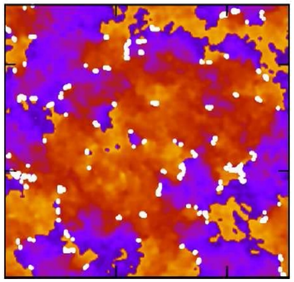
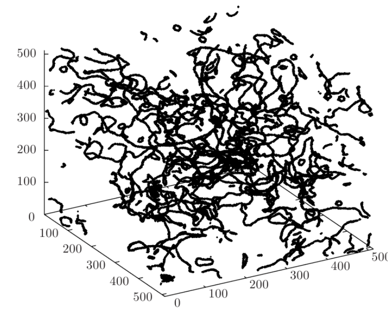


Non-thermal fixed points and superfluid turbulence in ultracold quantum gases



Boris Nowak

S. Erne, M. Karl, M. Schmidt, J. Schole, D. Sexty, T. Gasenzer

Institut für Theoretische Physik

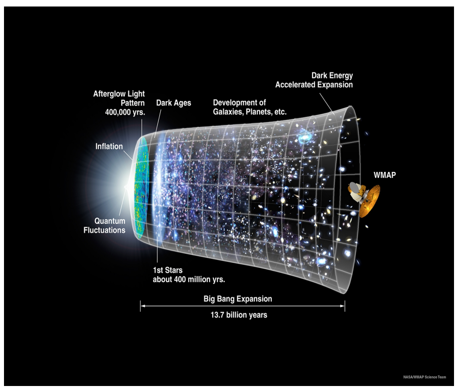
Ruprecht-Karls Universität Heidelberg

email: b.nowak@thphys.uni-heidelberg.de
www: www.thphys.uni-heidelberg.de/~gasenzer

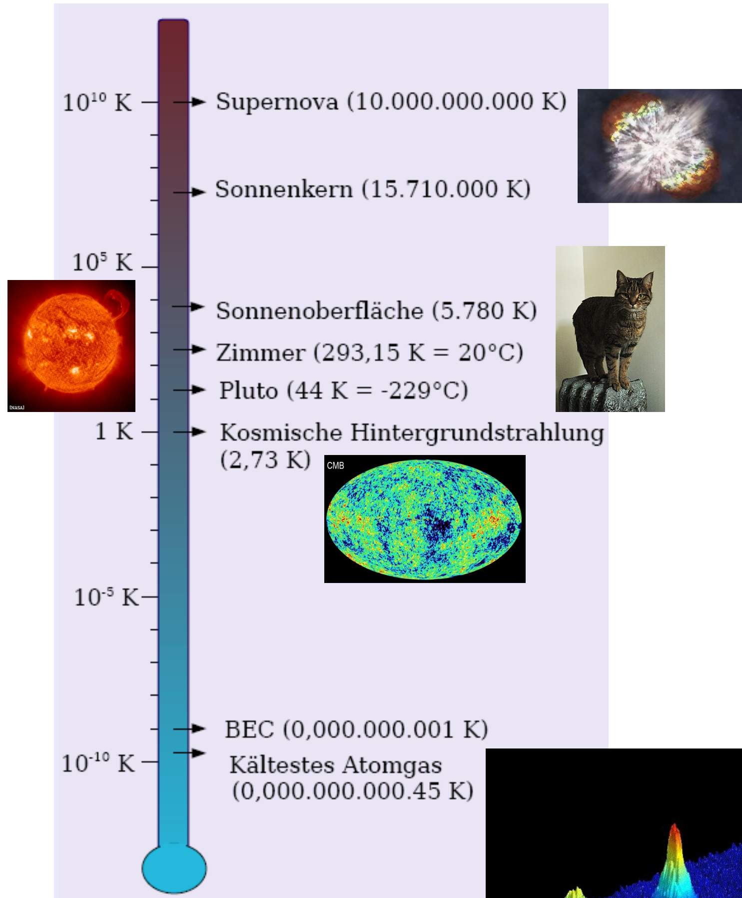
EMMI

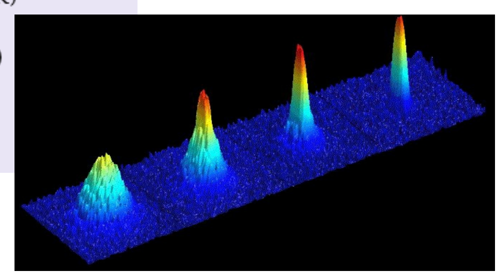
Center for Quantum Dynamics

Non-equilibrium quantum gases

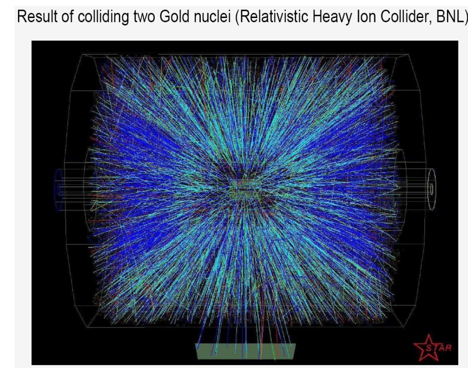


Early universe





Ultracold gases



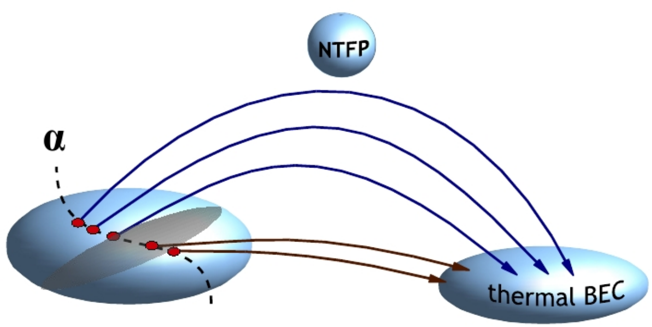
Heavy-ion collisions

Non-equilibrium dynamics

Initial state:
Far from equilibrium

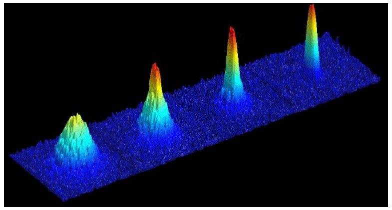
Transient state:
e. g. Turbulence
(Nonthermal fixed point)

Final state:
Thermal equilibrium

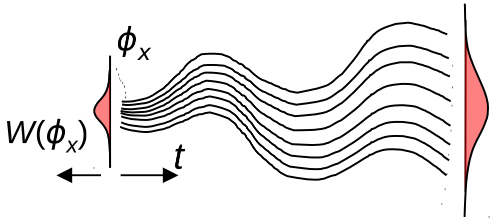


Semi-classical simulations

Classical field equation for ϕ(x,t):



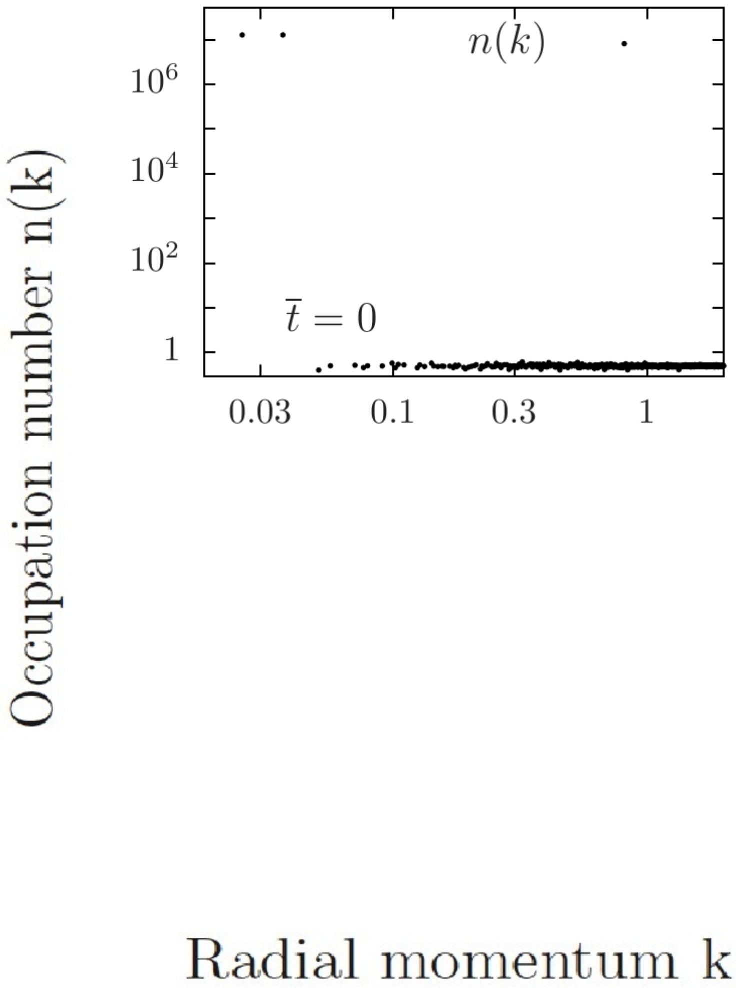
$$\mathrm{i}\partial_t \phi(\mathbf{x},t) = \left[-\frac{\nabla^2}{2m} + g|\phi(\mathbf{x},t)|^2\right]\phi(\mathbf{x},t)$$



Observables: e. g. Momentum distribution

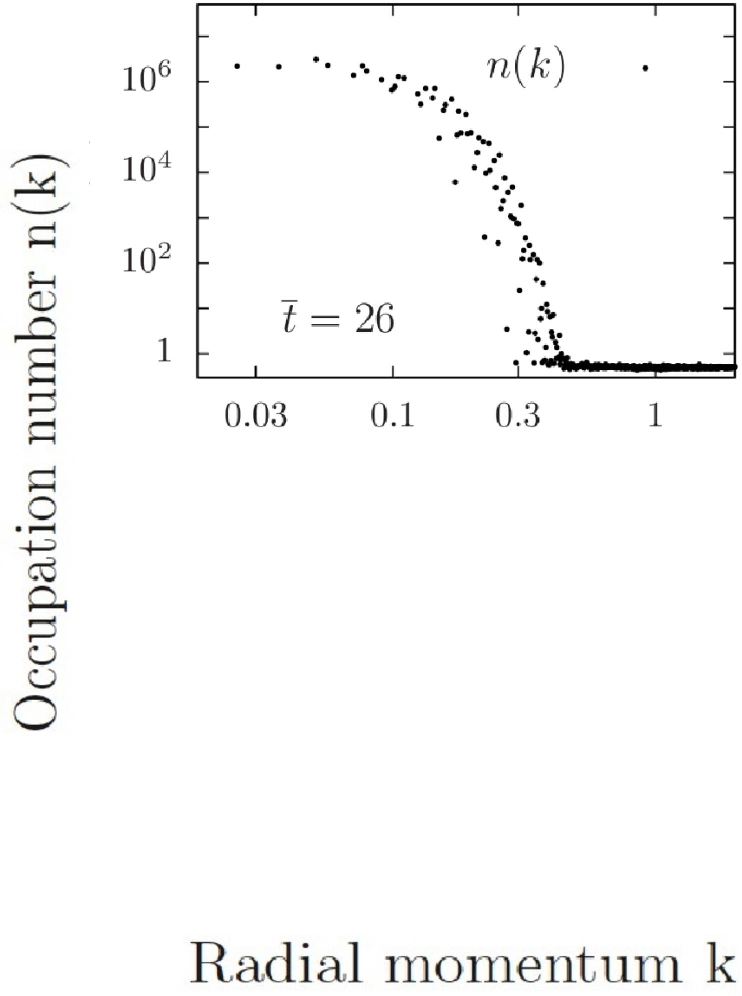
$$n(k) = \int \mathrm{d}^{d-1}\Omega_k \, \langle \phi^*(\mathbf{k})\phi(\mathbf{k})\rangle_{\mathrm{ensemble}}$$

2D: Quench dynamics



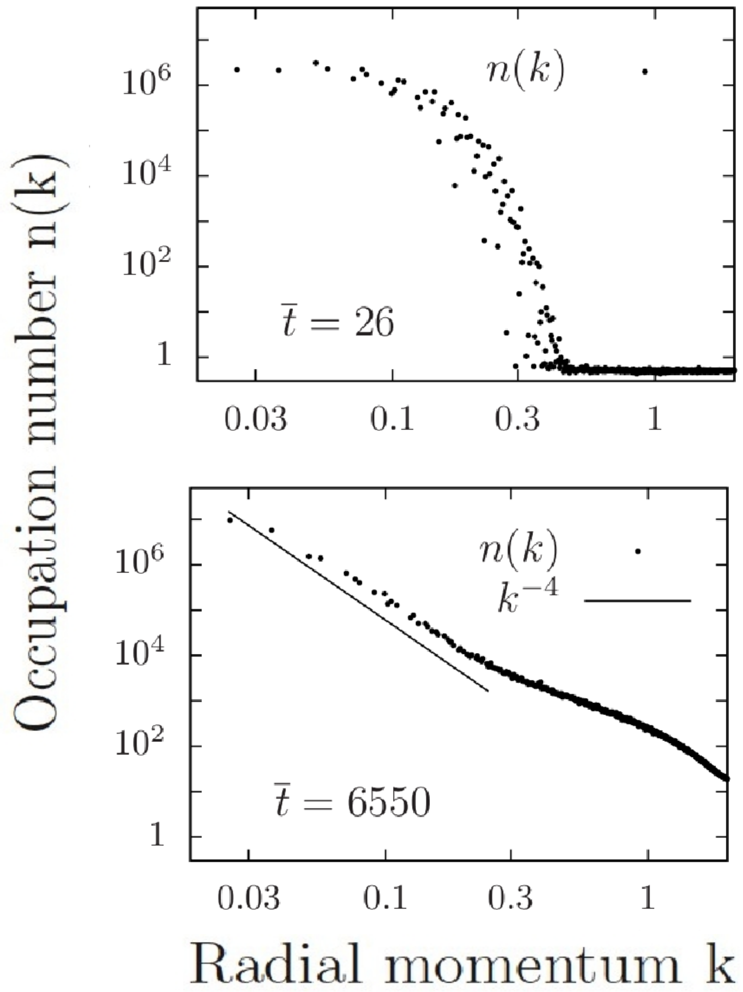
BN, D. Sexty, T. Gasenzer PRB 84(R) (2011), BN, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

2D: Quench dynamics



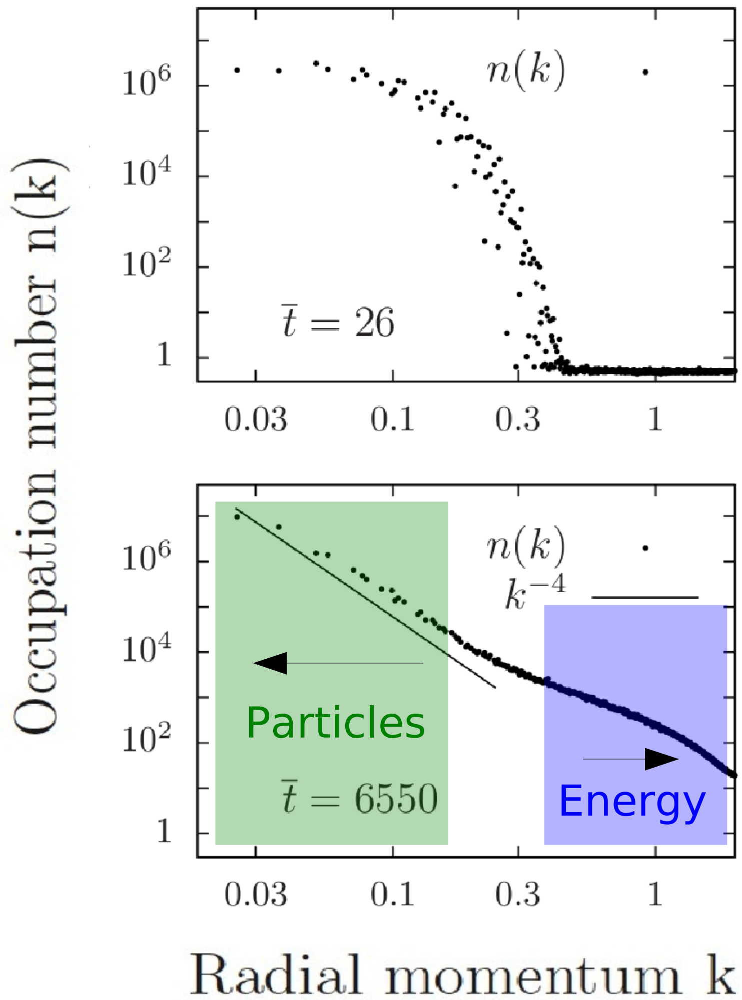
BN, D. Sexty, T. Gasenzer PRB 84(R) (2011), BN, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

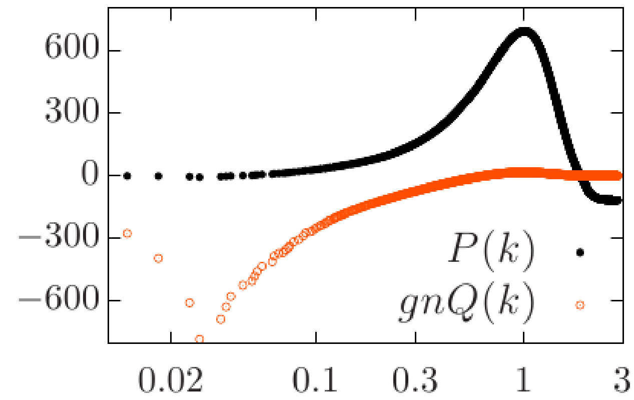
2D: Quench dynamics



BN, D. Sexty, T. Gasenzer PRB 84(R) (2011), BN, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

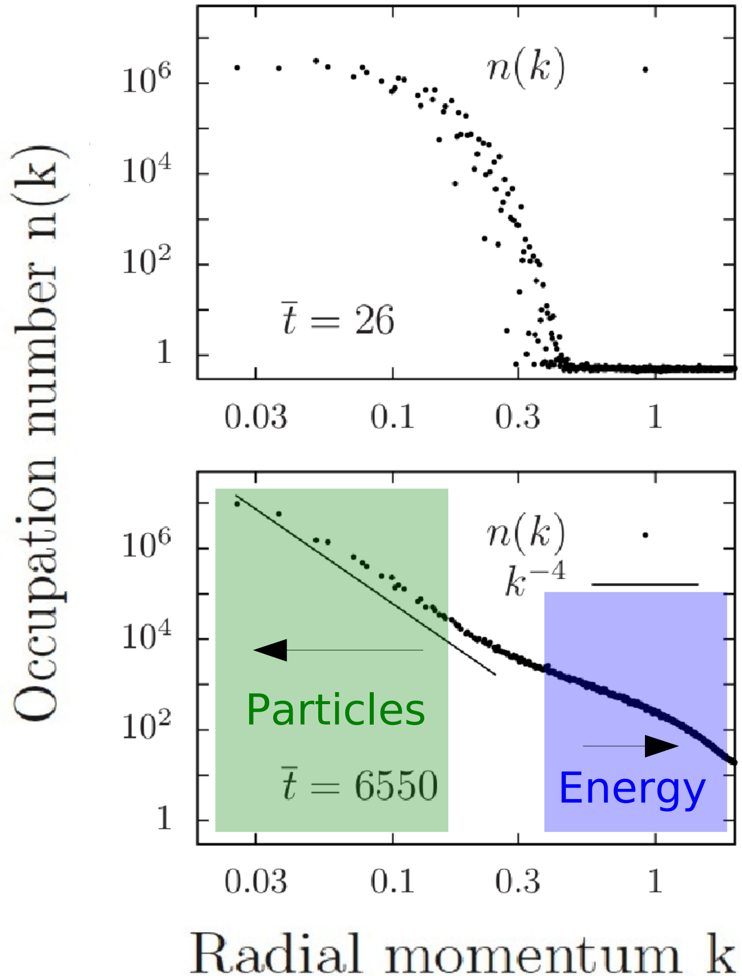
2D: Quench dynamics

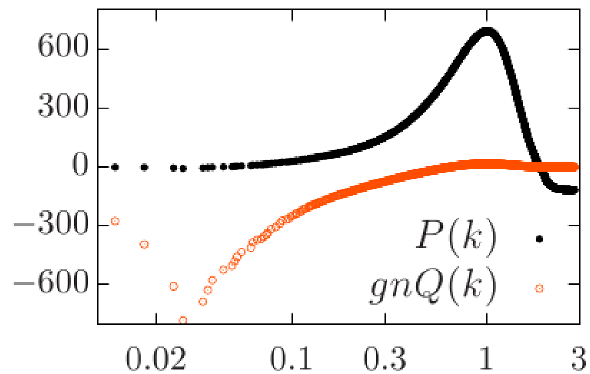


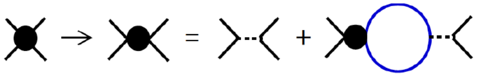


BN, D. Sexty, T. Gasenzer PRB 84(R) (2011), BN, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

2D: Quench dynamics



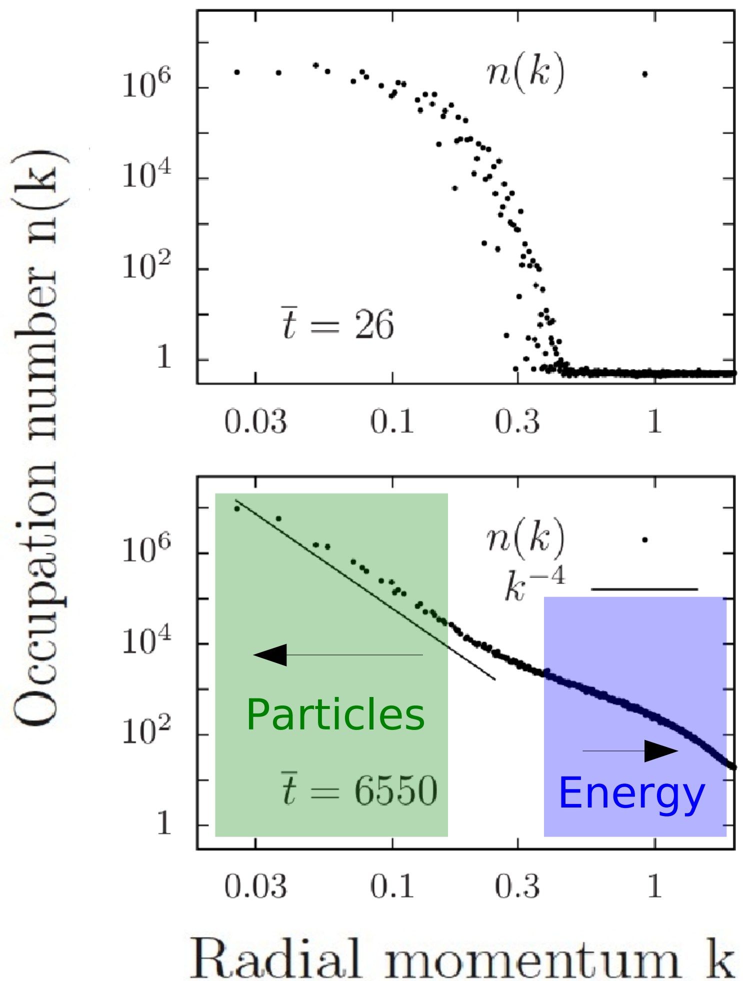


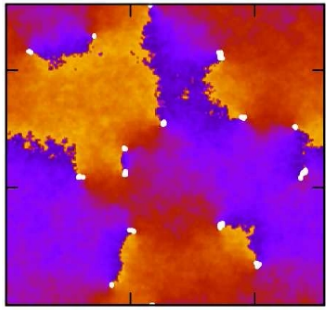


J. Berges, A. Rothkopf, J. Schmidt PRL (2008)
C. Scheppach, J. Berges, T. Gasenzer, PRA (2010)

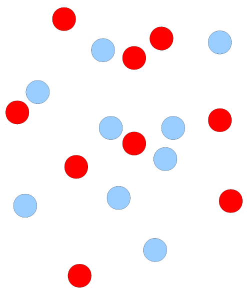
BN, D. Sexty, T. Gasenzer PRB 84(R) (2011), BN, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

2D: Quench dynamics





Phase φ



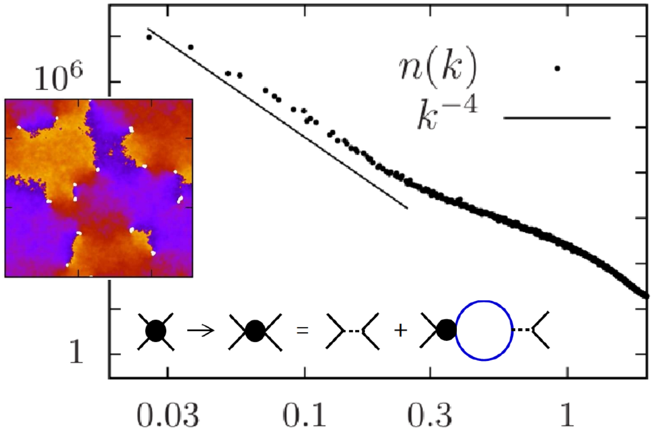
$n_k \sim k^{-4}$

BN, D. Sexty, T. Gasenzer PRB 84(R) (2011), BN, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)

2D: Wave turbulence / vortex dynamics - duality

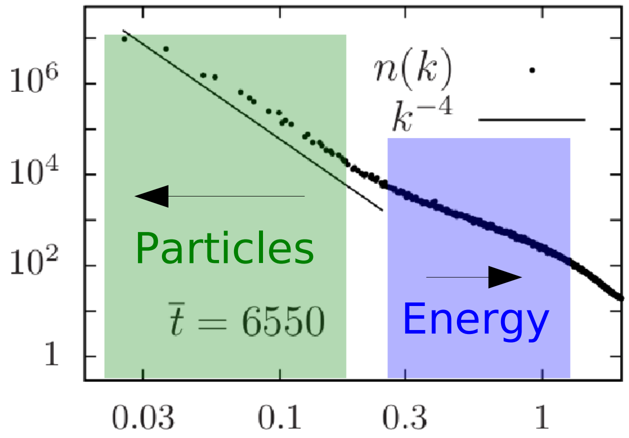
Old woman / young woman

Wave turbulence / vortex dynamics

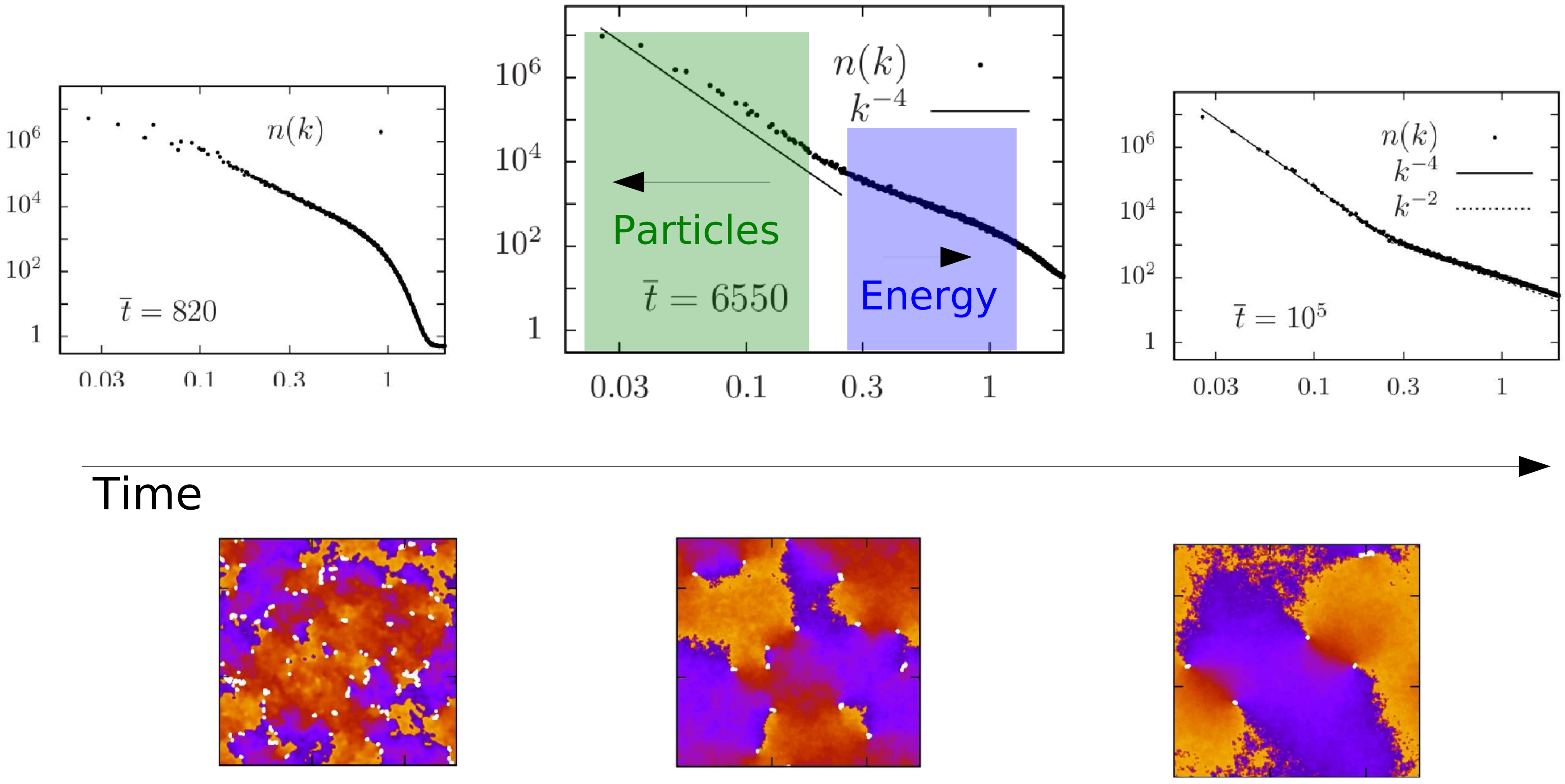


BN, D. Sexty, T. Gasenzer PRB 84(R) (2011), BN, J. Schole, D. Sexty, T. Gasenzer PRA 85 (2012)
J. Schole, BN, T. Gasenzer, PRA (2012), BN., T. Gasenzer arxiv: 1206.3181

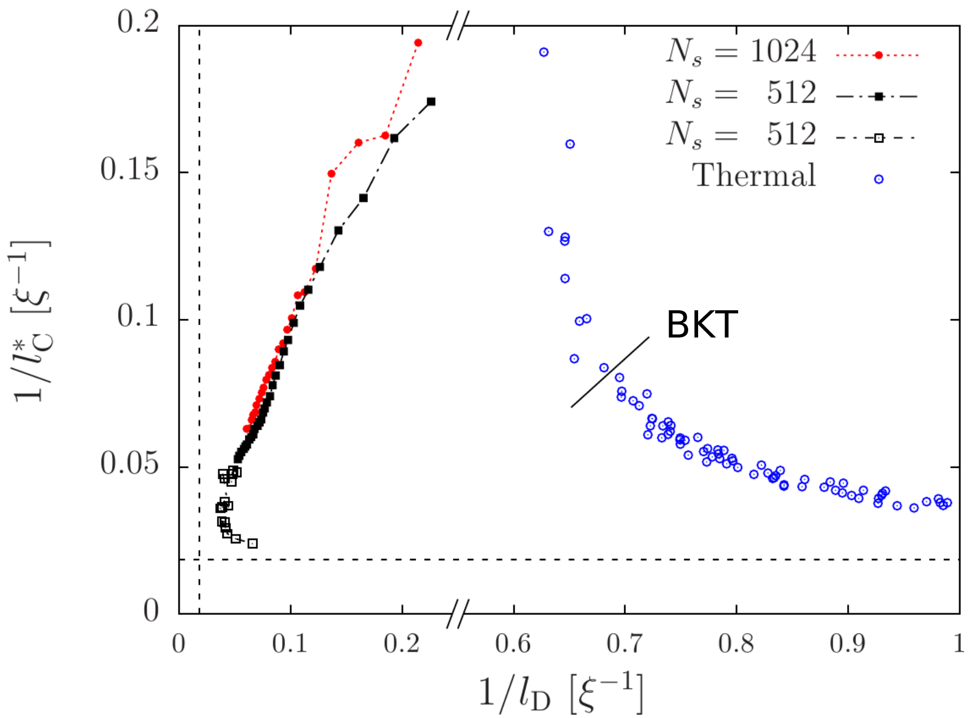
2D: Phase ordering dynamics

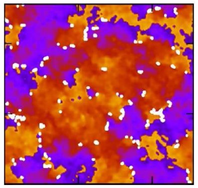


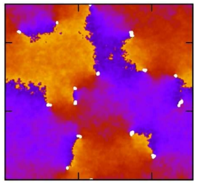
2D: Phase ordering dynamics

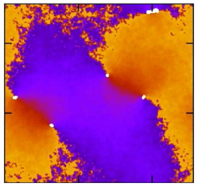


Correlations near the NTFP







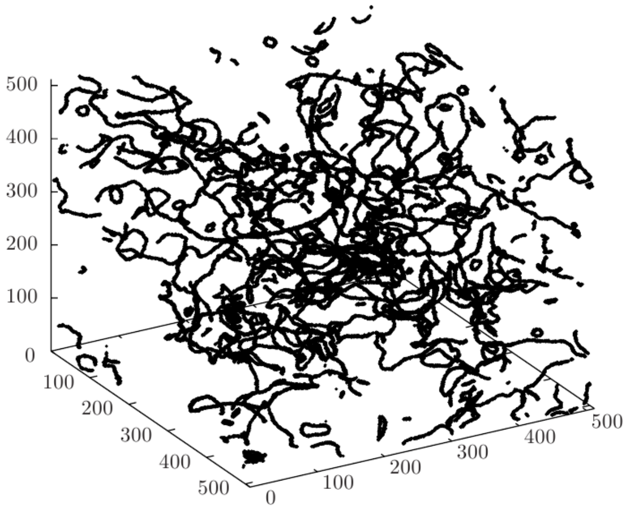


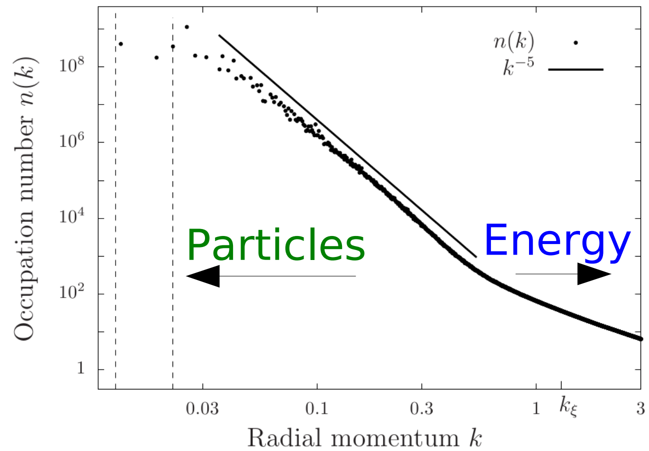
$l_C^\star$ Phase coherence length

l_D Vortex-antivortex pair distance

J. Schole, BN, T. Gasenzer, PRA (2012)

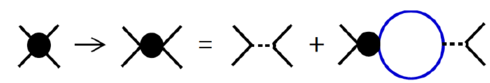
3D: Non-thermal fixed point





IR: $\varsigma = d+2$

UV: $\varsigma = d$

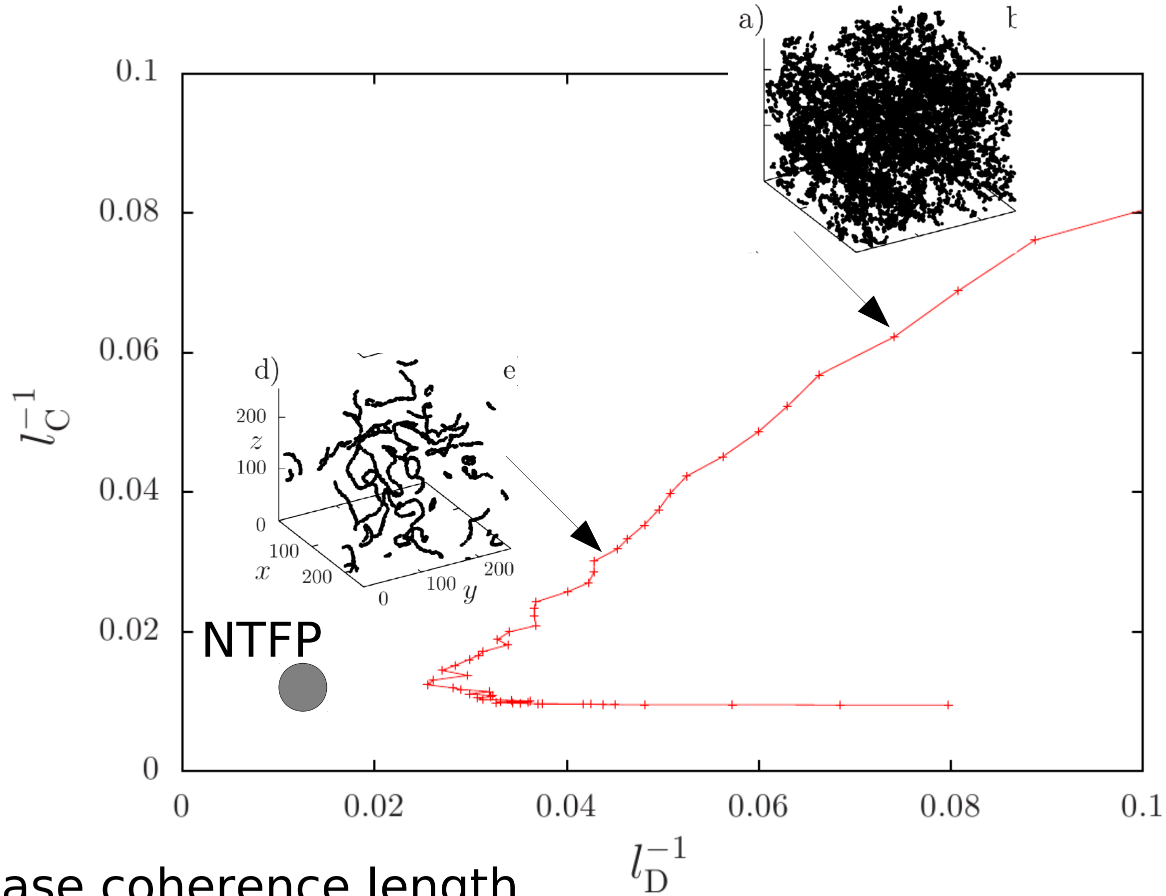


Berges, Rothkopf, Schmidt PRL (2008)
Scheppach, Berges, Gasenzer PRA (2010)

Vortices ← Spectrum n(k) → QFT

BN, D. Sexty, T. Gasenzer PRB (2011), BN, J. Schole, D. Sexty, T. Gasenzer PRA (2012)
BN., T. Gasenzer arxiv: 1206.3181

3D: Non-thermal fixed point

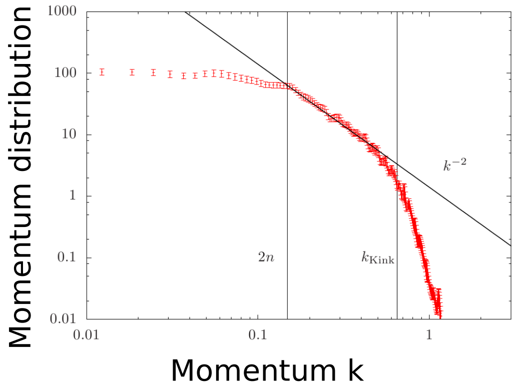


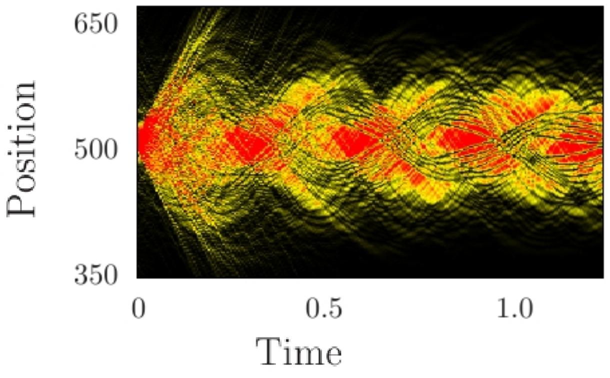
l_c Phase coherence length

l_D Vortex ring radius

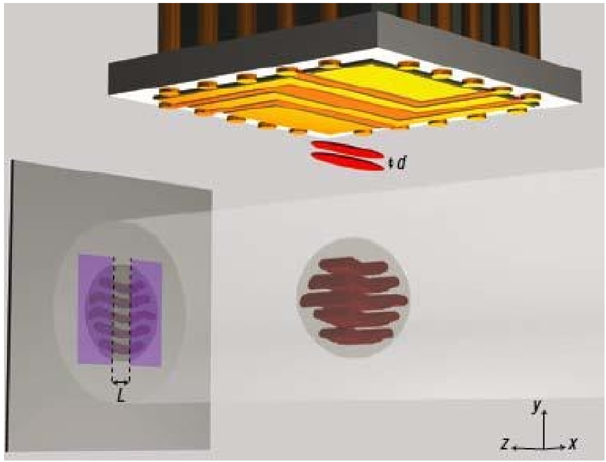
BN, J. Schole, T. Gasenzer in preparation

1D: Quench dynamics and solitons





^{87}Rb-Experiment:
Schmiedmeyer group (Vienna)

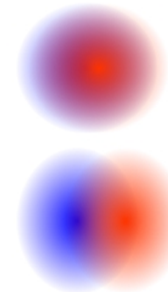


M. Schmidt, S. Erne, BN, D. Sexty, T. Gasenzer NJP (2012)

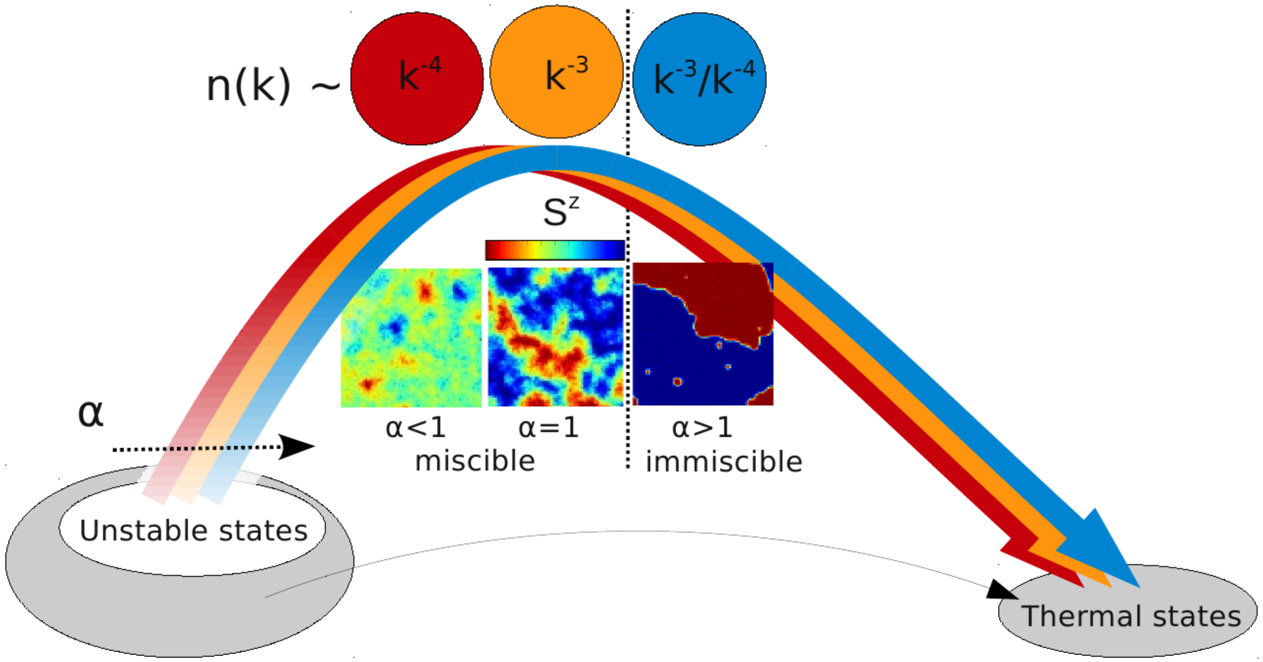
2D: 2-component BEC

miscible
$g_{12} < g$

immiscible
$g_{12} > g$



$\alpha = g_{12}/g$

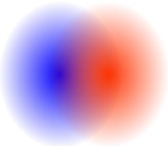


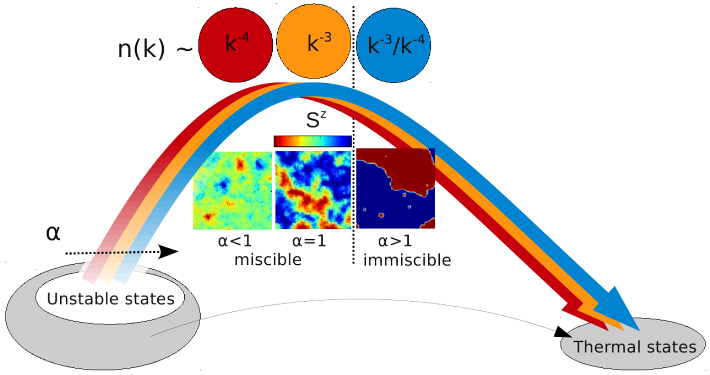
M. Karl, BN, T. Gasenzer in preparation

2D: 2-component BEC

miscible
$g_{12} < g$

immiscible
$g_{12} > g$



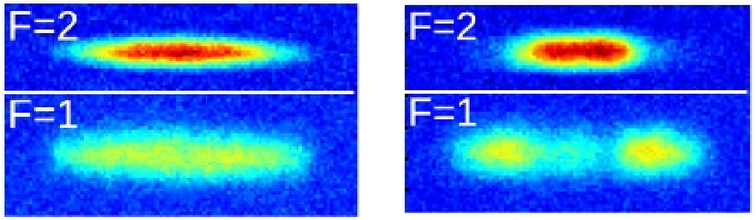


M. Karl, BN, T. Gasenzer in preparation

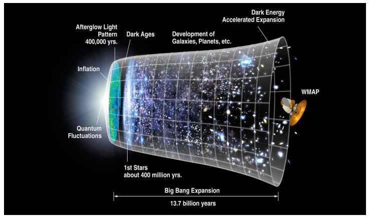
^{87}Rb-Experiment:
Oberthaler group (Heidelberg)

B=9.17 G

B=9.03 G



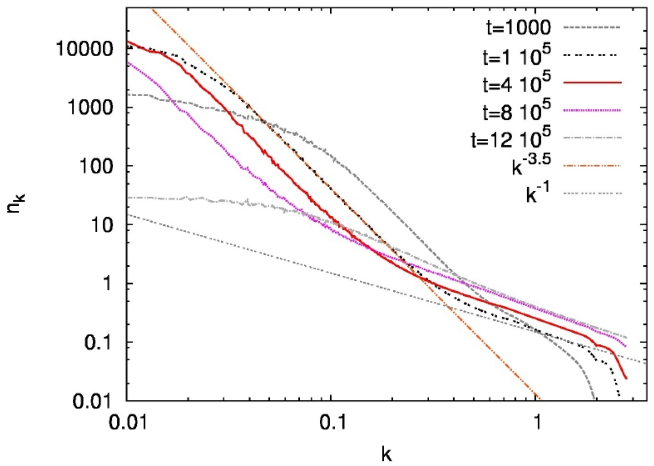
E. Nicklas et al. PRL (2012)



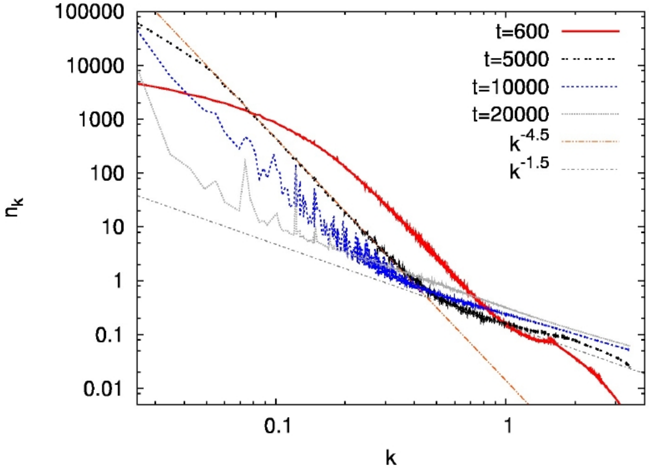
Relativistic simulations O(2)

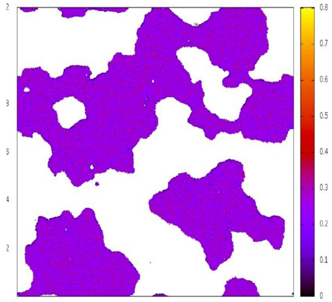
Classical field equation: $\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$

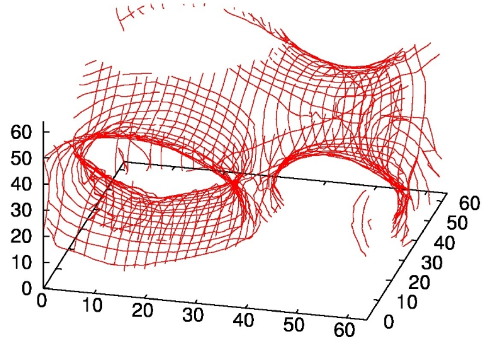
d=2



d=3



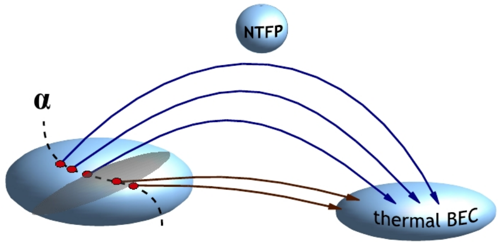




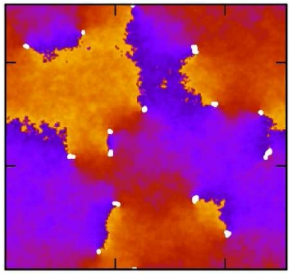
T. Gasenzer, BN, D. Sexty PLB (2012)

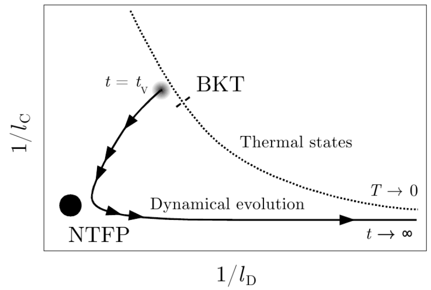
Summary

Non-thermal fixed points (NTFP)

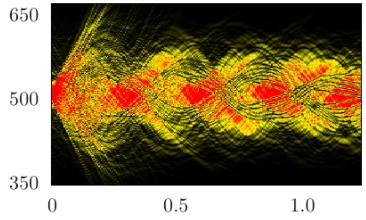


Superfluid turbulence in 2D

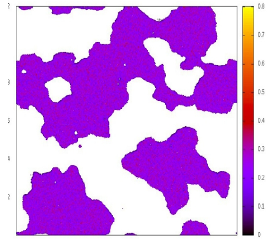




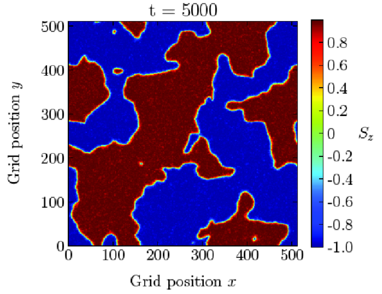
Dynamics near the NTFP in 2D



Solitonic state as a NTFP

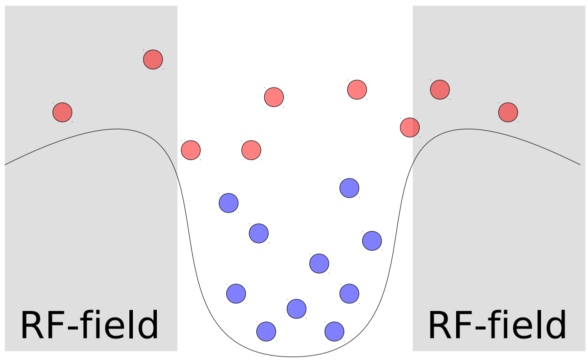


Charge Separation/Domain Walls

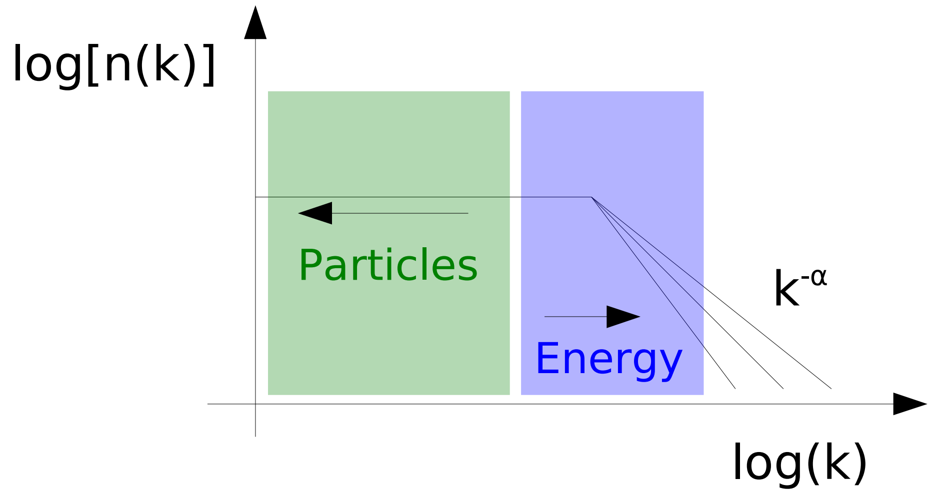


Supplementary Slides

3D: Bose condensation



Evaporative cooling

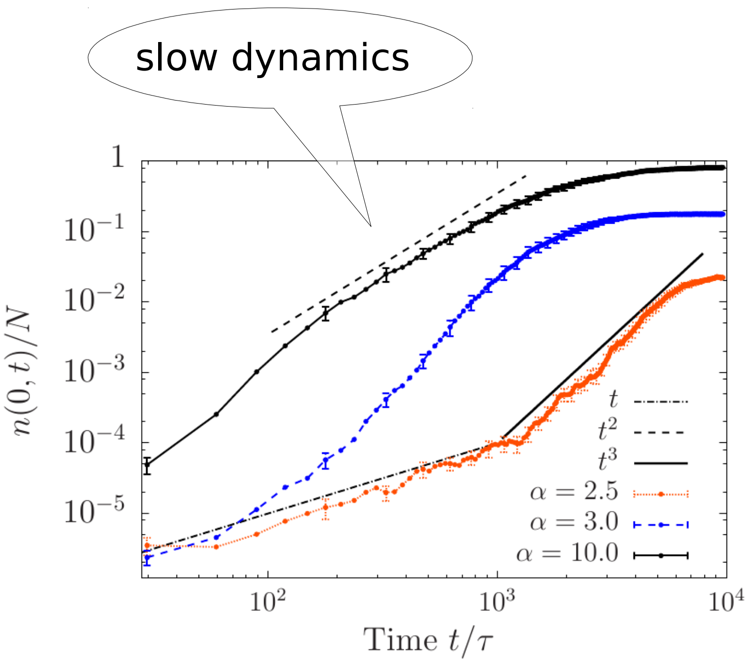


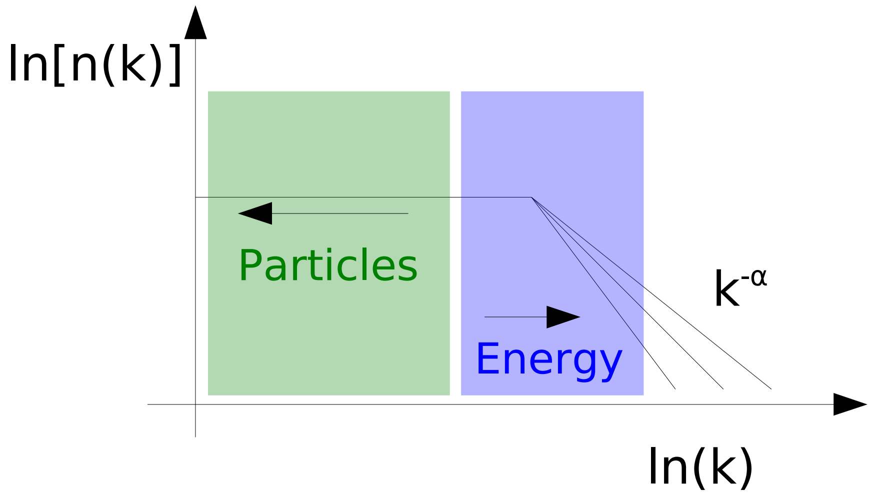
Experiments: Hänsch, Esslinger (2002), Esslinger (2007), Hadzibabic (2012)

Condensation dynamics: Kagan, Svistunov, Shlyapnikov ('90s), Semikoz, Tkachev (1995), Berloff (2002), Anderson, Davis (2008), Blaizot, McLerran(2012), Berges, Sexty (2012)

BN., T. Gasenzer arxiv: 1206.3181

3D: Bose condensation

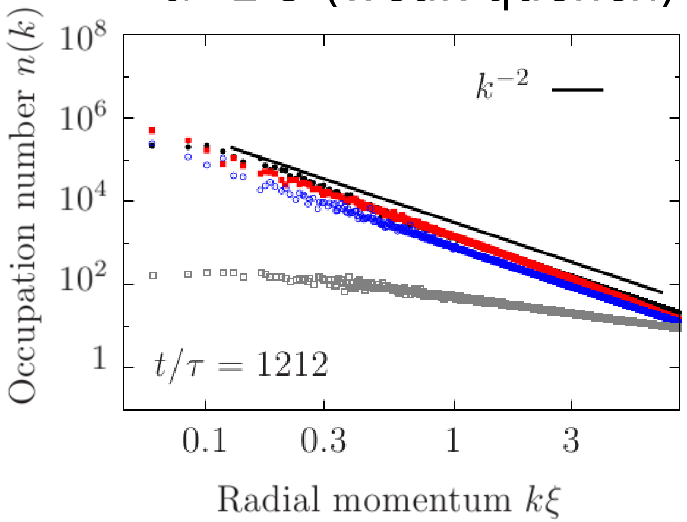




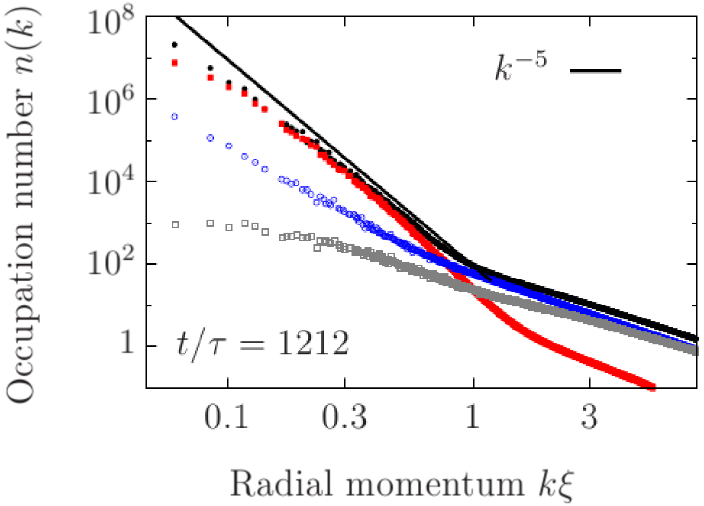
BN., T. Gasenzer arxiv: 1206.3181

3D: Bose condensation

α=2.5 (weak quench)

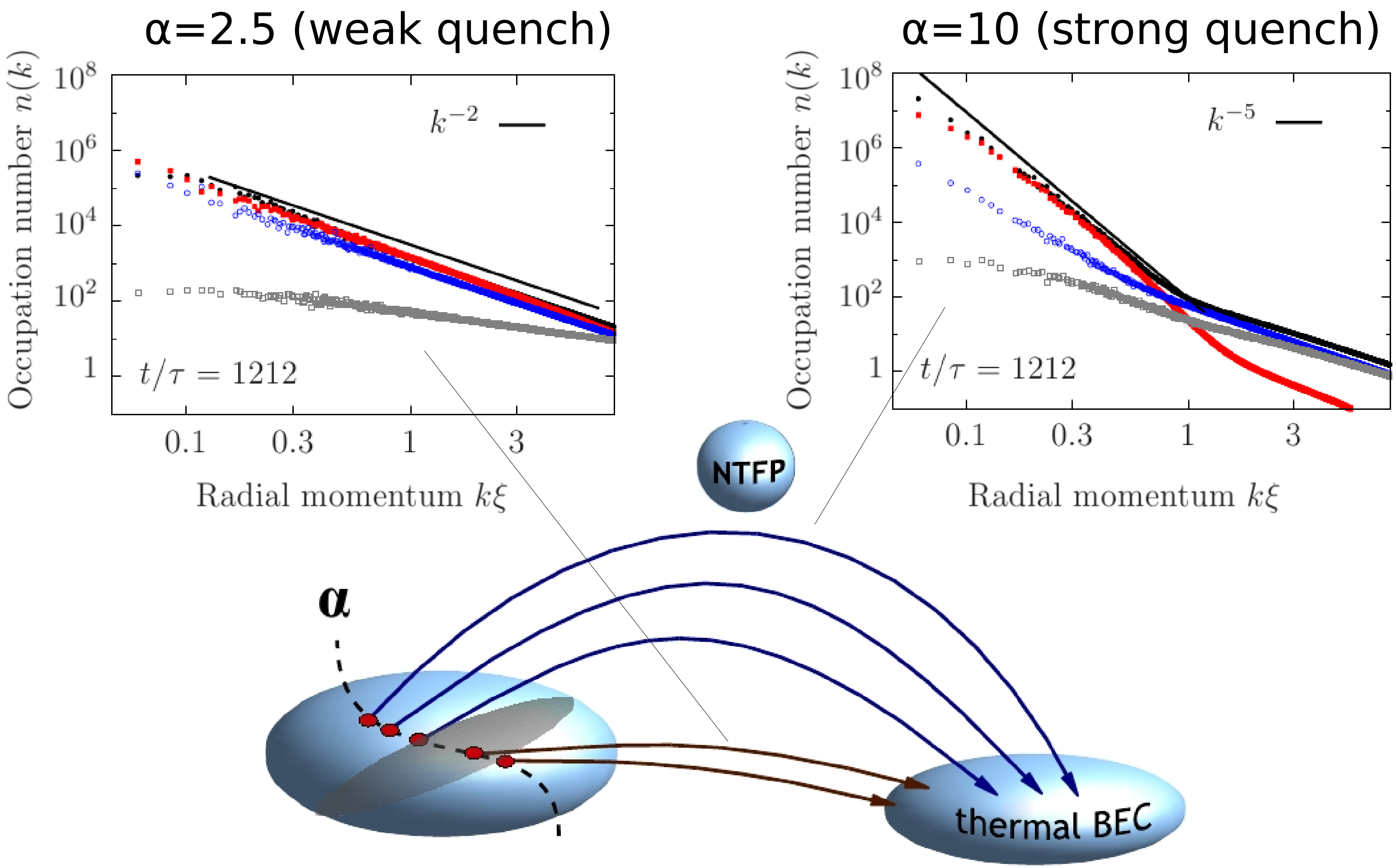


α=10 (strong quench)



BN., T. Gasenzer arxiv: 1206.3181

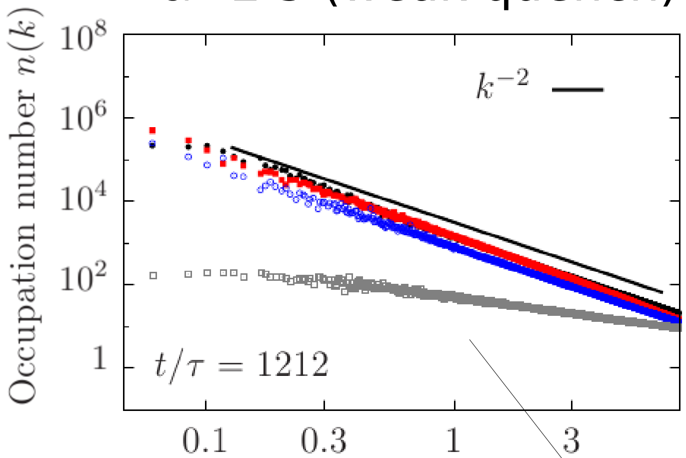
3D: Bose condensation



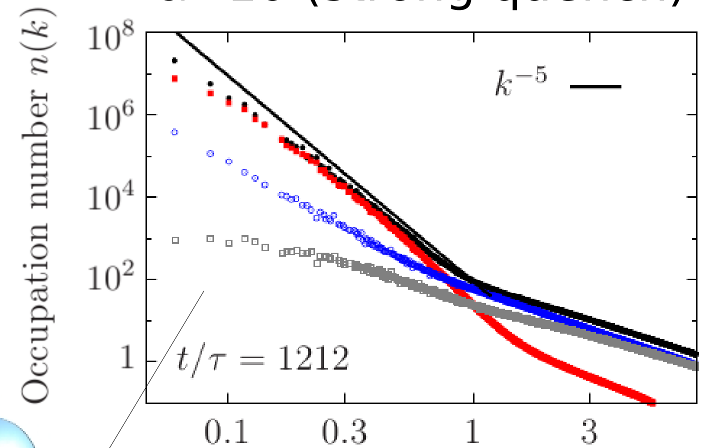
BN., T. Gasenzer arxiv: 1206.3181

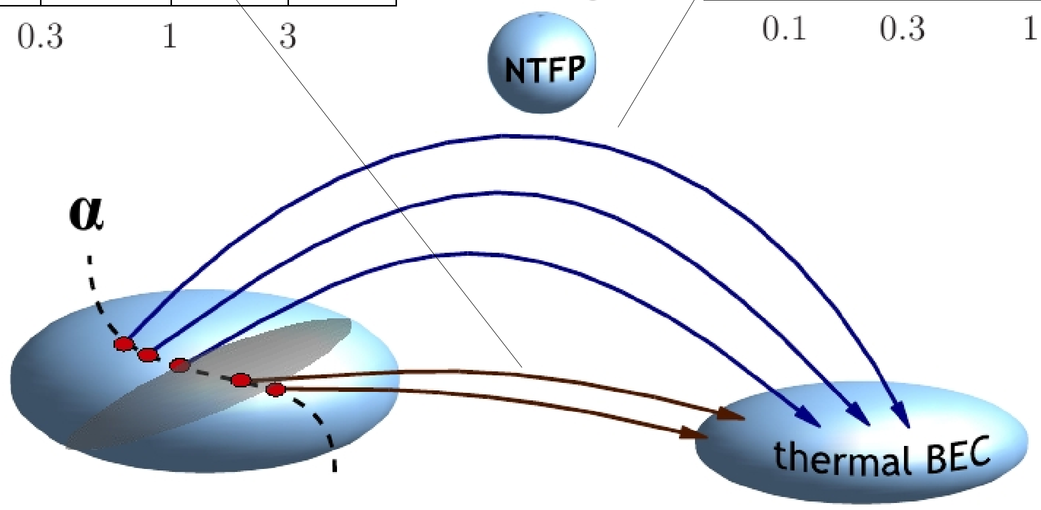
3D: Bose condensation

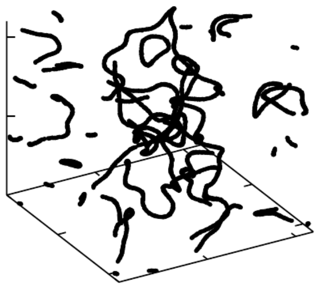
α=2.5 (weak quench)



α=10 (strong quench)

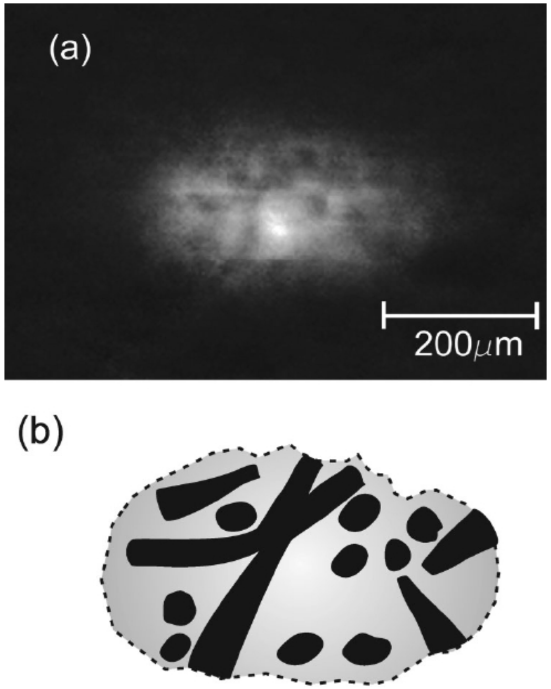




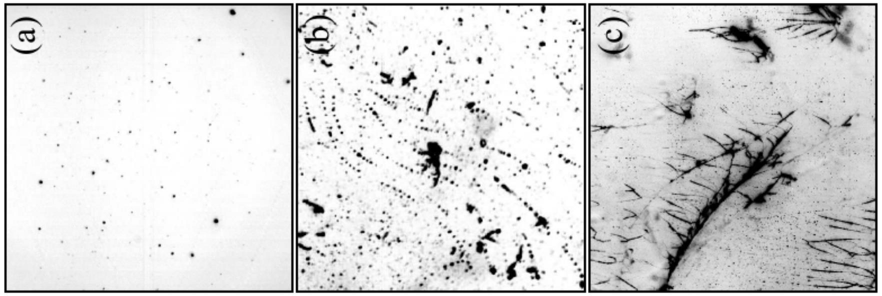


BN., T. Gasenzer arxiv: 1206.3181

Turbulence experiments

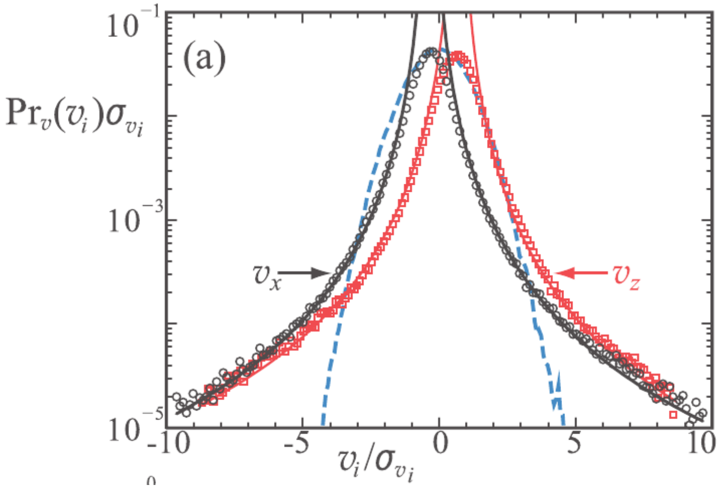


V. Bagnato (Brazil)



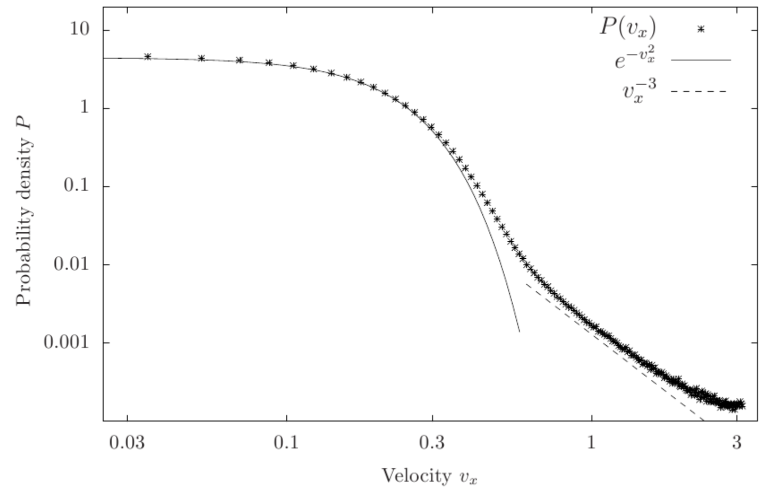
D. Lathrop (USA)

Velocity distribution

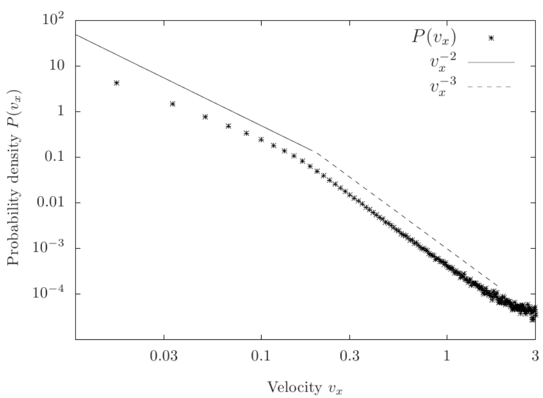


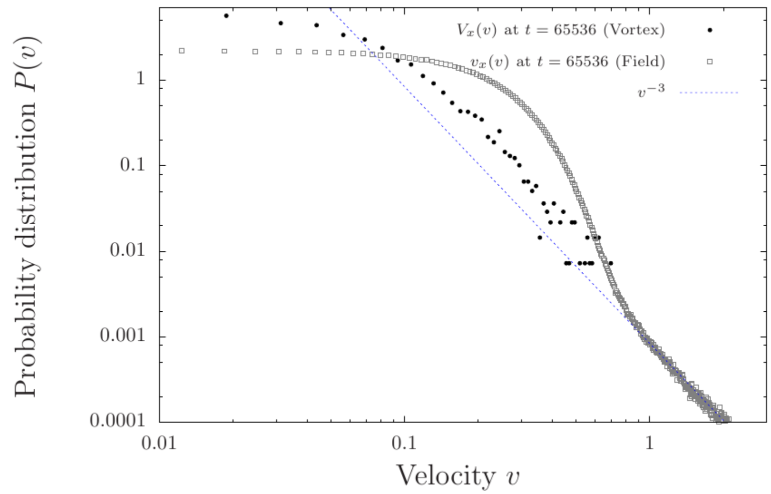
M. S. Paoletti, M. E. Fisher,

K. R. Sreenivasan, D. P. Lathrop: PRL (2008)



Pairing:





Quantum turbulence

$$E_{tot} = \int \left(\frac{1}{2} |\nabla \sqrt{n} e^{-i\varphi}|^2 + \frac{1}{2} g n^2 \right) d\boldsymbol{\rho}$$

$$= E_{kin} + E_q + E_{int}$$

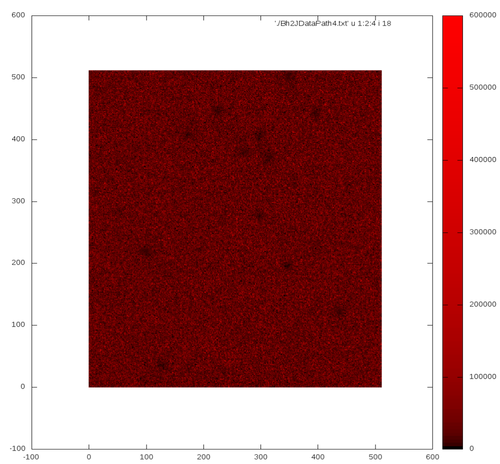
$$E_{kin} = \frac{1}{2} \int |\sqrt{n}\mathbf{u}|^2 d\boldsymbol{\rho} = E_{kin}^i + E_{kin}^c$$

$$\mathbf{u}(\boldsymbol{\rho},t) = \nabla \varphi(\boldsymbol{\rho},t)$$

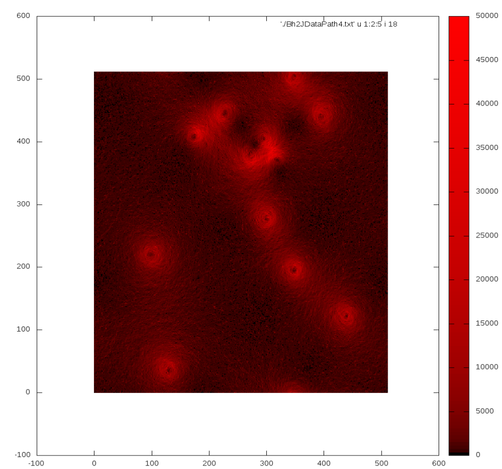
$$\nabla \times (\sqrt{n}\mathbf{u})^c = 0$$

$$\nabla \cdot (\sqrt{n}\mathbf{u})^i = 0$$

C. Nore, M. Abid, M. Brachet: Phys. Fluids (1997)



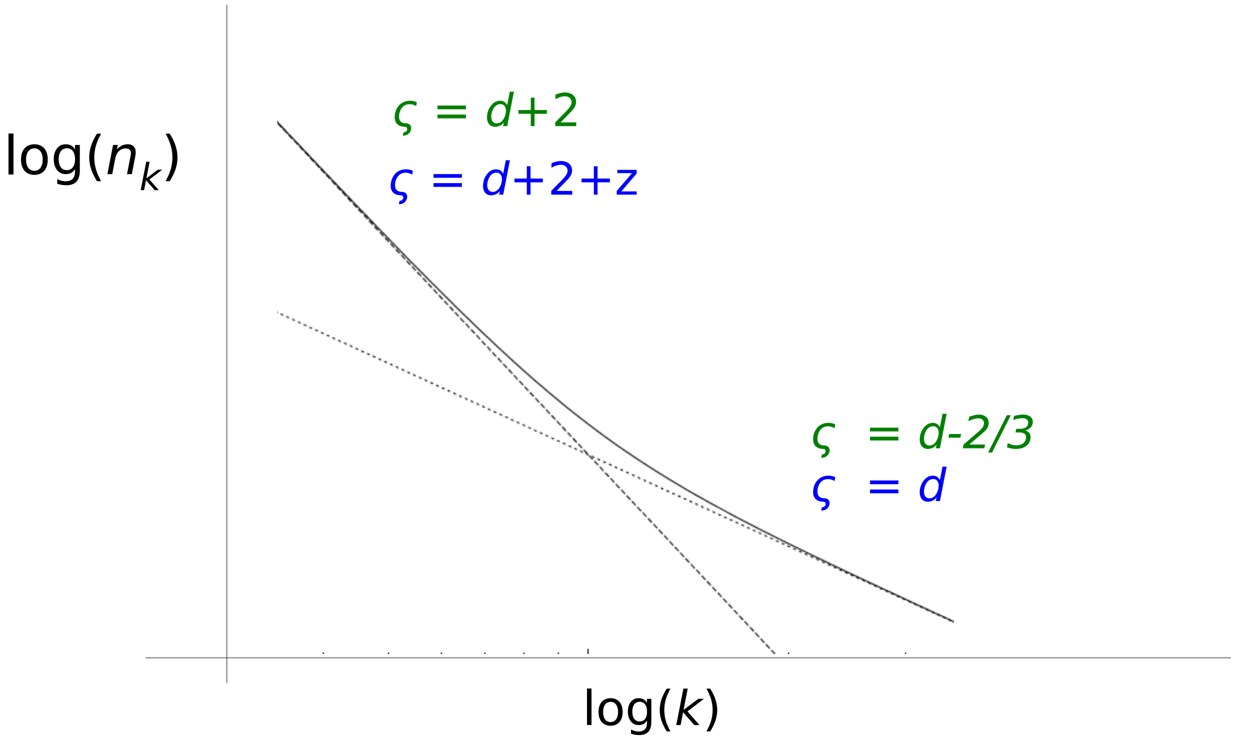
E_{kin}^c

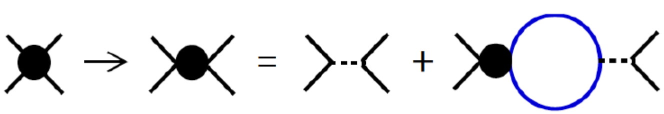


E_{kin}^i

Nonthermal fixed points $n_k \sim k^{-\varsigma}$

from dynamical quantum field theory

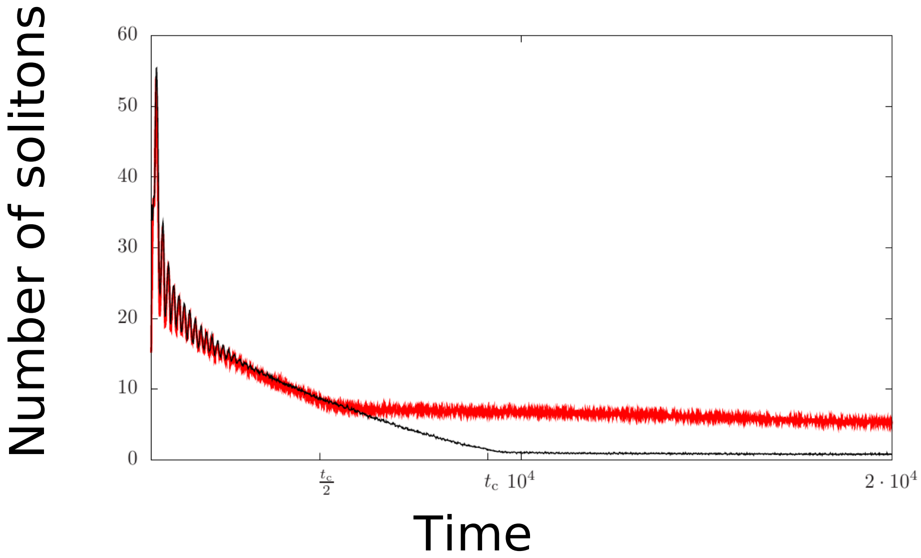




J. Berges, A. Rothkopf, J. Schmidt, PRL (2008)

C. Scheppach, J. Berges, T. Gasenzer, PRA (2010)

Soliton density decay



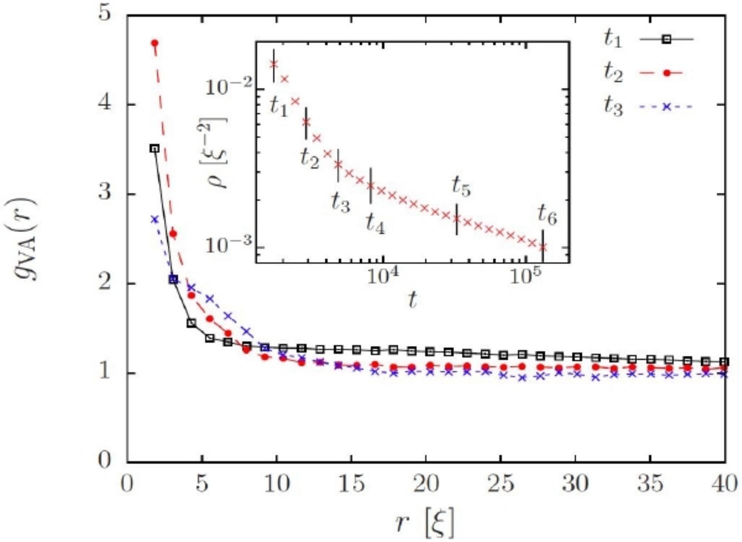
<u>Time Scales</u>:

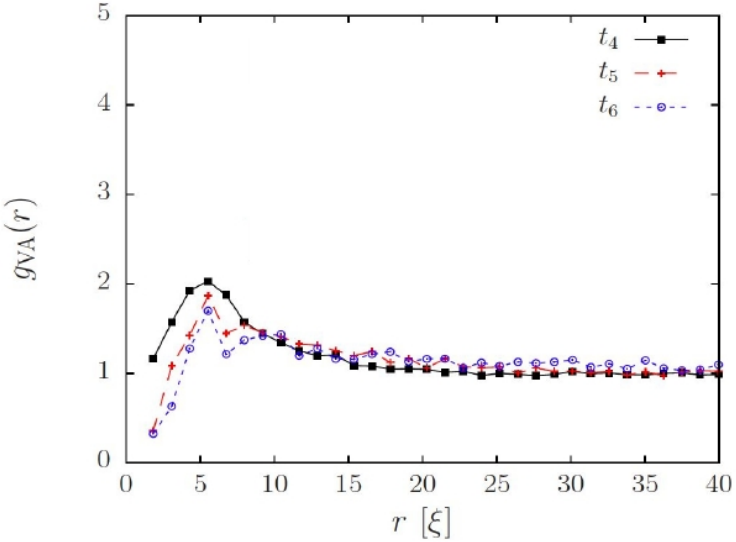
- Sound propagation a_{ho}/c_s ~ 10
- Inverse trap frequency 1/ω~ 10^2
- Damping of breathing mode ~ 10^3
- Cooling time ~ 10^4
- Soliton lifetime > 10^6

2D: Vortex correlations

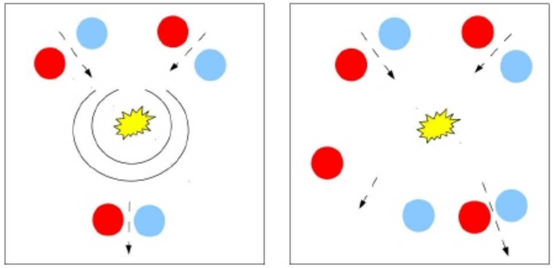
B. N. et al. PRA 85, 043627 (2012)

B. N. et al. PRB 84, 020506(R) (2011)





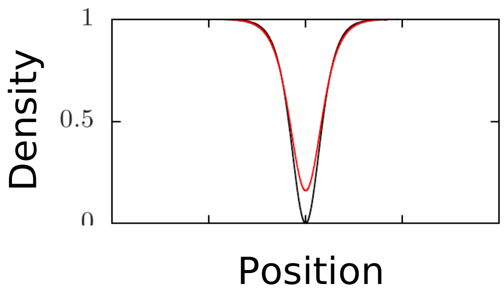
$$g_{\mathrm{VA}}(\mathbf{x},\mathbf{x}',t) = \frac{\langle \rho^{\mathrm{V}}(\mathbf{x},t)\rho^{\mathrm{A}}(\mathbf{x}',t)\rangle}{\langle \rho^{\mathrm{V}}(\mathbf{x},t)\rangle\langle \rho^{\mathrm{A}}(\mathbf{x}',t)\rangle}$$

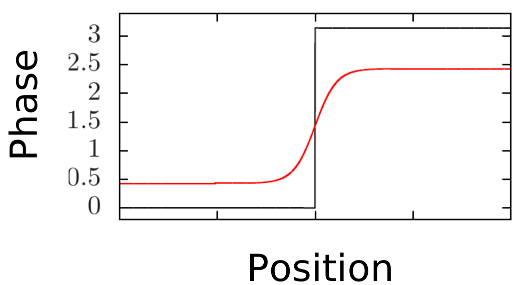


1D: Solitons replace vortices

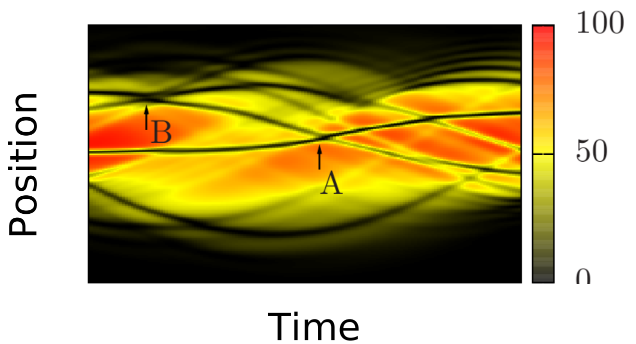
M. Schmidt, S. Erne, B. N., D. Sexty, T. Gasenzer, arXiv:1203.3651 (2012)

- Stationary solutions of non-linear wave equation:





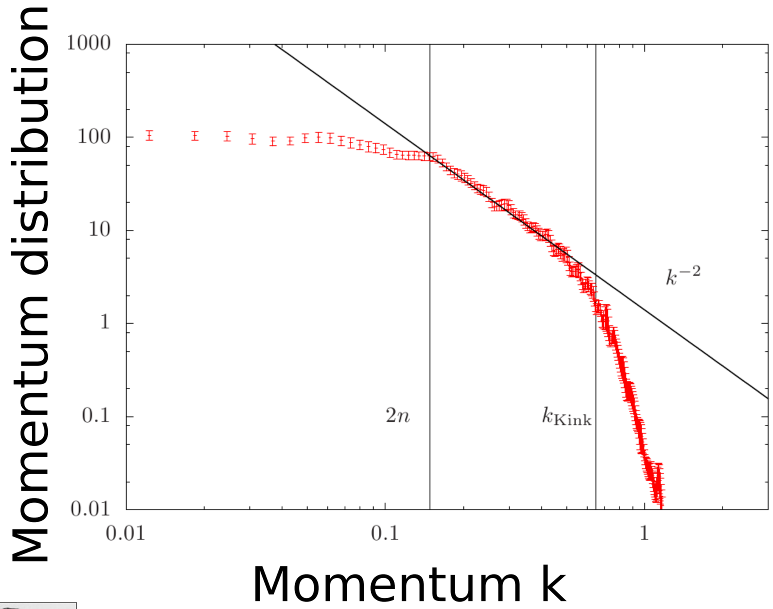
- Soliton-Soliton interactions:

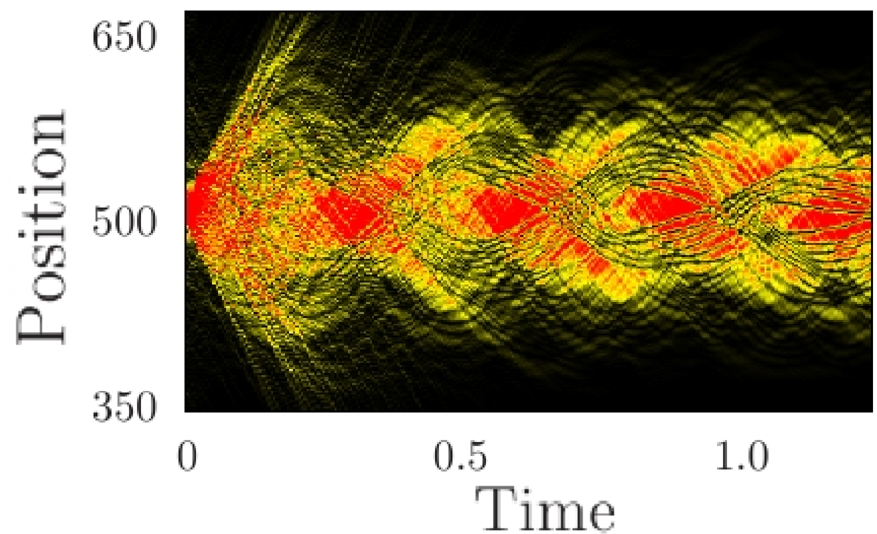


→ Turbulence (NTFP) as a random state of solitons

1D: Quench dynamics

Features:
- Quasi-stationary profile
- Scaling



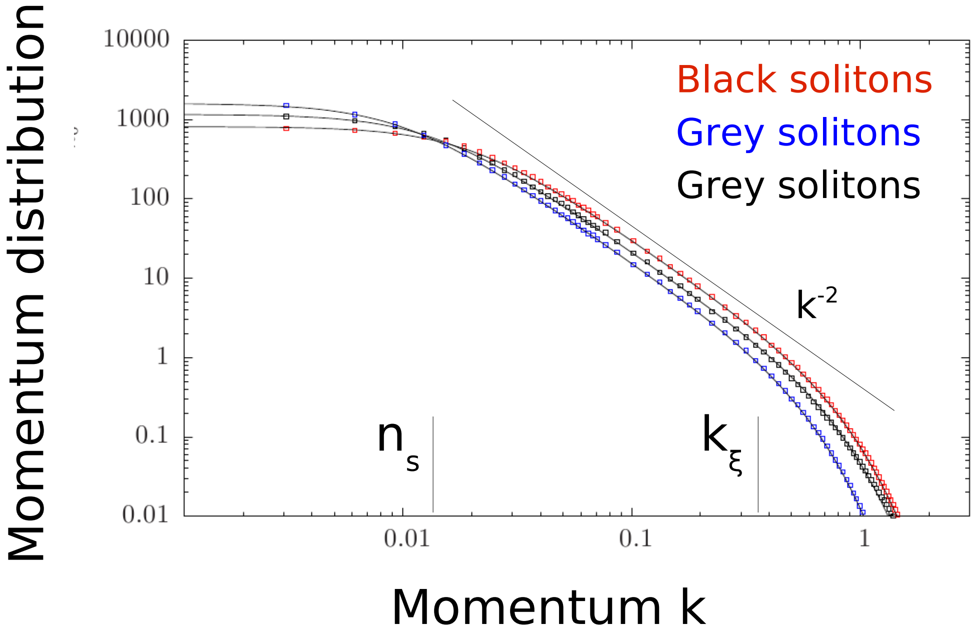


M. Schmidt, S. Erne, BN, D. Sexty, T. Gasenzer NJP (2012)

1D: Momentum distribution

- Random soliton model for black/grey solitons (in a trap):

e. g. $$n(k)|_{\nu=0} = \frac{4n_s n}{4n_s^2 + k^2} \frac{(\pi k \xi)^2/2}{\sinh^2\left(\pi k \xi/\sqrt{2}\right)}$$ (Black Solitons)

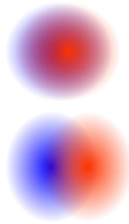


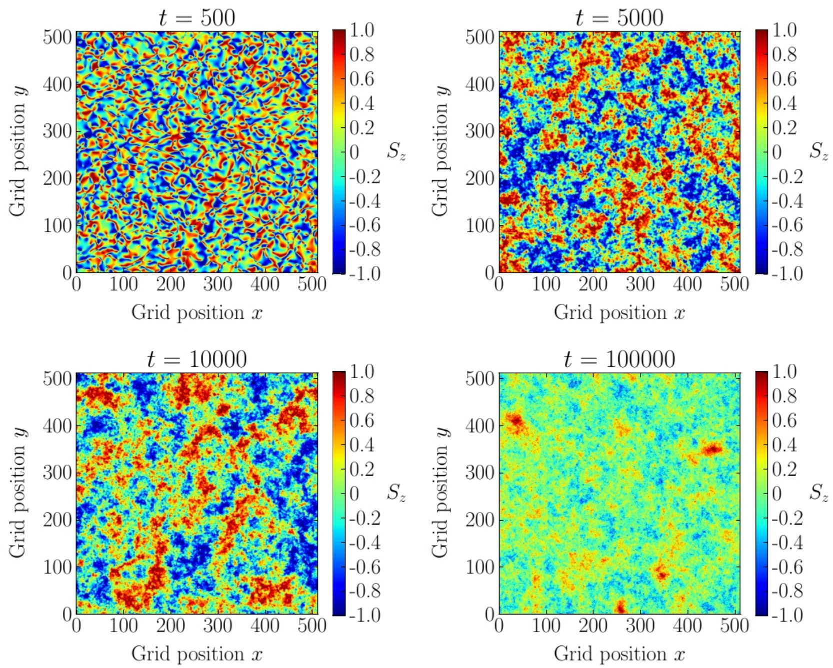
M. Schmidt, S. Erne, BN, D. Sexty, T. Gasenzer NJP (2012)

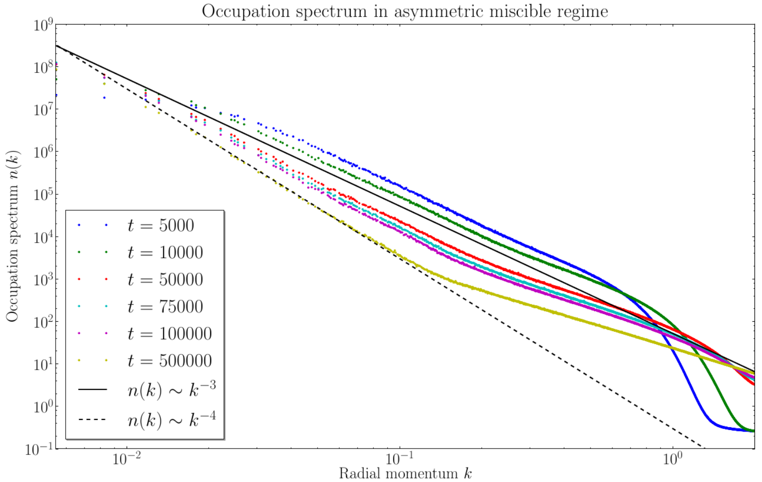
2-component BEC

miscible
$g_{12} < g$

immiscible
$g_{12} > g$

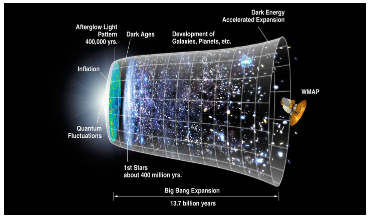






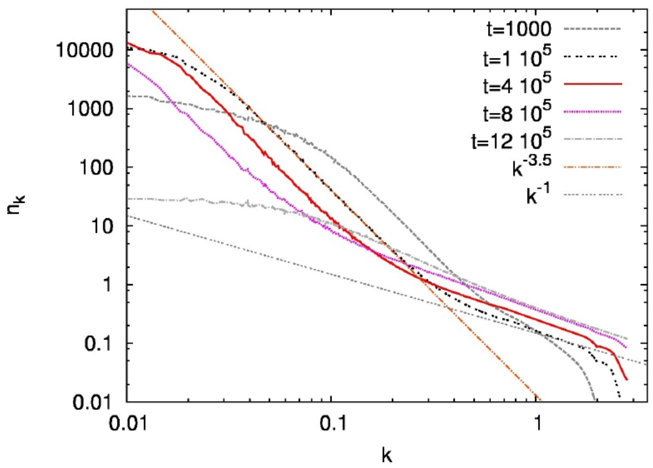
Miscible regime

Relativistic simulations O(2)

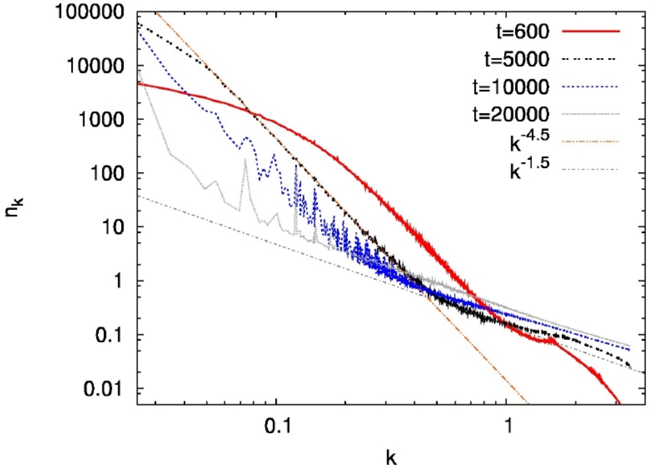


Classical field equation: $\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$

d=2



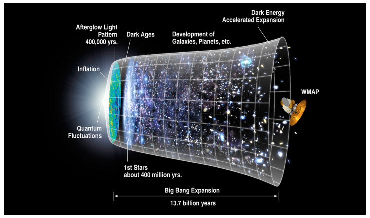
d=3



S. Khlebnikov, I. Tkachev PRL (1996)
J. Berges, A. Rothkopf, J. Schmidt PRL (2008)
J. Berges, D. Sexty PRD (2011)

T. Gasenzer, BN, D. Sexty PLB (2012)

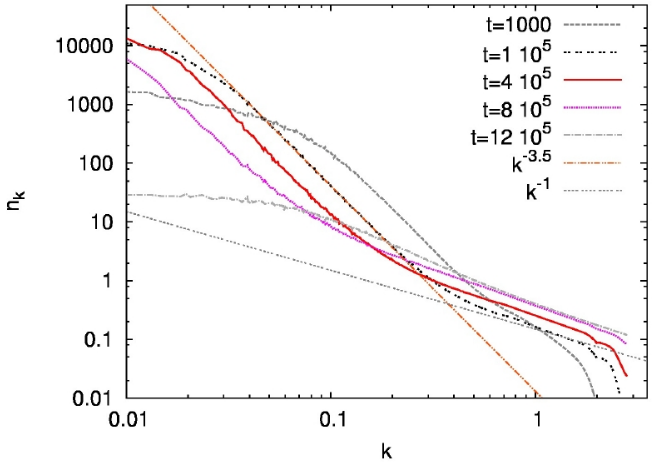
Relativistic simulations O(2)



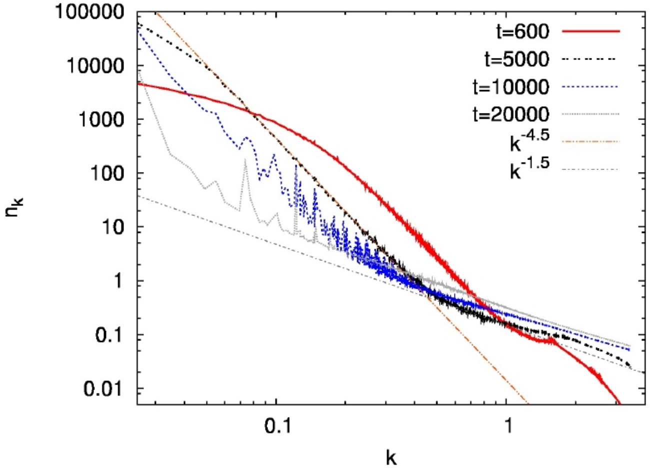
Classical field equation: $$\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$$

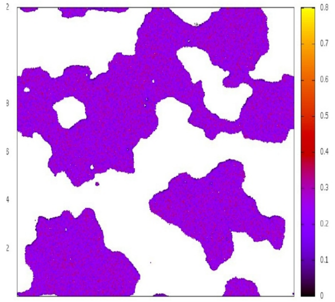
$-m^2$

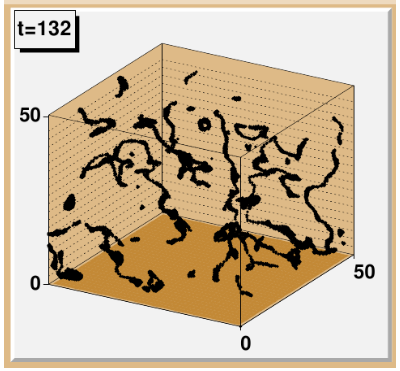
d=2

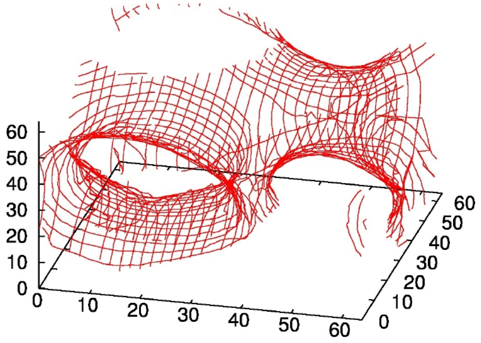


d=3



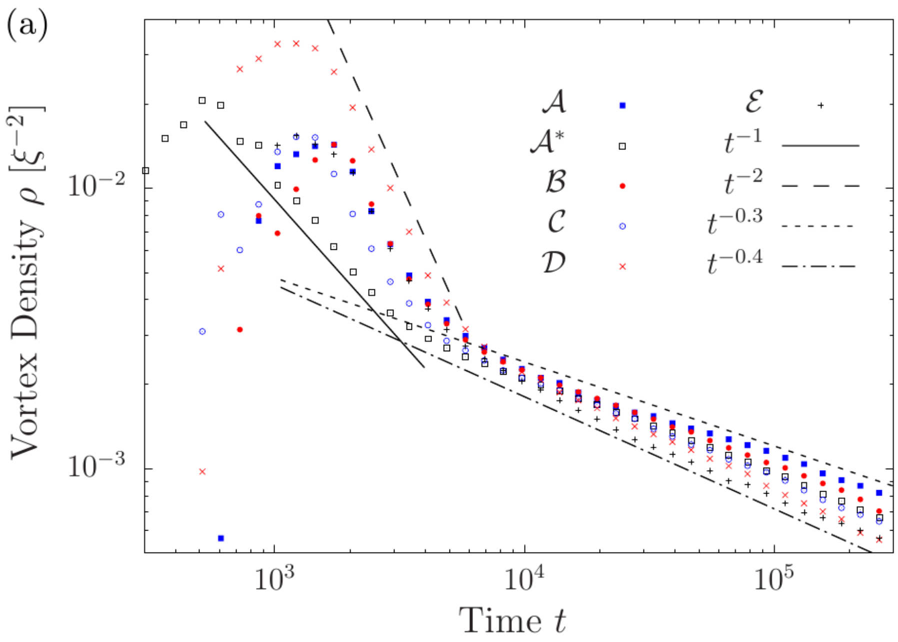






T. Gasenzer, BN, D. Sexty PLB (2012) I. Tkachev, S. Khlebnikov, L. Kofman, A. Linde PL (1998)

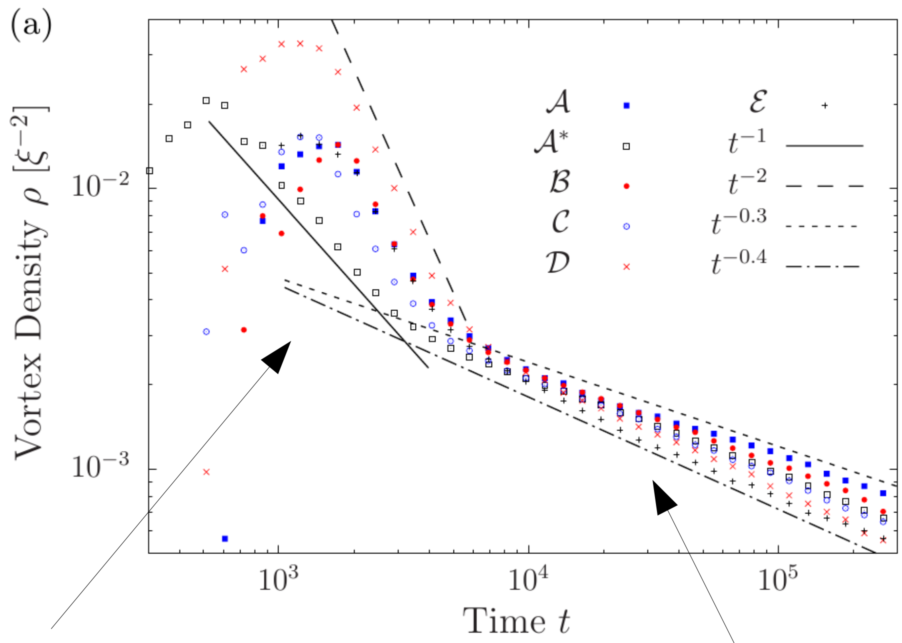
2D: Phase ordering dynamics



J. Schole, BN, T. Gasenzer, arXiv:1204.2487

2D: Phase ordering dynamics

Scaling needs vortex unbinding

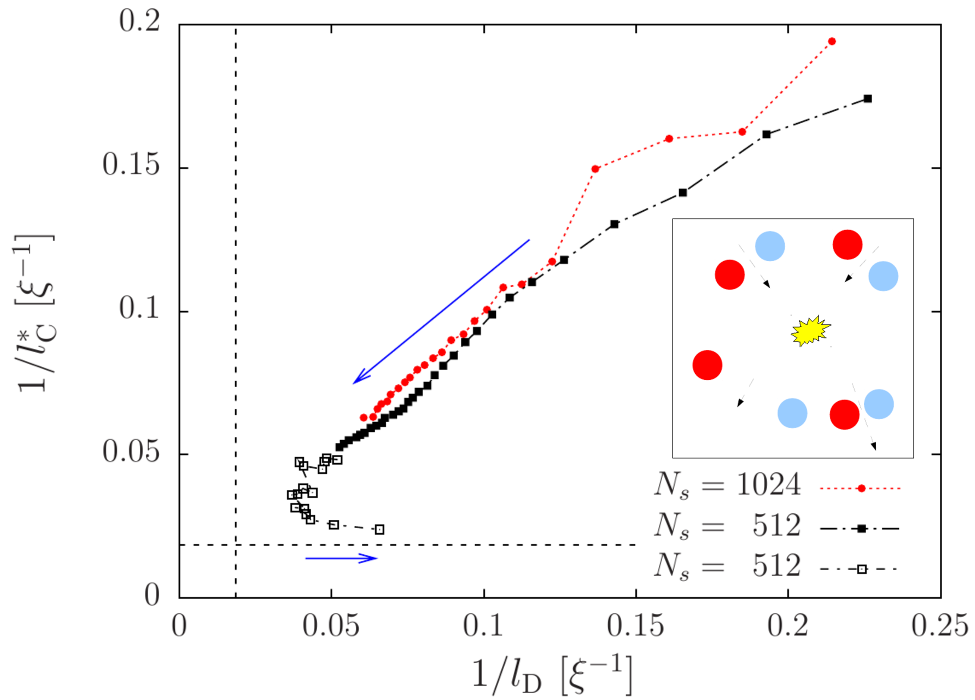


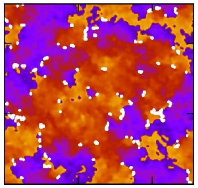
Non-universal decay law
(Initial vortex distribution dependent)
Kinetic gas theory for dipoles

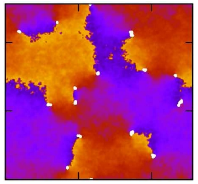
Universal decay regime
Strongly correlated, dilute vortex gas
Scaling $n(k) \sim k^{-4}$

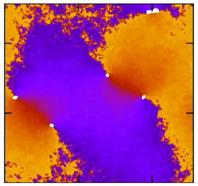
J. Schole, BN, T. Gasenzer, arXiv:1204.2487

Correlations near the NTFP







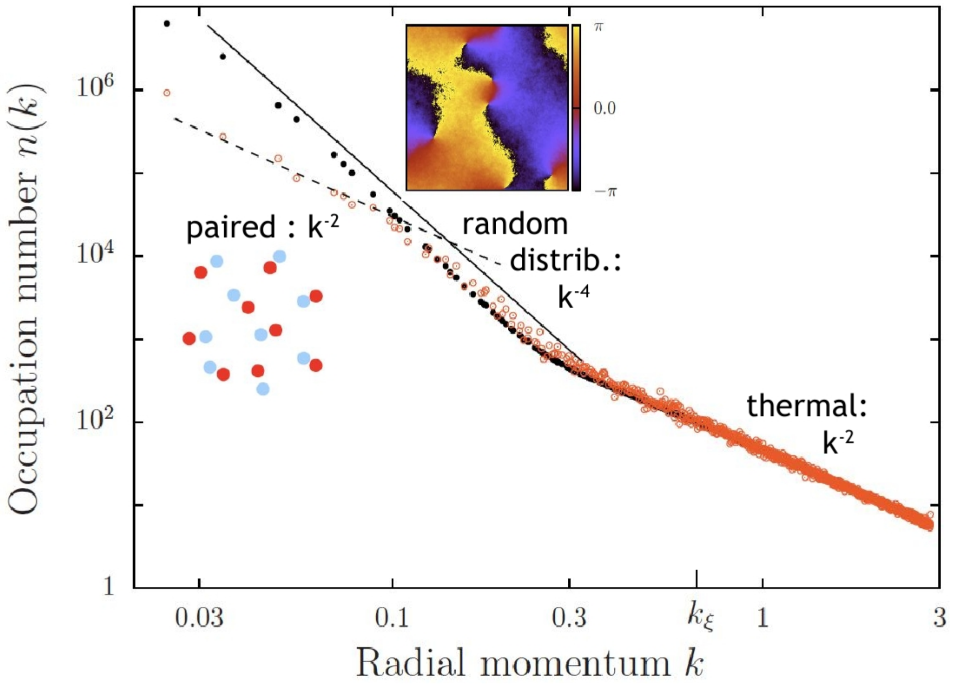


$l_C^\star$ Phase coherence length

l_D Vortex-antivortex pair distance

J. Schole, BN, T. Gasenzer, PRA (2012)

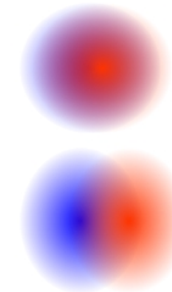
Vortex Pairing Spectrum in2D

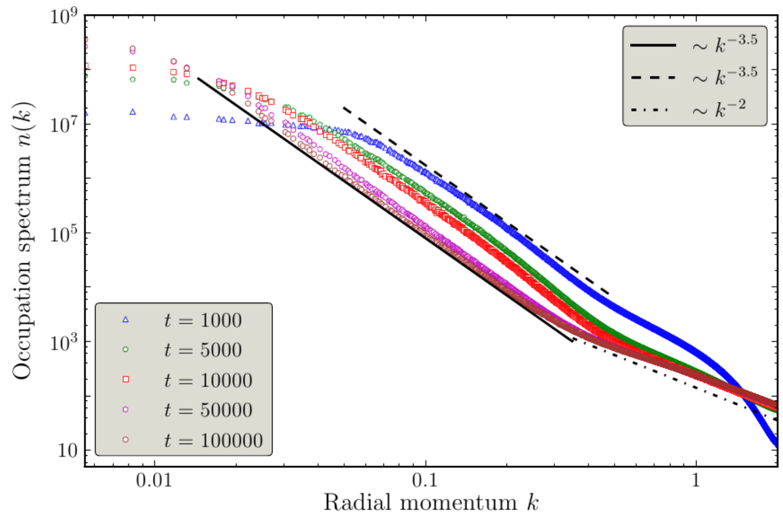


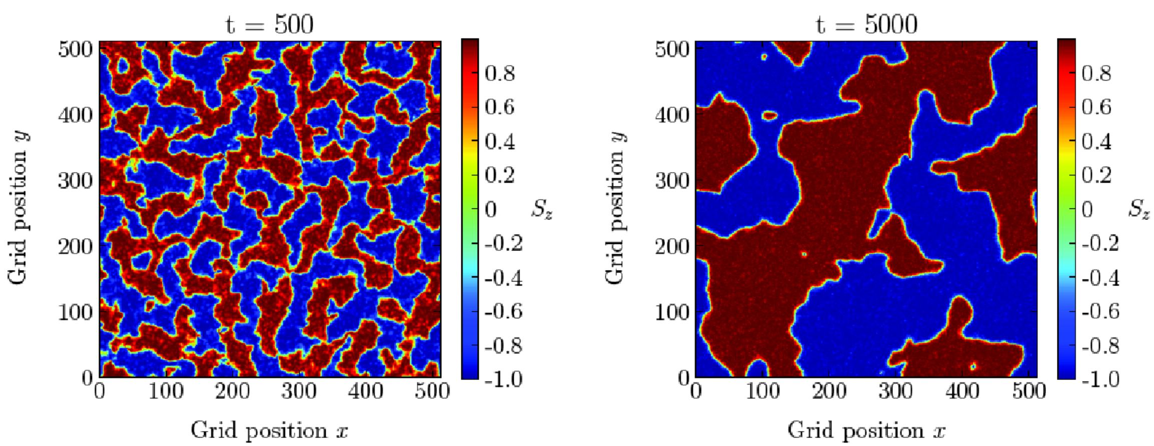
2D: 2-component BEC

miscible
$g_{12} < g$

immiscible
$g_{12} > g$







M. Karl, BN, T. Gasenzer in preparation