UNIVERSITÄT BIELEFELD
Fakultät für Physik

CRC-TR 211

Quarkonium spectral functions and thermal static quark-antiquark potential from Lattice QCD

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Hirschegg 2024-Strong interaction physics of heavy flavors

Motivation

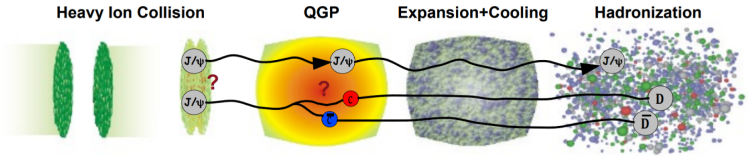
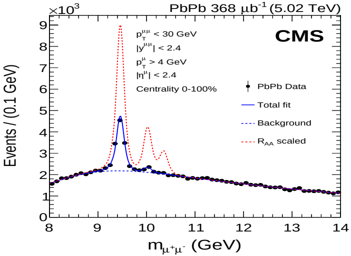


Figure: Steffen Bass

- Experimentally this QGP phase is recreated at RHIC and LHC in heavy ion collisions.
- QGP causes suppression of Quarkonia (bound states of heavy $q\bar{q}$), an important probe to study properties of QGP.



CMS Collaboration, PLB 790 (2019) 270

Correlators and spectral functions

- Heavy $q\bar{q}$: a thermometer of QGP in heavy ion collisions
- The spectral functions $\rho_H(\omega)$ contains information about the in-medium hadron properties

$$\sum_{\vec{x}} \left\langle \bar{\psi}\Gamma_H \psi(\tau,\vec{x})(\bar{\psi}\Gamma_H \psi(0,\vec{0}))^{\dagger} \right\rangle \equiv G_H(\tau) = \int_0^{\infty} \frac{d\omega}{\pi} \rho_H(\omega) \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

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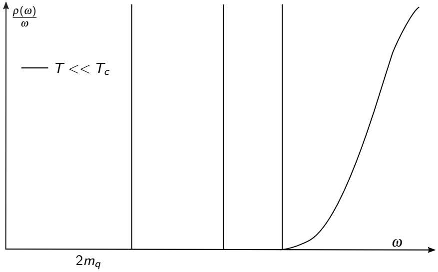
Strategy:

- $G_H(\tau)$ on the lattice
- Extract spectral function
- Estimate in-medium hadronic properties
- In addition transport coefficients, like heavy quark diffusion coefficients, are encoded in the vector meson spectral function

The spectral function



[Sandmeyer's thesis]

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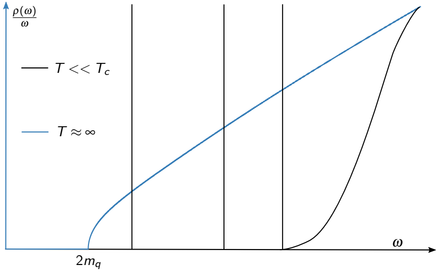
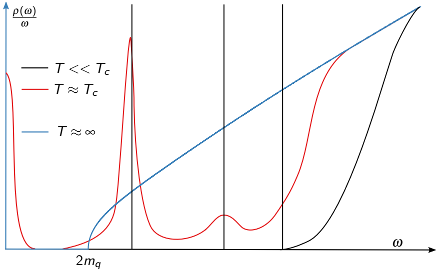


[Sandmeyer's thesis]

- At infinite temperature there cannot be bound states

The spectral function



[Sandmeyer's thesis]

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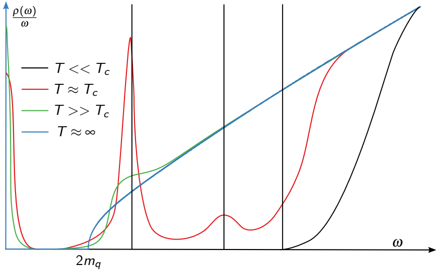

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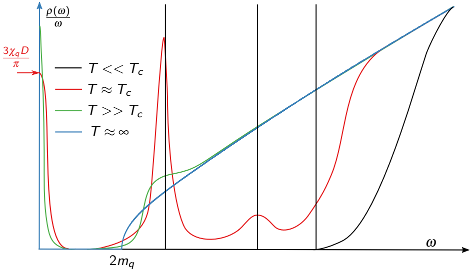


- At infinite temperature there cannot be bound states
- Melting of states visualizes in **shrinking and broadening of bound peaks**
- **Heavy quark diffusion constant** can be read off in vector channel

$$D = \frac{\pi}{3\chi_q} \lim_{\omega \to 0} \sum_{i=1}^{3} \frac{\rho_V(\omega, T)}{\omega}$$

Extraction of spectral function is ill-posed problem $\rightarrow$ large lattices needed. [Sandmeyer's thesis]

SPF's contribution to correlators

$$G_H(\tau) = \int_0^\infty \frac{\omega}{\pi} \rho_H(\omega) \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

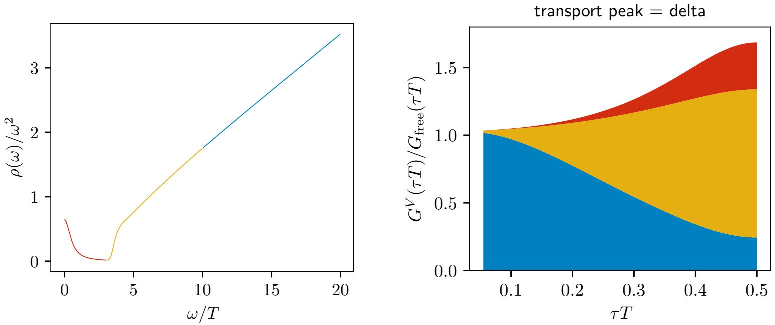
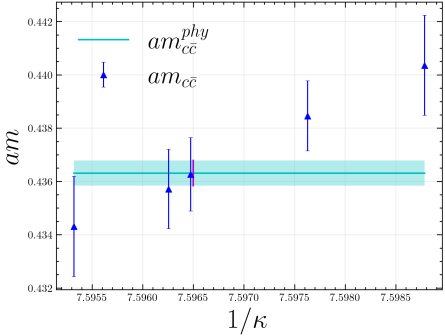


Figure: Contribution of the spectral function to the correlator.
[Sandmeyer's thesis]

Mass tuning on mixed action (Full QCD)



- Mixed action approach (Wilson Clover fermions on HISQ configurations)
- Tadpole improved tree-level, $c_{SW} = \frac{1}{{u_0}^3}$, $u_0 = (tr[U_{\mu\nu}])^{\frac{1}{4}}$
- Quark mass tuning
- Tune spectrum to experimental values

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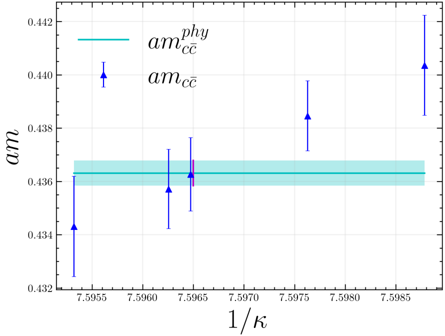

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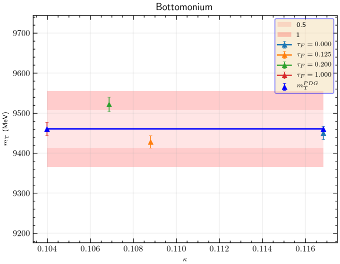
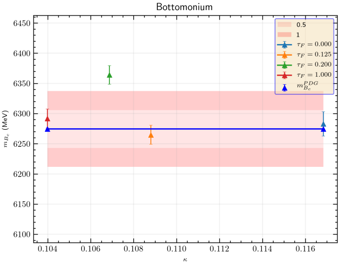
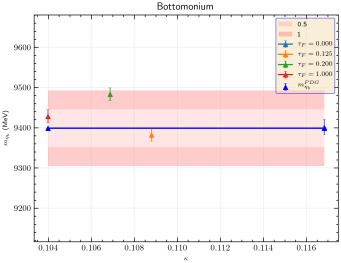
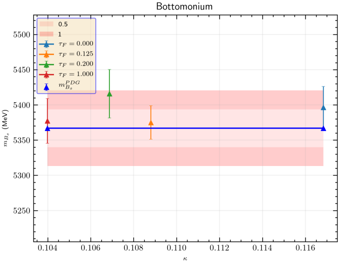
- HISQ lattices from HotQCD $(m_l = m_s/5)$; 64^3x64, 96^3x32, new temperatures at 96^3x56 and 96^3x28
- Gradient flow (renormalizes the operators, removes cut-off and mixed action effects and improves signal-to-noise ratio)

Mass spectrum

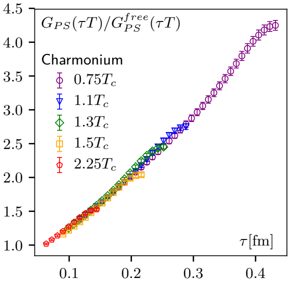


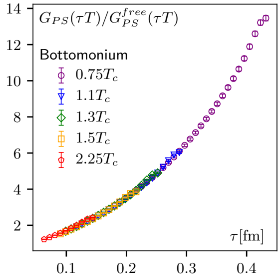






Correlators: Quenched VS Unquenched





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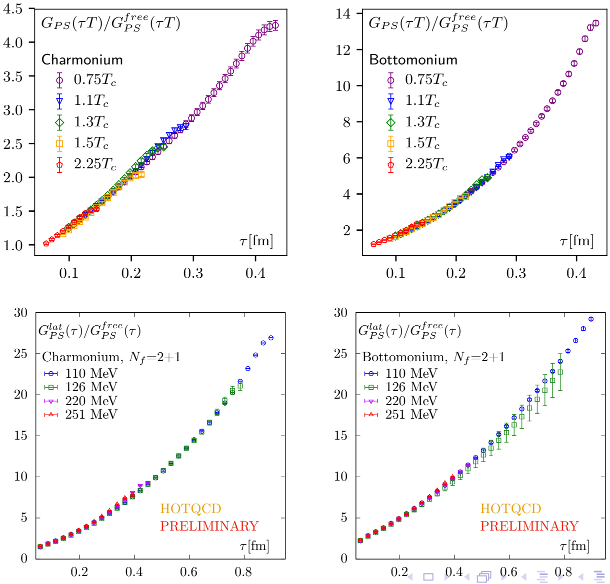

Correlators and spectral functions

$$G_H(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_H(\omega) \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

- $G_H(\tau)$ from lattice
- $\rho_H(\omega)$ = Ansatz

Perturbative SPF (Full QCD)

$$\rho_V^{NRQCD}(\omega) = \frac{1}{2}\left(1 - e^{-\frac{\omega}{T}}\right)\int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t}\, C_>^V$$

where $C_>$ is solved from

$$\left\{ i\partial_t - \left[2M + V(r) - \frac{\nabla^2}{M} \right] \right\} C_>^V = 0,$$

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For $T > 0$ the potential is

$$V_T(r) = -\alpha_s C_F \left[m_D + \frac{\exp(-m_D r)}{r}\right] - i\alpha_s C_F T\, \phi(m_D r) + O(\alpha_s^2)\,,$$

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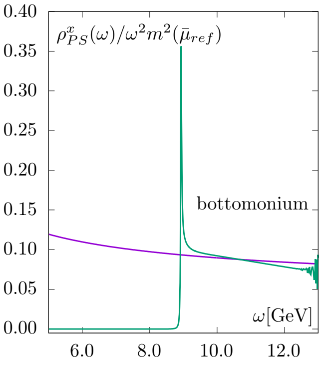
At short separations, $r \ll 1/m_D$, $V(r)$ is replaced by V_0

$$V_0(r) = -\frac{C_F \alpha_s(\frac{e^{-\gamma_E}}{r})}{r} \left\{ 1 + \frac{\alpha_s(\frac{e^{-\gamma_E}}{r})}{4\pi} a_1 + \frac{\alpha_s^2(\frac{e^{-\gamma_E}}{r})}{(4\pi)^2} \left[a_2 + \frac{\pi^2 \beta_0^2}{3} \right] \right\} + O(\alpha_s^4$$

[JHEP 11 (2017) 206]

Perturbative SPF (Full QCD)

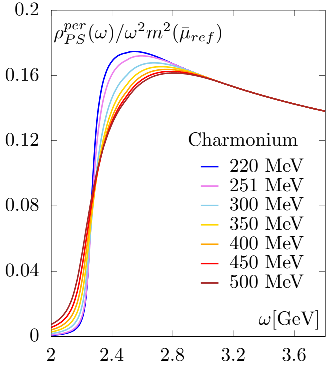
- $\omega \gg 2M$: Thermal effects are suppressed. Vacuum perturbation theory will work.
- $\omega \sim 2M$: Thermal effects are important. Spectral function needs to be calculated using thermal potential.
- $\omega \ll 2M$: For the pseudoscalar channel, the spectral weights are exponentially suppressed.
- For the vector channel, there is a contribution around $\omega \sim 0$ due to transport.



$$\rho_{PS}^{pert}(\omega) = \rho_{PS}^{VAC}(\omega) + A^{match}\rho_{PS}^{THERM}(\omega)$$

[JHEP 11 (2017) 206]

Perturbative SPF (Full QCD)



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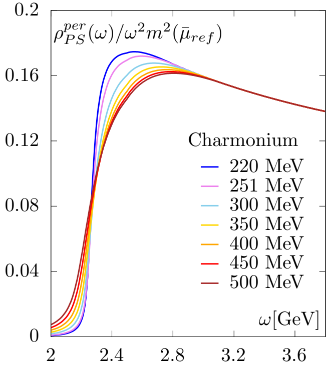
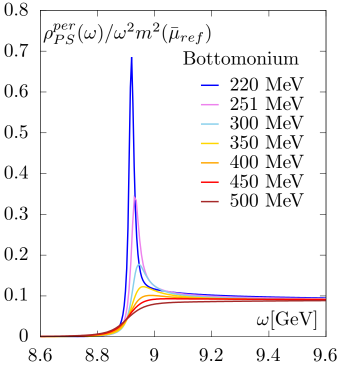
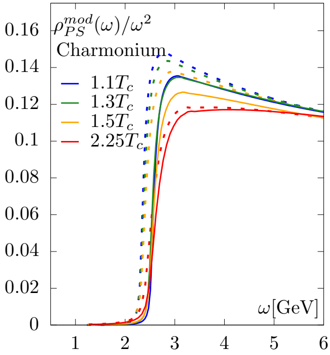


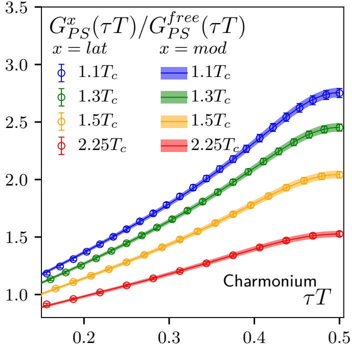


Spectral reconstruction (Quenched)

$$\rho_{PS}^{pert}(\omega) = \rho_{PS}^{VAC}(\omega) + A^{match}\rho_{PS}^{THERM}(\omega)$$

$$\rho_{PS}^{mod}(\omega) = A\,\rho_{PS}^{pert}(\omega - B)$$





[JHEP 11 (2017) 206]

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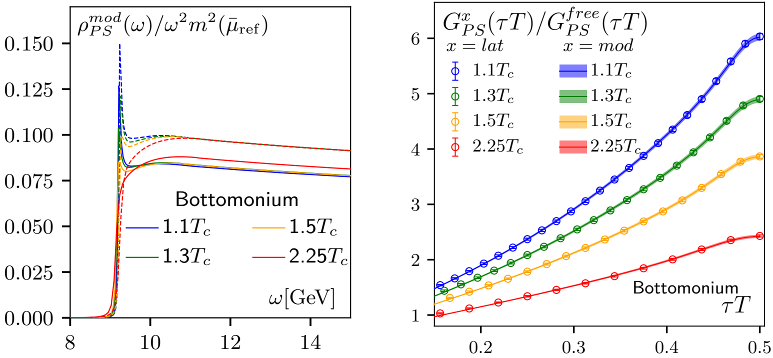
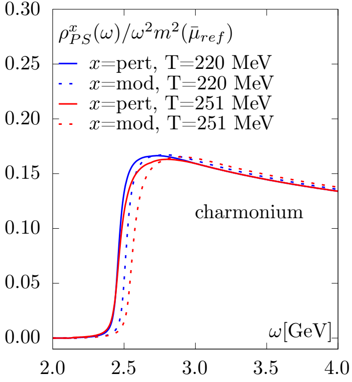
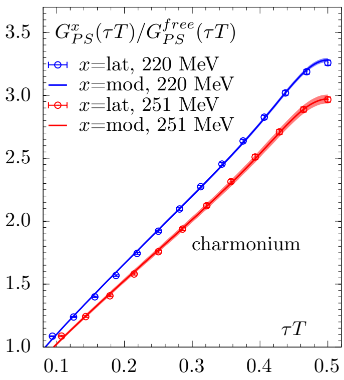


[JHEP 11 (2017) 206]

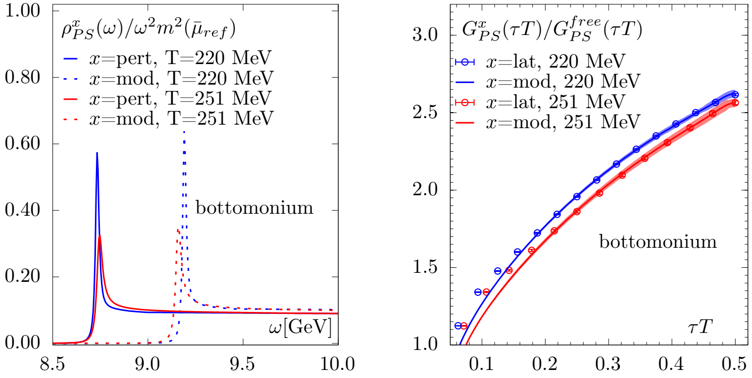
Spectral reconstruction (Full QCD)





[S. Ali et al, Few Body Syst. 64 (2023) 3, 52]

Spectral reconstruction (Full QCD)



[S. Ali et al, Few Body Syst. 64 (2023) 3, 52]

Thermal static potential

- Non-perturbative potential is significantly different from the perturbative potential.
- Non-perturbative formulation,

  A. Rothkopf et al., PRL. 108 (2012) 162001

$$W(r,\tau) = \int_{-\infty}^{\infty} d\omega\, \rho(\omega, T) \exp(-\omega\tau)$$

$$W(r,t) = \int_{-\infty}^{\infty} d\omega\, \rho(\omega, T) \exp(-i\omega t)$$

- $\rho(\omega, T)$ should have a form which is consistent with potential, $i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t}$ should exist.
- Gaussian spectral function does not have this limit.
- Simple Lorentzian has this limit but results depend on the lower cut-off.
- Bayesian analysis has a higher systematic error.

Thermal static potential

$$\log(W(r,\tau)) = -V_{re}(r)\tau - \int_{-\infty}^{\infty} du\,\sigma(r,u)\left[\exp(u\,\tau)+\exp(u\,(\beta-\tau))\right] + \ldots$$

HTL like τ dependence.

- $i\,\lim_{t\to\infty}\frac{\partial \log W(r,t)}{\partial t} = \text{finie} \implies \lim_{u\to 0}\sigma(r,u) \sim \frac{1}{u^2}$
- Following HTL PT, $\sigma(r,u) = n_B(u)\left[\frac{V_{im}}{u} + c_1\,u + c_3\,u^3 + \ldots\right]$
- Parametrization

$$W(r,\tau) = A\exp[-V_{re}(r)\tau - \frac{\beta\,V_{im}(r)}{\pi}\log(\sin(\frac{\pi\tau}{\beta})) + \ldots]$$

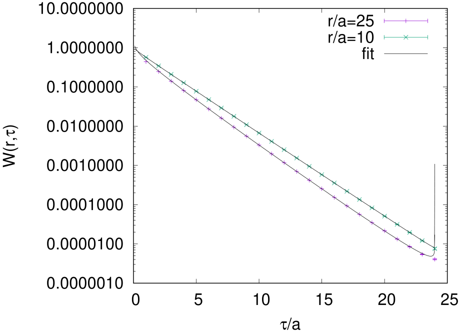
D. Bala et al, PRD 101, 034507

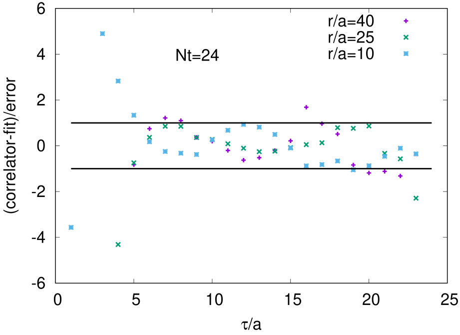
D. Bala et al, PRD 103, 014512

D. Bala et al, PRD 105, 054513

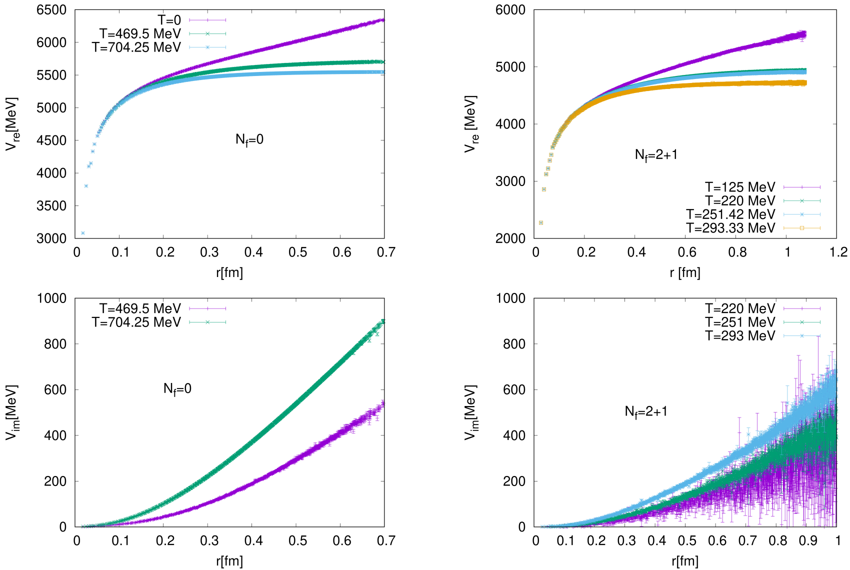
Thermal static potential

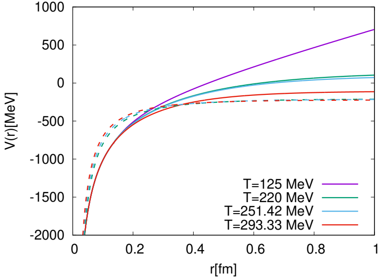
Three parameters fit ($\chi^2/dof \sim 1$) of Wilson line correlator for different distances.

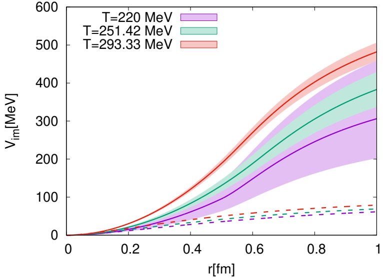




Color screening supported by the lattice data

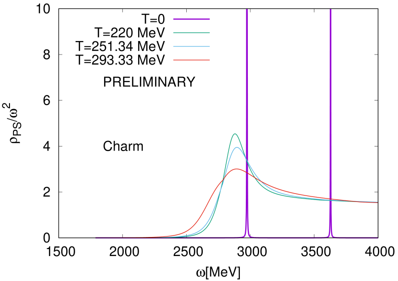


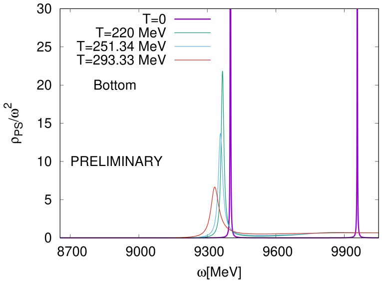




- Non-perturbative thermal potential is very much different from the HTL perturbative potential.

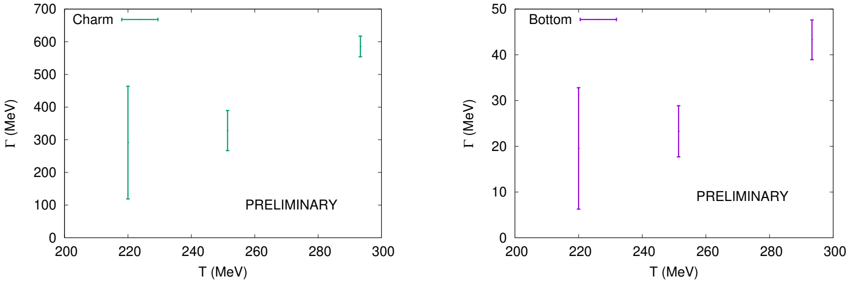
Description of the lattice data





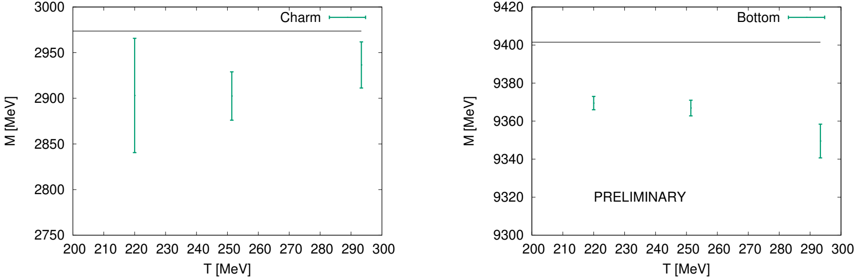
- (1S) state for bottom melts much after T_c ($T_c = 180 MeV$)
- Significant thermal effects on charmonium state.

Thermal width

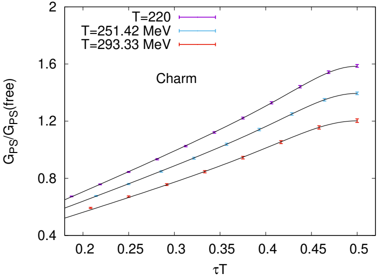


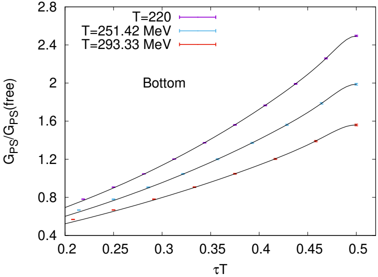
- We performed skewed Lorentzian fit near the peak.
- $\Gamma_c(1S) \gg \Gamma_b(1S)$

Thermal mass shift



- Mass is identified with peak position of the spectral function.
- Finite mass shift is observed





- Value of B is consistent with zero within error except for charm at $T = 293.33$ MeV with a value $((\sim 30 \pm 15)\text{MeV})$.

Outlook

- Extend the studies on spectral and transport properties from quenched to dynamical QCD
- Study light quark mass effects by comparing $m_l = m_s/5$ and $m_l = m_s/27$
- Study cut-off effects and perform continuum extrapolation
- Improve on perturbative and non-perturbative spectral function models
- Spectral reconstruction based on spectral function model fits and other reconstruction methods
- Estimate in-medium hadronic and transport properties (Kubo relation)
- Lattice data supports color screening of the non-perturbative thermal potential (careful analysis).

Thank you for your attension !