Transport coefficients for heavy quarks and quarkonia

Jacopo Ghiglieri, SUBATECH, Nantes

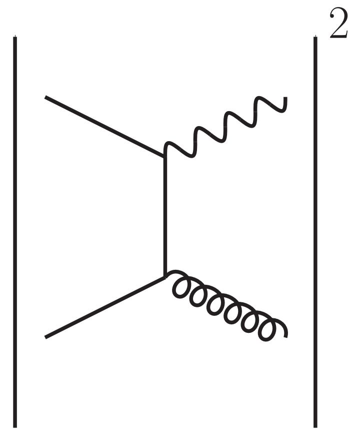
EMMI workshop, Hirschegg, January 19th 2024

In this talk

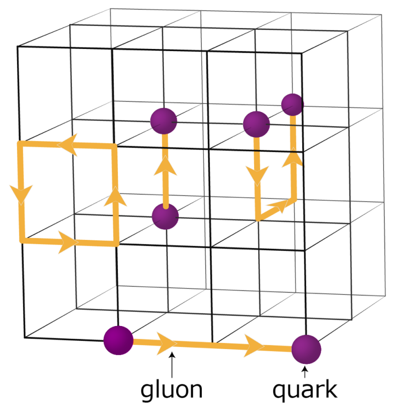
- Heavy quark diffusion in the quark-gluon plasma and force-force correlators

- Heavy quarkonia in the quark-gluon plasma: open quantum systems and force-force correlators

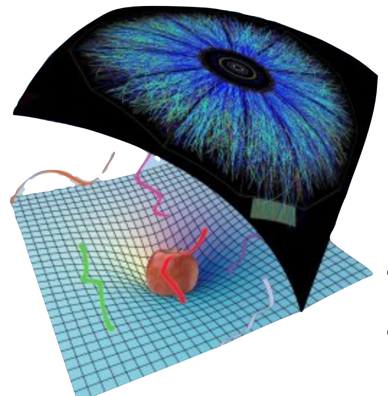
- Force-force correlators, their similarities and differences



in perturbation theory



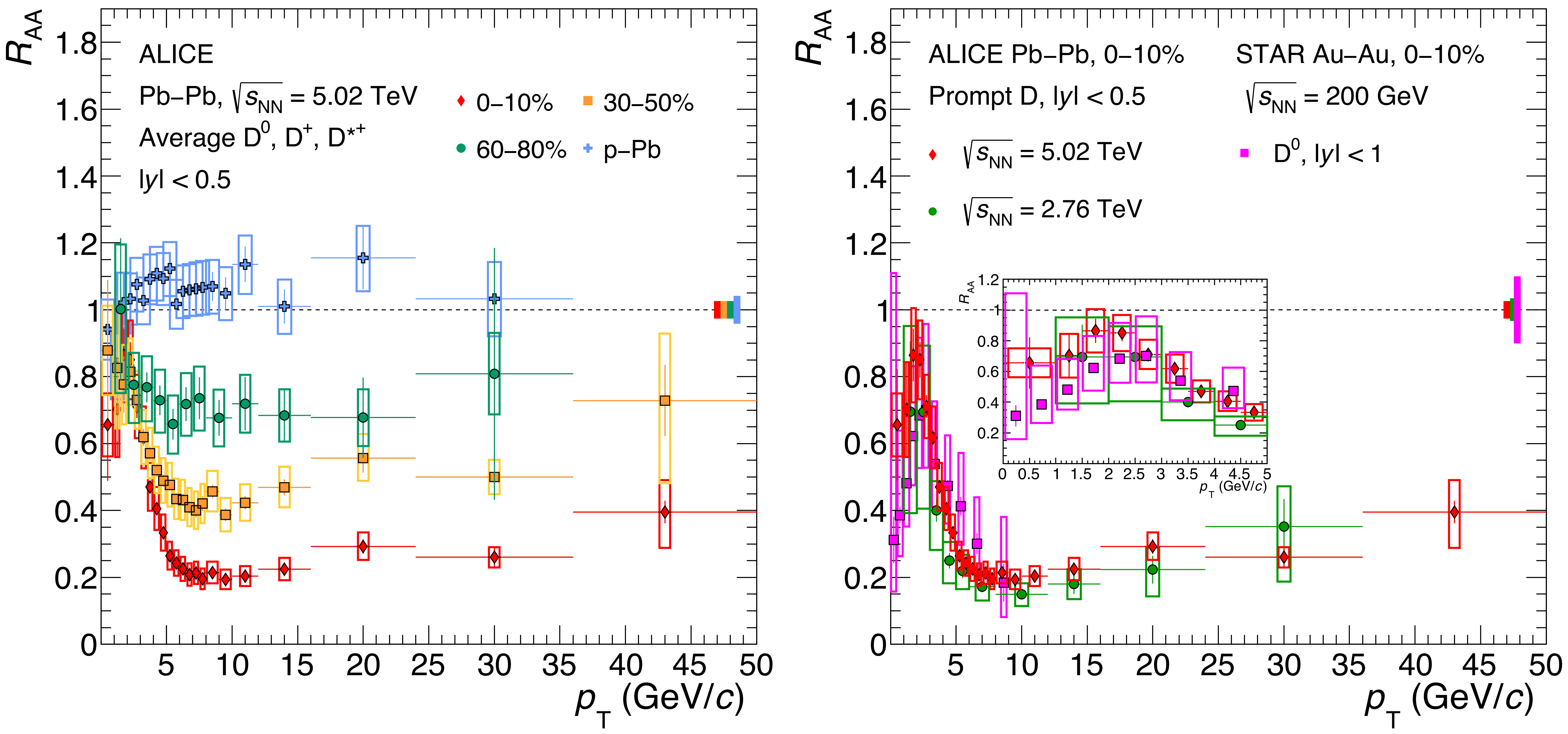
on the lattice



in holography

Heavy-quark diffusion

Heavy quark diffusion



ALICE, **JHEP0122** (2022)

Heavy quark diffusion, *ab initio*

- Long story, see Pol-Bernard's talk later for the many approaches, phenomenological implementations and consequences
- Main idea: rare, isolated heavy quarks undergoing diffusion (Brownian motion) in the quark-gluon plasma

"contemplator enim, cum solis lumina cumque
inserti fundunt radii per opaca domorum:
multa minuta modis multis per inane videbis
corpora misceri radiorum lumine in ipso
et vel ut aeterno certamine proelia pugnas
edere turmatim certantia nec dare pausam,
conciliis et discidiis exercita crebris;
conicere ut possis ex hoc, primordia rerum
quale sit in magno iactari semper inani."

"behold whenever the sun's light and the rays, let in, pour down
Across dark halls of houses: thou wilt see
The many mites in many a manner mixed
Amid a void in the very light of the rays,
And battling on, as in eternal strife,
And in battalions contending without halt,
In meetings, partings, harried up and down.
From this thou mayest conjecture of what sort
The ceaseless tossing of primordial seeds
Amid the mightier void."

Lucretius, De Rerum Natura, **II 110** (~55 BC)

Heavy quark diffusion: Langevin approach

- Long story, see Pol-Bernard's talk later
- Main idea: rare, isolated heavy quarks undergoing diffusion (Brownian motion) in the quark-gluon plasma
- In the non-relativistic limit $\dot{\mathbf{p}} = -\eta_D \mathbf{p} + \mathbf{f}(t)$ ~~Newton~~ Langevin equation with a drag coefficient and a random force with $\langle \mathbf{f}(t) \rangle = 0$ and a momentum broadening coefficient

$$\langle f_i(t) f_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

Svetitsky **PRD37** (1988) Moore Teaney **PRC71** (2005)

Heavy quark diffusion: Langevin approach

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$$\langle f_i(t) f_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

- Equilibration $\left(\lim_{t\to\infty} \langle \mathbf{p}(t)^2 \rangle = 3MT = \frac{3\kappa}{2\eta_D} \right)$ entails an Einstein relation $\eta_D = \frac{\kappa}{2MT}$

Svetitsky **PRD37** (1988) Moore Teaney **PRC71** (2005)

Heavy quark diffusion: broadening coefficient

- In the non-relativistic limit $\dot{\mathbf{p}} = -\eta_D \mathbf{p} + \mathbf{f}(t)$ ~~Newton~~ Langevin equation with a drag coefficient and a random force with $\langle \mathbf{f}(t) \rangle = 0$ and a momentum broadening coefficient

$$\langle f_i(t) f_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

- 3κ can then be understood as the momentum picked up per unit time

Svetitsky **PRD37** (1988) Moore Teaney **PRC71** (2005)
Bouttefeux Laine **JHEP1220** (2020): detailed analysis of subtleties & relativistic corrections

Heavy quark diffusion: broadening coefficient

$$\langle f_i(t) f_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

- From the (un)correlation relation we have

$$\kappa = \frac{1}{3} \int_{-\infty}^{+\infty} dt \, \langle \mathbf{f}(t) \cdot \mathbf{f}(0) \rangle$$

👋 We can identify the random force with the Lorentz force $\mathbf{F}$. For a NR quark $\mathbf{F} = g\mathbf{E}$ and thus

$$\kappa \sim \frac{g^2}{3} \int_{-\infty}^{+\infty} dt \, \langle \mathbf{E}(t) \cdot \mathbf{E}(0) \rangle + \mathcal{O}(\mathbf{v}^2)$$

Casalderrey-Solana Teaney **PRD74** (2006) Caron-Huot Laine Moore **JHEP0409** (2009)
Bouttefeux Laine **JHEP1220** (2020): detailed analysis of subtleties & relativistic corrections

Heavy quark diffusion: broadening coefficient

- Color anyone surprised? Color, anyone?

$$\kappa_Q = \frac{g^2}{3N_c}\int_{-\infty}^{+\infty} dt\, \mathrm{Tr}\langle U(-\infty, t)E_i(t)U(t,0)E_i(0)U(0,-\infty)\rangle$$

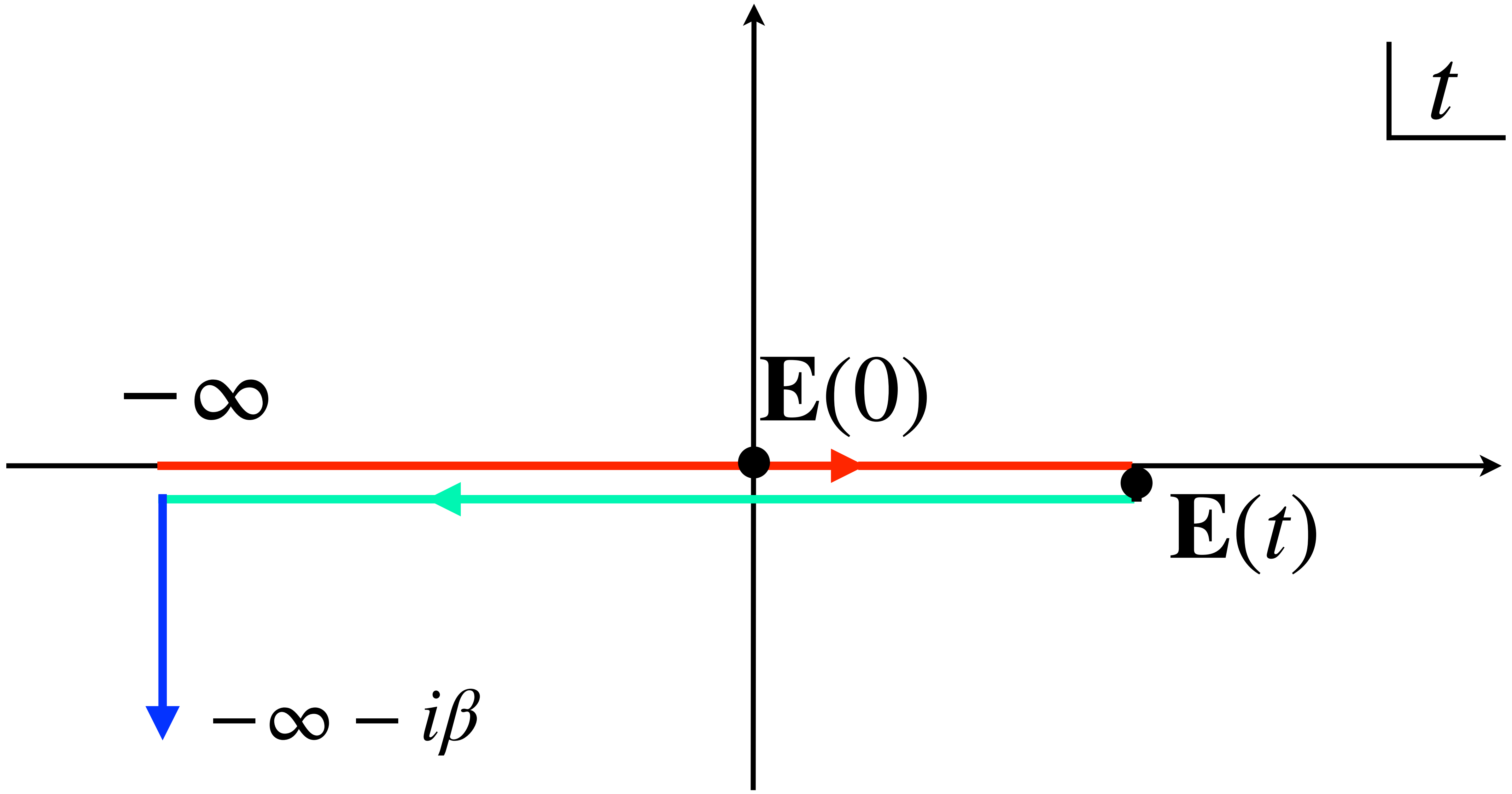
- Wilson lines along the time axis: connection to the density matrix of the system at early time
- Physically: color rotation and interactions always possible for the heavy-quark probe

Casalderrey-Solana Teaney **PRD74** (2006) Caron-Huot Laine Moore **JHEP0409** (2009)

Heavy quark diffusion: broadening coefficient

$$\kappa_Q = \frac{g^2}{3N_c}\int_{-\infty}^{+\infty} dt\,\mathrm{Tr}\langle U(-\infty,t)E_i(t)U(t,0)E_i(0)U(0,-\infty)\rangle$$

- Operator living on the Schwinger-Keldysh contour: naturally emerges from having to evolve the initial density matrix



Casalderrey-Solana Teaney **PRD74** (2006) Caron-Huot Laine Moore **JHEP0409** (2009)

Heavy quark diffusion: broadening coefficient

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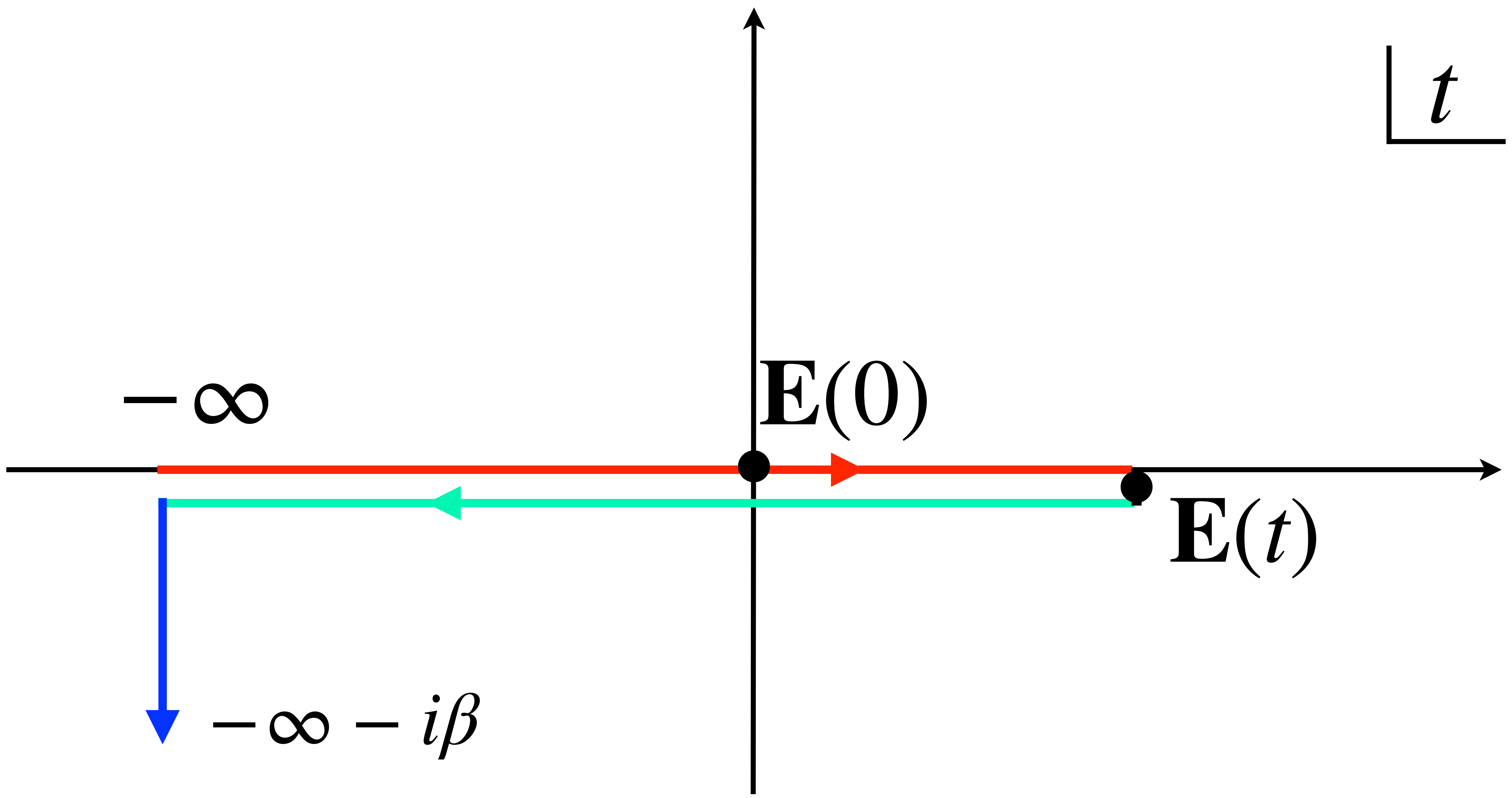
- Contour-ordered fields

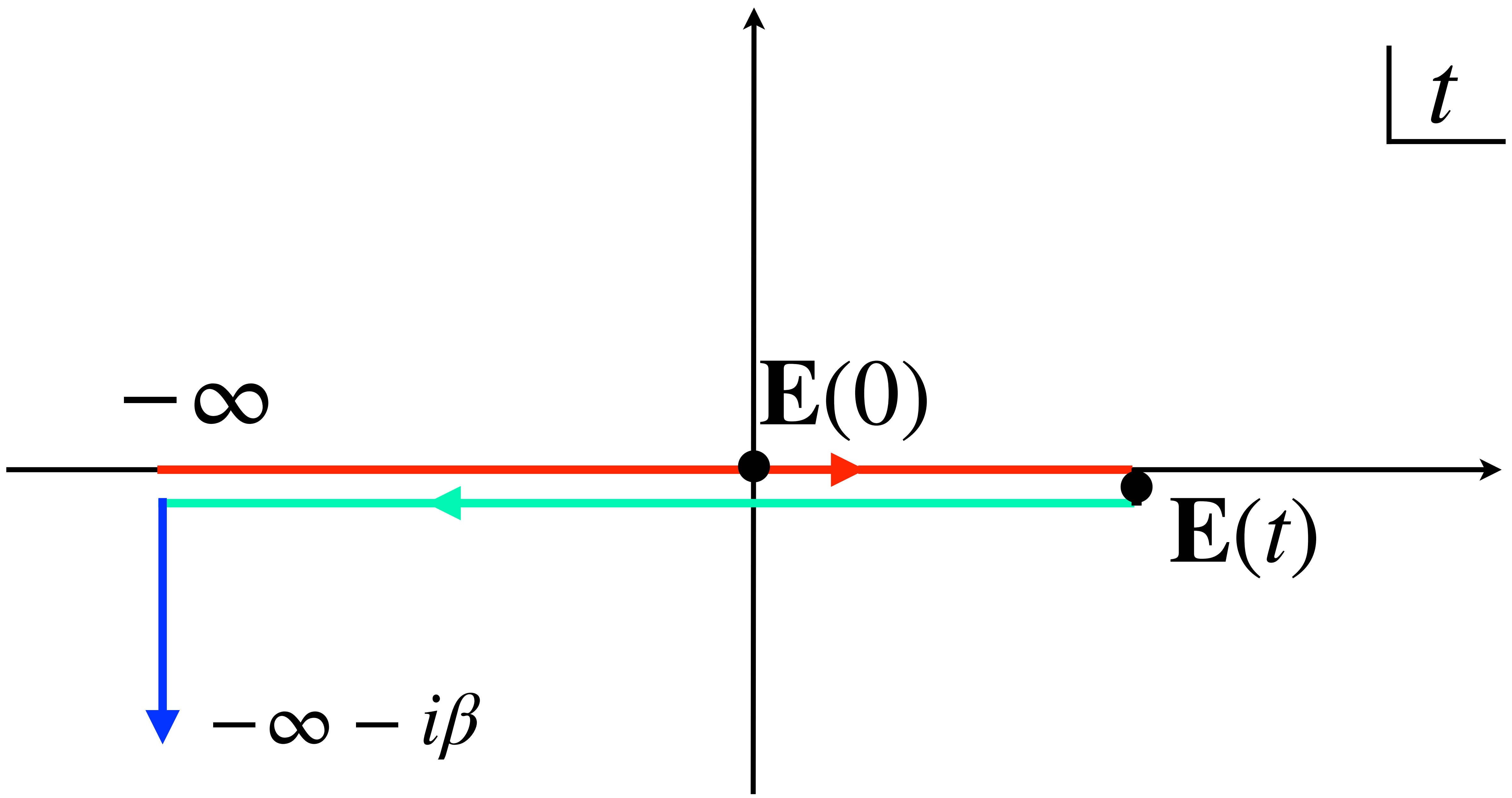


Casalderrey-Solana Teaney **PRD74** (2006) Caron-Huot Laine Moore **JHEP0409** (2009)

Heavy quark diffusion: broadening coefficient

$$\kappa_Q = \frac{g^2}{3N_c}\int_{-\infty}^{+\infty} dt\, \mathrm{Tr}\langle U(-\infty, t)E_i(t)U(t,0)E_i(0)U(0,-\infty)\rangle$$

- Amplitude $\langle \vec{p}\,|\,\vec{0}\rangle$ times conjugate amplitude in a medium

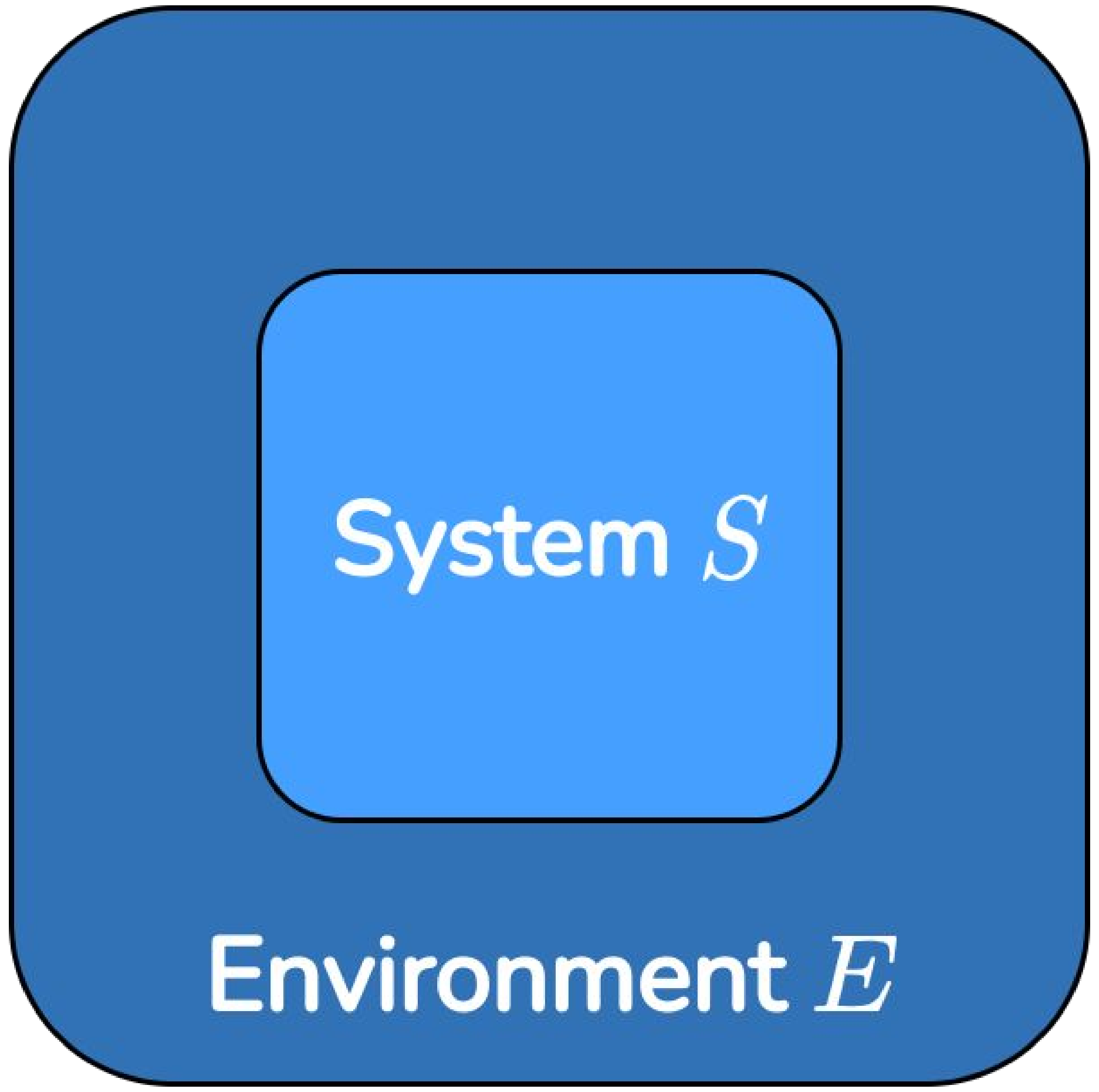


Casalderrey-Solana Teaney **PRD74** (2006) Caron-Huot Laine Moore **JHEP0409** (2009)

Heavy quarkonium suppression

Heavy quarkonium: EFTs and OQS

Open Quantum Systems



- Time evolution by Von-Neumann Equation

$$\frac{d}{dt}\rho = -i[H,\rho]$$

- Environmental d.o.f. not needed Trace out!

$$\rho_S = \mathrm{Tr}_E[\rho]$$

- “Master equation” for the System: **Lindblad Equation**

$$\frac{d\rho_S}{dt} = -i[H_S,\rho_S] + \sum_n \left(C_n \rho_S C_n^\dagger - \frac{1}{2}\left\{C_n^\dagger C_n, \rho_S\right\}\right)$$

Tom Magorsch, Monday

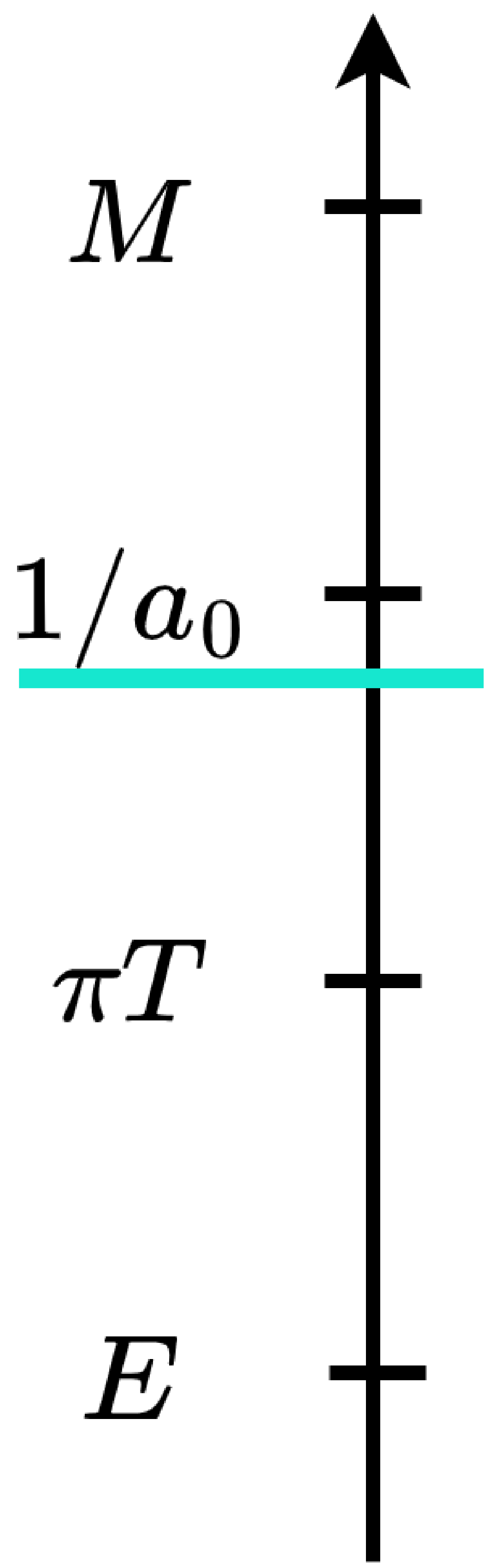
Heavy quarkonium: EFTs and OQS

- After integrating out $mv \sim 1/a_0$ you have pNRQCD

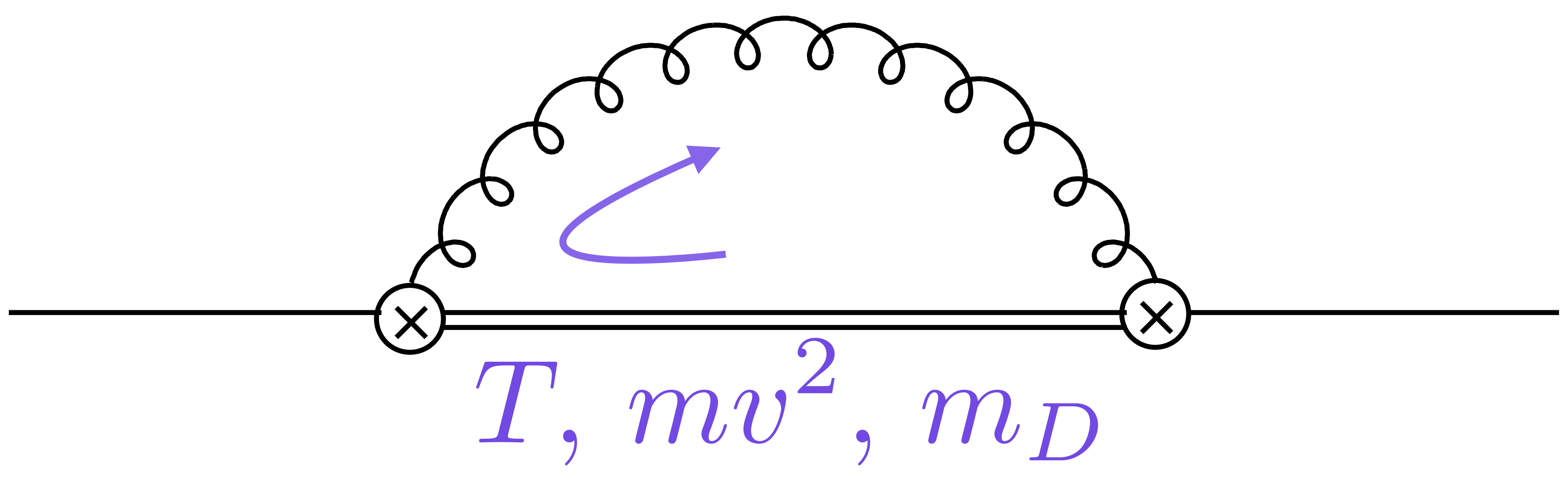
$$\mathcal{L} = \mathcal{L}_{\rm light} + {\rm Tr}\left\{ {\rm S}^\dagger \left[i\partial_0 + \frac{\nabla^2}{m} - V_s \right] {\rm S} + {\rm O}^\dagger \left[iD_0 + \frac{\nabla^2}{m} - V_o \right] {\rm O} \right\}$$

$$+{\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S} + {\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm O} \right\} + \frac{1}{2}{\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm O} + {\rm O}^\dagger {\rm O}\mathbf{r}\cdot g\mathbf{E} \right\} + \dots$$

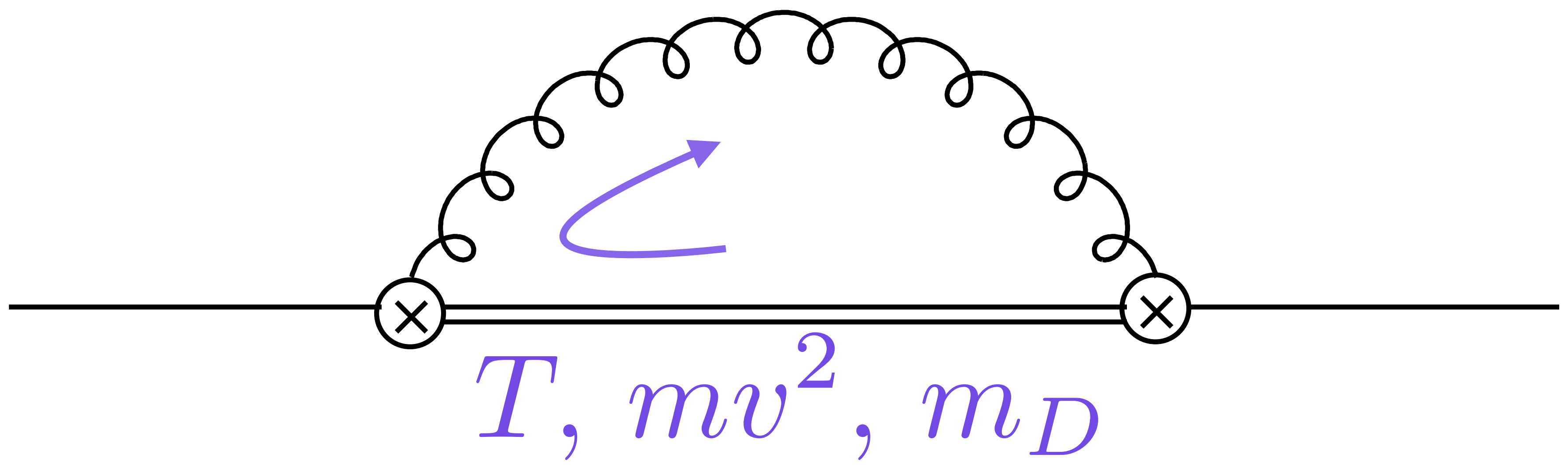
Pineda Soto **NPPS64** (1998) Brambilla Pineda Soto Vairo **NPB566** (2000)



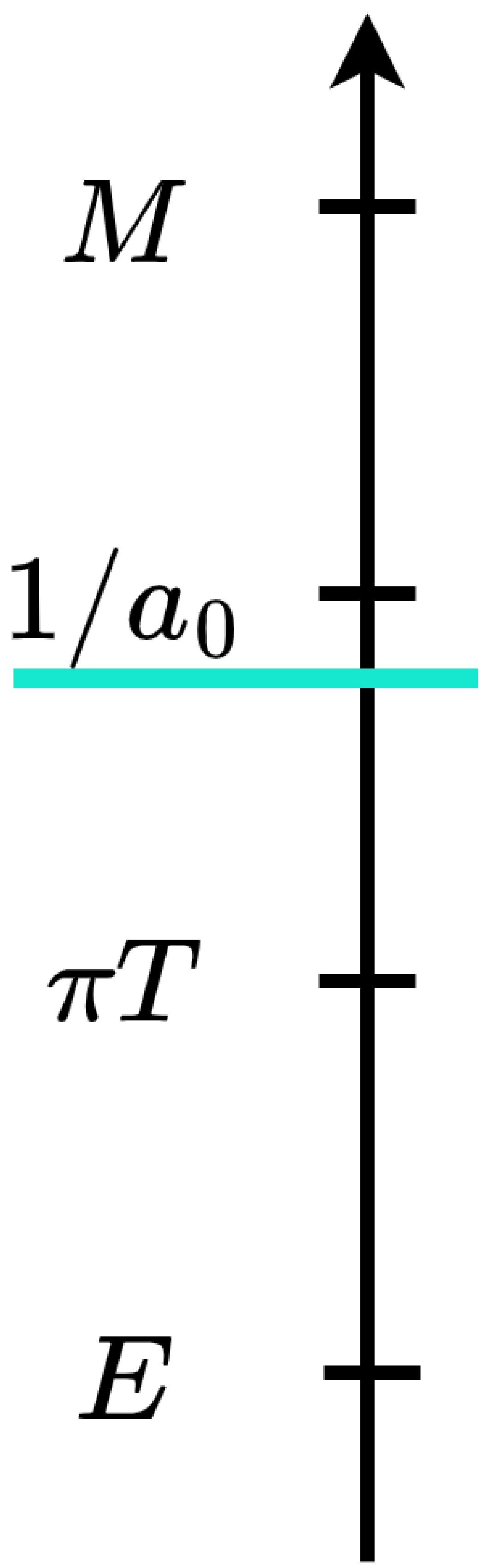
- Contribution from the medium encoded in pNRQCD correlators. Dipole interaction at first order



Heavy quarkonium: EFTs and OQS



$$A_i^{uv} = \frac{g^2}{6N_c}\int_0^\infty \mathrm{d}s\, e^{-ih_u s} r_i e^{ih_v s}\left\langle \tilde{E}_j^a(0,\vec{0})\tilde{E}_j^a(s,\vec{0})\right\rangle$$



- Exploiting $T \gg E$ the phases drop out at first order

$$\kappa_{Q\bar{Q}} = \frac{g^2 T_F}{3N_c}\mathrm{Re}\int_{-\infty}^{+\infty} dt\, \langle \mathrm{T}\, E_i^a(t) U_{ab}(t,0) E_i^b(0)\rangle$$

$$\gamma_{Q\bar{Q}} = \frac{g^2 T_F}{3N_c}\mathrm{Im}\int_{-\infty}^{+\infty} dt\, \langle \mathrm{T}\, E_i^a(t) U_{ab}(t,0) E_i^b(0)\rangle$$

Brambilla Escobedo Soto Vairo (2016-17)

Heavy quarkonium: EFTs and OQS

$$\kappa_{Q\bar{Q}} = \frac{g^2 T_F}{3N_c}\mathrm{Re}\int_{-\infty}^{+\infty} dt\,\langle \mathrm{T}\, E_i^a(t) U_{ab}(t,0) E_i^b(0)\rangle \qquad \gamma_{Q\bar{Q}} = \frac{g^2 T_F}{3N_c}\mathrm{Im}\int_{-\infty}^{+\infty} dt\,\langle \mathrm{T}\, E_i^a(t) U_{ab}(t,0) E_i^b(0)\rangle$$

pNRQCD master equation

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \sum_n \left[C_i^n \rho(t) C_i^{n\dagger} - \frac{1}{2}\left\{ C_i^{n\dagger} C_i^n, \rho(t) \right\} \right],$$

$$H = \begin{pmatrix} h_s + \frac{r^2}{2}\gamma & 0 \\ 0 & h_o + \frac{N_c^2-2}{2(N_c^2-1)}\frac{r^2}{2}\gamma \end{pmatrix} \qquad C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r_i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix}$$

$$C_i^1 = \sqrt{\frac{\kappa(N_c^2-4)}{2(N_c^2-1)}}\, r_i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$h_{s,o} = \vec{p}^{\,2}/M + V_{s,o}$$

Brambilla Escobedo Soto Vairo (2016-17), Tom Magorsch Monday

Heavy quarkonium: EFTs and OQS

$$\kappa_{Q\bar{Q}} = \frac{g^2 T_F}{3N_c} \mathrm{Re} \int_{-\infty}^{+\infty} dt \, \langle \mathrm{T} \, E_i^a(t) U_{ab}(t,0) E_i^b(0) \rangle \qquad \gamma_{Q\bar{Q}} = \frac{g^2 T_F}{3N_c} \mathrm{Im} \int_{-\infty}^{+\infty} dt \, \langle \mathrm{T} \, E_i^a(t) U_{ab}(t,0) E_i^b(0) \rangle$$

- γ plays the role of a mass shift. Recall that κ had amplitude x amplitude* structure for dissipation. γ is the dispersive counterpart, the two forces are both in amplitude or in amplitude*: first you get kicked away from your momentum, then back into it. This forward scattering shifts the dispersion relation

Brambilla Escobedo Soto Vairo (2016-17), Tom Magorsch Monday

Heavy quarkonium: EFTs and OQS

$$\kappa_{Q\bar{Q}} = \frac{g^2 T_F}{3N_c} \mathrm{Re} \int_{-\infty}^{+\infty} dt \, \langle \mathrm{T}\, E_i^a(t) U_{ab}(t,0) E_i^b(0) \rangle \qquad \gamma_{Q\bar{Q}} = \frac{g^2 T_F}{3N_c} \mathrm{Im} \int_{-\infty}^{+\infty} dt \, \langle \mathrm{T}\, E_i^a(t) U_{ab}(t,0) E_i^b(0) \rangle$$

- The Wilson line structure is different here: for the singlet mass shift and width, the adjoint self-energy corresponds to the octet state propagating between the dipole vertices. No interactions or color rotations are possible before the earliest and after the latest dipole interaction

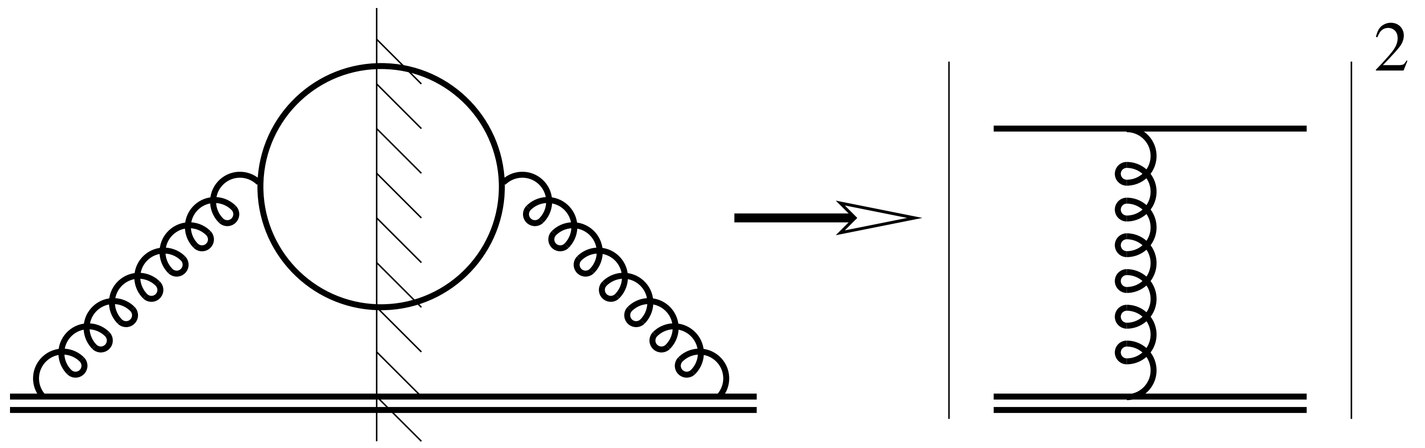
Brambilla Escobedo Soto Vairo (2016-17), Tom Magorsch Monday

Intermediate summary

- κ is a very important transport coefficient, encoding the dissipative dynamics of non relativistic heavy quarks and of quarkonia
- For quarkonia, the mass shift γ is another key ingredient
- What do we know about them? Perturbation theory, lattice, holography
- Are the heavy-quark and quarkonium coefficients the same?

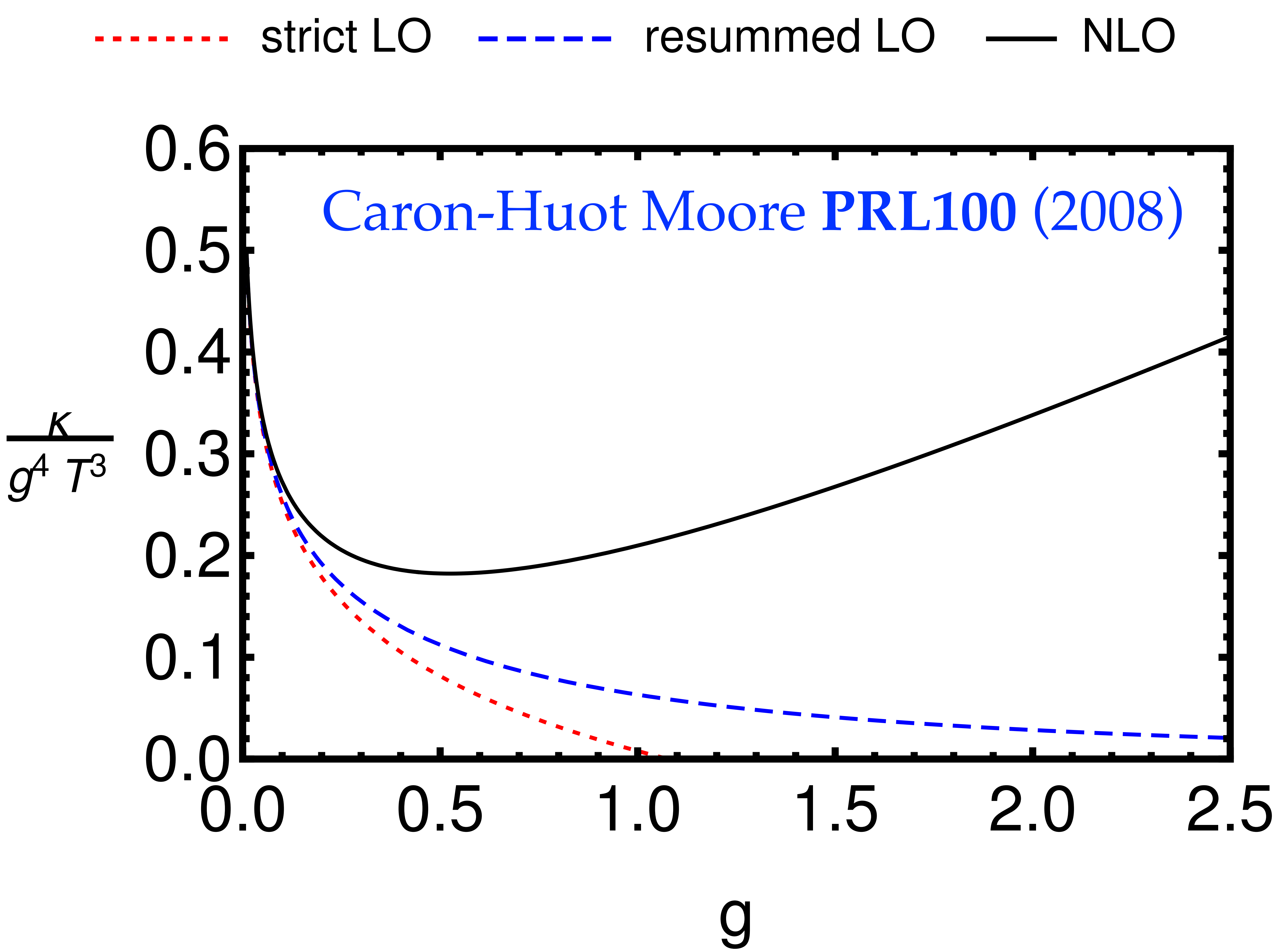
Determining force-force correlators

Perturbative results: κ

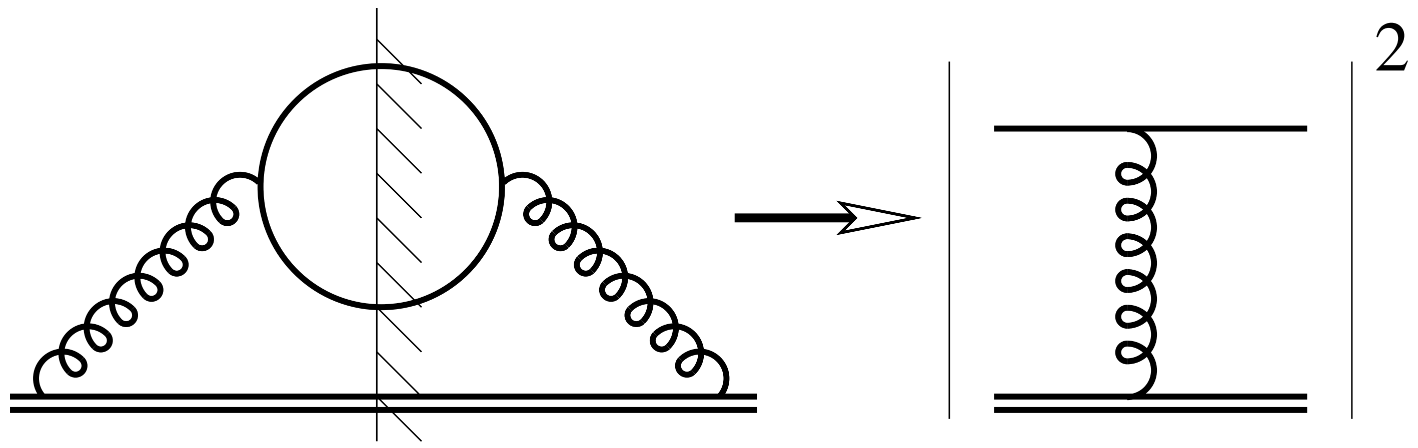


- Leading order from Coulomb scattering off medium constituents Moore Teaney (2004)

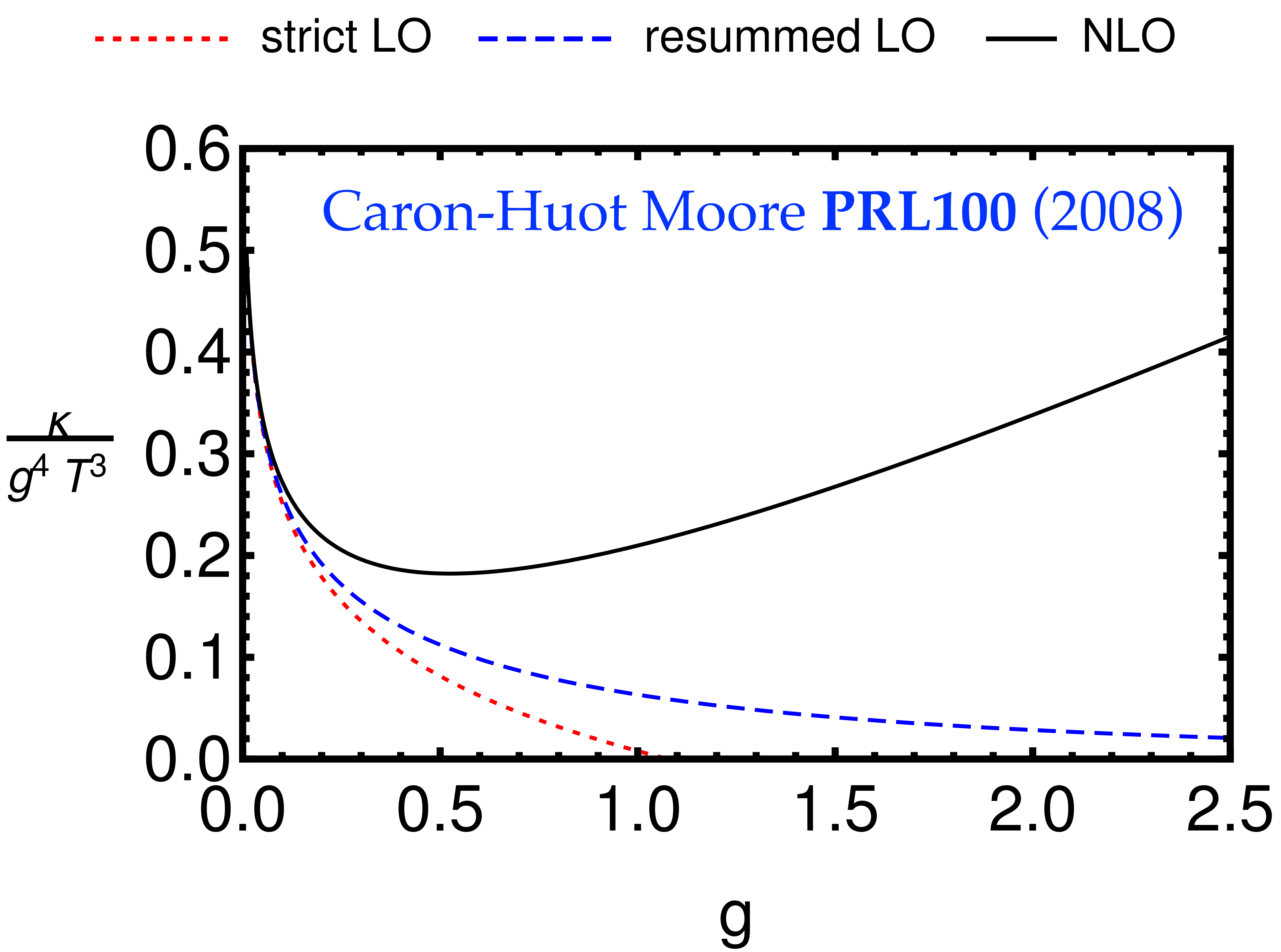
- NLO from soft, classical gluons in the HTL-resummed theory



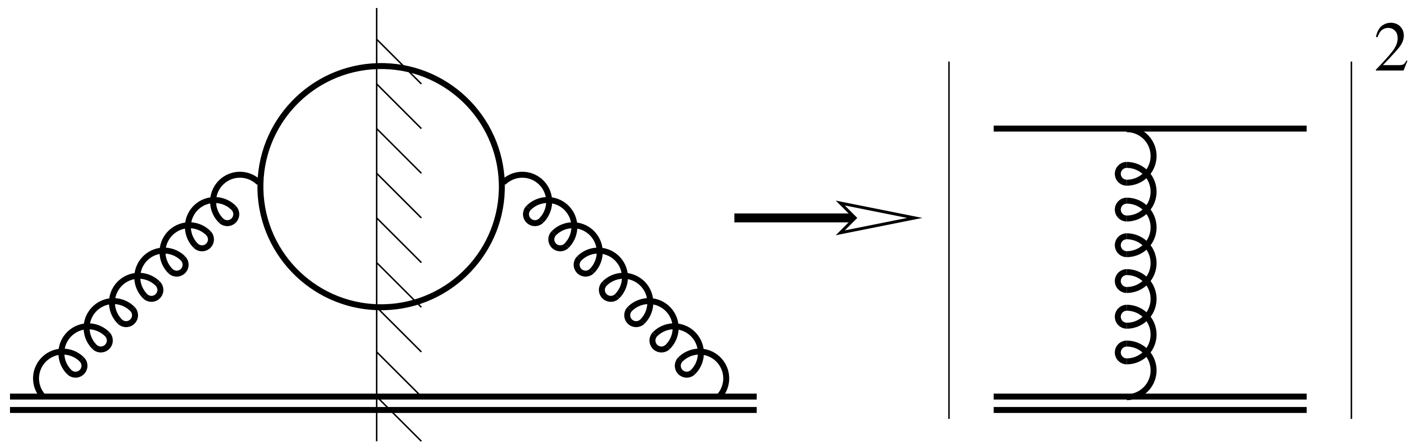
Perturbative results: κ



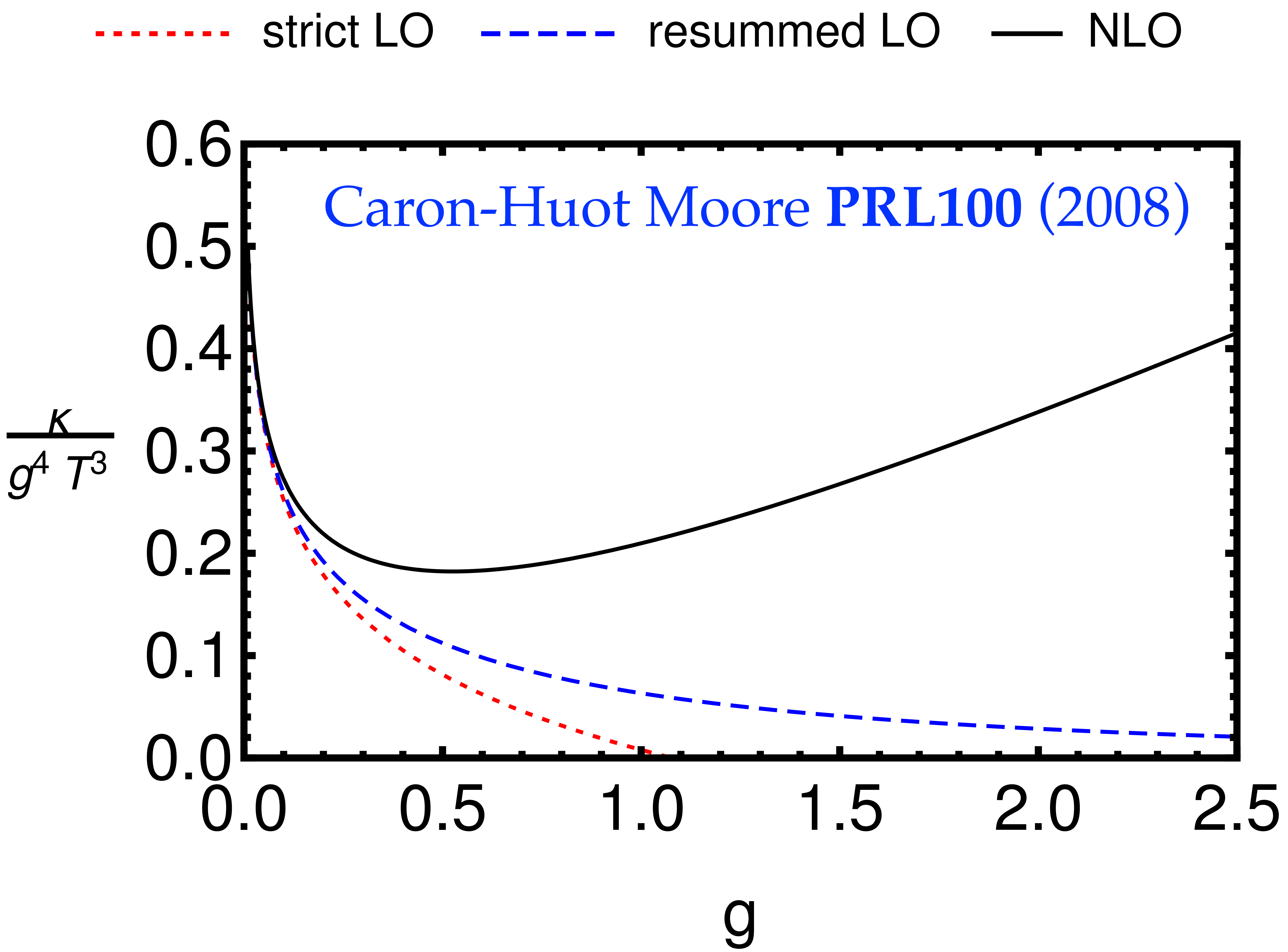
- Leading order from Coulomb scattering off medium constituents Moore Teaney (2004)
- Large $\mathcal{O}(g)$ correction from the classical modes



Perturbative results: κ



- Leading order from Coulomb scattering off medium constituents Moore Teaney (2004)
- NLO corrections are classical, difference between the two κ commutes away



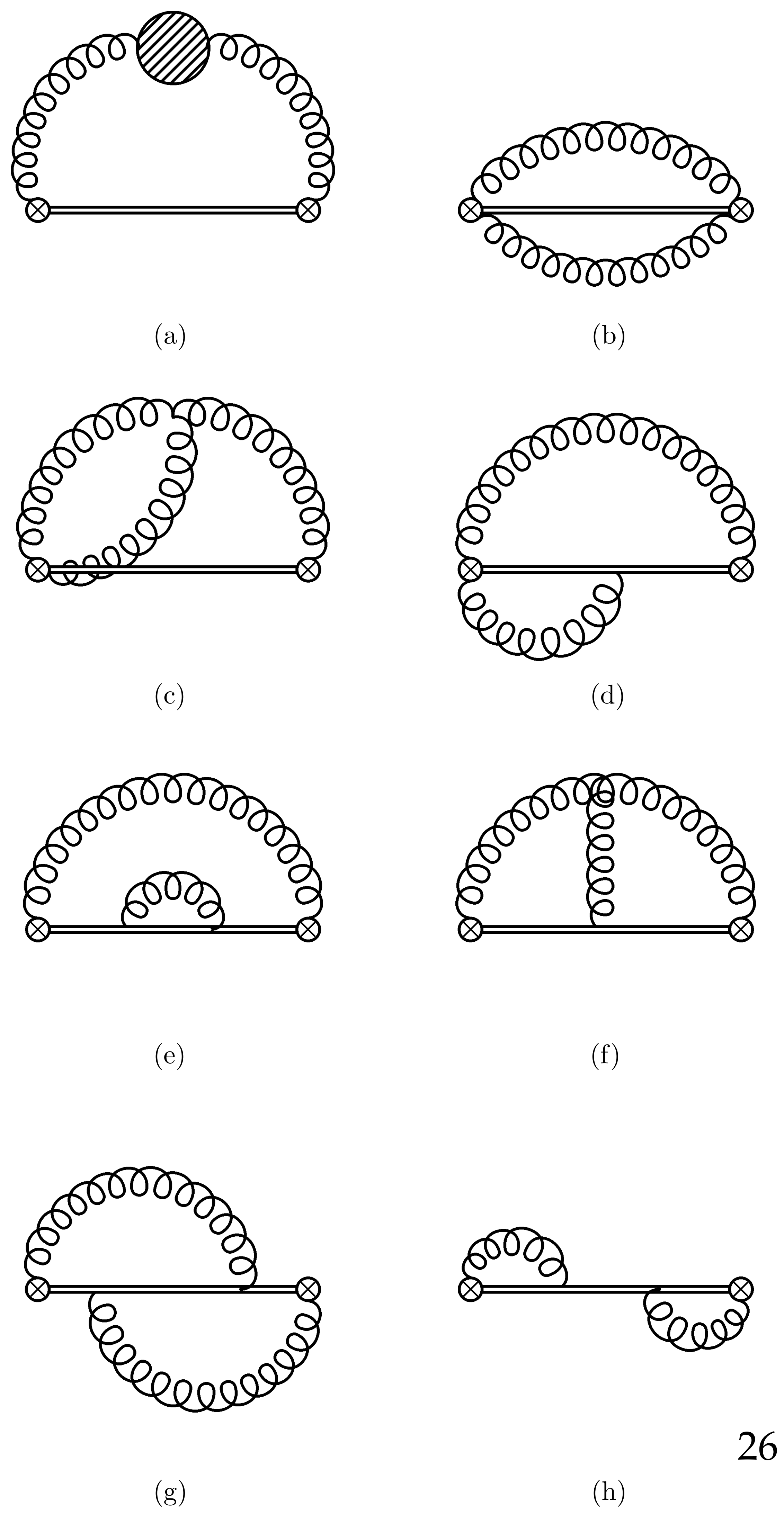
Perturbative results: γ

- Leading order from the forward-scattering contribution from these diagram and classical NLO correction

$$\gamma_{Q\bar{Q}} = -2\alpha_s^2 T^3 C_F \zeta(3)\left(\frac{4}{3}N_c + N_f\right) + \frac{\alpha_s C_F m_D^3}{3}$$

Brambilla JG Petreczky Vairo **PRD78** (2008)

- Is this consistent with the small $\hat{\gamma}$ resulting from comparisons with lattice? Can we determine this on the lattice?



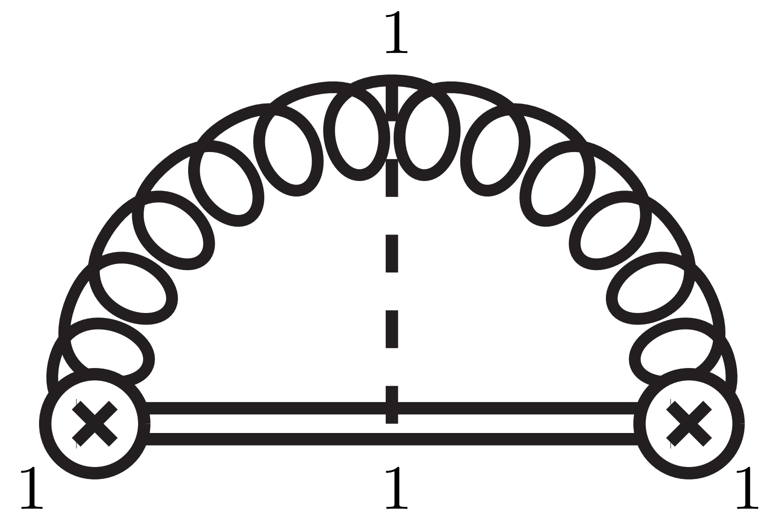
Perturbative results: γ

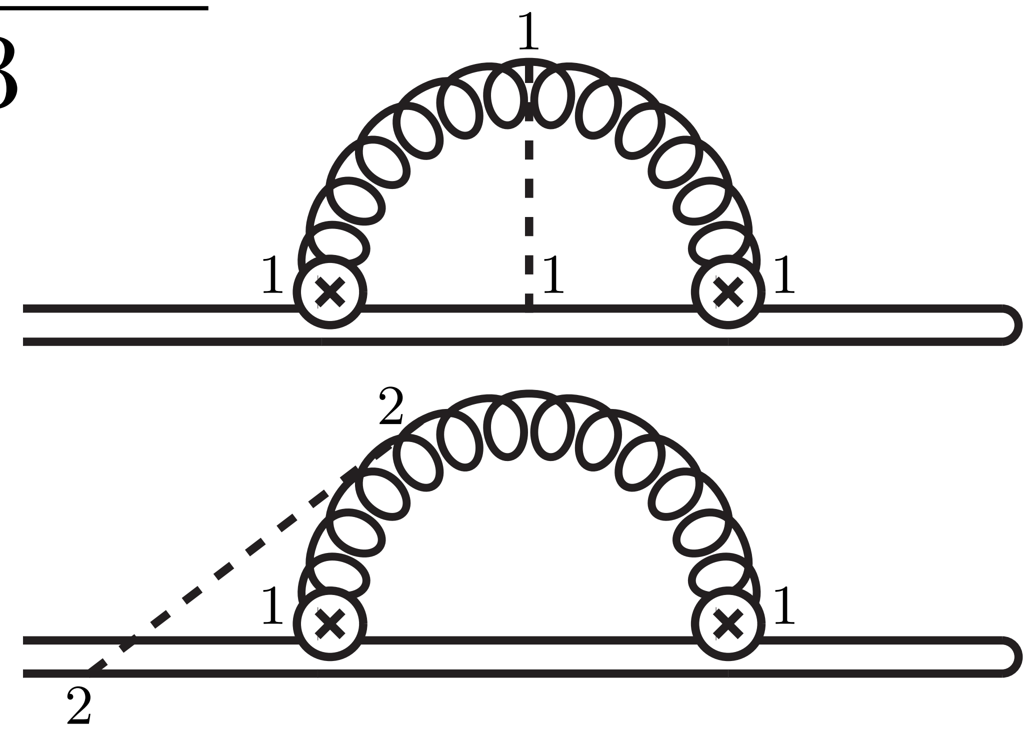
- Imagine defining a single-quark equivalent of $\gamma_{Q\bar{Q}}$ (connection to lattice to be clarified soon)

$$\gamma_Q = \frac{g^2}{3N_c}\mathrm{Im}\int_{-\infty}^{+\infty} dt\,\mathrm{Tr}\langle \mathrm{T}\, U(-\infty,t)E_i(t)(t,0)E_i(0)(0,-\infty)\rangle$$

- This is ~~the same~~ different from the $Q\bar{Q}$ version at leading order!

$$\gamma_Q = -2\alpha_s^2 T^3 C_F \zeta(3)\left(0 + N_f\right) + \frac{\alpha_s C_F m_D^3}{3}$$





Eller JG Moore **PRD99, erratum PRD102** (2019-20)

Perturbative results: γ

- This means that the naive temporal axial gauge cannot be used for observables like γ_Q where what happens at asymptotic times is very important
  Scheihing-Hitschfeld Yao **PRL130** (2023)

- The spectral function associated with the $Q\overline{Q}$ operator is not purely odd in the frequency, because the operators are not self-adjoint

$$\gamma_{Q\bar{Q}} \sim g^2 \mathrm{Im} \int_{-\infty}^{+\infty} dt \, \langle \mathrm{T}\, S^\dagger E_i^a O^a(t) \; O^{b\dagger} E_i^b S(0) \rangle = 2 \int_0^{+\infty} dt \int \frac{d\omega}{2\pi} e^{-i\omega t} [1 + n_{\mathrm{B}}(\omega)] \rho_{EQ\bar{Q}}(\omega)$$

  The even component is responsible for the difference between the two γ. Corresponding Euclidean correlator not periodic

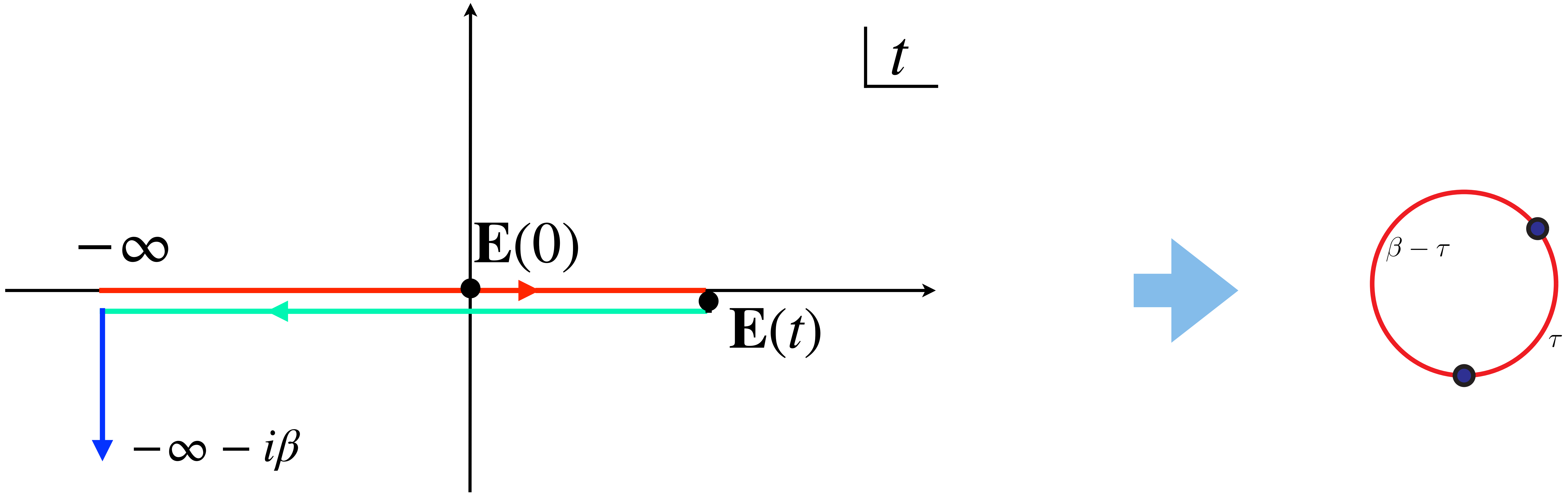
Eller JG Moore **PRD99, erratum PRD102** (2019-20) Scheihing-Hitschfeld Yao **PRD108** (2023)

Heavy quark transport coefficients on the lattice

Heavy quark diffusion on the lattice

- The story starts with κ_Q. Do analytical continuation to Euclidean

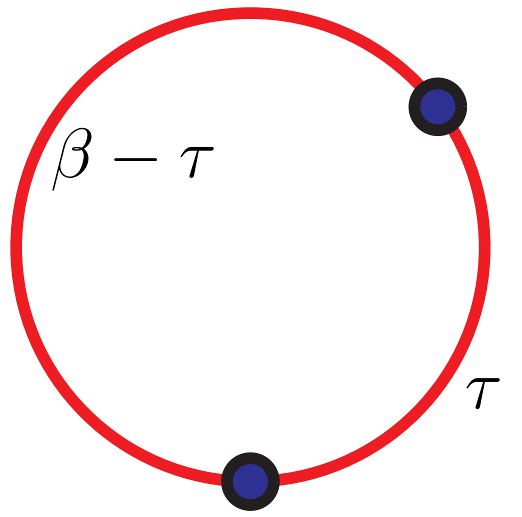
$$\kappa_Q = \frac{g^2}{3N_c}\int_{-\infty}^{+\infty} dt\,\mathrm{Tr}\langle U(-\infty,t)E_i(t)U(t,0)E_i(0)U(0,-\infty)\rangle$$



Heavy quark diffusion on the lattice

- The story starts with κ_Q. Do analytical continuation to Euclidean

$$G_E(\tau) = -\frac{g^2}{3}\frac{\langle \mathrm{ReTr}\,[U(1/T,\tau)E_i(\tau)U(\tau,0)E_i(0)]\rangle}{\langle \mathrm{ReTr}\,[U(1/T,0)]\rangle}$$



- What we want is the spectral function, which is hidden in the convolution integral on the right, and (after many non-trivial steps such as gradient flow) we have a few discrete datapoints on the left

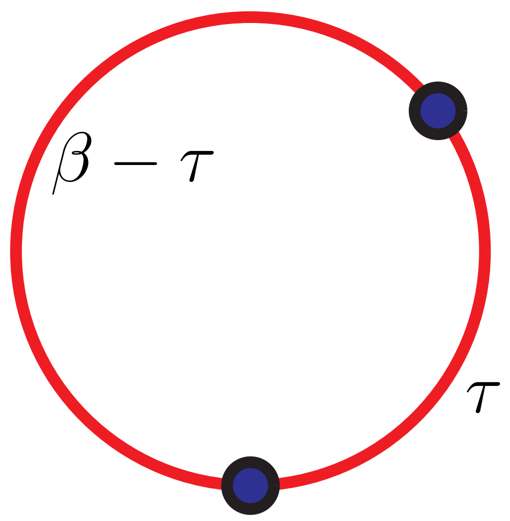
$$G_E(\tau) = \int \frac{d\omega}{2\pi} e^{-i\omega\tau}[1 + n_{\mathrm{B}}(\omega)]\rho_Q(\omega) \qquad \kappa = \lim_{\omega\to 0}\frac{\rho_Q(\omega)}{\omega}$$

- An inverse problem, perhaps with less of a peak to climb

Caron-Huot Laine Moore **JHEP0409** (2009)

Heavy quark diffusion on the lattice

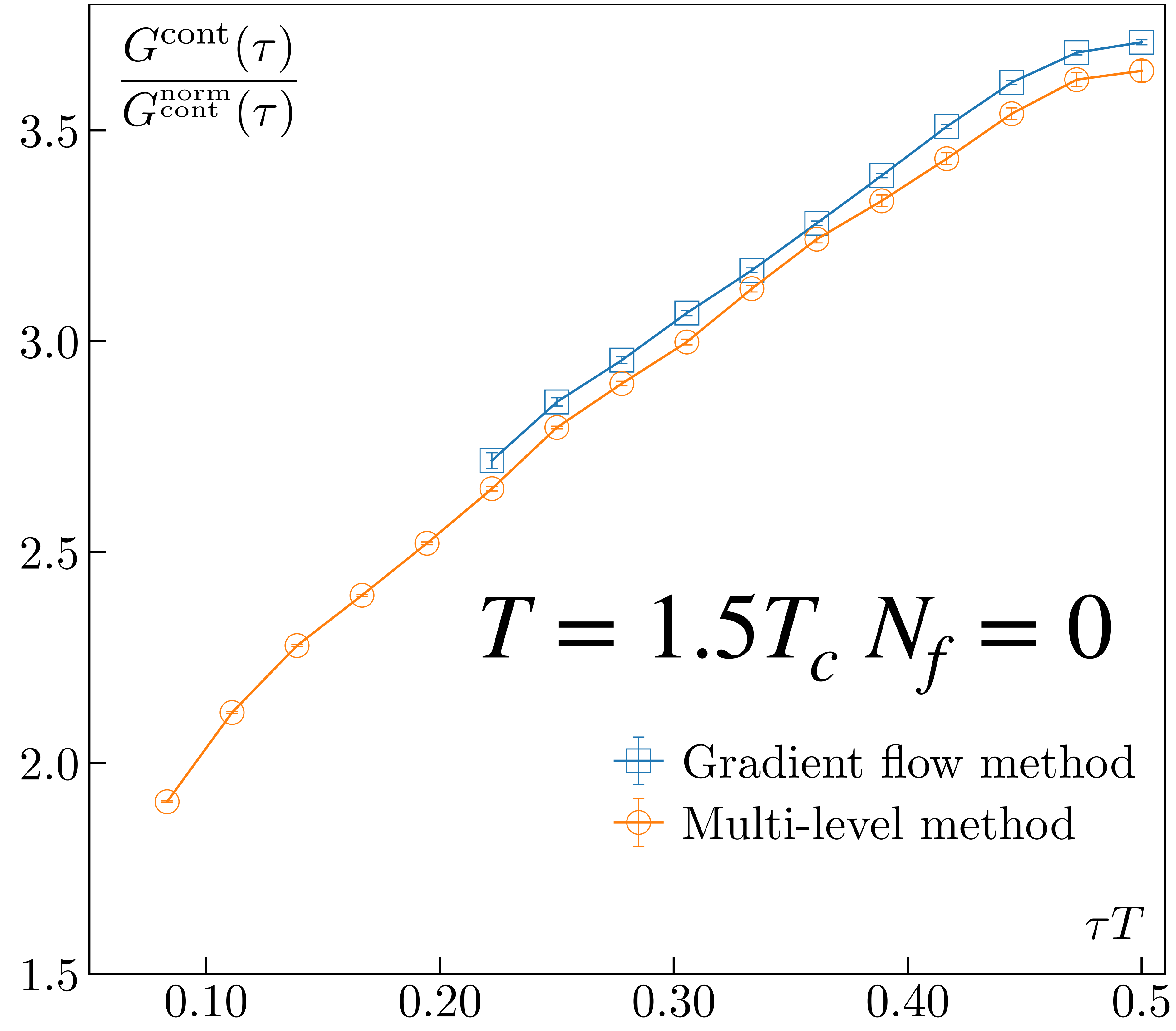
$$G_E(\tau) = -\frac{g^2}{3}\frac{\langle \mathrm{ReTr}\,[U(1/T,\tau)E_i(\tau)U(\tau,0)E_i(0)]\rangle}{\langle \mathrm{ReTr}\,[U(1/T,0)]\rangle}$$



$$G_E(\tau) = \int \frac{d\omega}{2\pi} e^{-i\omega\tau}[1 + n_B(\omega)]\rho_Q(\omega)$$

$$\kappa = \lim_{\omega\to 0}\frac{\rho_Q(\omega)}{\omega}$$

Altenkort *et al.* **PRD103** (2021)

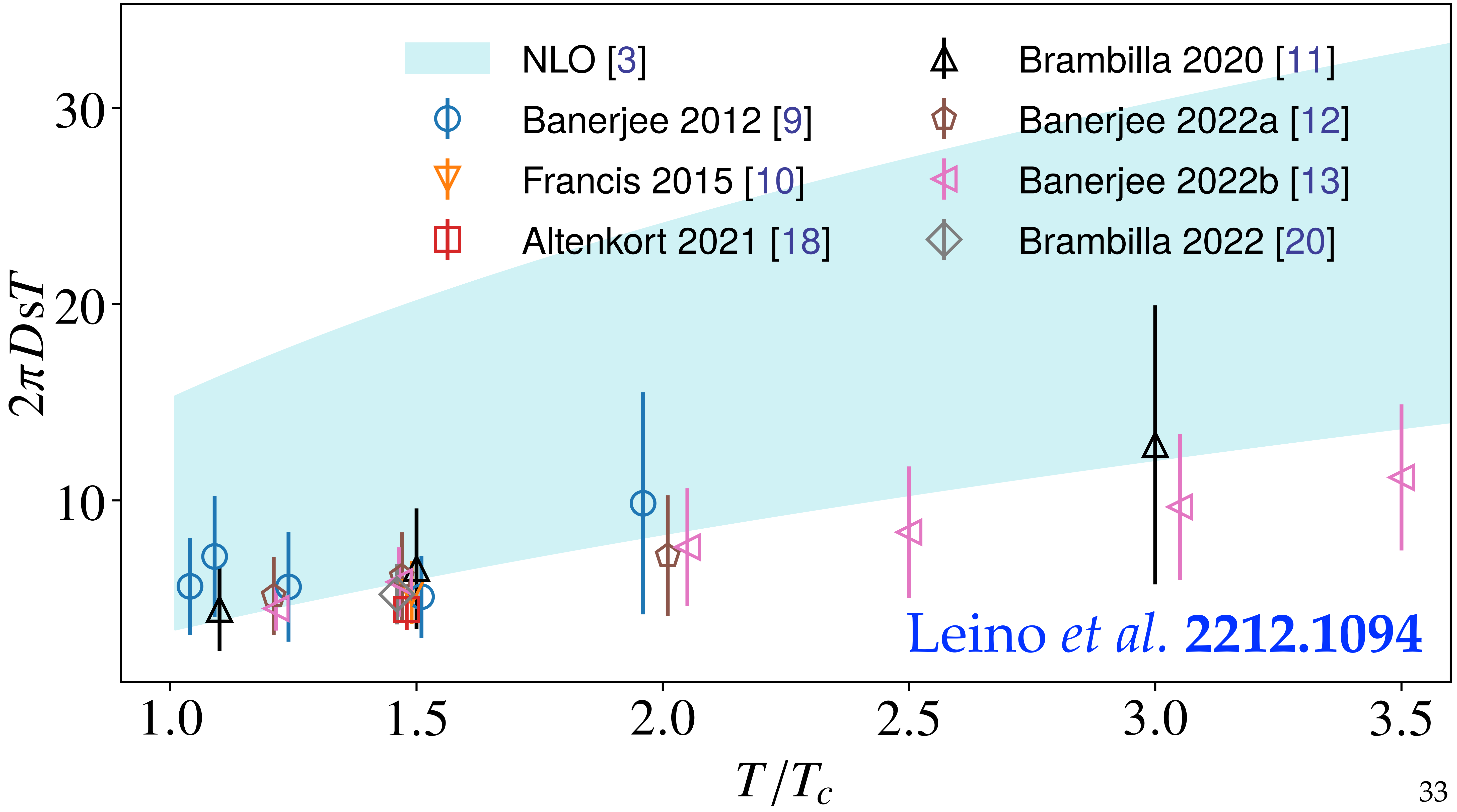


- Gradient flow reduces noise and renormalizes the chromo-E fields
  Altenkort *et al.* **PRD103** (2021) Brambilla *et al.* **2212.1094 2312.17321**
- Inverse problem in general tackled by fitting to specific Ansätze

Heavy quark diffusion on the lattice

- A long history in the quenched case, with v^2 corrections recently being determined

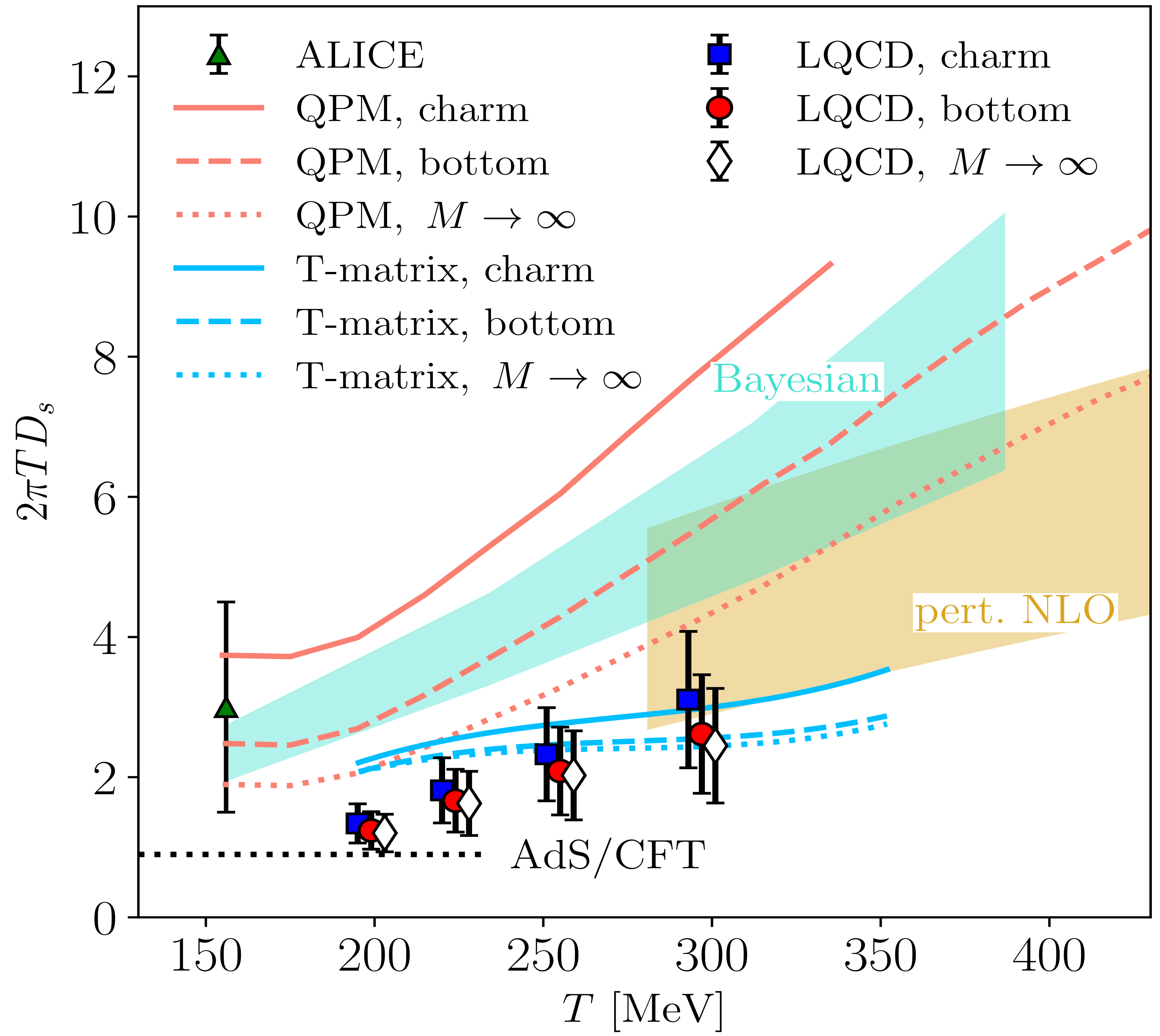
$$D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2 \rangle}{3MT}$$



Heavy quark diffusion on the lattice

- Recently, unquenched case, with v^2 corrections
- $\langle p^2 \rangle$ and $\langle v^2 \rangle$determined in a quasi-particle model. The latter multiplies the magnetic contribution
- Very mild mass dependence

$$D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2 \rangle}{3MT}$$



Altenkort *et al.* **2311.01525** (2021)

γ on the lattice?

- If we wanted the heavy-quark γ_Q, all would be well*

$$\gamma_Q = -\int_0^\beta d\tau\, G_E(\tau)\Big|_{\text{vacuum subtracted}}$$

Eller JG Moore **PRD99** (2019)

- Unfortunately, we really need the adjoint, $Q\bar{Q}$ version
  - Define $G_E^{Q\bar{Q}}(\tau) = \dfrac{g^2 T_F}{3N_c}\langle E_i^a(\tau) U_{ab}(\tau,0) E_i^b(0)\rangle$
  - Complicated on the lattice because perimeter divergence not compensated by Polyakov loop denominator
  - Need to reconstruct spf

γ on the lattice?

- If we wanted the heavy-quark γ_Q, all would be well*

$$\gamma_Q = -\int_0^\beta d\tau\, G_E(\tau)\Big|_{\text{vacuum subtracted}}$$

- Unfortunately, we really need the adjoint, $Q\bar{Q}$ version
  - Define $G_E^{Q\bar{Q}}(\tau) = \dfrac{g^2 T_F}{3N_c}\langle E_i^a(\tau) U_{ab}(\tau,0) E_i^b(0)\rangle$
  - Need $\gamma_{Q\bar{Q}} = -\displaystyle\int_0^\beta d\tau\, G_E^{Q\bar{Q}}(\tau) - \int \frac{d\omega}{2\pi}\frac{1+2n_{\rm B}(\omega)}{\omega}\rho_E^{Q\bar{Q}}(\omega)$

Eller JG Moore **PRD99** (2019) Scheihing-Hitschfeld Yao **PRD108** (2023)

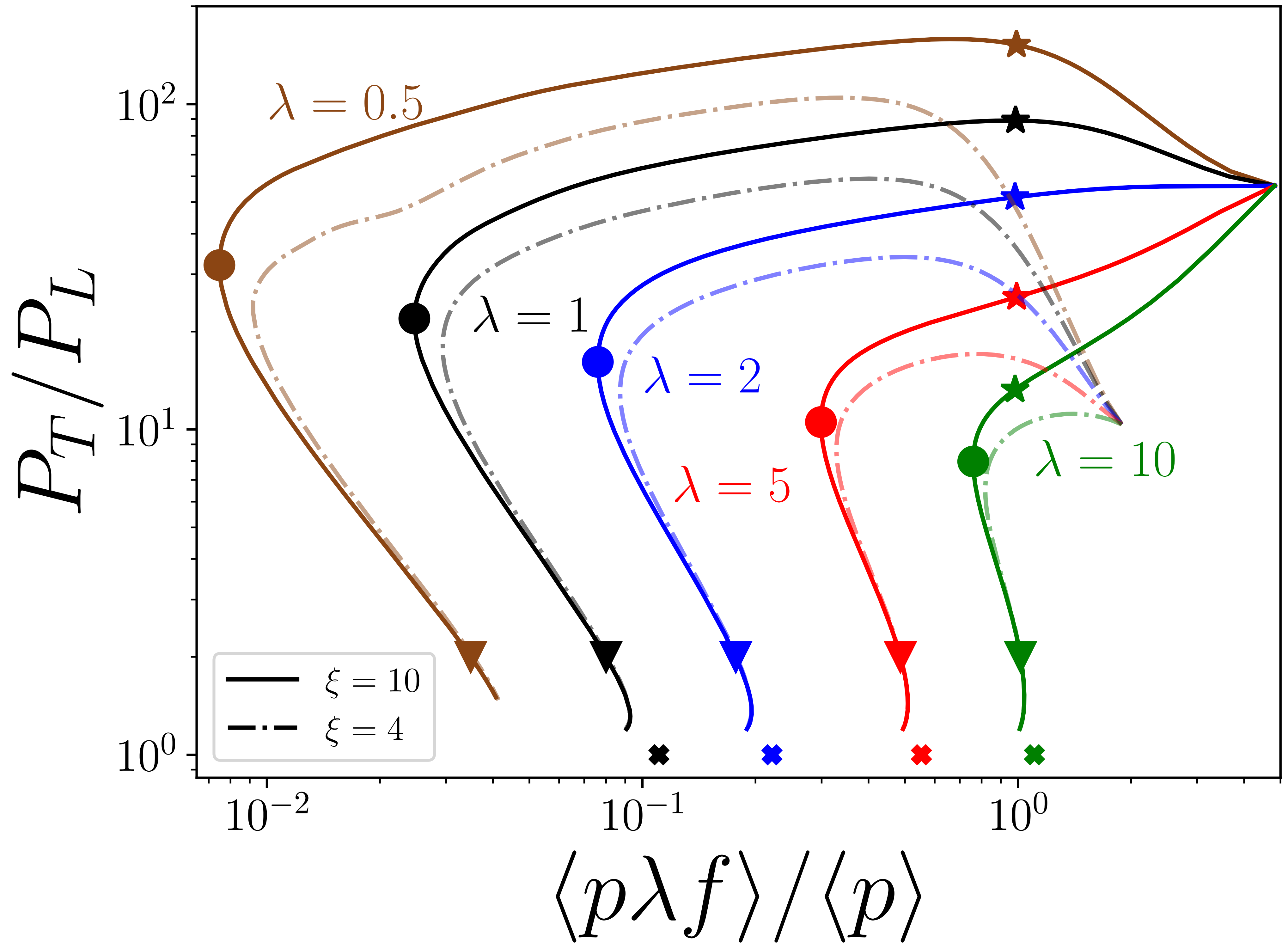
γ on the lattice?

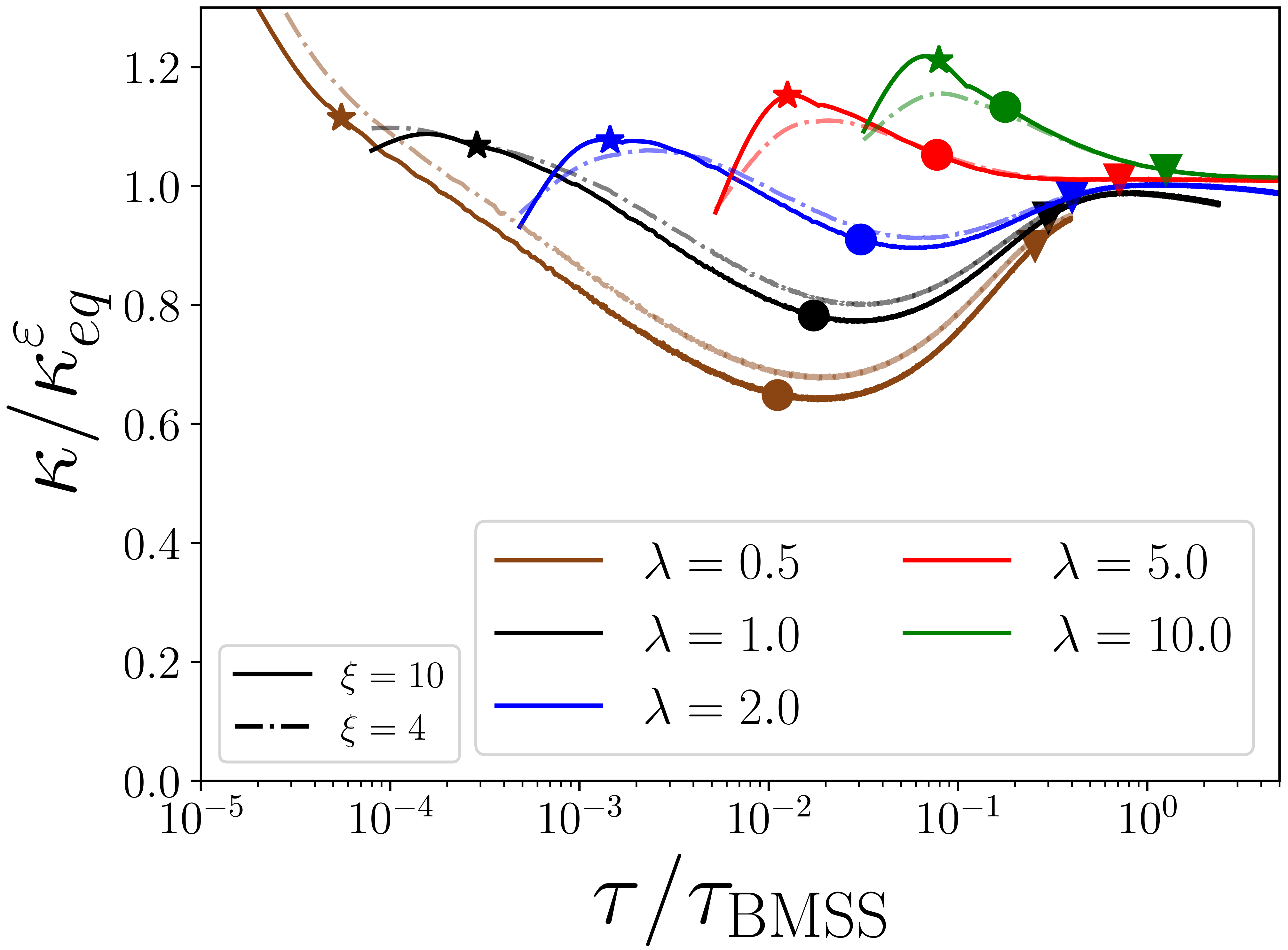
- Unfortunately, we really need the adjoint, $Q\bar{Q}$ version
  - Define $G_E^{Q\bar{Q}}(\tau) = \frac{g^2 T_F}{3N_c}\langle E_i^a(\tau)U_{ab}(\tau,0)E_i^b(0)\rangle$
- First attempts at lattice determinations of $G_E^{Q\bar{Q}}(\tau)$ are ongoing, using gradient flow and fitting or using the renormalization constant of Polyakov loops to subtract the perimeter divergence
  Leino **2401.06733**
- Not clear thus at the moment if the two κ are different in weakly or strongly-coupled QCD

A puzzle from holography

- It was recently determined using AdS/CFT that $\gamma_{Q\bar{Q}}^{\mathcal{N}=4} = \kappa_{Q\bar{Q}}^{\mathcal{N}=4} = 0$ at infinite λ
- This arises from a spectral function that vanishes at non-positive frequencies
  Nijs Scheihing-Hitschfeld Yao **JHEP0623** (2023)
- This is **different** from the holographic result $\kappa_{Q}^{\mathcal{N}=4} = \sqrt{\lambda}\pi T^3$
  Casalderrey-Solana Teaney **PRD74** (2006)
- Not clear why this striking difference. "$Q\overline{Q}$ operator more vacuum-like in holography" B. Scheihing-Hitschfeld, any misinterpretation/misphrasing my own

Heavy quark diffusion out of equilibrium

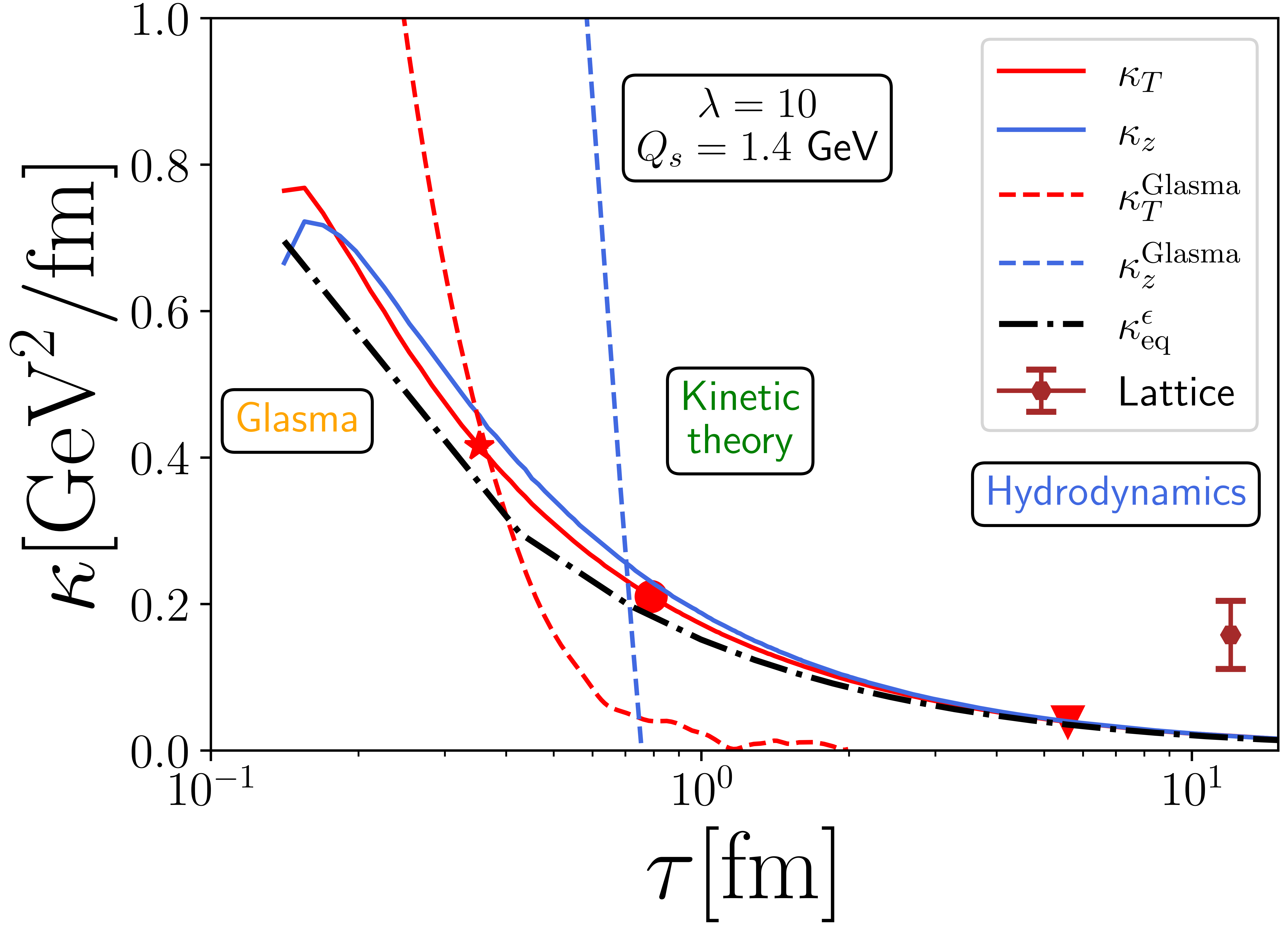




- Kinetic theory determination during bottom-up thermalization
  Boguslavski Kurkela Lappi Lindenbauer Peuron **2303.12520**

Heavy quark diffusion out of equilibrium

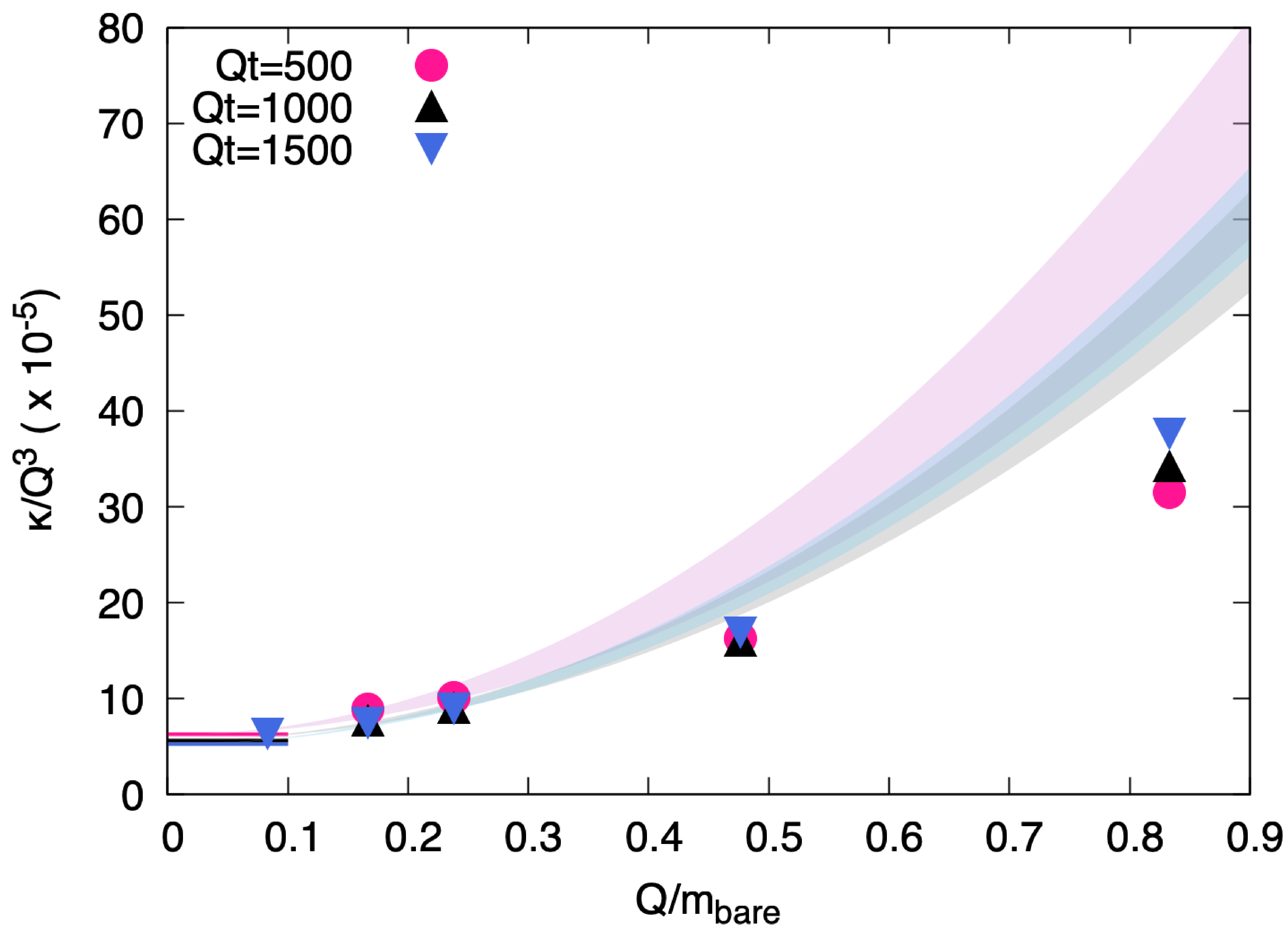
- Kinetic theory determination during bottom-up thermalization
  Boguslavski Kurkela Lappi Lindenbauer Peuron **2303.12520**
- Glasma Avramescu *et al.* **2307.07999**
  lattice Brambilla *et al.* **PRD107** (2023)



- Discrepancy with glasma due to non-diffusive evolution therein

Heavy quark diffusion out of equilibrium

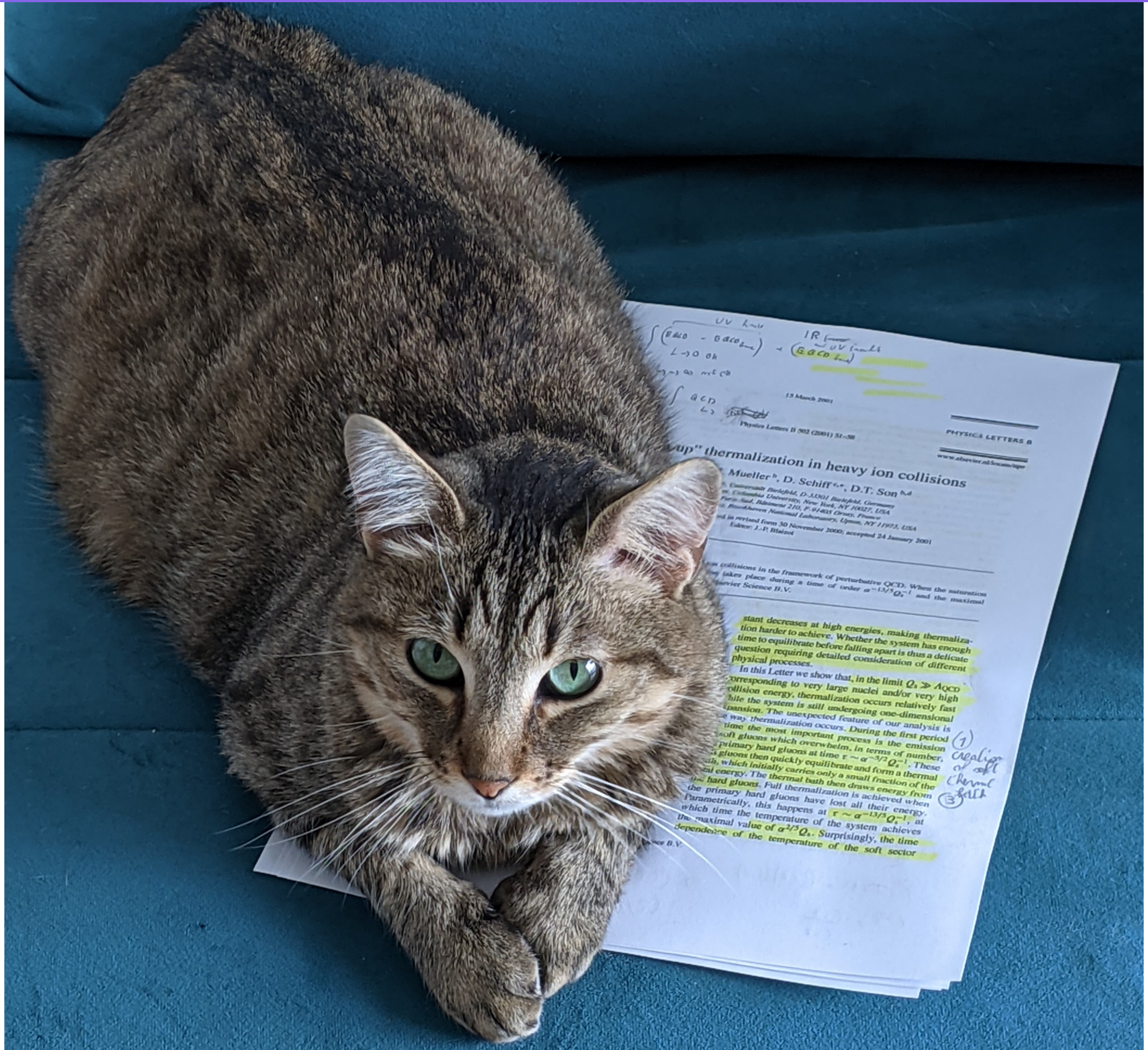
- Classical-statistical calculation with relativistic heavy quarks
  Pandey Schlichting Sharma **2312.12280**
- Infinite mass limit agrees well with Boguslavski *et al.* **JHEP0920** (2020)
- Charm not well described by quadratic fit: full mass dependence important



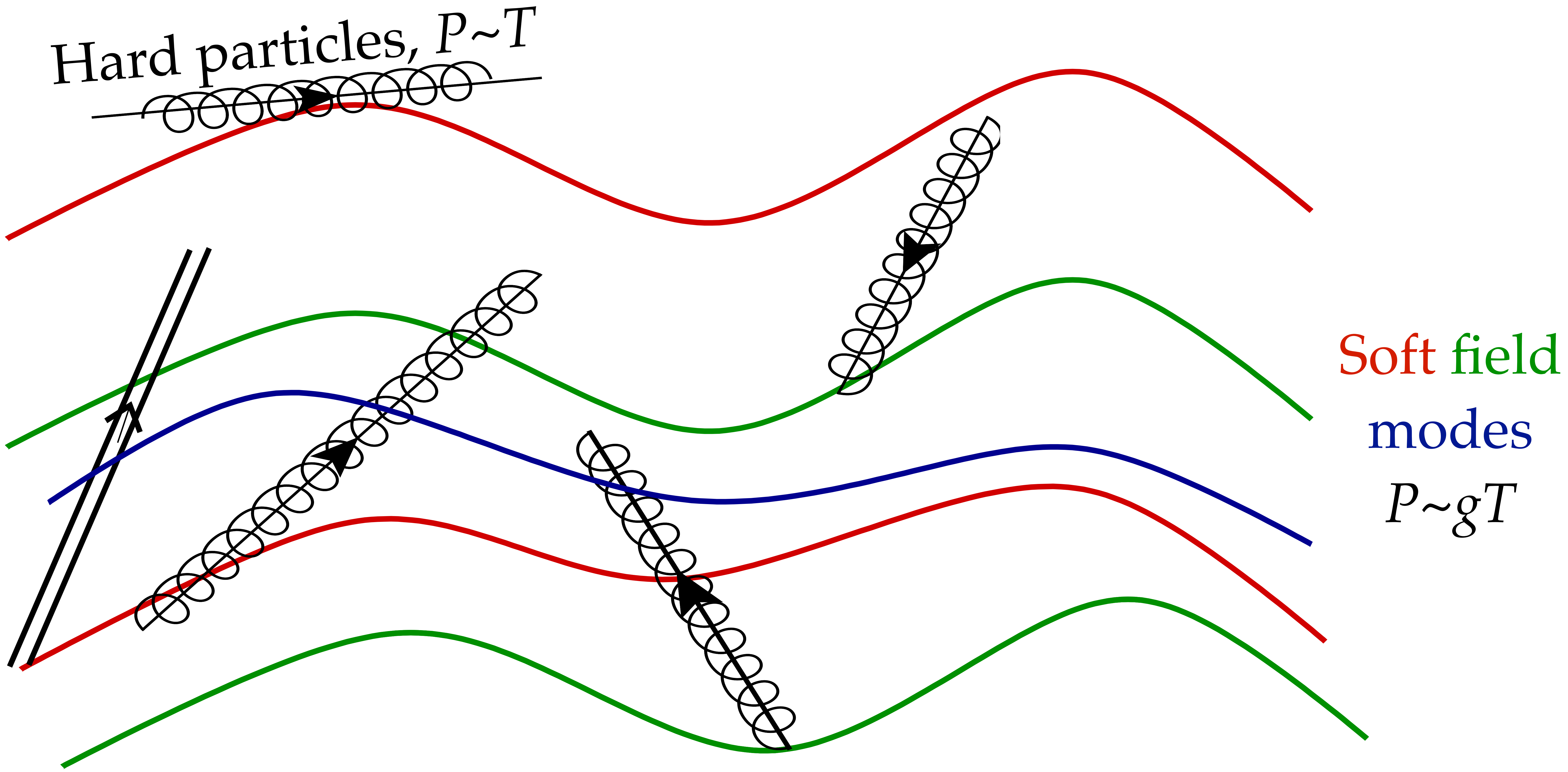
Conclusions

- Correlators of chromoelectric (and magnetic) fields on a timelike Wilson line are key ingredients for heavy quark diffusion and quarkonium modification
- Importance of proper definition of the Wilson line structure in the two cases. Unclear if the two definitions of κ are distinct
- Lots of recent lattice determinations, gradient flow very important. Mild (rescaled) temperature dependence
- Tackling early-stage dynamics

Backup



The weak-coupling picture

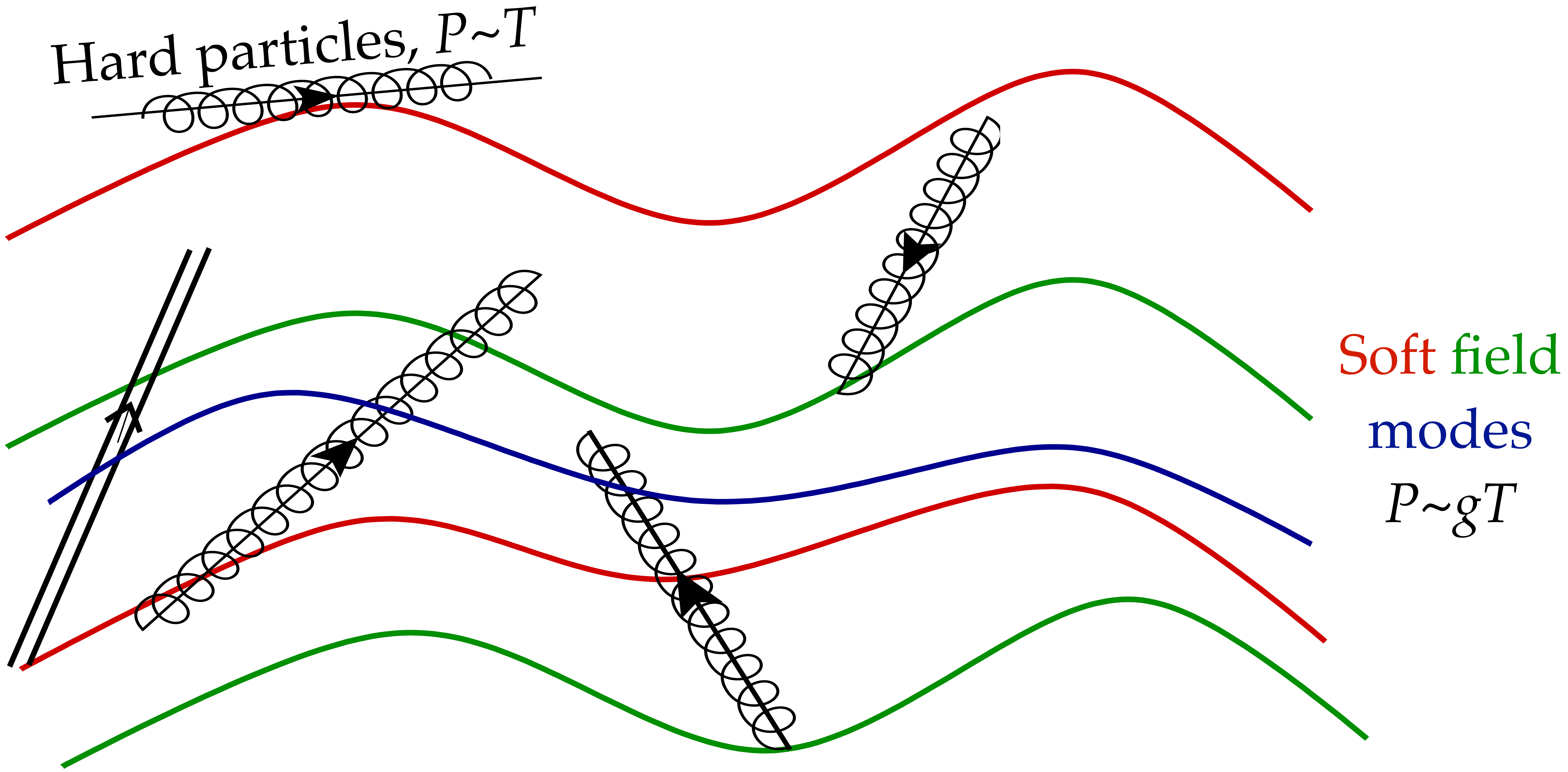


$$\alpha_s = \frac{g^2}{4\pi}$$

Figure by D. Teaney

- Hard (quasi)-particles carry most of the stress-energy tensor. (Parametrically) largest contribution to thermodynamics

The weak-coupling picture



$$\alpha_s = \frac{g^2}{4\pi}$$

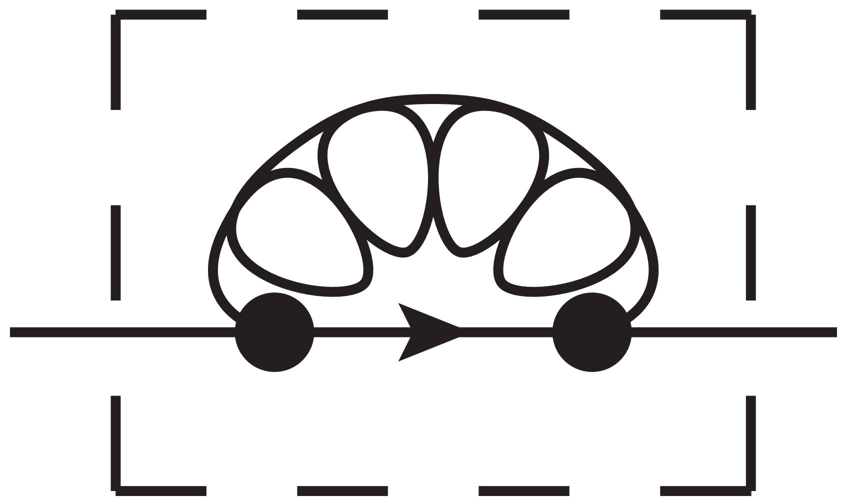
Figure by D. Teaney

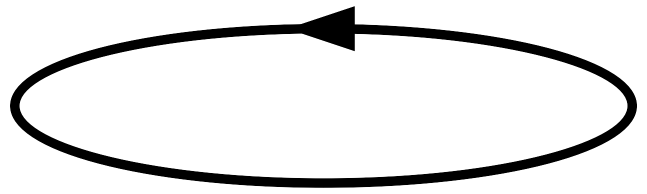
- The gluonic soft fields have large occupation numbers ⇒ they can be treated classically

$$n_B(\omega) = \frac{1}{e^{\omega/T} - 1} \overset{\omega \sim gT}{\simeq} \frac{T}{\omega} \sim \frac{1}{g}$$

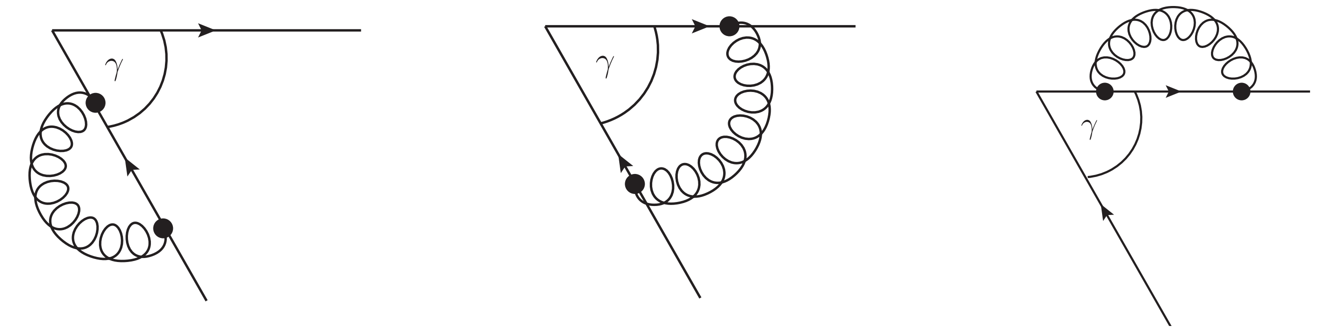
Renormalization of Wilson lines and loops

- All Wilson lines have a linear UV divergence proportional to their length ⇒A Wilson loop with a smooth, nonintersecting contour is finite in dimensional regularization after charge renormalization but needs multiplicative renormalization on the lattice





- Cusps in the contour introduce UV *cusp divergences*, renormalized multiplicatively through the *cusp anomalous dimension*, which only depends on the angle. Known in QCD at least to NLO



$$\frac{\alpha_s C_F}{2\pi\epsilon}\left(1+(\pi-\gamma)\cot\gamma\right)$$

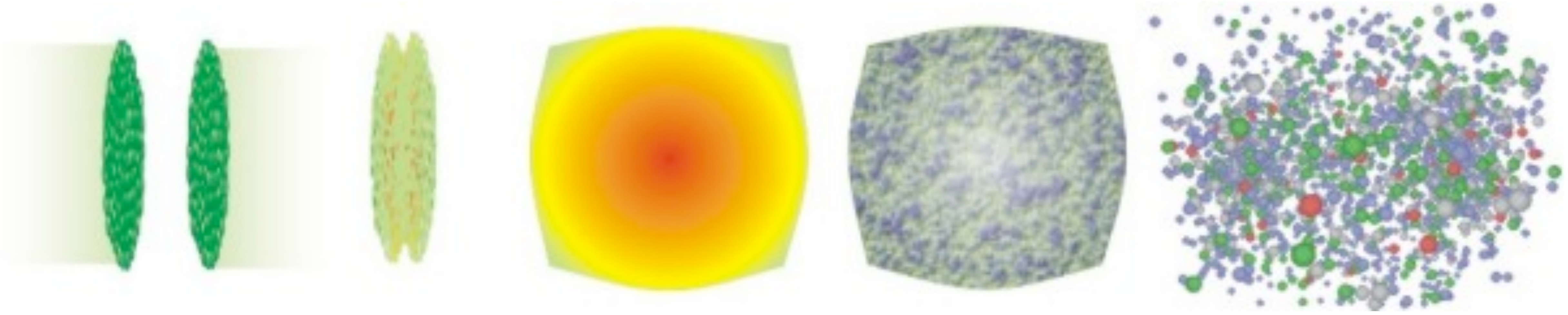
Polyakov **NPB84** (1980) Dotsenko Vergeles **NPB169** (1980) Brandt Neri Sato **PRD24** (1981) Korchemsky Radyushkin **NPB283** (1987)

Bottom-up thermalisation

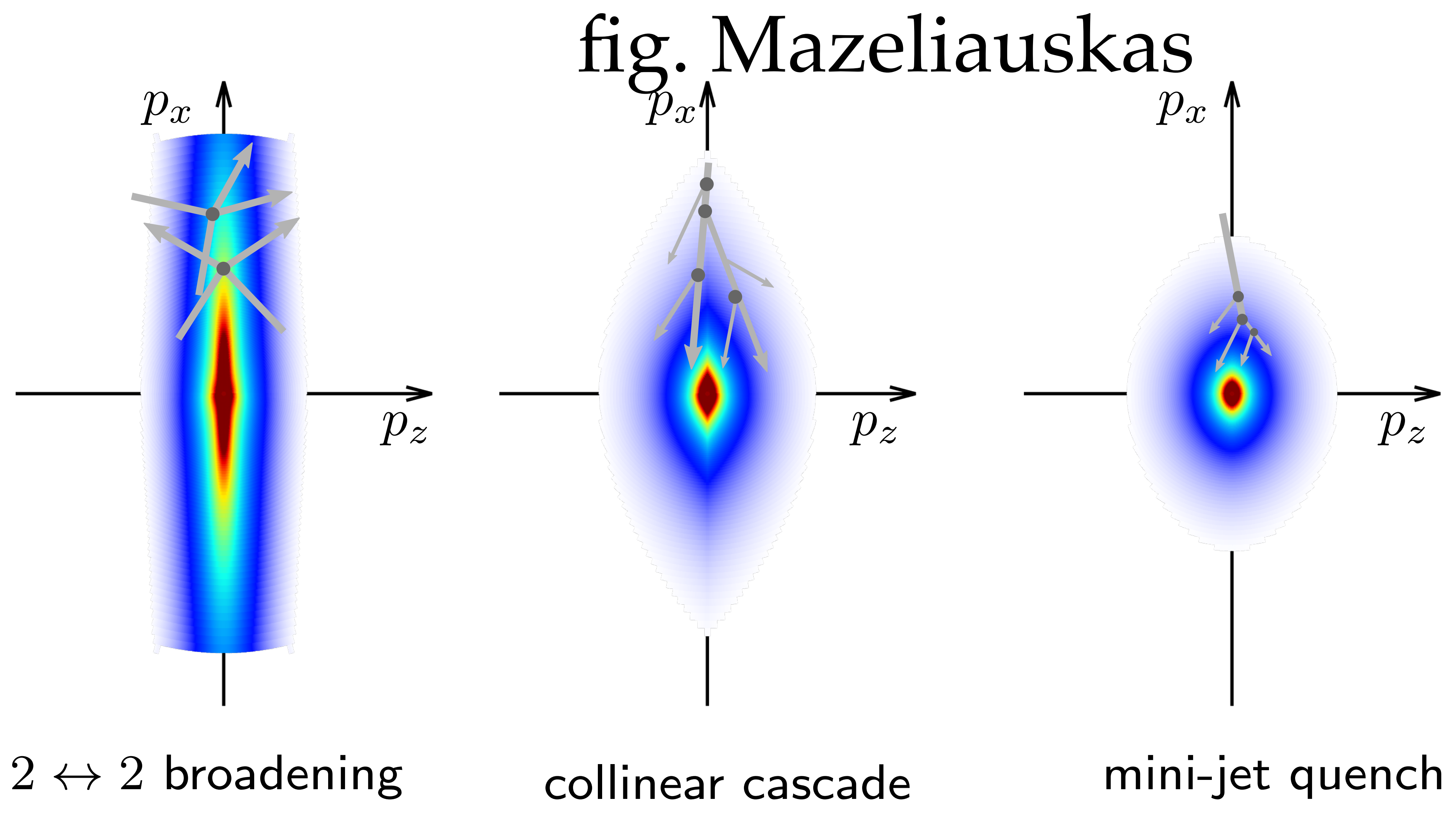
- Competition between expansion and interaction, attractor solution when they balance out

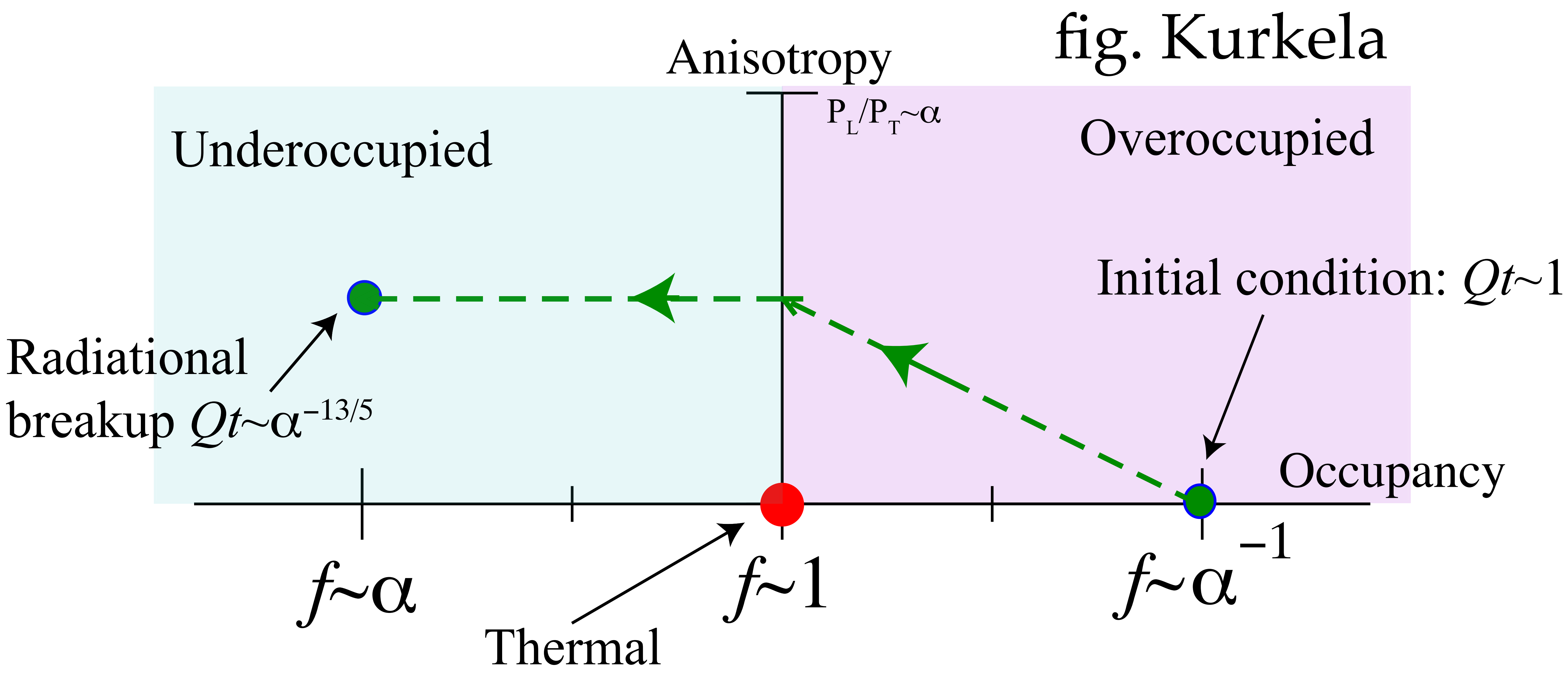
$$\left(\frac{\partial}{\partial t} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- Expansion is driven by the specifics of the heavy-ion collision and the initial state, drives the system away from equilibrium. Interaction among the constituents tends to isotropize the system.



Baier Mueller Schiff Son (2001) Kurkela Moore (2011)





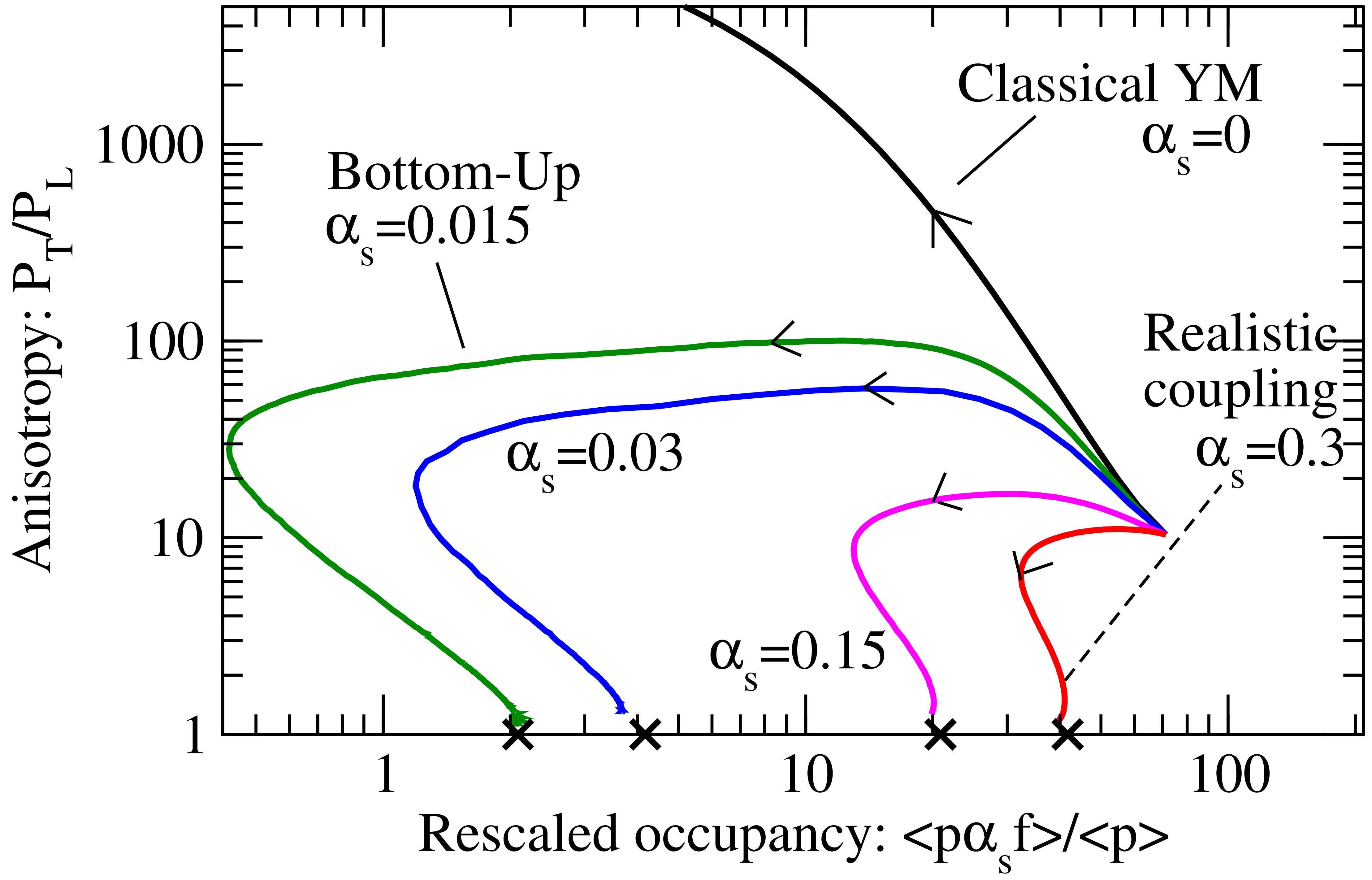
- Initially, strong isotropizing effect of transverse-momentum broadening $\propto \hat{q} \equiv \langle k_\perp^2 \rangle / t$
- Later, transverse-momentum broadening acts as the driver of bremsstrahlung in the cascade and mini-jet quench, rapid transfer of energy from UV to IR without intermediate accumulation

Baier Mueller Schiff Son (2001) Kurkela Moore (2011)

Bottom-up thermalisation: numerical solution

- From numerical solution of LO* kinetic theory

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$



Kurkela Zhu **PRL115** (2015)