Universality in Halo Systems

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Outline

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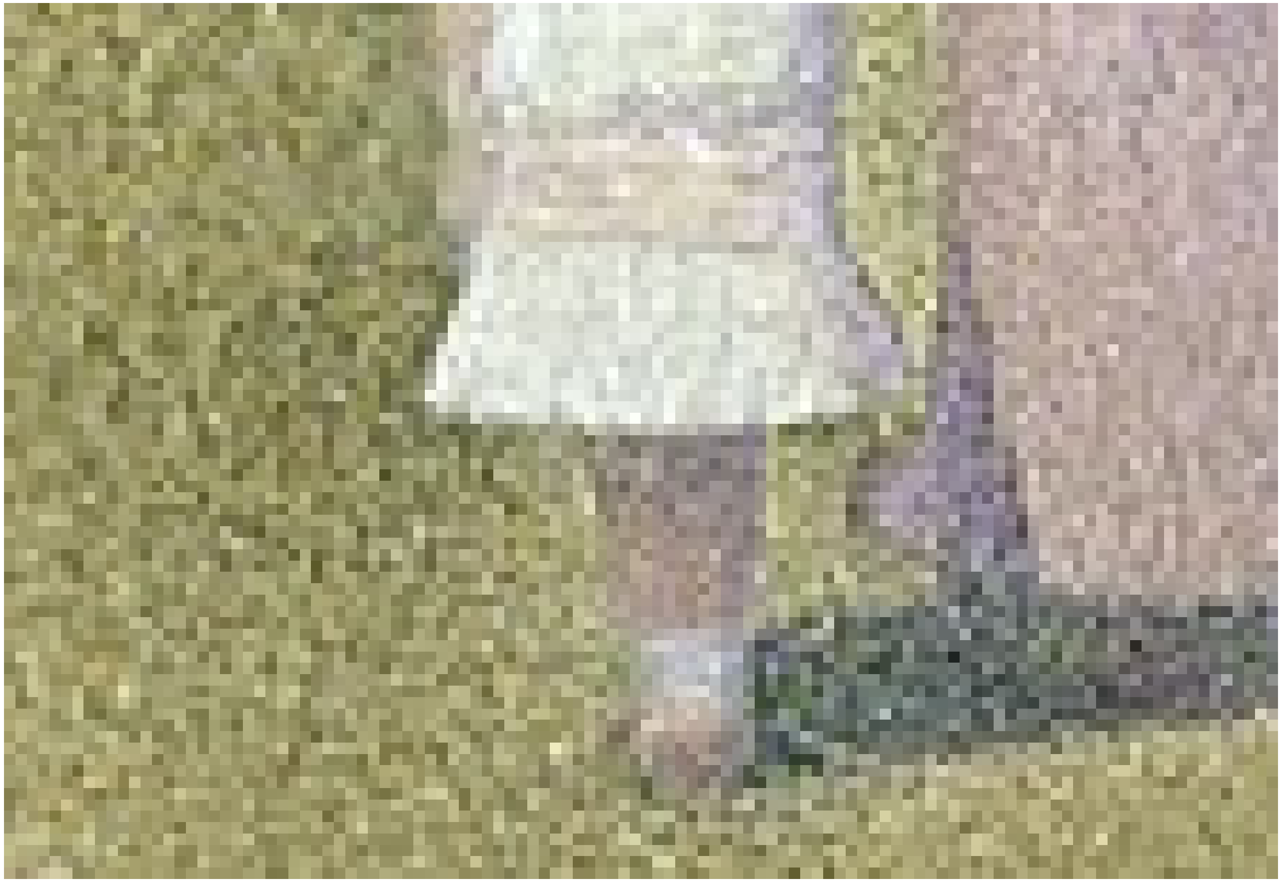
- Introduction
- Resonant interactions and the unitary limit
  - Efimov physics in ultracold atoms
  - Hadronic molecules
  - Shallow bound states in a box
  - Topological phases
  - Halo nuclei
- Summary and Outlook

Universality and Pointilism

- Painting at the limit of resolution of the human eye



Universality and Pointilism

- Painting at the limit of resolution of the human eye

G. Seurat, A Sunday on La Grande Jatte

Physics Near the Unitary Limit

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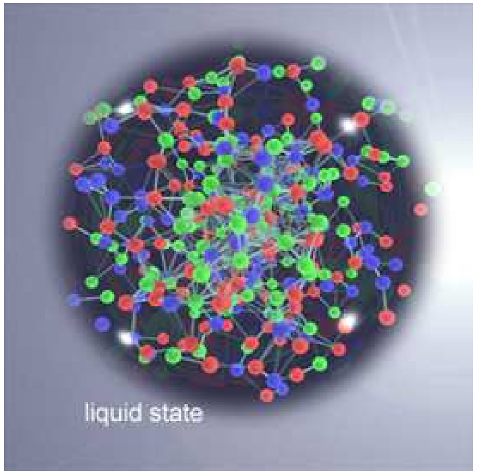
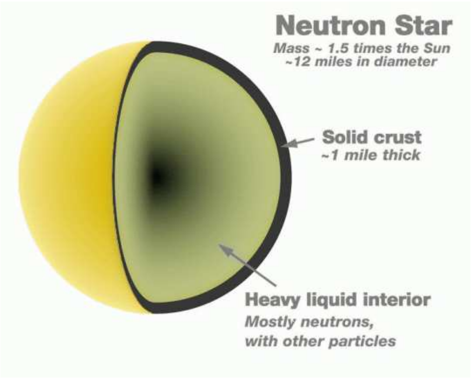
- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty$, $\ell \to 0$ (cf. Bertsch problem, 2000)

$$\mathcal{T}_2(k,k) \propto [\underbrace{k\cot\delta}_{-1/a+r_e k^2/2+\ldots} - ik]^{-1} \implies i/k$$

  - Scattering amplitude scale invariant, saturates unitarity bound
- Interesting many-body physics: BEC/BCS crossover, universal viscosity bound $\Rightarrow$ perfect liquid, ...





Physics Near the Unitary Limit

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  - Scattering amplitude scale invariant, saturates unitarity bound
- Interesting many-body physics: BEC/BCS crossover, universal viscosity bound $\Rightarrow$ perfect liquid, ...
- Use as starting point for description of few-body properties
  - Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
  - Natural expansion parameter: $\ell/|a|$, $k\ell$,...
  - Universal dimer with energy $E_d = -1/(ma^2)$ ($a > 0$)

    size $\langle r^2 \rangle^{1/2} = a/2 \quad \Rightarrow$ halo state

- Effective Lagrangian (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + \ldots$$

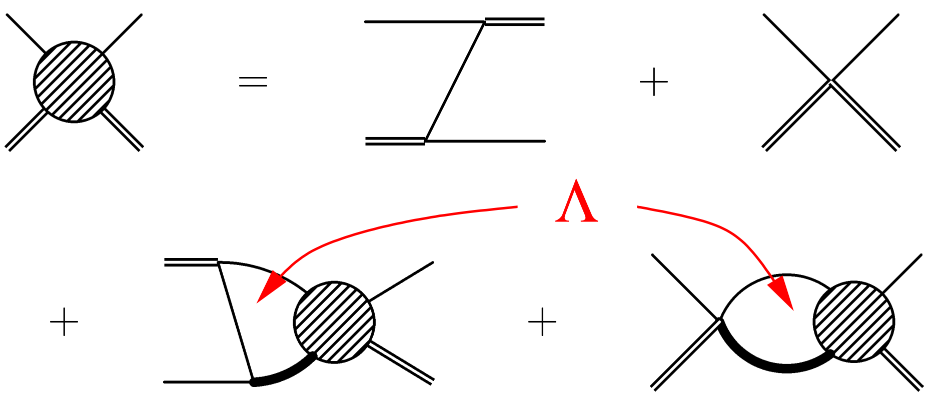
- 2-body amplitude:

- 2-body coupling g_2 near fixed point $(1/a = 0)$

  $\Rightarrow$ scale and conformal invariance $\Longleftrightarrow$ unitary limit

  (Mehen, Stewart, Wise, 2000; Nishida, Son, 2007; ...)

- 3-body amplitude:



$g_3(\Lambda)$ $\Rightarrow$ limit cycle

$\Rightarrow$ discrete scale inv.

Limit Cycle

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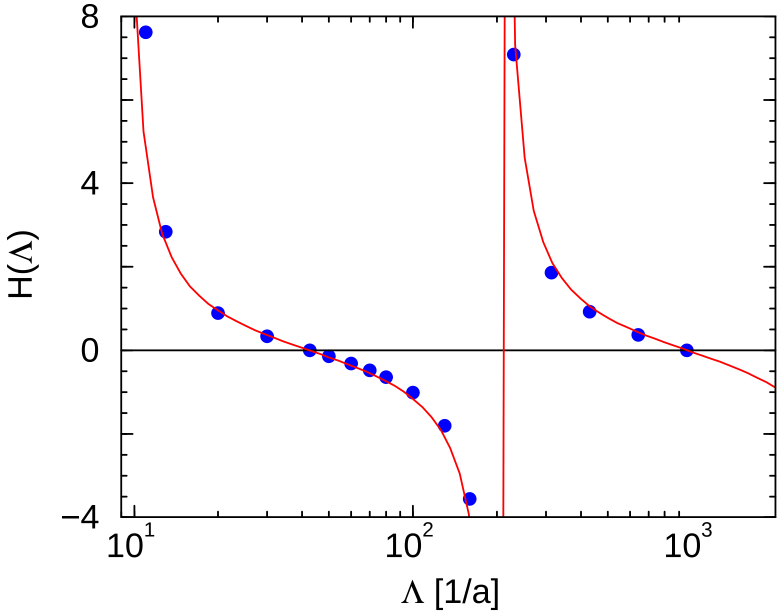
- RG invariance $\Longrightarrow$ running coupling $H(\Lambda) = g_3\Lambda^2/(9g_2^2)$

  - $H(\Lambda)$ periodic: limit cycle

    $$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

    (cf. Wilson, 1971)

  - Anomaly: scale invariance broken to discrete subgroup



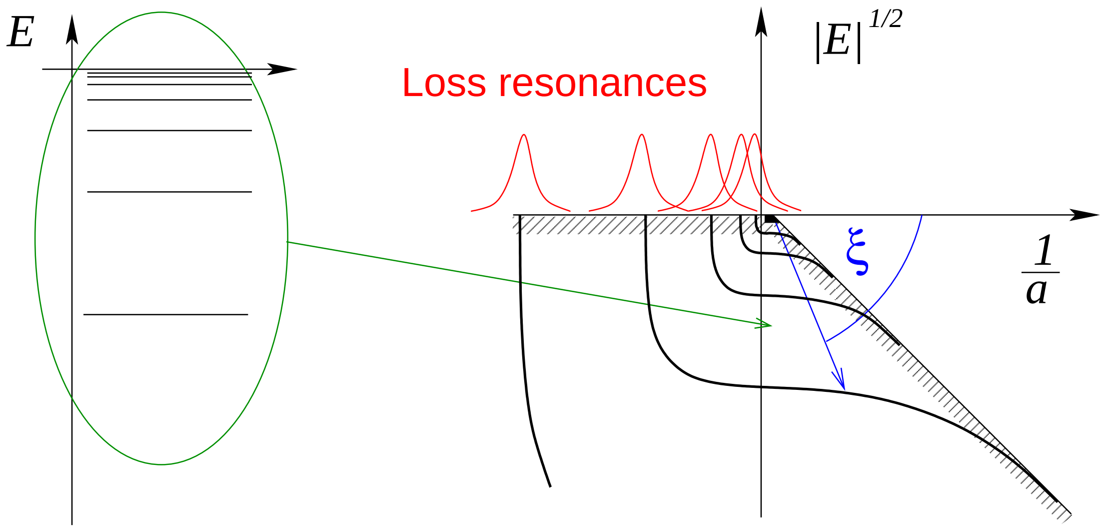
$$H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

(Bedaque, HWH, van Kolck, 1999)

- Three-body parameter: $\Lambda_*, \ldots$
- Limit cycle $\Longleftrightarrow$ Discrete scale invariance

Limit Cycle: Efimov Effect

- Universal spectrum of three-body states
  (Efimov, 1970)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \quad \xrightarrow{1/a\to 0} \quad \left(e^{\pi/s_0}\right)^2 = 515.035\ldots$$

- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances

Efimov States in Ultracold Atoms

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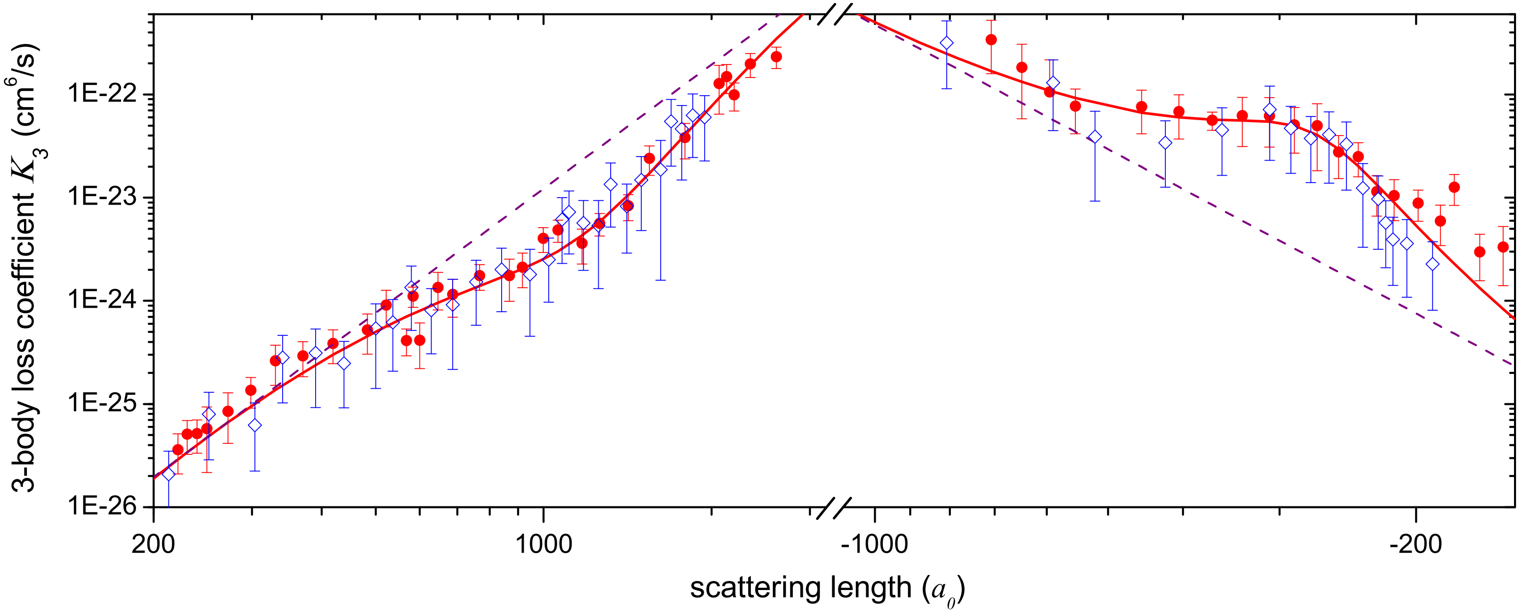
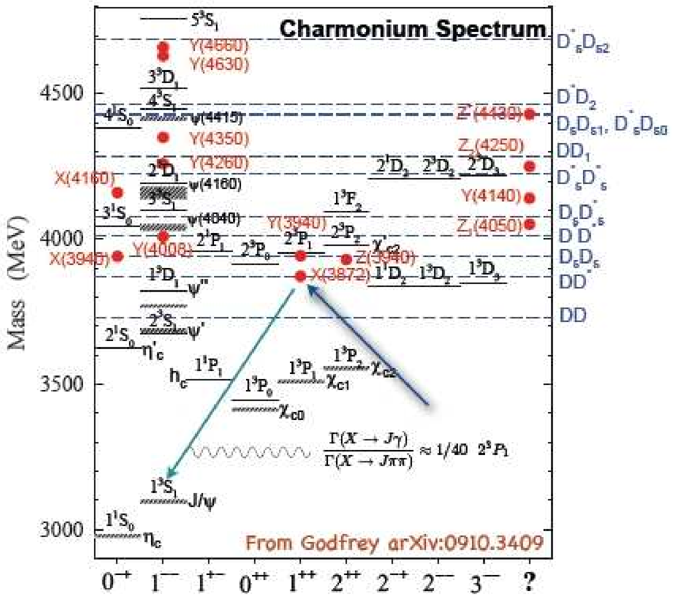
- Efimov physics has now been observed in ^{133}Cs, ^{6}Li, ^{7}Li, ^{39}K, ^{41}K/^{87}Rb
- Example: Efimov spectrum in ^{7}Li ($|m_F = 0\rangle$,$|m_F = 1\rangle$)
  (Gross et al. (Bar-Ilan Univ.), Phys. Rev. Lett. **105** (2010) 103203)



- Data described by universal theory (Braaten, HWH, 2000, ..., 2006)
- Future: dipolar interactions, confined systems, ...

Hadronic Molecules

- New $c\bar{c}$ states at B factories: X, Y, Z
- Example: $X(3872)$ (Belle, CDF, BaBar, D0)
- No ordinary $c\bar{c}$-state
  - Decays violate isospin
  - Measured mass depends on decay channel

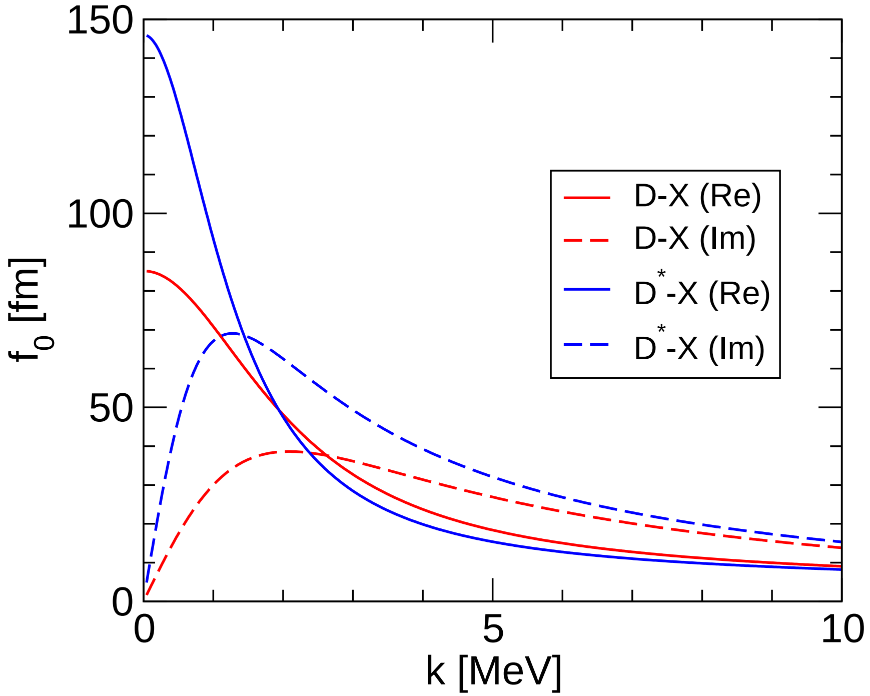


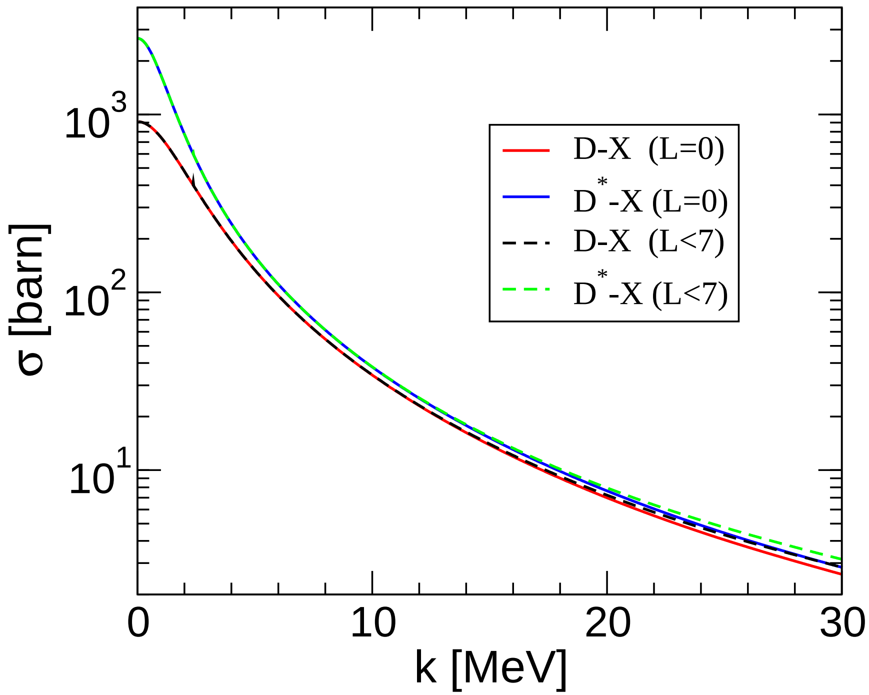
$m_X = (3871.68 \pm 0.17)$ MeV $\Gamma < 1.2$ MeV $J^{PC} = 1^{++}$ or 2^{-+}

- Nature of $X(3872)$?
  $\bar{D}^0 D^{0*}$-molecule, tetraquark, charmonium hybrid, ...
- Molecular nature $\Rightarrow$ interaction of $X(3872)$ with D^0, $\bar{D}^0$, D^{0*}, $\bar{D}^{0*}$ determined by large scattering length

X-D- and X-D^*-Scattering

- Predictions for scattering amplitude/cross section





Canham, HWH, Springer, Phys. Rev. D **80**, 014009 (2009)

- Three-body scattering lengths: $a_{D^0X} = -9.7a \approx -85$ fm
  $a_{D^{*0}X} = -16.6\,a \approx -146$ fm

Volume Dependence of Cluster States

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- Finite volume dependence required for ab initio lattice calculations (cubic box $\sim L^3$) (cf. Epelbaum et al., PRL **106** (2011) 192501)
  - L-dependence of 2-body halo states behaves as 2-body system
- Mass shift from overlap of copies from periodic boundary cond.

$$\Delta E_B = \sum_{|\mathbf{n}|=1} \int d^3\mathbf{r}\ \psi_B^*(\mathbf{r}) V(\mathbf{r}) \psi_B(\mathbf{r} + \mathbf{n}L) + \mathcal{O}(e^{-\sqrt{2}\kappa L})$$

- S-wave bound states (Lüscher, 1986)

$$\Delta E_B = E_B(\infty) - E_B(L) = -3|\gamma|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}(e^{-\sqrt{2}\kappa L})$$

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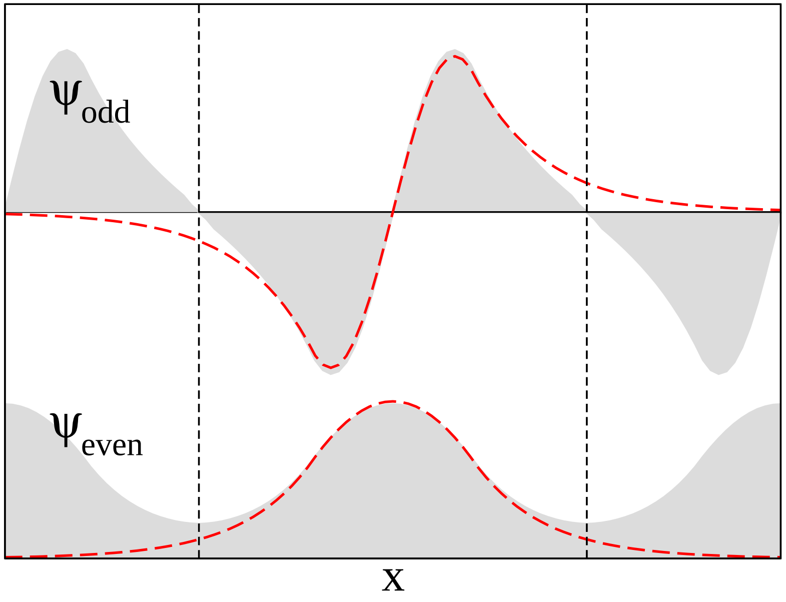
- Generalization to higher ℓ, e.g. P-waves (König, Lee, HWH, PRL **107** (2011) 112001)

$$\Delta E_B^{(1,0)} = \Delta E_B^{(1,\pm 1)} = +3|\gamma|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}(e^{-\sqrt{2}\kappa L})$$

Volume Dependence of Cluster States

- Different sign for even and odd partial waves

$\Rightarrow$ can be understood
from curvature
of wave function
at the boundary



- m-dependence for D- and higher waves, but

$$\sum_{m=-\ell}^{\ell} \Delta E_B^{(\ell,m)} = (-1)^{\ell+1}(2\ell+1)\cdot 3|\gamma|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}(e^{-\sqrt{2}\kappa L})$$

Topological Phase Corrections

- Topological phase for scattering of bound states
- Factorize center-of-mass motion

$$\psi_L\left(\vec{r}_1, \vec{r}_2\right) = e^{i2\pi\alpha\vec{k}\cdot\vec{r}_1/L} e^{i2\pi(1-\alpha)\vec{k}\cdot\vec{r}_2/L} \phi_L(\vec{r}_1 - \vec{r}_2)\,, \quad \alpha = m_1/(m_1+m_2)$$

- Periodicity of ψ_L induces topological phase for winding of ϕ_L around cube

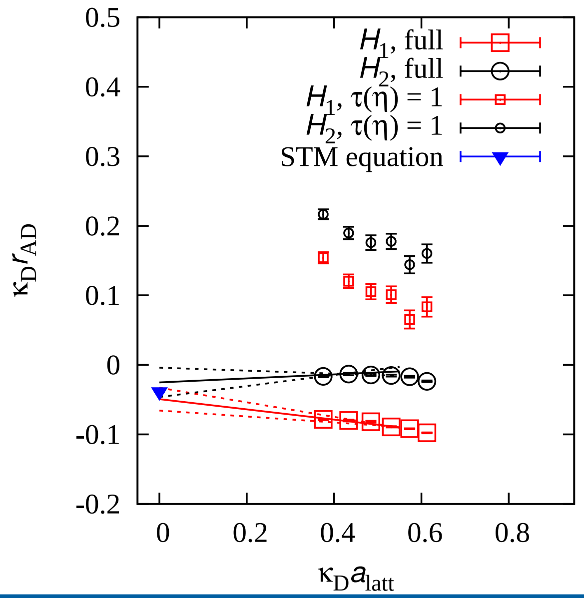
$$\phi_L(\vec{r} + \vec{n}L) = e^{-i2\pi\alpha\vec{k}\cdot\vec{n}} \phi_L(\vec{r})$$

- Energy correction:

$$\frac{\Delta E_{\vec{k}}(L)}{\Delta E_{\vec{0}}(L)} = \frac{1}{3} \sum_{l=1,2,3} \cos\left(2\pi\alpha k_l\right)$$

Bour, König, Lee, HWH, Meißner,

Phys. Rev. D **84** (2012) 091503



Topological Phase Corrections

- Fermion-dimer scattering (spin-1/2 fermions)
  - Lattice calculation:

$$\kappa a_{fd} = 1.174(9), \quad \kappa r_{fd} = -0.029(13)$$

  - Continuum EFT: (cf. Simenog et al. (1984))

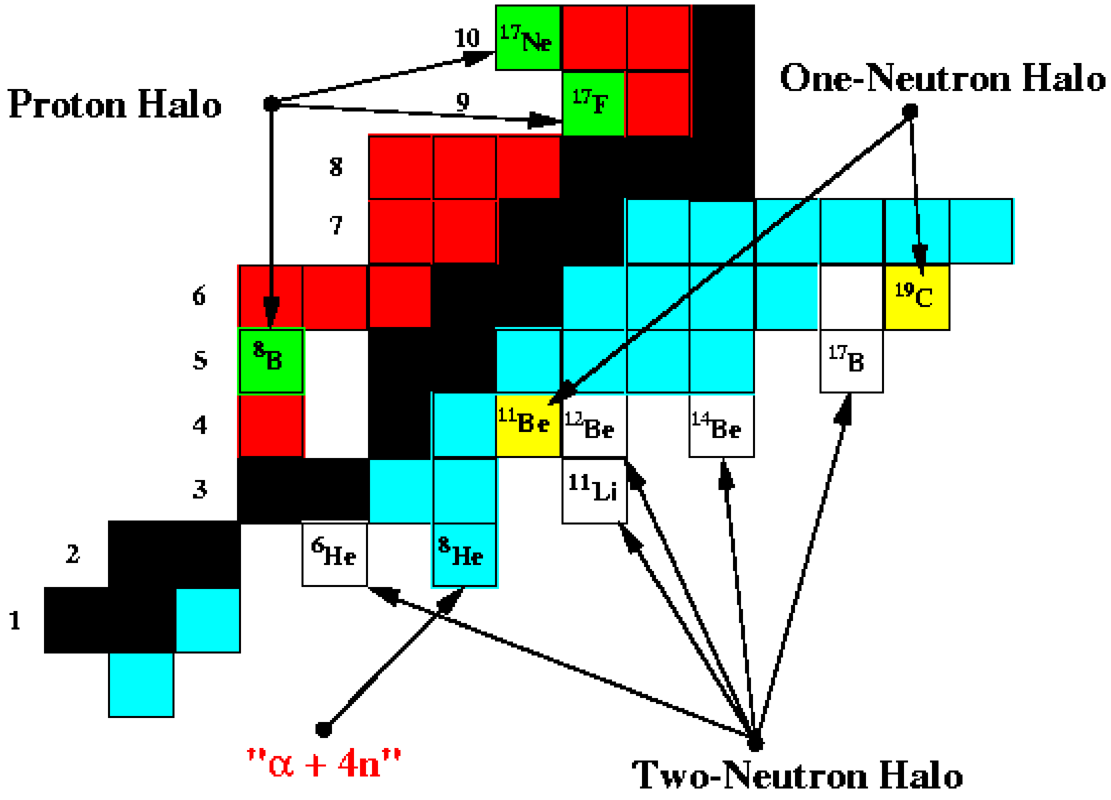
$$\kappa a_{fd} = 1.17907(1), \quad \kappa r_{fd} = -0.0383(3)$$

Bour, HWH, Lee, Meißner, Phys. Rev. C **86** (2012) 034003

- Small, negative effective range
- Result also applies to quartet neutron-deuteron scattering
- Future extension: 4-body scattering calculations, ...

Halo Nuclei

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$
  $\longrightarrow$ close to "nucleon drip line" $\longrightarrow$ scale separation $\longrightarrow$ EFT



http://www.nupecc.org

- EFT for halo nuclei (Bedaque, Bertulani, HWH, van Kolck, 2002)

Electromagnetic Structure of ^{11}Be

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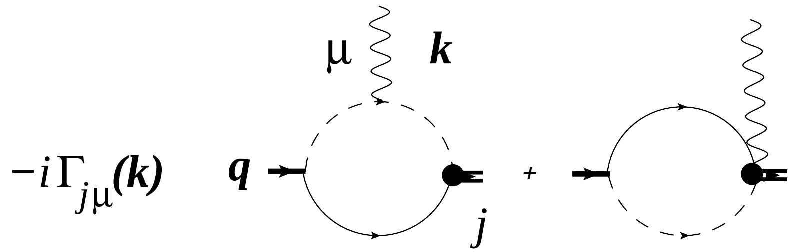
- Properties of ^{11}Be
  - Ground state: $J^P = 1/2^+$, neutron separation energy: 504 keV
  - Excited state: $J^P = 1/2^-$, neutron separation energy: 184 keV
- Properties of ^{10}Be
  - Ground state: $J^P = 0^+$
  - First excitation: 3.4 MeV above g.s.
- Separation of scales: $E_{lo}/E_{hi} \approx \frac{0.5}{3.5} = \frac{1}{7} \quad \Rightarrow \quad R_{core}/R_{halo} \approx 0.4$

$\Rightarrow$ one neutron halo picture for ^{11}Be appropriate

- Effective range theory (Typel, Baur, 2004, 2005, 2008)
- EFT $\Longrightarrow$ straightforward coupling to external currents
- Study EM properties in halo EFT picture (HWH, Phillips, NPA **865** (2011) 17)

S-to-P Transition



- Irreducible transition vertex

$$\Gamma_{ji} = \delta_{ji}\Gamma_E + (k_j q_i + \cancel{q_j k_i})\Gamma_M \qquad \text{for} \qquad \mathbf{k}\cdot\mathbf{q} = 0, \quad \mathbf{k}\cdot\epsilon = 0$$

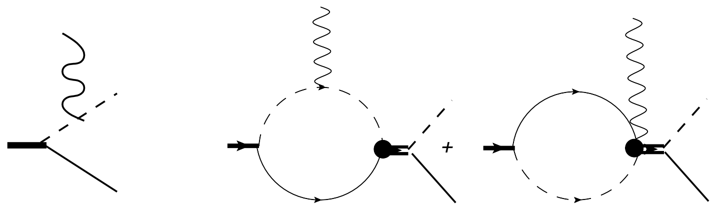
- Current conservation: $k_\mu \Gamma_{j\mu} = 0 \implies \omega\Gamma_{j0} = k_j \Gamma_E$
- $B(E1)$ transition strength:

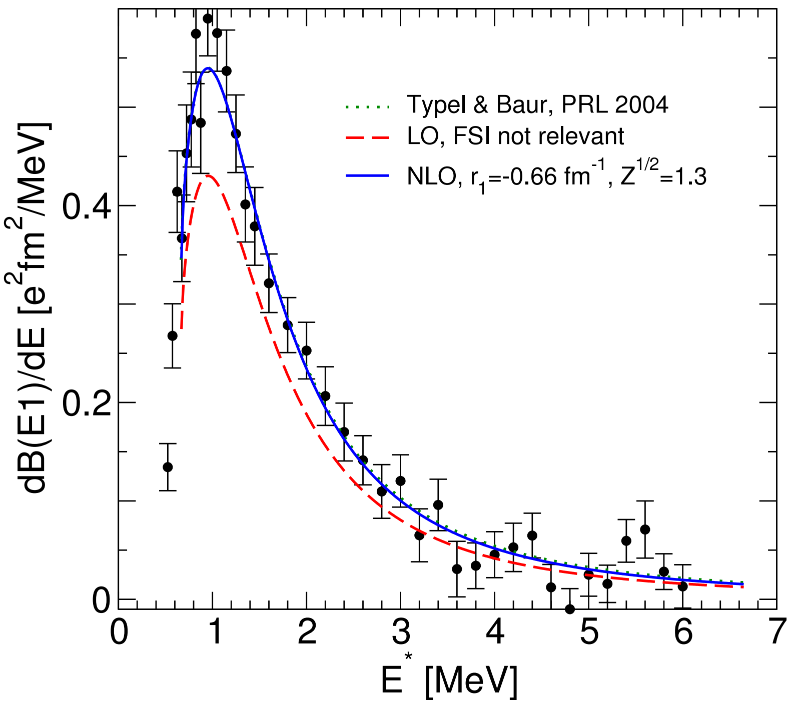
$$B(E1) = \frac{1}{4\pi}\left(\frac{\Gamma_E}{\omega}\right)^2 = \frac{Z_{eff}^2 e^2}{3\pi}\,\frac{\gamma_0}{-r_1}\left[\frac{2\gamma_1 + \gamma_0}{(\gamma_0+\gamma_1)^2}\right]^2 + \ldots$$

- No cutoff required: divergences cancel!
- Experiment: $B(E1) = 0.105...0.116\ e^2$ fm^2
  (Summers et al., PLB **650** (2007) 124; Millener et al., PRC **28** (1983) 497)
- Strategy: determine $r_1 = -0.66$ fm^{-1} at LO

Coulomb Dissociation

- Transition to the continuum:





- Reasonable convergence
- At LO: no FSI
- At NLO:

$$r_1 = -0.66 \text{ fm}^{-1} \quad [B(E1)]$$

$$\sqrt{Z_\sigma} = 1.3 \quad \Rightarrow \quad r_0 = 2.7 \text{ fm}$$

- Detector resolution folded in

Data: Palit et al., PRC **68** (2003) 034318

Universal Correlations

- EFT gives correlations between different observables
- Example: $B(E1)$ and radius of P-wave state

$$B(E1) = \frac{2e^2 Q_c^2}{15\pi} \left(\langle r_c^2 \rangle_{^{11}\mathrm{Be}^*} - \langle r_c^2 \rangle_{^{10}\mathrm{Be}} \right) x \left[\frac{1+2x}{(1+x)^2} \right]^2 + \ldots ,$$

where $x = \sqrt{B_1/B_0}$

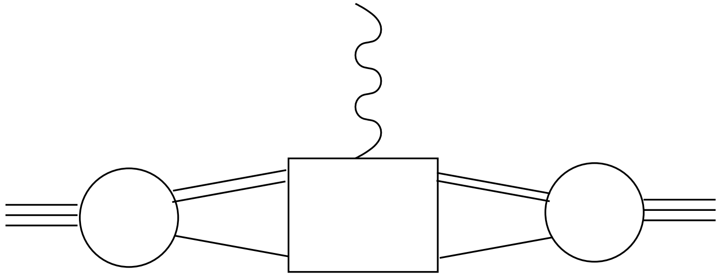
- Adapt strategy to experimental situation
- P-wave radius relative to ^{10}Be core from $B(E1)$

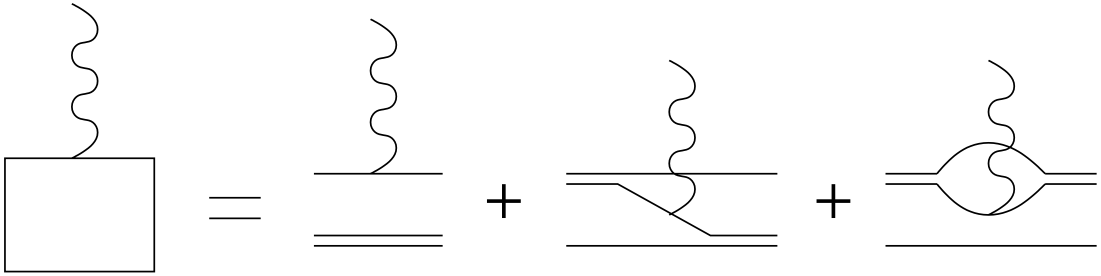
$$\langle r_c^2 \rangle_{^{11}\mathrm{Be}^*} - \langle r_c^2 \rangle_{^{10}\mathrm{Be}} = 0.35...0.39 \text{ fm}^2$$

Universality: can be applied to any one-neutron halo nucleus with shallow S- and/or P-Wave State

EM Form Factors

- Future extension: EM form factors of $2n$ halo nuclei
  (Hagen, HWH, Platter, in progress)





Summary and Outlook

- Effective field theory for large scattering length
  - Discrete scale invariance, universal correlations, ...
- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Few-body nuclei: halo nuclei, ...
  - Hadronic molecules: $X(3872)$, ...

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  - Cold atoms close to Feshbach resonance
  - Few-body nuclei: halo nuclei, ...
  - Hadronic molecules: $X(3872)$, ...
- Future directions: exploring the unitary limit at different scales
  - Finite volume effects in lattice simulations
  - Hadronic molecules: b-quark sector, three-body molecules?, ...
  - Halo nuclei: drip line systems, external currents, reactions, ...
  - Cold atoms: dipolar interactions, confined geometries, heteronuclear systems, 2d-systems, P-waves, ...

Additional Slides

Physics Near the Unitary Limit

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- Unitary limit relevant at different scales: threshold states
- Particle physics: hadronic molecules
  - $X(3872)$ as a $D^0\bar{D}^{0*}$ molecule? ($J^{PC} = 1^{++}$)

$$B_X = (0.3 \pm 0.4) \text{ MeV}$$

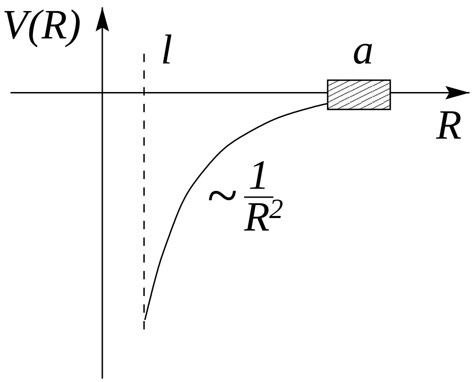
- Nuclear physics: drip line nuclei
  - $2N$-system: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - ^{11}Be $\Rightarrow$ ^{10}Be + n: n separation energy ≈ 0.5 MeV
- Atomic physics:
  - ^{4}He: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW} \longrightarrow B_d \approx 100$ neV
  - Feshbach resonances: a can be varied experimentally

$\Longrightarrow$ tune system to the unitary limit

Broken Scale Invariance

- Three-boson system near the unitary limit
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$



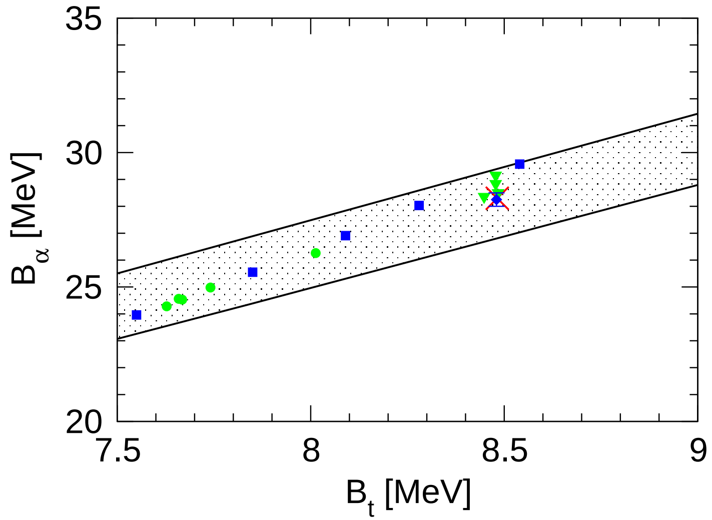
- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

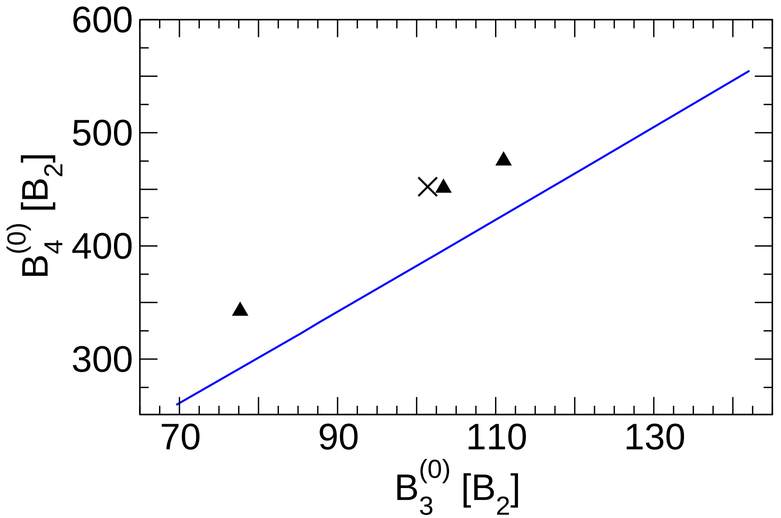
$\Longrightarrow$ scale invariance is anomalous
$\Longrightarrow$ observables depend on 3-body parameter (and a)

- EFT formulation: boundary condition $\Rightarrow$ 3-body interaction @ LO

Universal Correlations

- 2 Parameters at LO ⇒ 3-body observables are correlated
  $\Longrightarrow$ Phillips line (Phillips, 1968)
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)
  ⇒ 4-body observables are correlated $\Longrightarrow$ Tjon line



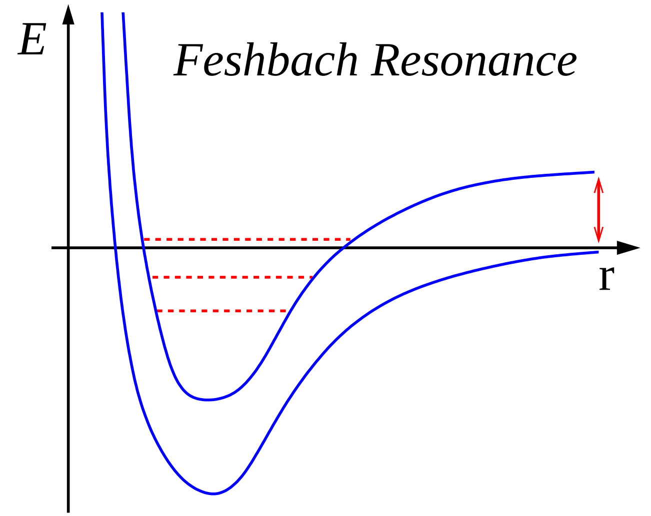


- Variation of 3-body parameter generates correlations
- Nuclear physics: Λ dependence of V_{low-k} (Bogner et al., 2004)
- Tjon line also at NLO (Kirscher et al., 2009)

Variable Scattering Length

- Feshbach Resonance:

  energy of molecular state in closed channel close to energy of scattering state



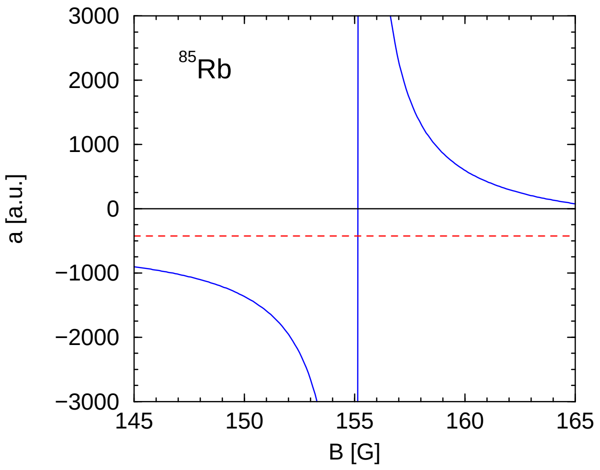
- Tune scattering length via external magnetic field

  (Tiesinga, Verhaar, Stoof, 1993)

- Example:

  ^{85}Rb atoms $\frac{a(B)}{a_0} = 1 + \frac{\Delta}{B_0 - B}$

  $a_0 = -422$ a.u., $B_0 = 155.2$ G, $\Delta = 11.6$ G

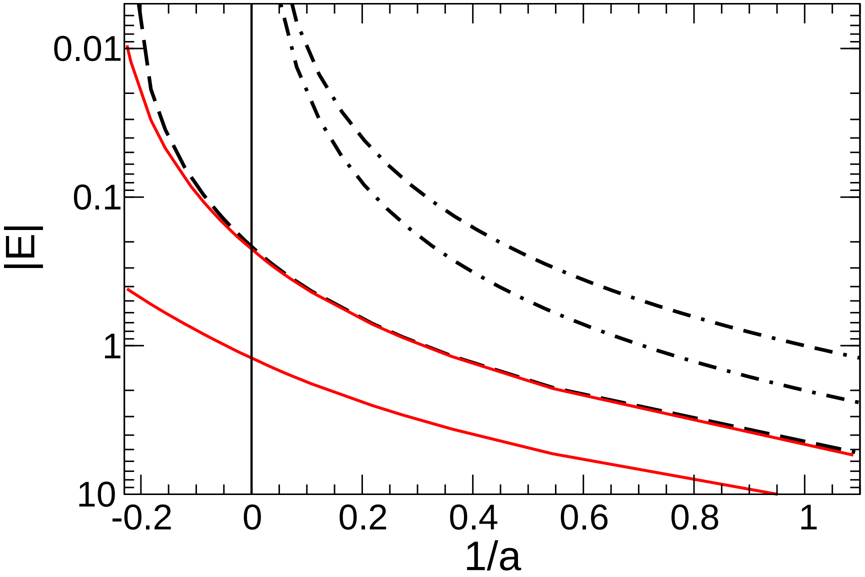


Universal Tetramers

- No four-body parameter at LO (Platter, HWH, Meißner, 2004)
- Universal properties of 4-body system with large a
  - Bound state spectrum, scattering observables, ...
- "Efimov-plot": 4-body bound state spectrum as function of $1/a$

$$B_4^{(0)} = 5B_3^{(0)} \qquad (1/a \equiv 0)$$
$$B_4^{(1)} = 1.01 B_3^{(0)}$$

(Platter, HWH, 2007)



- Extension to thresholds and signature in Cs loss data

  (von Stecher et al., 2009; Ferlaino et al. (Innsbruck), 2009)

Antisymmetrization Issues

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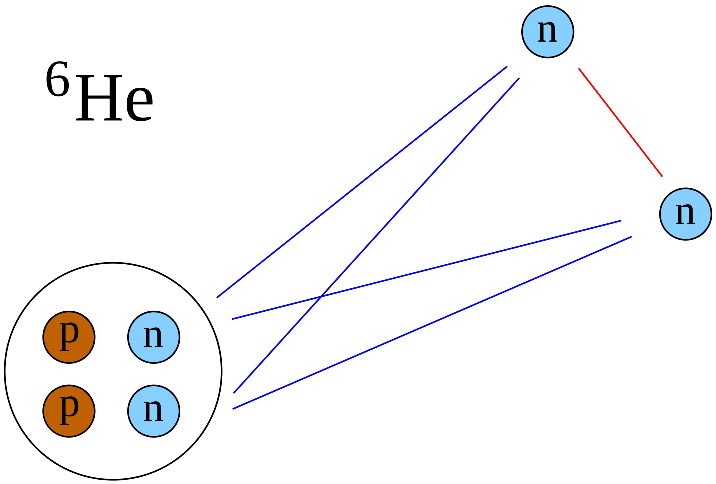
- Antisymmetrization with respect to neutrons in core?
- Core neutrons not active dof in halo EFT



- Physics: exchange of core nucleon and halo nucleon only contributes to observables if there is spatial overlap between wave functions of core and halo nucleon

$$\Longrightarrow \quad \text{small for } R_{core} \ll R_{halo}$$

- Effects subsumed in low-energy constants, included perturbatively in expansion in R_{core}/R_{halo}

Effective Field Theory for ^{11}Be

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- Introduce fields for neutron/core with S- and P-wave interactions
- Effective Lagrangian at NLO

  (cf. Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003)

$$\begin{aligned}\mathcal{L} \;=\;& c^\dagger\left(i\partial_t + \frac{\nabla^2}{2M}\right)c + n^\dagger\left(i\partial_t + \frac{\nabla^2}{2m}\right)n \\ &+\sigma^\dagger\left[\eta_0\left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_0\right]\sigma + \pi_j^\dagger\left[\eta_1\left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_1\right]\pi_j \\ &-g_0\left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c\right] - \frac{g_1}{2}\left[\pi_j^\dagger(n\, i\overleftrightarrow{\nabla}_j\, c) + (c^\dagger\, i\overleftrightarrow{\nabla}_j\, n^\dagger)\pi_j\right] + \ldots\end{aligned}$$

- Parameters:
  - Leading order: g_0, Δ_1, $g_1 \Leftarrow B_0$, B_1, a_1 or $B(E1)(1/2^+ \to 1/2^-)$
  - Next-to-leading order: $\Delta_0 \Leftarrow B(E1)(1/2^+ \to 1/2^-)$ or dB/dE

Effective Field Theory for ^{11}Be

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- EFT generates S- and P-wave states from core-neutron contact interactions
- Reproduces correct asymptotics of wave functions for S- and P-wave states

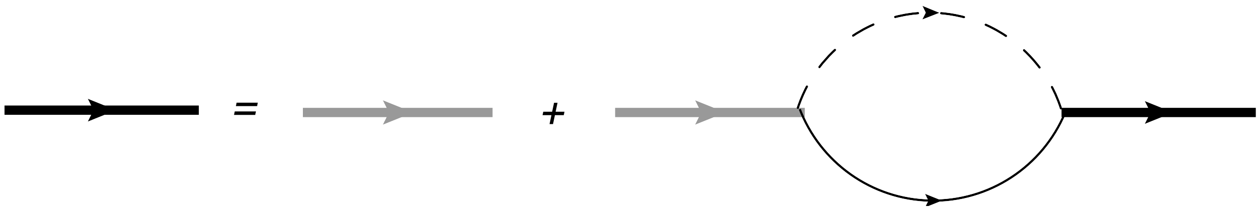
$$
\begin{aligned}
u_0(r) &= A_0 \exp(-\gamma_0 r) \\
u_1(r) &= A_1 \exp(-\gamma_1 r) \left(1 + \frac{1}{\gamma_1 r}\right)
\end{aligned}
$$

- Focus on observables: no discussion of n-core interaction at short distances, spectroscopic factors, ...
- Halo EFT: expansion in R_{core}/R_{halo}
- Generating the S- and P-wave states in Halo EFT:

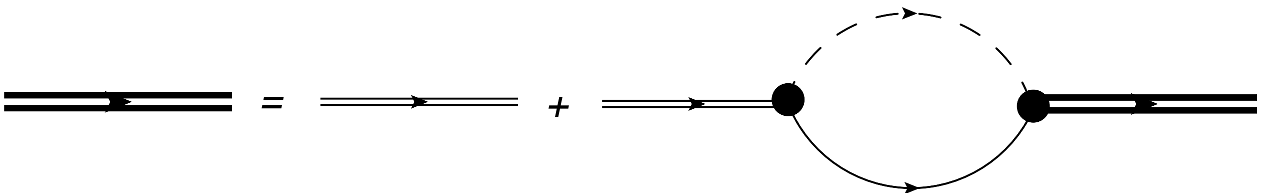
$\Longrightarrow$ sum the nc bubbles

Generating S- and P-Wave States

- S-wave state: $g_0^2/\Delta_0 \sim R_{halo}$, nc loop $\sim 1/R_{halo} \Rightarrow$ sum bubbles
  (van Kolck, 1997, 1999; Kaplan, Savage, Wise, 1998)



- P-wave state:



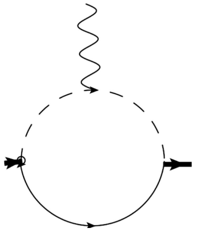
- Determine parameters from bound state pole and/or scattering parameters

$$D_\pi(p) \propto \frac{1}{r_1 + 3\gamma_1} \frac{1}{p_0 - \mathbf{p}^2/(2M_{nc}) + B_1} + \text{regular}$$

where $\gamma_1 = \sqrt{2m_R B_1}$

Including Photons

- Minimal substitution: $\partial_\mu \to D_\mu = \partial_\mu + ie\hat{Q}A_\mu$
- S-wave form factor (LO):



$$G_c(|\mathbf{q}|) = \frac{2\gamma_0}{f|\mathbf{q}|}\arctan\left(\frac{f|\mathbf{q}|}{2\gamma_0}\right) \qquad \text{where} \qquad f = m_R/M = 1/11$$

- Charge radius of ^{11}Be relative to ^{10}Be:

$$\langle r_c^2\rangle_{^{11}\mathrm{Be}} = \langle r_c^2\rangle_{^{10}\mathrm{Be}} + \frac{f^2}{2\gamma_0^2}\frac{1}{1-\gamma_0 r_0}$$

- Results:
  - At LO: $\langle r_c^2\rangle_{^{11}\mathrm{Be}} - \langle r_c^2\rangle_{^{10}\mathrm{Be}} = 0.19$ fm^2
  - At NLO: $\langle r_c^2\rangle_{^{11}\mathrm{Be}} - \langle r_c^2\rangle_{^{10}\mathrm{Be}} = 0.27...0.32$ fm^2

S-Wave Radii

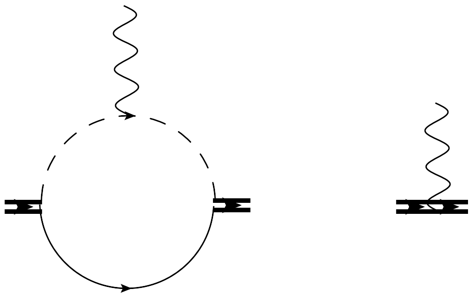
- Comparison to experimental values

  Nörteshäuser et al., Phys. Rev. Lett. **102** (2009) 062503

- Using the experimental value: $\sqrt{\langle r_c^2 \rangle_{^{10}\mathrm{Be}}} = 2.357(18)$ fm
  - At LO: $\sqrt{\langle r_c^2 \rangle_{^{11}\mathrm{Be}}} = 2.40$ fm
  - At NLO: $\sqrt{\langle r_c^2 \rangle_{^{11}\mathrm{Be}}} = 2.42$ fm
- Experimental value: $\sqrt{\langle r_c^2 \rangle_{^{11}\mathrm{Be}}} = 2.463(16)$ fm

P-wave Form Factors

- P-wave form factor (LO):



- Charge form factor:

$$G_c(|\mathbf{q}|) = \frac{1}{r_1 + 3\gamma_1}\left[r_1 + \frac{1}{qf}\left(2qf\gamma_1 + (q^2f^2 + 2\gamma_1^2)\arctan\left(\frac{f|\mathbf{q}|}{2\gamma_1}\right)\right)\right]$$

- Charge radius of ^{11}Be*: $\langle r_c^2\rangle_{^{11}\mathrm{Be}^*} = \langle r_c^2\rangle_{^{10}\mathrm{Be}} - \frac{5f^2}{2\gamma_1 r_1}$
- Using the experimental value (Nörteshäuser et al., PRL **102** (2009) 062503)
  - At LO: $\sqrt{\langle r_c^2\rangle_{^{11}\mathrm{Be}^*}} = (2.43 \pm 0.1)$ fm
  - At NLO: unknown counterterm
- Quadrupole form factor also predicted (not measurable in $J = 1/2$ state)

Counter Terms

- Counter term contributions (not generated by minimal substitution)

$$\mathcal{L}_{EM} = -L_{C0}^{(\sigma)}\sigma_l^\dagger \underbrace{\left(\nabla^2 A_0 - \partial_0(\nabla\cdot\mathbf{A})\right)}_{\nabla\cdot\mathbf{E}}\sigma_l - L_{E1}^{(1/2)}\sum_{ll'j}\sigma_l\pi_{l'}^\dagger\left(\frac{1}{2}l\frac{1}{2}l'\middle|1j\right)\underbrace{\left(\nabla_j A_0 - \partial_0 A_j\right)}_{\mathbf{E}_j} - L_{C0}^{(\pi)}\pi_l^\dagger \underbrace{\left(\nabla^2 A_0 - \partial_0(\nabla\cdot\mathbf{A})\right)}_{\nabla\cdot\mathbf{E}}\pi_l + \ldots$$

- Where do they come in?
  - $L_{C0}^{(\sigma)}$: $\langle r_c^2\rangle^{(\sigma)}$ at N3LO
  - $L_{C0}^{(\pi)}$: $\langle r_c^2\rangle^{(\pi)}$ at NLO $\Longrightarrow$ accuracy of models
  - $L_{E1}^{(1/2)}$: $B(E1)$ at NLO

Nature of $X(3872)$

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- Nature of $X(3872)$ not finally resolved
- Assumption: $X(3872)$ is weakly-bound D^0-$\bar{D}^{0*}$-molecule

$\Longrightarrow |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle)/\sqrt{2}\,, \qquad B_X = (0.26 \pm 0.41)$ MeV

$\Longrightarrow$ universal properties (cf. Braaten et al., 2003-2008, ...)

  - Explains isospin violation in decays of $X(3872) \Rightarrow$ superposition of $I = 1$ and $I = 0$
  - Different masses due to different line shapes in decay channels
- Large scattering length to LO determines interaction of $X(3872)$ with D^0 and D^{0*}
- Higher orders: EFT with perturbative pions

(Fleming, Kusunoki, Mehen, van Kolck, 2007; Fleming, Mehen, 2008)

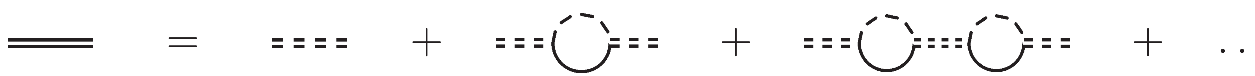
(Braaten, HWH, Mehen, 2010)

EFT for $X(3872)$

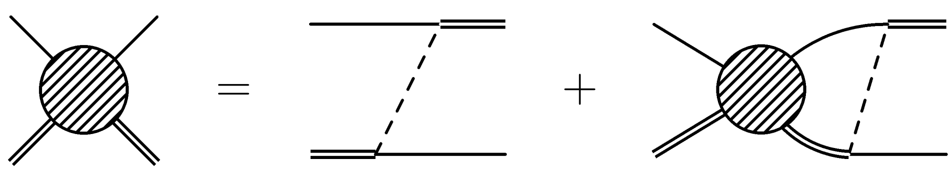
- Effective Lagrangian

$$\mathcal{L} = \sum_{j=D^0,D^{*0},\bar{D}^0,\bar{D}^{*0}} \psi_j^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_j} \right) \psi_j + \Delta X^\dagger X$$
$$- \frac{g}{\sqrt{2}} \left(X^\dagger (\psi_{D^0}\psi_{\bar{D}^{*0}} + \psi_{D^{*0}}\psi_{\bar{D}^0}) + \text{ H.c. } \right) + \ldots ,$$

- Propagator of the $X(3872)$



- Three-body integral equation



Experimental Observation ?

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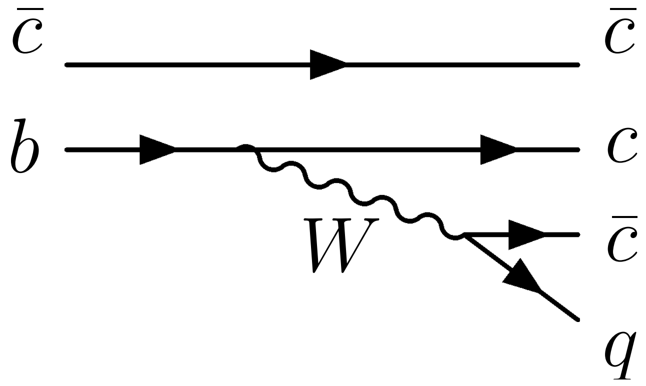
- Behavior of $X(3872)$ produced in isolation should be distinguishable from its behavior when in the presence of $D^0, D^{*0}, \bar{D}^0, \bar{D}^{*0}$
- Final state interaction of D, D^* mesons in B_c-decays
- Example: quark-level B_c decay yielding three charmed/anticharmed quarks in final state



- Process accessible at the LHC

Triton in a Finite Volume

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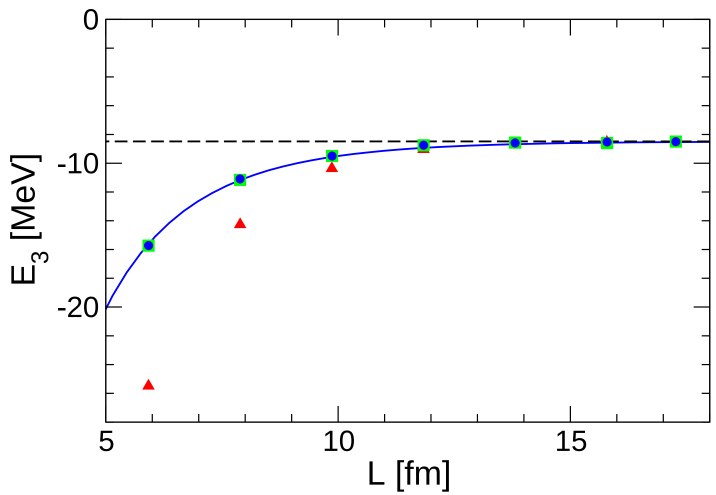
- Light nuclei can also be described in an expansion around the unitary limit (Efimov 1981) $\rightarrow$ pionless effective field theory
- Can be used to calculate volume dependence of triton binding energy
  $\Rightarrow$ Lattice QCD calculations of light nuclei (e.g. NPLQCD collaboration)
- Modification of spectrum by cubic box ($V = L^3$)
  - Box provides infrared cutoff $1/L$
    $\Rightarrow$ calculable in EFT
  - Box breaks rotational invariance
    $\Rightarrow$ partial wave mixing
  - Momenta quantized $\vec{p} = \vec{n}\,(2\pi/L)$
    $\Rightarrow$ 3d sum equation



Kreuzer, HWH, Phys. Lett B **694** (2011) 424