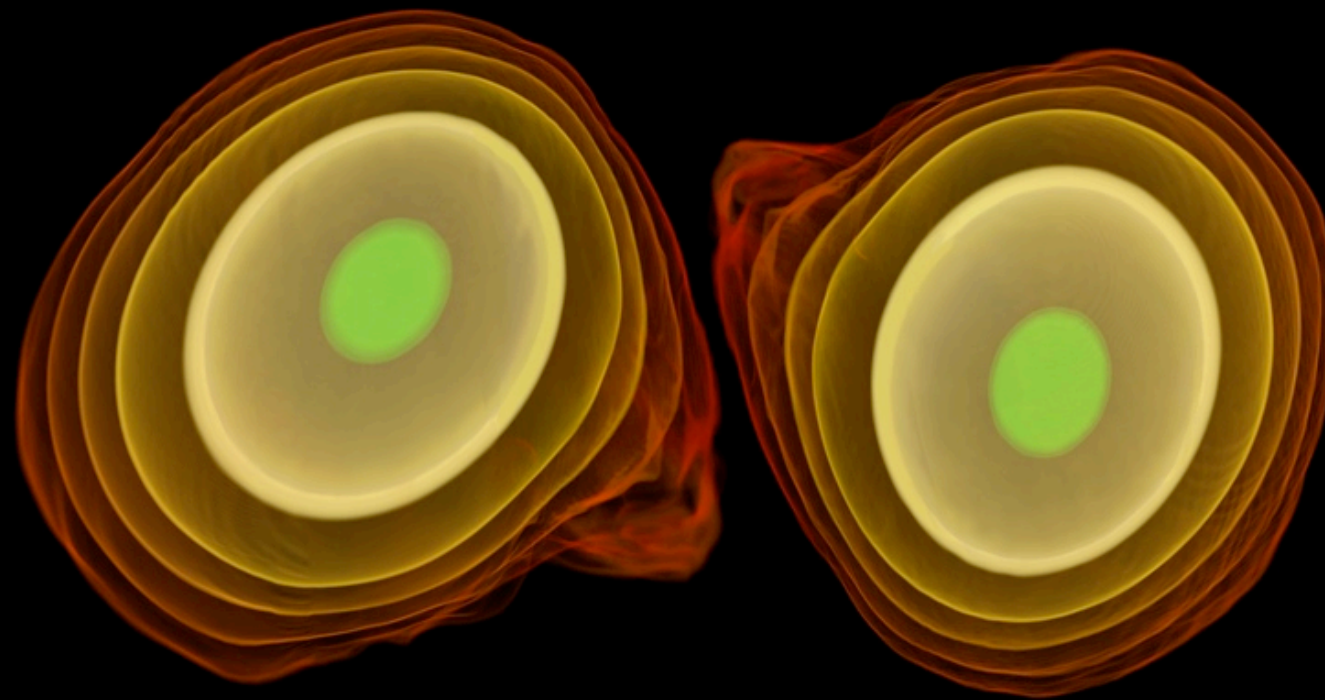


Binary neutron stars to explore nuclear physics and astrophysics

Luciano Rezzolla

Albert Einstein Institute, Potsdam, Germany



Tuebingen,
12/10/12



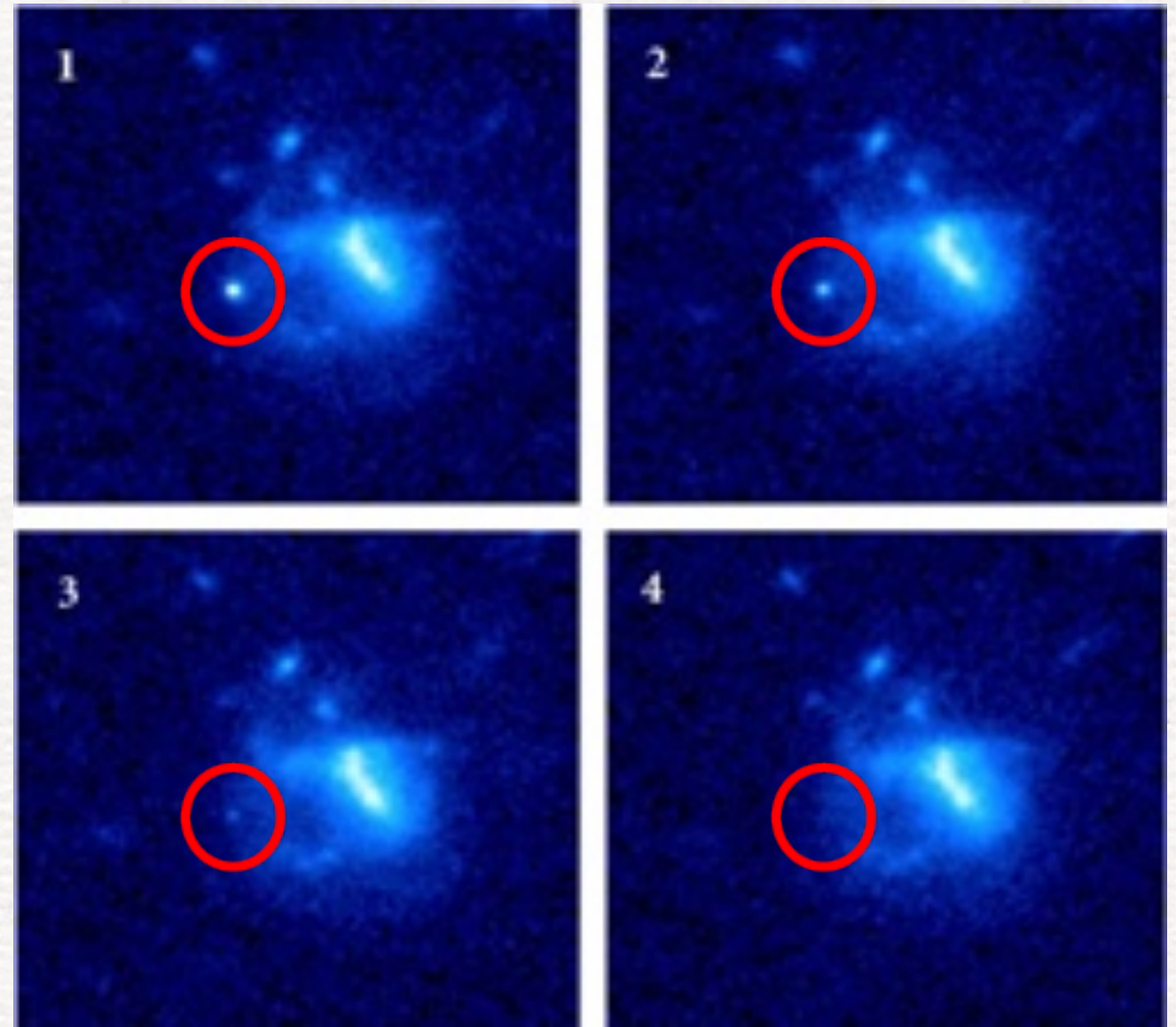
Plan of the talk

- Binary neutron stars in full GR:
 - * probes of fundamental physics
 - * probes of high-energy astrophysics

Baiotti, Giacomazzo, LR, PRD (2008); Baiotti, Giacomazzo, LR, CQG (2009); Giacomazzo, LR, Baiotti, MNRAS (2009); LR et al CQG (2010); Giacomazzo, LR, Baiotti, PRD (2010); Baiotti, LR, et al PRL (2010), LR et al, ApJL (2011); Baiotti et al, PRD (2011)

Why investigate binary neutron stars?

- We know they exist as opposed to binary BHs, whose existence is expected but never observed.
- Excellent sources of gravitational waves (GWs) and are expected to be most common source for advanced detectors



- We expect them related to SGRBs: energies released $\sim 10^{48-50}$ erg.
- Despite decades of observations no self-consistent model has yet been produced to explain them

Mathematical framework

Numerical relativity (NR) solves Einstein equations in those regimes in which no approximation holds: eg in the most nonlinear regimes of the theory. We build codes which we consider as “**theoretical laboratories**”.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} \quad (\text{field eqs : } 6 + 6 + 3 + 1)$$

$$\nabla_{\mu}T^{\mu\nu} = 0, \quad (\text{cons. en./mom. : } 3 + 1)$$

$$\nabla_{\mu}(\rho u^{\mu}) = 0, \quad (\text{cons. of baryon no : } 1)$$

$$p = p(\rho, \epsilon, \dots). \quad (\text{EoS : } 1 + \dots)$$

$$\nabla_{\nu}^*F^{\mu\nu} = 0, \quad (\text{Maxwell eqs. : induction, zero div.})$$

$$T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{em}} + \dots$$

It's our approximation to “**reality**” and it can be continuously improved:
microphysics, magnetic fields, viscosity, radiation transport, resistive effects, ...

The two-body problem in GR

- For BHs we know what to **expect**:

BH + BH \longrightarrow BH + gravitational waves (GWs)

- For NSs the question is more **subtle**: the merger leads to an hyper-massive neutron star (HMNS), ie a metastable equilibrium:

NS + NS \longrightarrow HMNS + ... ? \longrightarrow BH + torus + ... ? \longrightarrow BH

All complications are in the intermediate stages; the rewards high:

- studying the HMNS will show strong and precise imprint on the EOS
- studying the BH+torus will tell us on the central engine of GRBs

NOTE: with advanced detectors we expect to have a **realistic** rate of \sim **40 BNSs** inspirals a year, ie \sim 1 a week (Abadie+ 2010)

“merger \longrightarrow HMNS \longrightarrow BH + torus”

Quantitative differences are produced by:

- the gravitational mass:

a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time

- the EOS (“cold” or “hot”):

a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later

Here:

“cold” is a polytropic EOS:

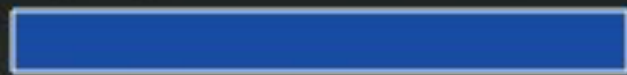
$$p = K \rho^\Gamma$$

“hot” is an ideal-fluid EOS:

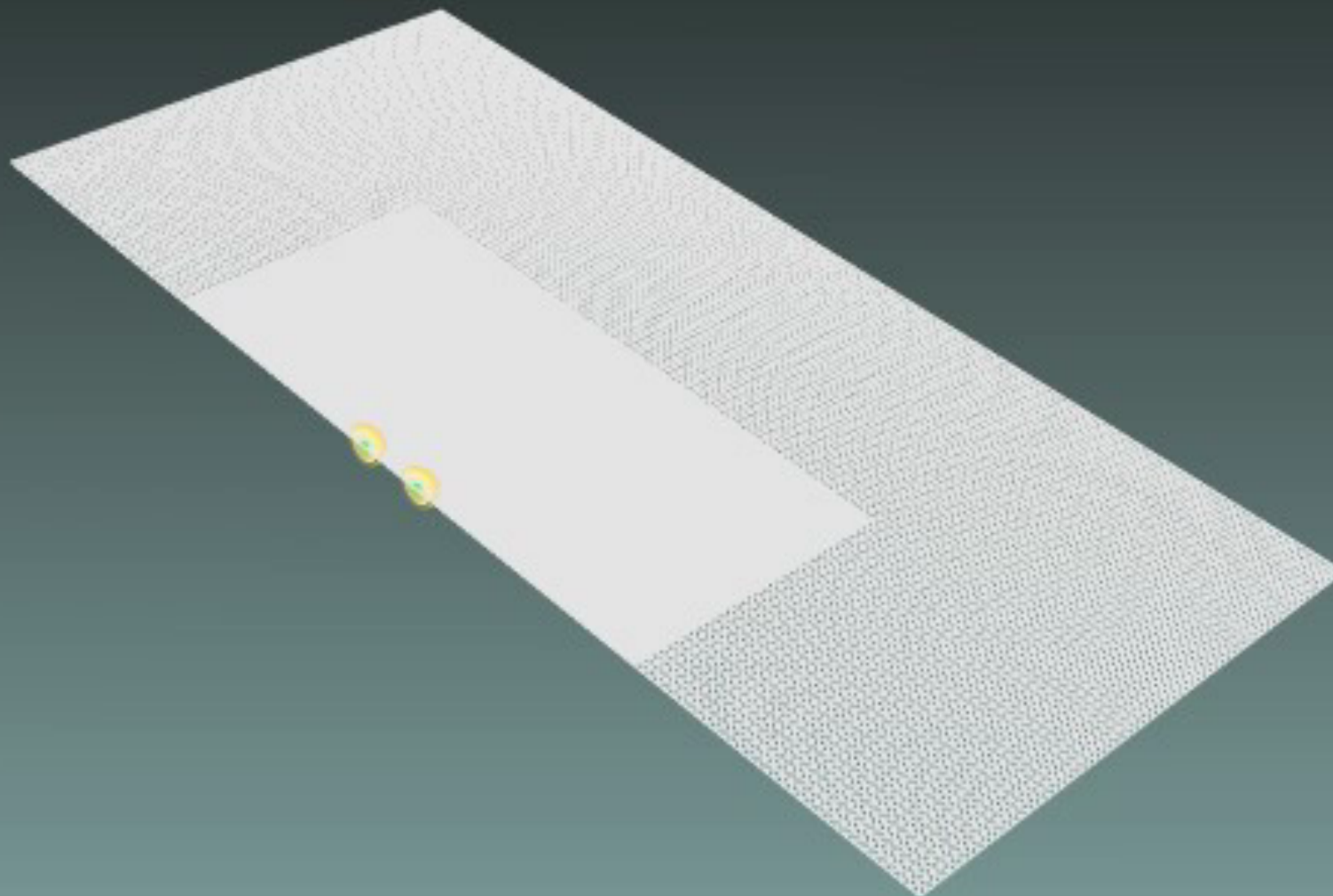
$$p = \rho \epsilon (\Gamma - 1)$$

Animations: Kaehler, Giacomazzo, Rezzolla

$T[\text{ms}] = 0.00$



$T[M] = 0.00$



Ideal-fluid EOS: high-mass binary

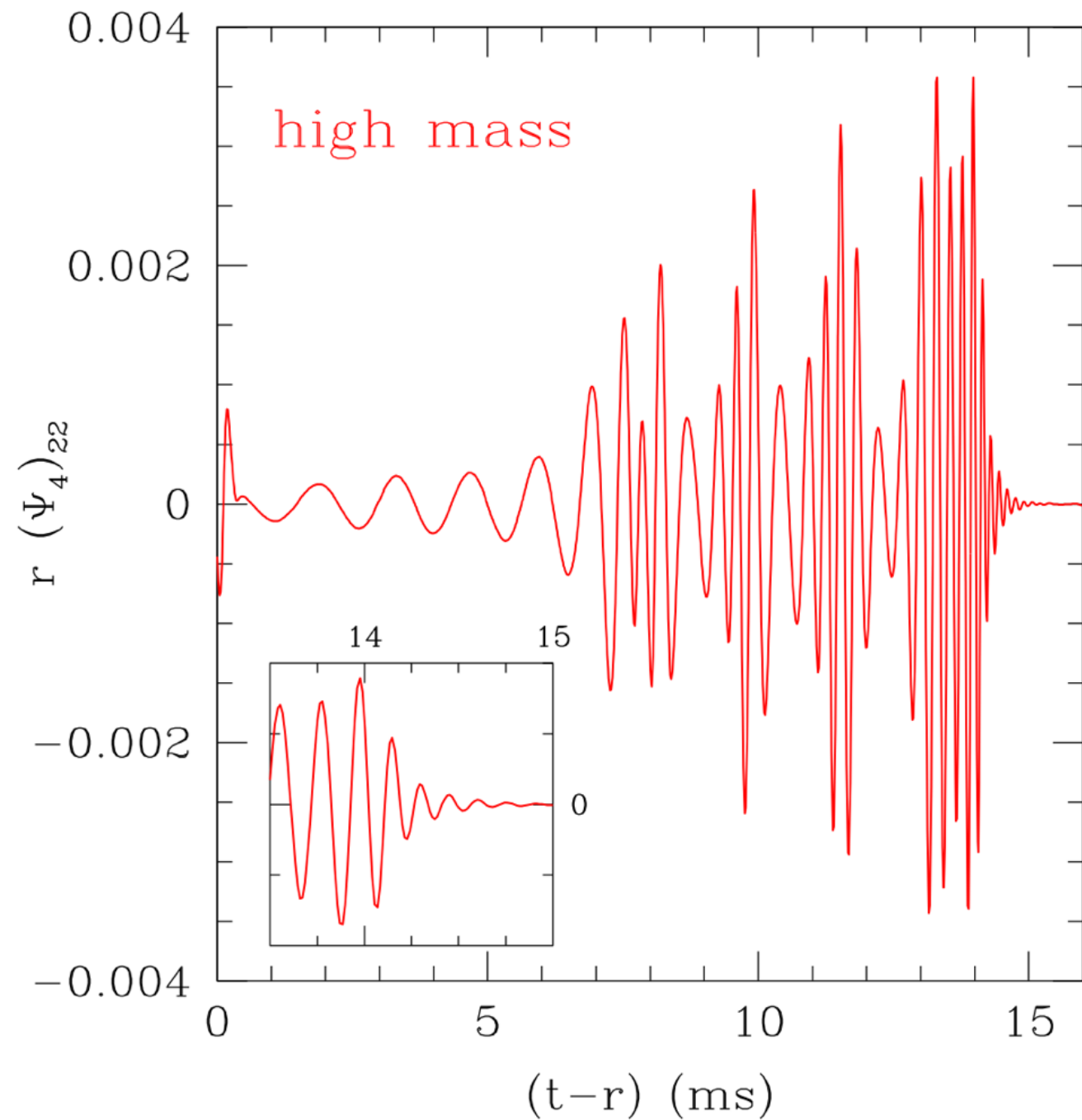
0.0 6.1E+14



Density [g/cm^3]

$M = 1.49 M_{\odot}$

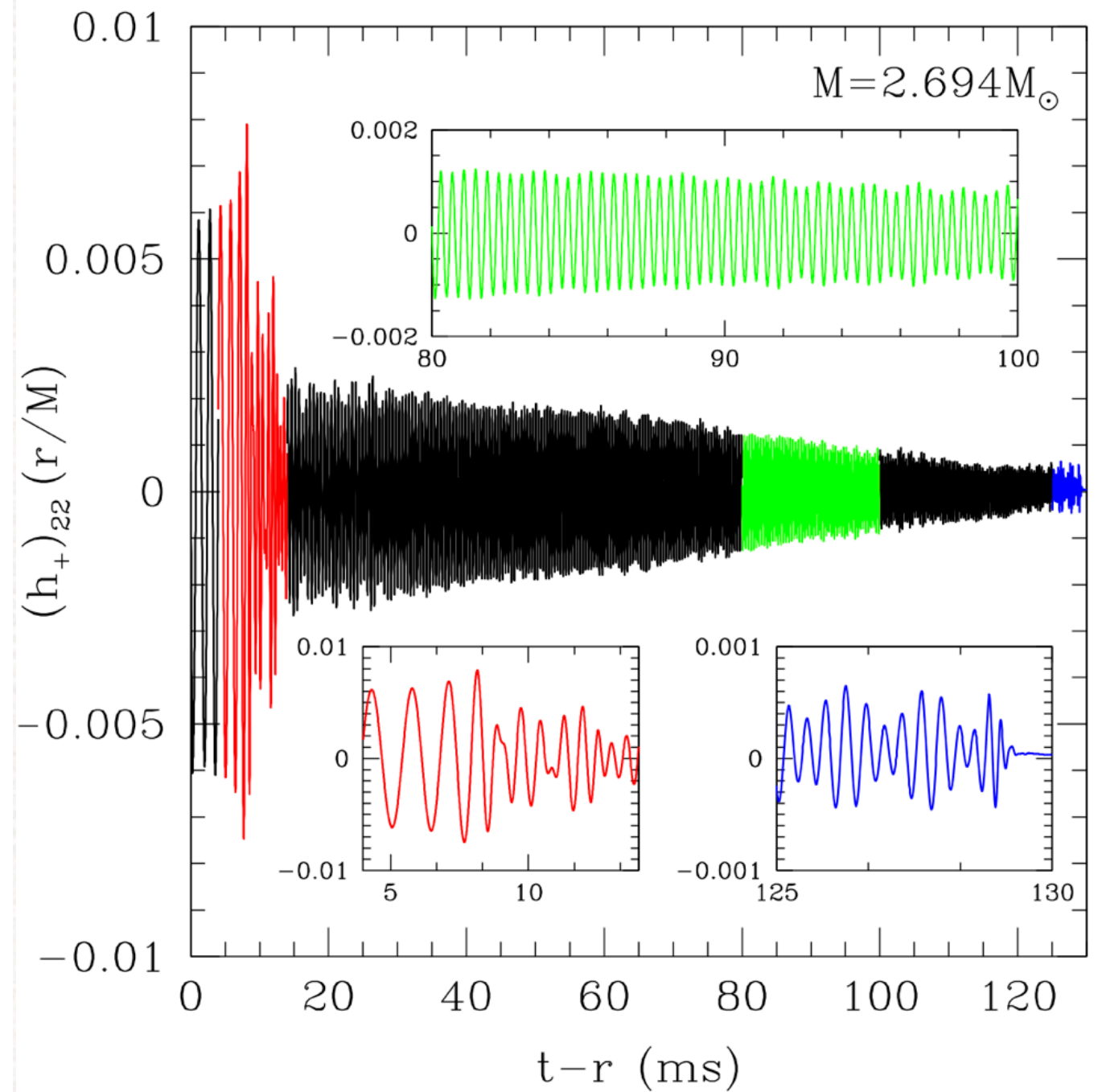
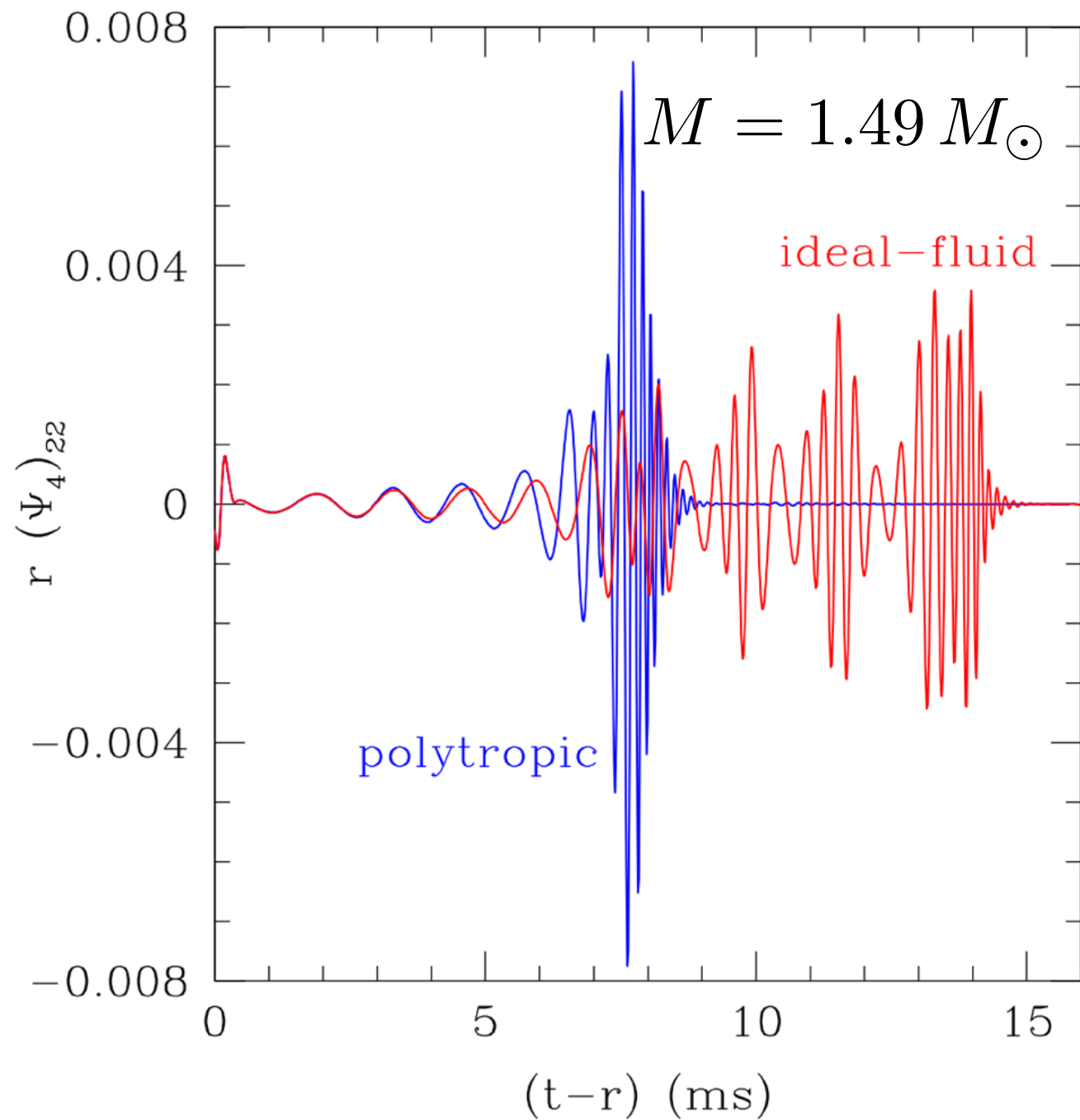
Imprint of the mass and EOS



the high internal energy (temperature) of the HMNS prevents a prompt collapse

Imprint of the mass and EOS

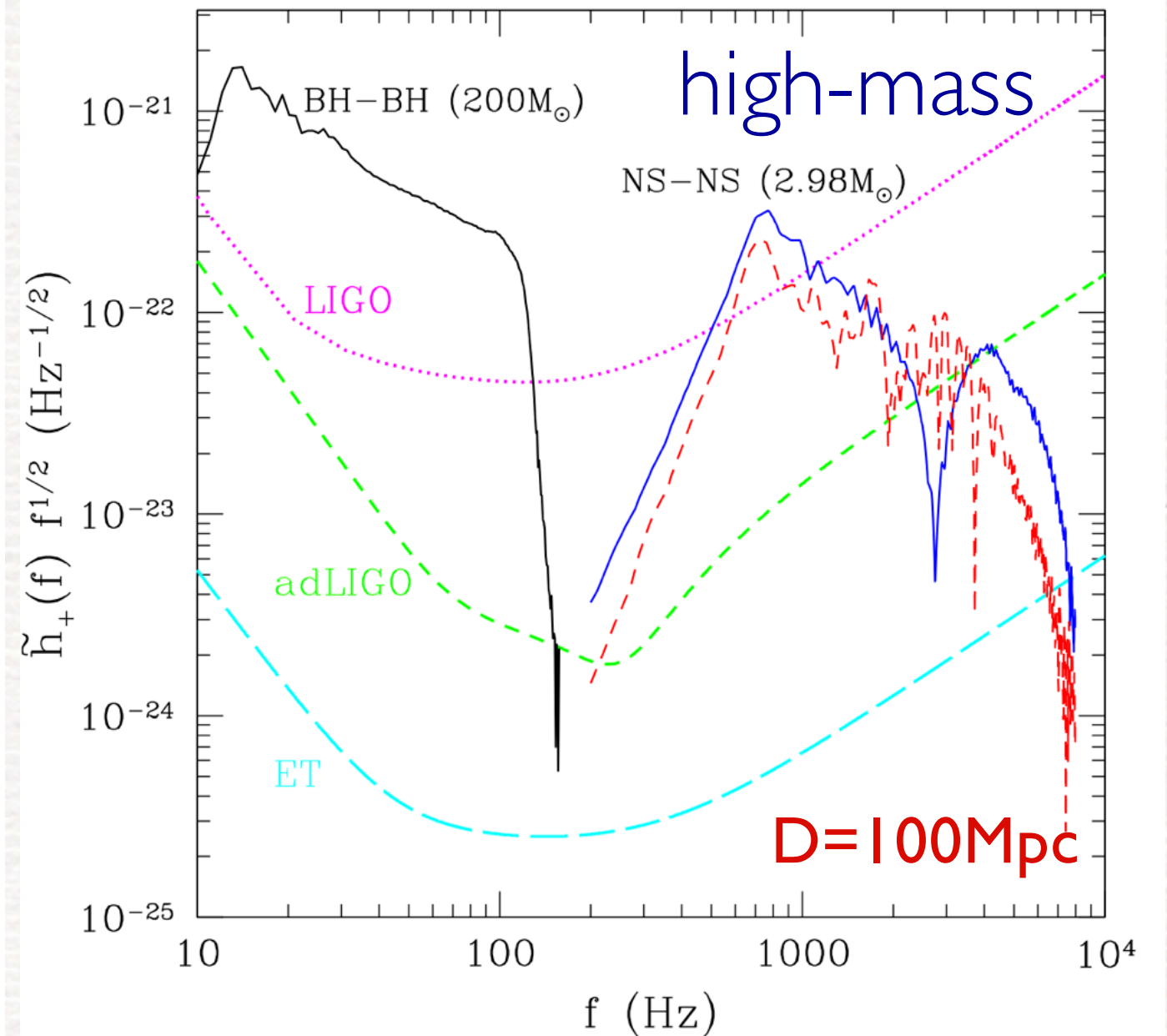
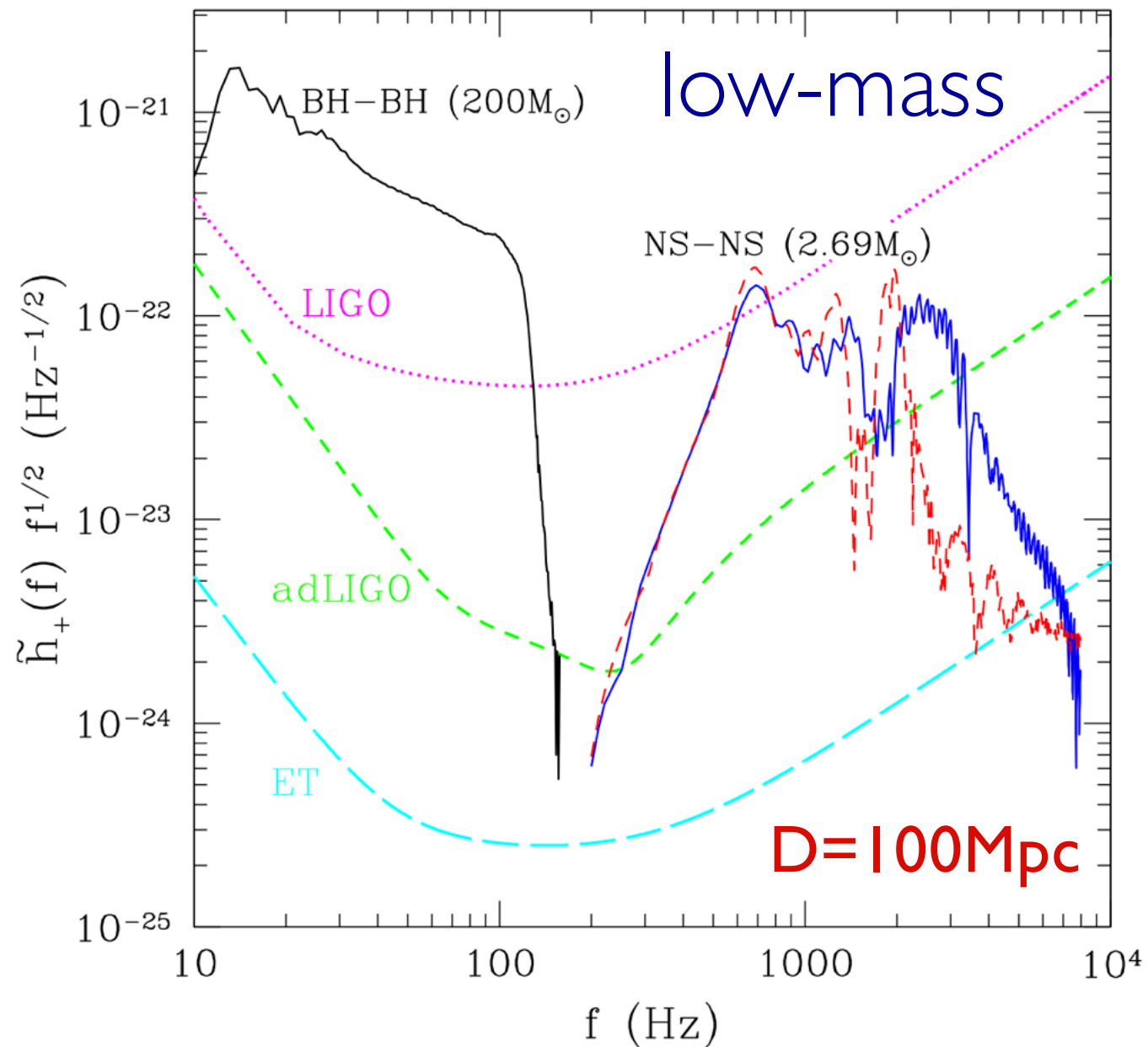
Baiotti, Giacomazzo, LR (2008)



There are clear differences for the **same mass** and for the **same EOS**: multidimensional parameter space

Imprint of the EOS: frequency domain

Andersson, LR, + (2010)



With sufficiently sensitive detectors, GWs will work as the **Rosetta stone** to decipher the NS interior

“merger → HMNS → BH + torus”

Quantitative differences are produced by:

- **the gravitational mass:**

a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time

- **the EOS (“cold” or “hot”):**

a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later

- **mass asymmetries:**

tidal disruption before merger; may lead to prompt BH

- **radiative processes:**

radiative losses will alter the equilibrium of the HMNS

- **magnetic fields:**

the angular momentum redistribution via magnetic braking or MRI can increase/decrease time to collapse

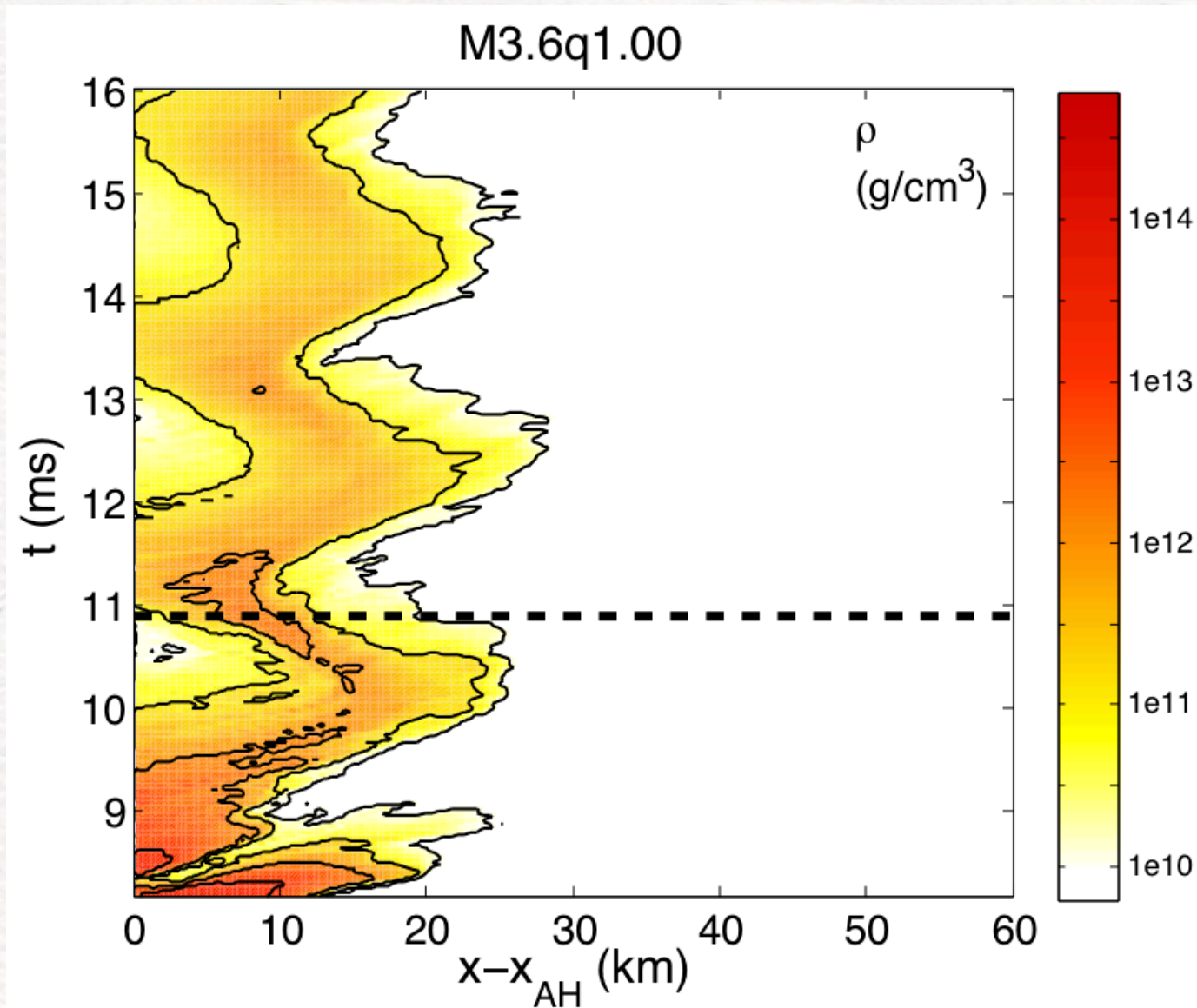
Total mass : $3.37 M_{\odot}$; mass ratio :0.80;



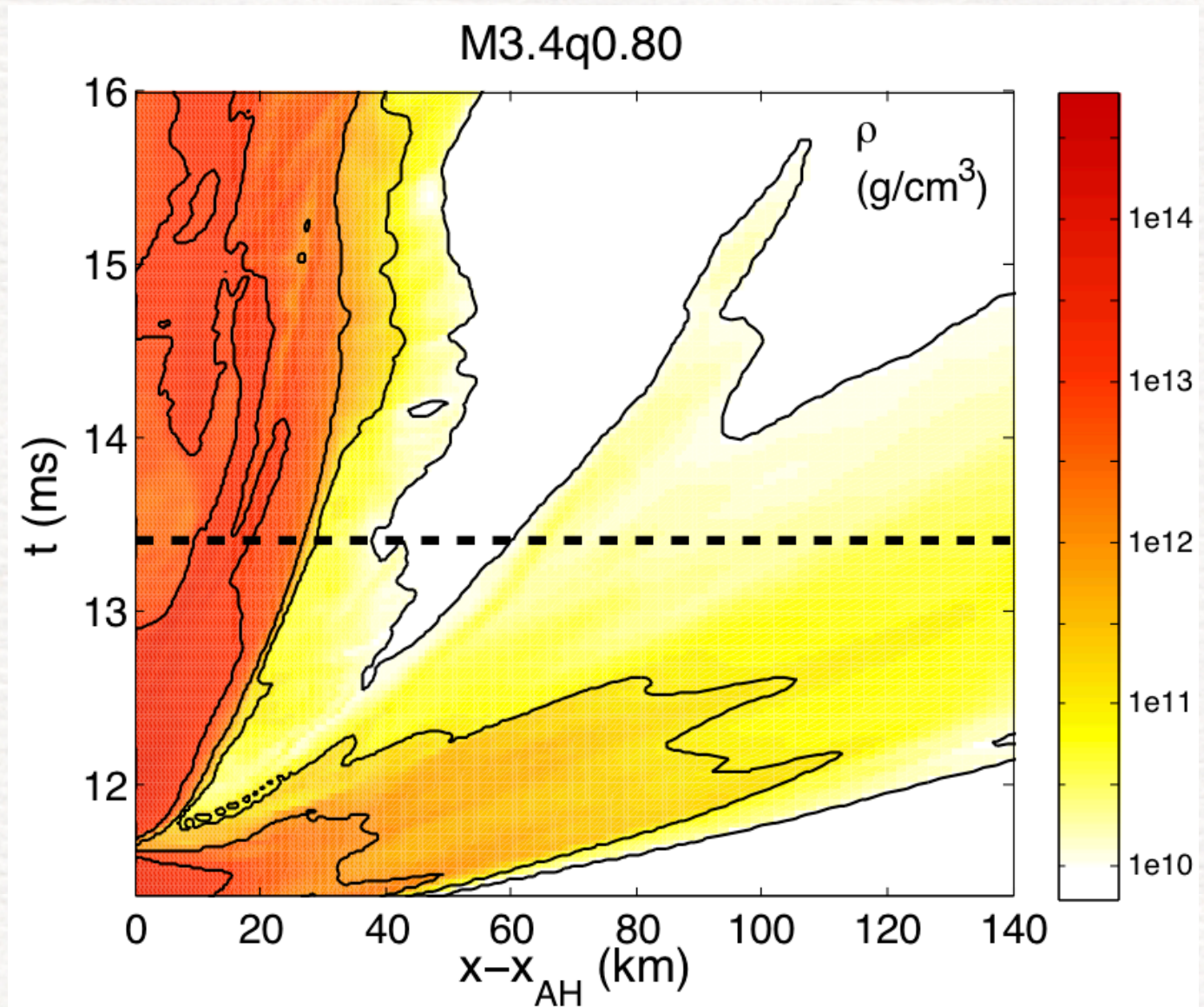
- * the torii are generically **more massive**
- * the torii are generically **more extended**
- * the torii tend to stable **quasi-Keplerian** configurations
- * overall unequal-mass systems have all the ingredients needed to create a GRB

Torus properties: density

spacetime diagram of rest-mass density along x-direction



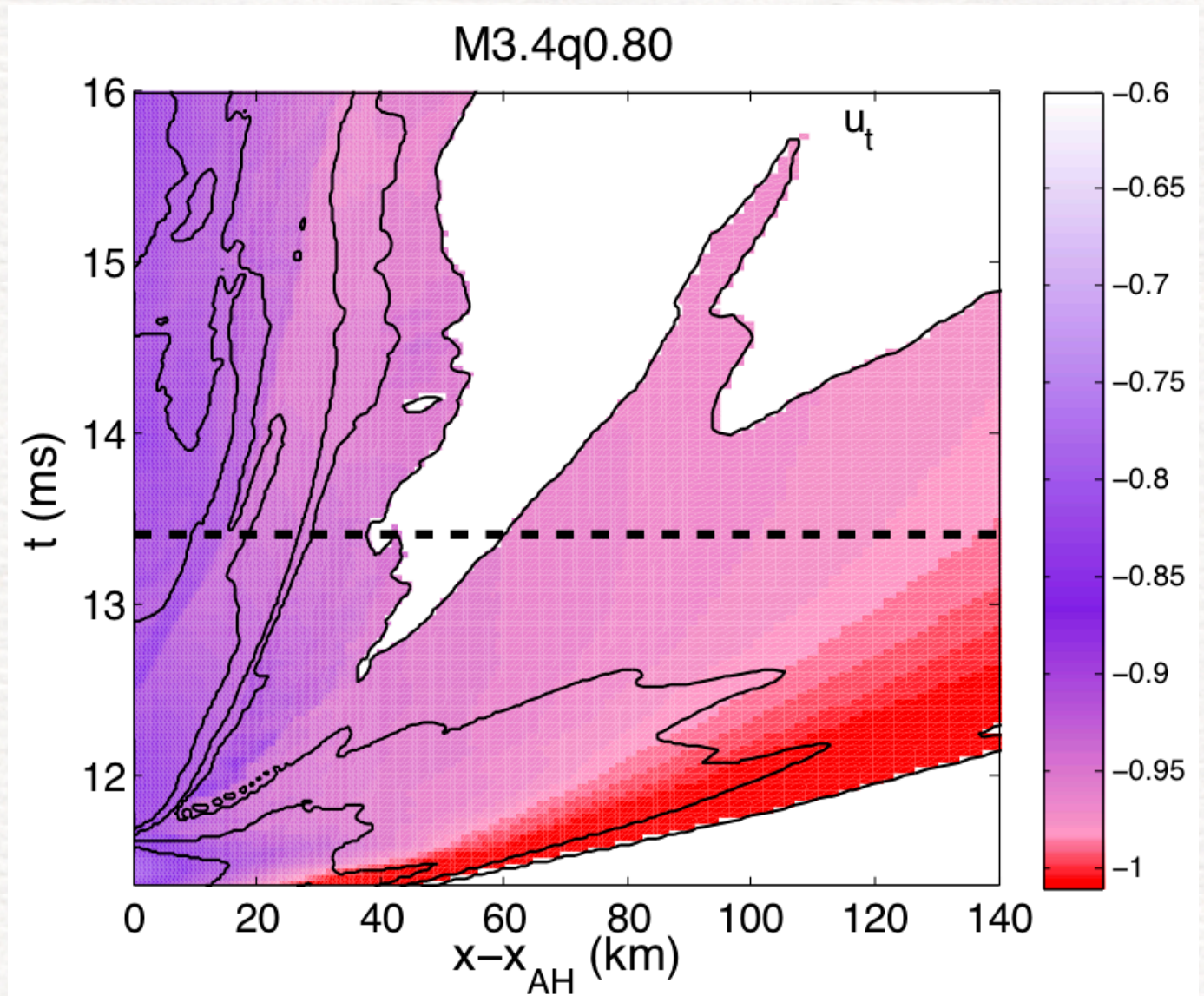
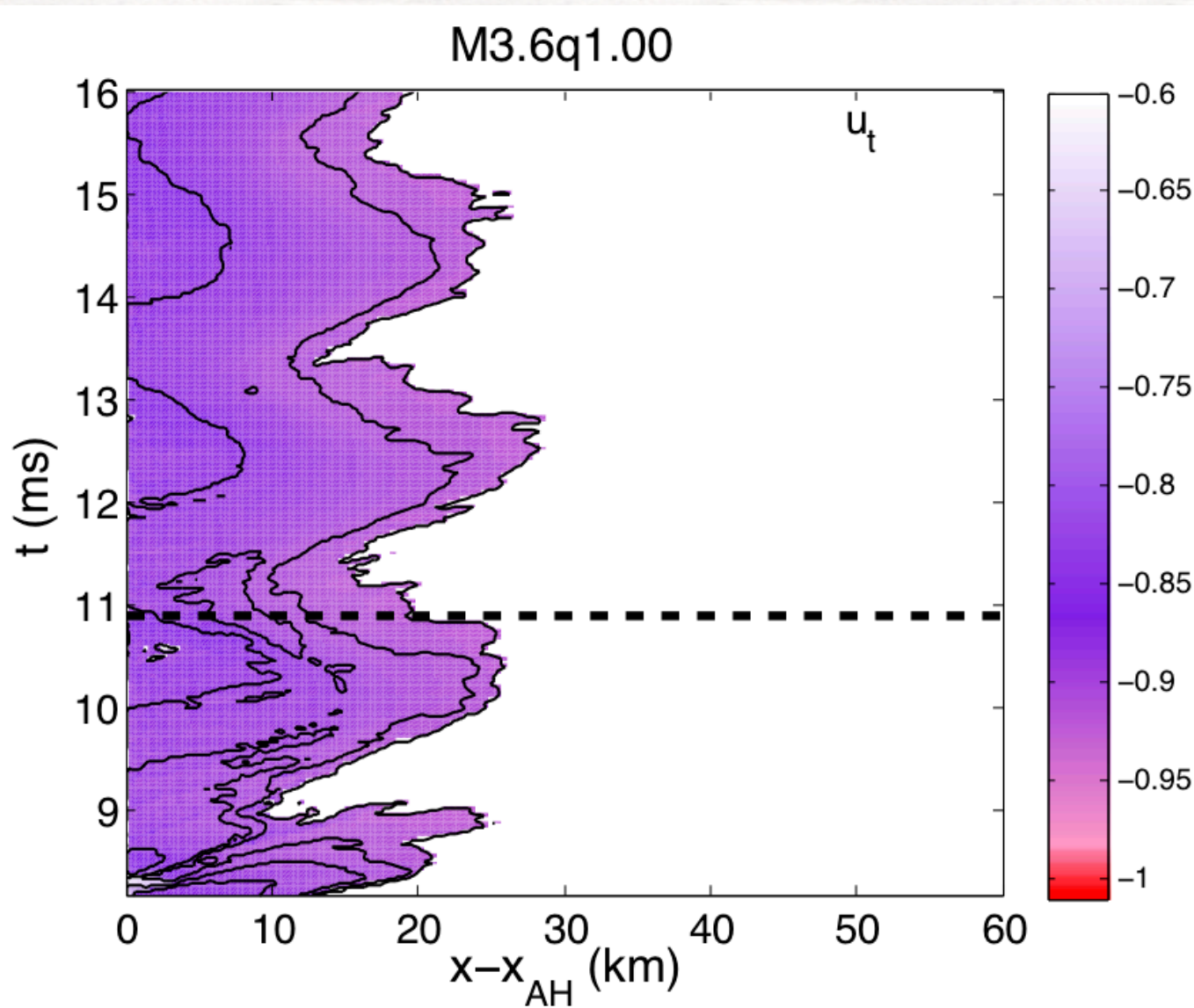
equal mass binary: note the **periodic** accretion and the **compact** size; densities are not very high



unequal mass binary: note the **continuous** accretion and the very **large** size and densities (temperatures)

Torus properties: bound matter

spacetime diagram of local fluid energy: u_t



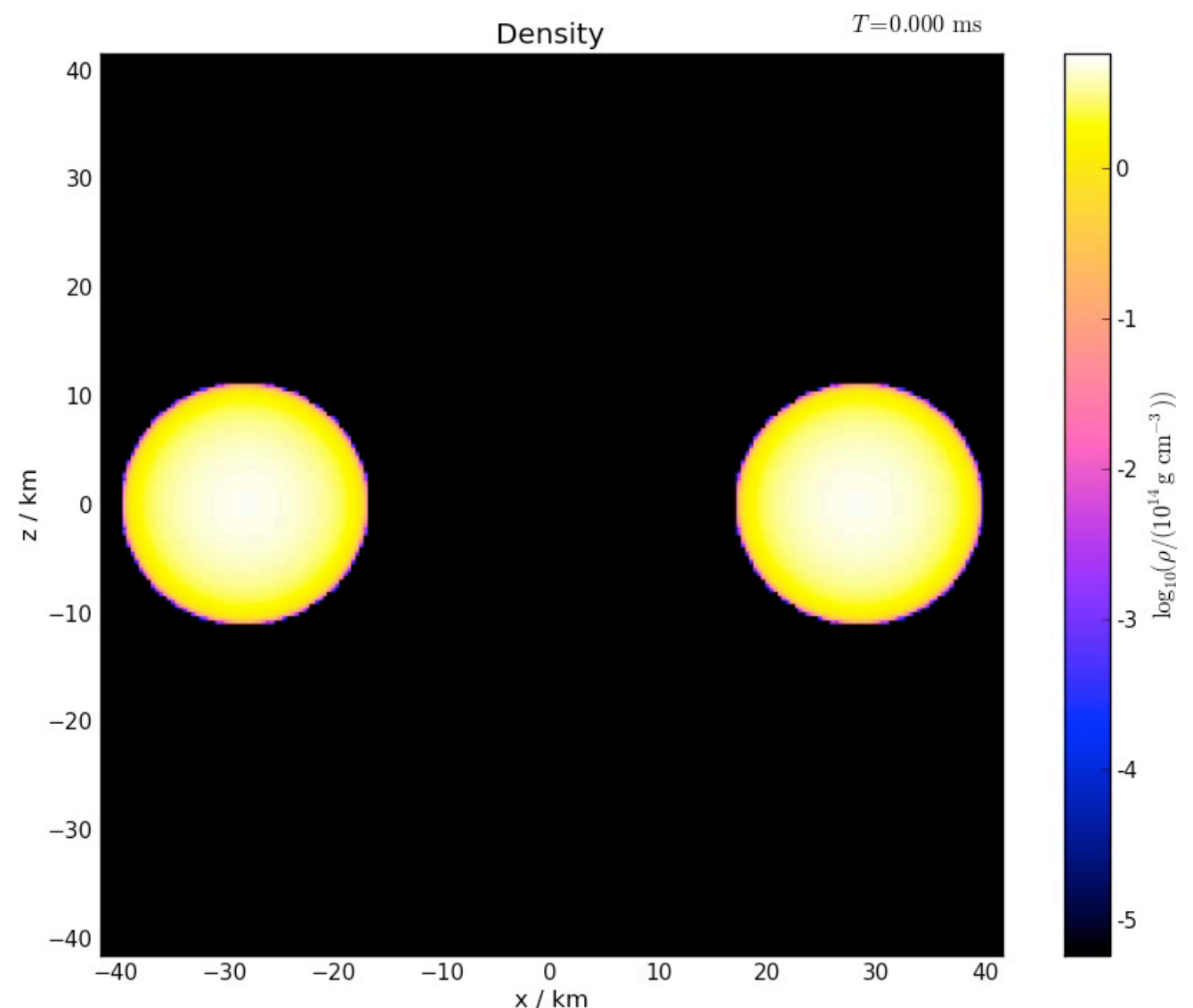
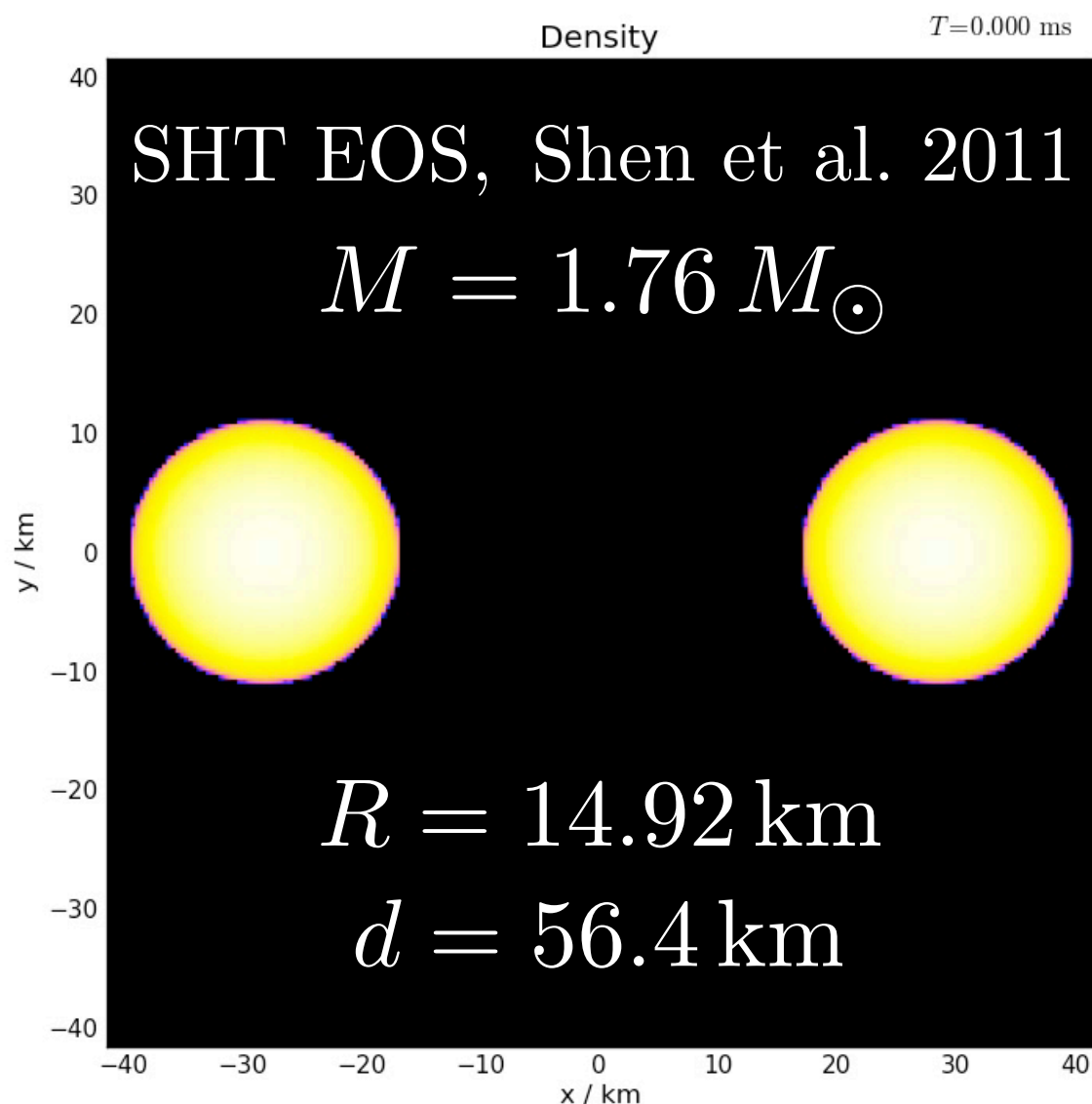
equal mass : all matter is clearly bound, i.e. $u_t < -1$
Note the accretion is quasi-periodic

unequal mass: some matter is unbound while other is ejected at large distances (cf. scale). In these regions r-processes can take place

Extending the work to hot realistic EOSs

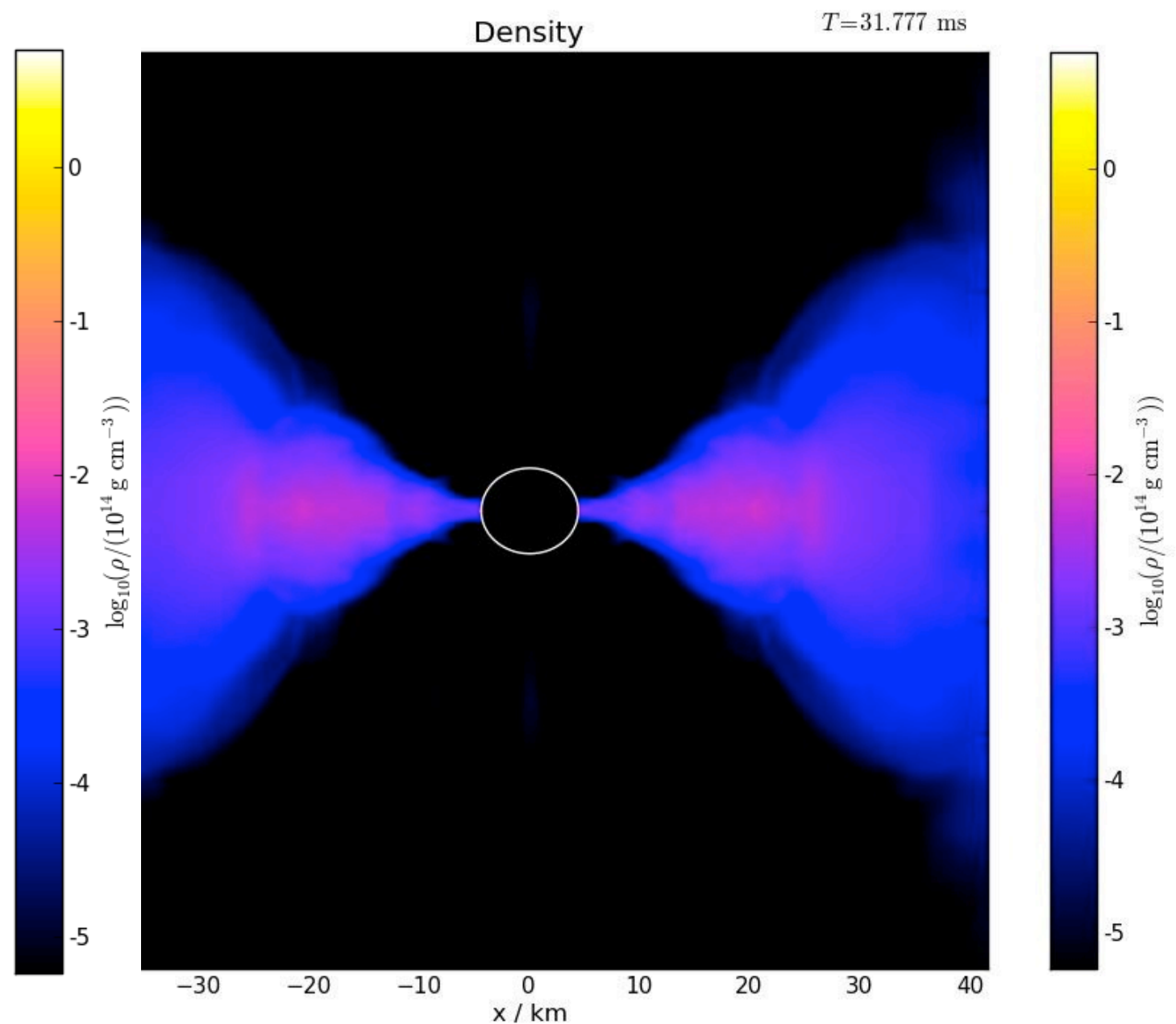
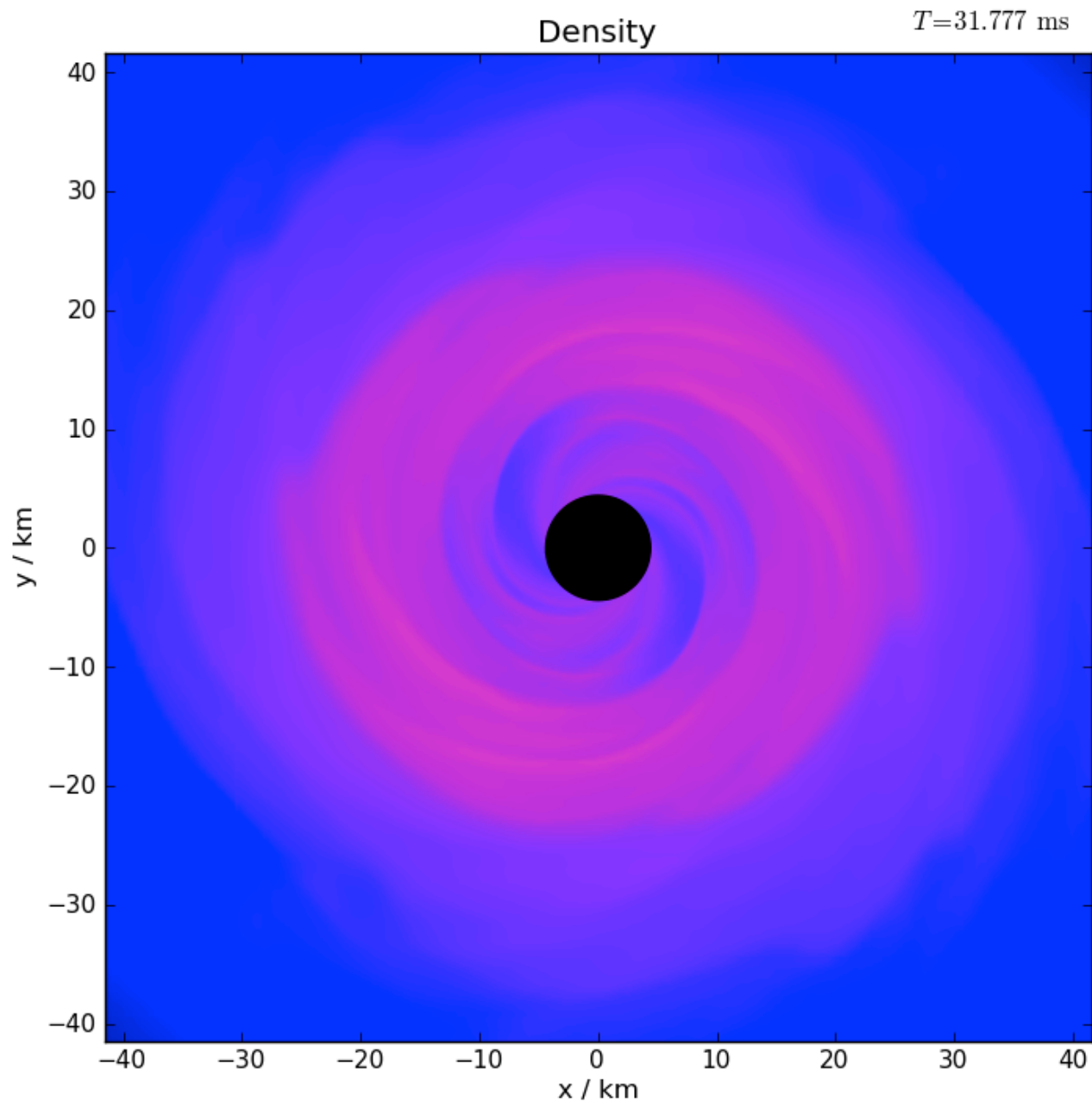
Galeazzi, Kastaun, LR

We are now able to perform simulations also with realistic hot EOSs (Lattimer-Swesty, Shen-et-al, Shen-Horowitz-Teige, etc.) and taking first steps towards modelling **radiative losses** (via “leakage” approach) and **r-process nucleosynthesis**.



Extending the work to hot realistic EOs

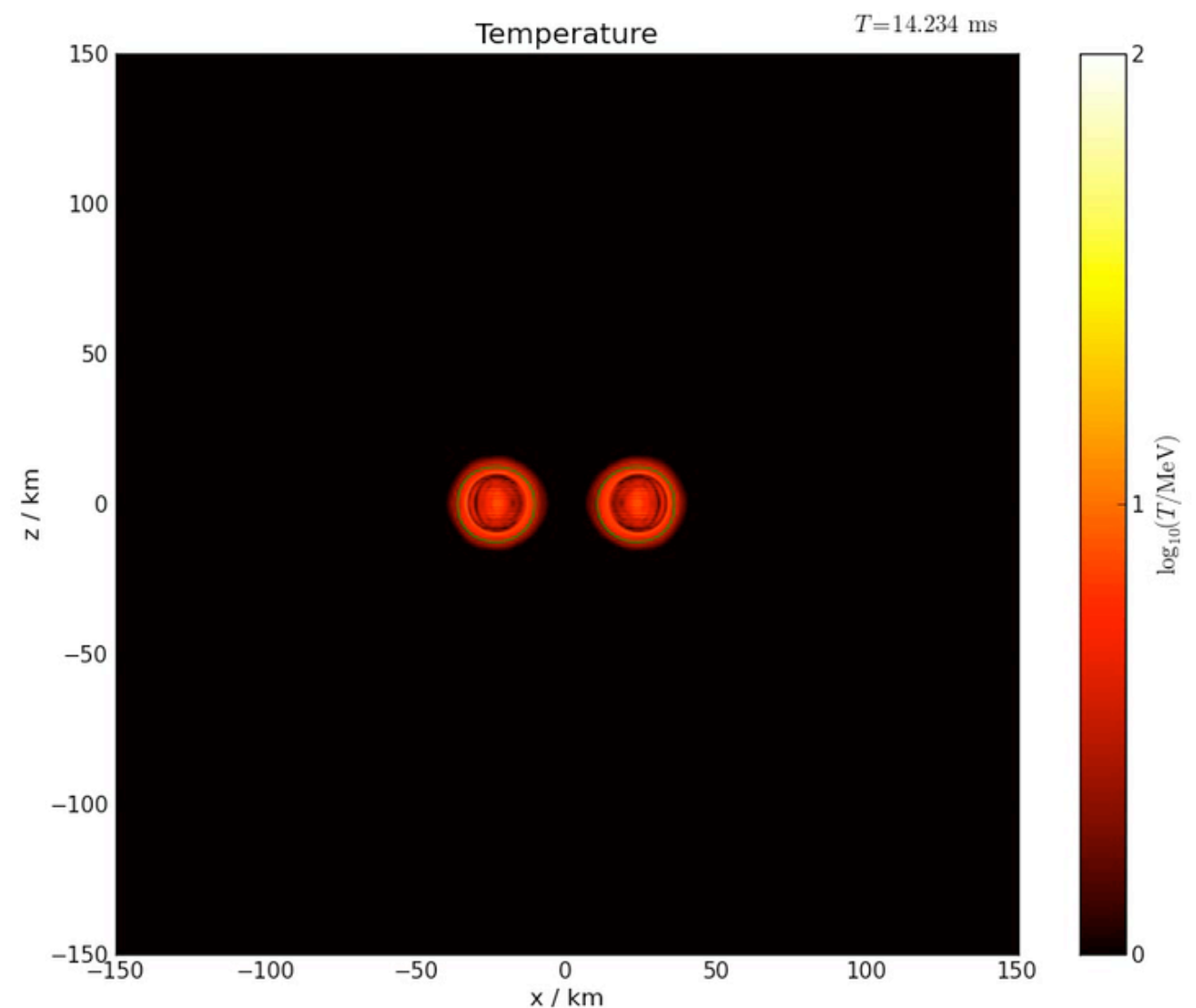
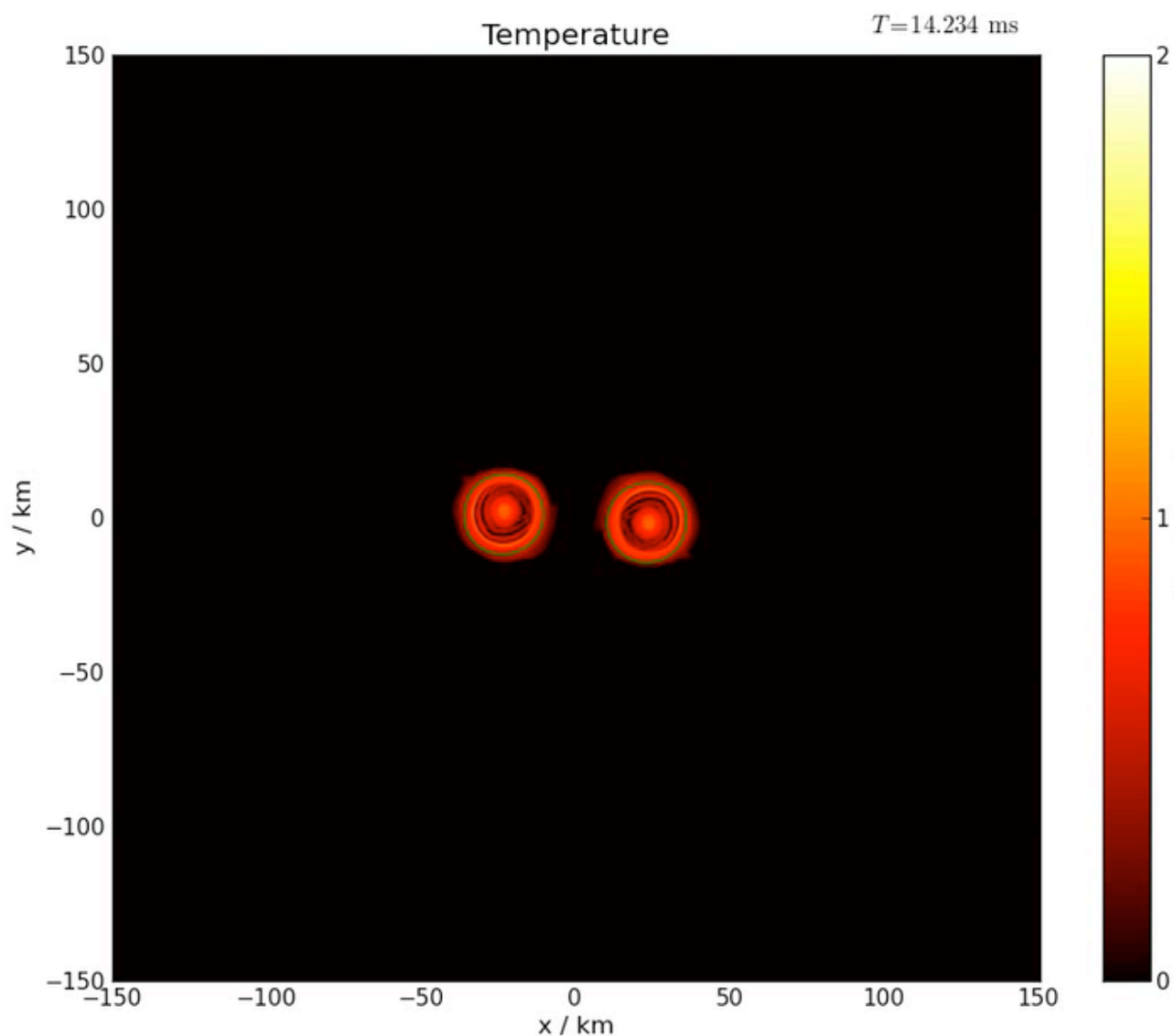
As expected, many of the **qualitative** features of analytic EOs (ideal-fluid) are present also when considering realistic EOs: merger \rightarrow HMNS \rightarrow BH + torus: $M_{\text{torus}} \simeq 0.024 M_{\odot} = 0.6\% M_0$
small but expected for equal-mass binaries

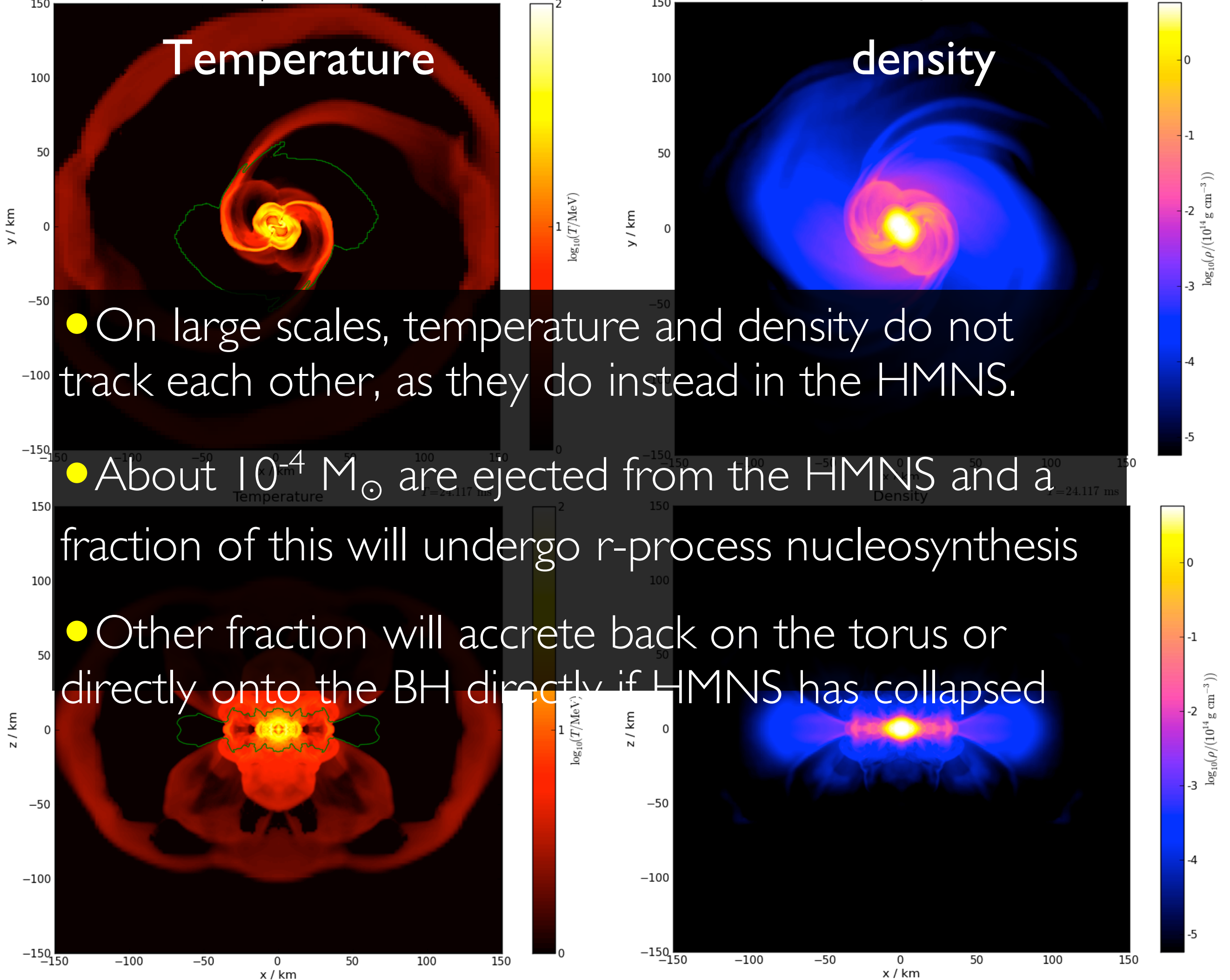


Extending the work to hot realistic EoSs

Particularly interesting are the evolutions of the **temperature** and of the **electron fraction**

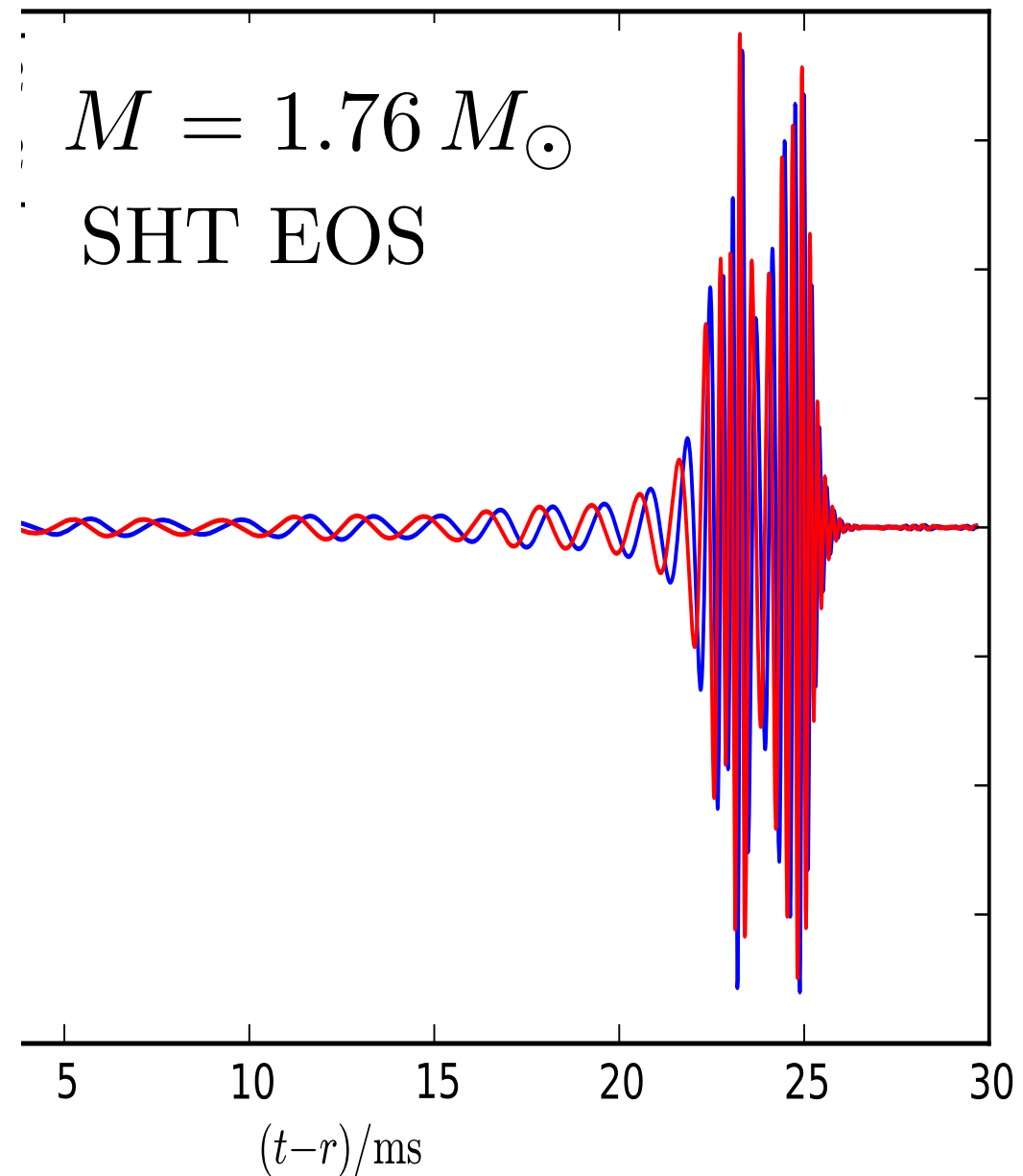
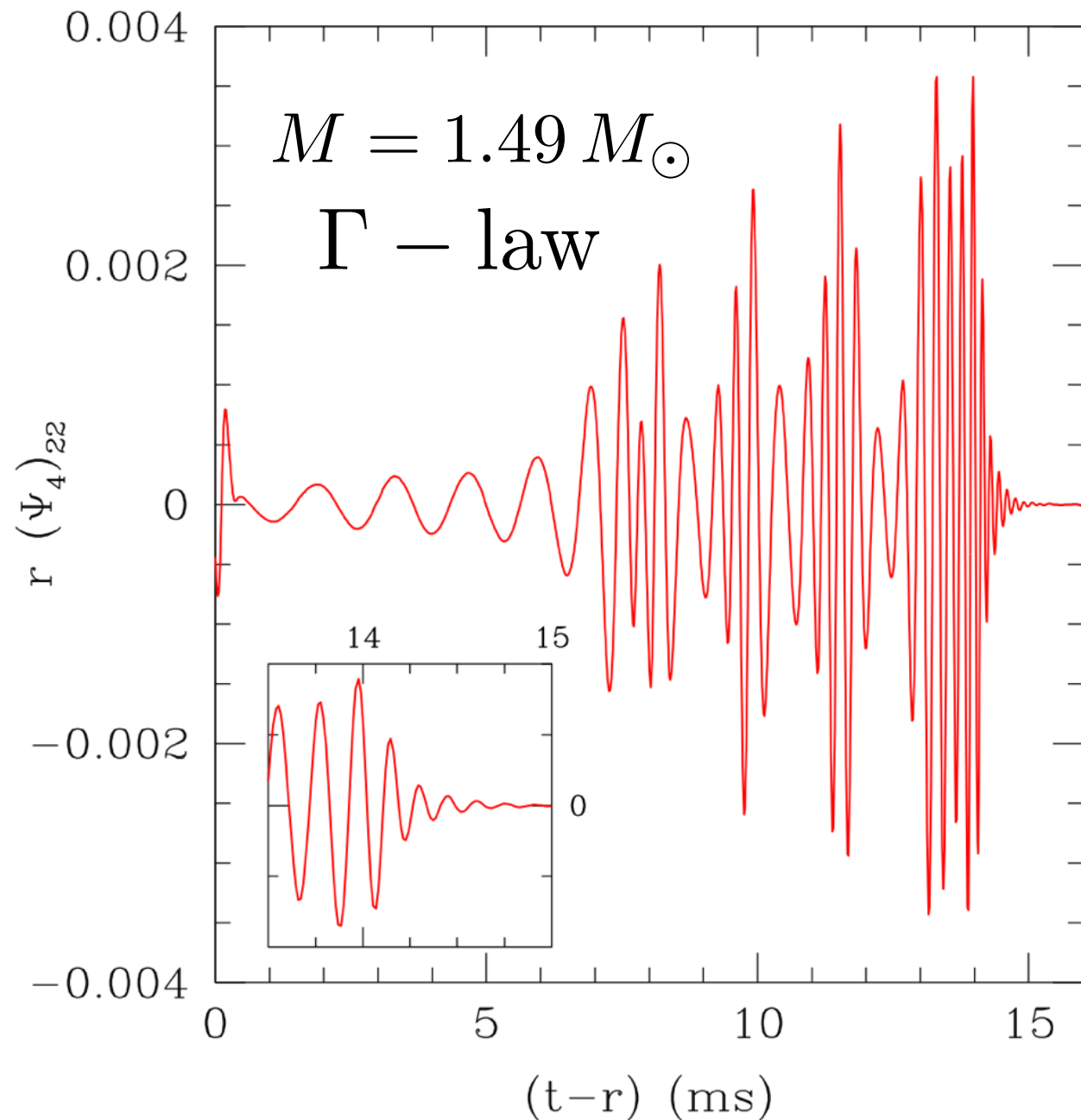
Color range in between 1 and ~ 200 MeV





Extending the work to hot realistic EOSs

As already observed in the case of ideal-fluid simulations, stiffening of the EOS at high densities leads to **bounces** of the two stellar cores and to **modulated emission**



Extending the work to ideal MHD

NSs have large magnetic fields and it is natural to ask:

- can B-fields be detected during the inspiral?

***NO**: present and future GW detectors will not be sensitive enough to measure the small differences

Giacomazzo, LR, Baiotti (2009)

- can B-fields be detected in the HMNS?

***YES** (in principle): different B-fields change the survival time of the HMNS (effect may be degenerate)

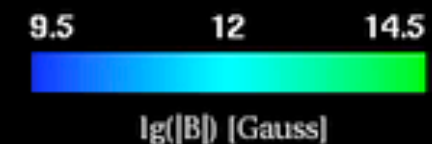
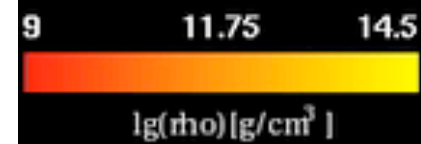
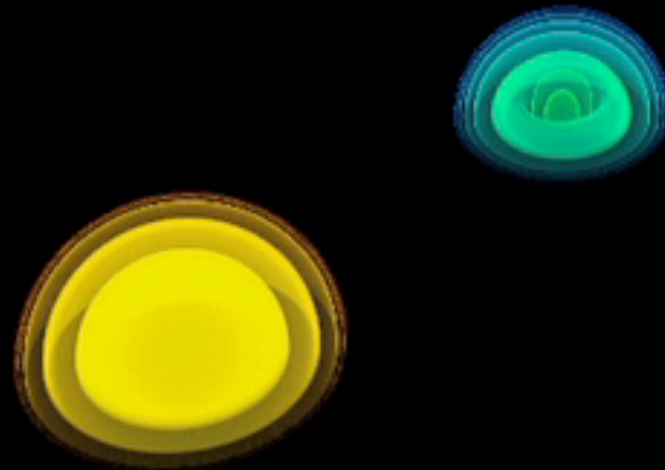
Giacomazzo, LR, Baiotti (2010)

- can B-fields grow after BH formation?

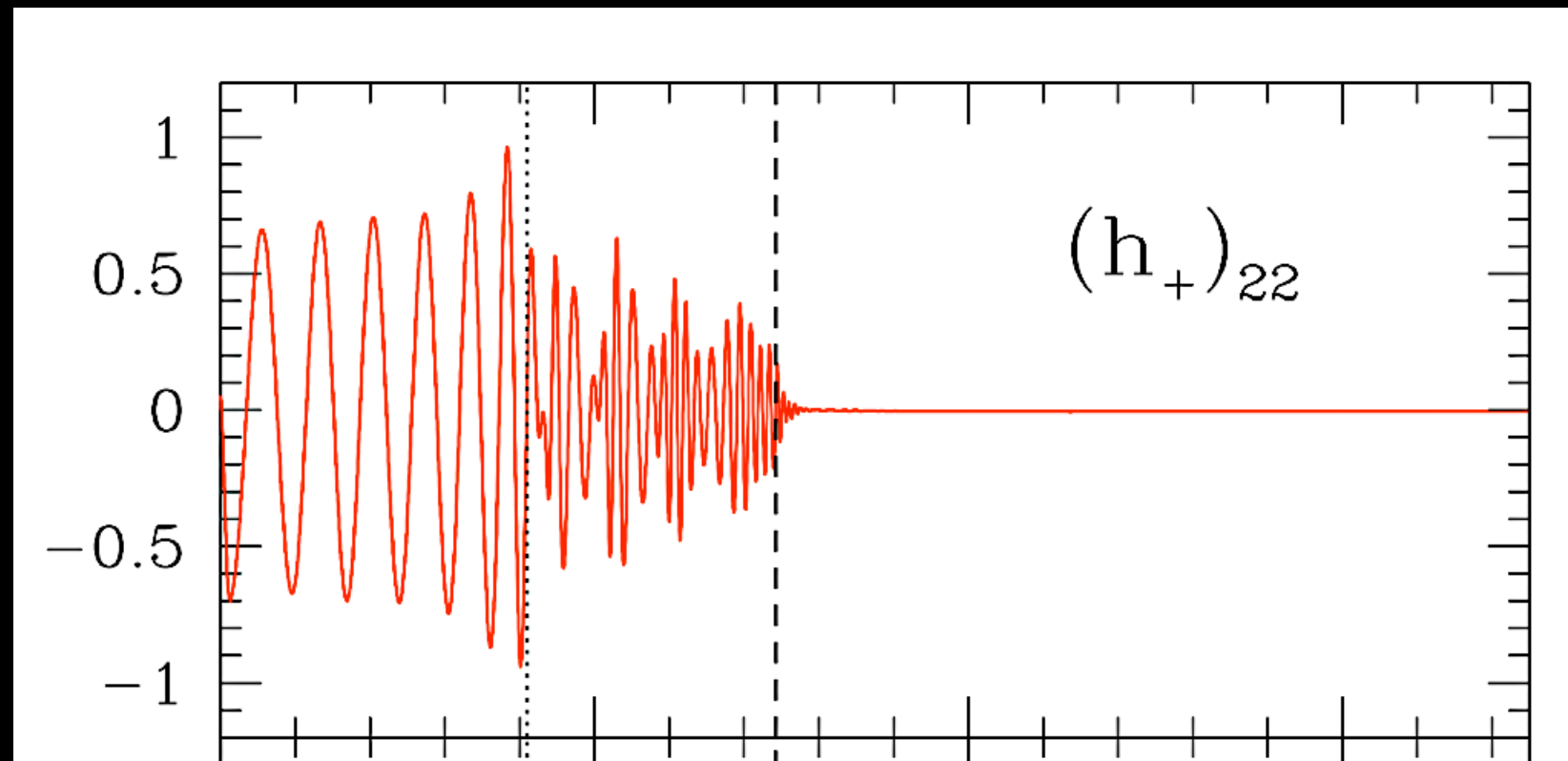
***YES**: B-fields are subject to instabilities and rotation of the BH introduces preferred direction for field geometry

LR, Giacomazzo, Baiotti, + (2011)

Typical evolution for a magnetized binary
(hot EOS) $M = 1.5 M_{\odot}, B_0 = 10^{12}$ G



Going beyond BH formation



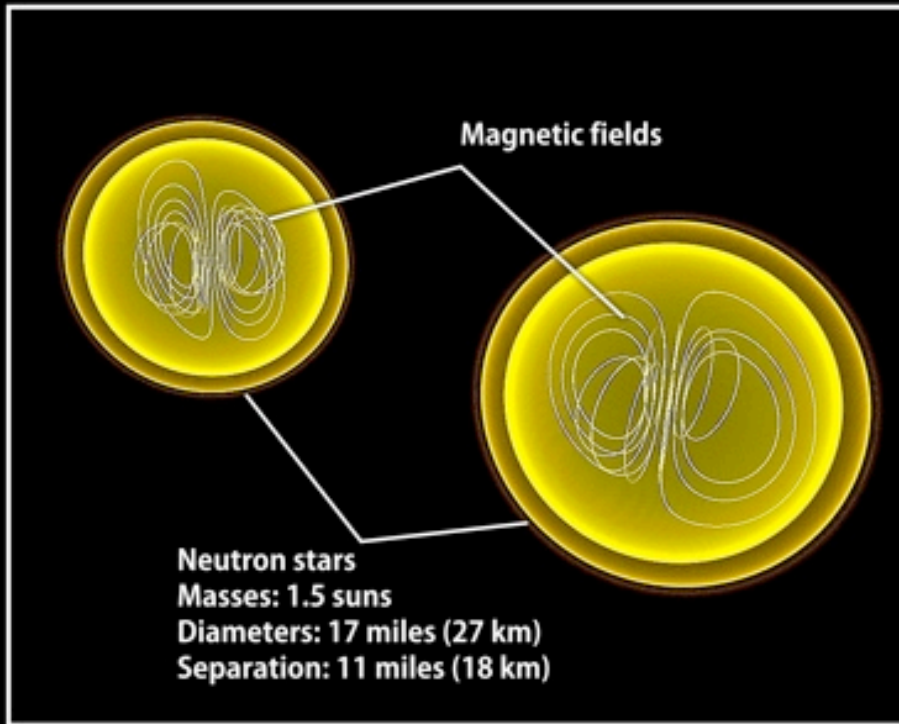
From a GW point of view,
the binary becomes silent
after BH formation and
ringdown.

Is this really the end of the story?

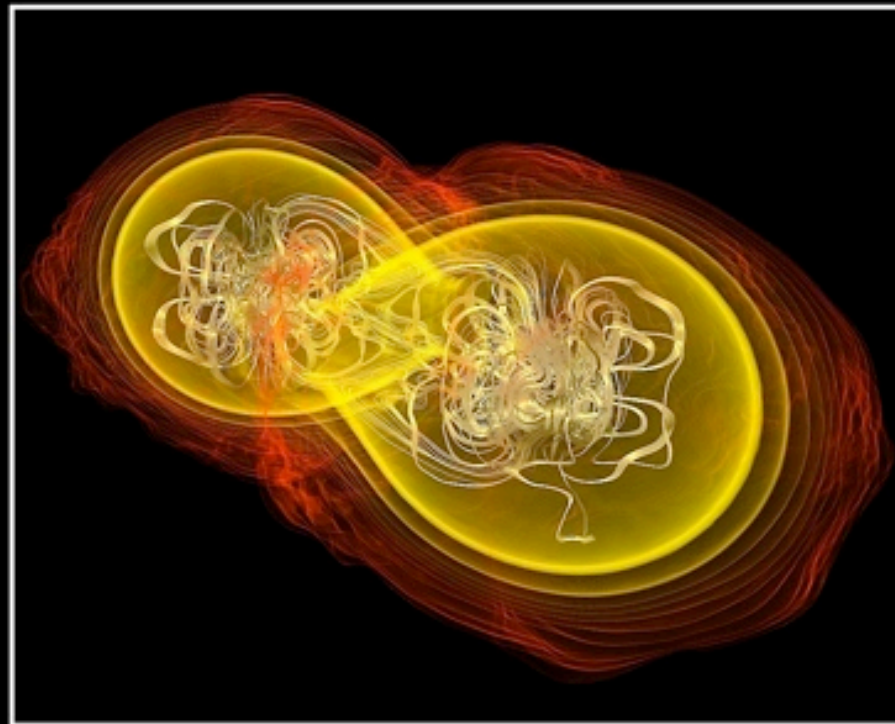
Animations:, LR, Koppitz



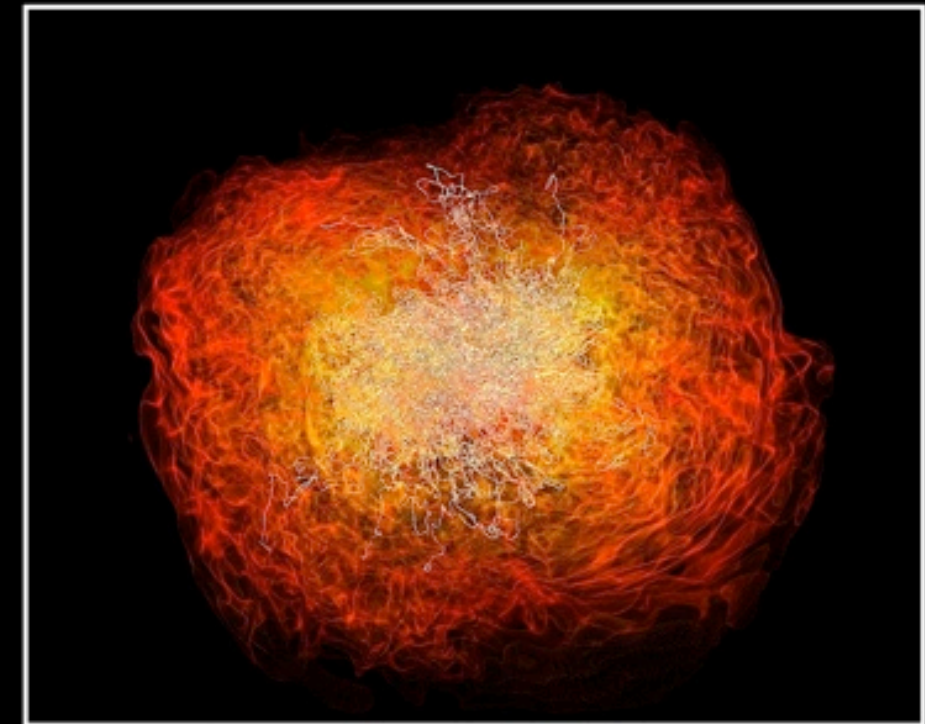
Crashing neutron stars can make gamma-ray burst jets



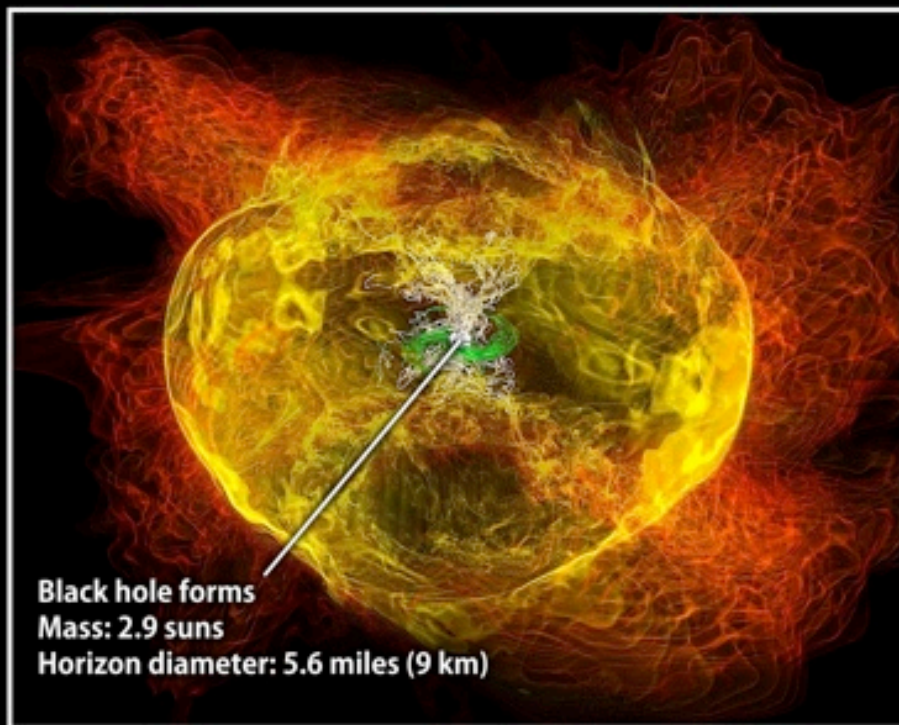
Simulation begins



7.4 milliseconds



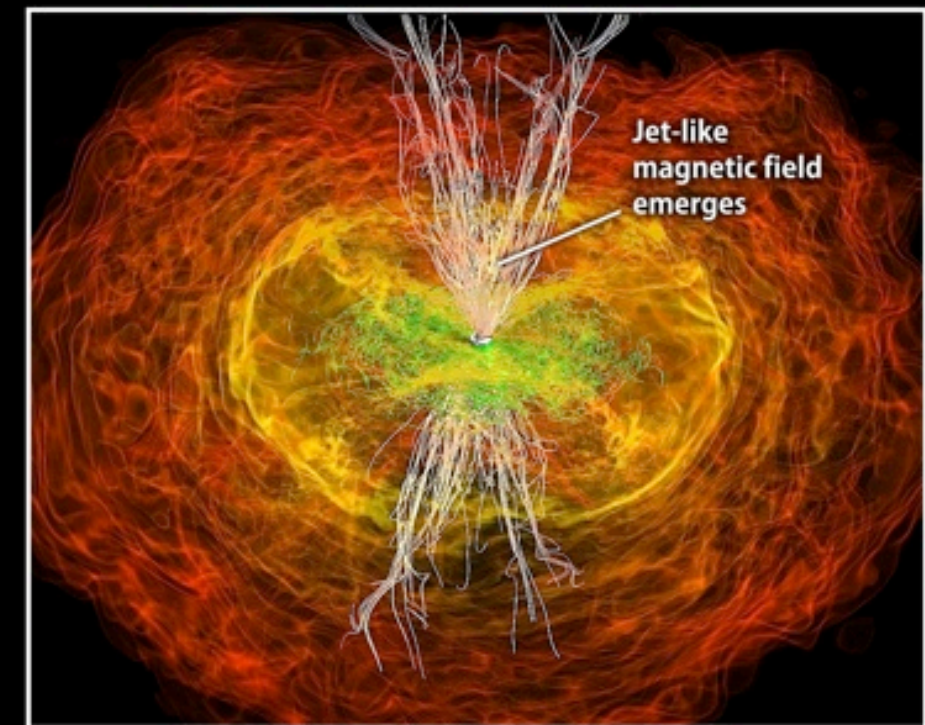
13.8 milliseconds



15.3 milliseconds



21.2 milliseconds



26.5 milliseconds

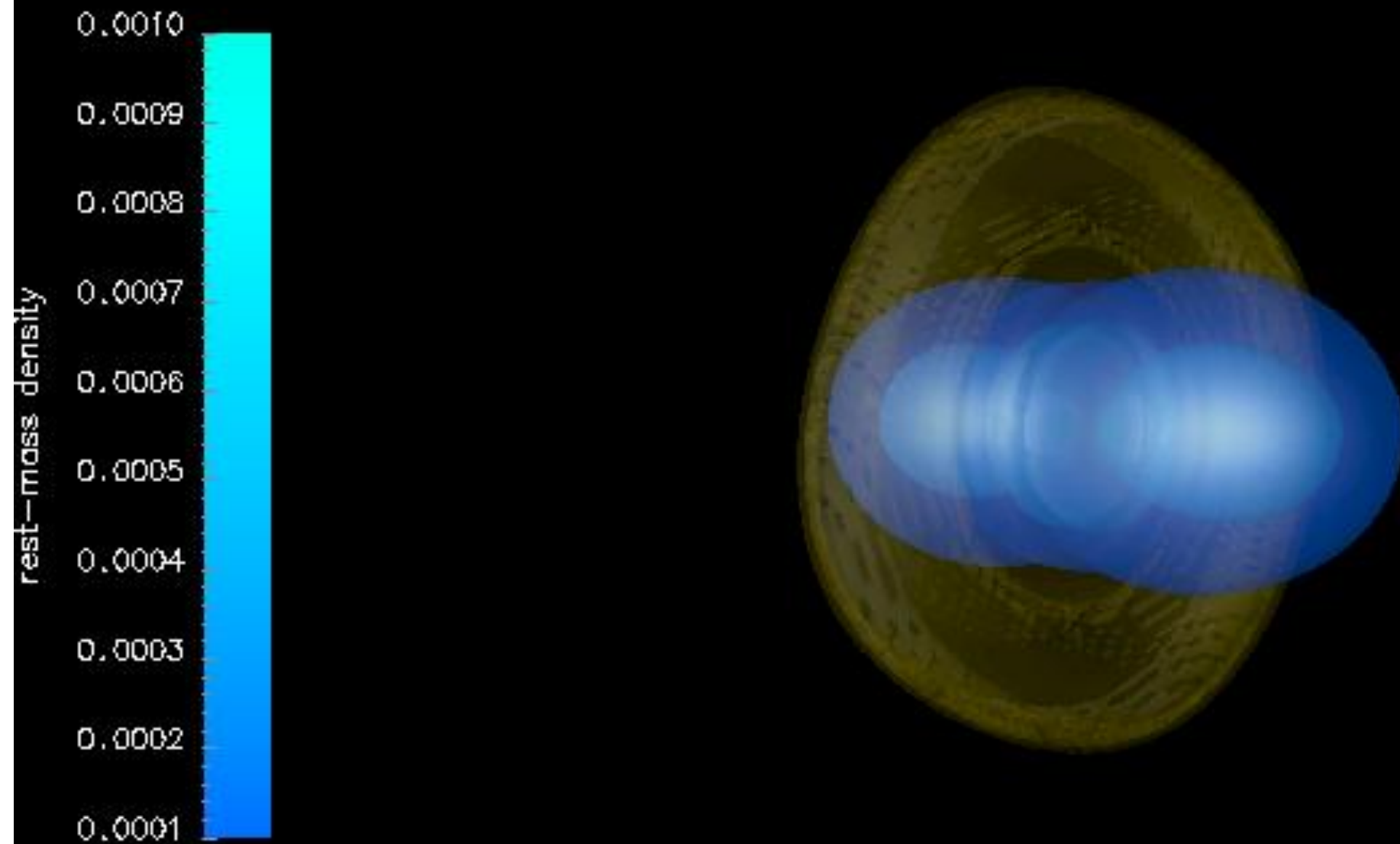
Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

$$J/M^2 = 0.83$$

$$M_{\text{tor}} = 0.063 M_{\odot}$$

$$t_{\text{accr}} \simeq M_{\text{tor}} / \dot{M} \simeq 0.3 \text{ s}$$

From star collisions to particle collisions



Time= 0.1539 ms

LR, K. Takami,

2012

The process in a cartoon

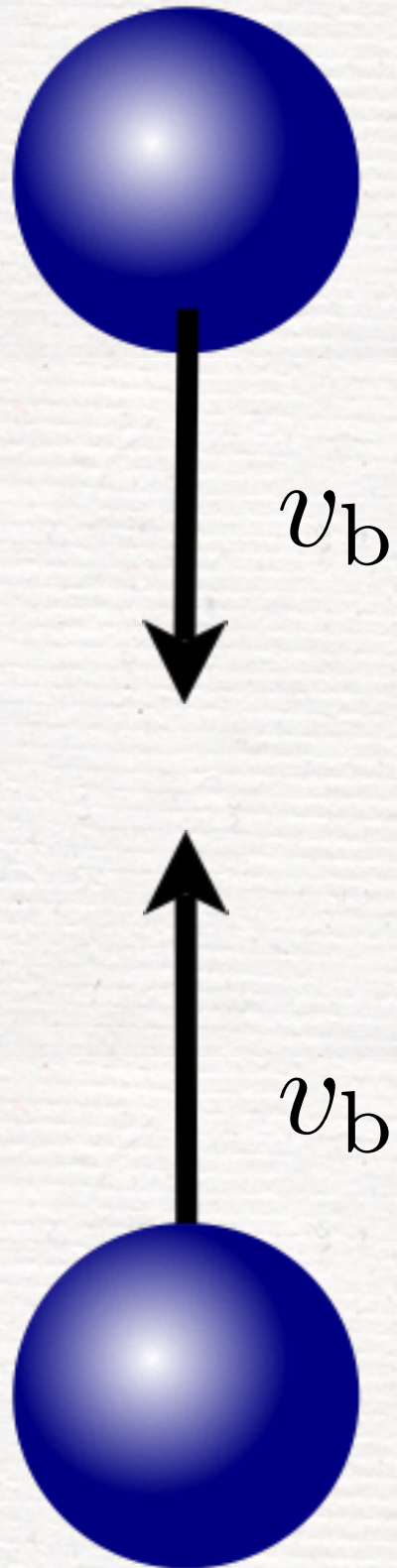
The question is very simple:
what are the conditions under which a black hole can be formed from the collision of two self-gravitating objects?

The answer does not exist yet:
no sufficient/necessary conditions are known.
Some guidance is offered by Thorne's *hoop conjecture*

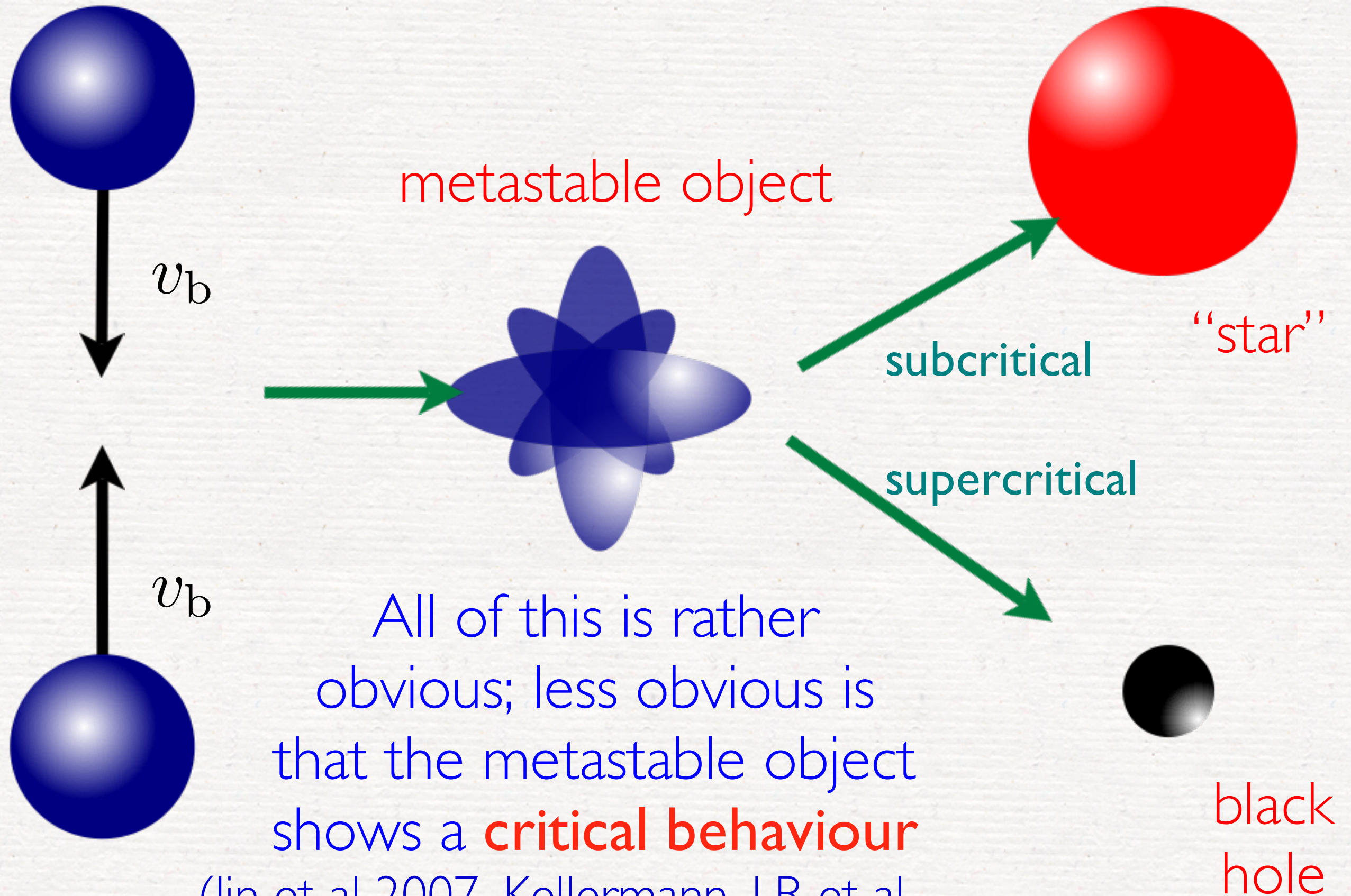
$$R_{\text{hoop}} \leq R_s = 2MG/c^2$$

Not a rigorous condition!
(difficult to measure energy in a volume in GR)

Numerical-relativity simulations can provide clues

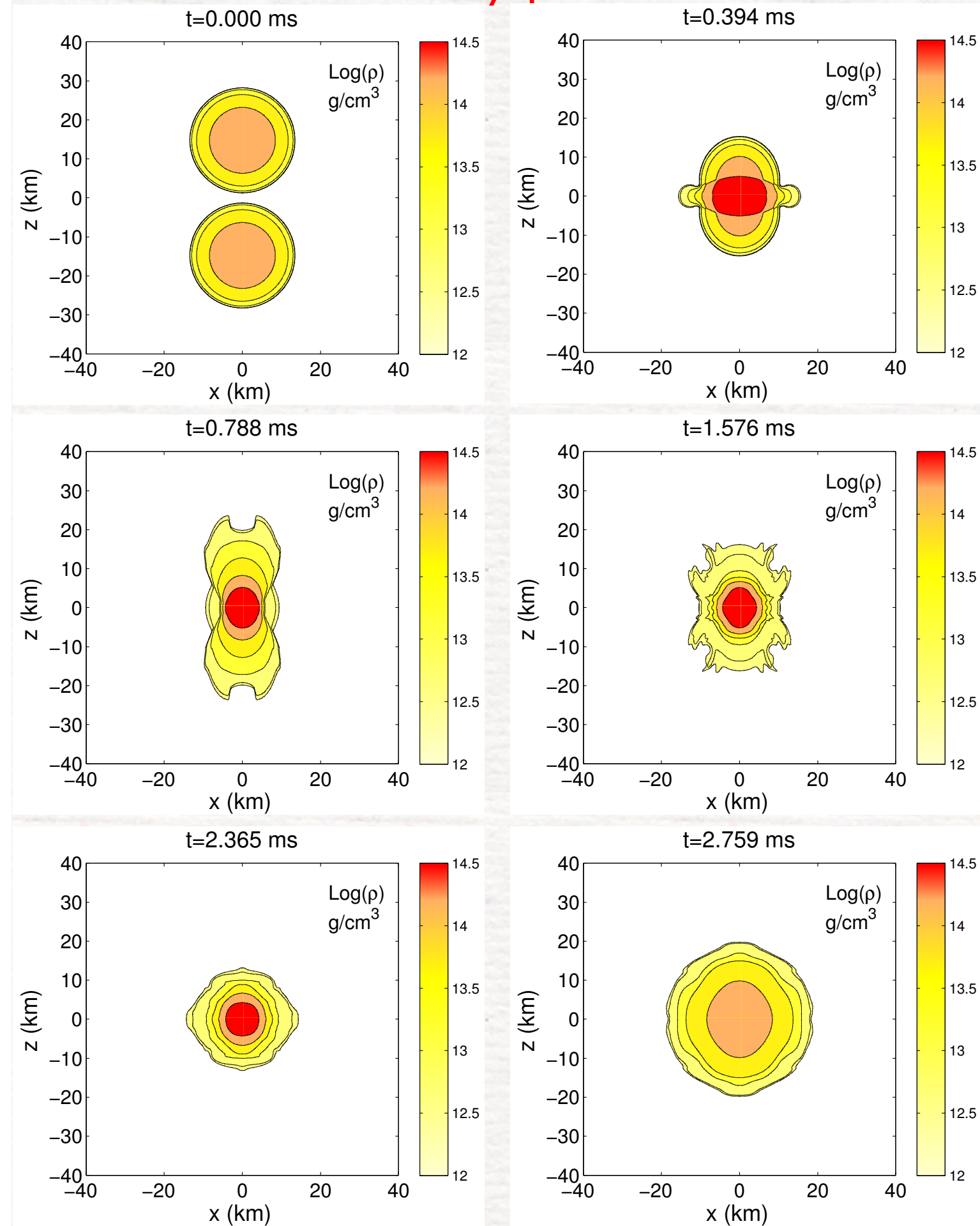


The process in a cartoon



(Jin et al 2007, Kellermann, LR et al

Typical subcritical collision



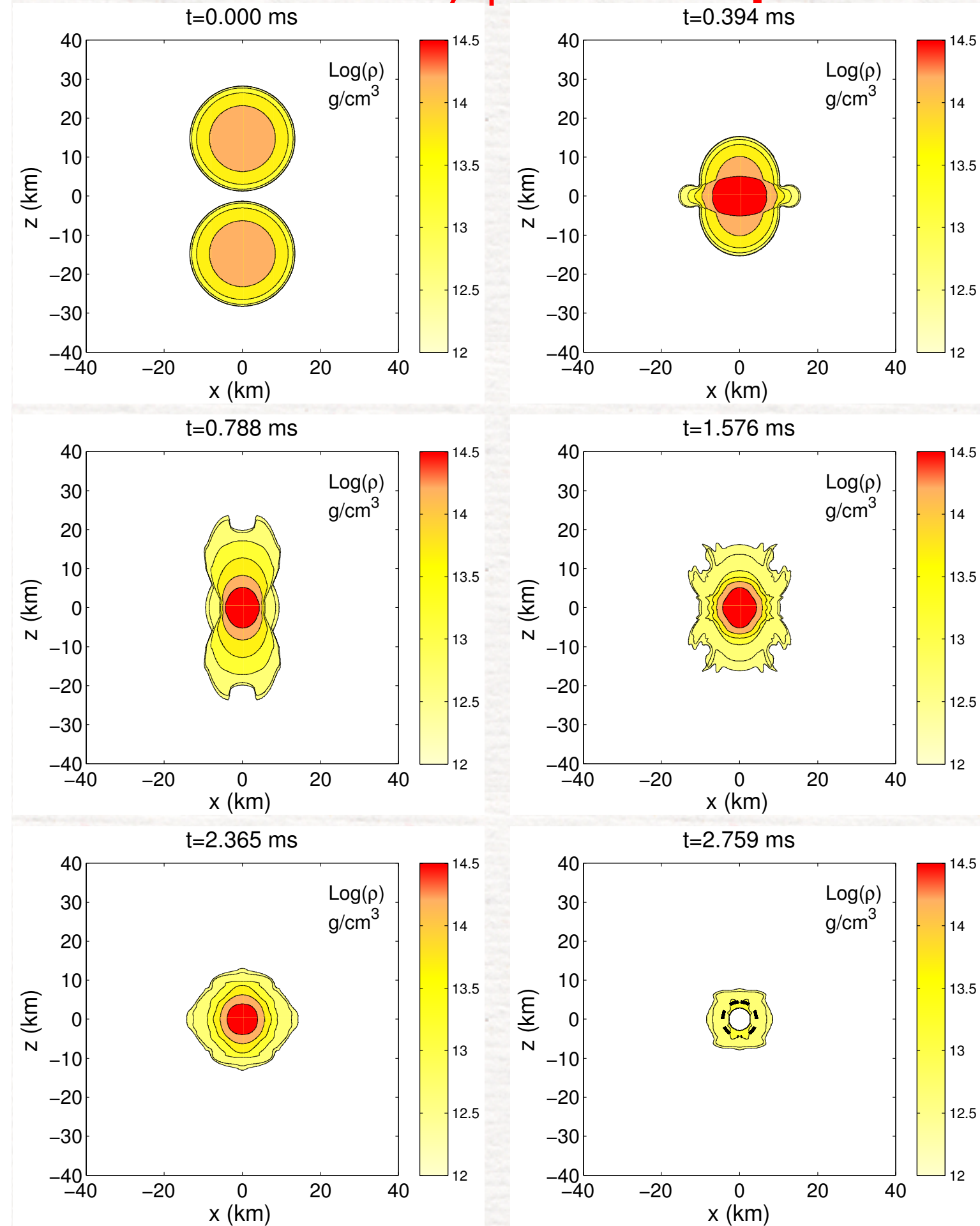
The different panels show snapshots of the rest-mass density at representative times for a **subcritical** binary.

Note the metastable object in panels 2-5.

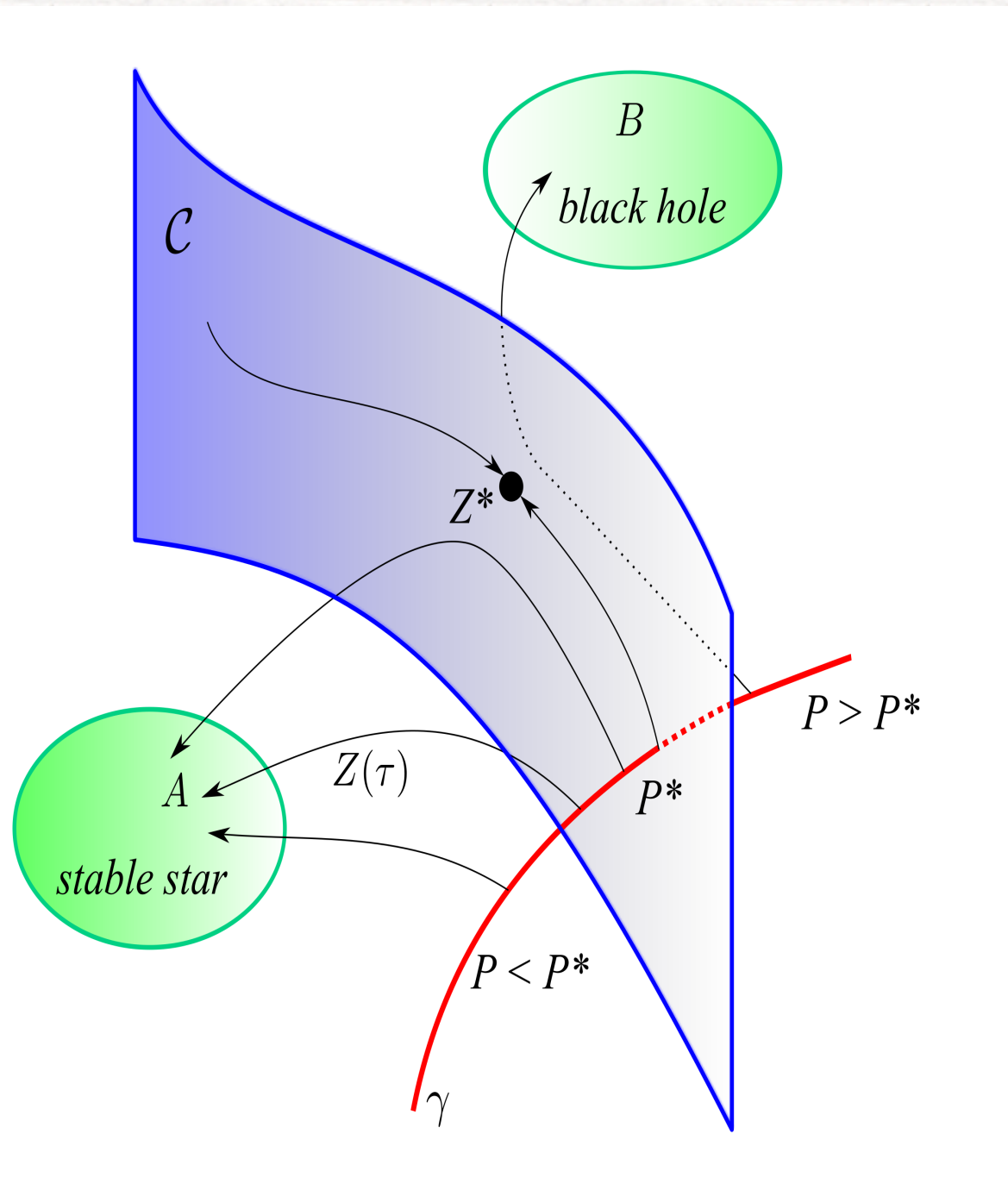
Typical supercritical collision

The different panels show snapshots of the rest-mass density at representative times for a **supercritical** binary.

Note the metastable object in panels 2-5.



A brief introduction to critical behaviour



Given a series of initial data parametrized by a scalar quantity P , the critical solution at P^* will separate two basins of attracting solutions.

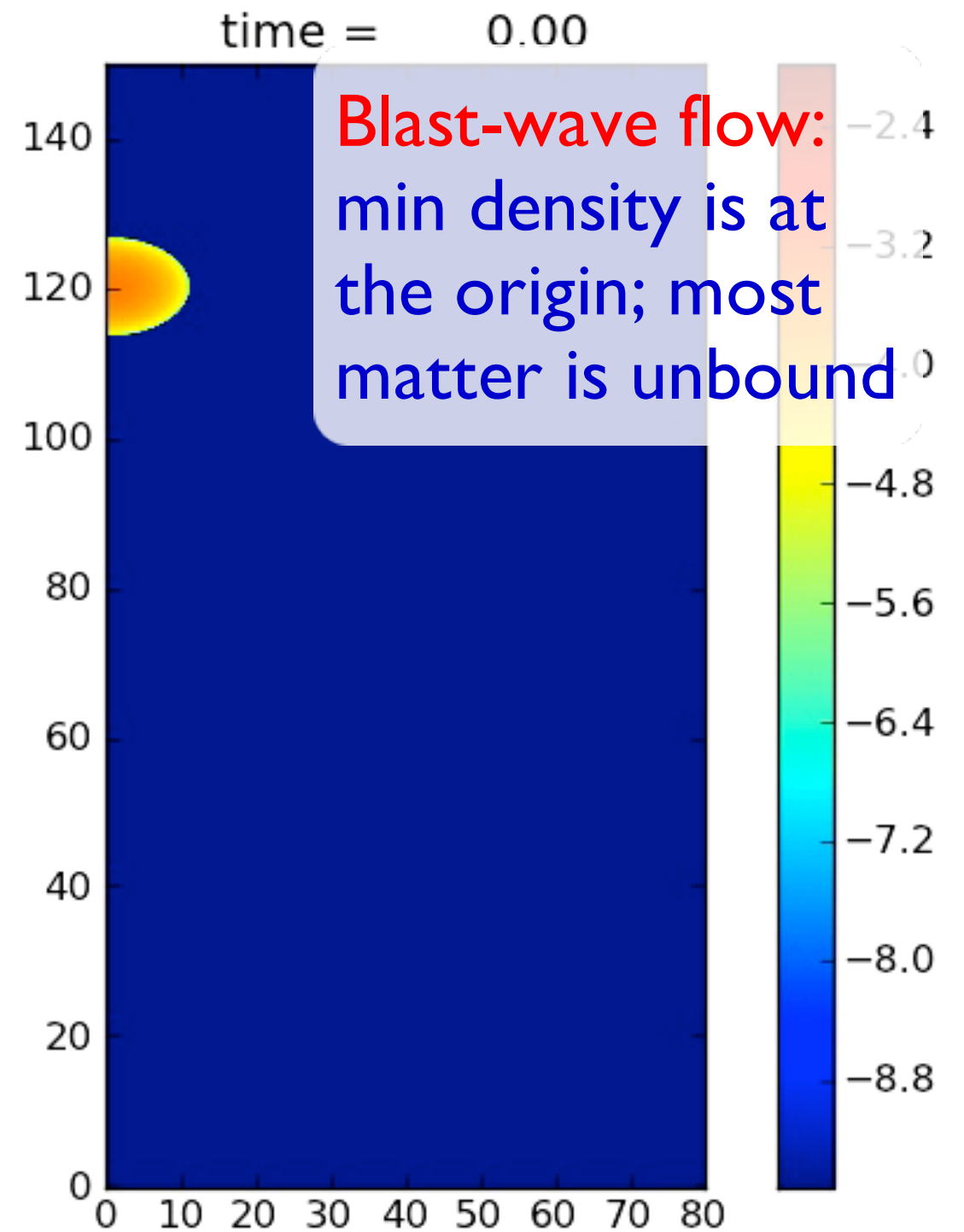
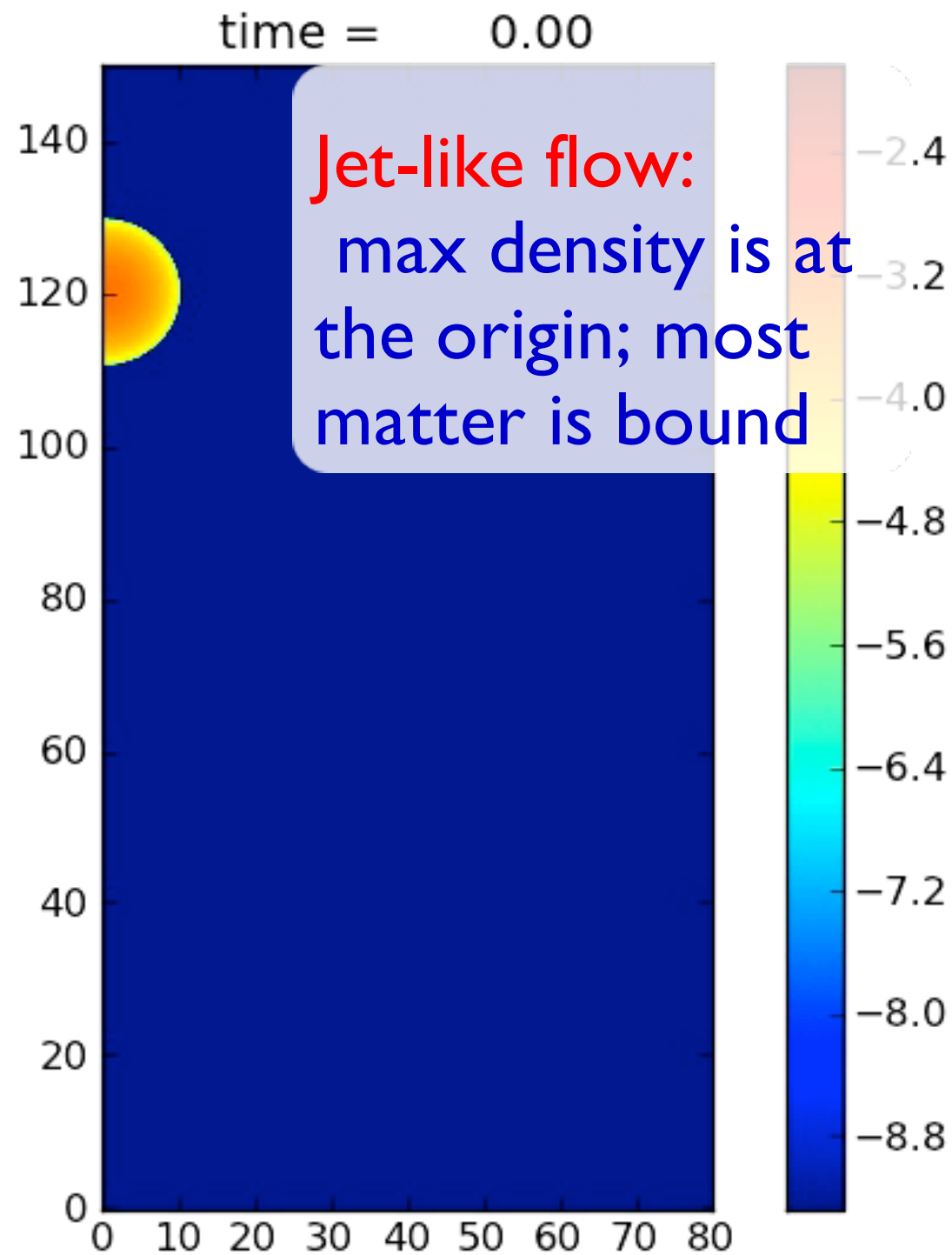
Solutions near the critical one will survive on the critical manifold for a certain time before evolving towards the corresponding basin

The critical solution is attractive on the critical manifold C , ie all but one mode converge towards Z^*

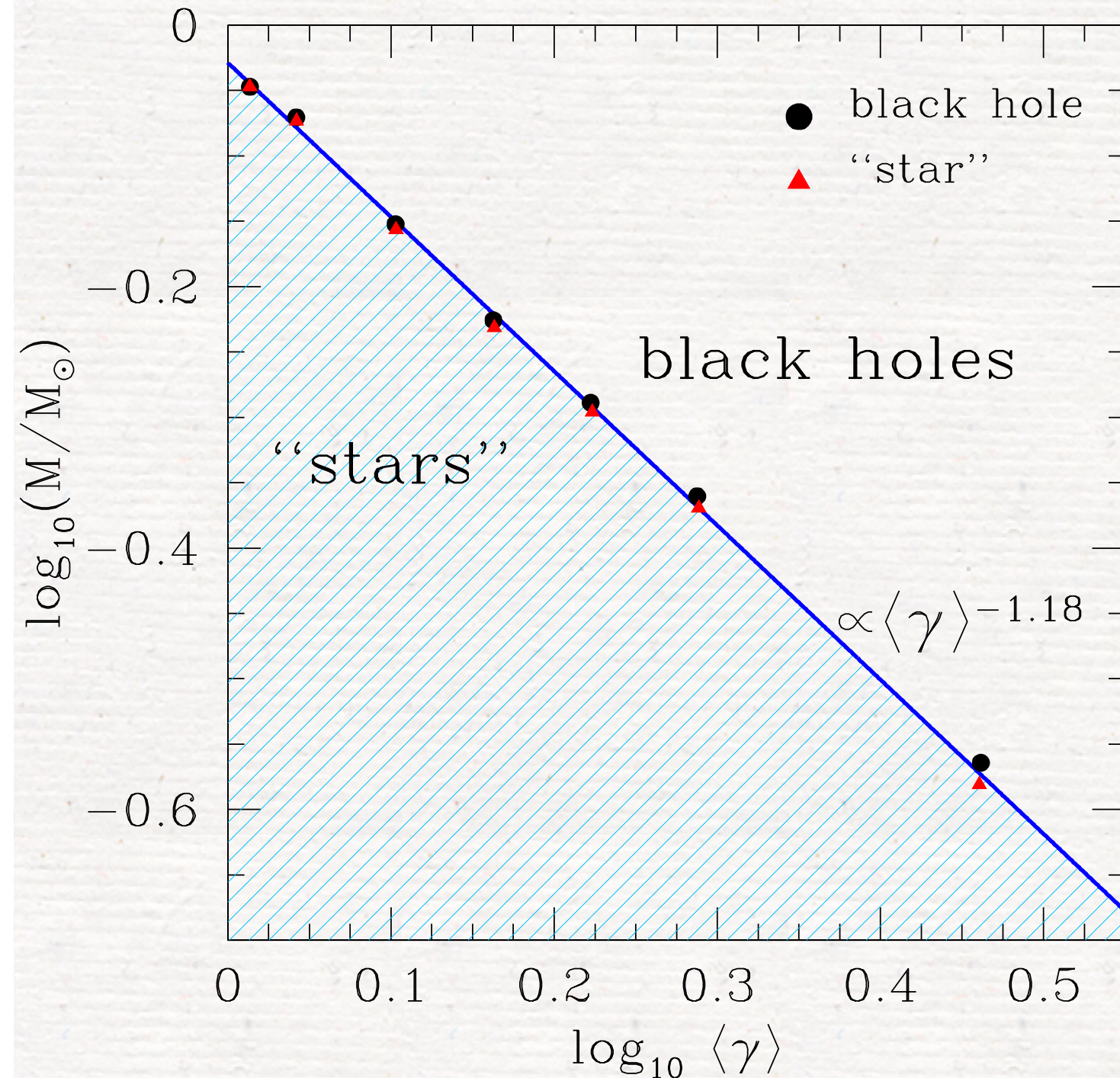
Different dynamics for different boosts

$$v_b/c = 0.3$$

$$v_b/c = 0.8$$



A simple scaling behaviour



For any value of the boost we can compute the threshold between BHs and NSs and find this follows a simple **scaling law**

$$\frac{M_{\text{th}}}{M_{\odot}} = K \langle \gamma \rangle^{-n} \approx 0.93 \langle \gamma \rangle^{-1.2}$$

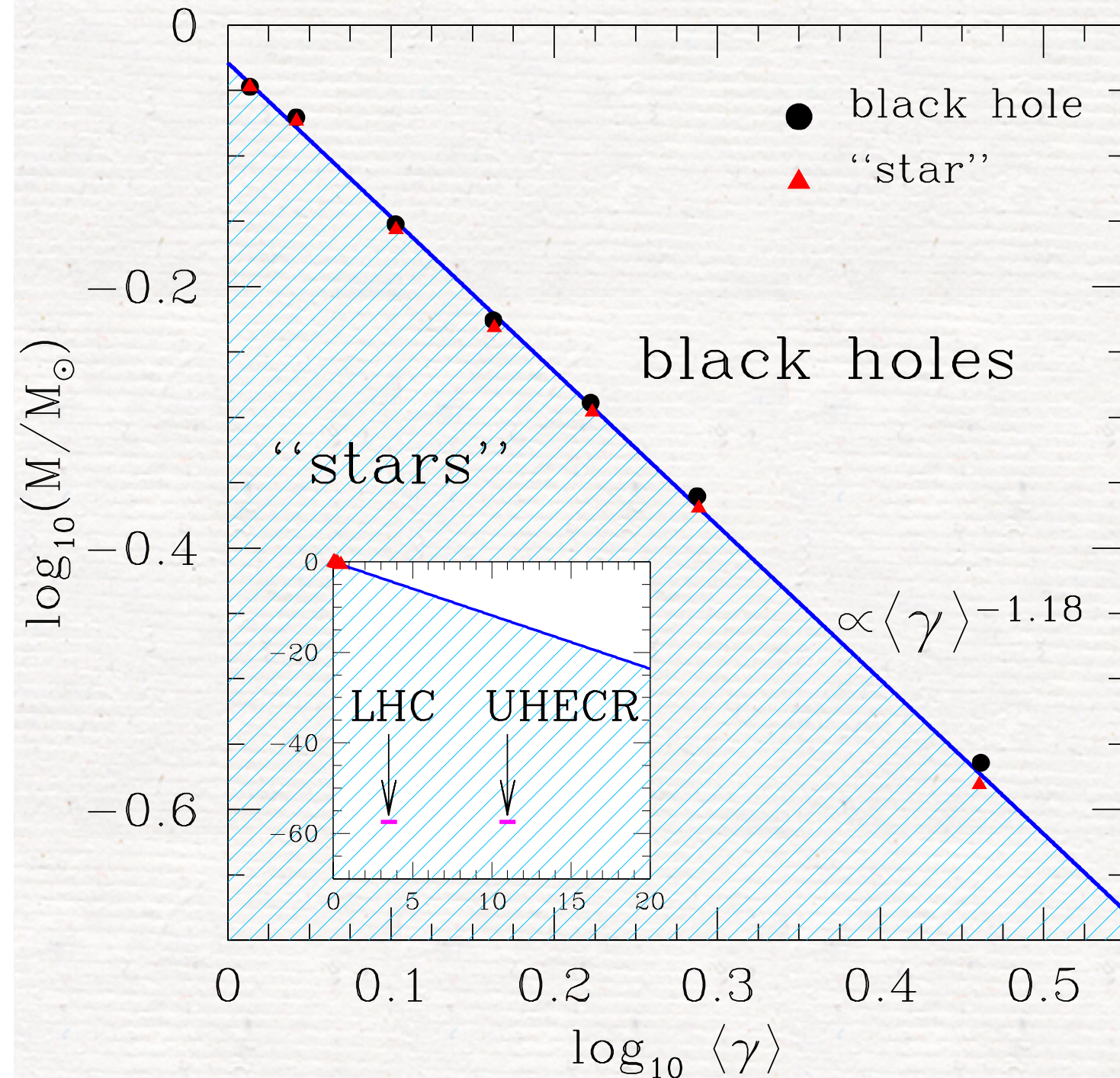
Relevant limits:

$$\langle \gamma \rangle \rightarrow 1 : M_{\text{th}} \rightarrow 0.93 M_{\odot}$$

$$\langle \gamma \rangle \rightarrow \infty : M_{\text{th}} \rightarrow 0$$

For divergent kinetic energies, the critical BH has infinitesimal mass

A simple scaling behaviour



For any value of the boost we can compute the threshold between BHs and NSs and find this follows a simple **scaling law**

$$\frac{M_{\text{th}}}{M_{\odot}} = K \langle \gamma \rangle^{-n} \approx 0.93 \langle \gamma \rangle^{-1.2}$$

Relevant limits:

$$\langle \gamma \rangle \rightarrow 1 : M_{\text{th}} \rightarrow 0.93 M_{\odot}$$

$$\langle \gamma \rangle \rightarrow \infty : M_{\text{th}} \rightarrow 0$$

For divergent kinetic energies, the critical BH has infinitesimal mass

Conclusions

- * Modelling of binary neutron stars is now **mature**. All aspects can be followed accurately: inspiral, merger, collapse to BH+torus.
- * GWs from BNSs are much more complex/rich than those from BBHs: can be the **Rosetta stone** to decipher the NS interior.
- * Magnetic fields unlikely to be detected during the inspiral but **important** after the merger (amplified by dynamos/instabilities).
- * Collisions of selfgravitating fluids show simple **scaling** behaviour and extrapolation to LHC scales suggests BHs are unlikely.
- * Binary neutron stars are **formidable laboratories** we are starting to explore. There is still a lot more to do: radiative transfer, resistive effects, nucleosynthesis, etc. Stay tuned!

EXTRAS:
BTRSS

Cold vs Hot EOSs

Simplest example of a **“cold”** EOS is the **polytropic** EOS. This **isentropic**: internal energy (temperature) increases/decreases only by mechanical work (compression/expansion)

$$p = K \rho^\Gamma, \quad \epsilon = \frac{K \rho^{\Gamma-1}}{\Gamma - 1}$$

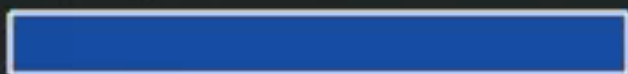
Simplest example of a **“hot”** EOS is the **ideal-fluid** EOS. This **non-isentropic** in presence of shocks: internal energy (i.e. temperature) can increase via shock heating.

$$p = \rho \epsilon (\Gamma - 1), \quad \partial_t \epsilon = \dots$$

A **cold** EOS is optimal for the inspiral; a **hot** EOS is essential after the merger. Take them as extremes of possible behaviours

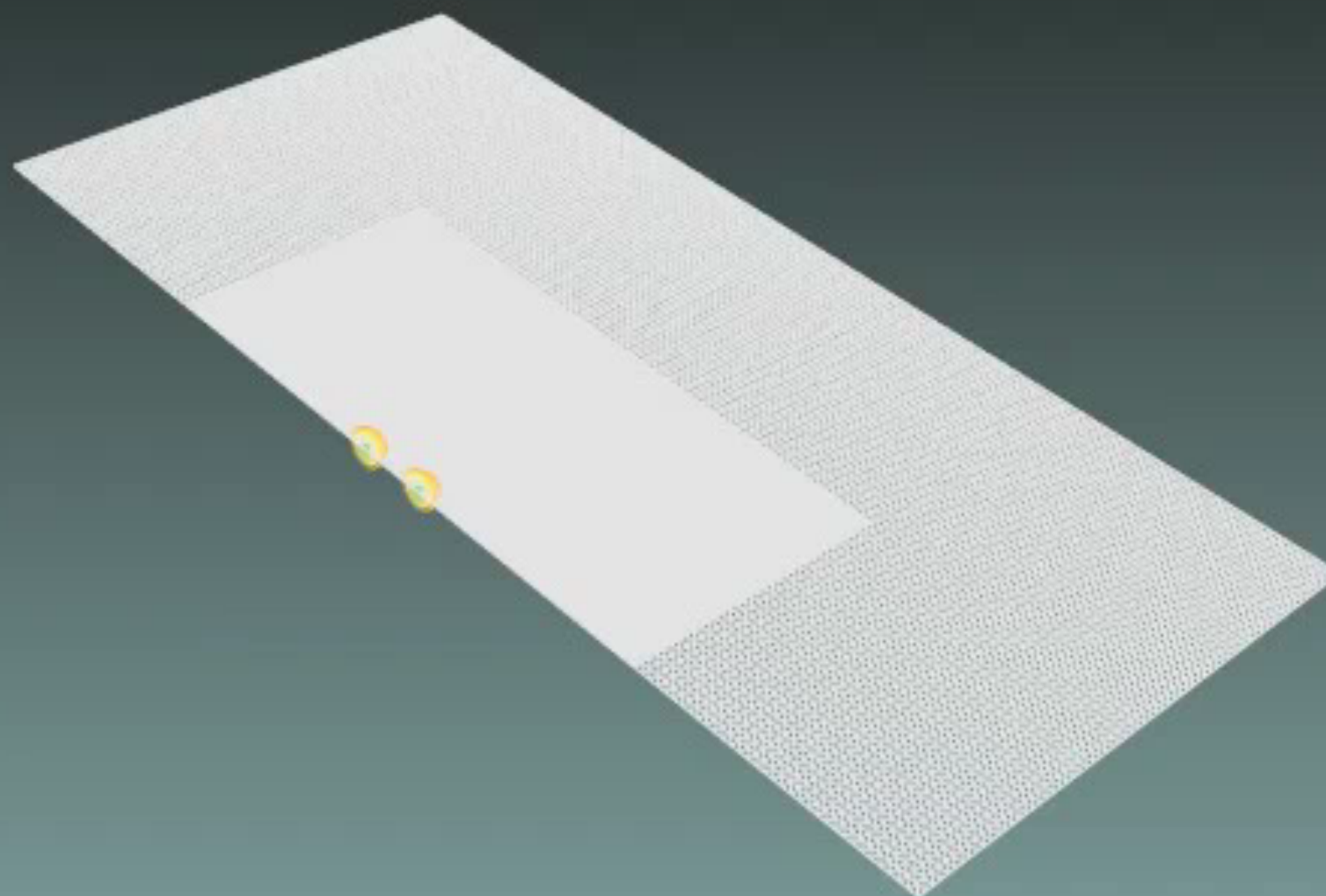
Animations: Kaehler, Giacomazzo, LR

T[ms] = 0.00



T[M] = 0.00

Baiotti, Giacomazzo, LR (PRD 2008, CQG 2008)



0.0 6.1E+14



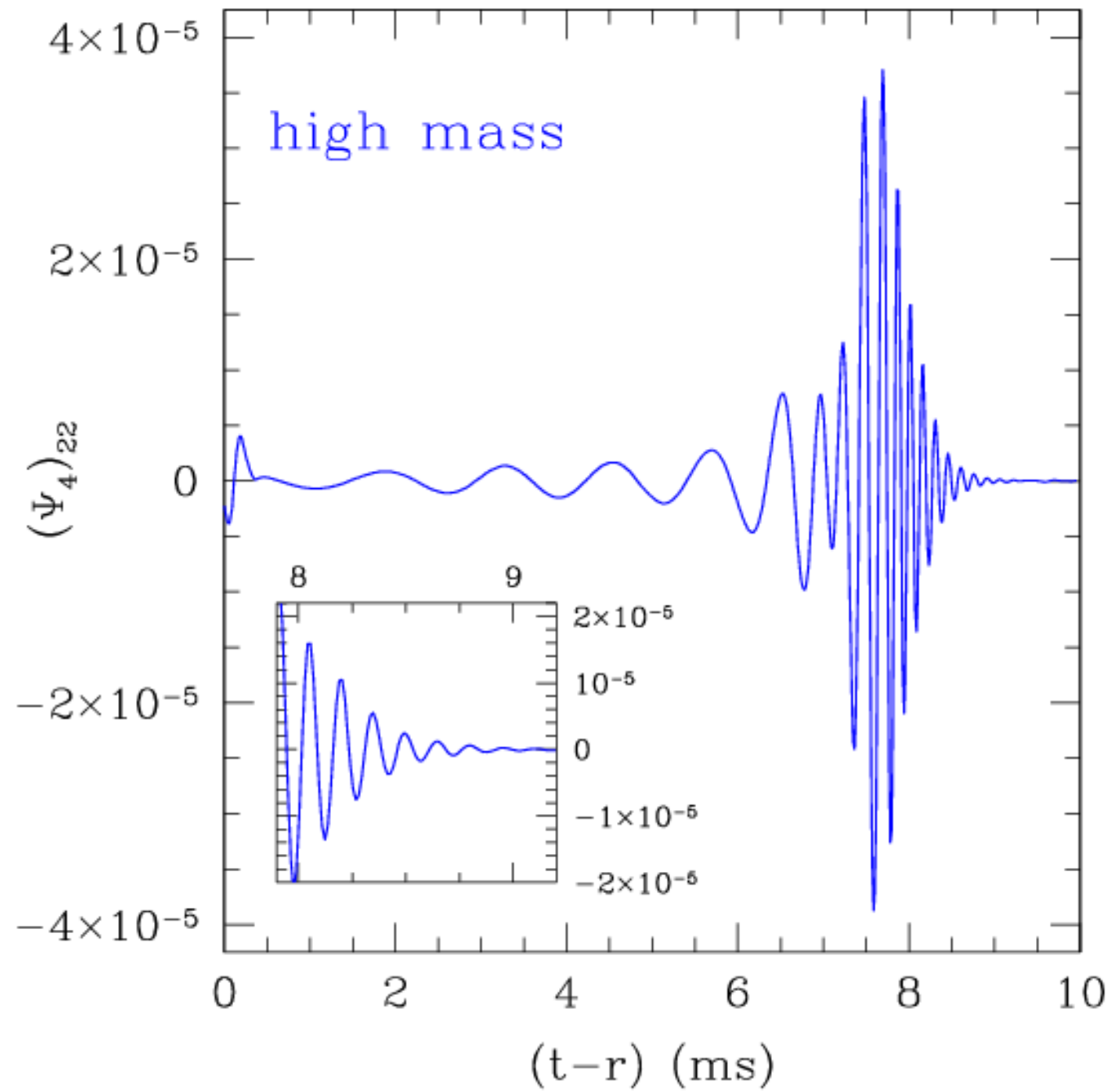
Density [g/cm³]

Cold EOS: high-mass binary

$$M = 1.6 M_{\odot}$$

Waveforms: cold EOS

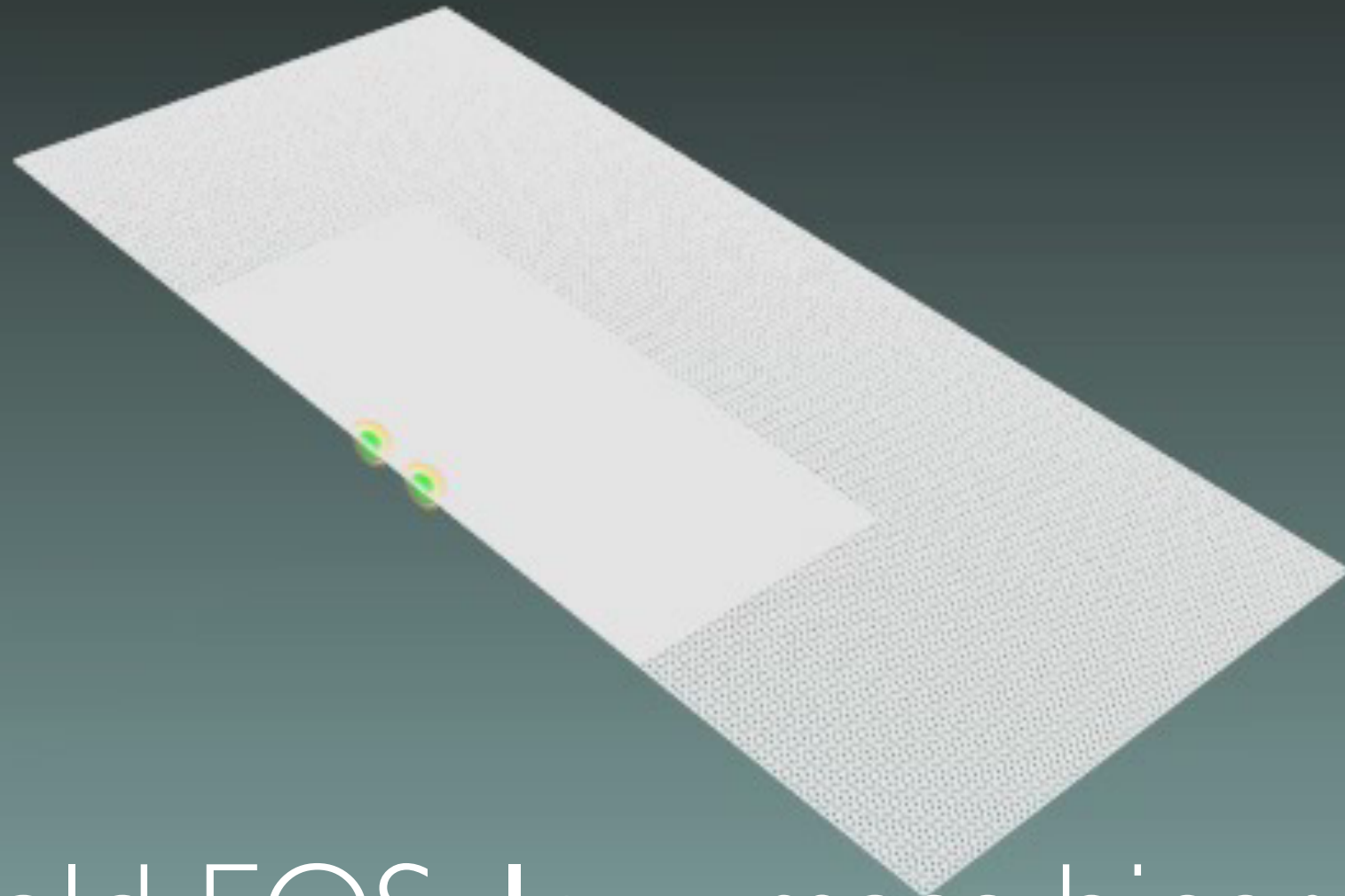
high-mass binary



T[ms] = 0.00



T[M] = 0.00



Cold EOS: low-mass binary

$$M = 1.4 M_{\odot}$$

0.0

6.1E+14

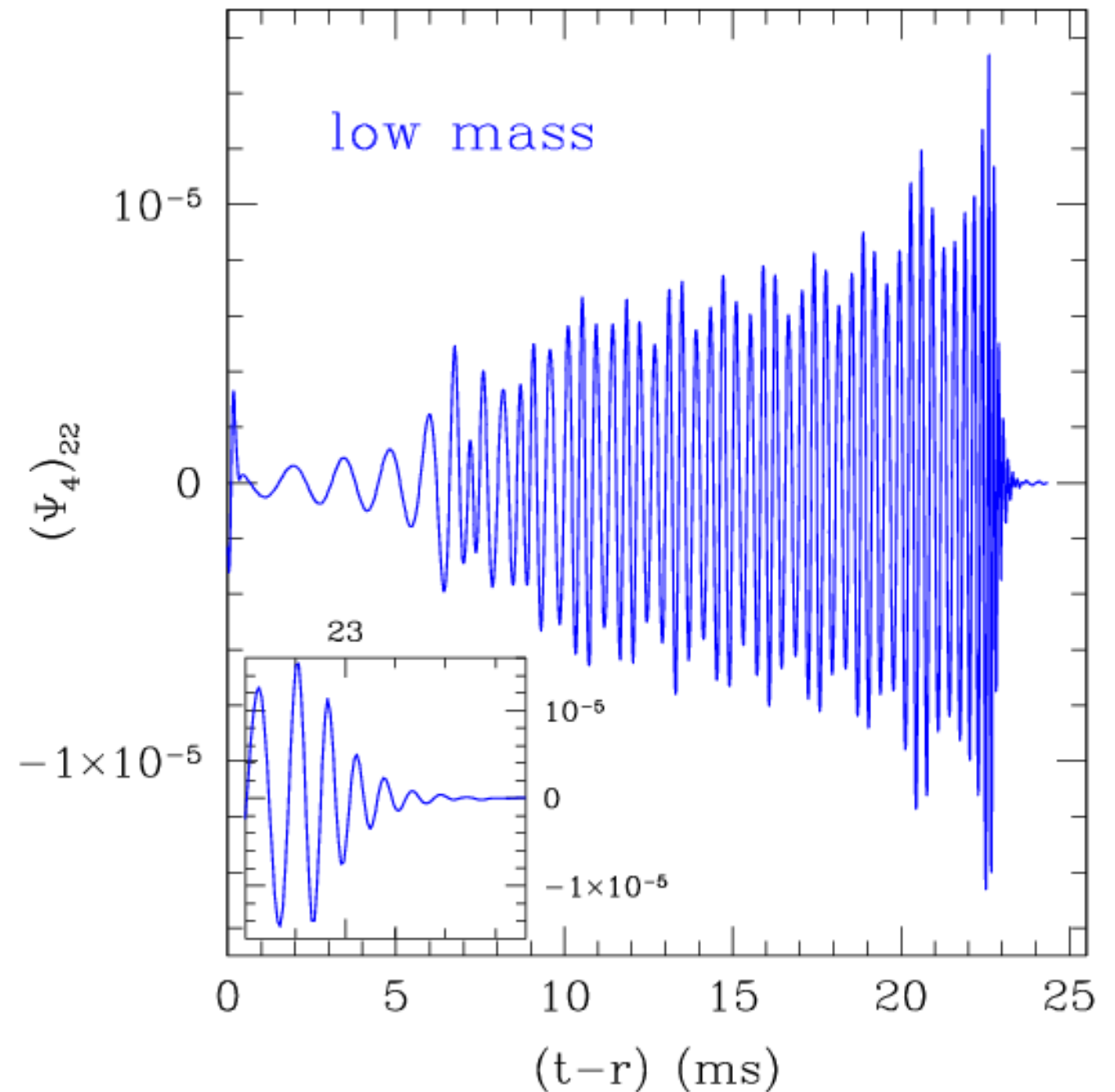
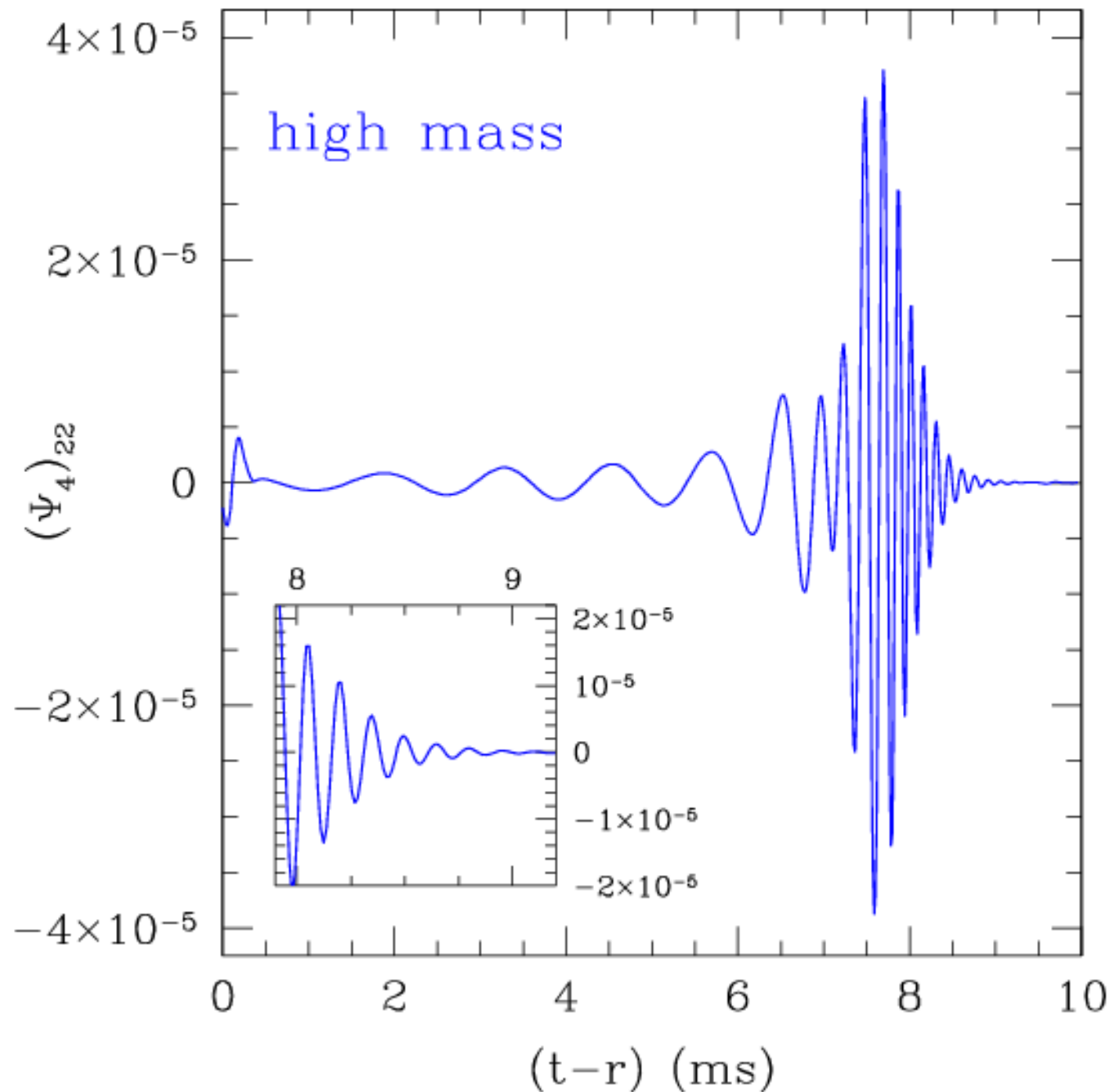


Density [g/cm³]

Waveforms: cold EOS

high-mass binary

low-mass binary



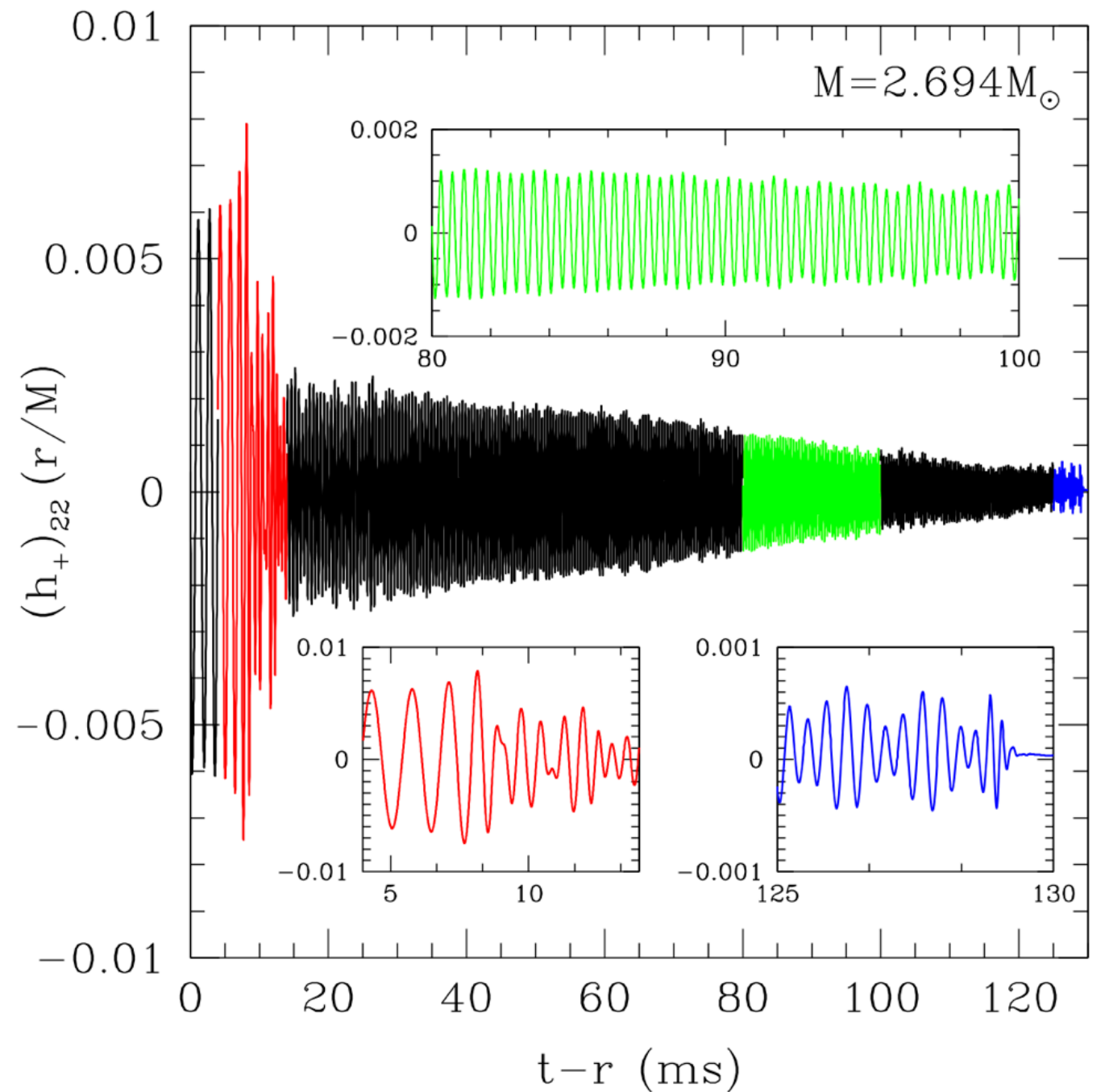
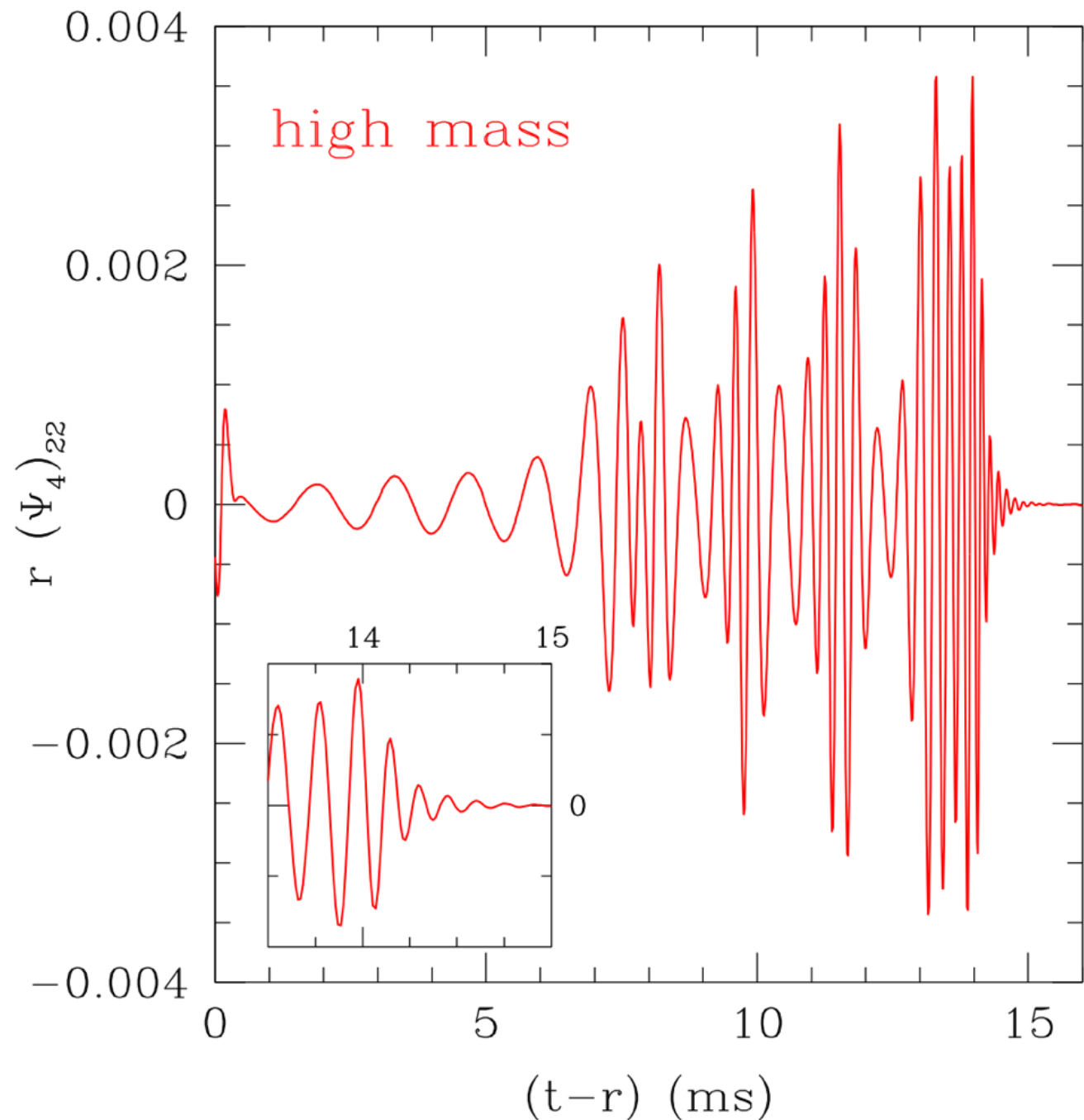
first time the full signal from the formation to a bh has been computed

development of a bar-deformed NS leads to a long gw signal

Waveforms: hot EOS

high-mass binary

low-mass binary

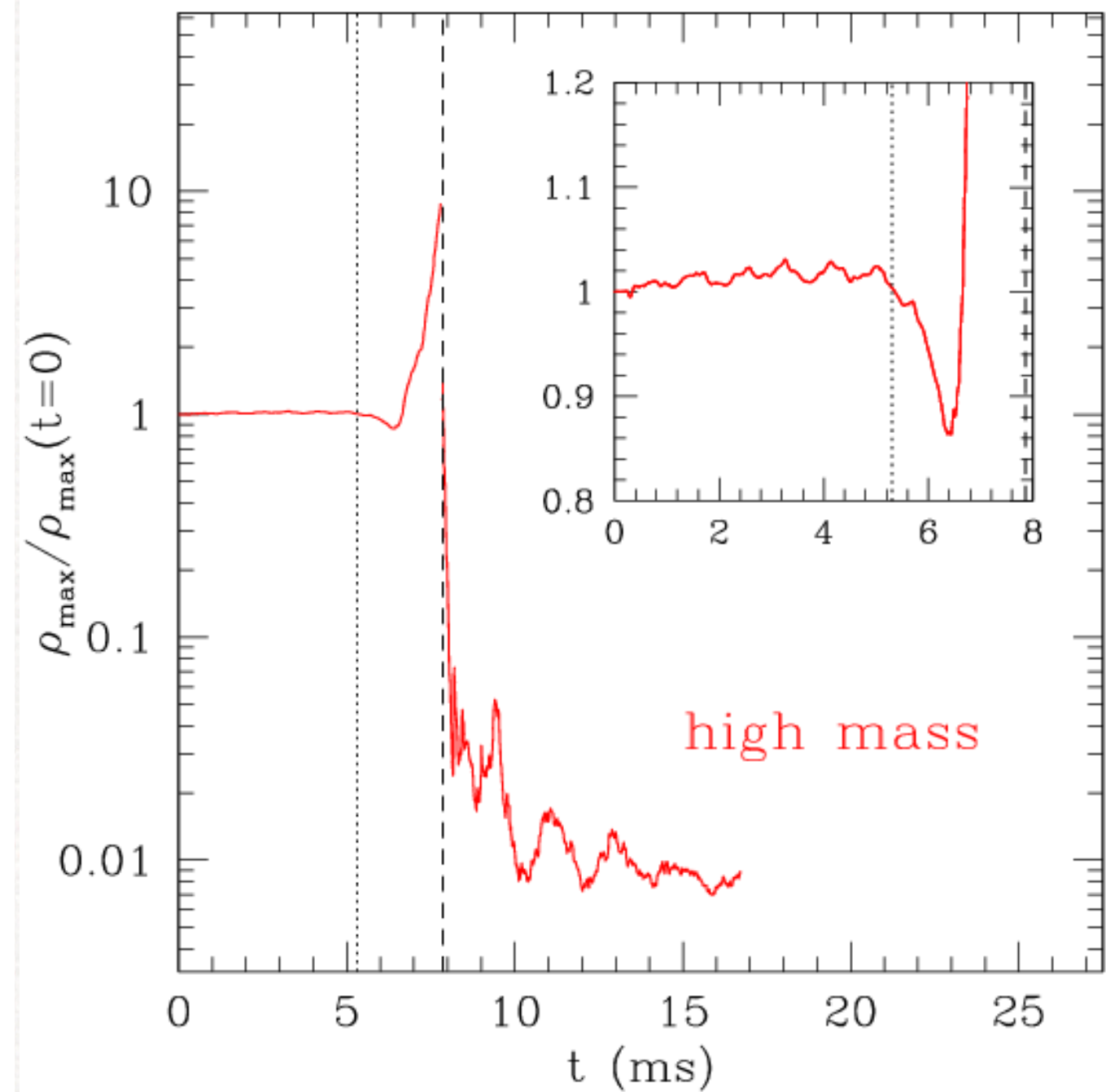


the high internal energy (temperature) of the HMNS prevents a prompt collapse

the HMNS evolves on longer (radiation-reaction) timescale

Matter dynamics

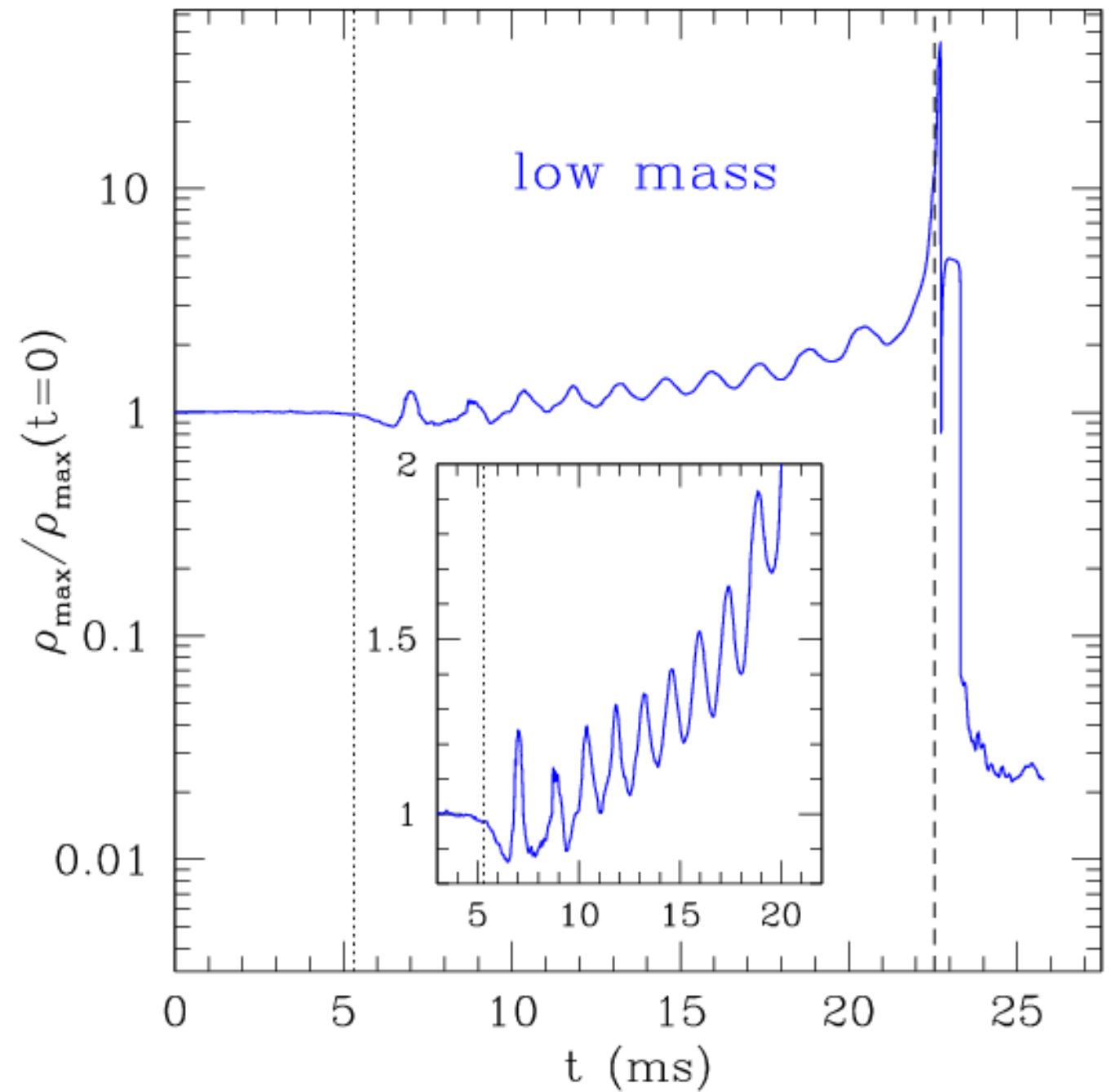
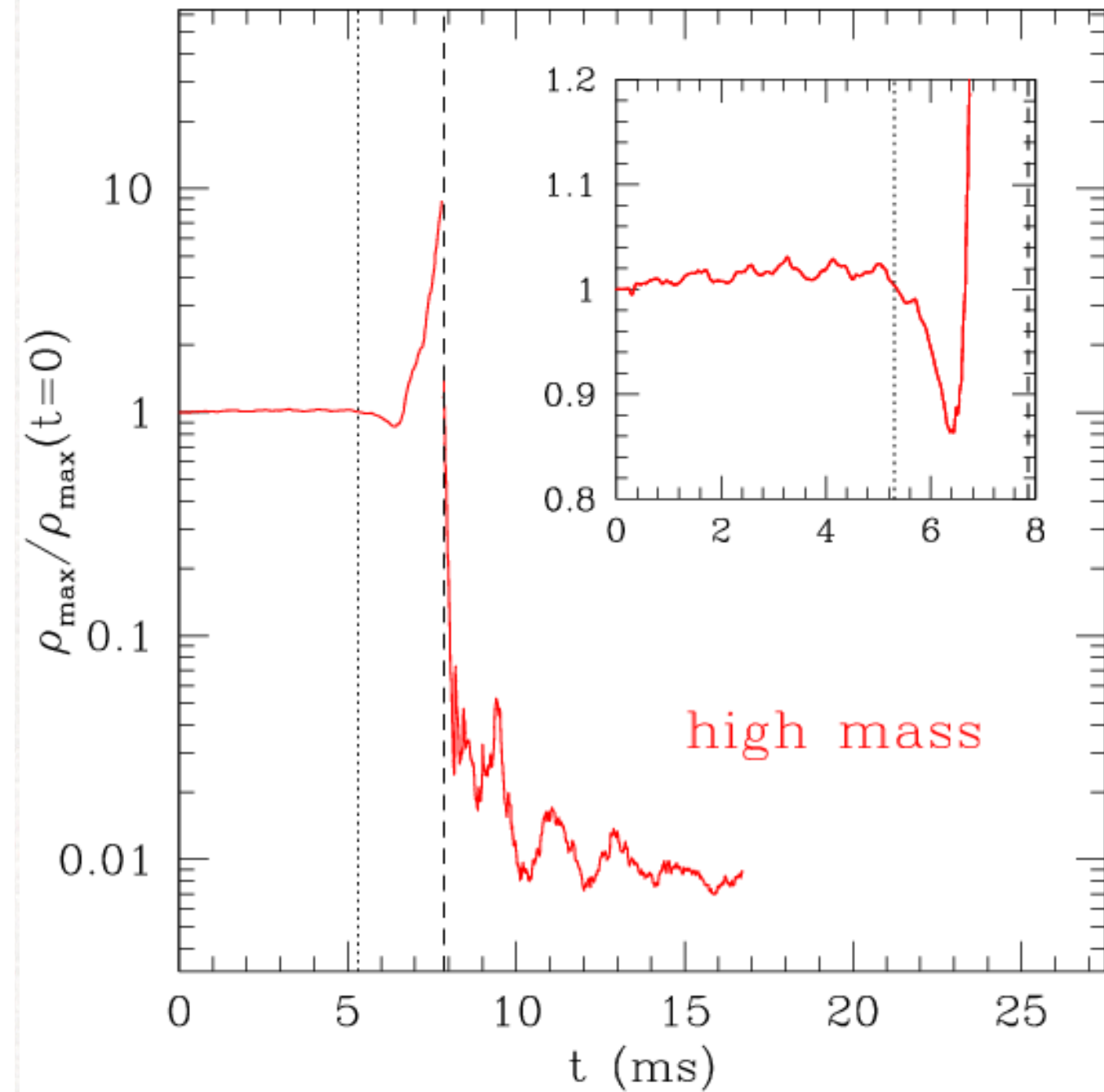
high-mass binary



Matter dynamics

high-mass binary

low-mass binary



Nonlinear hydrodynamics at work

Quite clearly, the two stars do not merge with a frontal (head-on) collision.

Rather, during the merger a shear interface forms across which the velocities are discontinuous.

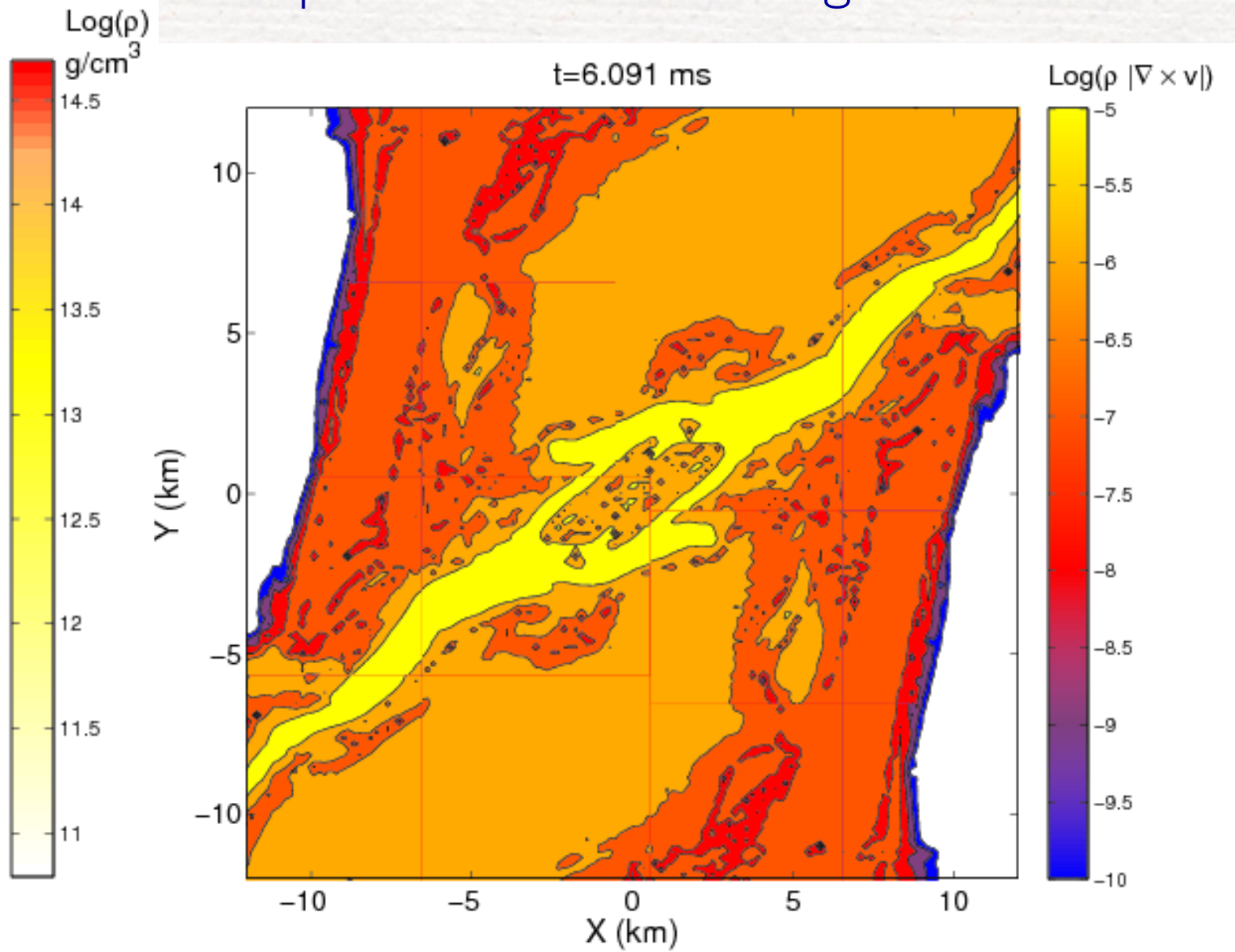
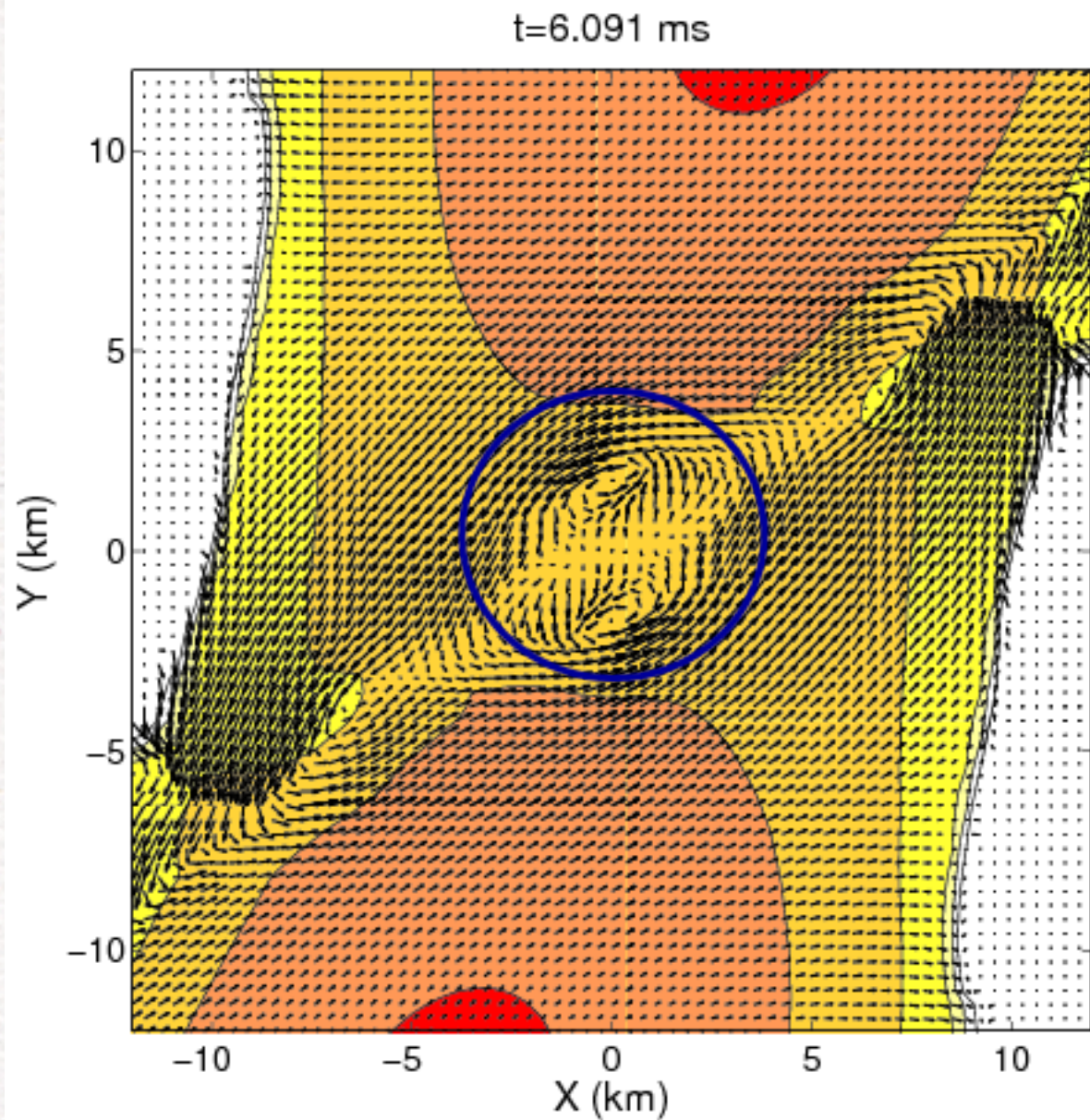
This leads to the formation of **vortices** and of a **Kelvin-Helmoltz** instability and a possible turbulent motion.

The instability can be quite important if the stars are magnetized

KH instability in the high-mass binary

Note the development of vortices in the shear boundary layer produced at the time of the merger

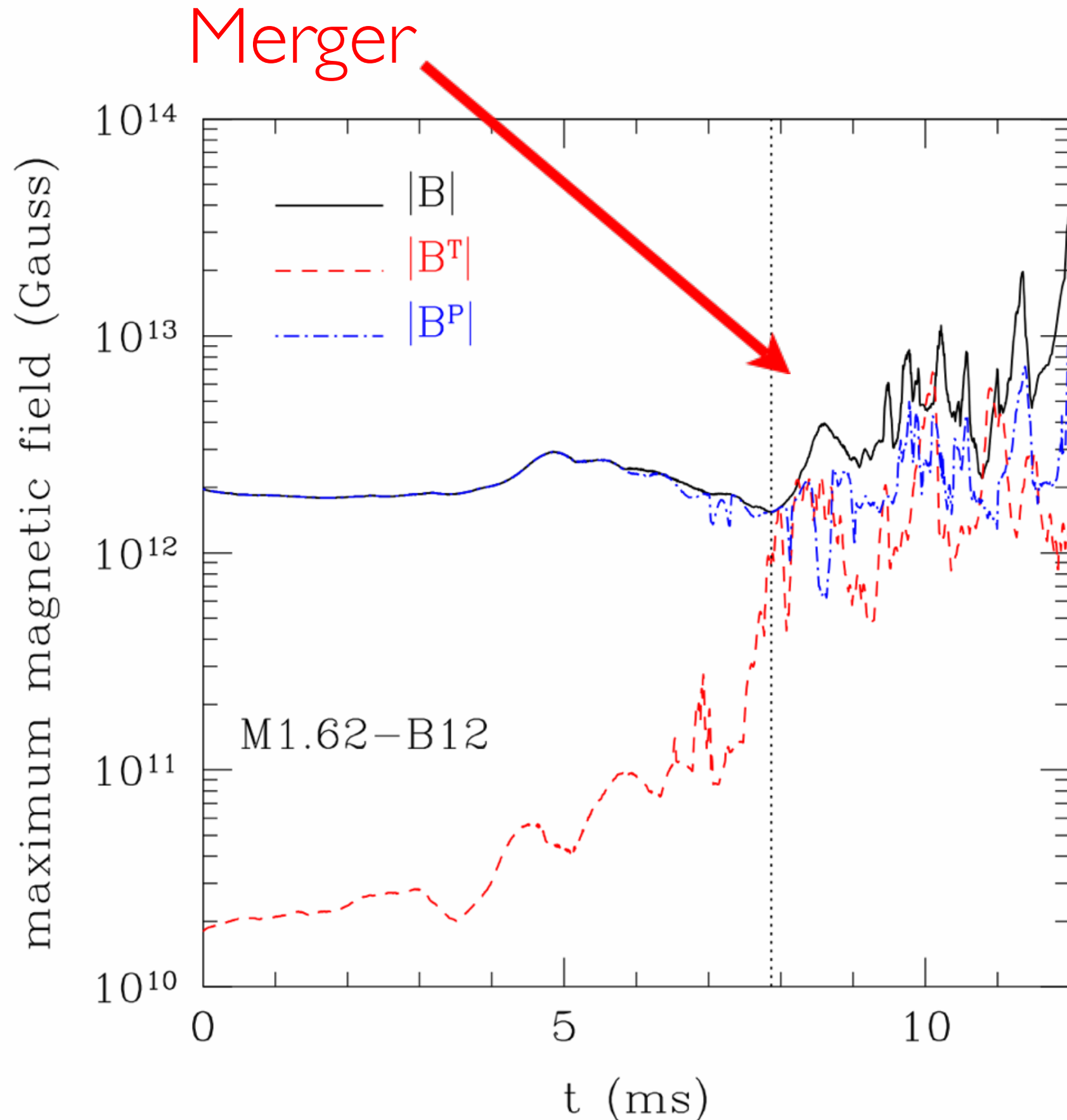
More evident in terms of the weighted vorticity In these regions one expects (and sees) large amplifications of the magnetic field.



(v^x, v^y) in "corotating" frame

$$\rho |\nabla \times v|^z$$

Magnetic field evolution



After merger the MF is amplified of one order of magnitude. The newly produced MF field is mostly **toroidal** and is clearly correlated with the increase in **vorticity**

First evidence in full GR that a MF field can be increased exponentially by the KH instability (Price & Rosswog, 2006)

Torus properties: unequal-masses

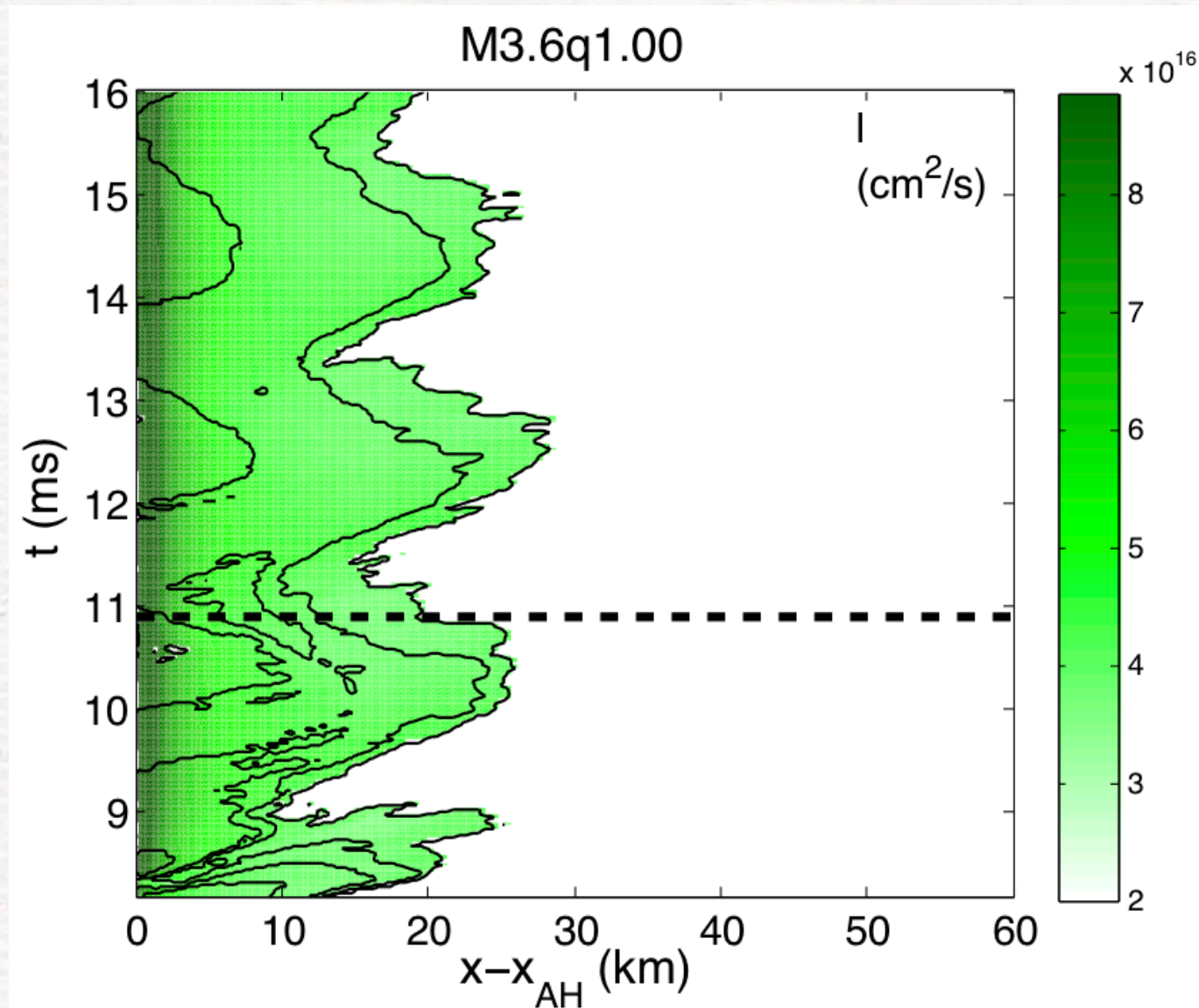
We have considered the inspiral and merger of 7 irrotational binaries with variable total mass and mass ratio (see table)

Model	M_{total} (M_{\odot})	q	J ($\text{g cm}^2/\text{s}$)	ν_{orbit} (Hz)	ρ_{max} (g/cm^3)	M_{torus} (M_{\odot})
M3.4q0.70	3.371	0.70	7.98×10^{49}	298.47	1.28×10^{15}	0.132
M3.4q0.80	3.375	0.80	8.36×10^{49}	303.62	9.21×10^{14}	0.120
M3.4q0.91	3.404	0.91	8.33×10^{49}	299.06	7.58×10^{14}	0.079
M3.5q0.75	3.464	0.75	8.40×10^{49}	300.84	1.27×10^{15}	0.097
M3.7q0.94	3.680	0.94	9.37×10^{49}	306.56	9.75×10^{14}	0.006
M3.6q1.00	3.558	1	8.92×10^{49}	303.32	7.58×10^{14}	0.001
M3.8q1.00	3.802	1	9.85×10^{49}	309.70	9.74×10^{14}	0.001

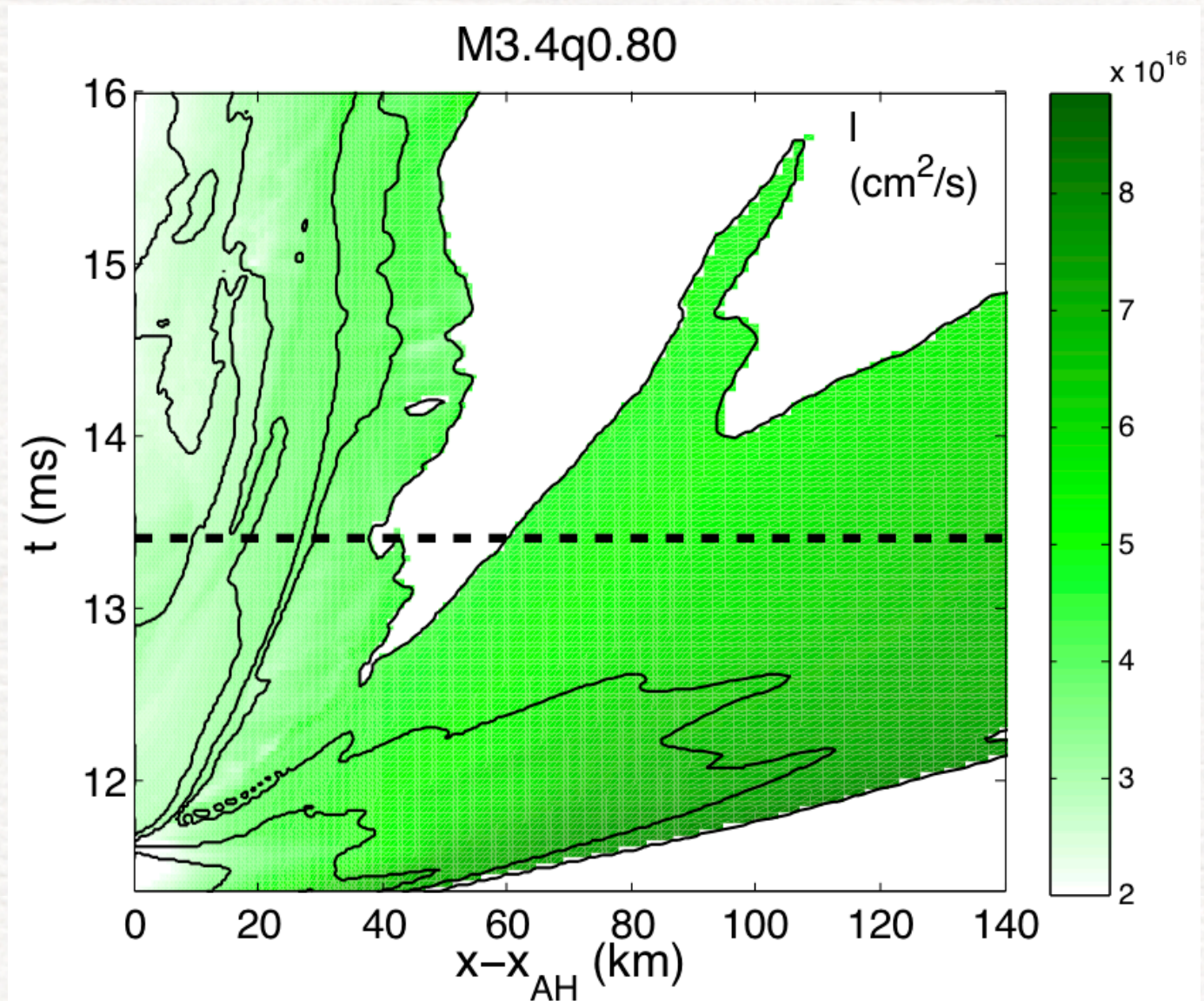
A lot to say about the torus properties but a movie summarizes most of them

Torus properties: specific ang. momentum

spacetime diagram of specific angular mom. $\ell \equiv -u_\phi/u_t$



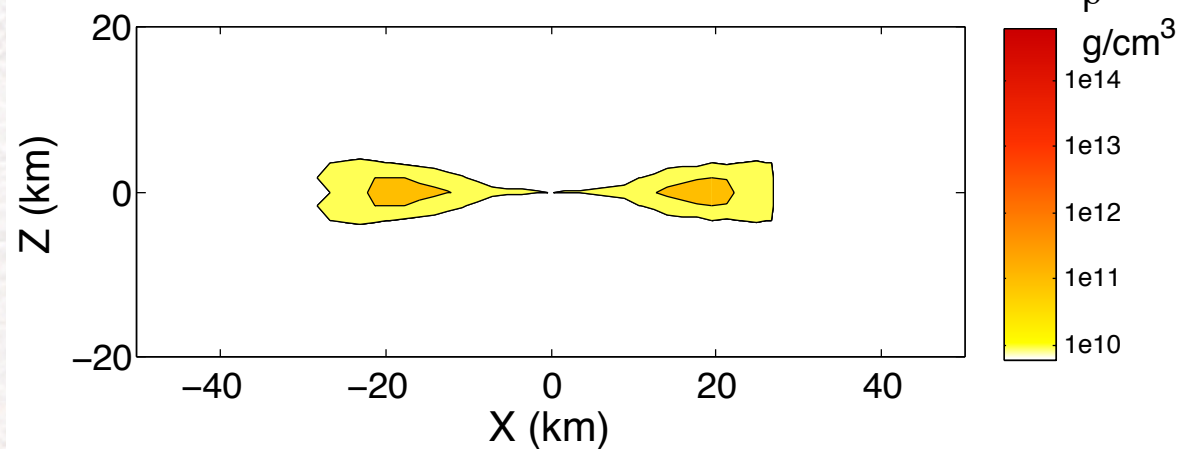
equal mass binary: specific angular momentum is larger at the inner edge and **decreases** outwards



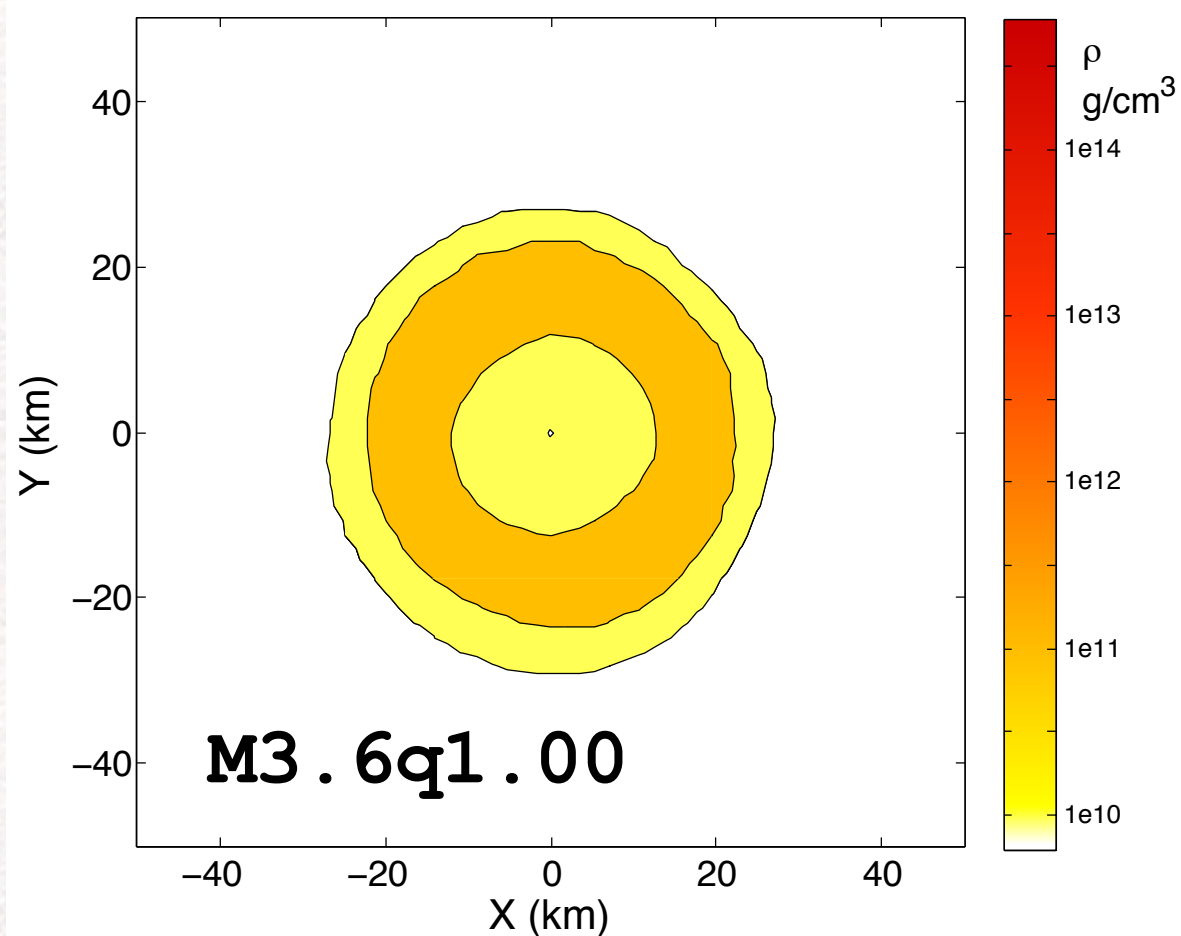
unequal mass binary: specific angular momentum is smaller at inner edge and **increases** outwards

Torus properties: size

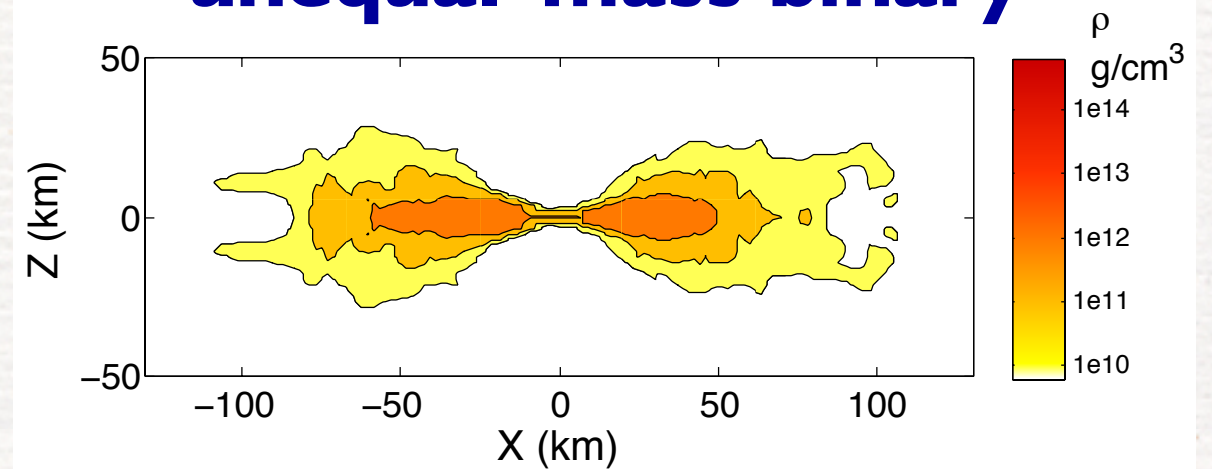
equal-mass binary



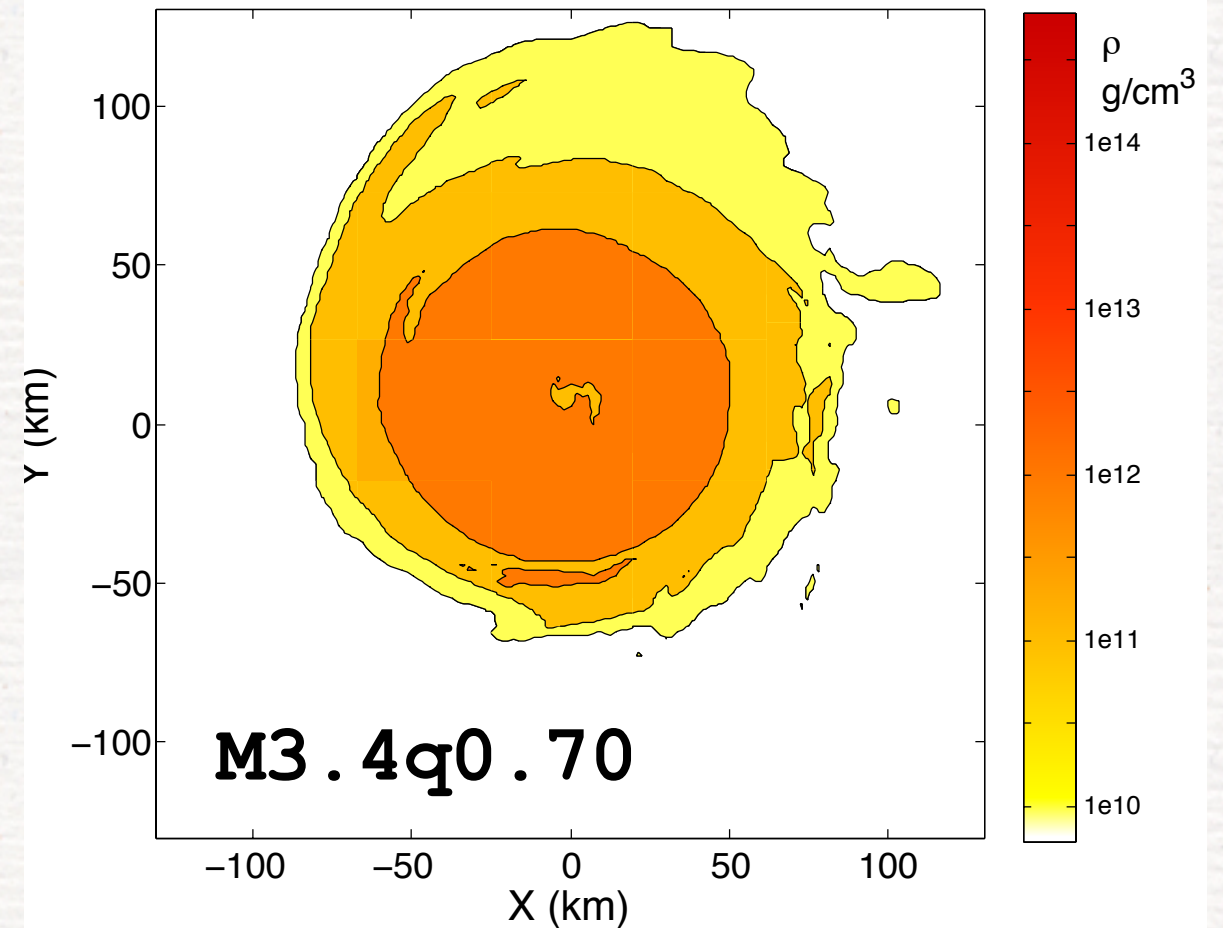
t=18.072 ms



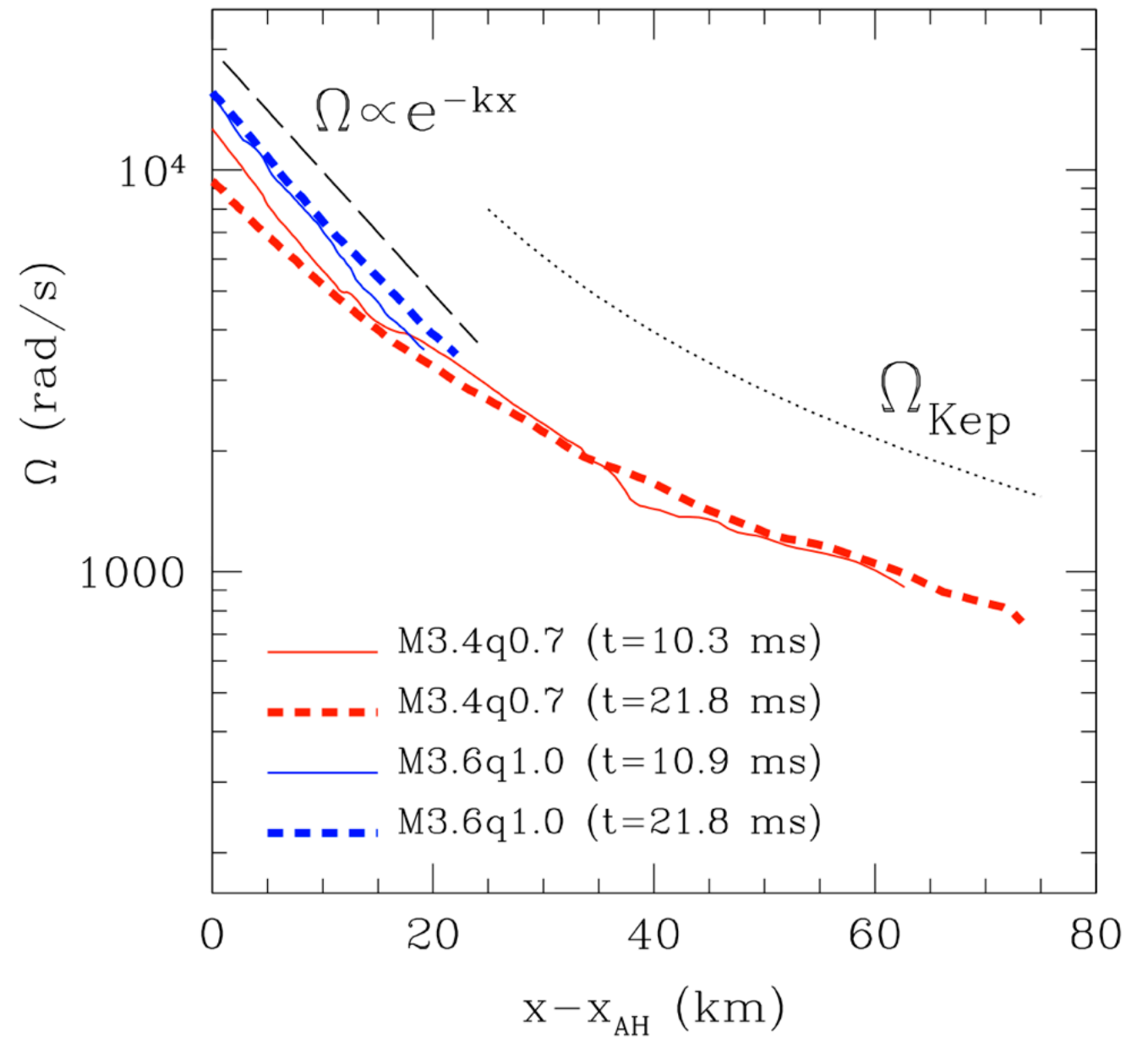
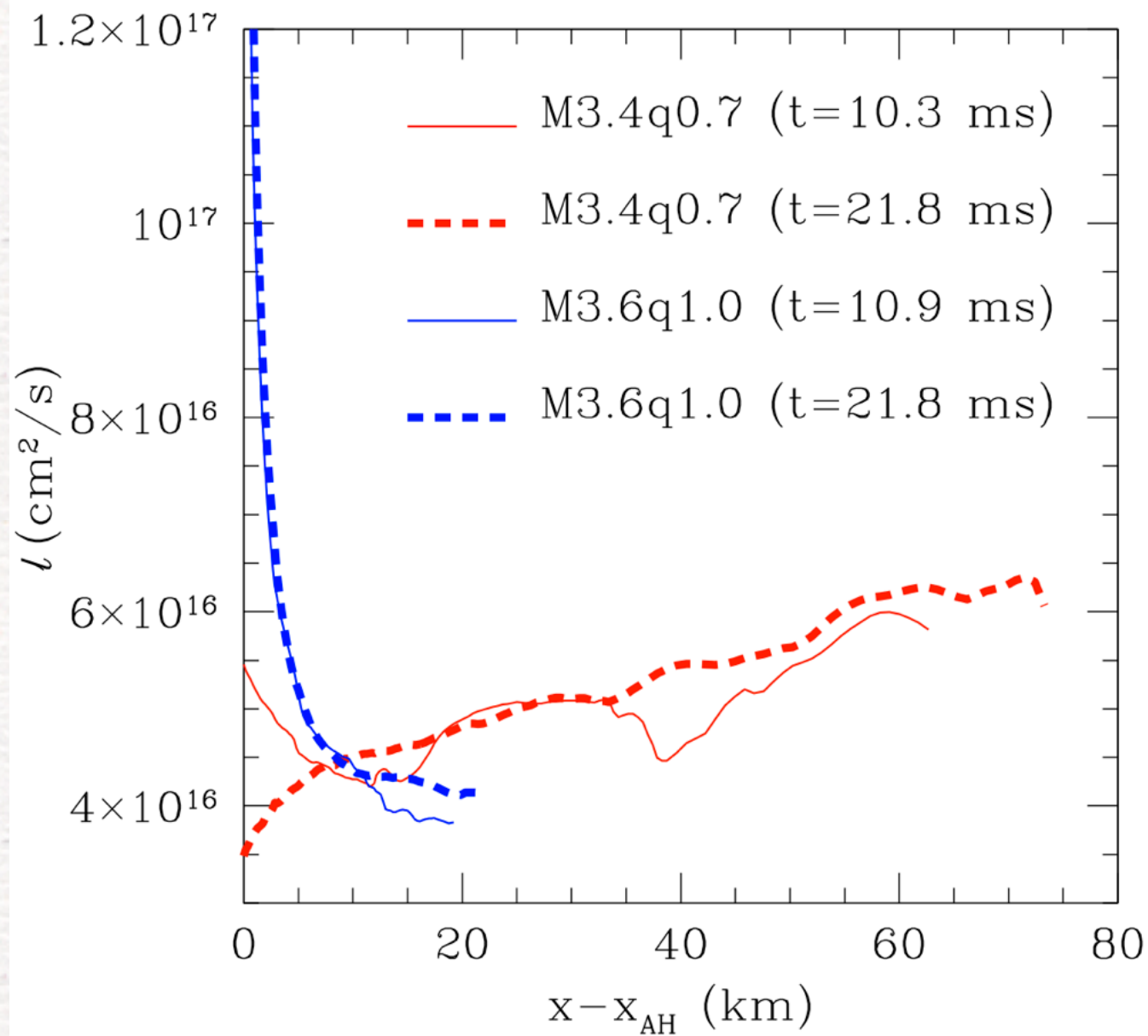
unequal-mass binary



t=18.072 ms

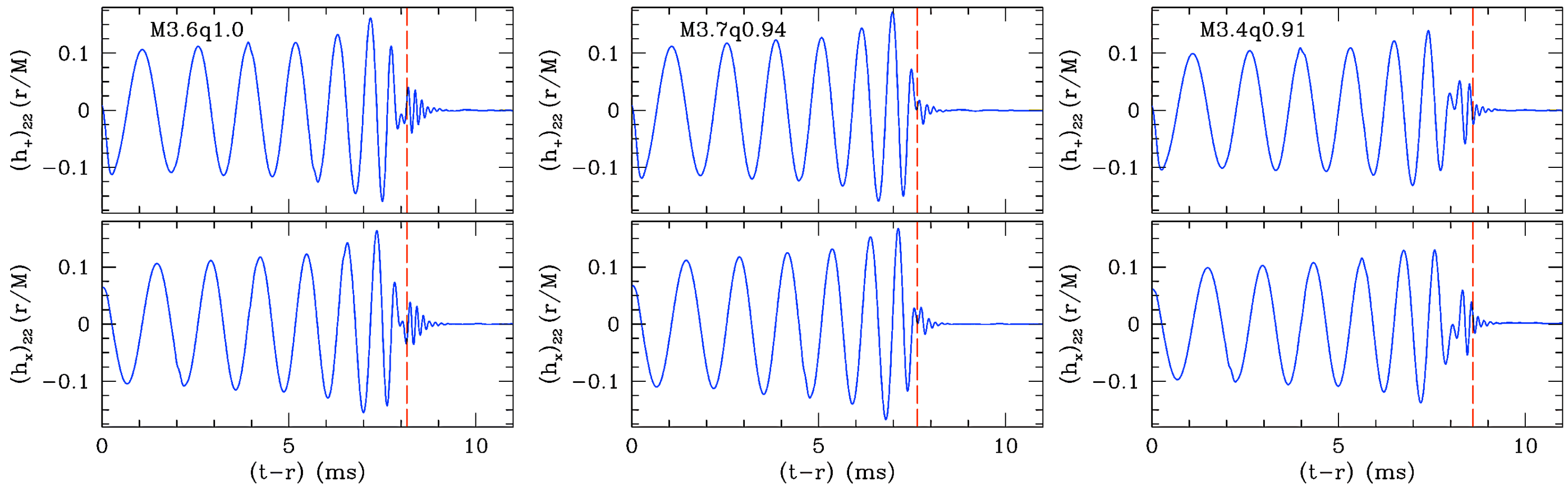


Note that although the total mass is very similar, the unequal-mass binary yields a torus which is about ~ 4 times larger and ~ 200 times more massive



- specific angular momentum has very different behaviour in the two cases: $d\ell/dx \geq 0$ for stability
- equal-mass binary has exponential differential rotation while the unequal-mass is essentially Keplerian

Gravitational waveforms



Note the waveforms are very simple with moderate modulation induced by mass asymmetry.

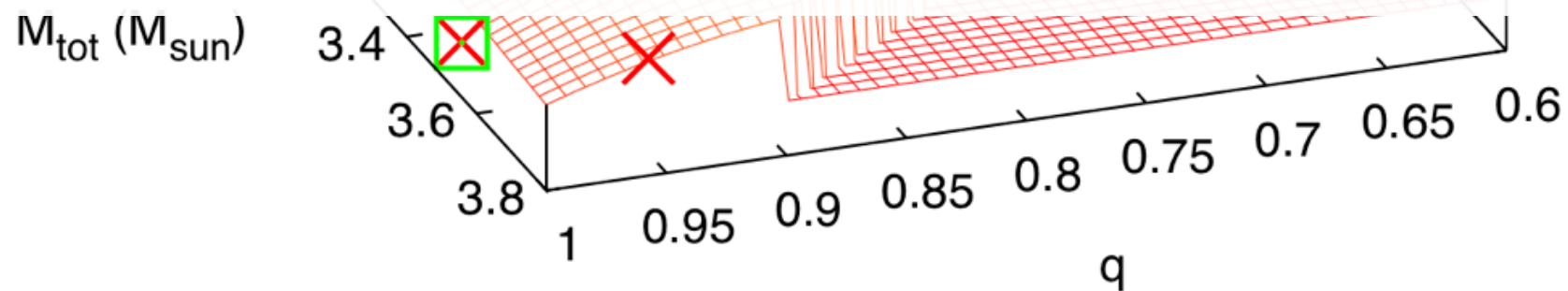
Furthermore, no HMNS is produced and the **QNM** ringing (shown by dashed vertical line) is **choked** by the intense mass accretion rate (the BH cannot ringdown...)

Torus properties: unequal-masses

$M_T (M_{\text{sun}})$

It's much harder to produce tori of such large masses with realistic BH-NS binaries.

Prospects for modelling GRBs from BNSs are promising



Model	$M_{\text{total}} (M_{\odot})$	q	$M_{\text{torus}} (M_{\odot})$
M3.6q1.00	3.558	1	0.0010
M3.7q0.94	3.680	0.94	0.0100
M3.4q0.91	3.404	0.91	0.0994
M3.4q0.80	3.375	0.80	0.2088
M3.5q0.75	3.464	0.75	0.0802
M3.4q0.70	3.371	0.70	0.2116

The torus mass **decreases** with the mass ratio and with the total mass; at lowest order:

$$\widetilde{M}_{\text{tor}}(q, M_{\text{tot}}) = (M_{\text{max}} - M_{\text{tot}}) [c_1(1 - q) + c_2]$$

where M_{max} is the maximum (baryonic) mass of the binary and c_1, c_2 are coefficients computed from the simulations.

Extending the work to MHD

The magnetic field is “added” by using the vector potential:

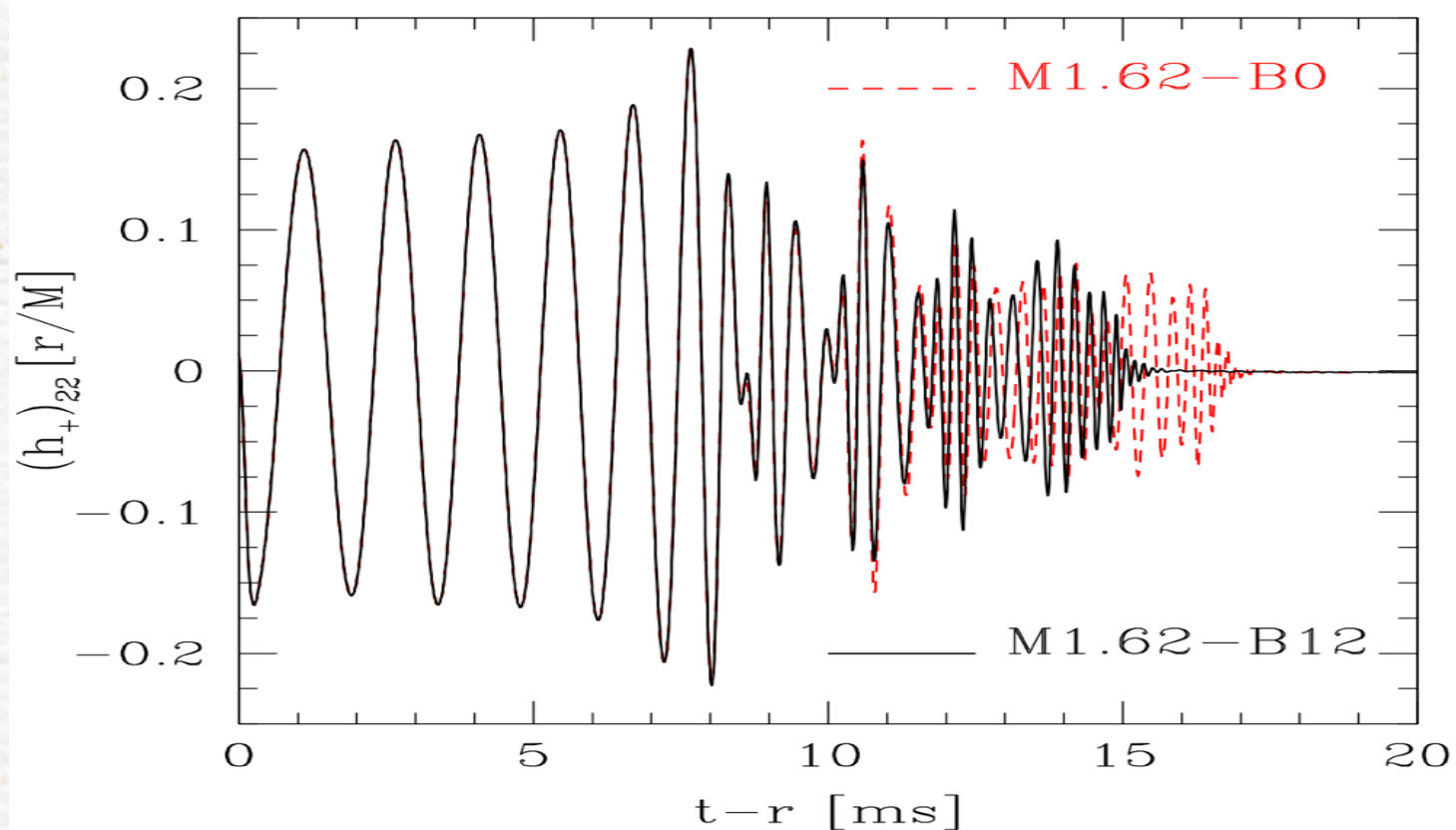
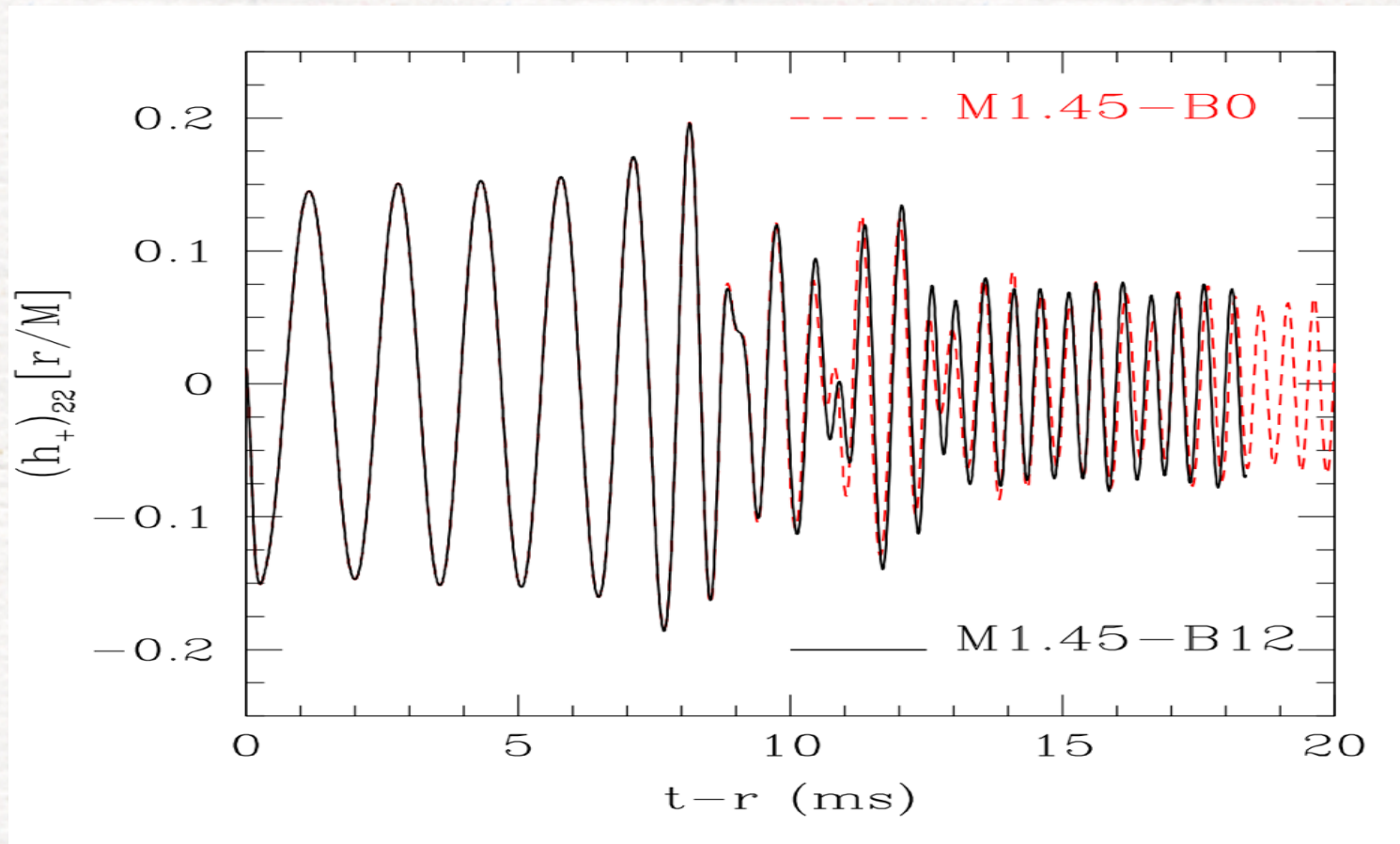
$$A_{\phi} = A_b r^2 [\max(P - P_{cut}, 0)]^n$$

where A_b and $P_{cut} = 0.04 \times \max(P)$ are two constants defining respectively the strength and the extension of the magnetic field inside the star. $n=2$ defines the profile of the initial magnetic field.

The magnetic fields are initially contained inside the stars: ie no magnetospheric effects. Overall we have considered 8 binaries (low/high mass) with MFs:

$$B=0, 10^{12}, 10^{14}, 10^{17} \text{ G}$$

Waveforms: comparing against magnetic fields



Compare B/no-B field:

- the evolution in the **inspiral** is different but only for ultra large B-fields (i.e. $B \sim 10^{17}$ G). For realistic fields the difference is not significant.
- the **post-merger** evolution is different for all masses; strong B-fields delay the collapse to BH

However, **mismatch** is too small for present detectors: influence of B-fields on the inspiral is **cannot be detected!**

Understanding the dependence on MF

To quantify the differences and determine whether detectors will see a difference in the inspiral, we calculate the **overlap**

$$\mathcal{O}[h_{B1}, h_{B2}] \equiv \frac{\langle h_{B1} | h_{B2} \rangle}{\sqrt{\langle h_{B1} | h_{B1} \rangle \langle h_{B2} | h_{B2} \rangle}}$$

where the scalar product

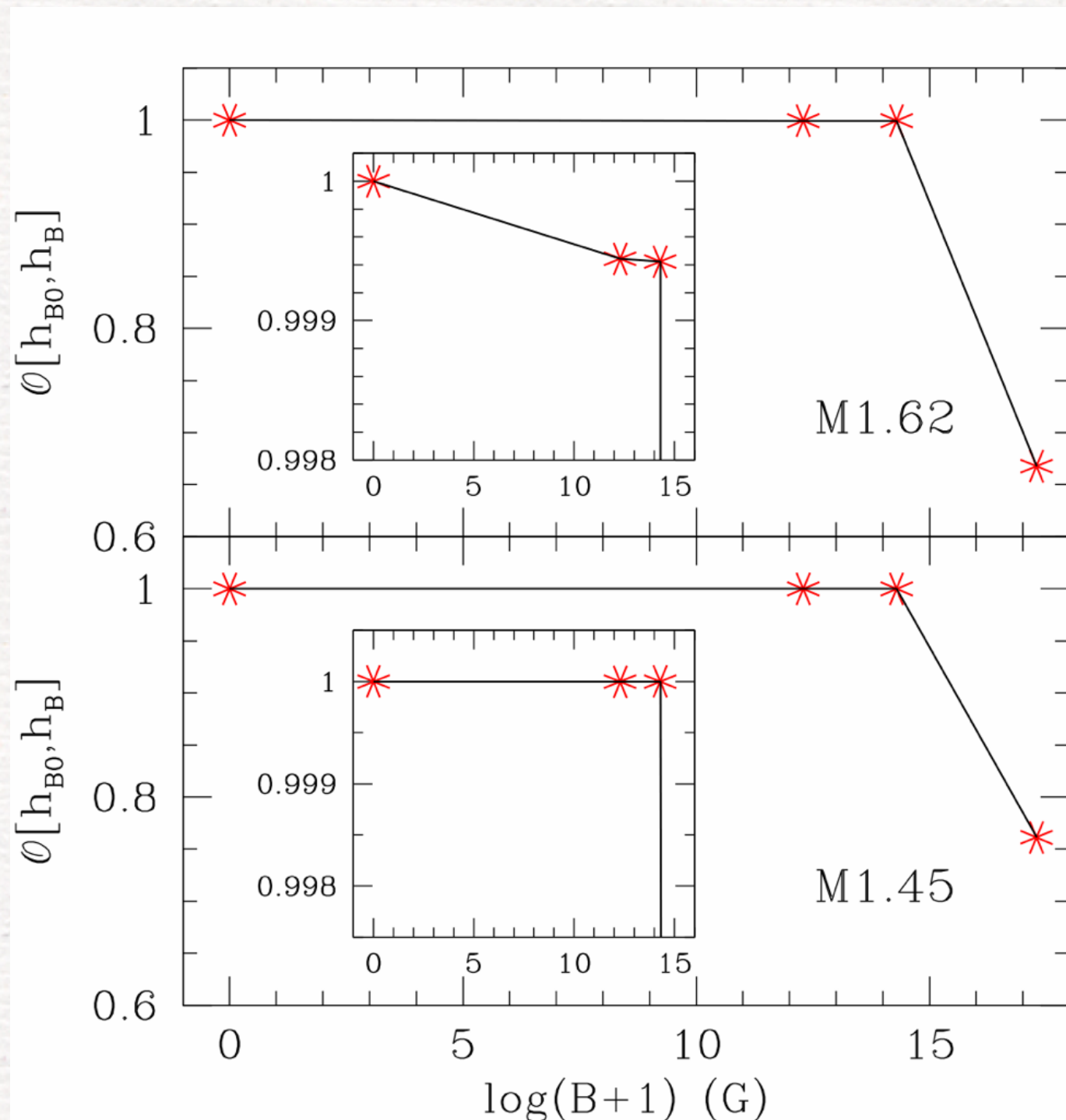
$$\langle h_{B1} | h_{B2} \rangle \equiv 4\Re \int_0^\infty df \frac{\tilde{h}_{B1}(f) \tilde{h}_{B2}^*(f)}{S_h(f)}$$

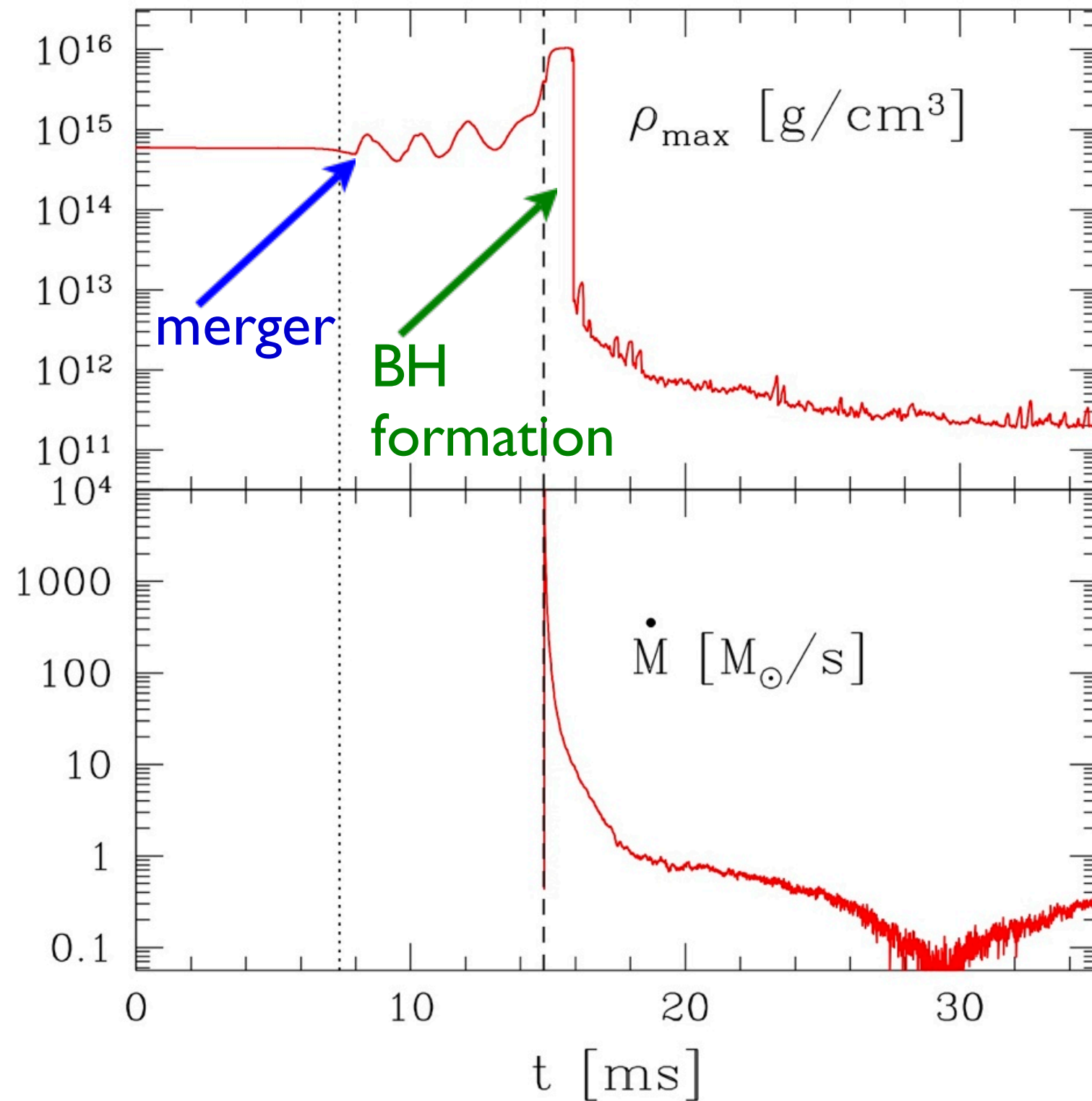
In essence, at these res:

$$\mathcal{O}[h_{B0}, h_B] \gtrsim 0.999$$

$$\text{for } B \lesssim 10^{17} \text{ G}$$

Because the match is even higher for lower masses, **the influence of MFs on the inspiral is unlikely to be detected!**





* Rest-mass density in the torus is still very high (only 3 orders of magnitude smaller). Ideal conditions to produce and diffuse neutrinos

* The BH spin and the torus mass are respectively:

$$J/M^2 = 0.83$$

$$M_{\text{tor}} = 0.063 M_{\odot}$$

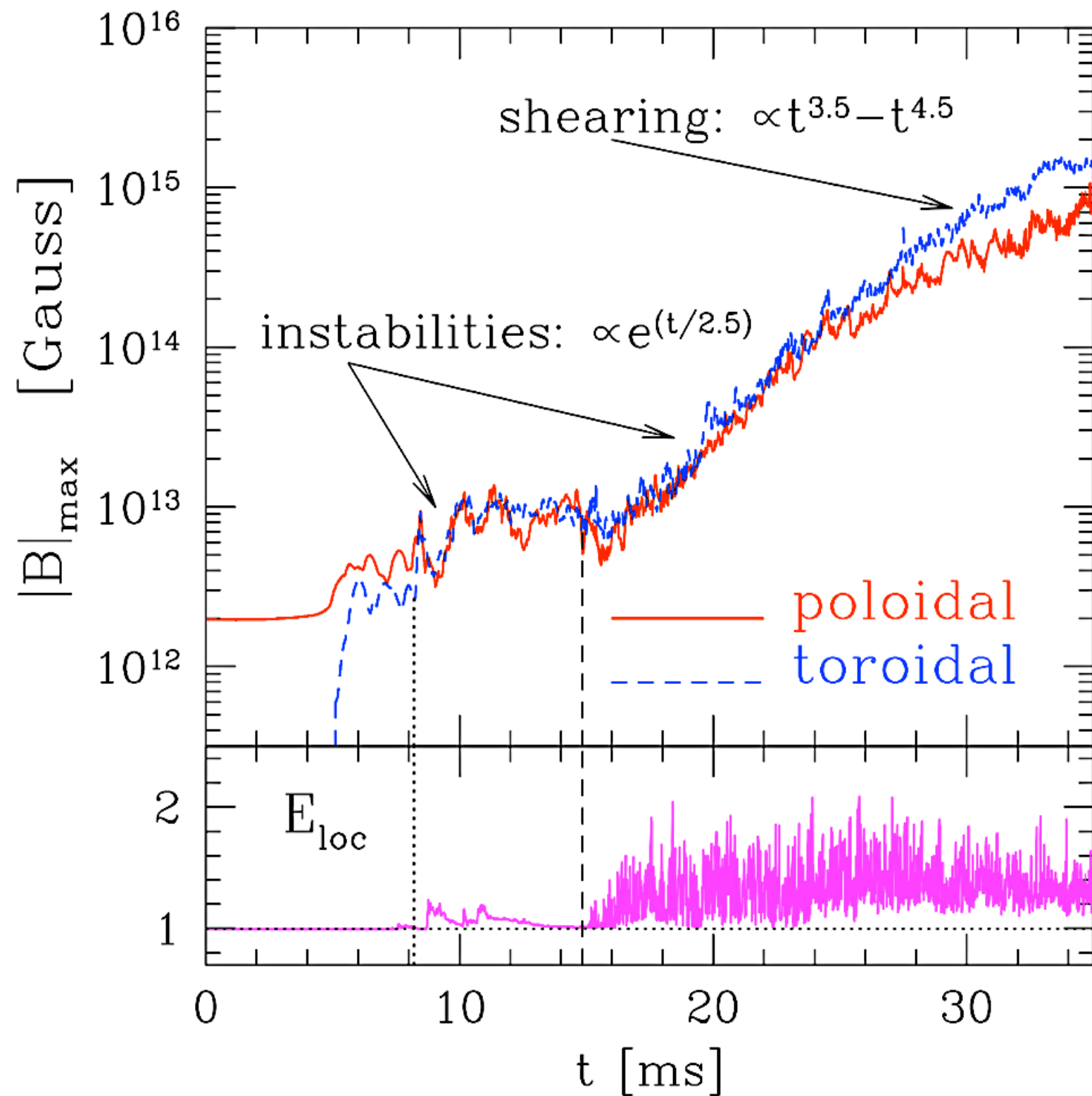
* After BH formation the mass accretion rate reaches quasi-stationary state:

$$\dot{M} \simeq 0.2 M_{\odot} \text{ s}^{-1}$$

* Assuming stationarity, the torus will be totally accreted over a timescale:

$$t_{\text{accr}} \simeq M_{\text{tor}} / \dot{M} \simeq 0.3 \text{ s}$$

matching very well the typical duration of SGRBs.



*B-field grows exponentially first because of the magnetorotational instability:

$$\tau_{\text{MRI}} = 2 \left(\frac{\partial \Omega}{\partial \varpi} \right)^{-1} \simeq 1 \Omega_3^{-1} \text{ ms}$$

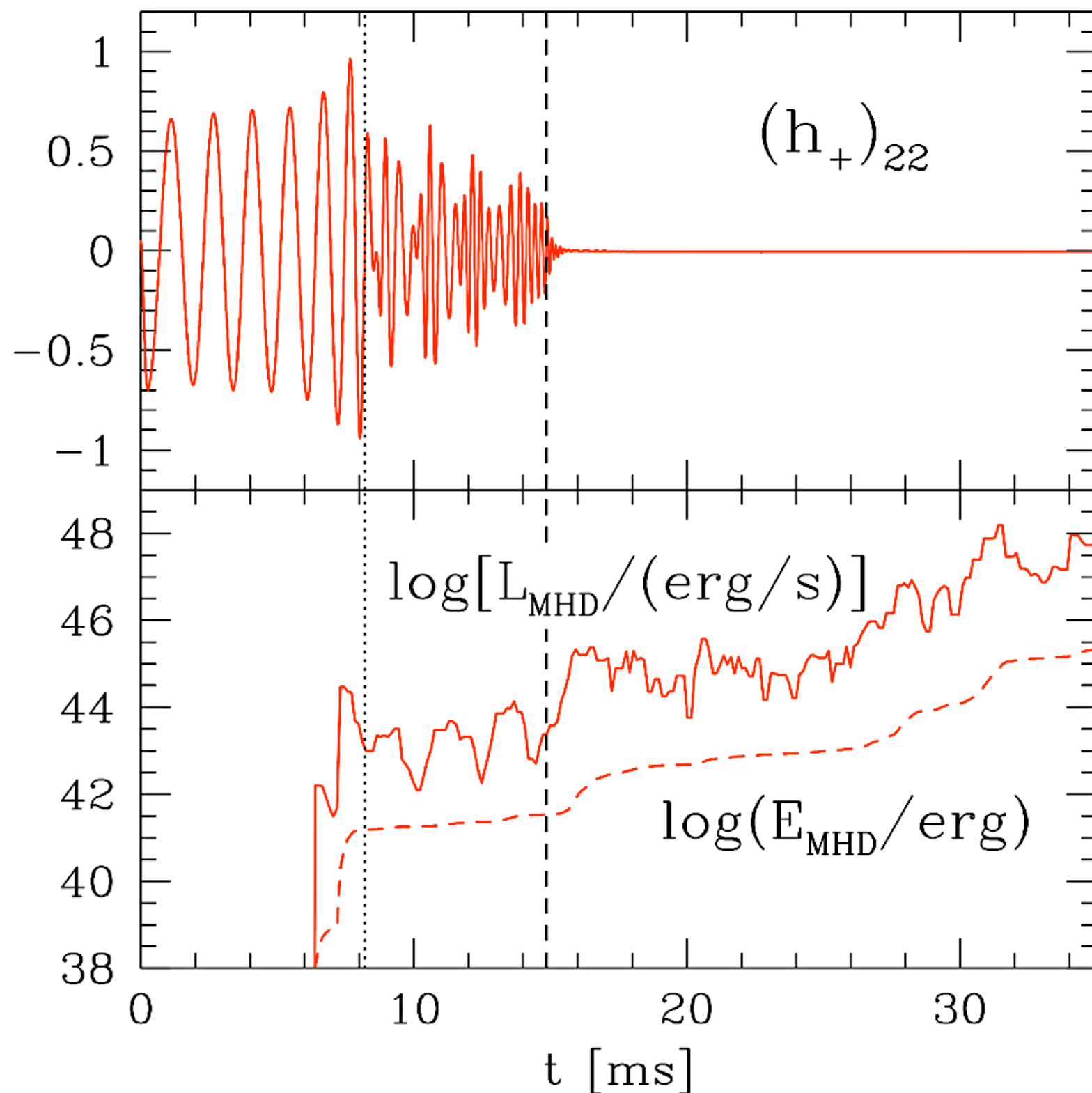
$$\lambda_{\text{max}} \simeq 2\pi v_A / \Omega \simeq 10^4 \Omega_3^{-1} B_{15} \text{ cm}$$

*Later on the growth is only a power law as the B-field reaches equipartition

*B-field is mostly toroidal in the torus and $\sim 10^{15}$ G. A poloidal component dominant along the BH spin axis.

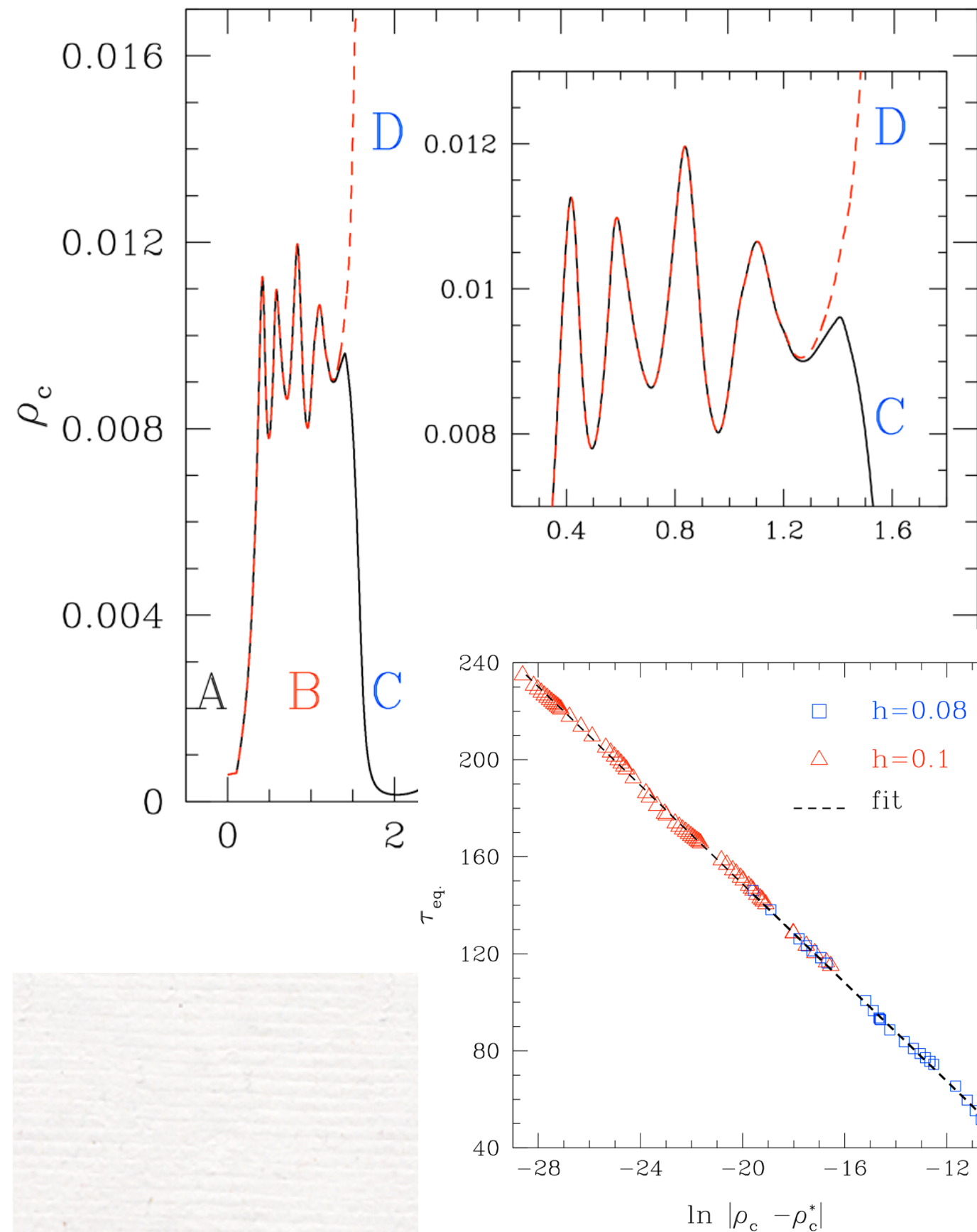
*Note that material becomes unbound soon after the BH is formed indicating that an outflow can be produced; mildly relativistic: $\gamma \lesssim 4$

Multimessenger signal



- *Note that the **GW signal** is essentially **shuts-off** after BH formation.
- *After the merger the **EM signal starts** but is essentially constant during the HMNS phase
- *After the BH formation, the **EM signal** starts to **grow exponentially**
- *At the end of the simulation the system has released a total **EM energy** of $\sim 10^{46}$ **erg** and reached an **EM luminosity** of $\sim 10^{48}$ **erg/s**
- *Despite the crudeness of the physics, the ball-park numbers match observations.

Different stages of the dynamics



The central density is a good marker for the evolution:

- (A) initial data;
- (B) metastable object
- (C) “star” in equilibrium
- (D) black hole.

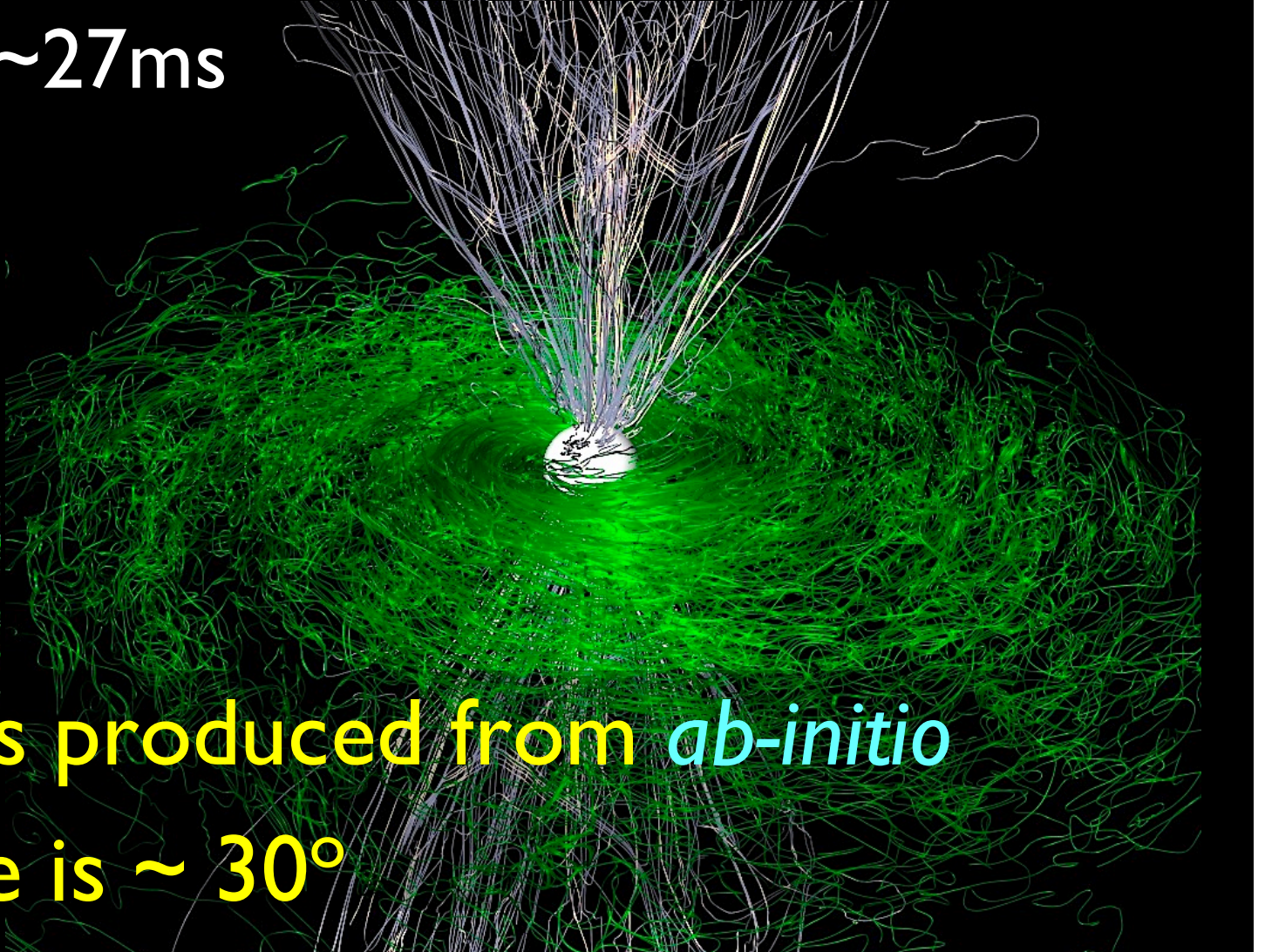
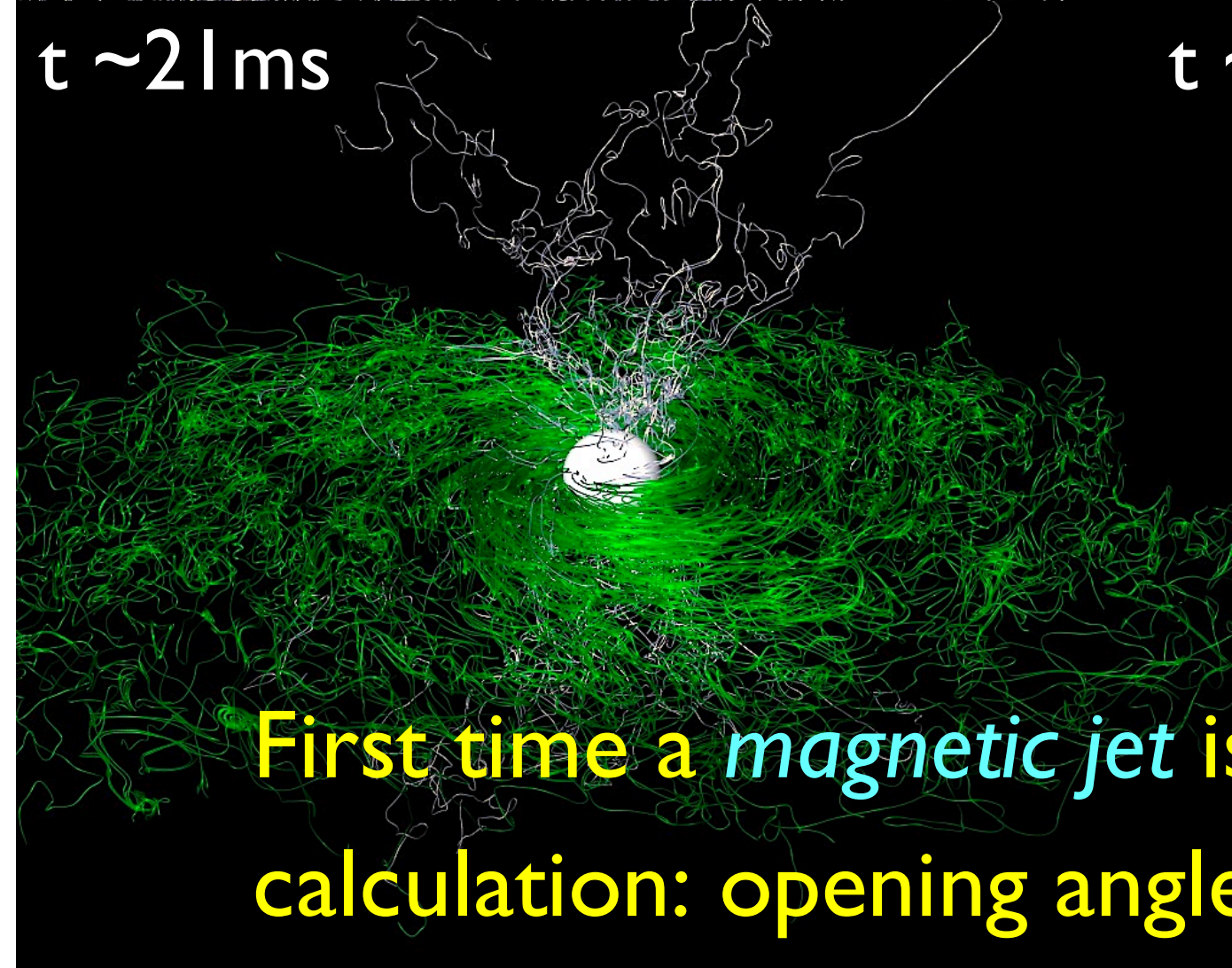
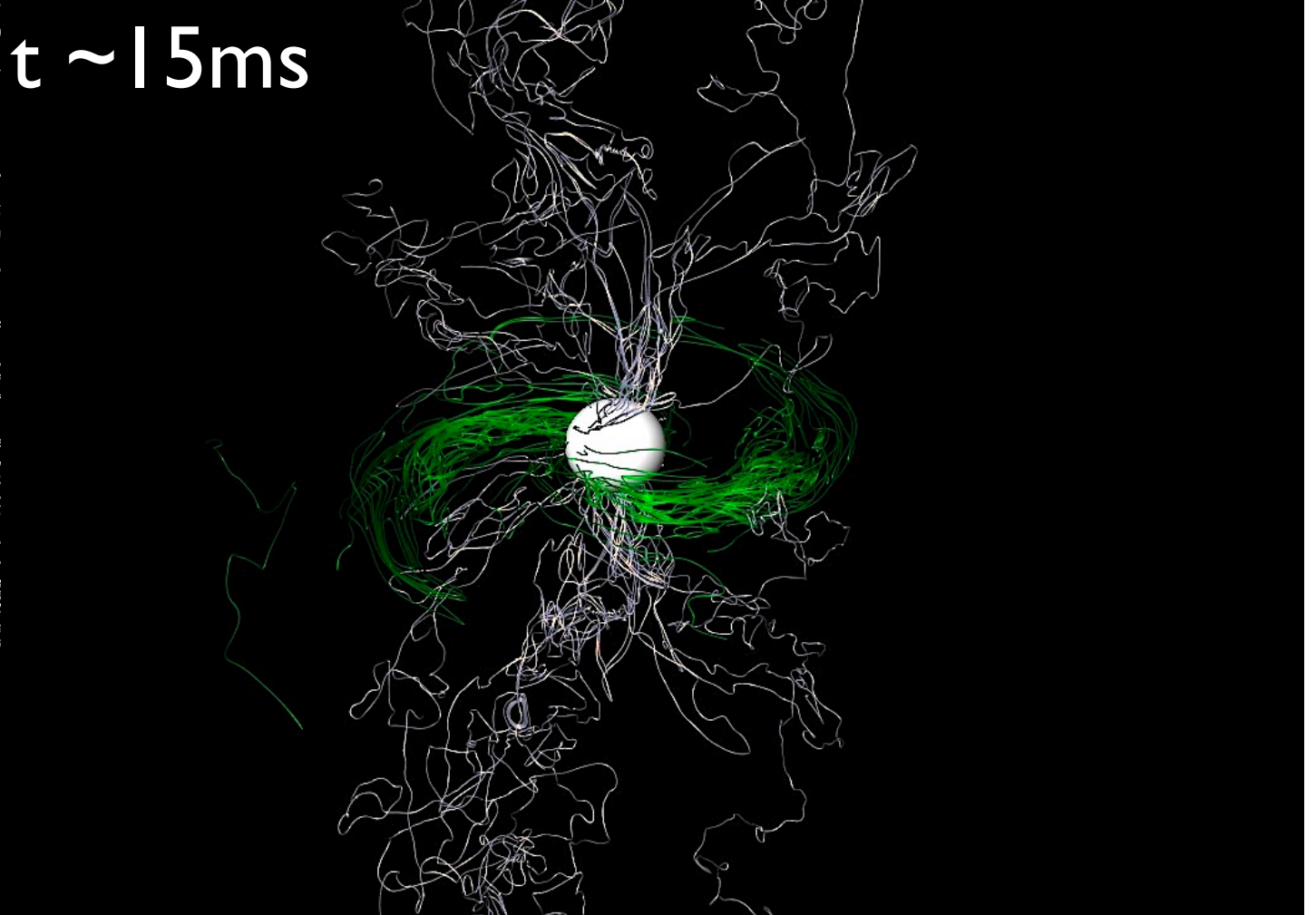
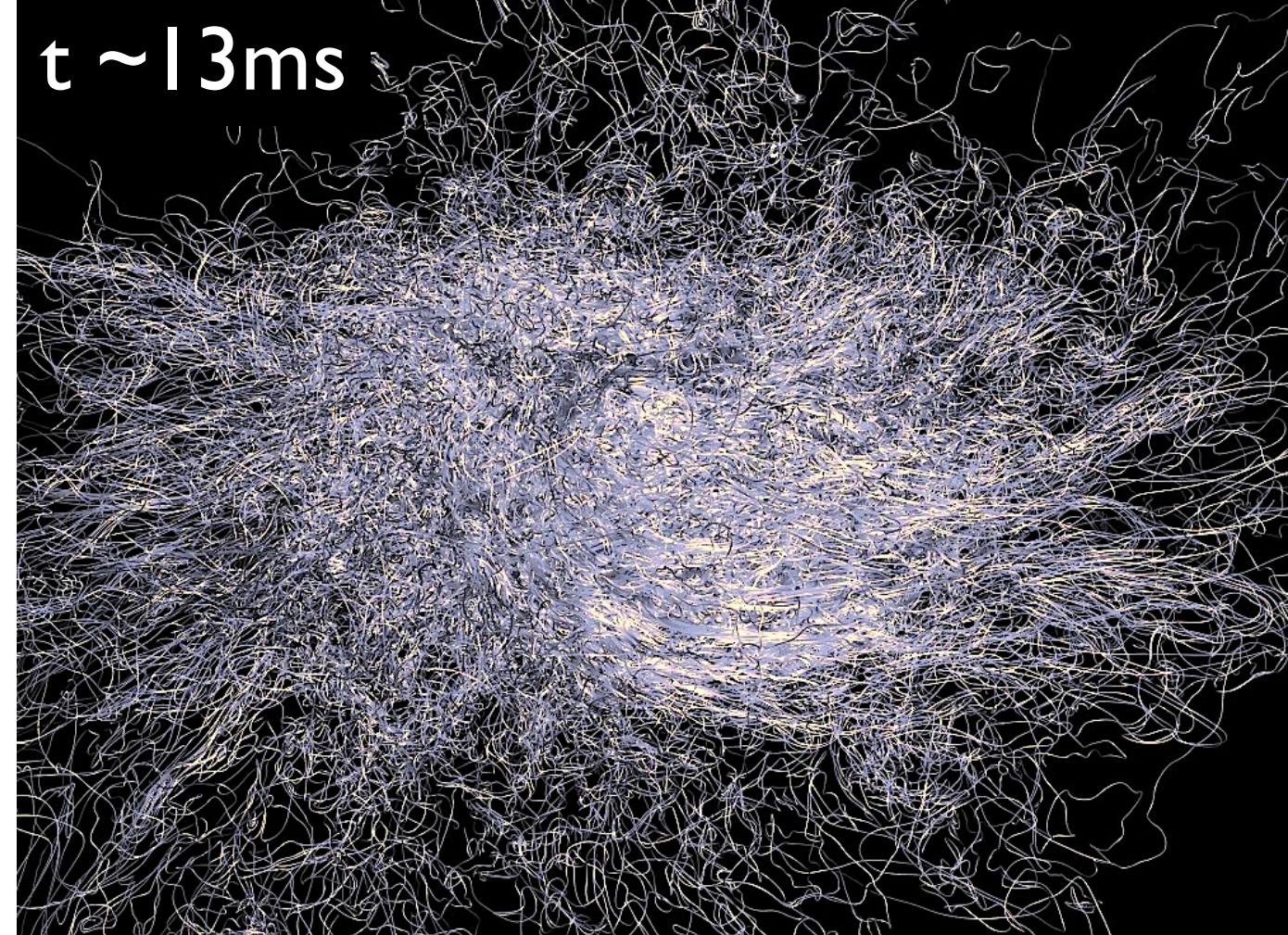
$$\rho_c^* = 5.790998966725 \times 10^{-4}$$

We can measure the time in the metastable state and see it increases as the critical solution is approached, (Jin et al 2007, Kellermann, LR et al 2010)

$$\tau_{\text{eq}} = -\lambda \ln |\rho_c - \rho_c^*|$$

$$\lambda \simeq 10.0$$

This is a type-I critical behaviour



First time a *magnetic jet* is produced from *ab-initio* calculation: opening angle is $\sim 30^\circ$