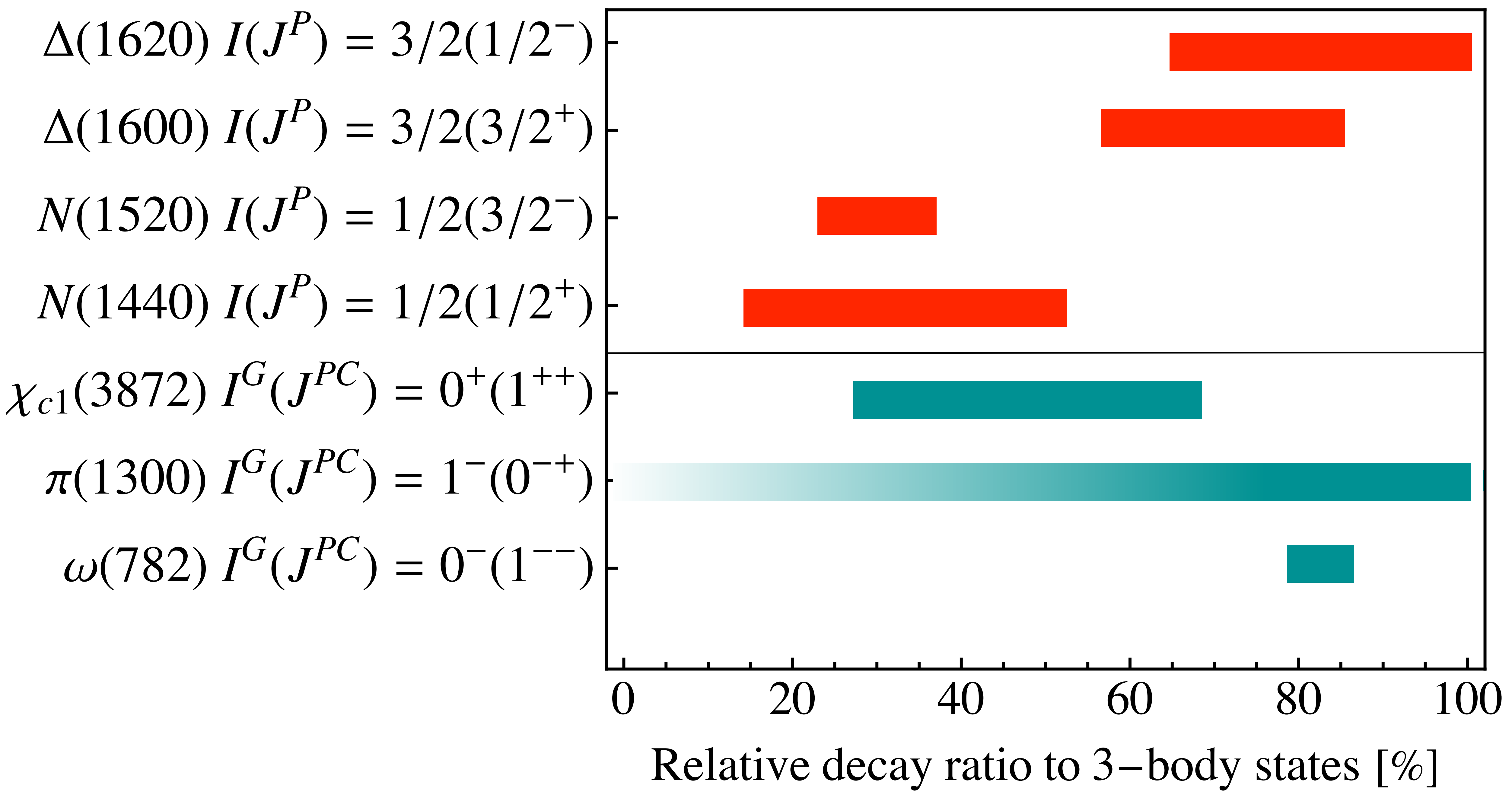
THREE HADRONS

THEORY AND APPLICATIONS



HADRONIC 3-BODY PROBLEM

- Many known states have large 3-body content
  - Roper(1440)
  - X(3872)
  - a_1(1260)/…
- Beyond Standard Model searches (τ-EDM/…)
- Exotic states of matter[1]

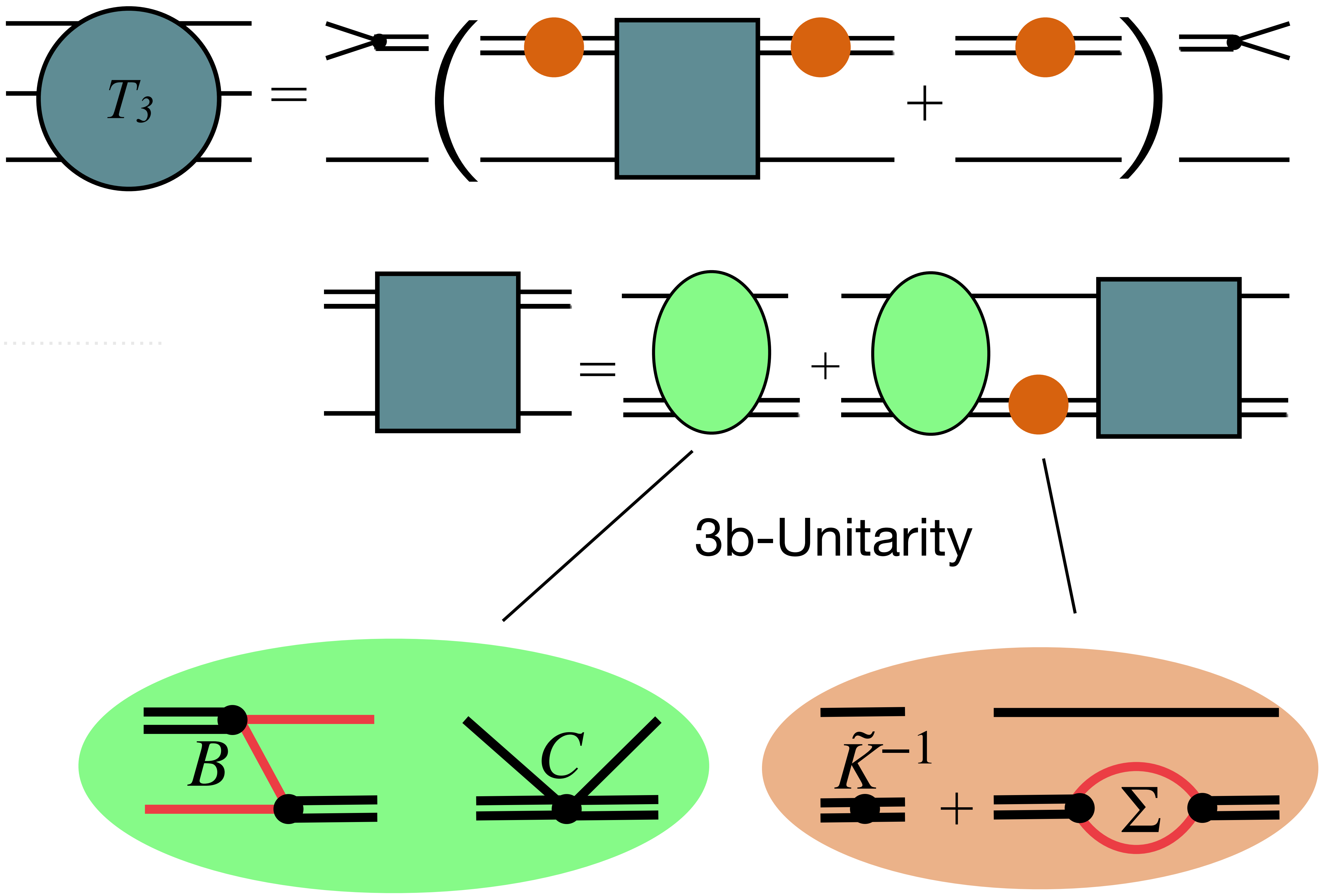


[1] Experimental programs: GlueX@JLAB; COMPASS@CERN;
[2] Figure data Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

TRANSITION AMPLITUDE

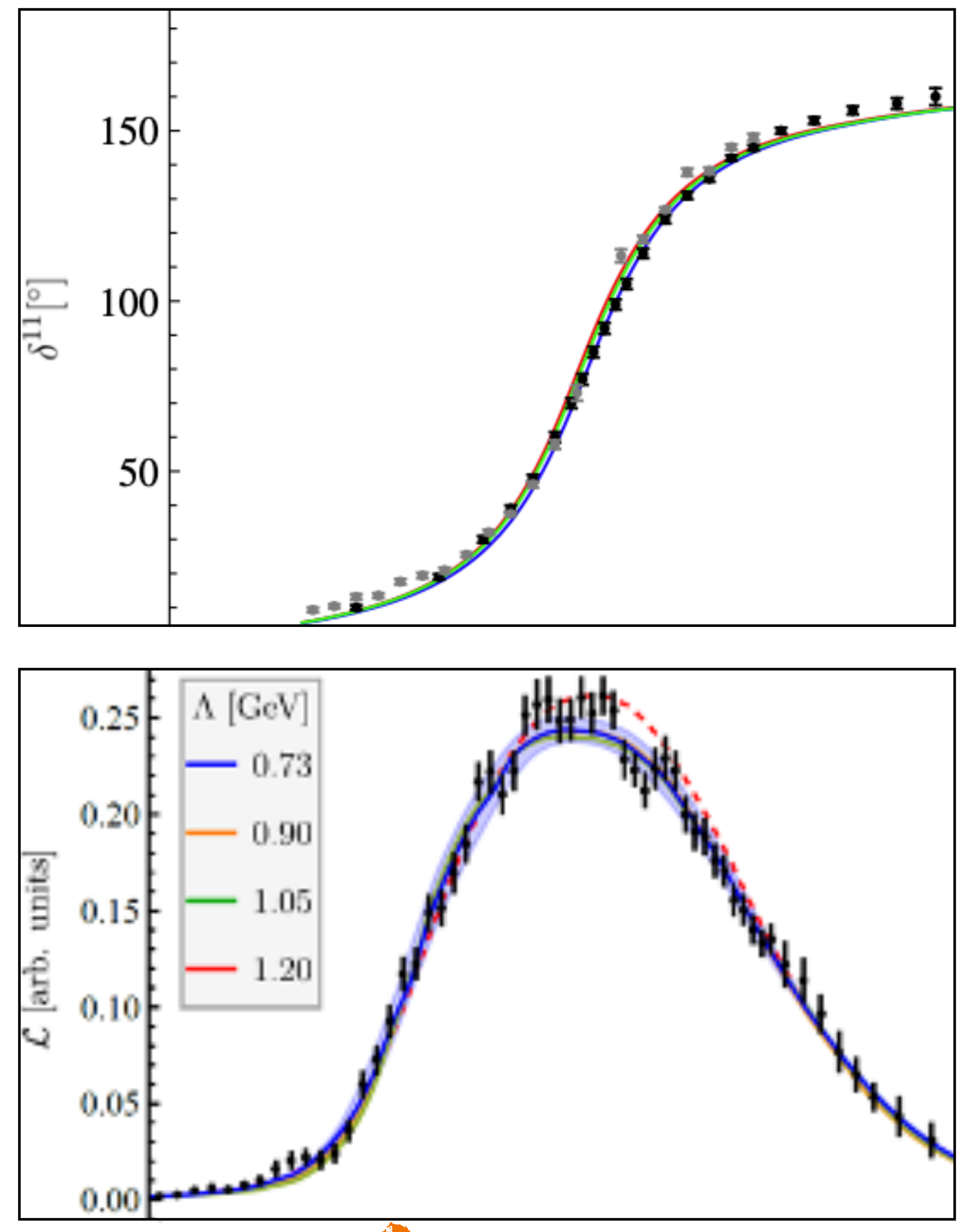
- **Three-body scattering amplitude[1][2]**
  - Express 3-body through 2+1 system
  - Genuine integral equation
  - On-shell configurations are fixed by Unitarity
  - Input: C and K

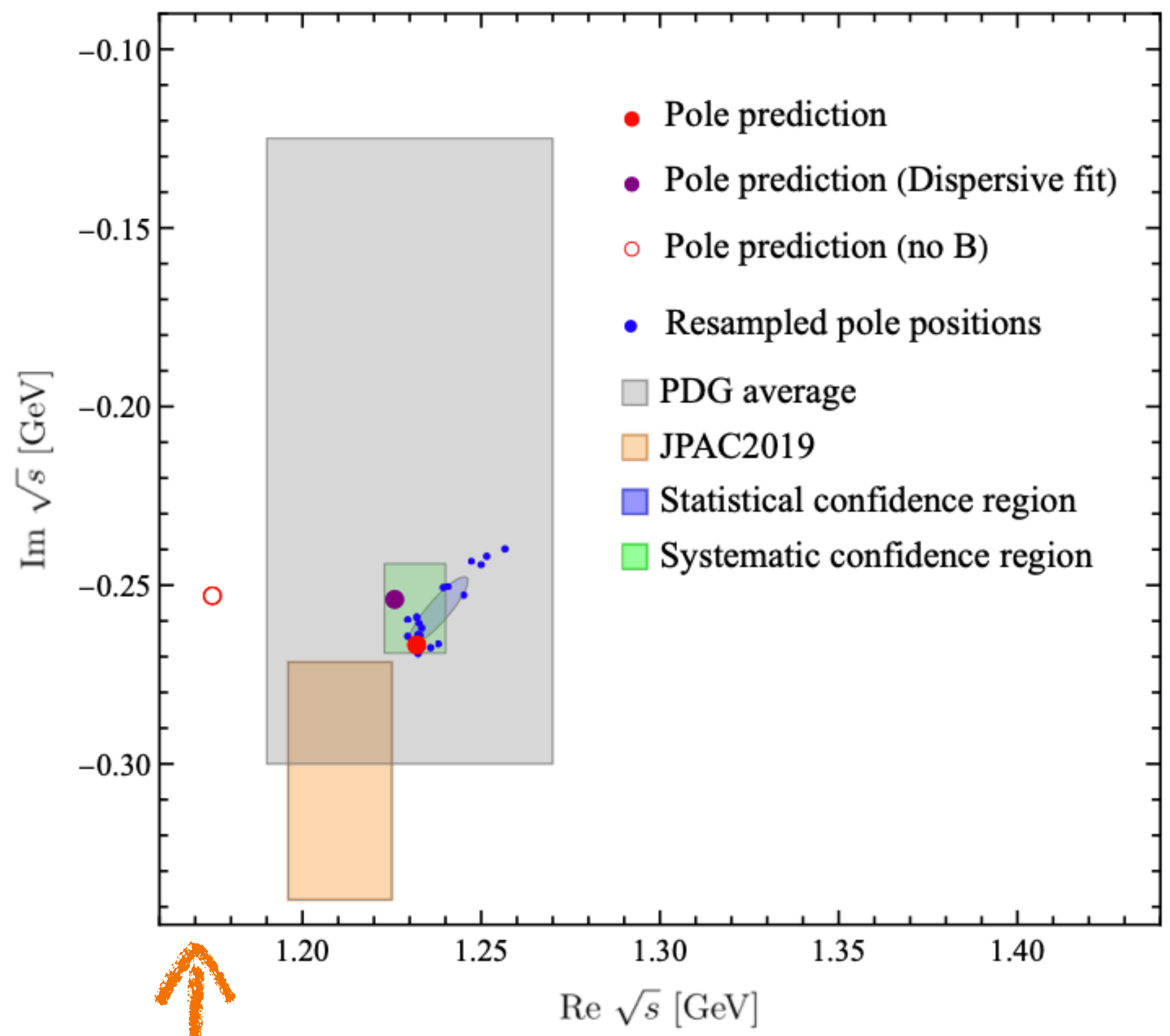
"Infinite Volume Unitarity" — IVU formalism



[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
[2] Related approaches: Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019);

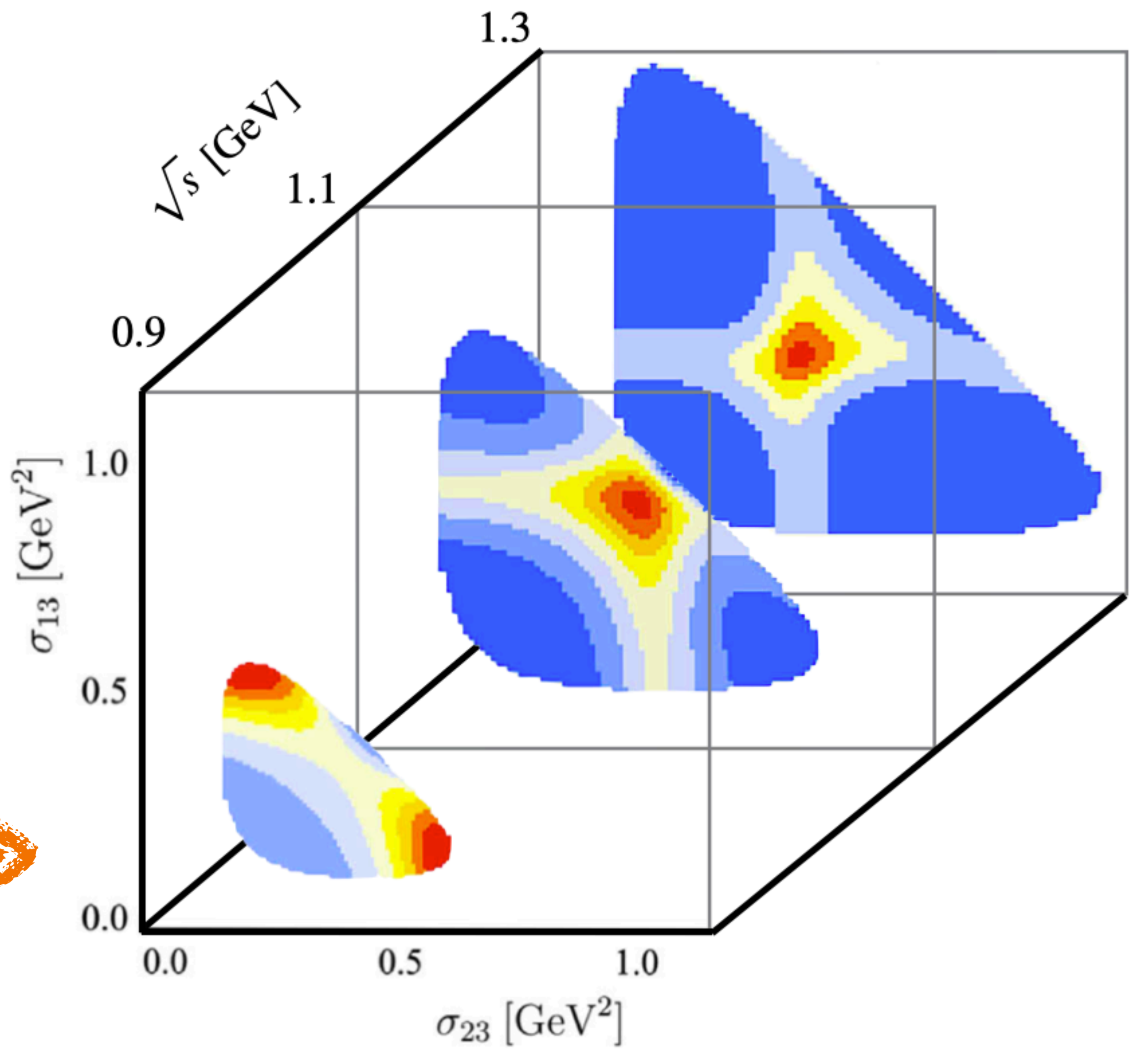
APPLICATION: $a_1(1260)$





- $\pi\rho$ dynamics dominates the $1^-(1^{++})$ system
- Integral equation solved
  - Helicity formalism
  - complex momentum mapping
- $\pi\rho/\pi\sigma/\pi(\pi\pi)_2$ extended…

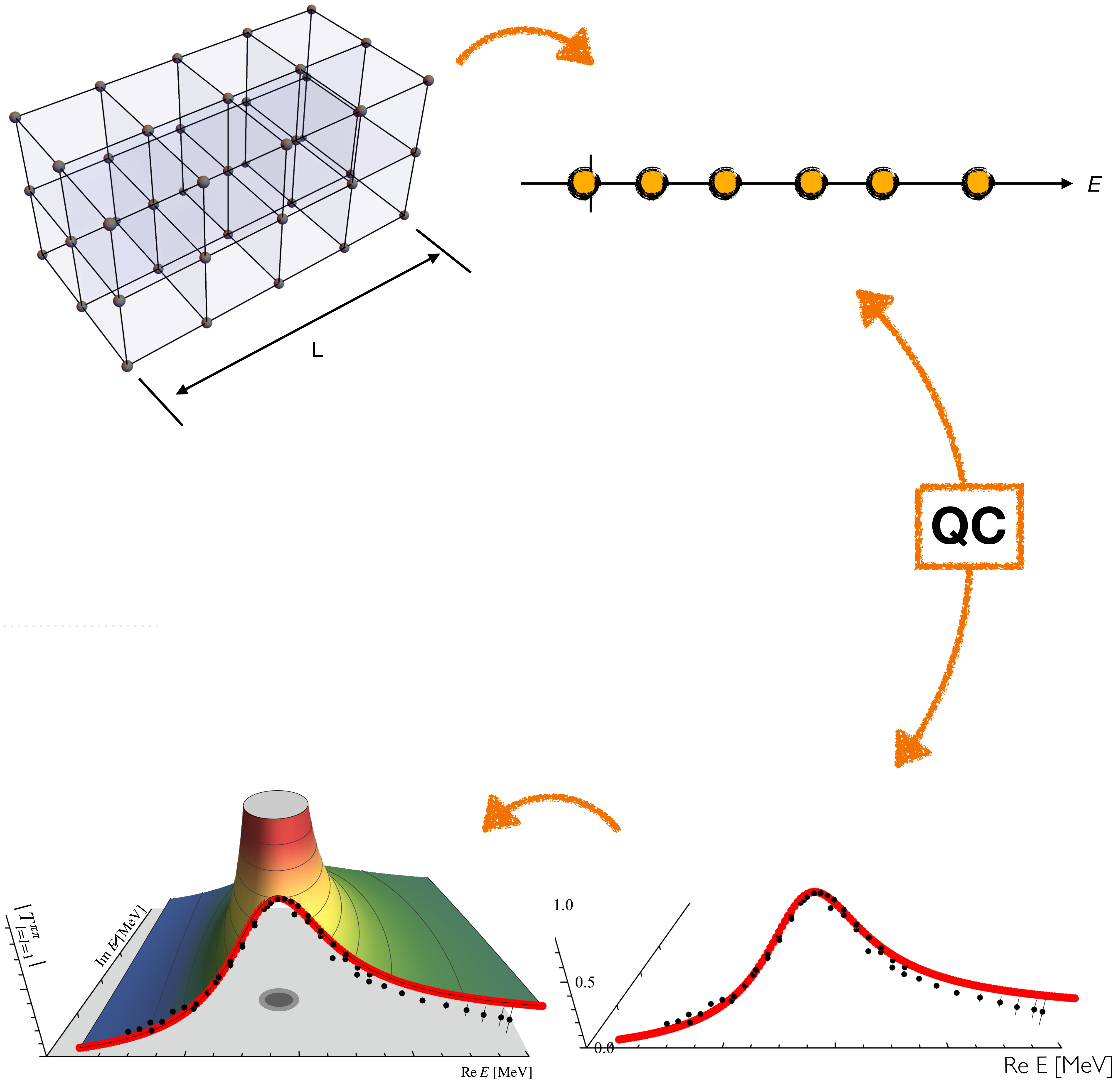
$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1}-\Sigma_n}T^c$$



Data: Schael [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7;
Pole extraction: Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020)

FINITE-VOLUME SPECTRUM

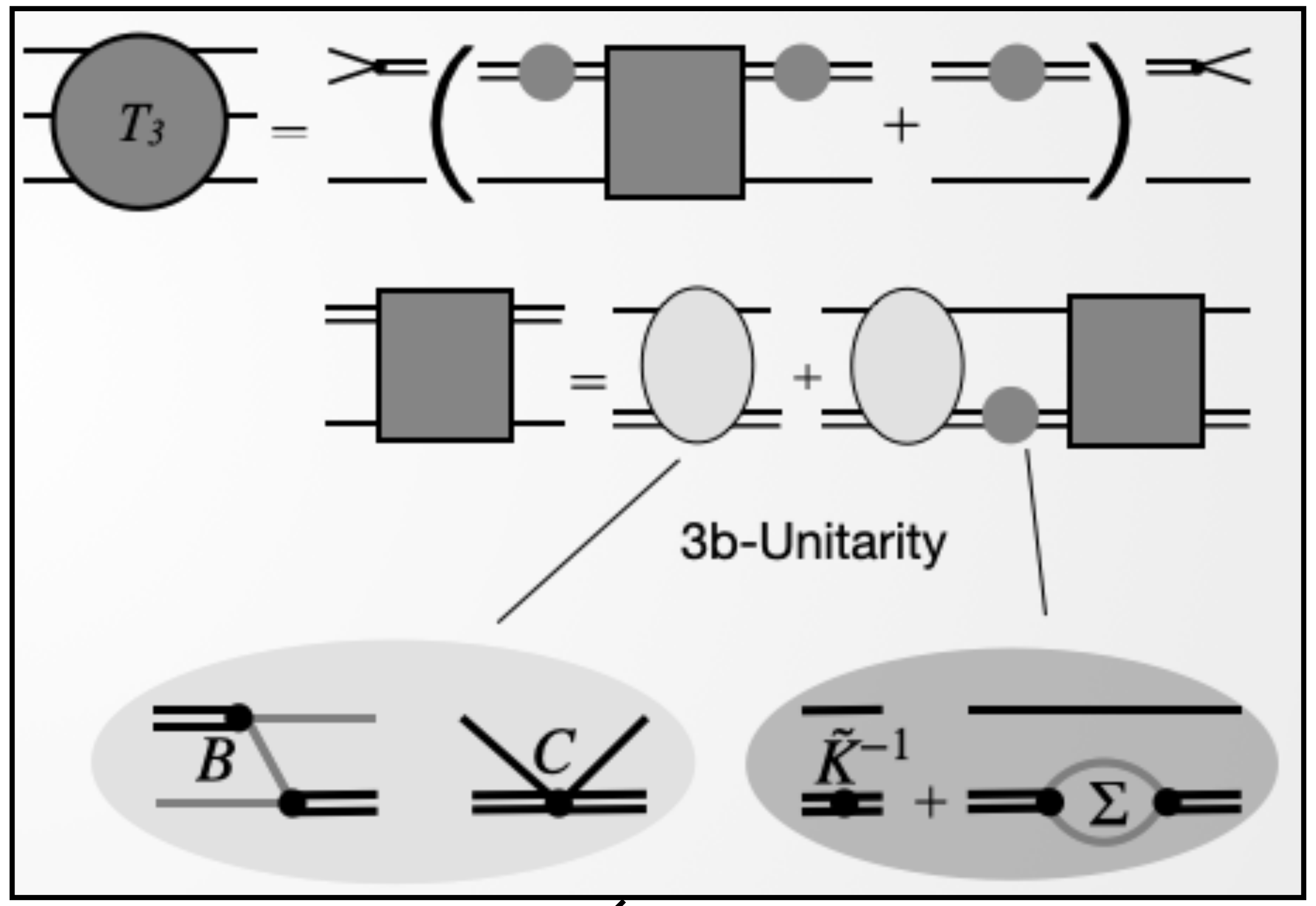
- LQCD provides numerical access to QCD Green's functions
  - in discretised Euclidean space-time
  - **in finite-volume**
    - **mapping through Quantization conditions[1]**



[1] Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, …
Reviews: Briceno/Dudek/Young (2017) Rev.Mod.Phys. 90 (2018) 2 Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

3-BODY QUANTIZATION CONDITION (FVU)

reminder



- **Finite-volume unitarity (FVU[1])**
  - heavily simplified:
    - on-shell particle-configurations: **Δ**E~mL
    - off-shell particle-configurations: **Δ**E~e^{-mL}
  - *Unitary 3-body amplitude separates these effects*
  - unknown volume independent quantities (K, C)

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

[1] MM/Döring Phys.Rev.Lett. 122 (2019) 6
Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

APPLICATION: $a_1(1260)$

$$0 = \det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}_2^{-1} - \Sigma_2^L\right) - B - C \,\right]^{T_{1g}}$$

FVU

IVU

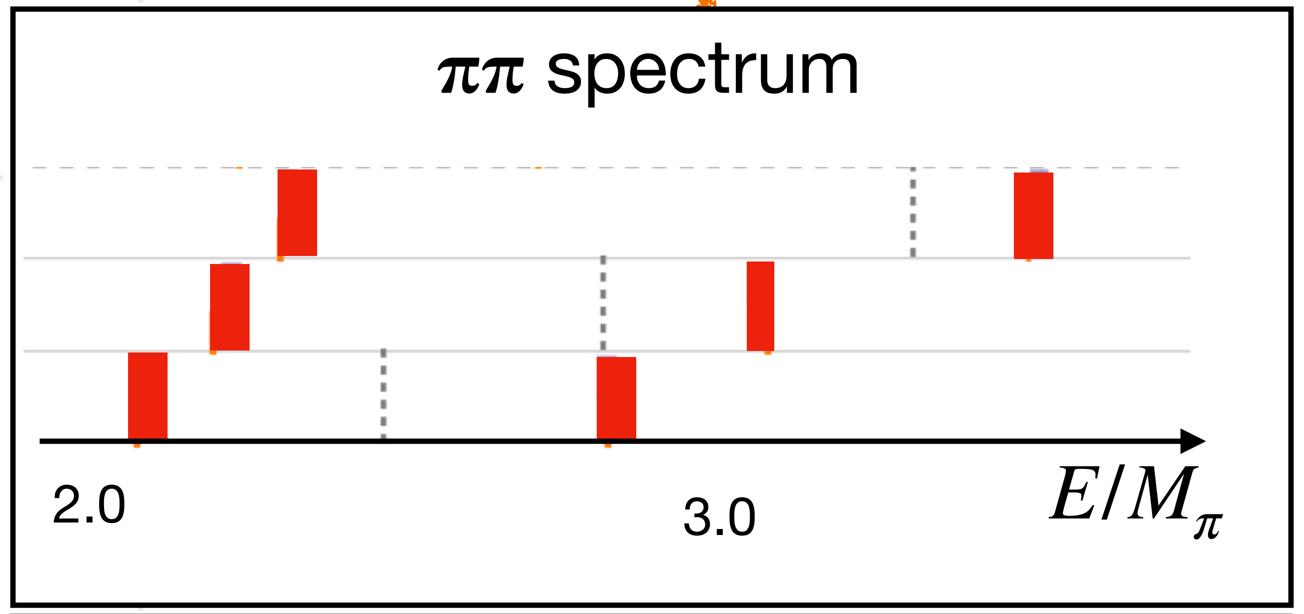
$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

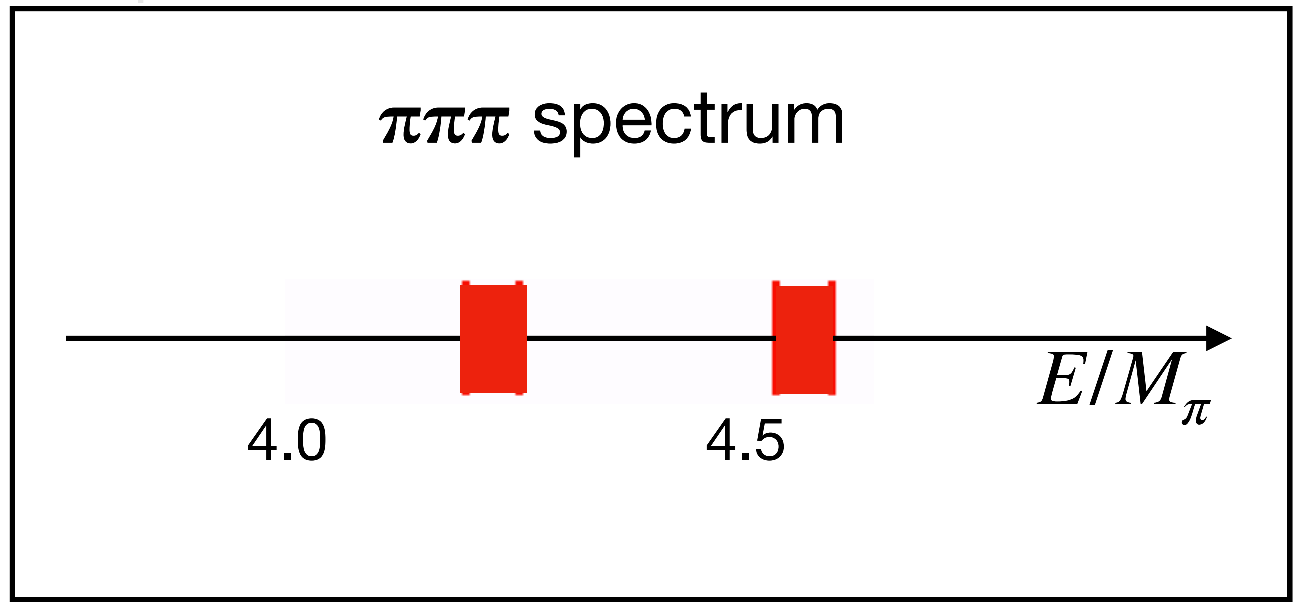
Input:

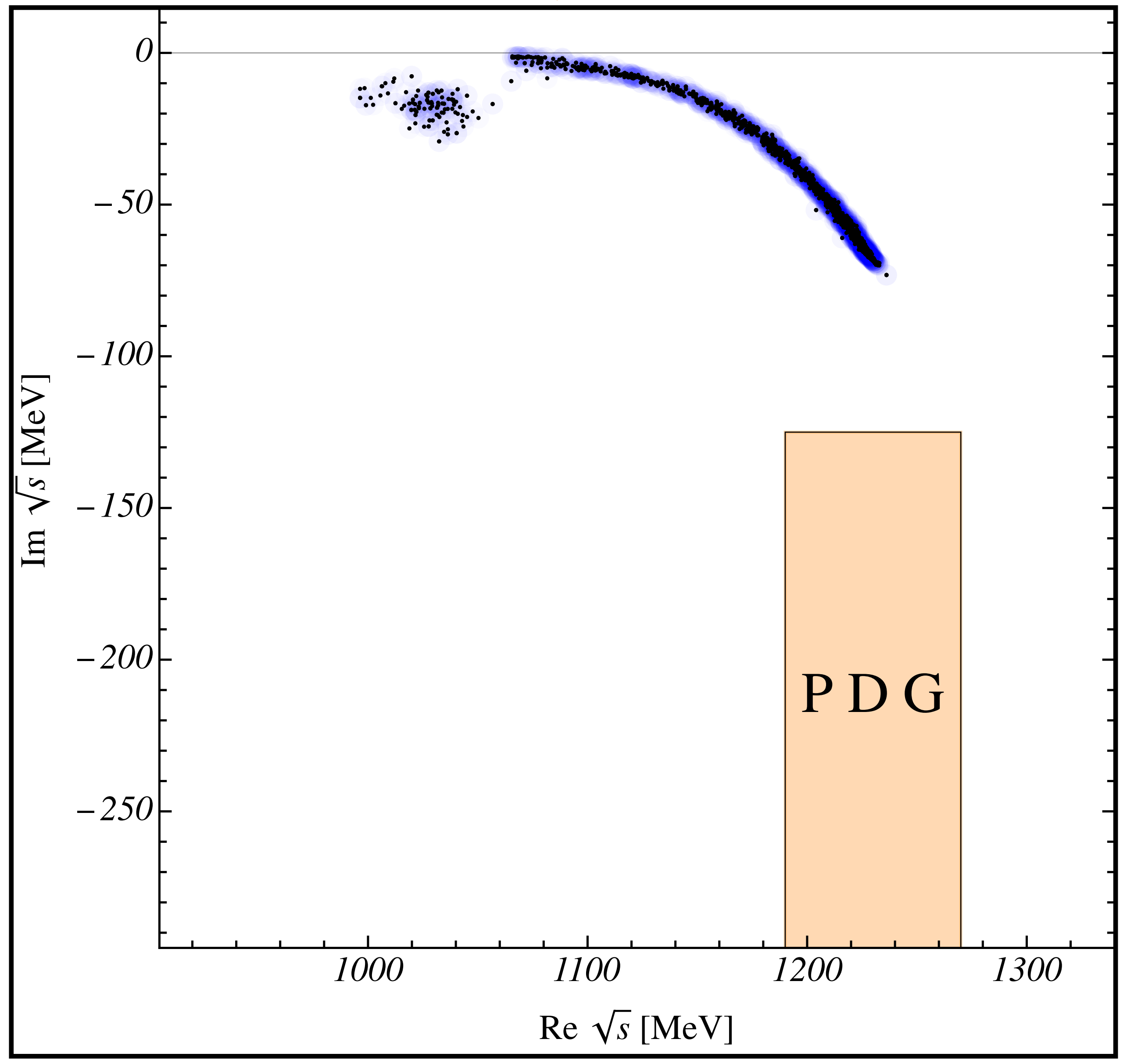
- 2- and 3-body lattice results with multi-hadron operators[1]
- Unphysical pion mass

Determine infinite-volume quantities

- Pole position of the *a₁(1260)*[2,3]
- Chiral trajectory







[1] [GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)
[2] MM/Culver/Sadasivan/Brett/Döring/Alexandru/Lee [GWQCD] PRL 127 (2022)

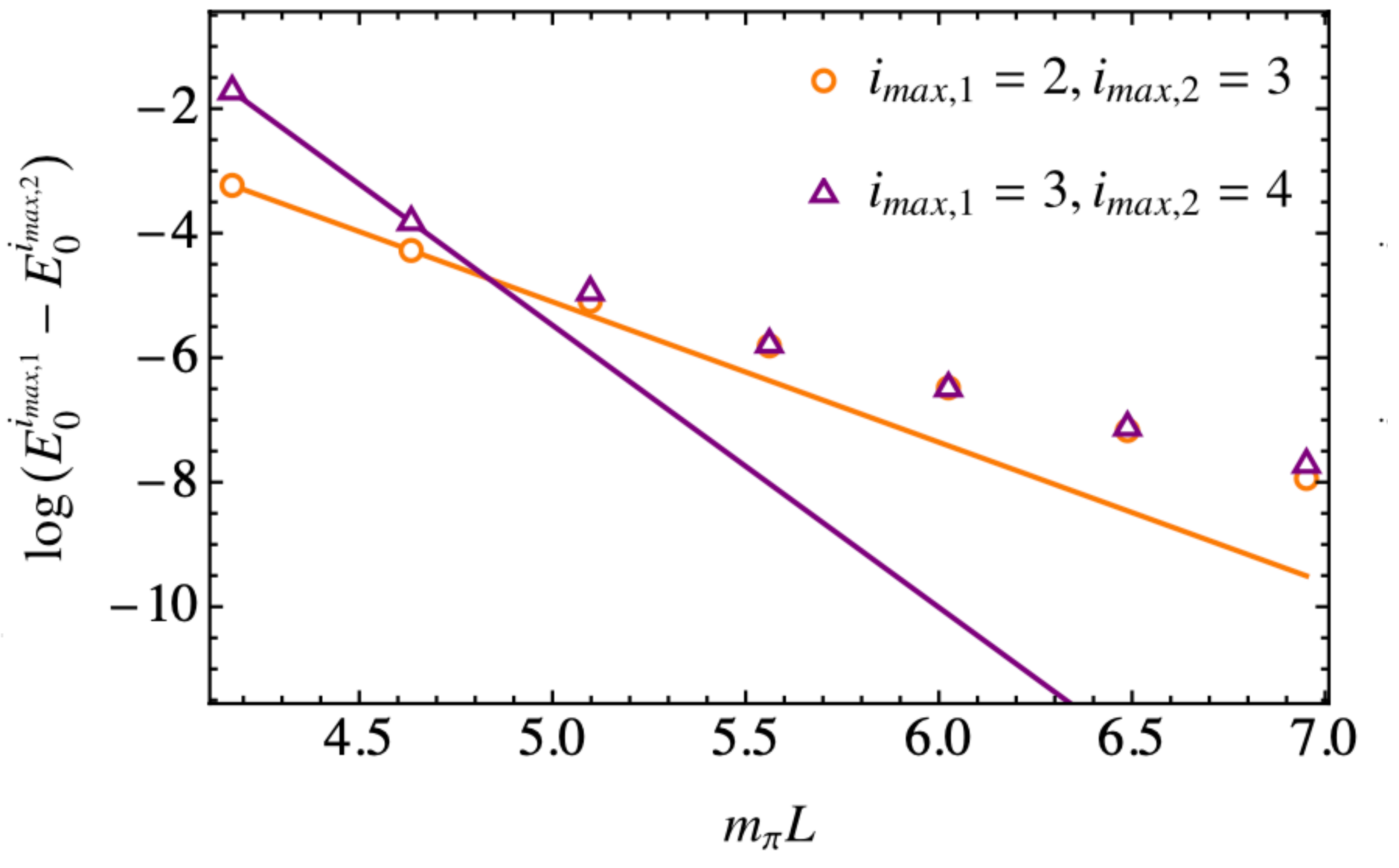
CUTOFF DEPENDENCE[1]

Consider fixed C, K then increase <u>hard</u> cutoff

- 3-body amplitude = genuine integral equation
  - spectator can carry arbitrary momentum away
  - cutoff required (form factors, hard cutoff,…)

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

$$B(\sqrt{s}) = \frac{1}{\sqrt{s} - \sqrt{s_{\rm on}} + i\epsilon}$$



- energy eigenvalues change slower than $\Delta E \sim e^{-mL}$
- one-particle exchange falls off not rapidly enough

[1] paper in preparation

CUTOFF DEPENDENCE[1]

Consider fixed C, K then increase <u>hard</u> cutoff

… over-subtract OPE

$$\overline{B}(\sqrt{s}) = B(0) + B'(0)\sqrt{s} + \frac{s}{s_{\rm on}}\frac{N}{2E_{p+p'}}\frac{1}{\sqrt{s}-\sqrt{s_{\rm on}}+i\epsilon}$$

- 3-body amplitude = genuine integral equation
  - spectator can carry arbitrary momentum away
  - cutoff required (form factors, hard cutoff,…)

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1}-\Sigma\right)-B-C\right]_{\mathbf{p}'\mathbf{p}}$$

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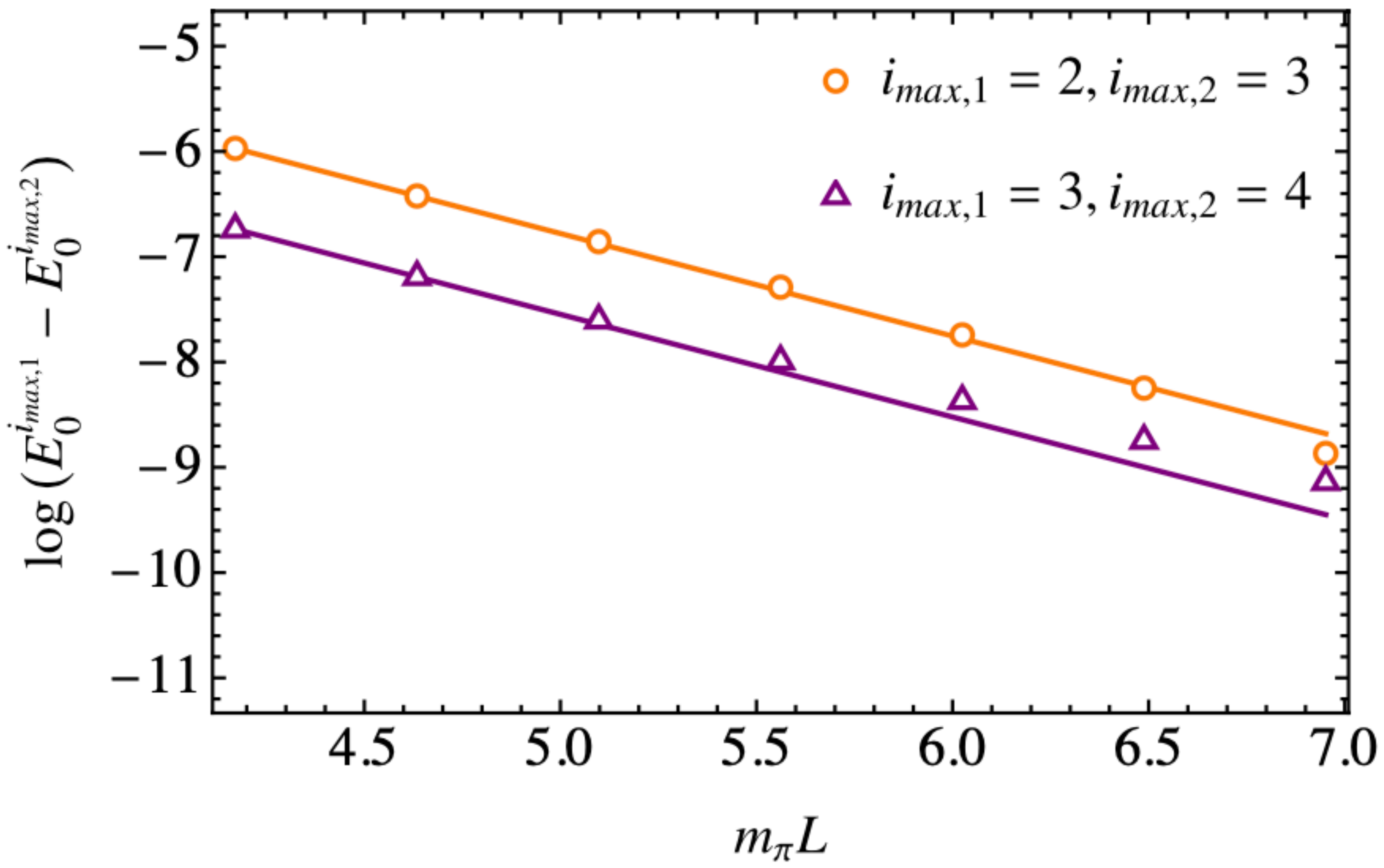


- energy eigenvalues change as $\Delta E \sim e^{-mL}$

[1] paper in preparation

CUTOFF DEPENDENCE[1]

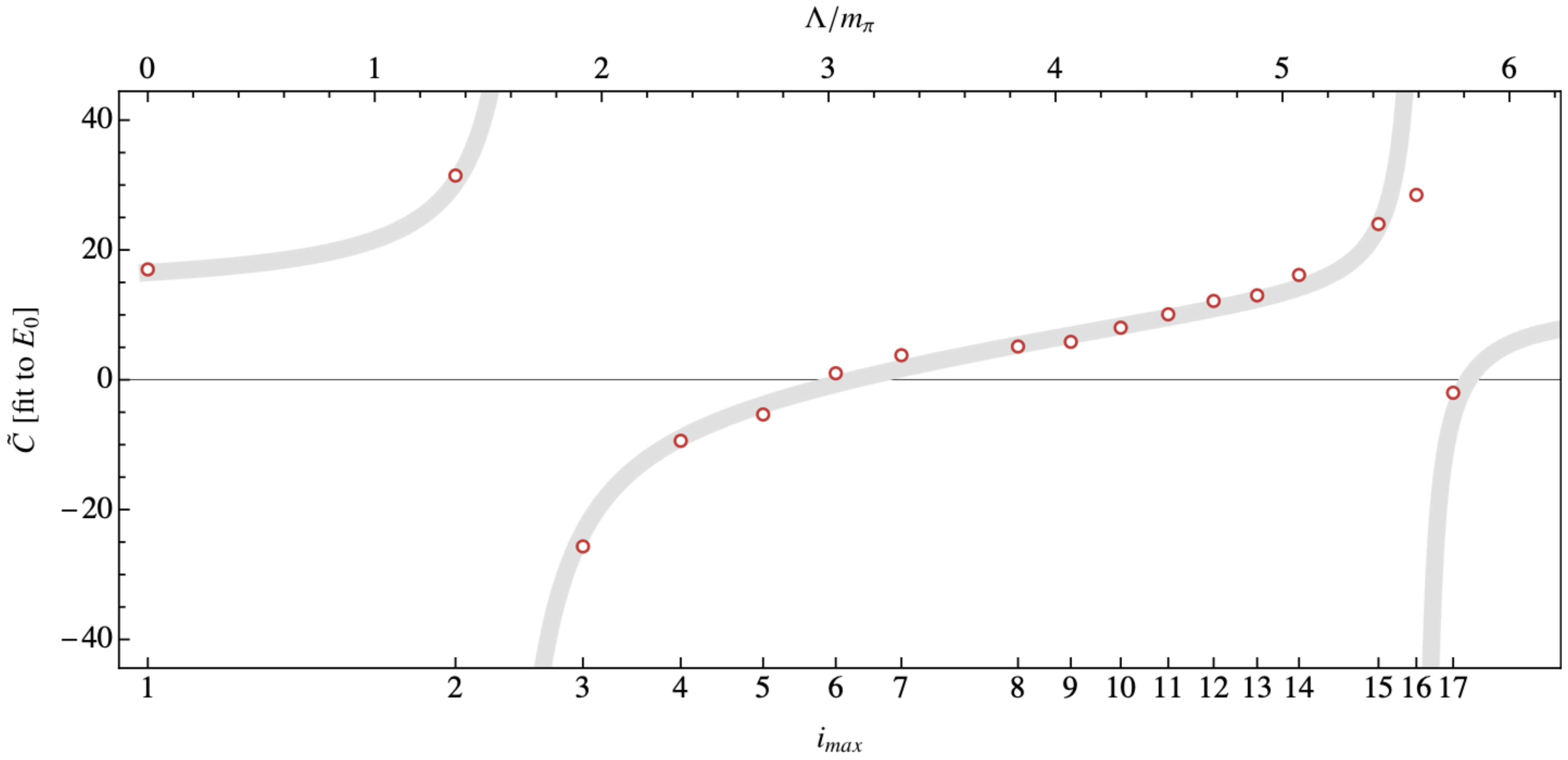
Consider fixed ground-state finite-volume level (E_0)

- change cutoff & refit C
- $\pi\rho/\pi(\pi\pi)_2$ repulsiv system
- $C(\Lambda)$ shows cyclic behaviour[2]

◉ 3-body amplitude = genuine integral equation

- spectator can carry arbitrary momentum away
- cutoff required (form factors, hard cutoff,…)

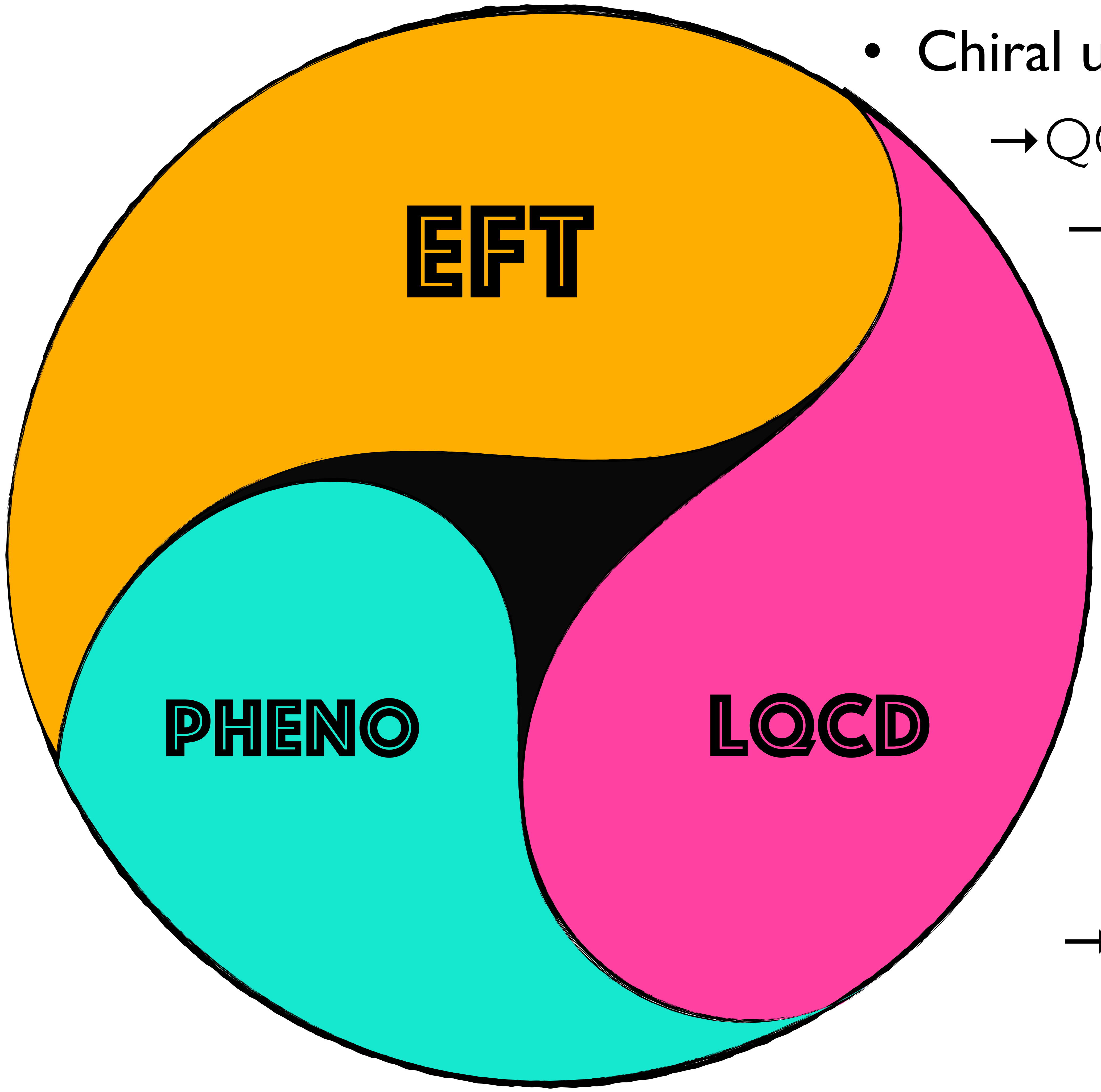
$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$



[1] paper in preparation
[2] Bedaque/Hammer/van Kolck, Phys. Rev. Lett. 82 (1999) 463; Bedaque/Hammer/van Kolck, Nucl.Phys. A 646 (1999) 444

SUMMARY

New synergetic approaches to universal parameters of resonance become available

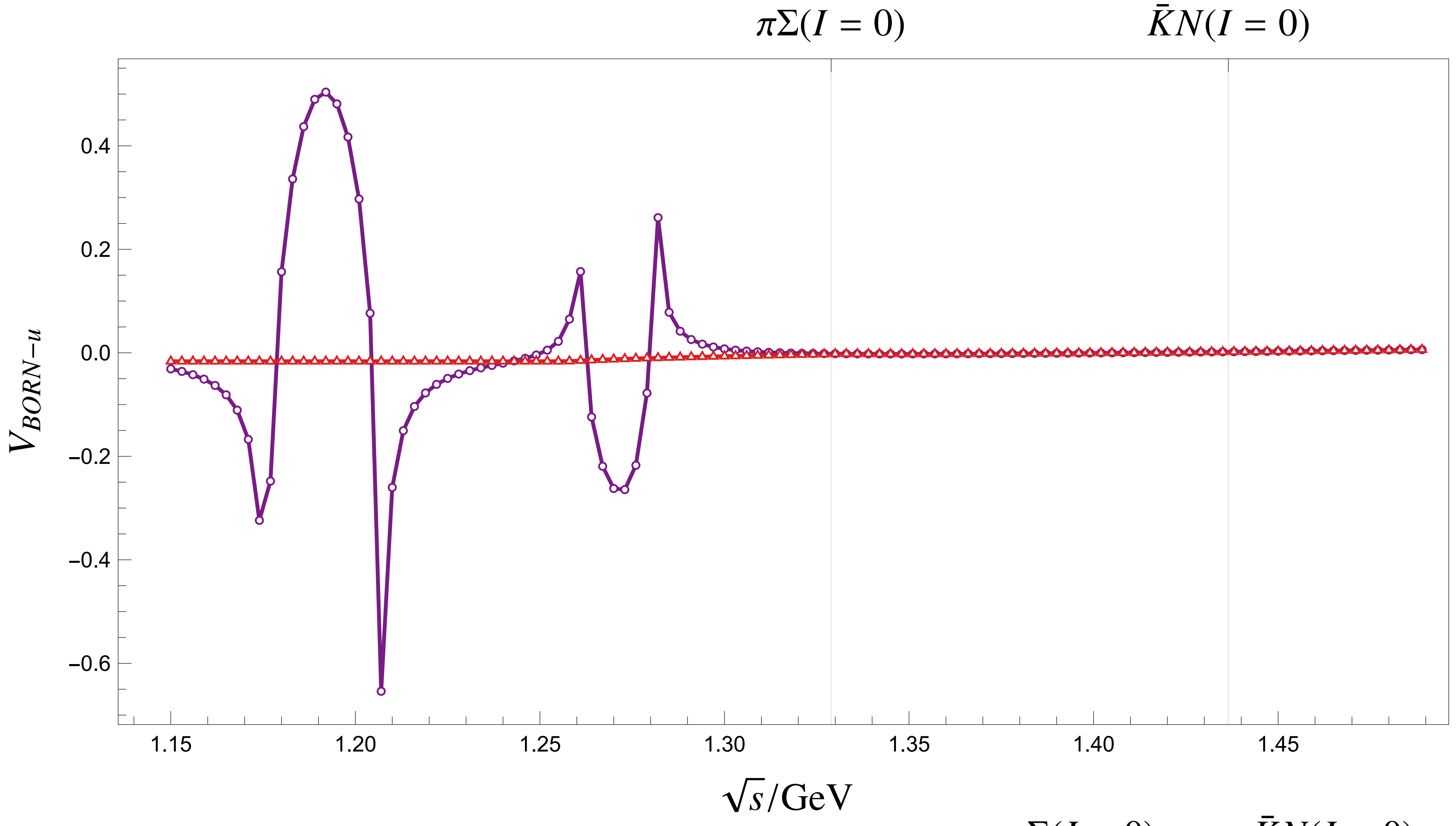


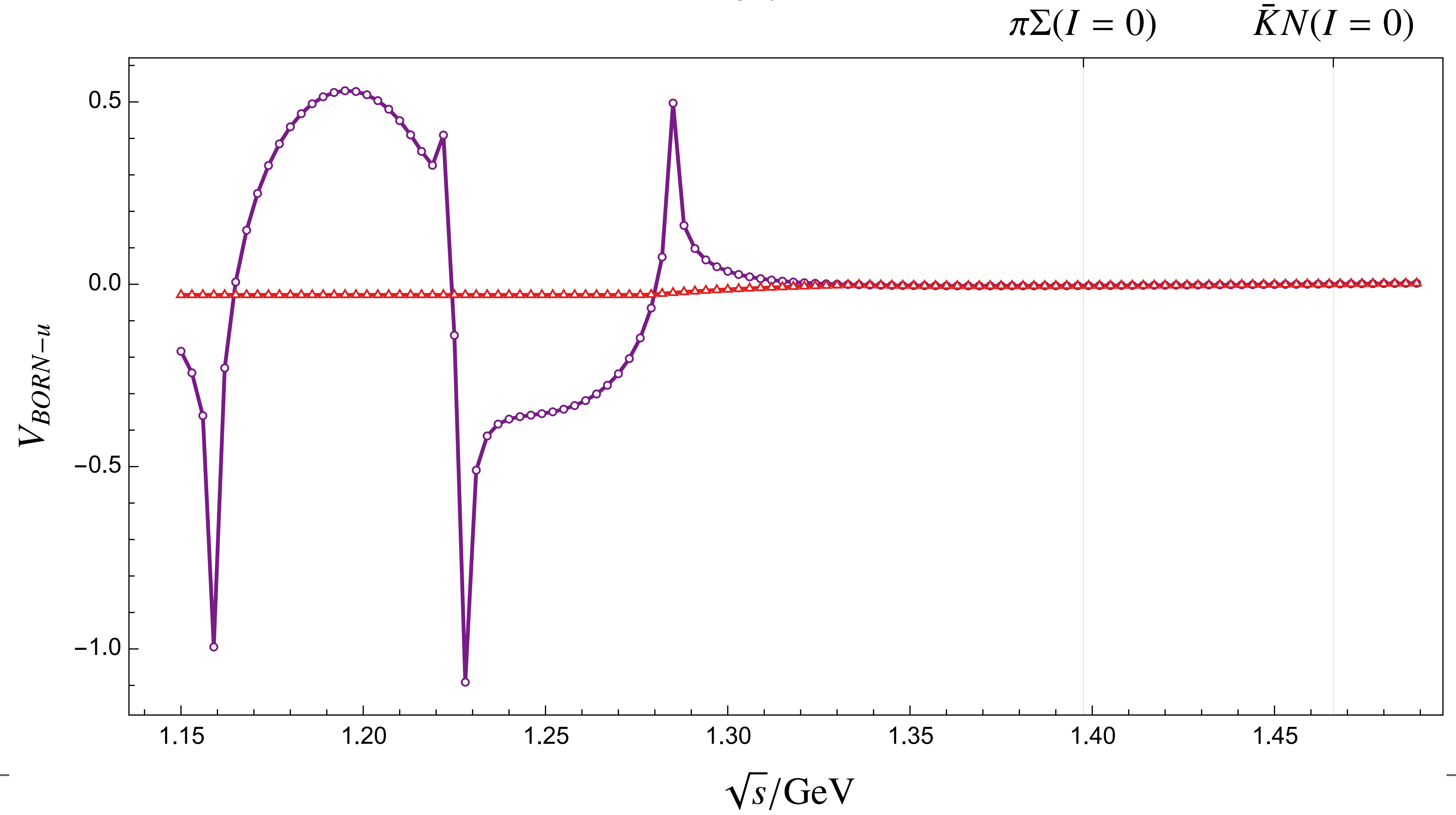
- **Chiral unitary models & LQCD**
  - →QCD symmetries constraints to hadron-hadron dynamics
  - →strong support for the two-pole scenario
  - →Novel 3-body methodology has matured
  - →EFTs and S-matrix theory: bridge to real world physics
- **Future directions**
  - →Heavy-light systems (chiral trajectory, LHC, …)
  - →DDpi
  - →Roper resonance

THANK YOU

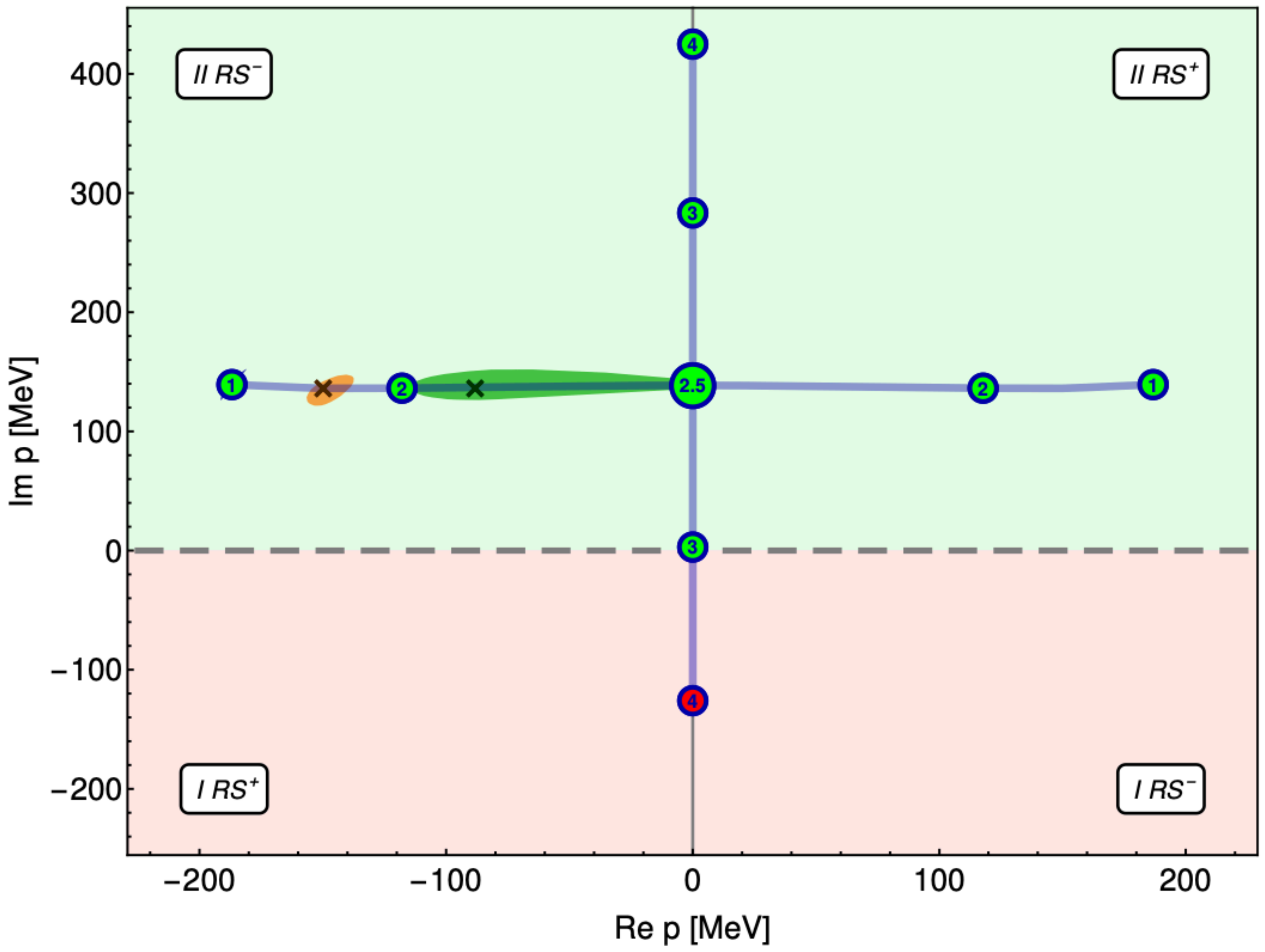
U-CHANNEL IN THE Λ(1405)

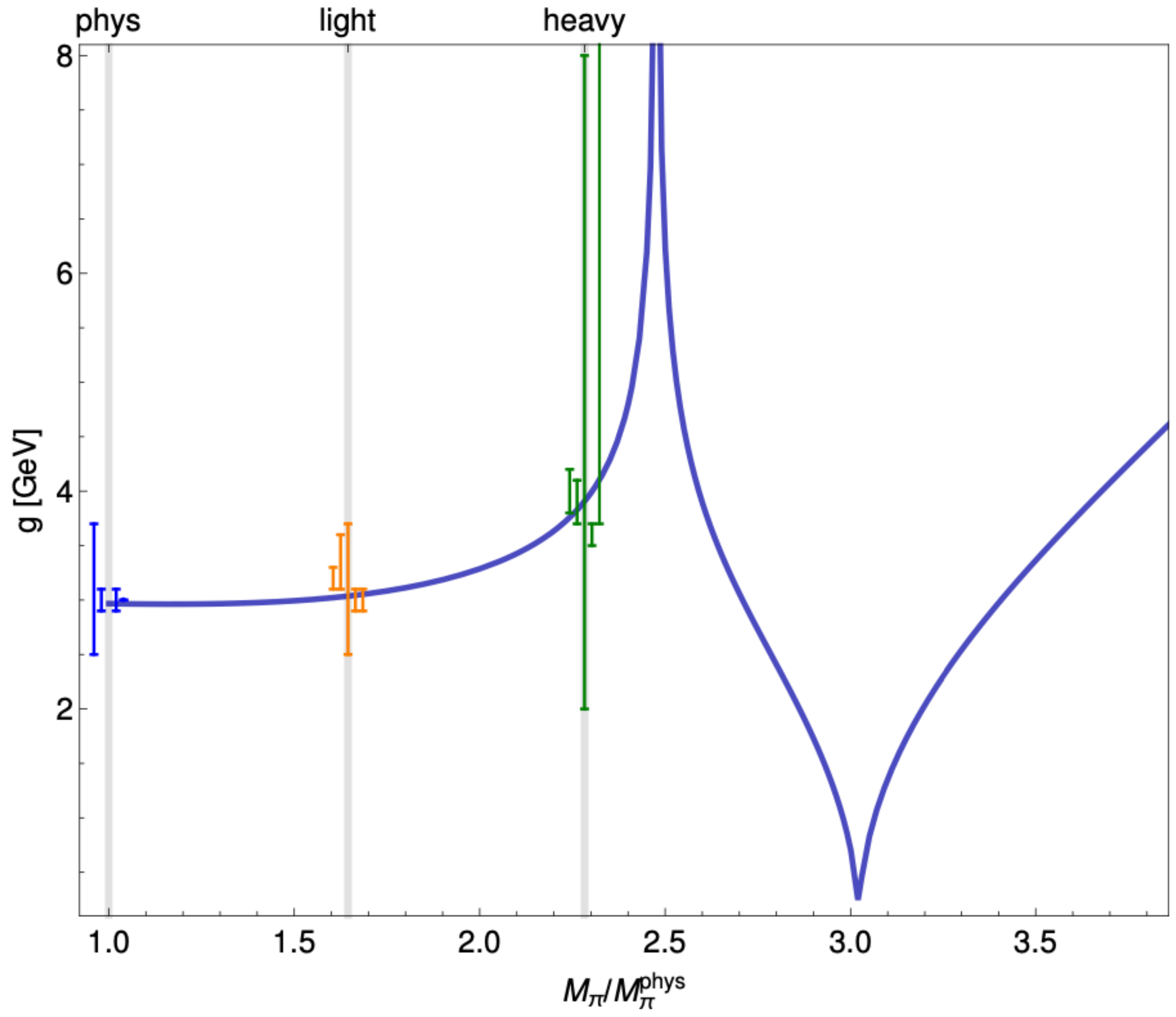
- New insights[1] from LQCD [next talk]
  - confirming two-pole scenario
- Chiral extrapolations (through UCHPT)[2]
  - u-channel baryon exchange may complicate the picture (3-body)
  - sub-leading effect

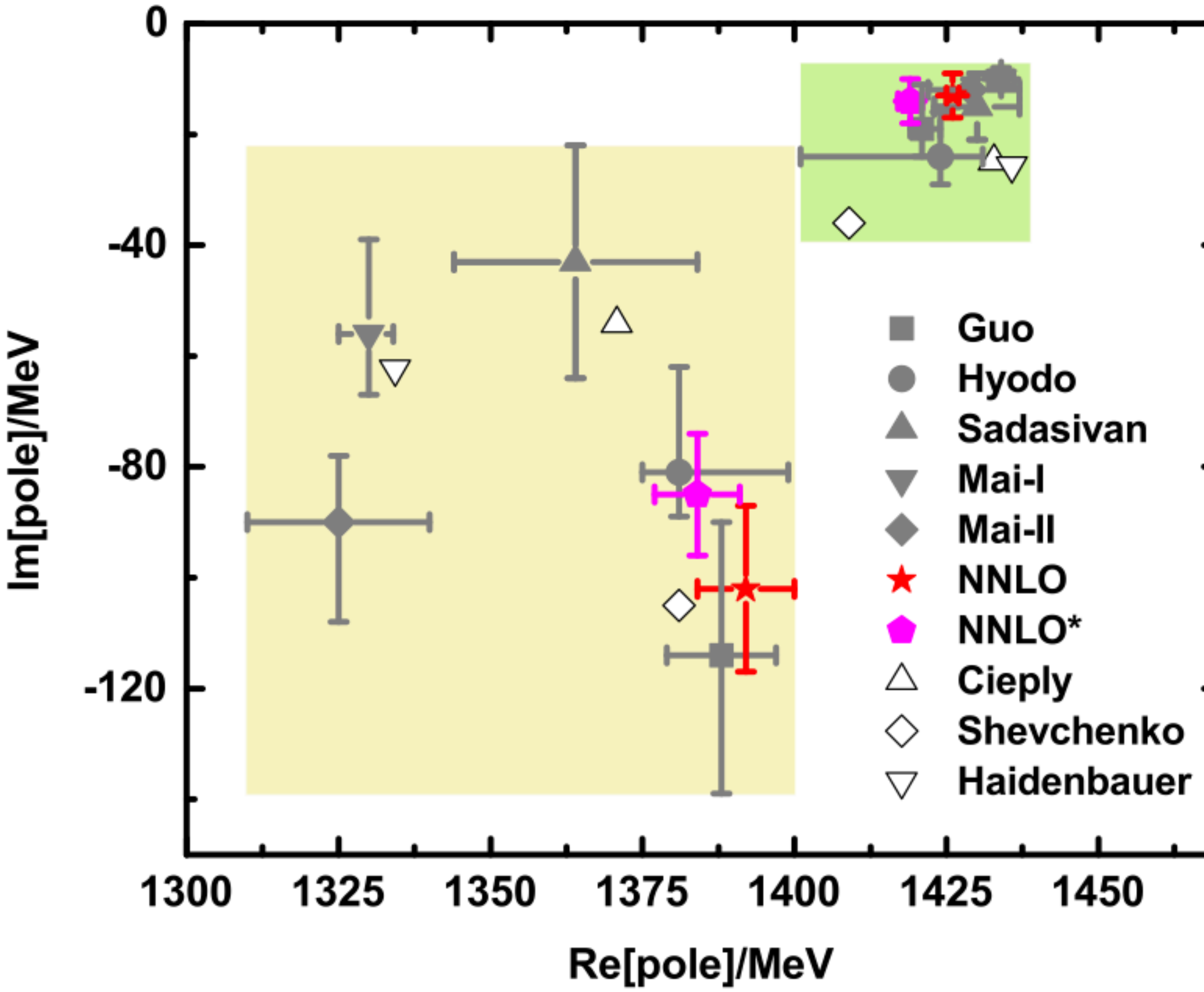




[1] [BaSc] Bulava et al. 2307.10413; 2307.13471
[2] Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)


II RS⁻
II RS⁺
I RS⁺
I RS⁻
Im p [MeV]
Re p [MeV]
400
300
200
100
0
−100
−200
−200
−100
0
100
200


phys
light
heavy
g [GeV]
M_π/M_π^{phys}
8
6
4
2
1.0
1.5
2.0
2.5
3.0
3.5


0
-40
-80
-120
Im[pole]/MeV
1300
1325
1350
1375
1400
1425
1450
Re[pole]/MeV
Guo
Hyodo
Sadasivan
Mai-I
Mai-II
NNLO
NNLO*
Cieply
Shevchenko
Haidenbauer

$$\{1, 8_s, 8_a, 10, \overline{10}, 27\}$$

$$\begin{pmatrix} |\pi\Sigma\rangle \\ |\bar{K}N\rangle \\ |\eta\Lambda\rangle \\ |K\Xi\rangle \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} \sqrt{15} & -\sqrt{24} & 0 & -1 \\ -\sqrt{10} & -2 & \sqrt{20} & -\sqrt{6} \\ -\sqrt{5} & -\sqrt{8} & 0 & 3\sqrt{3} \\ \sqrt{10} & 2 & 2\sqrt{5} & \sqrt{6} \end{pmatrix} \begin{pmatrix} |1\rangle \\ |8\rangle \\ |8'\rangle \\ |27\rangle \end{pmatrix},$$

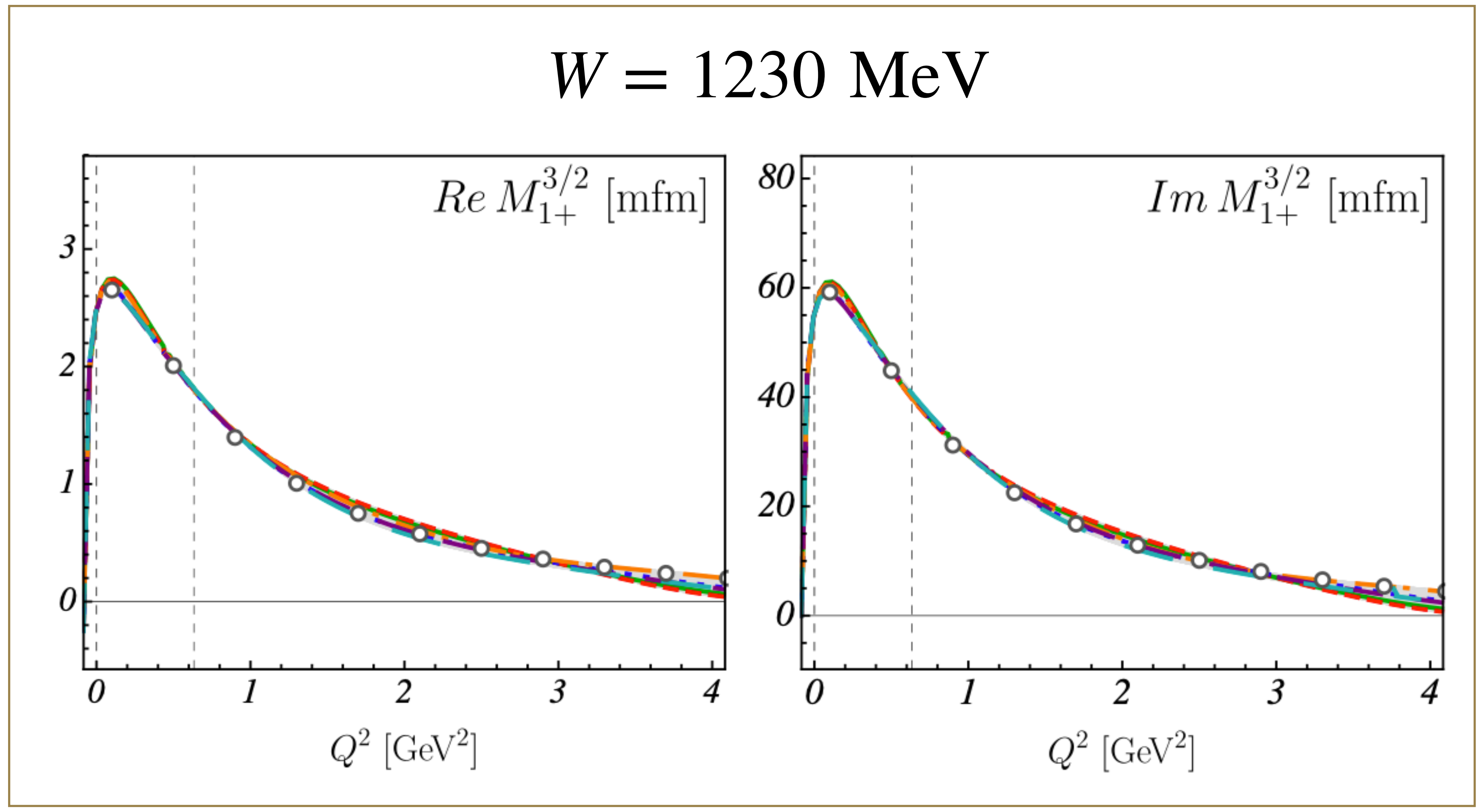
$$C_{\alpha\beta} = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad \text{for} \quad \alpha, \beta \in \{1, 8, 8', 27\}.$$

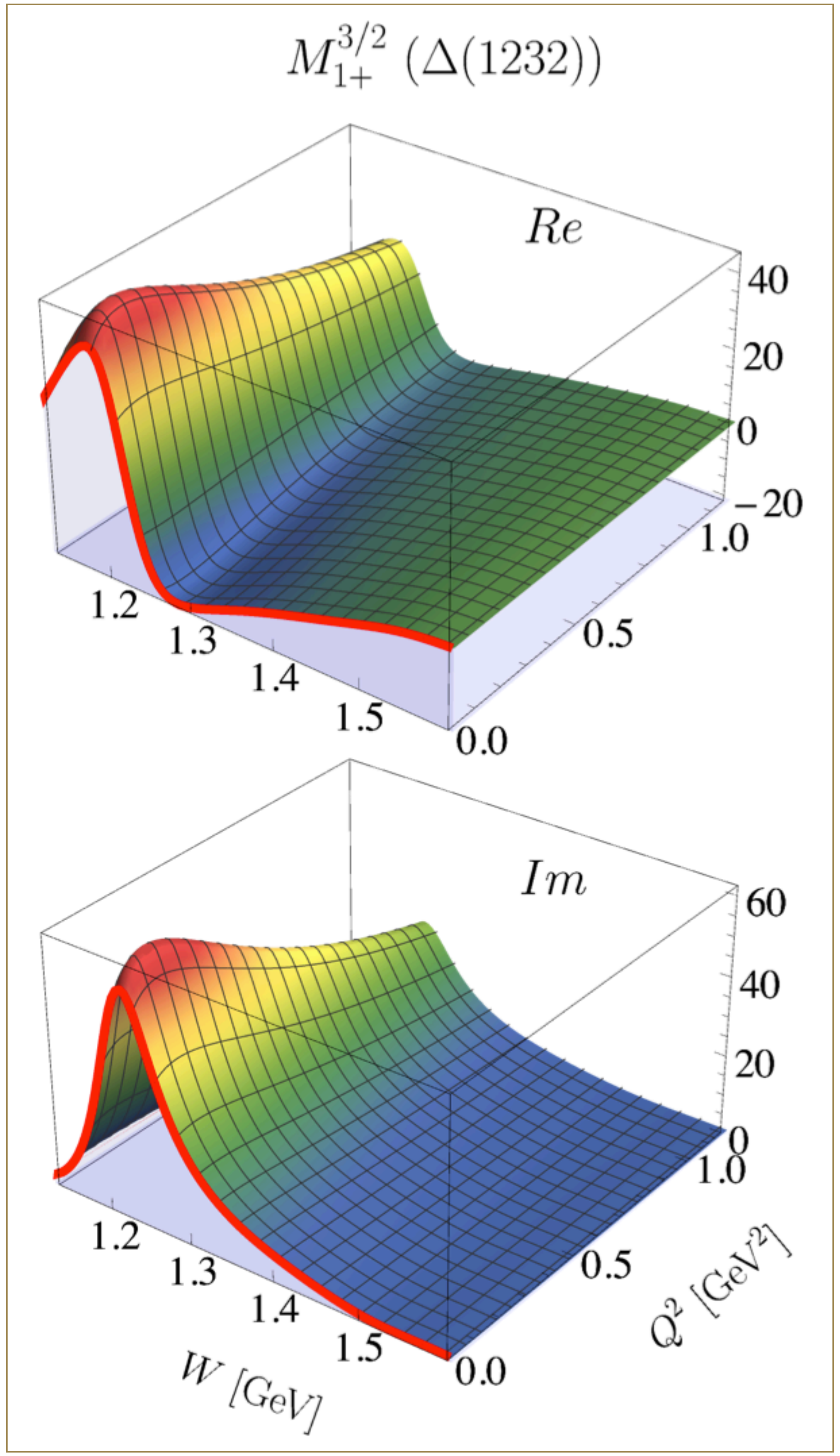
$$C_{\alpha\beta}^{\text{NLO1}} = \begin{pmatrix} \frac{4}{3}(3b_0 + 7b_D)m_q & 0 & 0 & 0 \\ 0 & \frac{2}{3}(6b_0 + b_D)m_q & -\sqrt{20}b_F m_q & 0 \\ 0 & -\sqrt{20}b_F m_q & 2(2b_0 + 3b_D)m_q & 0 \\ 0 & 0 & 0 & 4(b_0 + b_D)m_q \end{pmatrix},$$

$$C_{\alpha\beta}^{\text{NLO2}} = \begin{pmatrix} -3d_2 + \frac{9}{2}d_3 + d_4 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(-3d_2 + d_3 + 2d_4) & -\frac{\sqrt{5}}{2}d_1 & 0 \\ 0 & -\frac{\sqrt{5}}{2}d_1 & \frac{1}{2}(9d_2 - d_3 + 2d_4) & 0 \\ 0 & 0 & 0 & \frac{1}{2}(2d_2 + d_3 + 2d_4) \end{pmatrix}.$$

NLO breaks accidental octet symmetry

RESULTS





Delta(1232):

- Large multipoles well determined
- simple Q^2 dependence

[JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

HADRONIC 3-BODY PROBLEM: IMPACT

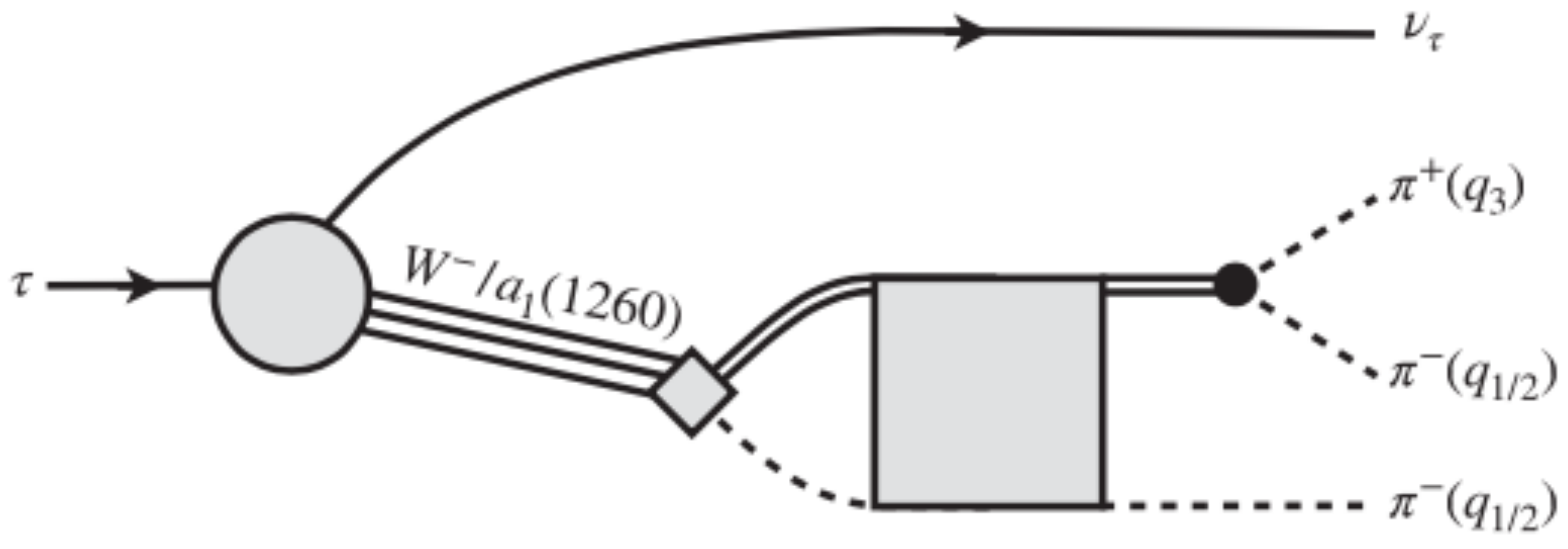
Intricate kinematics/dynamics

- 8 variables
- 2-body sub-channel dynamics

Hadron spectroscopy riddles

- Roper(1440) → **$\pi\pi$N** [first FV evaluations[1]]
- X(3872) → **DbarDπ**
- a_1(1260) → **$\pi\pi\pi$**
- …

- Beyond Standard Model: τ-EDM



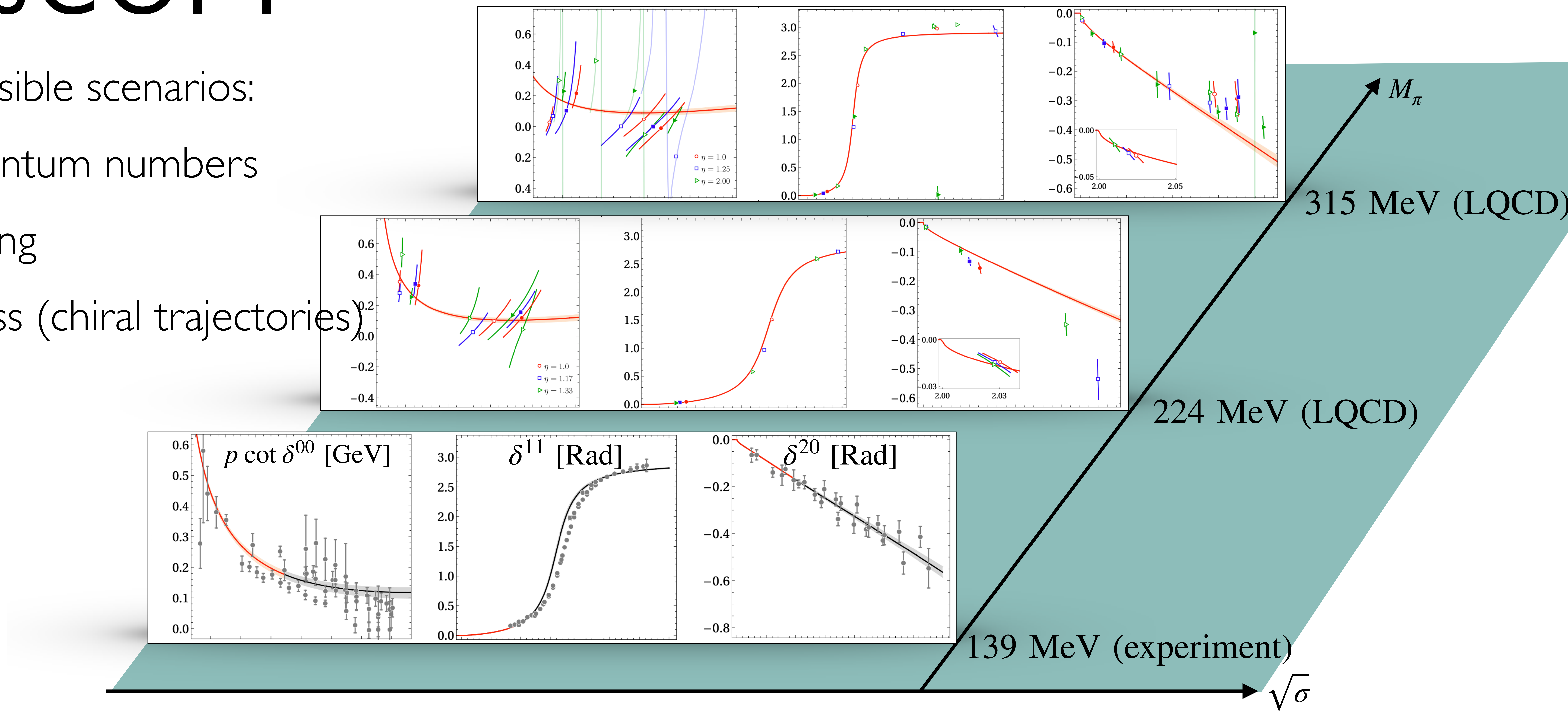
- Precision physics: rare hadronic W-decays[2]
- Exotic states of matter[3]

1) Severt/MM/Meißner JHEP04(2023) >>> PHD talk on Friday
2) Sirunyan et al. [CMS@CERN] PRL122
3) Experimental programs: GlueX@JLAB; COMPASS@CERN;

LATTICE HADRON SPECTROSCOPY

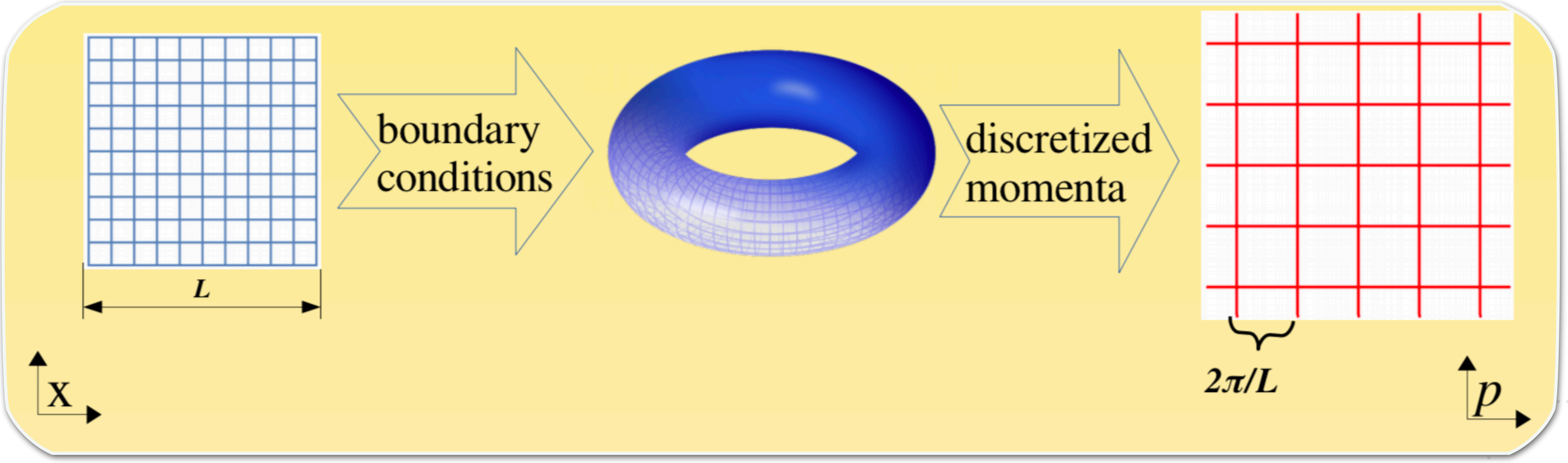
- Experimentally inaccessible scenarios:
  - Unconventional quantum numbers
  - Three-body scattering
  - Unphysical pion mass (chiral trajectories)

…



MM/Culver/Brett/Alexandru/Döring/Lee Phys.Rev.D 100 (2019)
Review: MM/Döring/Rusetsky EPJ ST (2021)

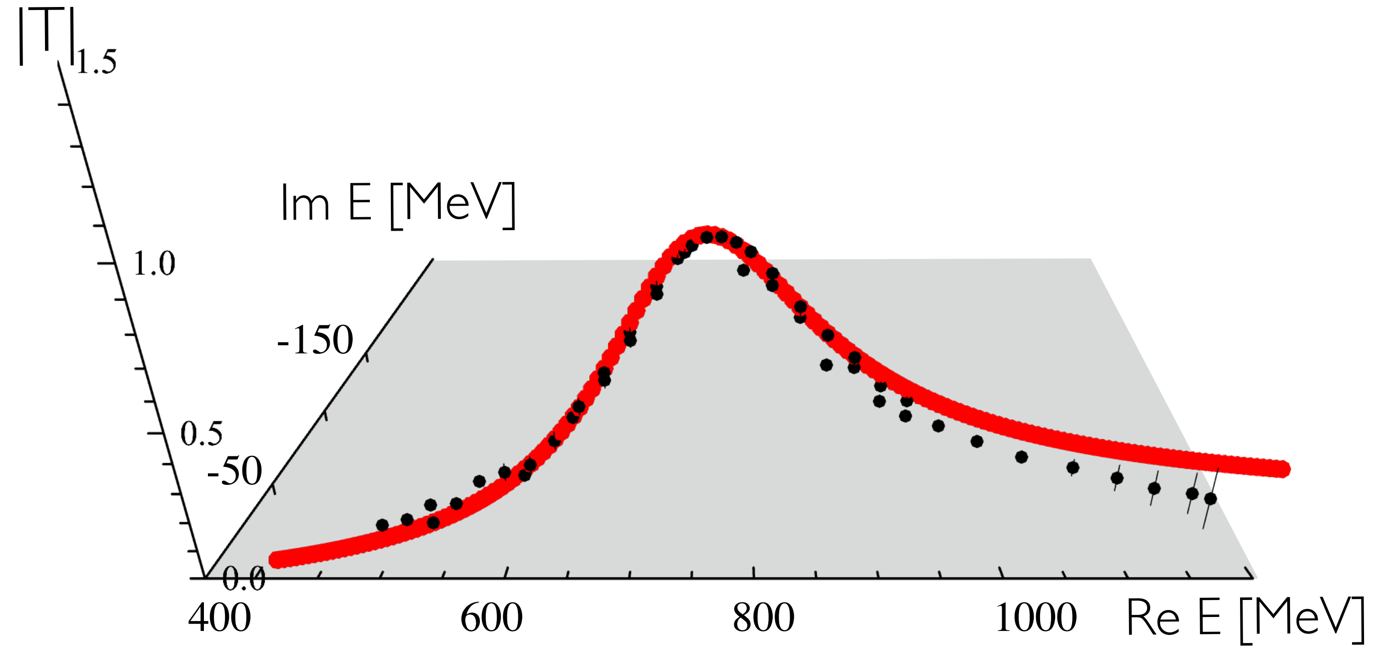
HADRONS IN A BOX



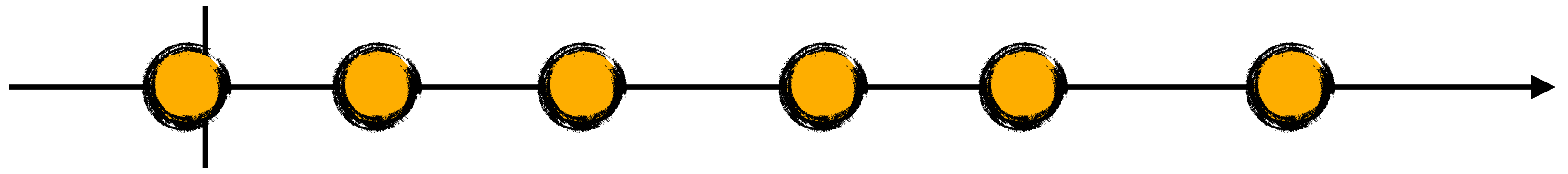
Heavily simplified:

on-shell particle-configurations: $\mathbf{\Delta}E\sim mL$

off-shell particle-configurations: $\mathbf{\Delta}E\sim e^{-mL}$

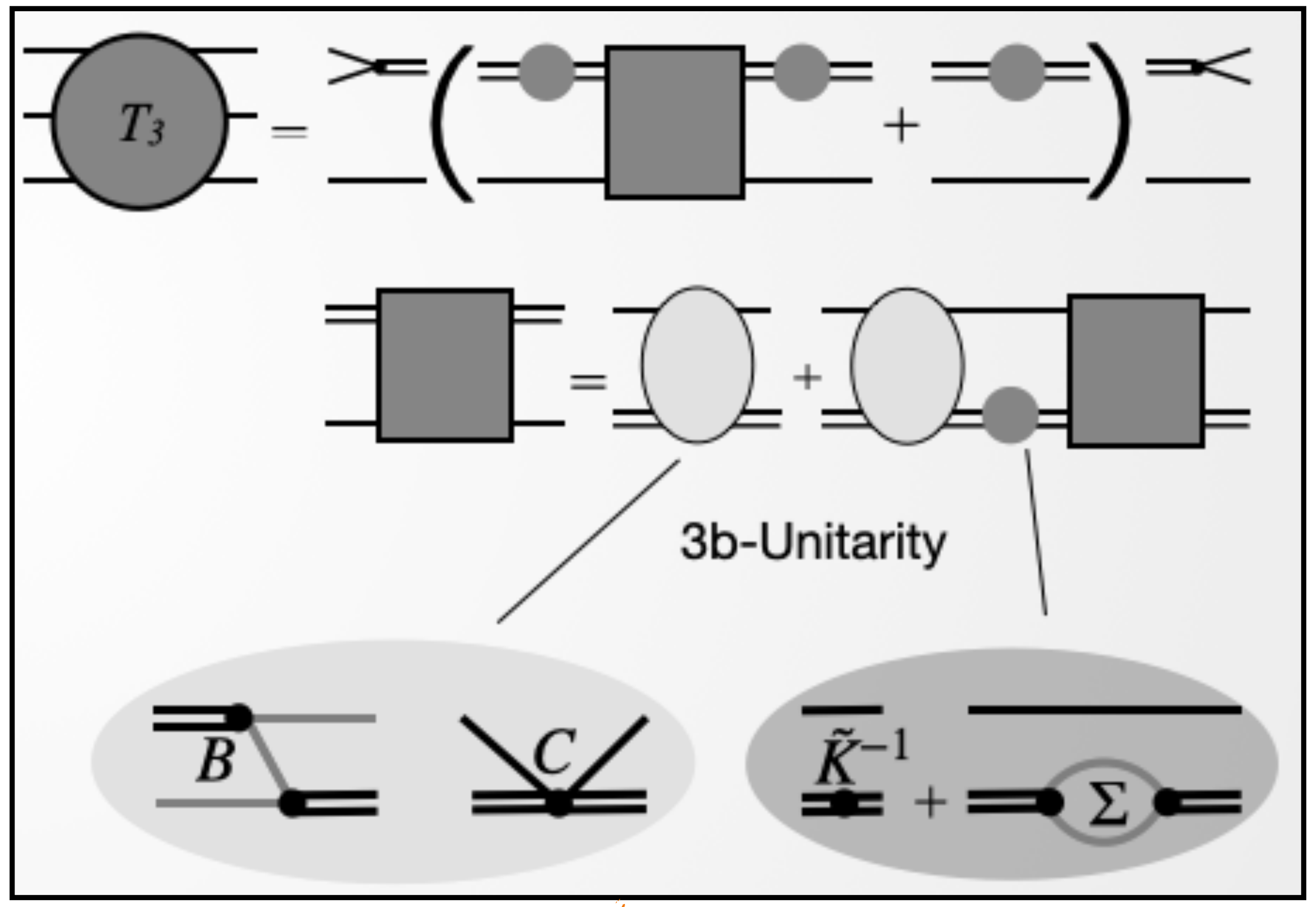


A unitary "T-matrix" accounts for all O(mL) effects!



1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...
2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

3-BODY QUANTISATION CONDITION



Reminder

Finite-volume unitarity (FVU)[1,2]

- separates volume dependent terms
- volume independent terms connect infinite/finite-volume spectra

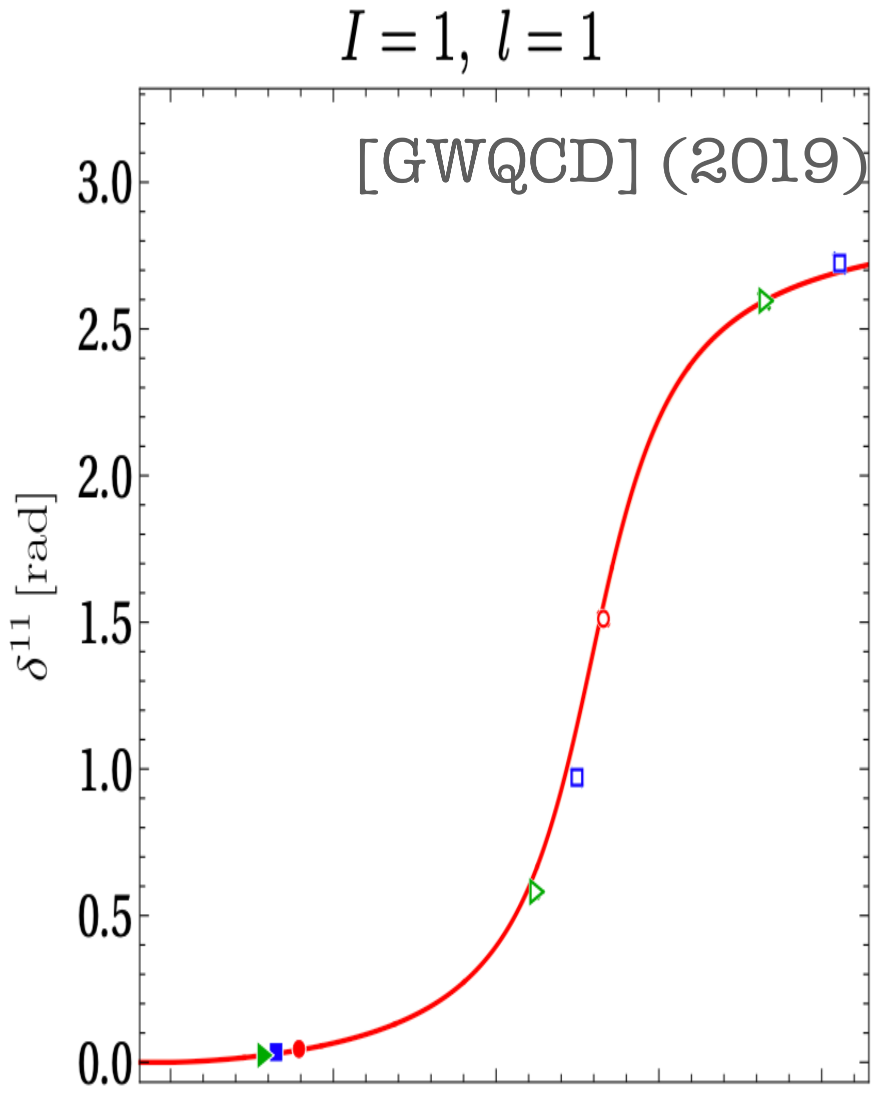
$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

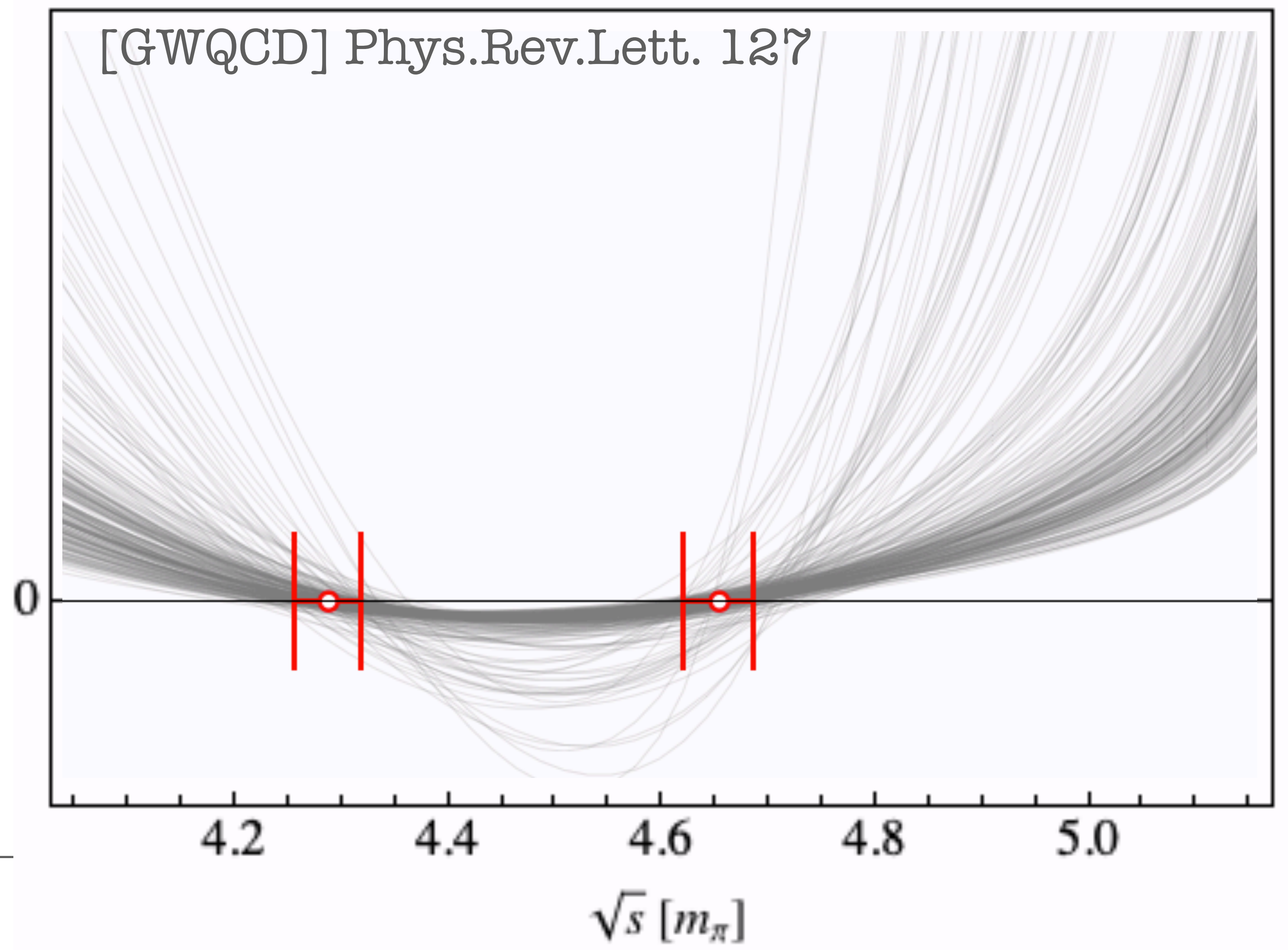
1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, …
2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

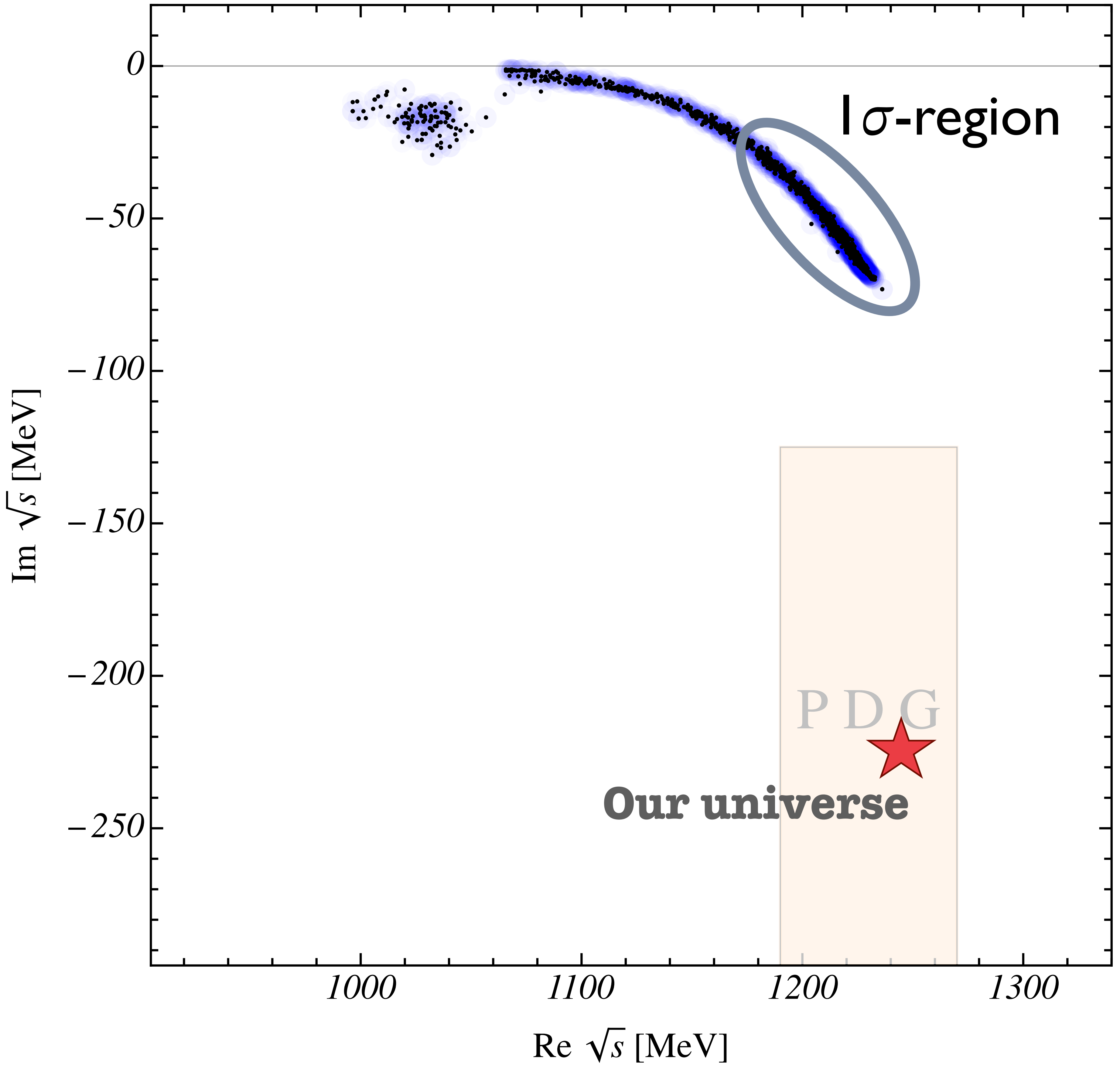
"Heavier Universe"

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$







1) MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127

HADRONS IN A BOX

Finite-volume spectrum is real and discrete!

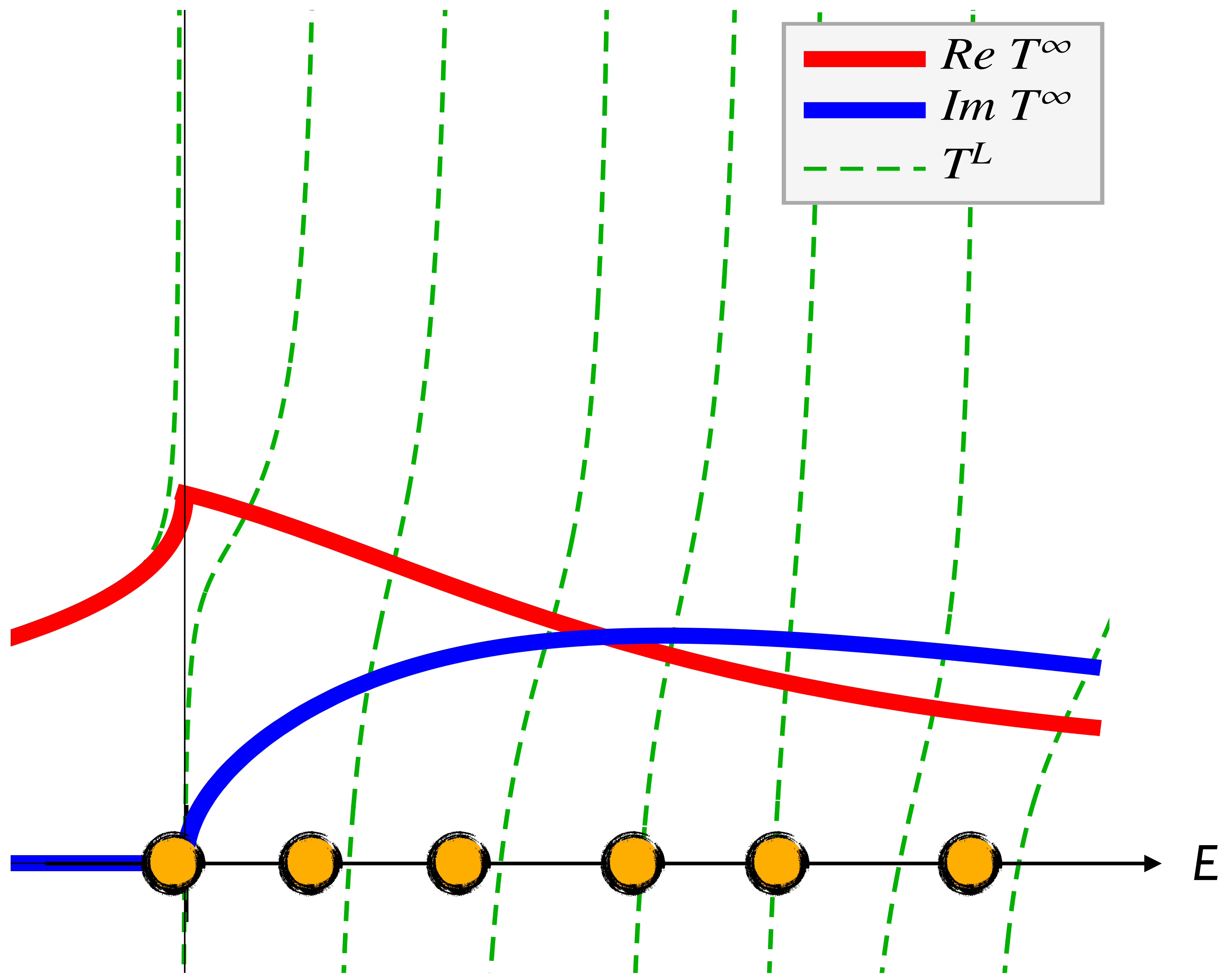
... requires mapping: Quantization condition[1,2]

🤗 Heavily simplified:

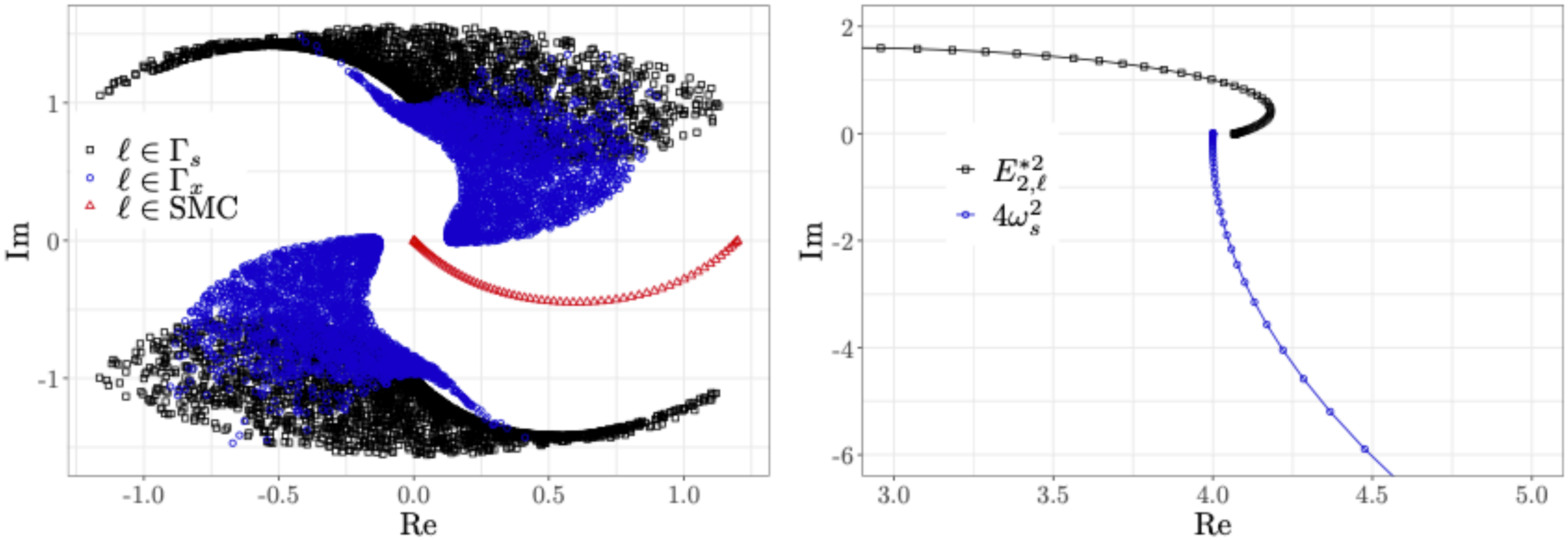
on-shell particle-configurations: $\Delta E \sim mL$

off-shell particle-configurations: $\Delta E \sim e^{-mL}$

🤩 *A unitary "T-matrix" accounts for all O(mL) effects!*



1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...
2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);


$\ell \in \Gamma_s$
$\ell \in \Gamma_x$
$\ell \in$ SMC
Im
Re
1
0
-1
-1.0
-0.5
0.0
0.5
1.0
$E^{*2}_{2,\ell}$
$4\omega_s^2$
2
0
-2
-4
-6
3.0
3.5
4.0
4.5
5.0

Current frontier: 3-body dynamics from LQCD

- ➥ 3-body Quantization Conditions[1]
  - ➥ RFT / FVU / NREFT
    - ➥ many perturbatively interacting systems are studied[2]

$$0 = \det\left(L^3\left(\tilde{F}/3 - \tilde{F}(\tilde{K}_2^{-1} + \tilde{F} + \tilde{G})^{-1}\tilde{F}\right)^{-1} + K_{\mathrm{df},3}\right)$$ RFT

$$0 = \det\left(B_0 + C_0 - E_L\left(K^{-1}/(32\pi) + \Sigma_L\right)\right)$$ FVU

3-body force

2-body interaction

one-particle exchange

2-body self-energy

1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo....

Reviews:

Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019);

MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

2) MM/Döring PRL122(2019); Blanton et al. PRL 124 (2020); Hansen et al. PRL 126 (2021); ....

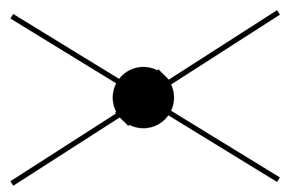
AVOIDED LEVEL CROSS

Variate g($\varphi_1 \rightarrow \varphi_0\varphi_0\varphi_0$) coupling:

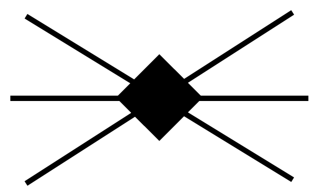
- avoided level crossing becomes wider
- RFT and FVU

g		a	m_1	c_0	c_1	m'_1	c'_0	c'_1	χ^2_{dof}
5	FVU	−0.1512(9)	3.0229(1)	−0.0188(35)	–	–	–	–	2.9
	RFT	−0.1522(12)	–	–	–	3.0232(2)	31.6(8.4)	–	2.5
	FVU	−0.1569(12)	3.0233(2)	−0.0297(57)	2.29(38)	–	–	–	1.5
	RFT	−0.1571(10)	–	–	–	3.0237(2)	37.6(9.0)	2789(540)	1.5
10	FVU	−0.1521(11)	3.0205(2)	−0.0475(66)	–	–	–	–	1.7
	RFT	−0.1531(13)	–	–	–	3.0212(3)	80(14)	–	1.6
	FVU	−0.1549(16)	3.0205(2)	−0.0595(99)	0.93(41)	–	–	–	1.5
	RFT	−0.1563(27)	–	–	–	3.0213(3)	97(16)	1773(980)	1.4
20	FVU	−0.1444(11)	3.0184(2)	−0.1136(77)	–	–	–	–	1.6
	RFT	−0.1450(17)	–	–	–	3.0199(2)	178(17)	–	1.6
	FVU	−0.1464(14)	3.0183(2)	−0.1363(148)	0.84(39)	–	–	–	1.3
	RFT	−0.1484(16)	–	–	–	3.0200(2)	210(23)	2227(600)	1.2

$$q^* \cot\delta = \frac{1}{aM_0}$$



$$C = \frac{c_0}{E_3^3 - m_1^2} + c_1$$



... same fit quality

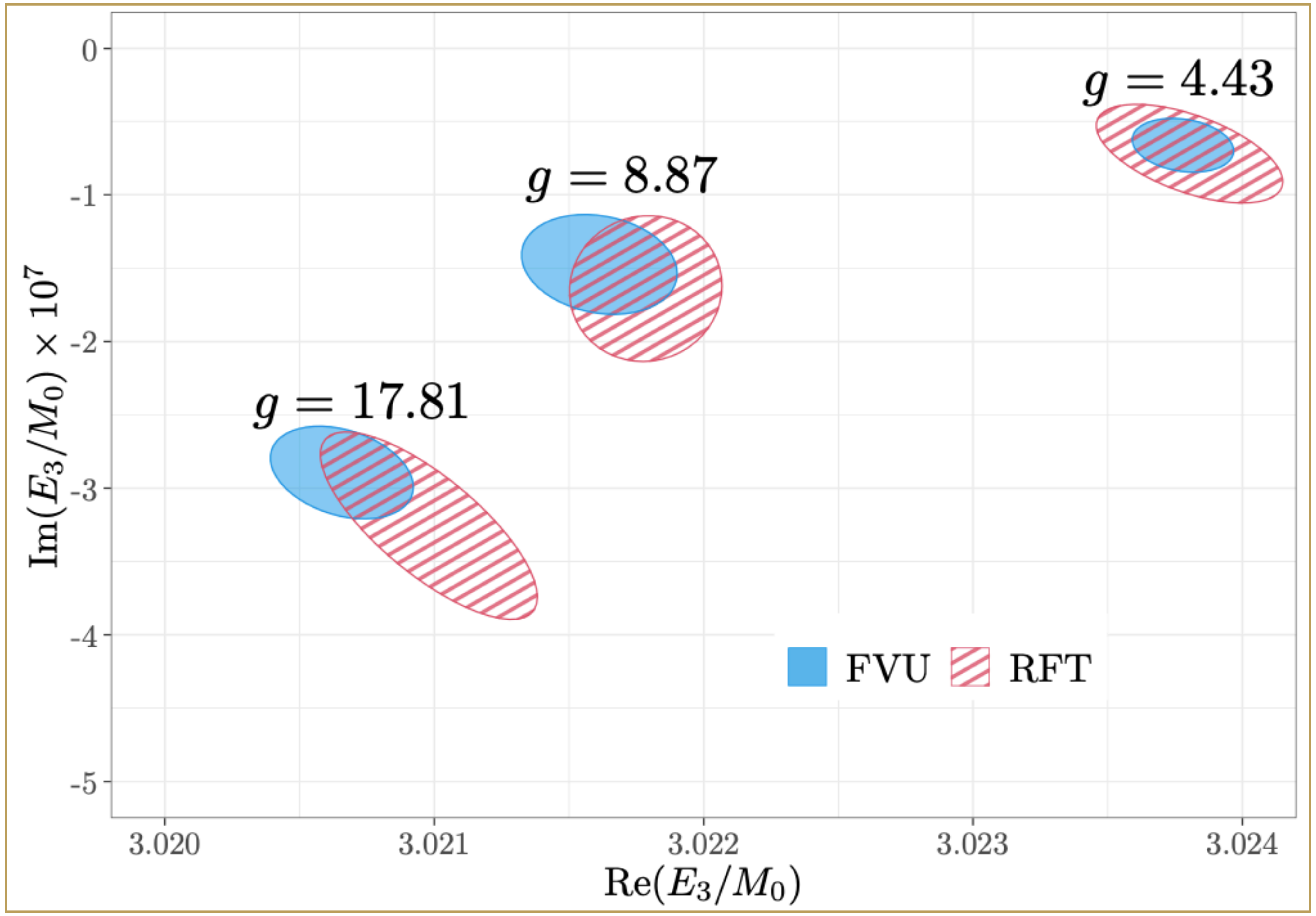
... observables determined consistently

Pole positions

- FVU: complex energy-plane analysis[1]

  -- resonance width grows ~ ***g²***

  -- avoided level crossing gap >> width
- Similarly from RFT with Breit-Wigner like approximation



1) Sadasivan/MM/.. Phys.Rev.D 101 (2020)